Contents

LIST OF FIGURES

PROBLEM STATEMENT

Double wish bone suspension needs to be investigated by considering 2 degrees of freedom system. Further, it is required to find the equations of motion and the damped frequency.

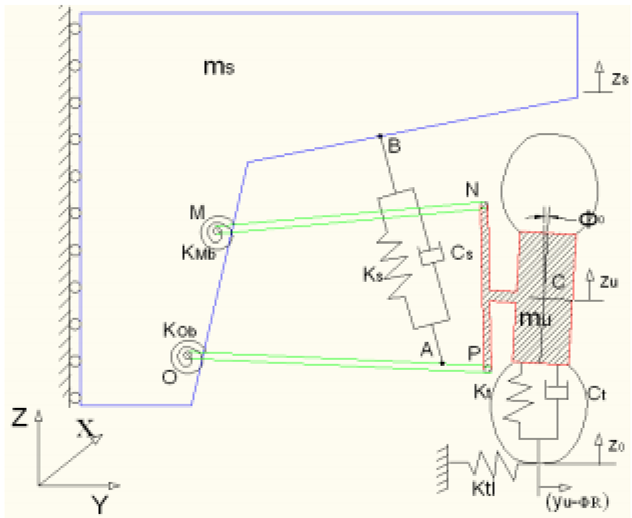

Figure 1 Double wishbone suspension system

A bump on the road has been given and for that acceleration responses need to be evaluated for symmetric and asymmetric damping capacity of the suspension damper.

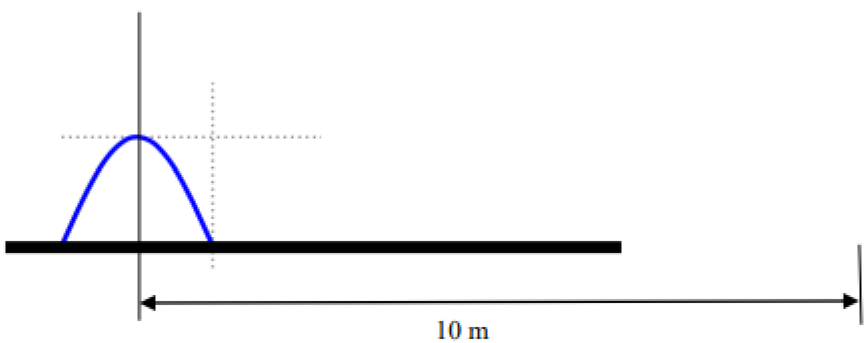

Figure 2 Bump on the ride

MODEL FORMULATION

Figure 3 shows the non-linear Simulink Model for damping capacity of 500 Ns/m and Figure 4 shows the non-linear Simulink Model for asymmetric damper having Ceq= 750 Ns/m during rebound and 250 Ns/m during compression.

