

RAHUL CHOPRA | BSC CS Hons |

20211449 | Practical- 8

Plot the integral surface of a given first order PDE with the initial data

Problem I: Obtain the solution of the linear equation

$u[(x,y),x]-u[(x,y),y]=1$ with

the Cauchy data $u(x,0) = x*x$.

Plot the integral surface with in the range $\{x,-4,4\}$ and $\{y,-5,5\}$.

Solution :

```
pde = D[u[x, y], x] - D[u[x, y], y] == 1
DSolve[{D[u[x, y], x] - D[u[x, y], y] == 1, u[x, 0] == (x * x)}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}, PlotLabel -> "Integral surface "]
```

$$-u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$$

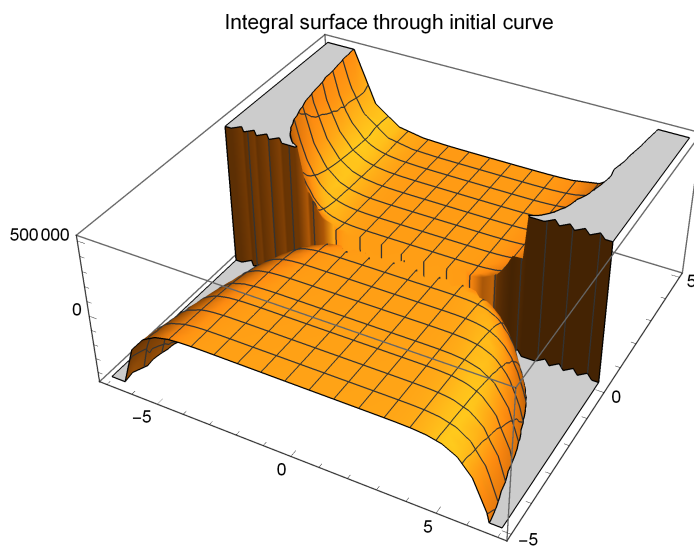
$$\{ \{ u[x, y] \rightarrow x^2 - y + 2xy + y^2 \} \}$$


```

pde = y * D[u[x, y], x] - 2 * x * y * D[u[x, y], y] == 2 * x * u[x, y]
sol3 = DSolve[{pde, u[0, y] == y * y * y}, u[x, y], {x, y}]
Plot3D[u[x, y] /. sol3, {x, -7, 7}, {y, -5, 5},
  PlotLabel -> "Integral surface through initial curve"
-2 x y u(0,1)[x, y] + y u(1,0)[x, y] == 2 x u[x, y]

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$$\left\{ \left\{ u[x, y] \rightarrow \frac{(x^2 + y)^4}{y} \right\} \right\}$$

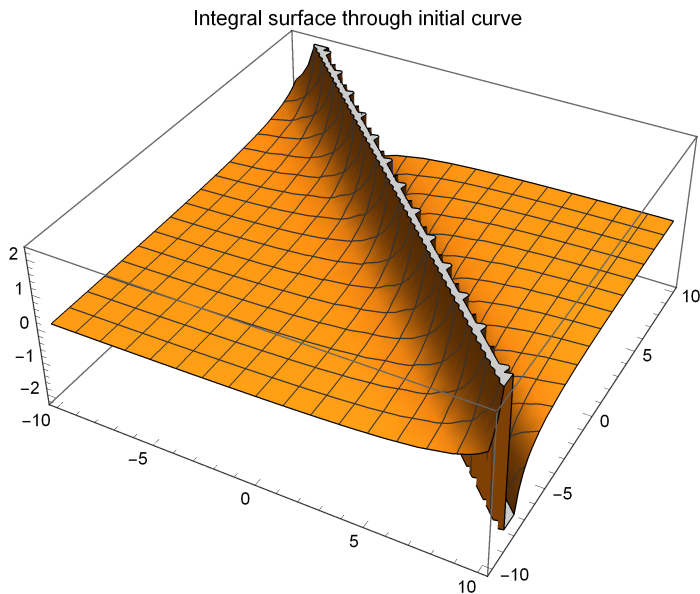


Problem 3 : Determine the integral surfaces of the equation $u[(x,y),x]+u[(x,y),y]=u[x,y]*u[x,y]$, with the data $x+y=0,u=1$.
Plot the integral surface with in the range $\{x,-10,10\}$ and $\{y,-10,10\}$.

Solution :

