VEHICLE DYNAMICS

MECH 6541

Assignment #1

MECH 6751

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Submitted by-

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1. PROBLEM STATEMENT

A large diameter soft tire has been used to provide the cushioning of the unsprung service vehicle. The tire model by considering as a single degree of freedom has been shown in figure 1. m is mass of the quarter-vehicle supported on one tire, k is effective dynamic stiffness of the tire and c is the equivalent viscous damping coefficient of the tire.

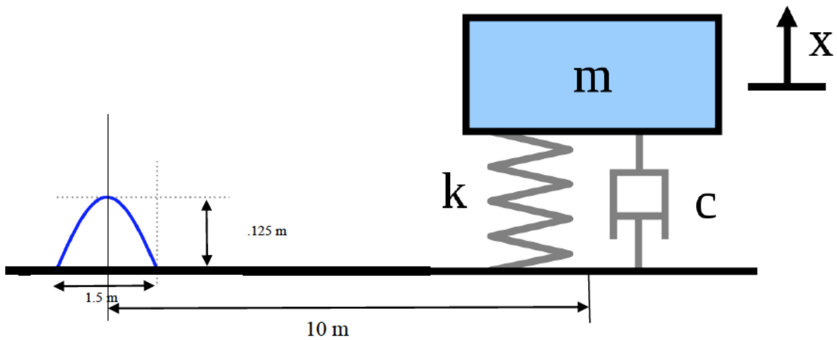

Figure 1. Simulation Model

The load carried by the tire is 4 KN, which has been dropped from a specific height by assuming that the tire-ground contact was maintained through the transient oscillations. The oscillation period of the tire has been measured to be 0.25 s and the ratio of consecutive oscillations amplitude of 1.7.

The stiffness and the damping properties need to be evaluated for the given model.

In the second part of the problem, the vehicle has been passed over a road bump, having a half sinusoidal waveform shape and a height of 15 cm and width of 1.2 m. Dynamic response in terms of acceleration, vertical and horizontal force has to be simulated using the Simulink.

Finally, wheel hop has to be identified in the model, if present and the effect of increased tire damping on the vehicle vibration and dynamic forces transmitted to the pavement has to be compared.

2. MODEL FORMULATION AND ASSUMPTIONS

Assumptions-

1. Point contact between the tyre and the ground
2. Predominant vertical forces

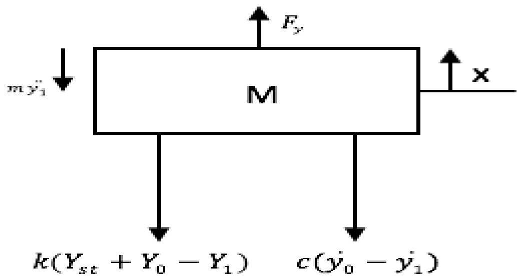

Figure 2. Free body diagram

Here, $F_v = k(Y_{st} + Y_0 - Y_1) + c(\dot{y}_0 - \dot{y}_1)$

The equation of motion is given by;

$$m\ddot{y}_1 + F_y = F_v \; ; \quad F_y = mg$$

$$m\ddot{y}_1 + mg = F_v$$

$$m\ddot{y}_1 + mg = [k(Y_{st} + Y_0 - Y_1) + c(\dot{y}_0 - \dot{y}_1)]$$

For linear point contact, Static deflection of the tire due to its own weight,

$$Y_{st} = \frac{mg}{k}$$

$$m\ddot{y}_1 = k(Y_0 - Y_1) - c(\dot{y}_0 - \dot{y}_1)$$

$$\boldsymbol{\ddot{y}_1 = \frac{k}{m}(Y_0 - Y_1) + \frac{c}{m}(\dot{y}_0 - \dot{y}_1)}$$

Vertical Force $\quad \boldsymbol{F_v = m\ddot{y}_1 + mg}$

Also, $\frac{F_h}{F_v} = \frac{dy_0}{dx}$; where $\frac{dy_0}{dx} = \frac{\dot{y}_0}{v}$

Therefore,

$$\boldsymbol{F_h = (F_v)(\frac{1}{v})(\frac{d\dot{y}_0}{dx})}$$

3. METHOD OF SOLUTION

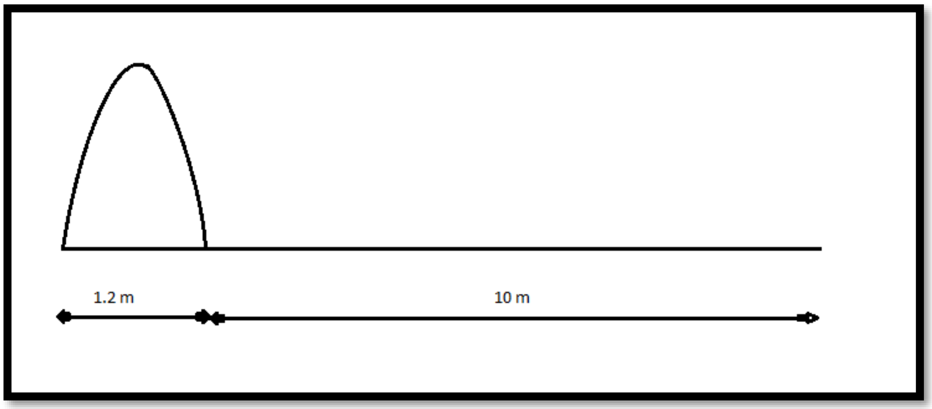

Figure 3. Road bump

1. $v = 20$ Km/hr = 5.55 m/s
   $1/v = .18018$
   For straight road, time $t_1 = 10 / 5.55 = 1.8001$ m/s
   For total road, time $t_2 = 11.2 / 5.55 = 2.018$ m/s

   Time for bumpy road = $t_2 - t_1 = 2.018 - 1.8001 = 0.2179$ m/s

   Hence, frequency of sine curve, $f = \pi / (t_2 - t_1) = 14.416$ rad/ s

2. $v = 40$ Km/hr = 11.1 m/s
   $1/v = .09$
   For straight road, time $t_1 = 10 / 11.1 = 0.9$ m/s
   For total road, time $t_2 = 11.2 / 11.1 = 1.009$ m/s

   Time for bumpy road = $t_2 - t_1 = 1.009 – 0.9 = 0.109$ m/s

   Hence, frequency of sine curve, $f = \pi / (t_2 - t_1) = 28.82$ rad/ s

3. $v = 60$ Km/hr = 16.67 m/s
   $1/v = .05998$
   For straight road, time $t_1 = 10 / 16.67 = 0.5999$ m/s
   For total road, time $t_2 = 11.2 / 16.67 = 0.6719$ m/s

   Time for bumpy road = $t_2 - t_1 = 0.6719 – 0.5999 = 0.07196$ m/s

   Hence, frequency of sine curve, $f = \pi / (t_2 - t_1) = 43.65$ rad/ s

(a) Load carried by the tire, W = 4 KN

Oscillation period, τ_d = 0.25 s

Ratio of consecutive oscillation amplitude = 1.7 = $\frac{X_1}{X_2}$

Mass = Load/g

= 4000/9.81 = 407.75 Kg

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \ln\left(\frac{X_1}{X_2}\right)$$

$$\frac{2\pi\xi}{\sqrt{1-\xi^2}} = \ln(1.7) = 0.5306$$

$$\xi = 0.084$$

Hence,

$$\omega_d = \frac{2\pi}{\tau_d} = \frac{2\pi}{0.25} = 25.13 = \omega_n \sqrt{1-\xi^2}$$

ω_n = 25.22 rad/s

It is known that $\omega_n = \sqrt{\frac{K}{m}}$

$k = \omega_n^2 \times m = (25.22)^2 \times 407.75$

k = 259348.74 N/m

For critical damping,

$c_c = 2\ m\ \omega_n = 2 \times 407.75 \times 25.22$

c_c = 20566.91 Ns/m

Damping coefficient, $c = \xi\ c_c$ = 1727.62 Ns/m

4. RESULTS AND DISCUSSIONS

(b) Plots for Bump

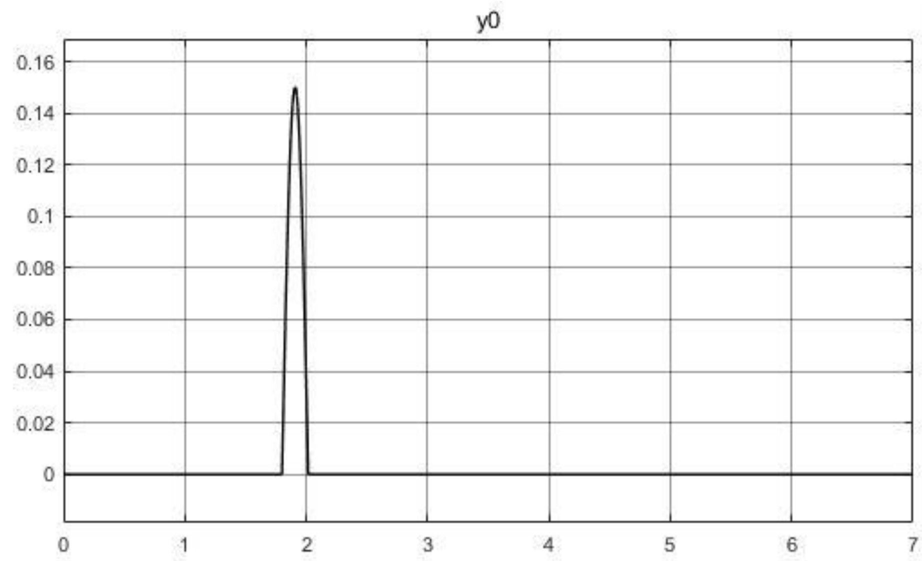

Figure 4. Bump at 20 km/hr (5.55 m/s)