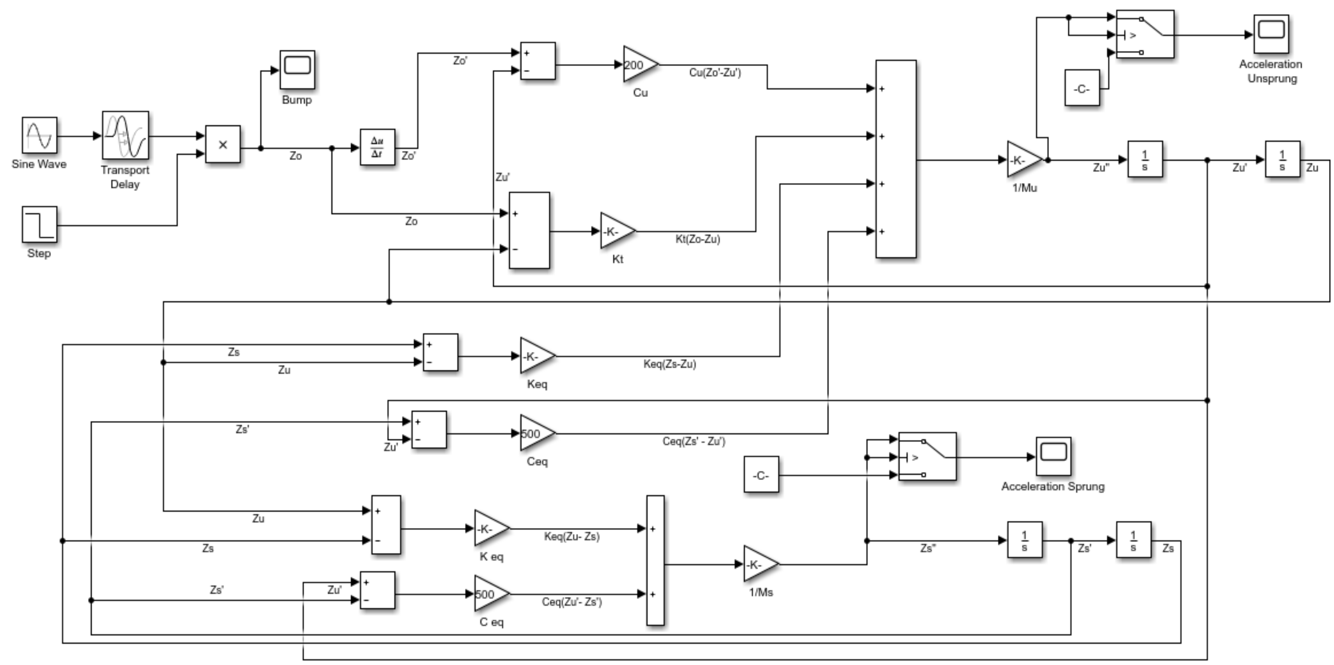

Figure 3 Simulink Model for Ceq= 500 Ns/m

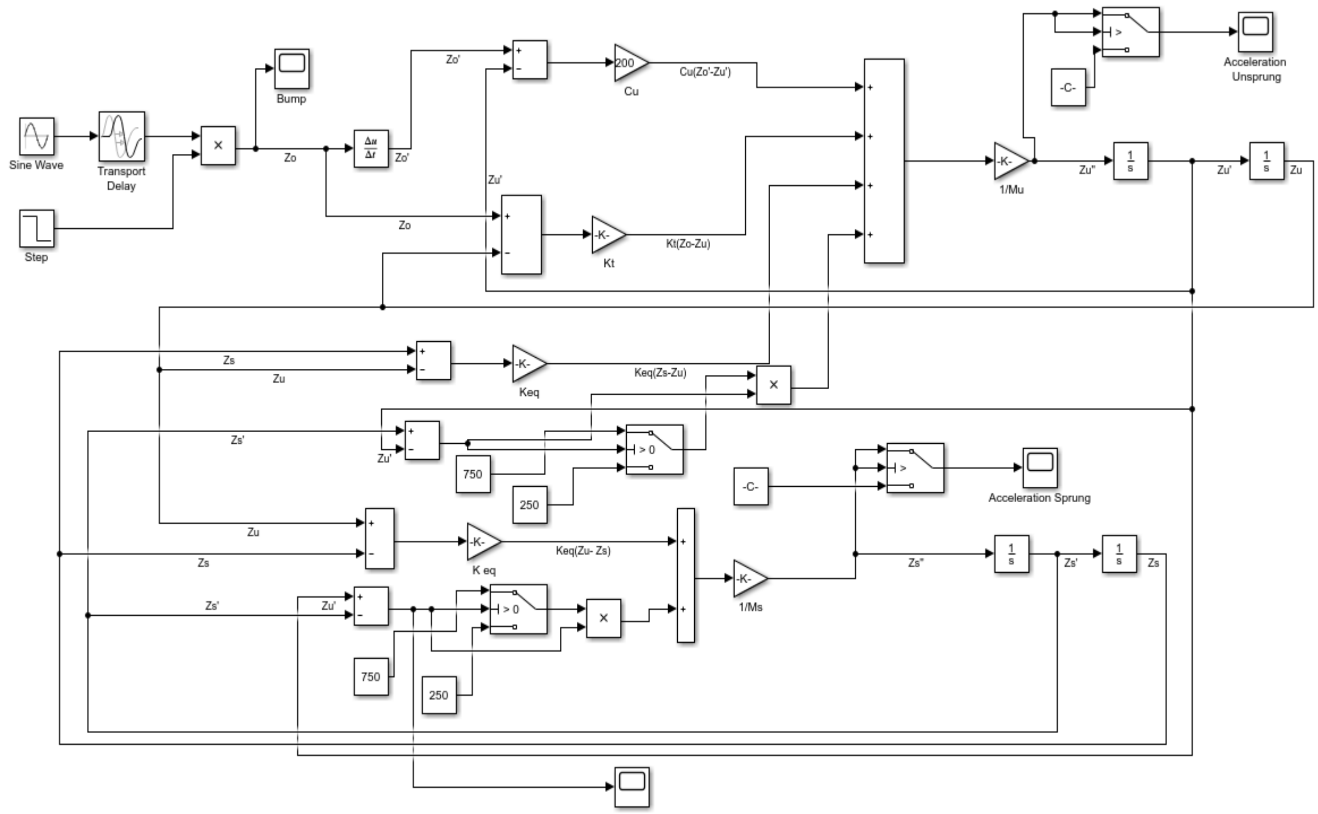

Figure 4 Simulink Model for asymmetric suspension damper with Cwq= 750 Ns/m during reboiund and 250 Ns/m during compression

RESULTS AND DISCUSSION

A. Equations of Motion

The 2 degrees of freedom system model for the double wishbone suspension system has been given in Figure 5. Based on this model equations of motion have been evaluated to construct the model in Simulink as shown in Figure 3-4.

