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COURSE: Bsc.(HONS) COMPUTER SCIENCE |

PRACTICAL -4

Method of Variation of Parameters

QUESTION 1 : Solve second order differential equation y''[x] + y[x] = tan[x] by method of variation of parameter

Solution :
Step – 1 : Find complementary function

... DSolve: Equation or list of equations expected instead of 0 in the first argument 0.

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```
In[6]:= eqn := y''[x] + y[x];
        f[x_] := Tan[x];
        P = DSolve[eqn == 0, y[x], x]
        {{y[x] → ℂ1 Cos[x] + ℂ2 Sin[x]}}
Out[8]= {{y[x] → C[1] Cos[x] + C[2] Sin[x]}}

Out[9]= {{y[x] → ℂ1 Cos[x] + ℂ2 Sin[x]}}
```

Step -2 Consider fundamental solution function u(x) and v(x)

In[10]:=
```
u[x_] := Cos[x];
v[x_] := Sin[x];
```

Step – 3 Find Wronskian W = ({u[x], v[x]}, {u'[x], v'[x]})

In[12]:=
```
w = Simplify[Det[{{u[x], v[x]}, {u'[x], v'[x]}}]]
```

Out[12]= 1

Step – 4 Find g[x] = (–v[x] f[x]) / w and h[x] = (u[x] f[x]) / w

In[30]:=
```
g[x_] := (-v[x] * f[x]) / w
h[x] := (u[x] * f[x]) / w
```

Step – 5 Find G = Integrate[g[x], x] and H = Integrate[h[x], x]

In[28]:=
```
G = Integrate[g[x], x]
H = Simplify[Integrate[h[x], x]]
```

In[33]:= $\int g[x]\, dx$

Out[33]= $\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]-\mathrm{Sin}\left[\frac{x}{2}\right]\right]-\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]+\mathrm{Sin}\left[\frac{x}{2}\right]\right]+\mathrm{Sin}[x]$

In[32]:= $\int h[x]\, dx$

Out[32]= $-\mathrm{Cos}[x]$

Step – 6 Find PI = u[x] G + v[x] H

In[34]:=
```
PI = u[x] G + v[x] H
```

Out[34]= $-\mathrm{Cos}[x]\,\mathrm{Sin}[x]+\mathrm{Cos}[x]\left(\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]-\mathrm{Sin}\left[\frac{x}{2}\right]\right]-\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]+\mathrm{Sin}\left[\frac{x}{2}\right]\right]+\mathrm{Sin}[x]\right)$

QUESTION 2 : Solve second order differential equation y''[x]-2y'[x]=e^x Sin[x] by method of variation of parameter

Step – I : Find complementary function

In[38]:=
```
eqn := y''[x] - 2 y'[x];
f[x_] := e^x * Sin[x];
P = DSolve[eqn == 0, y[x], x]
```

Out[40]= $\left\{\left\{y[x] \to \frac{1}{2} e^{2x} C[1] + C[2]\right\}\right\}$

Step – 2 Consider fundamental solution function u (x) and v (x)

In[41]:=
```
u[x_] := 1 / 2 Exp[2 x]
v[x_] := 1
```

Step – 3 Find Wronskian W = ({u[x], v[x]}, {u'[x], v'[x]})

In[43]:=
```
w = Simplify[Det[{{u[x], v[x]}, {u'[x], v'[x]}}]]
```

Out[43]= $-e^{2x}$

Step – 4 Find g[x] = (–v[x] f[x]) / w and h[x] = (u[x] f[x]) / w

```
g[x_] := (-v[x] × f[x]) / w
h[x_] := (u[x] × f[x]) / w
```

Step – 5 Find G = Integrate[g[x], x] and H = Integrate[h[x], x]

In[44]:=
```
G = Integrate[g[x], x]
H = Simplify[Integrate[h[x], x]]
```

Out[44]=
$$\frac{e^{x}\, e^{-2x}\left(-\mathrm{Cos}[x]+(-2+\mathrm{Log}[e])\,\mathrm{Sin}[x]\right)}{5-4\,\mathrm{Log}[e]+\mathrm{Log}[e]^{2}}$$

Out[45]=
$$\frac{e^{x}\left(\mathrm{Cos}[x]-\mathrm{Log}[e]\,\mathrm{Sin}[x]\right)}{2\left(1+\mathrm{Log}[e]^{2}\right)}$$

Step – 6 Find PI = u[x] G + v[x] H

In[46]:=
```
PI = u[x] G + v[x] H
```

Out[46]=
$$\frac{e^{x}\left(-\mathrm{Cos}[x]+(-2+\mathrm{Log}[e])\,\mathrm{Sin}[x]\right)}{2\left(5-4\,\mathrm{Log}[e]+\mathrm{Log}[e]^{2}\right)}+\frac{e^{x}\left(\mathrm{Cos}[x]-\mathrm{Log}[e]\,\mathrm{Sin}[x]\right)}{2\left(1+\mathrm{Log}[e]^{2}\right)}$$