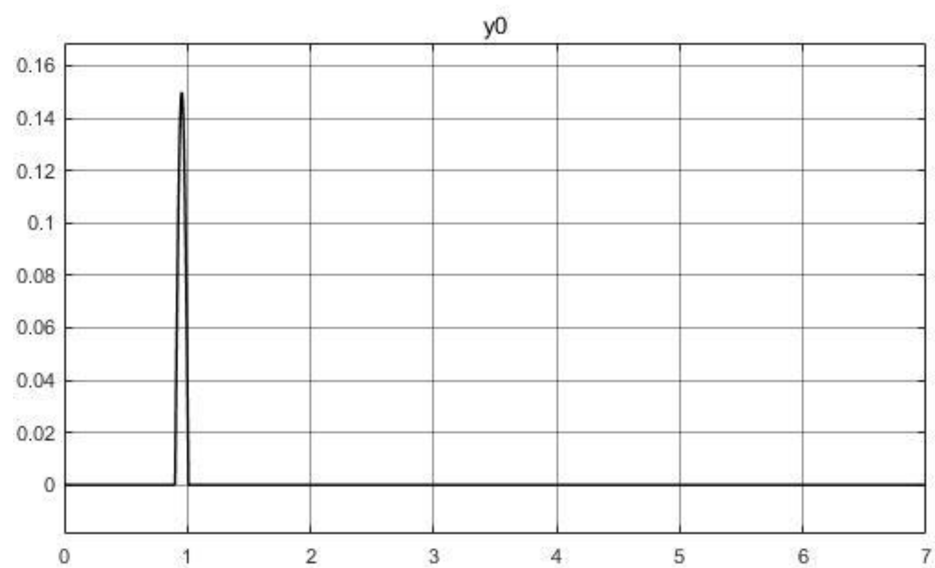

Figure 5. Bump at 40 km/hr (11.11 m/s)

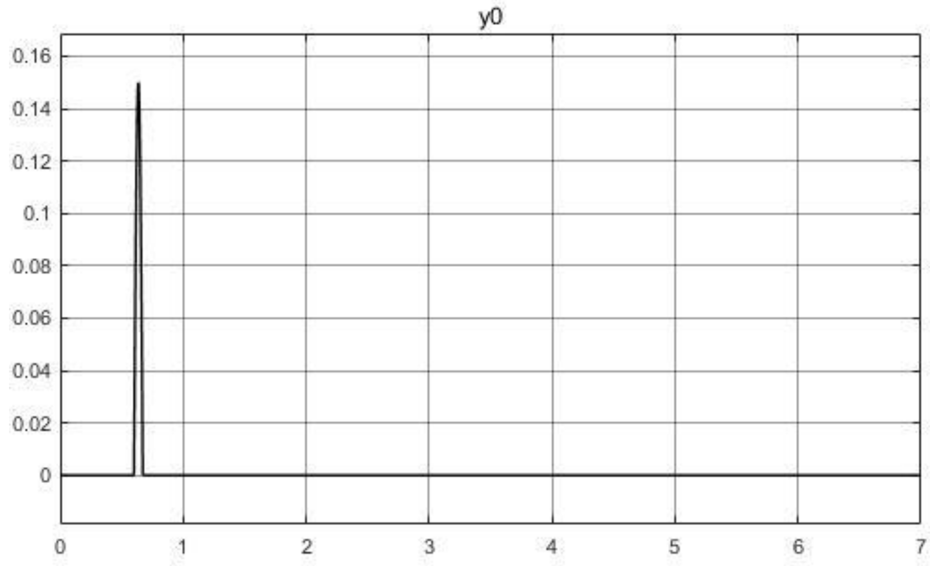

Figure 6. Bump at 60 km/hr (16.67 m/s)

From Figure 4-6, it can be observed that high-velocity vehicle covers the bump faster and gives a narrow bump with respect to time.

There can be many ways to produce the required model, among them, two types of Simulink model (Figure.7-8) have been generated and the results were verified.

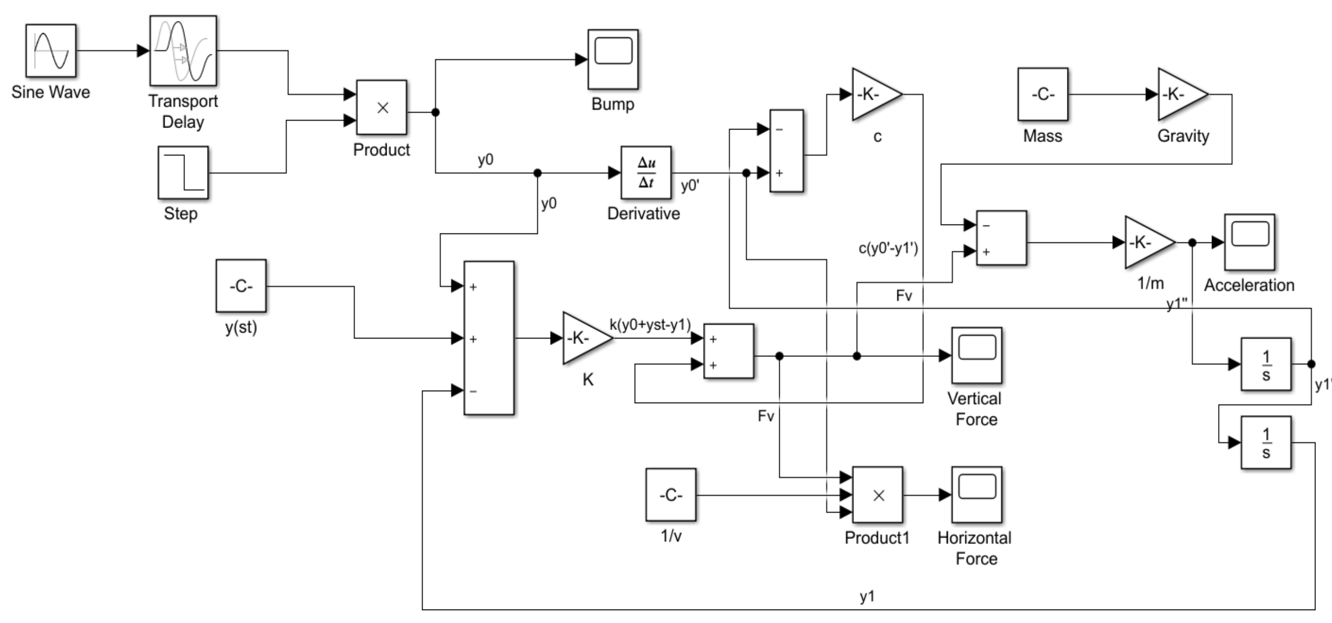

Figure 7. Linear Model on Simulink (I Method)

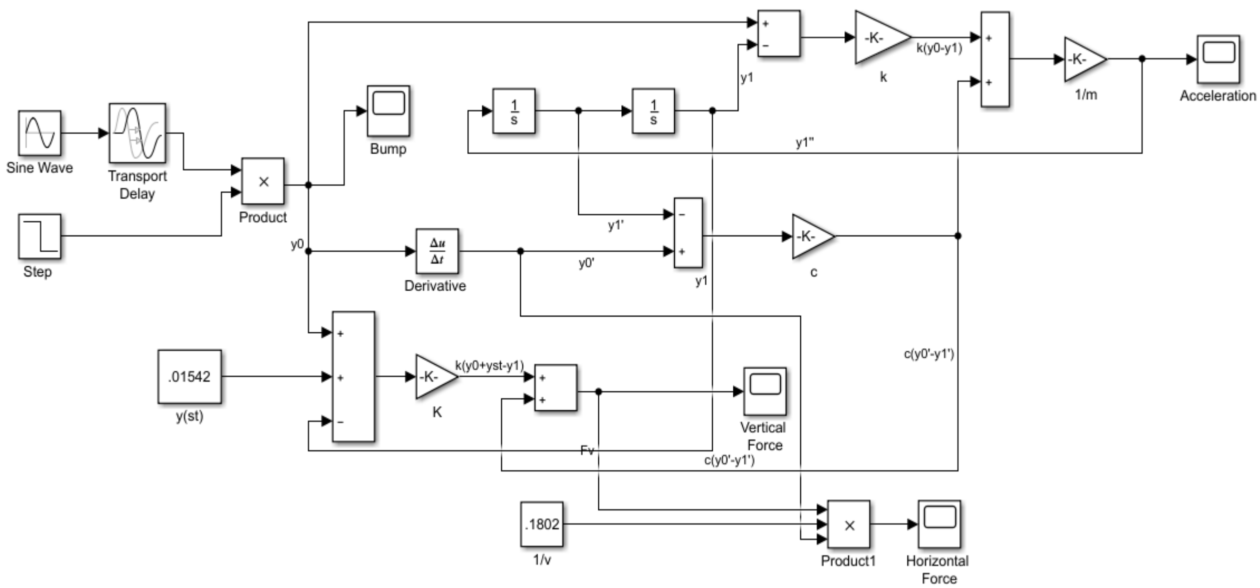

Figure 8. Linear Model on Simulink (II Method)

