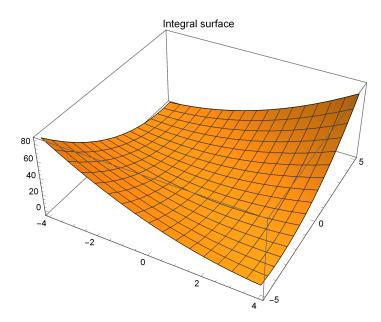
RAHUL CHOPRA | BSC CS Hons | 20211449 | Practical- 8

Plot the integral surface of a given first order PDE with the initial data

Problem I: Obtain the solution of the linear equation u[(x,y),x]-u[(x,y),y]=1 with the Cauchy data $u(x,0)=x^*x$. Plot the integral surface with in the range $\{x,-4,4\}$ and $\{y,-5,5\}$.

Solution:

```
 \begin{aligned} &\text{pde} = D[u[x,y],x] - D[u[x,y],y] &== 1 \\ &\text{DSolve}[\{D[u[x,y],x] - D[u[x,y],y] &== 1,u[x,\theta] &== (x*x)\},u[x,y],\{x,y\}] \\ &\text{Plot3D}[u[x,y] /. \%, \{x,-4,4\}, \{y,-5,5\}, \text{PlotLabel} \rightarrow \text{"Integral surface "}] \\ &-u^{(\theta,1)}[x,y] + u^{(1,\theta)}[x,y] &== 1 \\ &\left\{ \left\{ u[x,y] \rightarrow x^2 - y + 2 \times y + y^2 \right\} \right\} \end{aligned}
```



... DSolve: "Equation or list of equations expected instead of \!\(\(-圈\)\\\\\ *SuperscriptBox[\\\"u\\\", TagBox[\\n $+ \star SuperscriptBox[\\"u\\", TagBox[\\ "A\\", \\ "BowBox[{\\"1\\", \\", \\", \\"]$ $Derivative], \\ \n Multiline Function -> None][x, y] \) in the first argument \\ \| (\{(-\underline{B})), \\ \) \\ \xspace{The superscriptBox[\\"u\\\", \\ \]} \\ \n Multiline Function -> None][x, y] \\ \n Multiline Function -> N$ $\star SuperscriptBox[(x\), (2\)]\\\\ u[x, 0]}\)."$

DSolve $\left[\left\{ - \boxtimes u^{(0,1)}[x,y] + u^{(1,0)}[x,y], \boxtimes x^2 u[x,0] \right\}, u[x,y], \{x,y\} \right]$

Problem 2: Find the solution of the equation y*u[(x,y),x]-2*x*y*u[(x,y),y]=2*x*u[x,y] with the Cauchy data u(0,y)=y*y*y. Plot the integral surface with in the range $\{x,-7,7\}$ and $\{y,-5,5\}$. Solution

... Plot3D: Options expected (instead of PlotLabel → Integral surface) beyond position 3 in Plot3D[u[x, y] /. %, $\{x, -4, 4\}$, $\{y, -5, 5\}$, PlotLabel \rightarrow Integral surface]. An option must be a rule or a list of rules.

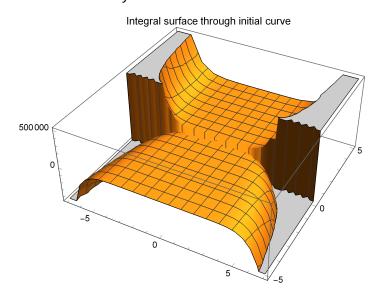
Plot3D[$u[x, y] /. %, \{x, -4, 4\}, \{y, -5, 5\}, PlotLabel <math>\rightarrow$ Integral surface]

pde = y * D[u[x, y], x] - 2 * x * y * D[u[x, y], y] == 2 * x * u[x, y]
sol3 = DSolve[{pde, u[0, y] == y * y * y}, u[x, y], {x, y}]
Plot3D[u[x, y] /. sol3, {x, -7, 7}, {y, -5, 5},

PlotLabel → "Integral surface through initial curve"]

$$-\,2\,x\,y\,u^{\,(\vartheta,\textbf{1})}\,\left[\,x\,,\,y\,\right]\,+\,y\,u^{\,(\textbf{1},\vartheta)}\,\left[\,x\,,\,y\,\right]\,=\,2\,x\,u\,[\,x\,,\,y\,]$$

$$\left\{\left\{u\left[x,\,y\right]\,\rightarrow\,\frac{\left(x^2\,+\,y\right)^4}{y}\right\}\right\}$$