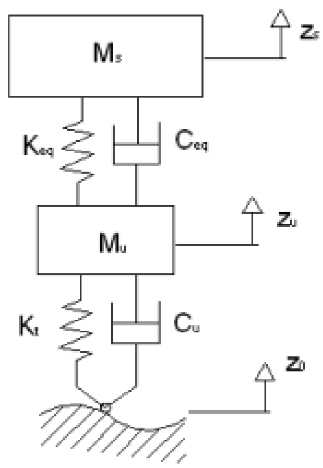

Figure 5 2-degrees of Freedom ride model

Sprung mass, M_s = 240 kg

Unsprung mass, M_u=42 kg

Equivalent suspension spring rate, K_{eq}=20,000 N/m

Equivalent viscous damping, C_{eq}=500 Ns/m or 750 Ns/m

Tire vertical stiffness, K_t = 180 kN/m

Tire viscous damping constant, C_t = 200 Ns/m

Equations of motions for the 2 degrees of freedom system can be written as,

For Mass M_s,

$$M_s\ddot{Z}_s + C_{eq}(\dot{Z}_s - \dot{Z}_u) + K_{eq}(Z_s - Z_u) = 0$$

$$\ddot{Z}_s = \frac{1}{M_s}\left[C_{eq}(\dot{Z}_u - \dot{Z}_s) + K_{eq}(Z_u - Z_s)\right]$$

For Mass M_u,

$$\boldsymbol{M_u\ddot{Z}_u - C_{eq}(\dot{Z}_s - \dot{Z}_u) + C_u(\dot{Z}_u - \dot{Z}_o) - K_{eq}(Z_s - Z_u) + K_t(Z_u - Z_0) = 0}$$

$$\boldsymbol{\ddot{Z}_u = \frac{1}{M_u}\left[C_{eq}(\dot{Z}_s - \dot{Z}_u) + C_u(\dot{Z}_0 - \dot{Z}_u) + K_{eq}(Z_s - Z_u) + K_t(Z_0 - Z_u)\right]}$$

B. Undamped Natural Frequency and Mode Shapes

For undamped frequency,

The above equations of motion can also be written as,

$$M_u\ddot{Z}_u - C_{eq}(\dot{Z}_s - \dot{Z}_u) + C_u(\dot{Z}_u - \dot{Z}_o) - K_{eq}(Z_s - Z_u) + K_t(Z_u - Z_0) = 0$$

$$M_u\ddot{Z}_u + Z_u(K_{eq} + K_t) + Z_s(-K_{eq}) = Z_0(K_t)$$

Also,

$$M_s\ddot{Z}_s + C_{eq}(\dot{Z}_s - \dot{Z}_u) + K_{eq}(Z_s - Z_u) = 0$$

$$M_s\ddot{Z}_s + Z_u(-K_{eq}) + Z_s(K_{eq}) = 0$$

By combining both the equations in Matrix form,

$$\begin{bmatrix} M_u & 0 \\ 0 & M_s \end{bmatrix}\begin{pmatrix} \ddot{Z}_u \\ \ddot{Z}_s \end{pmatrix} + \begin{bmatrix} K_{eq} + K_t & -K_{eq} \\ -K_{eq} & K_{eq} \end{bmatrix}\begin{pmatrix} Z_u \\ Z_s \end{pmatrix} = \begin{pmatrix} Z_0 K_t \\ 0 \end{pmatrix}$$

Assuming a harmonic solution of the form

$$\begin{pmatrix} Z_u \\ Z_s \end{pmatrix} = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} e^{j\omega t}$$

Where Z_1 and Z_2 are constants, yields

$$\begin{bmatrix} K_{eq} + K_t - M_u\omega^2 & -K_{eq} \\ -K_{eq} & K_{eq} - M_s\omega^2 \end{bmatrix}\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