Acceleration Plots

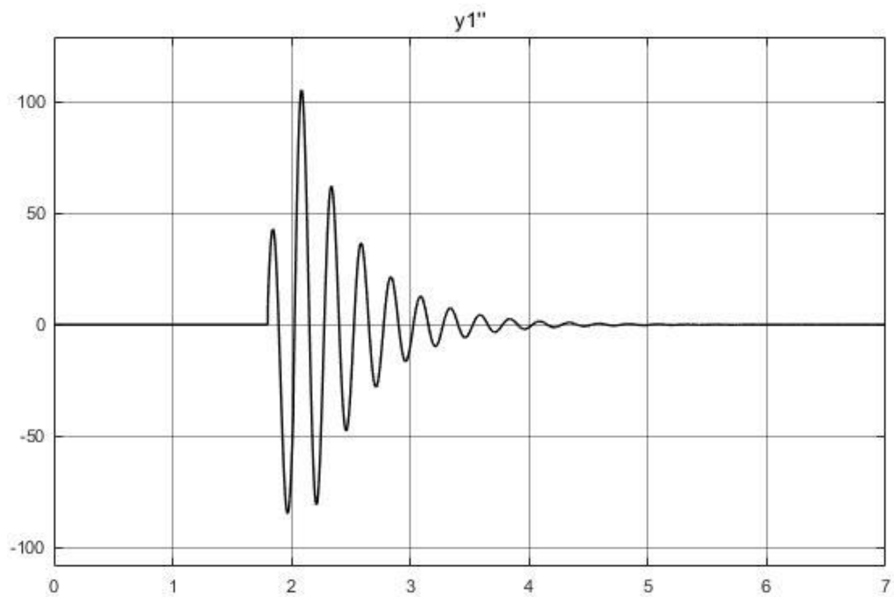

Figure 9. Linear Acceleration response (m/s2) vs Time for 20 km/hr (5.55 m/s)

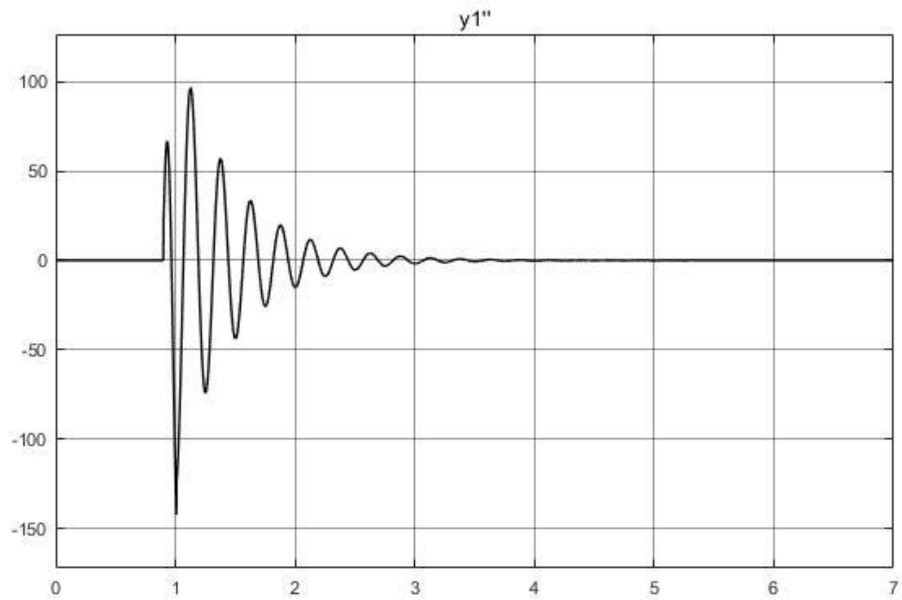

Figure 10. Linear Acceleration response (m/s2) vs Time for 40 km/hr (11.11 m/s)

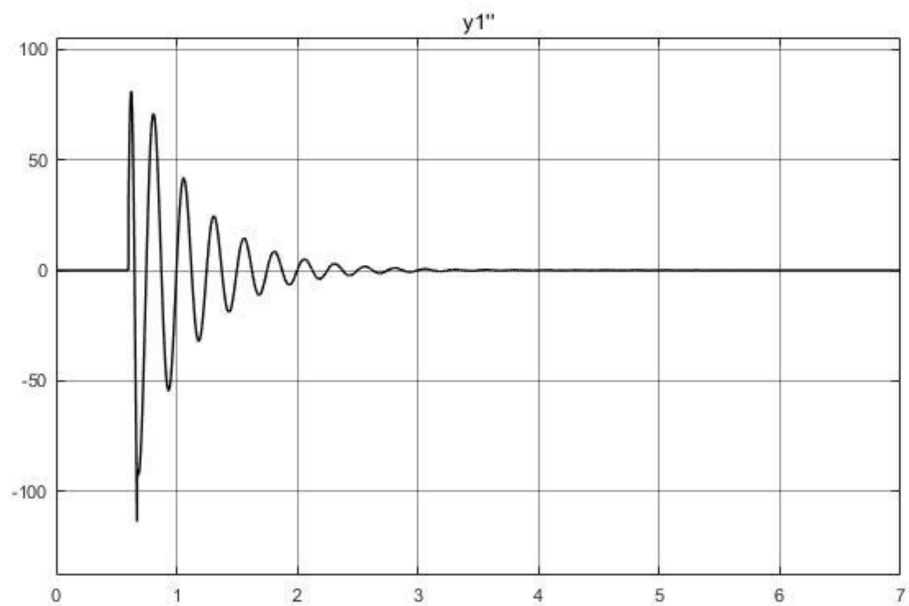

Figure 11. Linear Acceleration response (m/s2) vs Time for 60 km/hr (16.67 m/s)

From the above plots (Figure.9-11) it can be noticed that the acceleration magnitude decreases as the velocity increases and the oscillations tend to start early for the high velocity vehicle.

Vertical Force Plots

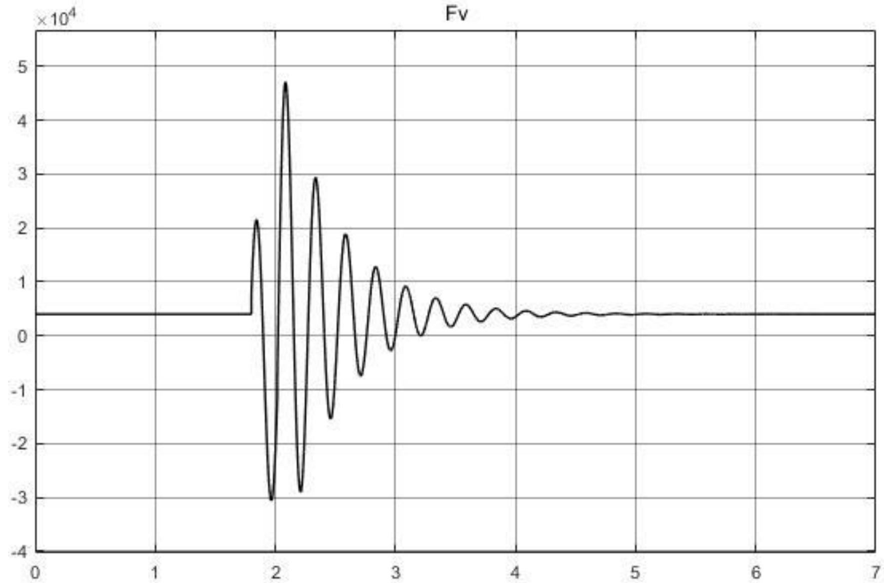

Figure 12. Linear Vertical Force vs Time for 20 Km/hr (5.55 m/s)