Problem 3 : Determine the integral surfaces of the equation u[(x,y),x]+u[(x,y),y]=u[x,y]*u[x,y], with the data x+y=0,u=1. Plot the integral surface with in the range $\{x,10,10\}$ and $\{y,-10,10\}$. Solution :

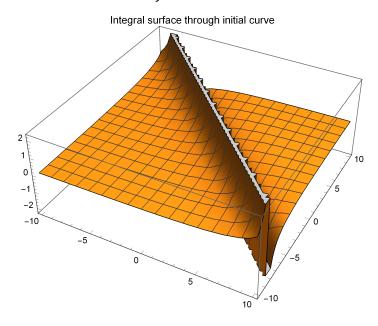
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Eqn = D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y] 

DSolve[{D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y], u[x, -x] == 1}, u[x, y], {x, y}] 

Plot3D[u[x, y] /. %, {x, -10, 10}, {y, -10, 10}, 

PlotLabel \rightarrow "Integral surface through initial curve "] 

u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == u[x, y]^2
\left\{\left\{u[x, y] \rightarrow -\frac{2}{-2 + x + y}\right\}\right\}
```



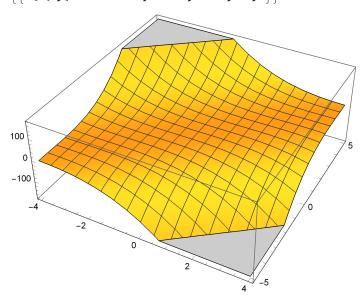
Problem 4 : Obtain the solution of the linear equation u[(x,y),x]+u[(x,y),y]=1with

the Cauchy data $u(x,2x)=x^*x^*x$.

Plot the integral surface with in the range $\{x,-4,4\}$ and $\{y,-5,5\}$.

Solution:

```
D[u[x, y], x] + D[u[x, y], y] == 1
DSolve[\{D[u[x, y], x] + D[u[x, y], y] == 1, u[x, 2x] == x * x * x\}, u[x, y], \{x, y\}]
Plot3D[u[x, y] /. %, \{x, -4, 4\}, \{y, -5, 5\}]
u^{(0,1)}[x, y] + u^{(1,0)}[x, y] = 1
\left\{ \left\{ u\,[\,x\,\text{, }y\,]\,\rightarrow 2\,x\,-\,x^{3}\,-\,y\,+\,3\,\,x^{2}\,y\,-\,3\,x\,\,y^{2}\,+\,y^{3}\,\right\} \right\}
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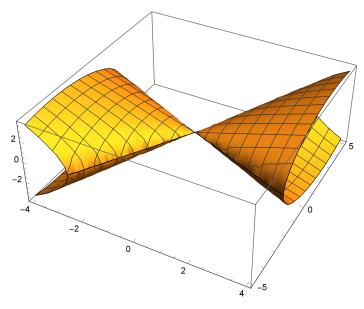


Problem 5: Obtain the solution of the linear equation $u(x+y)*u[(x,y),x]+u(x\otimes y)*u[(x,y),y]=x*x+y*y$ with the Cauchy data u(x,2x)=0. Plot the integral surface with in the range $\{x,-4,4\}$ and $\{y,-5,5\}$. Solution:

eqn =
$$u[x, y] * (x+y) * D[u[x, y], x] + u[x, y] * (x-y) * D[u[x, y], y] == $x * x + y * y$
DSolve[{ $u[x, y] * (x+y) * D[u[x, y], x] + u[x, y] * (x-y) * D[u[x, y], y] == (x * x) + (y * y), u[x, 2x] == 0$ }, $u[x, y], \{x, y\}$]
Plot3D[$u[x, y] /. %, \{x, -4, 4\}, \{y, -5, 5\}$]
 $(x-y) u[x, y] u^{(0,1)}[x, y] + (x+y) u[x, y] u^{(1,0)}[x, y] == x^2 + y^2$$$

... Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

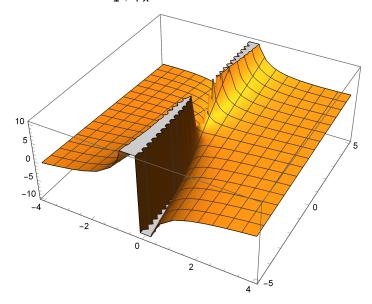
$$\begin{split} & \left\{ \left\{ u \left[x \text{, } y \right] \right. \right. \rightarrow - \sqrt{\frac{2}{7}} \, \sqrt{2 \, x^2 + 3 \, x \, y - 2 \, y^2} \, \right\} \text{, } \left\{ u \left[x \text{, } y \right] \right. \rightarrow \sqrt{\frac{2}{7}} \, \sqrt{2 \, x^2 + 3 \, x \, y - 2 \, y^2} \, \right\} \text{,} \\ & \left\{ u \left[x \text{, } y \right] \right. \rightarrow - \sqrt{\frac{2}{7}} \, \sqrt{2 \, x^2 + 3 \, x \, y - 2 \, y^2} \, \right\} \text{, } \left\{ u \left[x \text{, } y \right] \right. \rightarrow \sqrt{\frac{2}{7}} \, \sqrt{2 \, x^2 + 3 \, x \, y - 2 \, y^2} \, \right\} \right\} \end{split}$$



Problem 6: Obtain the solution of the linear equation u[(x,y),x]+u[x,y]*u[(x,y),y]=Iwith the Cauchy data u(0,y)=4*y. Plot the integral surface with in the range $\{x,-4,4\}$ and $\{y,-5,5\}$. Solution

D[u[x, y], x] + u[x, y] * D[u[x, y], y] == 1 $DSolve[\{D[u[x, y], x] + u[x, y] * D[u[x, y], y] == 1, u[0, y] == 4 * y\}, u[x, y], \{x, y\}]$ Plot3D[$u[x, y] /. %, \{x, -4, 4\}, \{y, -5, 5\}$] $u[x, y] u^{(0,1)}[x, y] + u^{(1,0)}[x, y] = 1$

 $\Big\{ \Big\{ u \, [\, x \, , \, y \,] \, \rightarrow \, \frac{x + 2 \, x^2 + 4 \, y}{1 + 4 \, x} \Big\} \Big\}$

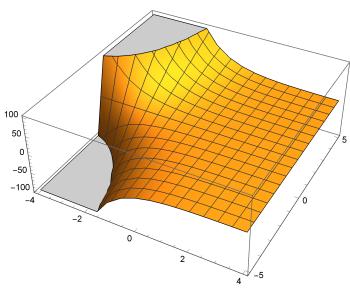


Problem 7 : Obtain the solution of the linear equation u[(x,y),x]+y*u[(x,y),y]=0with the Cauchy data u(0,y)=4*y.

Plot the integral surface with in the range $\{x,-4,4\}$ and $\{y,-5,5\}$.

Solution:

```
D[u[x, y], x] + y * D[u[x, y], y] == 0
DSolve[\{D[u[x, y], x] + y * D[u[x, y], y] == 0, u[0, y] == 4 * y\}, u[x, y], \{x, y\}]
Plot3D[u[x, y] /. %, \{x, -4, 4\}, \{y, -5, 5\}]
y u^{(0,1)} [x, y] + u^{(1,0)} [x, y] = 0
\left\{ \left\{ u\left[\,x\,,\;y\,\right]\right. \rightarrow 4\,\,\mathrm{e}^{-x}\,y\,\right\} \right\}
```



Problem 8 : Obtain the solution of the linear equation u[(x,y),x]+2*u[(x,y),y]=0with the Cauchy data u(0,y)=Exp[-y*y].

Plot the integral surface with in the range $\{x,-4,4\}$ and $\{y,-5,5\}$.

Solution:

```
D[u[x, y], x] + 2 * D[u[x, y], y] == 0
DSolve[\{D[u[x, y], x] + 2 * D[u[x, y], y] == 0, u[0, y] == Exp[-y * y]\}, u[x, y], \{x, y\}]
Plot3D[u[x, y] /. %, \{x, -4, 4\}, \{y, -5, 5\}]
2u^{(0,1)}[x,y] + u^{(1,0)}[x,y] = 0
```

 $\left\{ \left\{ u\left[\,x\,\text{, }y\,\right]\,\rightarrow\,\text{$\mathbb{e}^{-\,\left(\,-2\,x+y\right)\,^{2}}\,\right\}\,\right\}$

