NAME :
RAHUL CHOPRA | ROLL NO : 20 2 1 1 449
COURSE : Bsc.(HONS) COMPUTER SCIENCE | PRACTICAL – 6

SOLUTION OF CAUCHY PROBLEM FOR FIRST ORDER PDE

QUESTION 1 :

Obtain the solution of the linear equation u[(x, y), x] – u[(x, y), y] = 1 with the Cauchy data u (x, 0) = x * x
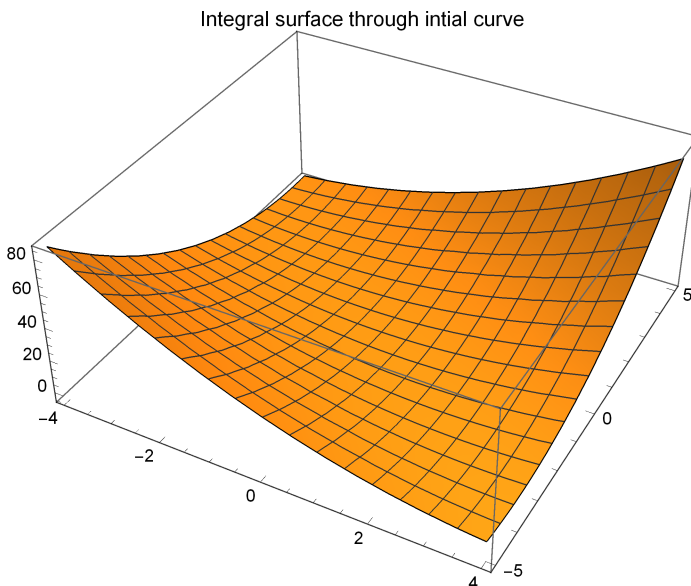
```
pde = D[u[x, y], x] - D[u[x, y], y] == 1
```
$-u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$

```
sol = DSolve[{pde, u[x, 0] == x * x}, u[x, y], {x, y}]
```
$\{\{u[x, y] \to x^2 - y + 2\,x\,y + y^2\}\}$

```
Plot3D[u[x, y] /. sol, {x, -4, 4}, {y, -5, 5},
 PlotLabel → "Integral surface through intial curve "]
```


Integral surface through intial curve

QUESTION 2 : Obtain the solution of the linear equation y * u[(x, y), x] – 2 * x * u[x, y] with the Cauchy data u (0, y) = y * y * y
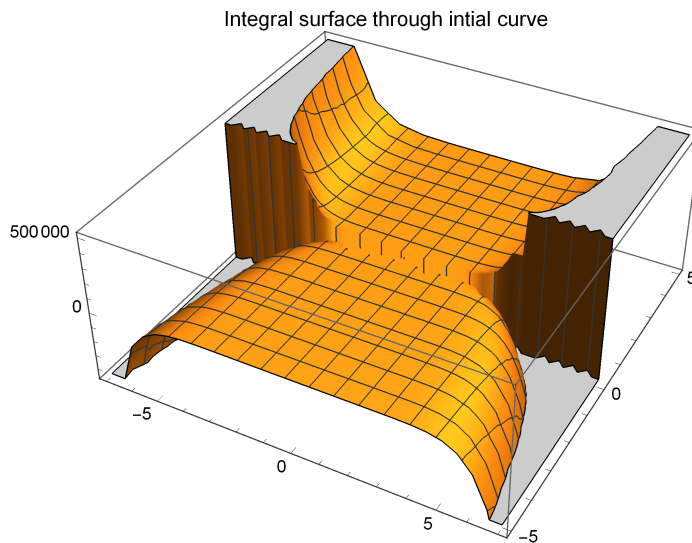## SOLUTION :

```
pde = y * D[u[x, y], x] - 2 * x * y * D[u[x, y], y]  ==  2 * x * u[x, y]
sol3 = DSolve[{pde, u[0, y]  ==  y * y * y}, u[x, y], {x, y}]
Plot3D[u[x, y] /. sol3, {x, -7, 7}, {y, -5, 5},
 PlotLabel → "Integral surface through intial curve "]
```

$- 2 \, x \, y \, u^{(0,1)}[x, y] + y \, u^{(1,0)}[x, y] \;==\; 2 \, x \, u[x, y]$

$$\left\{\left\{u[x, y] \;\to\; \frac{\left(x^2 + y\right)^4}{y}\right\}\right\}$$