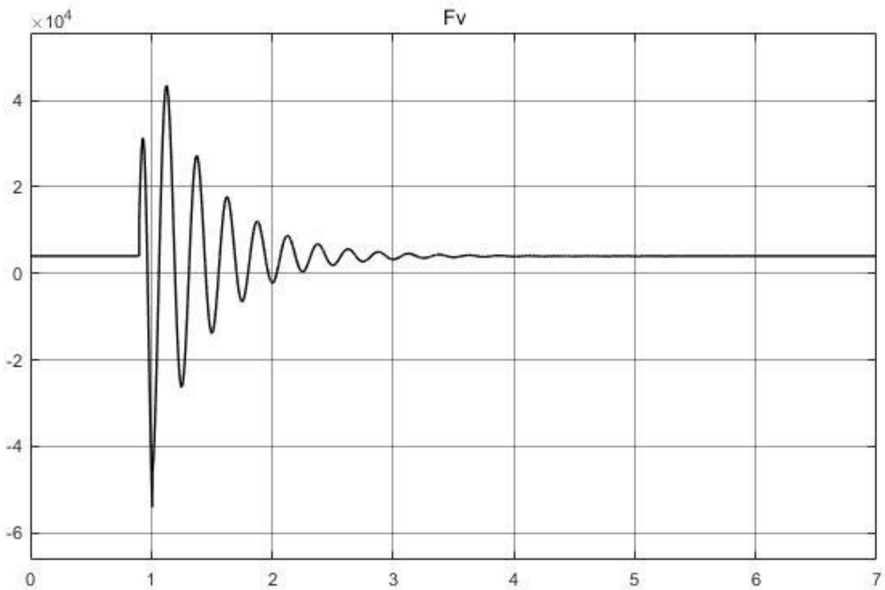

Figure 13. Linear Vertical Force vs Time for 40 Km/hr (11.11 m/s)

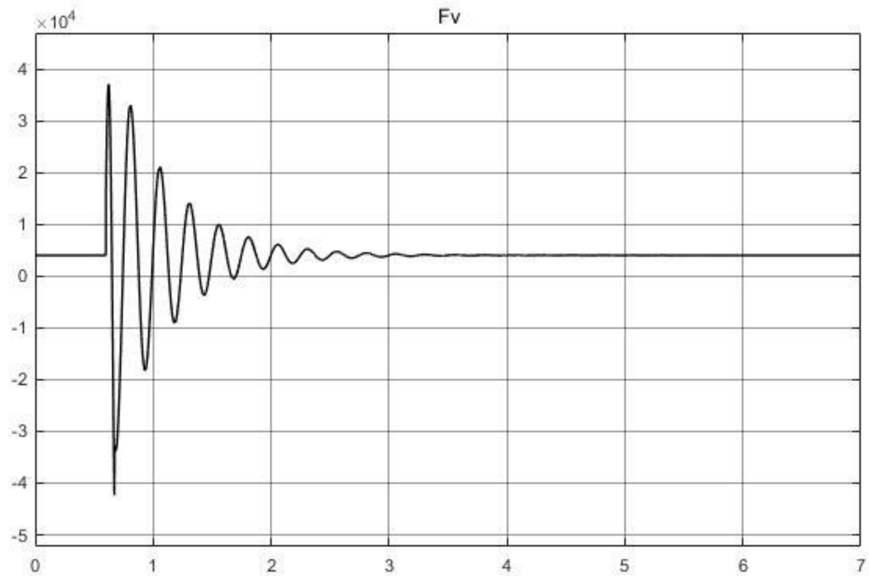

Figure 14. Linear Vertical Force vs Time for 60 km/hr (16.67 m/s)

By referring to the Figure. 12-14, it can be stated that the Vertical Forces on the vehicle decreases for high-velocity vehicles. Maximum Vertical Force of 4.6 x 10^4 was observed for the velocity of 20 km/hr, which decreased to 3.75 x 10^4 for 60 km/hr. One thing to note that the vertical force concept lies on the assumption of the point contact between the tire and ground.

Horizontal Force Plots

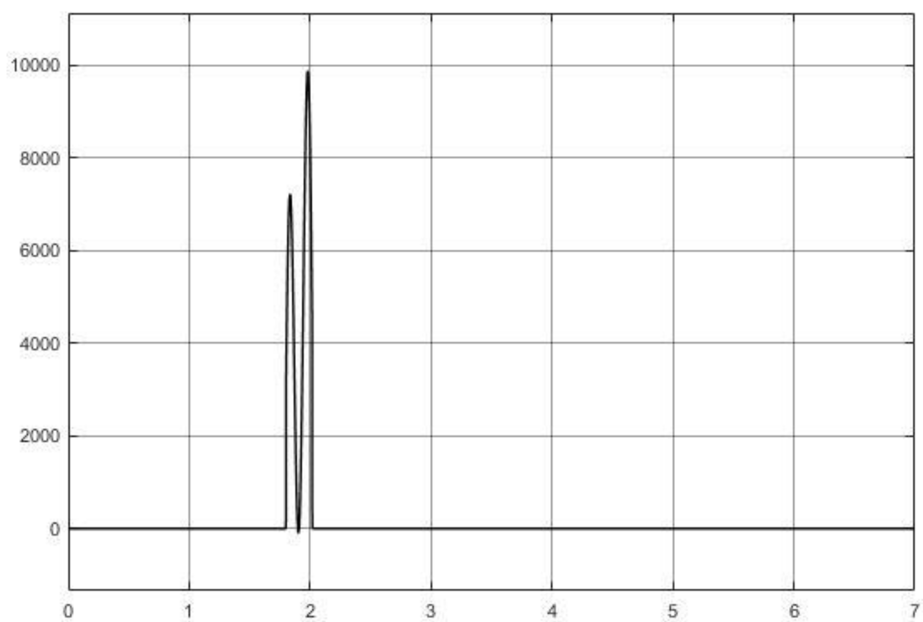

Figure 15. Linear Horizontal Force vs Time for 20 km/hr (5.55 m/s)

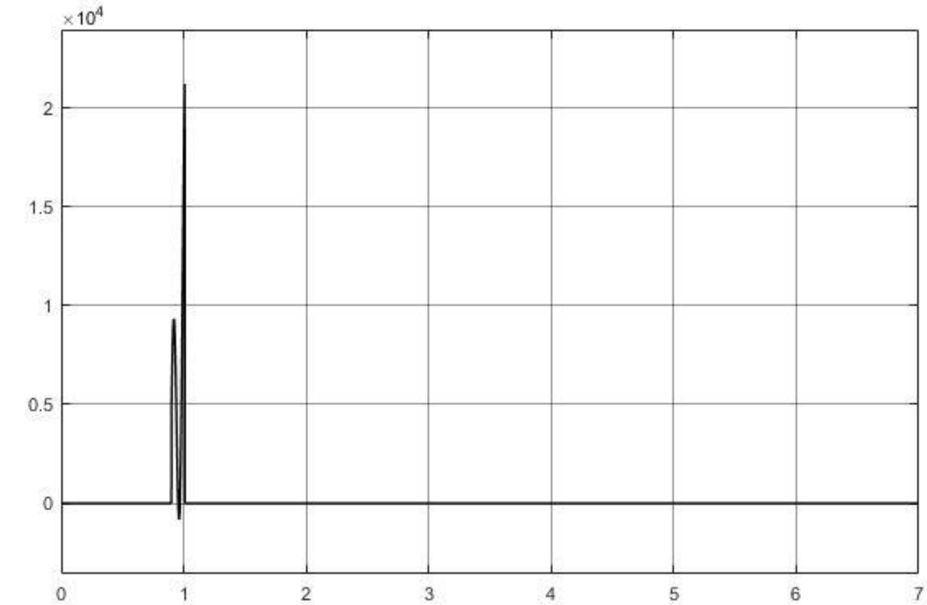

Figure 16. Linear Horizontal Force vs Time for 40 Km/hr (11.11 m/s)

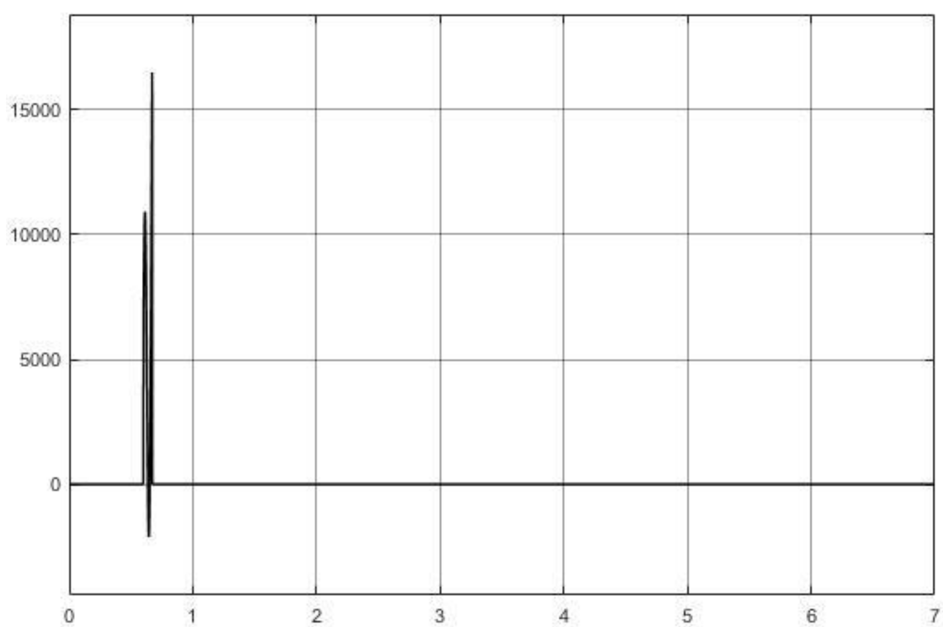

Figure 17. Linear Horizontal Force vs Time for 60 Km/hr (16.67 m/s)

Figure. 15-17 shows the transmitted horizontal force with respect to time. A different trend can be seen here where the horizontal force increases from 9800 N to 21,000 N as the velocity is increased from 20 to 40 km/hr. However, it again decreases to 17,000 at 60 km/hr, together with the presence of negative horizontal forces.

