

RAHUL CHOPRA |

BSC (Hons) COMPUTER SCIENCE |

20211449 | Practical 7

Find the Characteristics for the first order PDE and
Plotting them

Example I : Find the Characteristics of the equation $(u - y) u_x + y u_y = x + y$ and plot them.**Solution :**

The characteristics system is

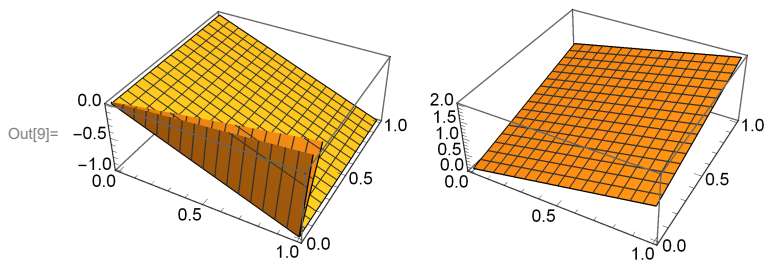
$$dx / (u - y) = dy / y = du / (x + y)$$

using (i) + (ii) + (iii), we have $v = (u + x) / y = c_1$, is a first integral. using (i) + (ii) = (iii),

we have $w = (x + y)^2 - u * u = c_2$, is a second first integral

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In[1]:= f0 = Plot3D[-x, {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10];  
f1 = Plot3D[5 y - x, {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10];  
f2 = Plot3D[10 y - x, {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10];  
g1 = Show[f0, f1, f2];  
h0 = Plot3D[x + y, {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10];  
h1 = Plot3D[Sqrt[(x + y)^2 + 5], {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10];  
h2 = Plot3D[Sqrt[(x + y)^2 + 10], {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10];  
g2 = Show[h0, h1, h2];  
Show[GraphicsArray[{g1, g2}]]
```

*** GraphicsArray: GraphicsArray is obsolete. Switching to GraphicsGrid.



Example 2 : The solution of the equation $u[(x, y), y] + u[x, y] * u[(x, y), x] = 0$, can be interpreted as a vector field on the $x -$ axis varying with time y . Find the integral satisfying the initial condition $u(s, 0) = h(s)$, where h is a given function. **Solution :**

We plot the curves

$\{C_t : x = s + t(s^3 - 3s^2 + 4), u = s^3 - 3s^2 + 4\}$

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In[10]:= u[s_] := s^3 - 3 s^2 + 4;
x[s_, t_] := s + t * u[s];
h0 = ParametricPlot[{x[s, 0], u[s]}, {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0"];
h1 = ParametricPlot[{x[s, 0.2], u[s]}, {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0.2"];
h2 = ParametricPlot[{x[s, 0.3], u[s]}, {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0.3"];
h3 = ParametricPlot[{x[s, 0.33], u[s]}, {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0.33"};
h4 = ParametricPlot[{x[s, 0.333], u[s]},
    {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0.333 "];
h5 = ParametricPlot[{x[s, 0.4], u[s]}, {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0.4"];
Show[GraphicsArray[{{h0, h1, h2}, {h3, h4, h5}}].FrameTicks -> None, Frame -> False]
    
```

GraphicsArray: GraphicsArray is obsolete. Switching to GraphicsGrid.

Show: No graphical objects to show.