```
Eqn = D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y]
DSolve[{D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y], u[x, -x] == 1}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -10, 10}, {y, -10, 10},
  PlotLabel -> "Integral surface through initial curve "]
u(0,1)[x, y] + u(1,0)[x, y] == u[x, y]2
```

$$\left\{ \left\{ u[x, y] \rightarrow -\frac{2}{-2 + x + y} \right\} \right\}$$



Problem 4 : Obtain the solution of the linear equation $u[(x,y),x]+u[(x,y),y]=1$ with

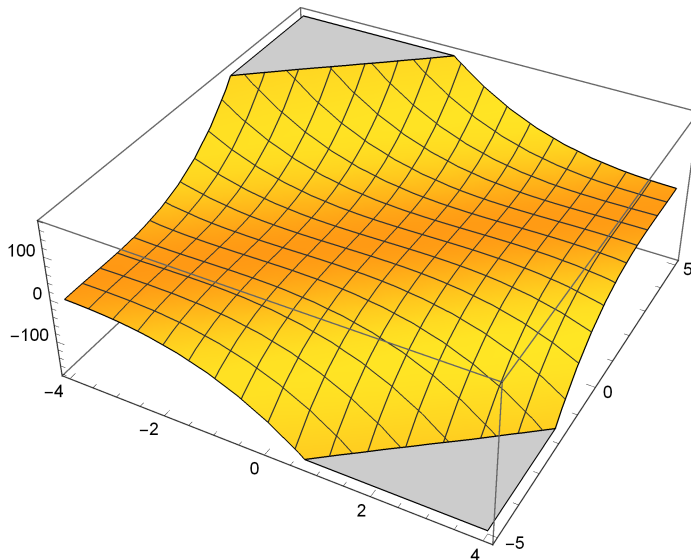
the Cauchy data $u(x,2x)=x*x*x$.

Plot the integral surface with in the range $\{x,-4,4\}$ and $\{y,-5,5\}$.

Solution :

```

D[u[x, y], x] + D[u[x, y], y] == 1
DSolve[{D[u[x, y], x] + D[u[x, y], y] == 1, u[x, 2 x] == x * x * x}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}]
 $u^{(0,1)}[x, y] + u^{(1,0)}[x, y] = 1$ 
 $\{ \{ u[x, y] \rightarrow 2x - x^3 - y + 3x^2y - 3xy^2 + y^3 \} \}$ 
    
```



Problem 5 : Obtain the solution of the linear equation

$u(x+y)u[(x,y),x]+u(x-y)u[(x,y),y]=x^2+y^2$ with the Cauchy data $u(x,2x)=0$.

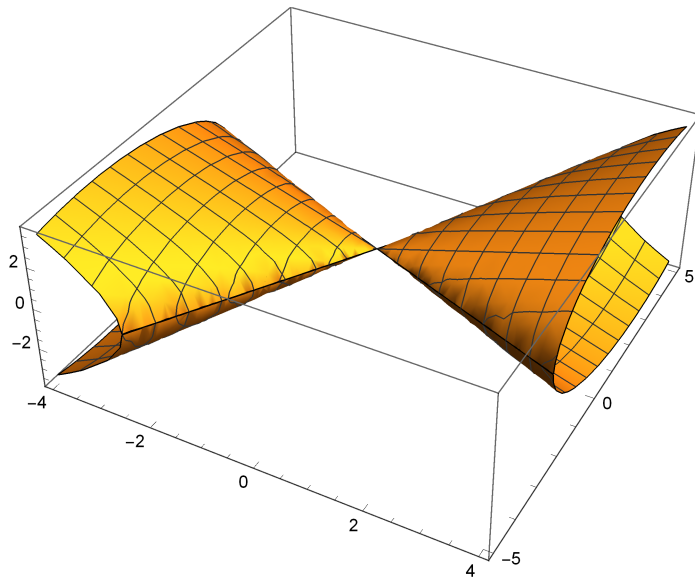
Plot the integral surface with in the range $\{x,-4,4\}$ and $\{y,-5,5\}$.

Solution :

```
eqn = u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y] == x * x + y * y
DSolve[{u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y] == (x * x) + (y * y),
  u[x, 2 x] == 0}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}]
(x - y) u[x, y] u(0,1)[x, y] + (x + y) u[x, y] u(1,0)[x, y] == x2 + y2
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

$$\left\{ \left\{ u[x, y] \rightarrow -\sqrt{\frac{2}{7}} \sqrt{2x^2 + 3xy - 2y^2} \right\}, \left\{ u[x, y] \rightarrow \sqrt{\frac{2}{7}} \sqrt{2x^2 + 3xy - 2y^2} \right\}, \right. \\ \left. \left\{ u[x, y] \rightarrow -\sqrt{\frac{2}{7}} \sqrt{2x^2 + 3xy - 2y^2} \right\}, \left\{ u[x, y] \rightarrow \sqrt{\frac{2}{7}} \sqrt{2x^2 + 3xy - 2y^2} \right\} \right\}$$



Problem 6 : Obtain the solution of the linear equation

$u[(x,y),x]+u[x,y]u[(x,y),y]=1$

with the Cauchy data $u(0,y)=4*y$.