(c) Non- Linear Model

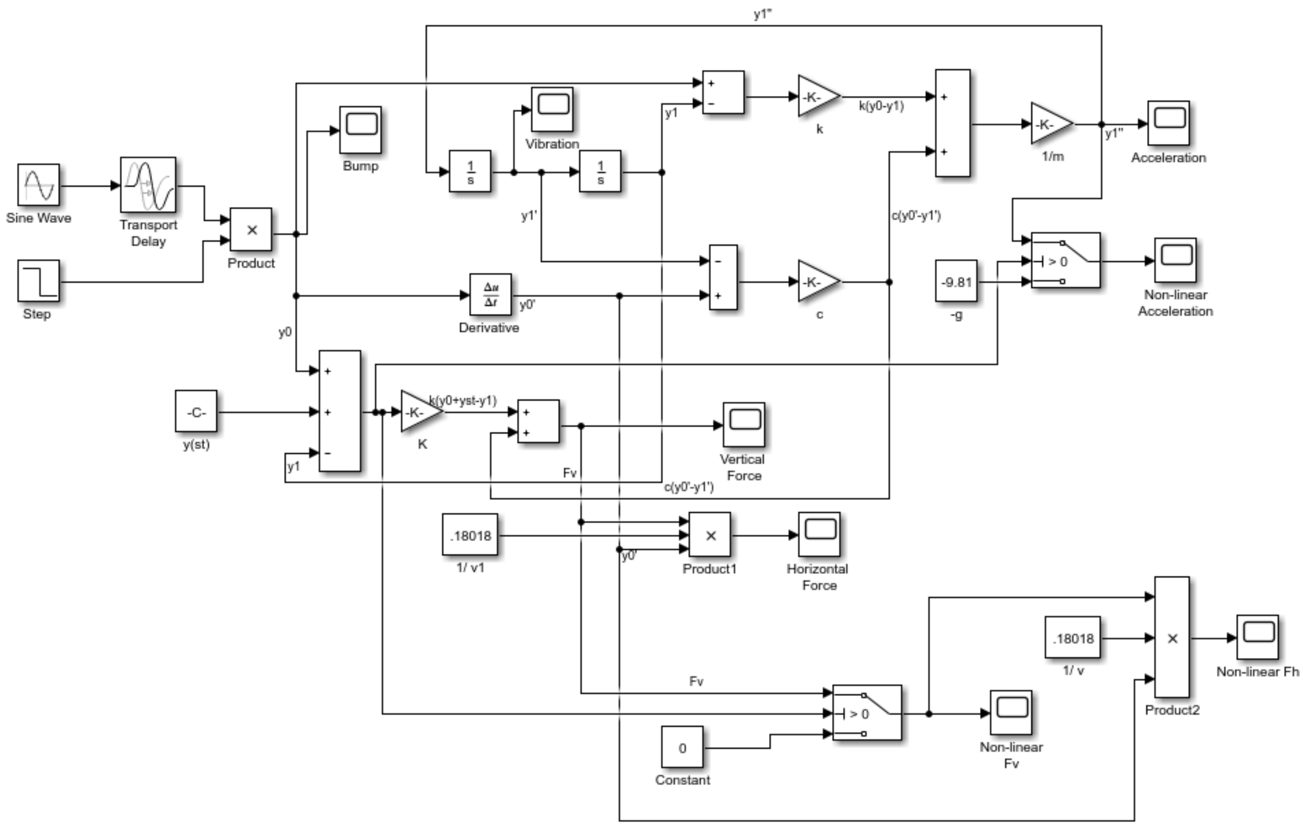

Figure 18. Non-Linear Model on Simulink

Vertical Force Plots

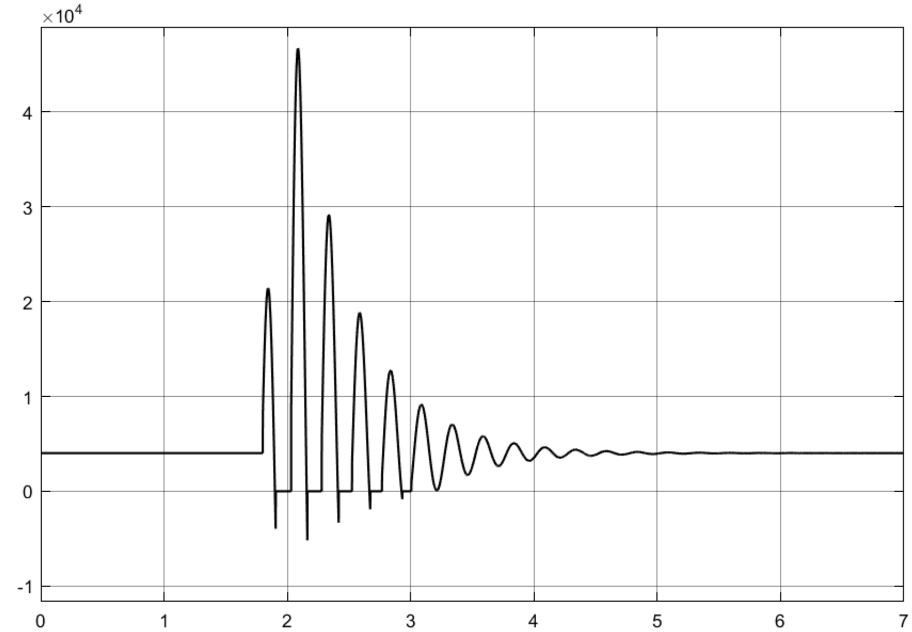

Figure 19. Non-Linear Vertical Force vs Time for 20 km/hr (5.55 m/s)

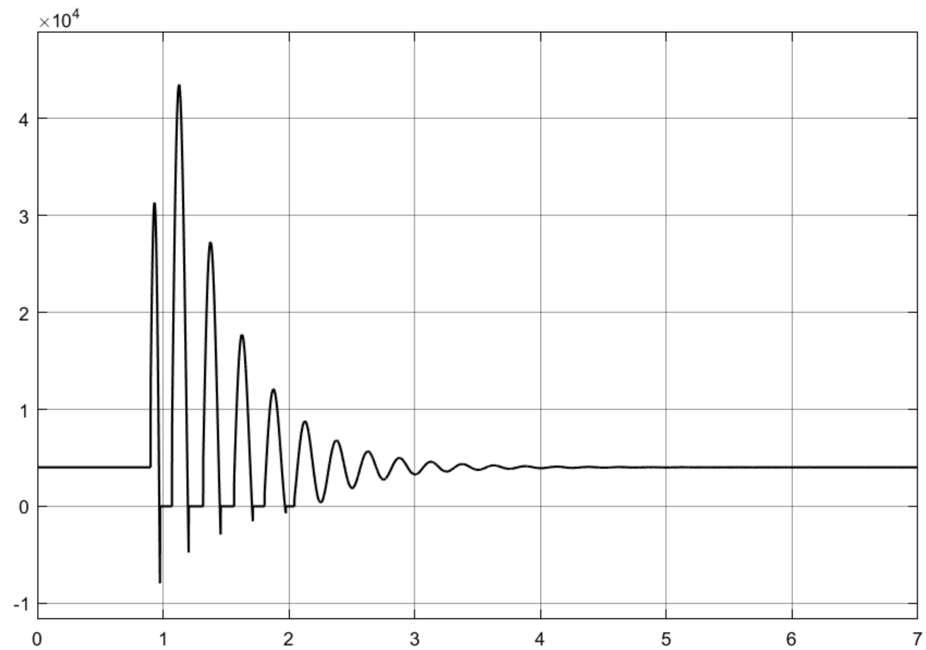

Figure 20. Non-Linear Vertical Force vs Time for 40 km/hr (11.11 m/s)