From the linear algebra we know that this equation can only be valid for nontrivial solution of Z_1 and Z_2, if the determinant of the coefficient matrix vanishes

$$det\begin{bmatrix} K_{eq}+K_t-M_u\omega^2 & -K_{eq} \\ -K_{eq} & K_{eq}-\ M_s\omega^2 \end{bmatrix}=0$$

Therefore,

$$M_uM_s\omega^4-\left(M_uK_{eq}+\ M_s\left(K_t+K_{eq}\right)\right)\omega^2+K_tK_{eq}=0$$

$$\omega_{1,2}=\ \sqrt{\frac{\left(M_uK_{eq}+\ M_s\left(K_t+K_{eq}\right)\right)\ \underline{+}\sqrt{\left(M_uK_{eq}+\ M_s\left(K_t+K_{eq}\right)\right)^2-4M_uM_sK_tK_{eq}}}{2M_uM_s}}$$

$\omega_{1,2}=$

$$\sqrt{\frac{((42*20{,}000)+240(180{,}000+20{,}000))\ \underline{+}\ \sqrt{((42*20{,}000+240(180{,}000+20{,}000))^2-4(42*240*20{,}000*180{,}000)}}{2*42*240}}$$

$$\omega_{1,2}=\ \sqrt{\frac{48{,}840{,}000\ \underline{+}\ \ 47{,}330{,}201.7}{20{,}160}}$$

$$\omega_1=69.06\,rad/s$$

$$\omega_2=8.654\,rad/s$$

Mode shape can be calculated with the mentioned relations which have been derived from the equations of motion,

$$\frac{Zs}{Zu}=\frac{Keq}{(Keq-ms.wn^2)}$$

$$\frac{Zs}{Zo}=\frac{Keq*Kt*M_u}{(Keq+Kt-mu.wn^2)(Keq-ms.wn^2)-(Keq^2)}$$

$$\frac{Zu}{Zo}=\frac{(Keq-ms.wn^2)(Kt)M_u}{((Keq-ms.wn^2)(Keq+Kt-mu.wn^2)-(Keq^2)}$$

For $\omega_1=69.06\,rad/s$

By putting all the given values, we will get,

$$\frac{Zs}{Zu} = -0.0178$$

$$\frac{Zs}{Zo} = -0.3356$$

$$\frac{Zu}{Zo} = 3.025$$

Also, for $\omega_2 = 8.654\, rad/s$

$$\frac{Zs}{Zu} = 9.8717$$

$$\frac{Zs}{Zo} = 13068$$

$$\frac{Zu}{Zo} = 11013$$

C. Frequency response characteristics relating Sprung mass acceleration

Figure 6 compares the acceleration response values for two different dampers of 750 Ns/m and 500 Ns/m. It can be observed that the damper with damper coefficient of 500 Ns/m gives the maximum acceleration response of around 5.4 m/s^2 whereas with the increased damping capacity to 750 Ns/m, the acceleration significantly reduced to 3.6 m/s^2. It justifies the use of high capacity damper for the vehicle as it reduces the vibrational energy and make the ride smoother and more comfortable.