Plot the integral surface with in the range $\{x,-4,4\}$ and $\{y,-5,5\}$.

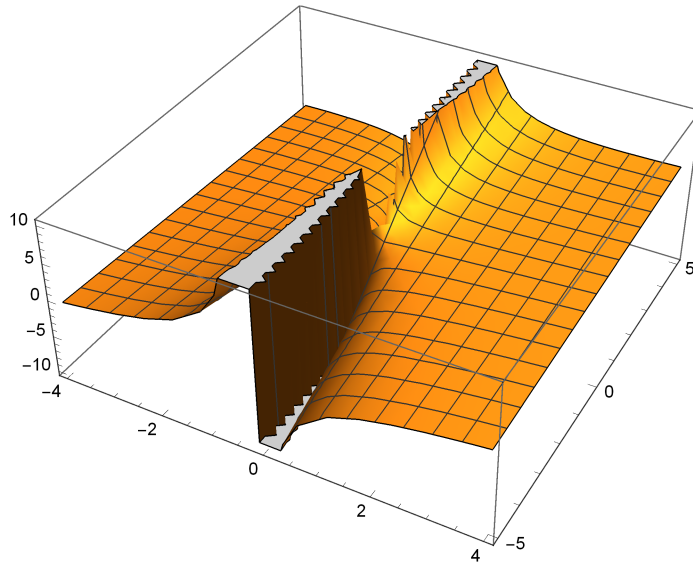
Solution

```

D[u[x, y], x] + u[x, y] * D[u[x, y], y] == 1
DSolve[{D[u[x, y], x] + u[x, y] * D[u[x, y], y] == 1, u[0, y] == 4 * y}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}]
u[x, y] u(0,1)[x, y] + u(1,0)[x, y] == 1

```

$$\left\{ \left\{ u[x, y] \rightarrow \frac{x + 2x^2 + 4y}{1 + 4x} \right\} \right\}$$



Problem 7 : Obtain the solution of the linear equation $u[(x,y),x]+y*u[(x,y),y]=0$ with the Cauchy data $u(0,y)=4*y$.

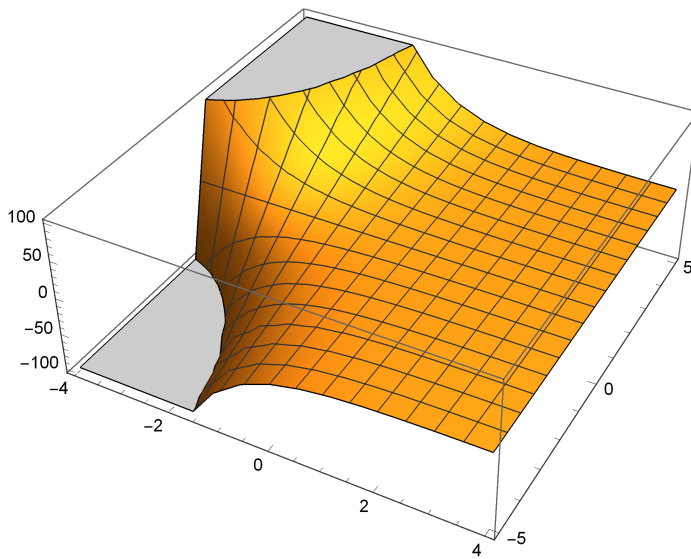
Plot the integral surface with in the range $\{x,-4,4\}$ and $\{y,-5,5\}$.

Solution :

```

D[u[x, y], x] + y * D[u[x, y], y] == 0
DSolve[{D[u[x, y], x] + y * D[u[x, y], y] == 0, u[0, y] == 4 * y}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}]
 $y u^{(0,1)}[x, y] + u^{(1,0)}[x, y] = 0$ 
 $\{ \{ u[x, y] \rightarrow 4 e^{-x} y \} \}$ 

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Problem 8 : Obtain the solution of the linear equation $u[(x,y),x]+2*u[(x,y),y]=0$ with the Cauchy data $u(0,y)=\text{Exp}[-y*y]$.

Plot the integral surface with in the range $\{x,-4,4\}$ and $\{y,-5,5\}$.

Solution :

```
D[u[x, y], x] + 2 * D[u[x, y], y] == 0
DSolve[{D[u[x, y], x] + 2 * D[u[x, y], y] == 0, u[0, y] == Exp[-y * y]}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}]
```

```
2 u(0,1)[x, y] + u(1,0)[x, y] == 0
```

```
{ {u[x, y] -> e-(-2 x+y)2 } }
```