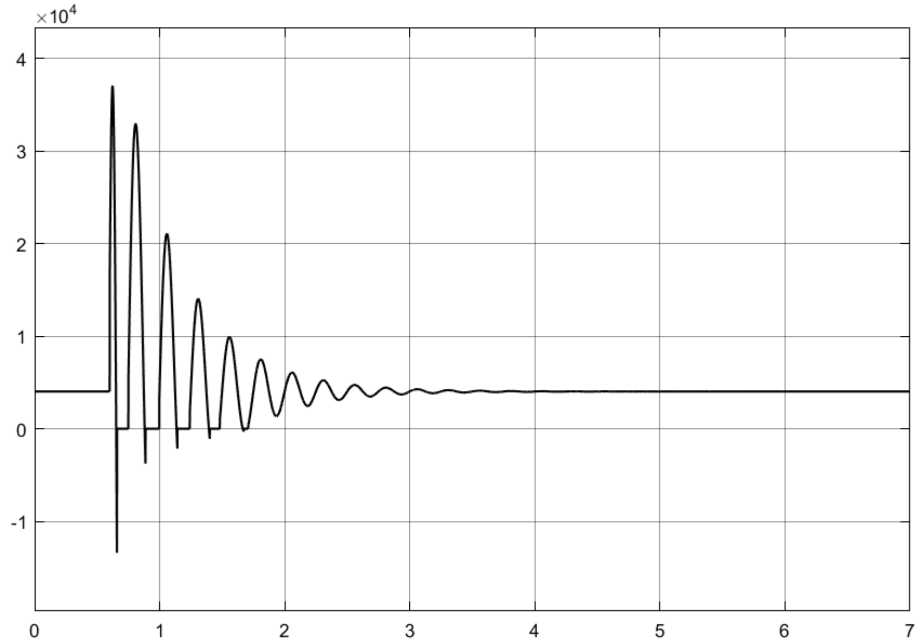

Figure 21. Non-Linear Vertical Force Vs time for 60 km/hr (16.67 m/s)

As can be seen in Figure.19-21, multiple hops are present in the non-linear vehicular motion where vertical force becomes zero. The number of hops present is 6 which are constant for all the three velocities considered, whereas a difference in start and end hop timings could be observed. Also, the magnitude of the vertical forces tends to decrease as the velocity of the vehicle increases.

Horizontal Force Plots

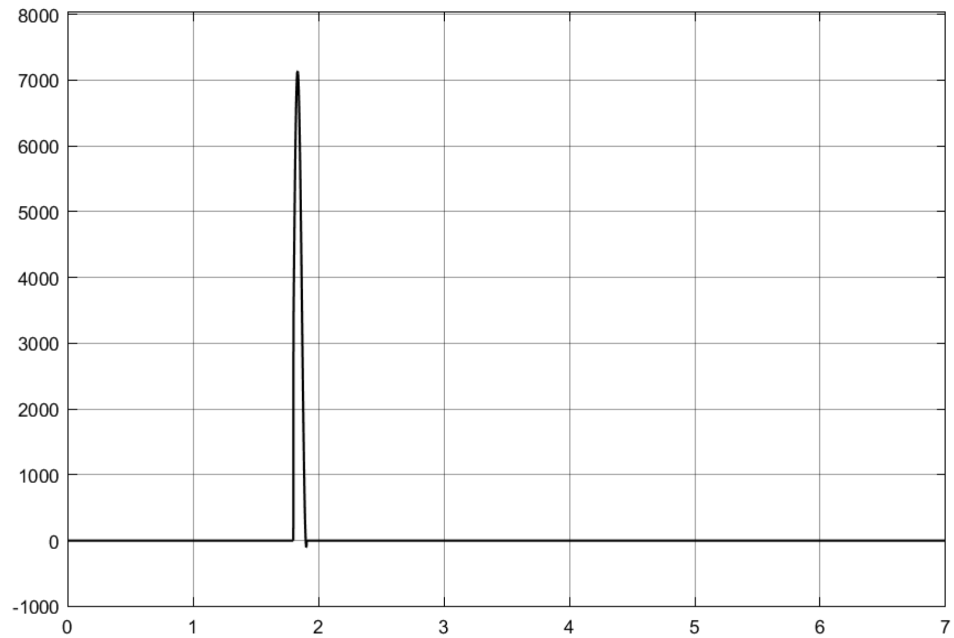

Figure 22. Non-Linear Horizontal Force vs Time for 20 km/hr (5.55 m/s)

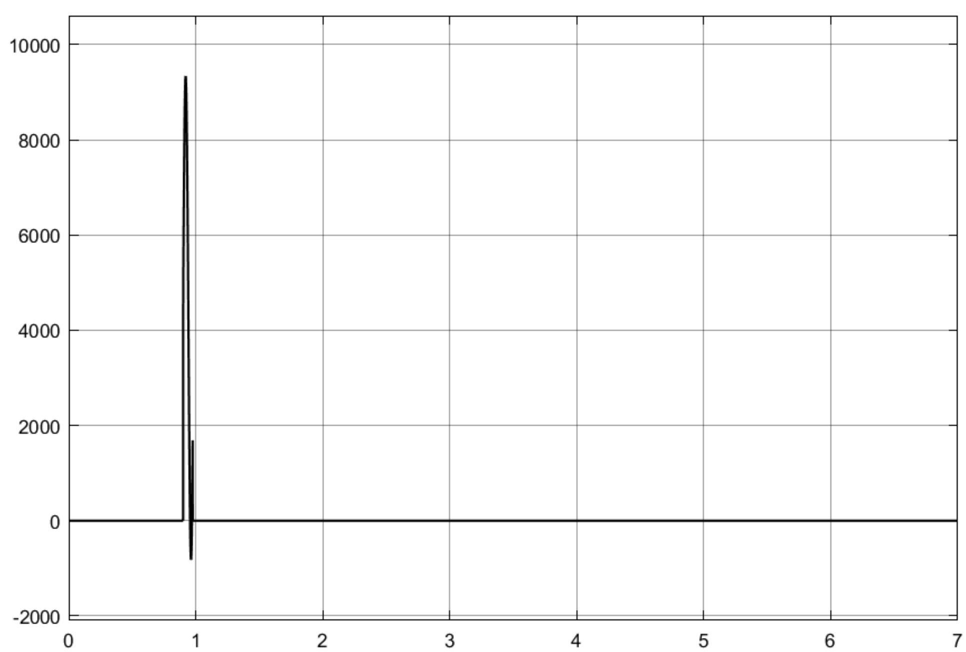

Figure 23. Non-Linear Horizontal Force vs Time for 40 km/hr (11.11 m/s)

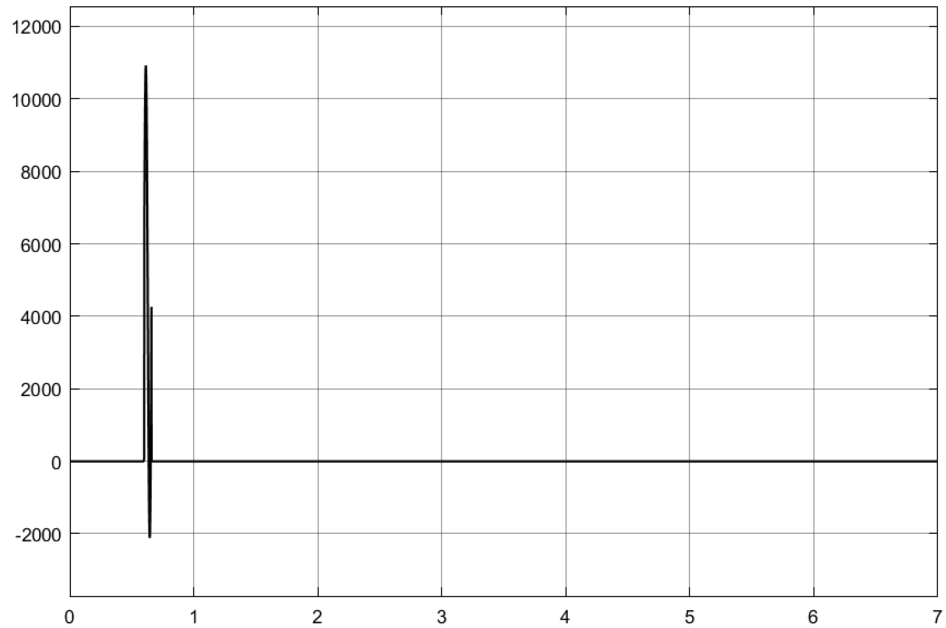

Figure 24. Non-Linear Horizontal Force vs Time for 60 km/hr (16.67 m/s)

An opposite behaviour for horizontal forces could be seen compared with the vertical forces. The horizontal force increases from 7100 N to 11,500 N as the velocity increases from 20 to 60 km/hr.

(d) Plots for Transmitted vibration on linear model for increased damping at v= 20 km/hr (5.55 m/s)

To simulate the model with increased damping, values of c=1727.62, c_1= 5183 (3 times of c), c_2= 10,366 Ns/m (6 times of c) have been considered.