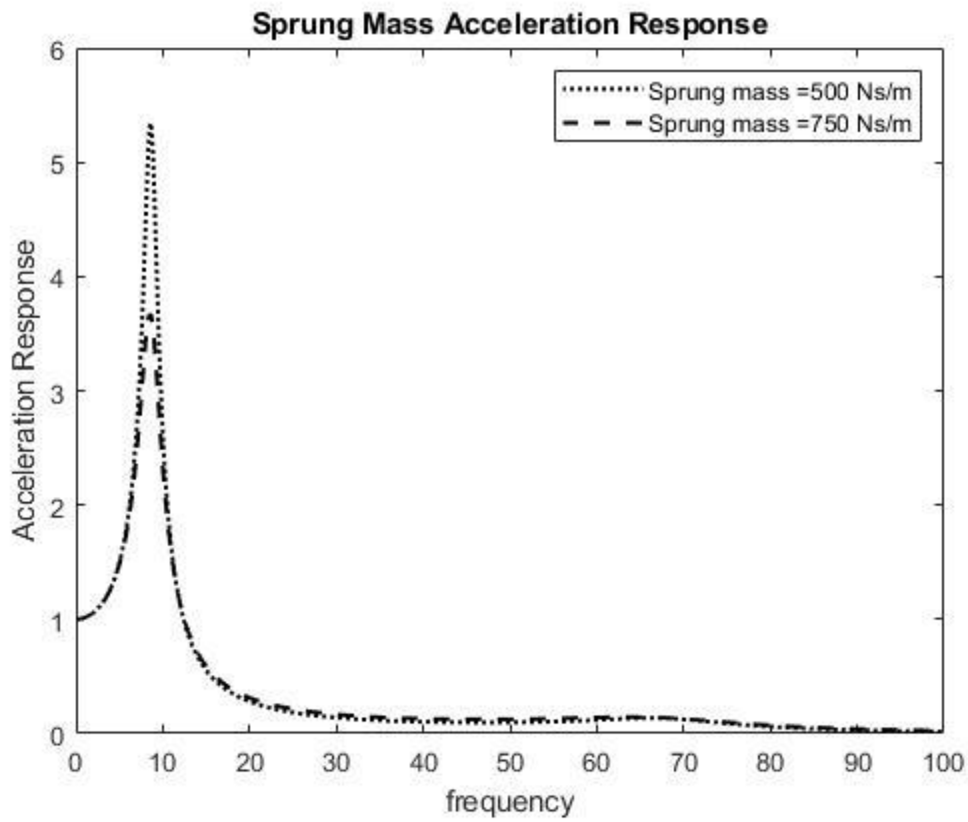

Figure 6 Acceleration Response for Sprung Mass

The MATLAB code for generating the Figure 6 has been given below,

---

```
f=0.1:0.1:100;
z1=nan(1,1000);
z2=nan(1,1000);
z3=nan(1,1000);
z4=nan(1,1000);


% for the case of damping Coefficient=500 Ns/m
for i=1:1000
   matrix=[(-240*f(i)^2)+(500i*f(i))+20000 ((-500i*f(i))-20000);((-500i*f(i))-20000)...
      (-42*f(i)^2)+(700i*f(i))+200000];
   invm=inv(matrix);
   k=invm*[0;(200i*f(i))+180000];
   z1(i)=abs(k(1,1));
   z2(i)=abs(k(2,1));
```

```
    i=i+1;
end

 % for the case of damping Coefficient= 750 Ns/m
for i=1:1000
    matrix2=[(-240*f(i)^2)+(750i*f(i))+20000 ((-750i*f(i))-20000);((-750i*f(i))-20000) ...
        (-42*f(i)^2)+(950i*f(i))+200000];
    invm2=inv(matrix2);
    k2=invm2*[0;(200i*f(i))+180000];
    z3(i)=abs(k2(1,1));
    z4(i)=abs(k2(2,1));
    i=i+1;
end



figure(1); clf
plot(f,z1,'k:',f,z3,'k--','linewidth',1.5);
xlabel('frequency');
ylabel('Acceleration Response');
legend('Sprung mass =500 Ns/m','Sprung mass =750 Ns/m');
title('Sprung Mass Acceleration Response')
```

D. Acceleration Response for Ceq= 500 Ns/m

Based on the Simulink model shown in Figure 3, the mass acceleration plots have been plotted for damping coefficient of 500 Ns/m. Figure 7 shows the acceleration response for unsprung mass for the non-linear condition in which the vehicle may lose contact from the road surface. In this condition the acceleration acting on the vehicle will be just due to gravity which can been verified from the plot. The maximum acceleration for unsprung mass reaches about 72 m/s^2.