Integral surface through intial curve

QUESTION 3 : Determine the integral surfaces of the equation u[(x, y), x] + u[(x, y), y] = u[x, y] * u[x, y], (a) with the data x + y = 0, u = 1. (b) with the data u (x, 0) = tanh (x).
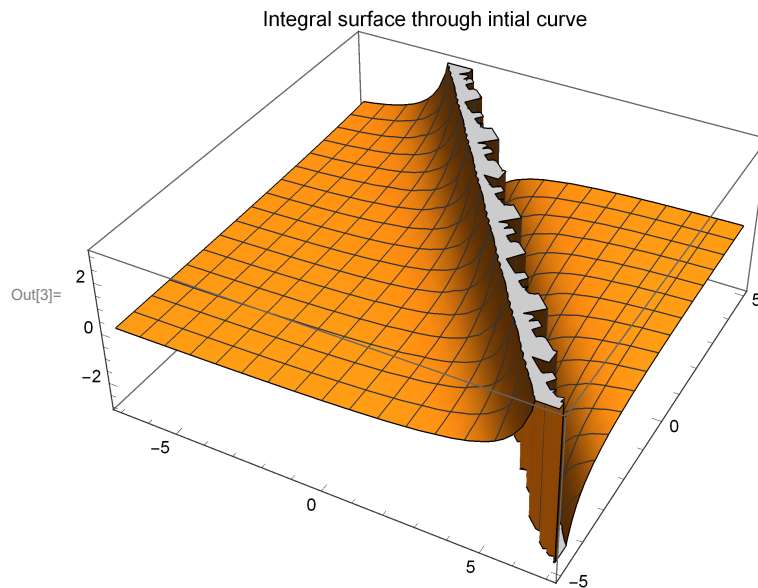
In[1]:= `Eqn = D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y]`
`sol4 =`
`DSolve[{D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y], u[x, -x] == 1}, u[x, y], {x, y}]`
`Plot3D[u[x, y] /. sol4, {x, -7, 7}, {y, -5, 5},`
`PlotLabel → "Integral surface through intial curve "]`

Out[1]= $u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == u[x, y]^2$

Out[2]= $\left\{\left\{u[x, y] \rightarrow -\dfrac{2}{-2 + x + y}\right\}\right\}$

Integral surface through intial curve

Out[3]=

(b)

In[4]:= `D[u[x, y], x] + D[u[x, y], y] ⊞ u[x, y] * u[x, y]`

Out[4]= $\boxed{}\, u[x, y]^2\, u^{(0,1)}[x, y] + u^{(1,0)}[x, y]$
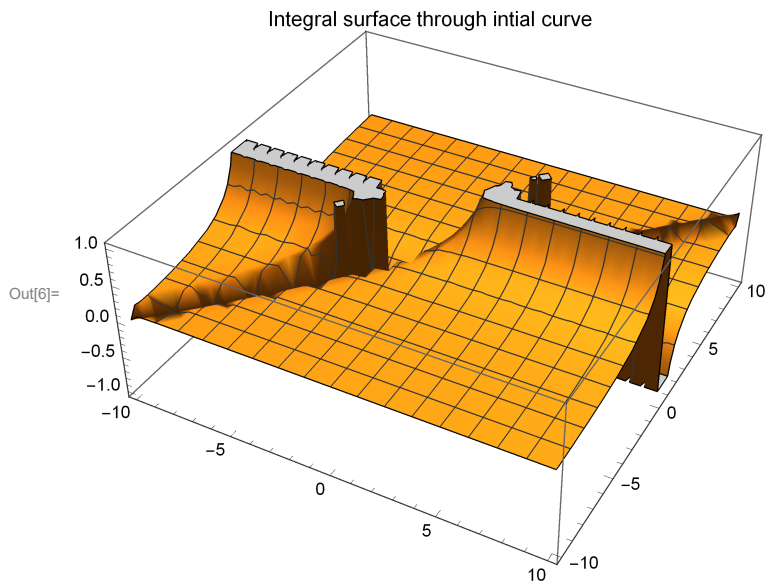
In[5]:= `sol5 = DSolve[`
`{D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y], u[x, 0] == Tanh[x]}, u[x, y], {x, y}]`