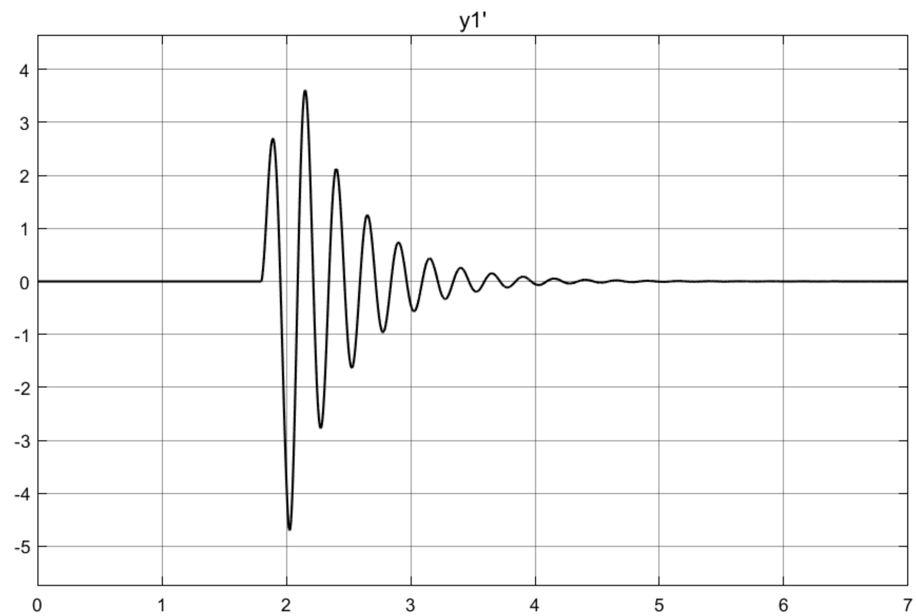

Figure 25. Linear Vibration Magnitude vs Time for c= 1727.62 Ns/m

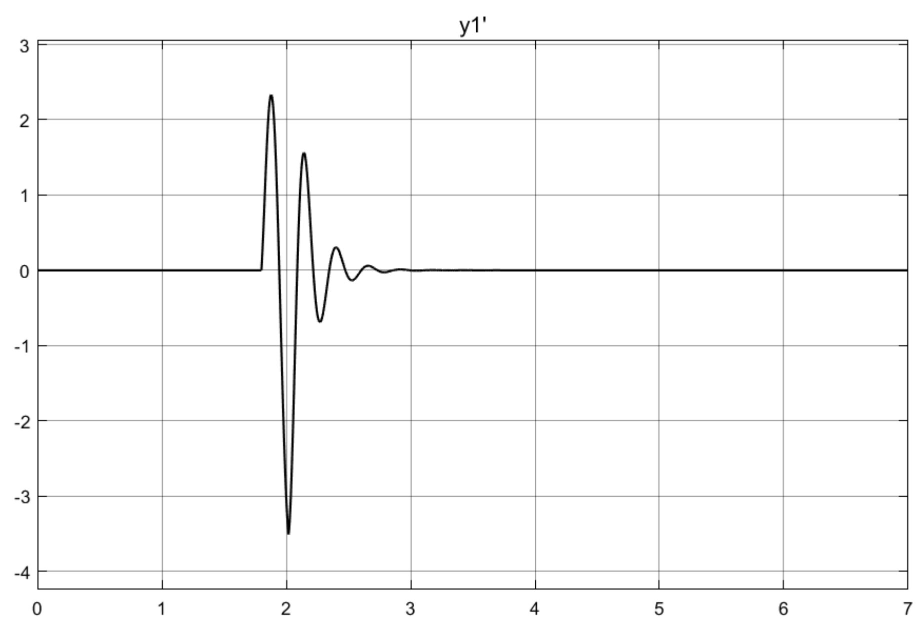

Figure 26. Linear Vibration Magnitude vs Time for c= 5183 Ns/m (3 times)

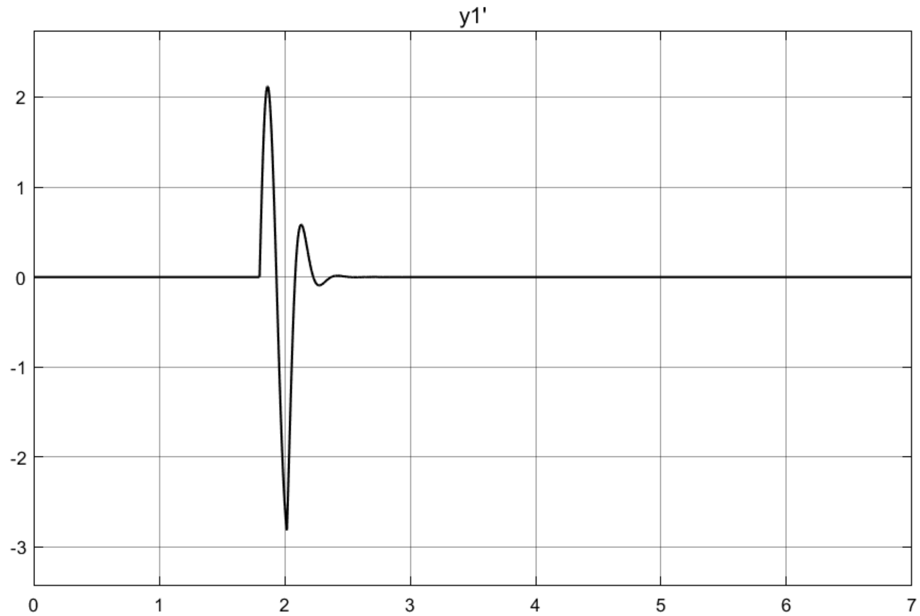

Figure 27. Linear Vibration Magnitude vs Time for c = 10,366 Ns/m (6 times)

The simulation was performed for a constant velocity of 5.55 m/s on a linear model, and it was observed that the vibration transmitted to the vehicle decreased as the damping capacity was increased in the model because of the fact that damper tends to absorb the vibration. As can be seen in the above figures.25-27, for damping capacity of 1727.62 Ns/m the vibration magnitude was around 3.5 which decreased to around 2.3 for c= 5183 Ns/m and to 2.1 for c= 10,366 Ns/m. Hence, a more comfortable ride can be possible with increased damper.

It was also observed that with the increase in damping capacity, there was a significant reduction in the vibration oscillations.

Plots for Vertical Force Vs time for increased damping at v= 20 km/hr (5.55 m/s))

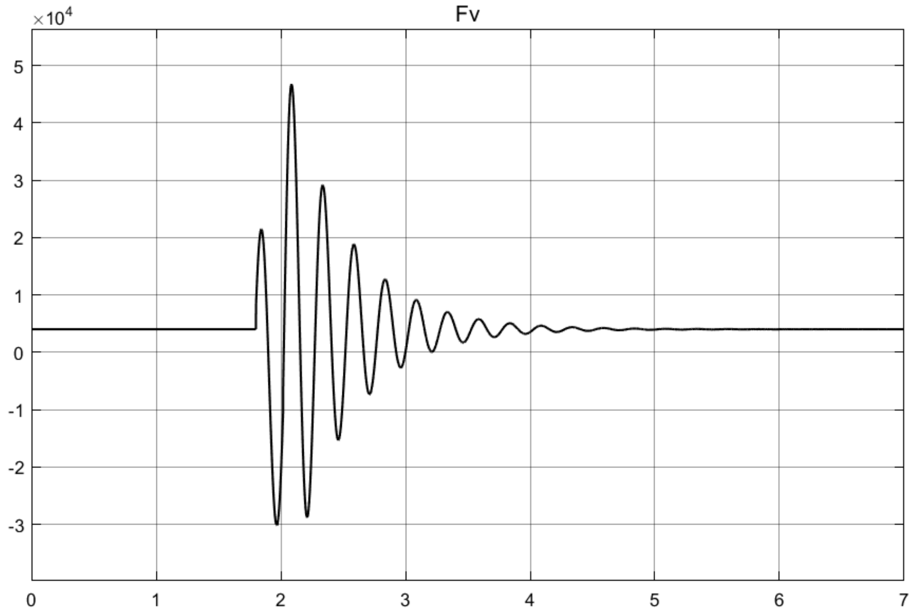

Figure 28. Vertical Force vs Time for c= 1727.62 Ns/m

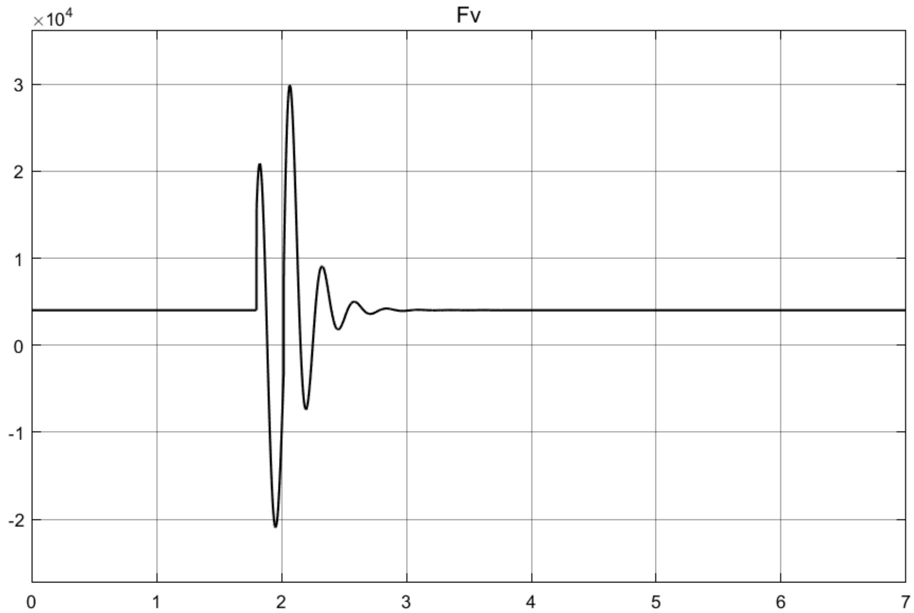

Figure 29. Vertical Force vs Time for c= 5183 Ns/m (3 times)