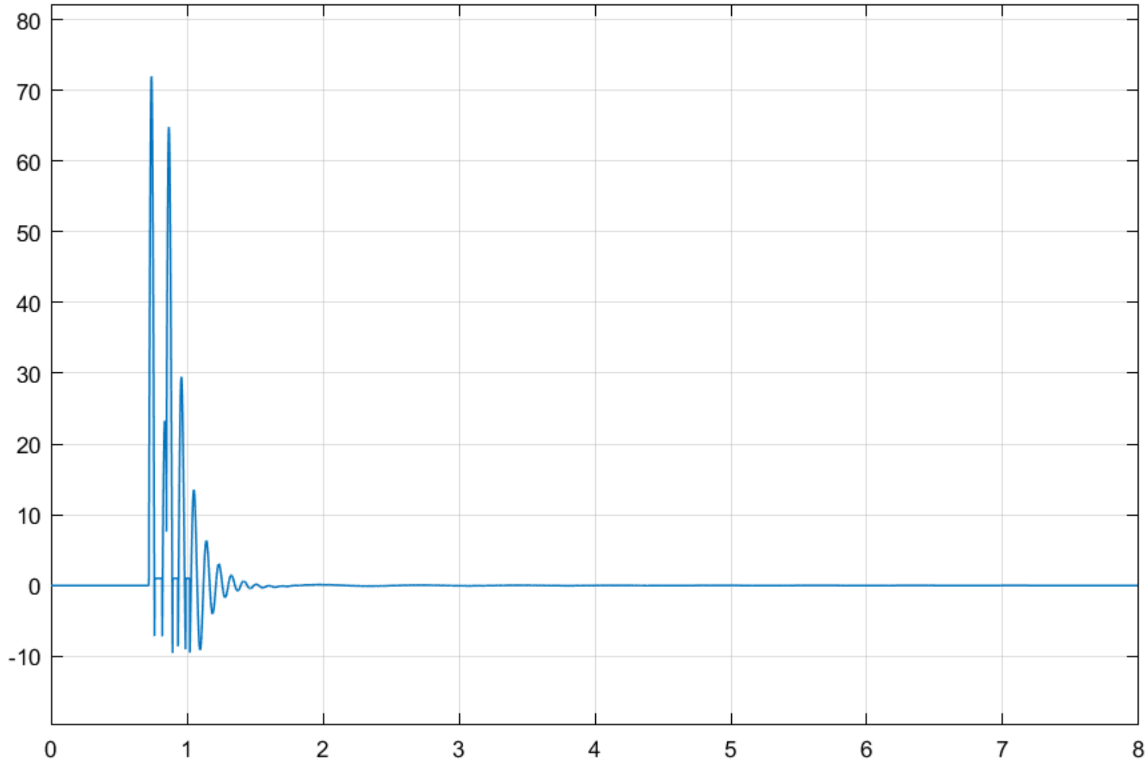

Figure 7 Unsprung Acceleration response vs time for Ceq= 500 Ns/m

The mass acceleration for the sprung mass reduced significantly to about 7.5 m/s^2 as shown in Figure 8 compared to the unsprung mass which was around 72 m/s^2. It can also be observed that the ride has become more convenient as the frequency of the vibrations also decreased due to the sprung mass damper and spring

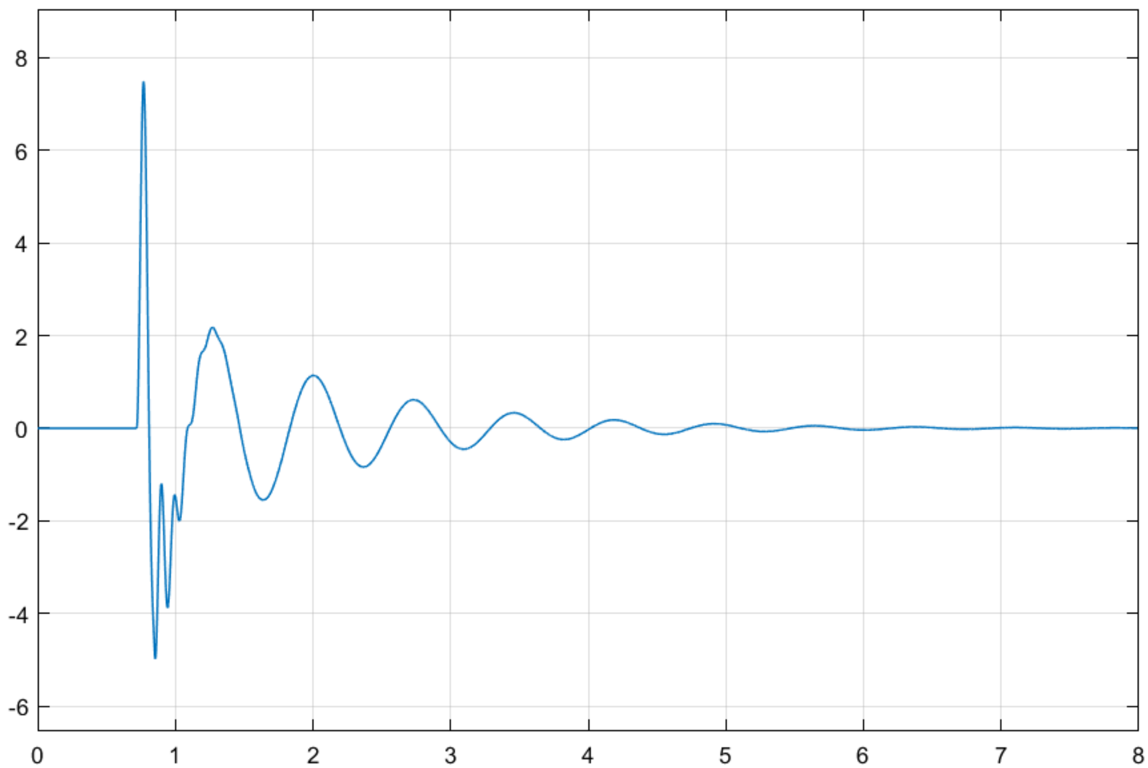

Figure 8 Sprung Acceleration response vs Time for Ceq= 500 Ns/m

E. Acceleration Response for Asymmetric Suspension Damper

For this case asymmetric damper has been considered which gives different damping coefficient values for rebound and compression of the damper. For rebound Ceq = 750 Ns/m and for compression it is 250 Ns/m. Due to this varying damping effect and high value for rebound gave rise to increased mass acceleration on the unsprung mass of about 77 m/s^2 as shown in Figure 9, compared to the case where damper was symmetric having value of Ceq= 500 Ns/m.