Out[5]= $\left\{\left\{u[x, y] \rightarrow \dfrac{1}{-y + \text{Coth}[x - y]}\right\}\right\}$

In[6]:= **Plot3D[u[x, y] /. sol5, {x, -10, 10}, {y, -10, 10},**
    PlotLabel → "Integral surface through intial curve "]

Integral surface through intial curve

Out[6]=

QUESTION 4 : Obtain the solution of the linear equation

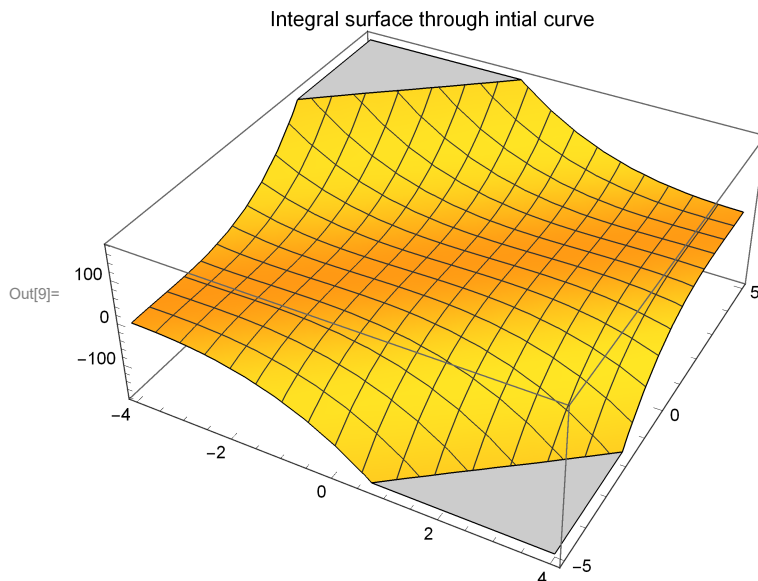## u[(x, y), x] + u[(x, y), y] = 1 with the Cauchy data u (x, 2 x) = x ∗ x ∗ x

In[7]:= **D[u[x, y], x] + D[u[x, y], y] == 1**

Out[7]= $u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$

In[8]:= **sol6 = DSolve[{D[u[x, y], x] + D[u[x, y], y] == 1, u[x, 2 x] == x ∗ x ∗ x}, u[x, y], {x, y}]**

Out[8]= $\left\{\left\{u[x, y] \rightarrow 2 x - x^3 - y + 3 x^2 y - 3 x y^2 + y^3\right\}\right\}$

In[9]:= `Plot3D[u[x, y] /. sol6, {x, -4, 4}, {y, -5, 5},`
`PlotLabel → "Integral surface through intial curve "]`

Integral surface through intial curve

Out[9]=

QUESTION 5 : Obtain the solution of the linear equation

u (x + y) * u[(x, y), x] + u (x − y) * u[(x, y), y] =

x * x + y * y with the Cauchy data

u (x, 2 x) = 0

SOLUTION :

In[10]:= `u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y]  ==  x * x + y * y`

Out[10]= $(x - y)\, u[x, y]\, u^{(0,1)}[x, y] + (x + y)\, u[x, y]\, u^{(1,0)}[x, y] == x^2 + y^2$

In[11]:= `DSolve[{u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y]  ==  (x * x) + (y * y),`
`u[x, 2 x]  ==  0}, u[x, y], {x, y}]`

... Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[11]= $\left\{\left\{u[x, y] \to -\sqrt{\frac{2}{7}}\ \sqrt{2\, x^2 + 3\, x\, y - 2\, y^2}\right\}, \left\{u[x, y] \to \sqrt{\frac{2}{7}}\ \sqrt{2\, x^2 + 3\, x\, y - 2\, y^2}\right\}\right.$,

$\left.\left\{u[x, y] \to -\sqrt{\frac{2}{7}}\ \sqrt{2\, x^2 + 3\, x\, y - 2\, y^2}\right\}, \left\{u[x, y] \to \sqrt{\frac{2}{7}}\ \sqrt{2\, x^2 + 3\, x\, y - 2\, y^2}\right\}\right\}$

In[12]:= **Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5},**
    PlotLabel → "Integral surface through intial curve "]

Integral surface through intial curve

Out[12]=