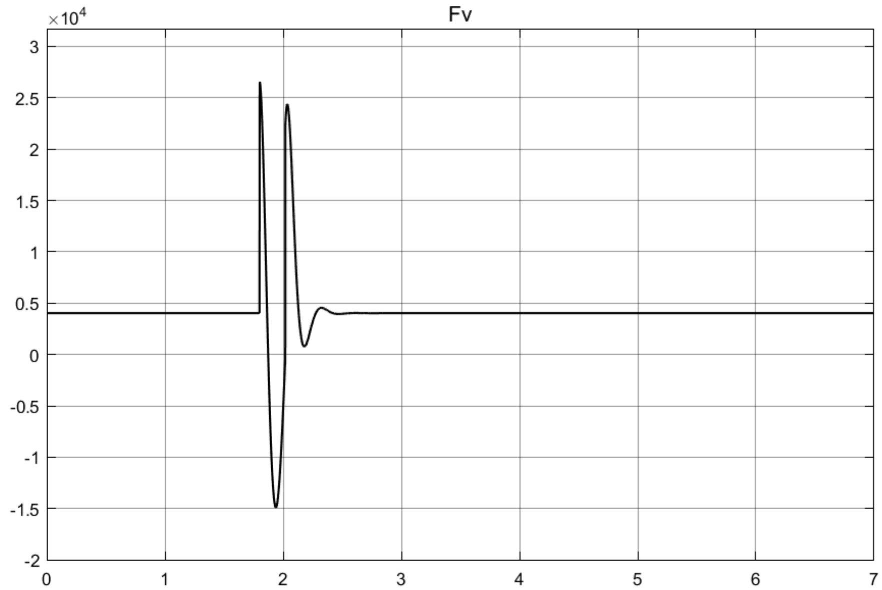

Figure 30. Vertical Force vs Time for c= 10,366 Ns/m (6 times)

The simulation was performed for a constant velocity of 5.55 m/s on a linear model, and it was observed that the Vertical force transmitted to the pavement also followed the same behaviour as the vibration. As can be seen in the above figures.28-30, for damping capacity of 1727.62 Ns/m the transmitted Vertical force was around 4.7 x 10^4 N which decreased to approximately 3 x 10^4 N when damping capacity increased to 3 times and to 2.6 x 10^4 N when increased to 6 times.

It was also observed that with the increase in damping capacity, there was a significant reduction in the transmitted vertical force, similar to the behaviour of vibration magnitude.

Plots for Horizontal Force Vs time for increased damping at v= 20 km/hr (5.55 m/s)

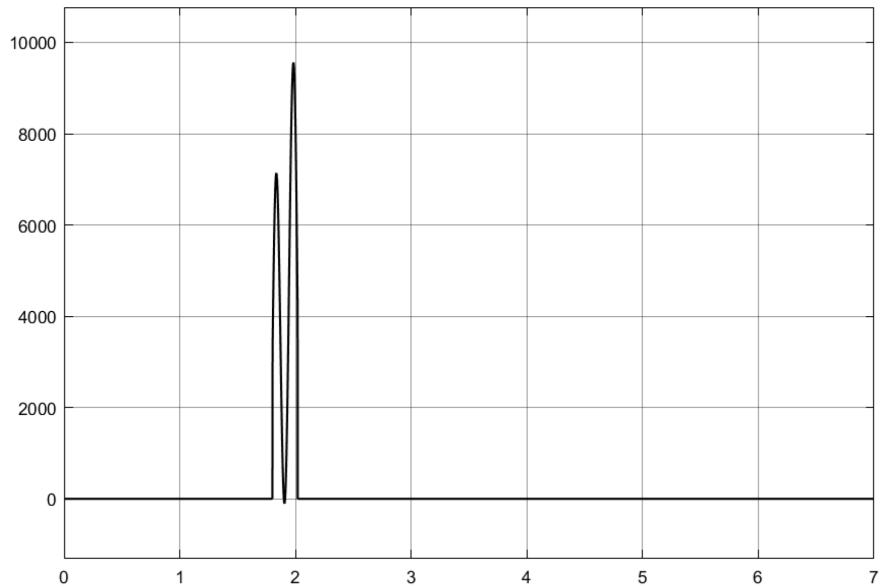

Figure 31. Horizontal Force vs Time for c= 1727.62 Ns/m

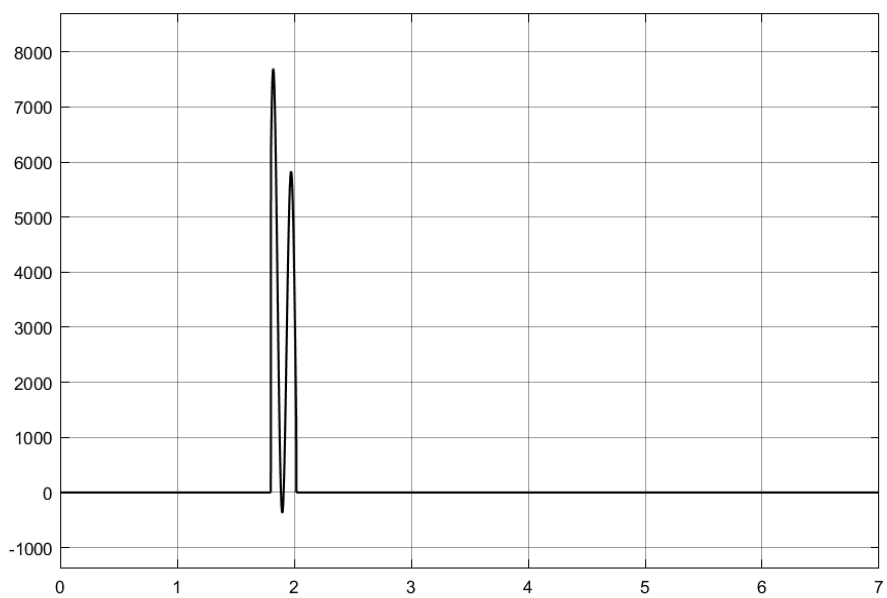

Figure 32. Horizontal Force vs Time for c= 5183 Ns/m (3 times)

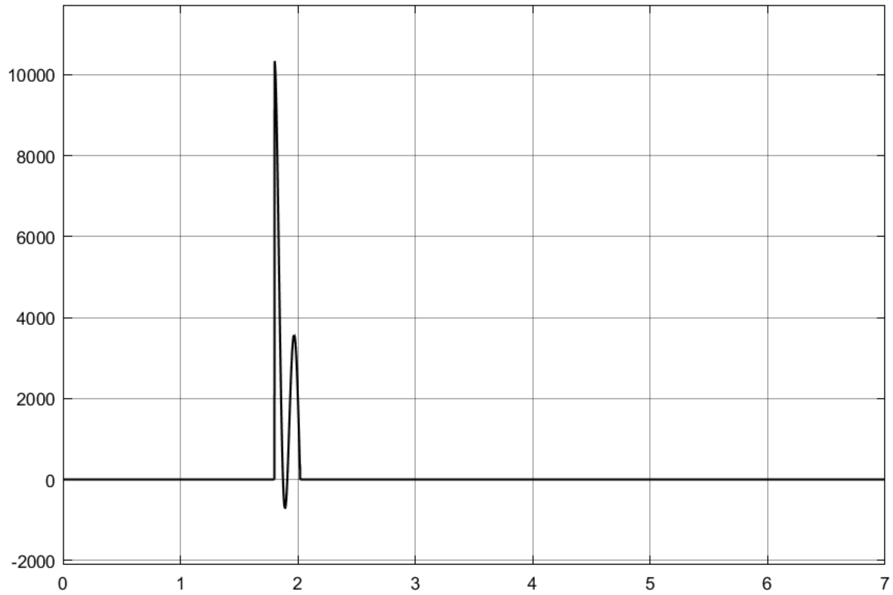

Figure 33. Horizontal Force vs Time for c= 10,366 Ns/m (6 times)

The expected results similar to the previous two cases were noticed in Figure.31-32, the maximum transmitted horizontal force decreased from 9500 N to 7600 N when the damping capacity increased from 1727.62 Ns/m to 3 times, however, it increased back to about 10,050 N when further increased to 6 times.

With increased damper, a significant reduction in wheel hop can be achieved leading to a more comfortable ride on the vehicle.

CONCLUSION

The linear mathematical model was created for the given problem on Simulink and plots for acceleration and dynamics forces were shown for different velocities. From the plots, it was observed that with the increase in vehicle velocity, the magnitude of the acceleration and vertical forces followed a decreasing trend, however, for horizontal forces, no specific trend could be seen with respect to the velocity. In the later part, non- linear model was taken into consideration and the number of hops was identified. And finally, the effect of increased damping capacity on the vehicle suspension was studied and a significant decrement in vibration magnitude and dynamic forces was observed with the increase in damping.