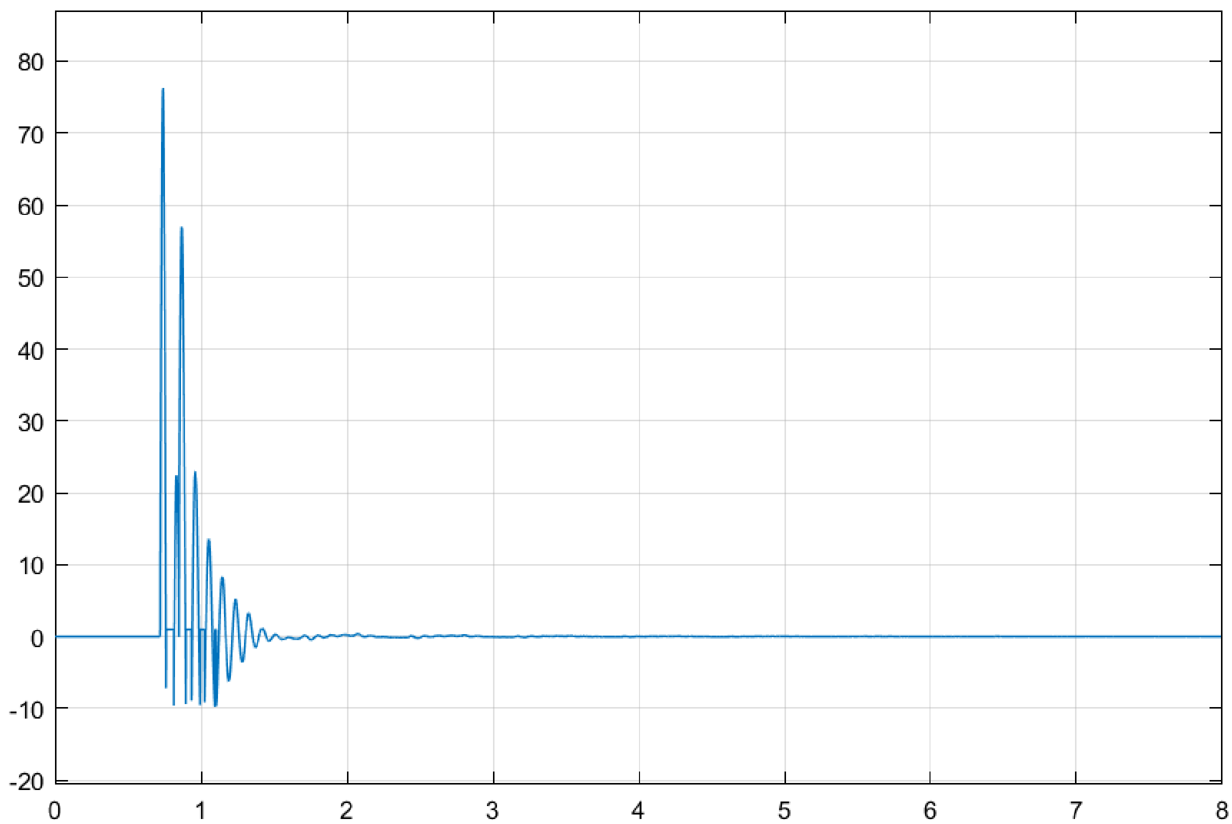

Figure 9 Unsprung Acceleration response for asymmetric damping having Ceq= 750 Ns/m during rebound and 250 Ns/m during compression

Similar to the unsprung case, the maximum acceleration measured for sprung mass was around 9.1 m/s_2 for asymmetric damper as shown in Figure 10. However overall behavior of the acceleration frequency is close to the case for symmetric damper.

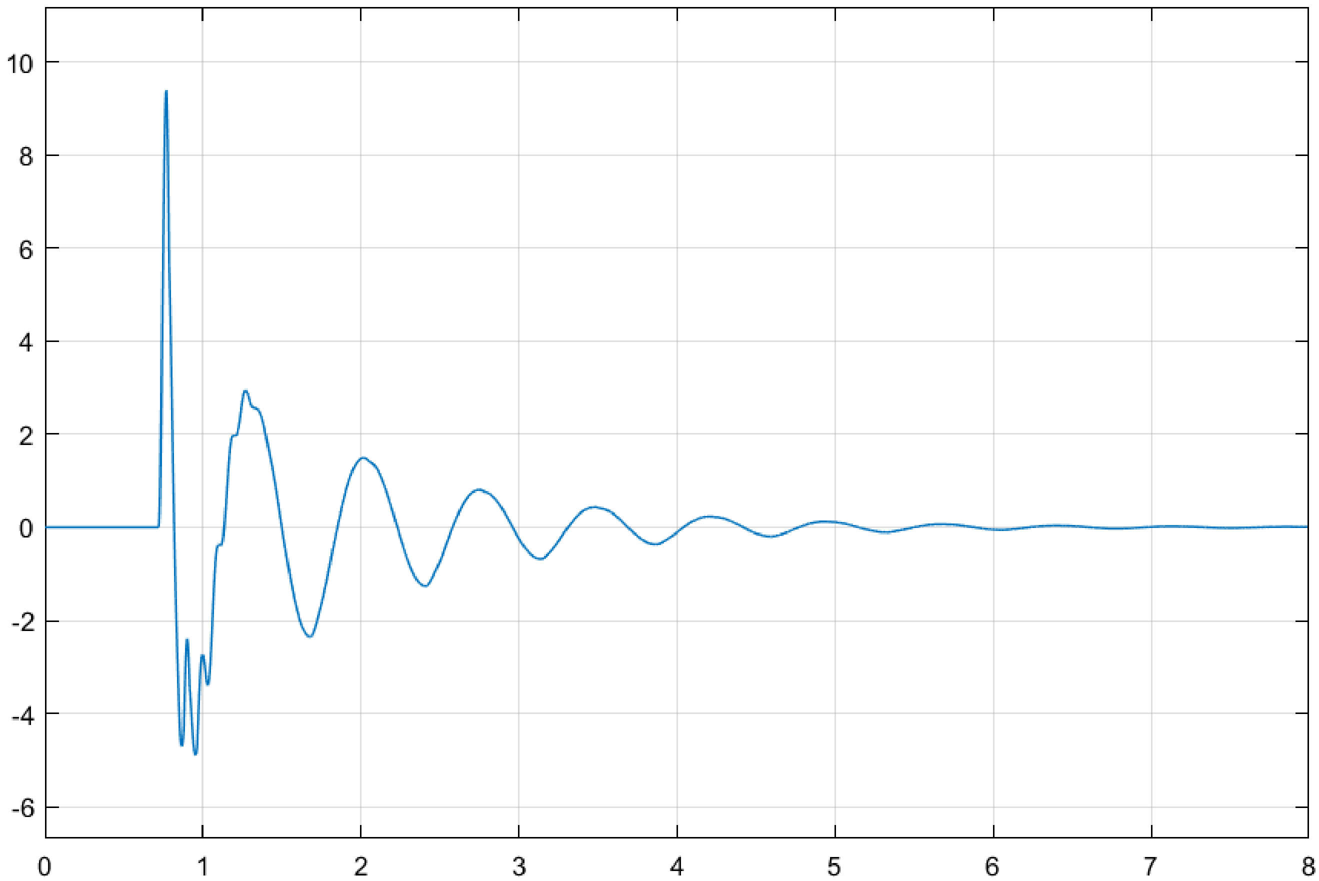

Figure 10 Sprung Acceleration response for asymmetric damping having Ceq= 750 Ns/m during rebound and 250 Ns/m during compression

F. CONCLUSION

Nature of suspension plays an important role in reducing the vibration effects on the vehicle occupants. In the previous two cases, symmetric suspension damper having Ceq = 500 Ns/m and an asymmetric damper having Ceq= 750 Ns/m during rebound and Ceq= 250 Ns/m during compression were studied. And it can be concluded that the second case yields undesirable results as it is increasing the peak mass acceleration of both the unsprung and sprung mass. This is due to the fact that the damping capacity during compression has been reduced to 250 from 500 Ns/m, which correspondingly decreases its tendency to absorb vibrational energy thus leading to high acceleration. Further, a tendency to wheel hop can also be observed for unsprung mass where acceleration reaches a value of -9.81 m/s^2.

VEHICLE DYNAMICS

MECH 6541

Assignment #4

MECH 6751

For: Prof Subhash Rakheja

Submitted by-

Rahul Chug

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