

Day - 7

Saturday, 17 January 2026 11:04 AM

Bit-manipulation

decimal NS $\Rightarrow \{0, 1, \dots, 9\} \Rightarrow 10 \text{ digits} \Rightarrow \text{Base } 10$

0	10	20
1	11	
2	12	
9	19	29

$$\text{eg } (531)_{10} \Rightarrow 5 \cancel{10^3} + 3 \cancel{10^2} + 1 \cancel{10^0}$$

$$7845 \Rightarrow 7 \cancel{10^3} + 8 \cancel{10^2} + 4 \cancel{10^1} + 5 \cancel{10^0}$$

binary NS $\Rightarrow \{0, 1\}$

base $\cancel{2}$

$$\begin{array}{r} 0-0 \\ 1-1 \\ \hline 10-1 \\ 11-0 \\ 10-1 \\ 11-0 \\ 11-1 \end{array}$$

$$\text{eg } (110)_2 = 1 * \cancel{2^2} + 1 * \cancel{2^1} + 0 \cancel{2^0} \\ = 4 + 2 + 0 \Rightarrow (6)_{10}$$

$$(1101)_2 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 \\ = 8 + 4 + 0 + 1 \Rightarrow (13)_{10}$$

$$\begin{array}{r} 4 \\ 2 \\ 2 \\ 2 \\ \hline 1 \\ 0 \\ \cancel{x} \\ \hline 1 \\ 1 \\ 0 \\ \hline \end{array} \Rightarrow (22)_{10}$$

16 & 4 2
10 0 1 0

$$\begin{array}{r} 16 \\ 8 \\ 4 \\ \hline 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ \hline \end{array} \Rightarrow (18)_{10}$$

Decimal to binary

$$\begin{array}{r} 2 | 15 \\ 2 | 7 \quad 1 \\ 2 | 3 \quad 1 \\ 2 | 1 \quad 1 \\ \hline \end{array}$$

$$(15)_{10} = (111)_2$$

$$\begin{array}{r} 2 | 22 \\ 2 | 11 \quad 1 \\ 2 | 5 \quad 1 \\ 2 | 2 \quad 1 \\ 2 | 1 \quad 1 \\ \hline \end{array} \Rightarrow (10)_{10}$$

$$(10110)_2$$

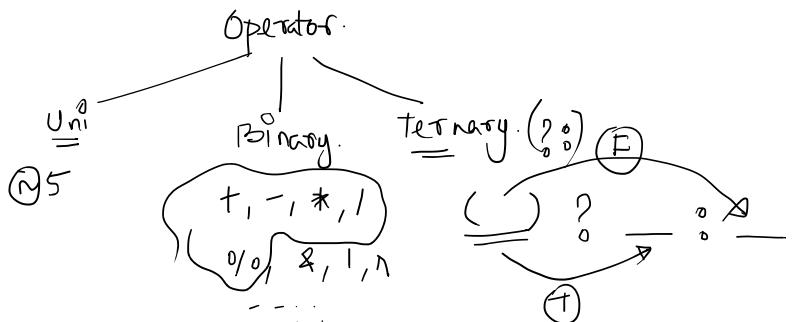
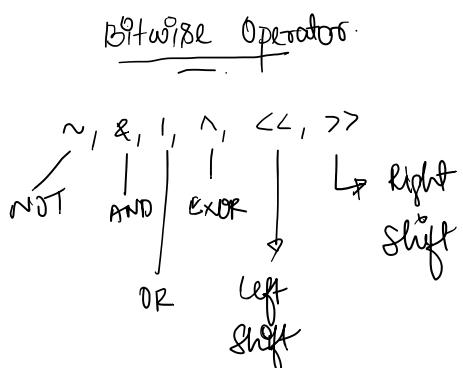
$$\begin{array}{r} \text{Add}^n \\ \hline \text{eg} \end{array}$$

$$\begin{array}{r} 1 \\ 1 \\ \hline 1 \cancel{+} 8 \end{array}$$

$$\begin{array}{r} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ \hline \end{array} \Rightarrow (5)$$

$$\begin{array}{r} 1 \\ + 234 \\ \hline 415 \end{array} \quad \times$$

$$\begin{array}{r} 0110 \\ + 1011 \\ \hline 1111 \end{array} \quad \times$$



$$\text{NOT } (\sim !) \Rightarrow 1 \rightarrow 0 \\ 0 \rightarrow 1$$

$$\text{eg } \sim(5)_{10} \Rightarrow \sim(0\underline{1}\underline{0}1)_2$$

$$(\sim 5)_0 \Rightarrow (1010)_2 \quad \checkmark$$

$$\text{AND } \Rightarrow (\&) \quad a = 5 \Rightarrow \begin{array}{r} 0101 \\ \times 0110 \\ \hline 0100 \end{array} \Rightarrow 4 \quad \times$$

$$\text{OR } (|) \quad a = 5 \Rightarrow 0101 \\ b = 6 \Rightarrow \begin{array}{r} 0110 \\ + 0111 \\ \hline 1101 \end{array} \Rightarrow 7 \quad \times$$

$$\text{EXOR } \quad a = 5 \Rightarrow 0101 \\ b = 6 \Rightarrow \begin{array}{r} 0110 \\ + 0011 \\ \hline 0101 \end{array} \Rightarrow 5 \quad \times$$

a	b	$a \& b$	$a \oplus b$	$a \text{!} b$
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

Properties:

$$(i) A \& 1 = ?$$

eg. $A = 5 (0101)$

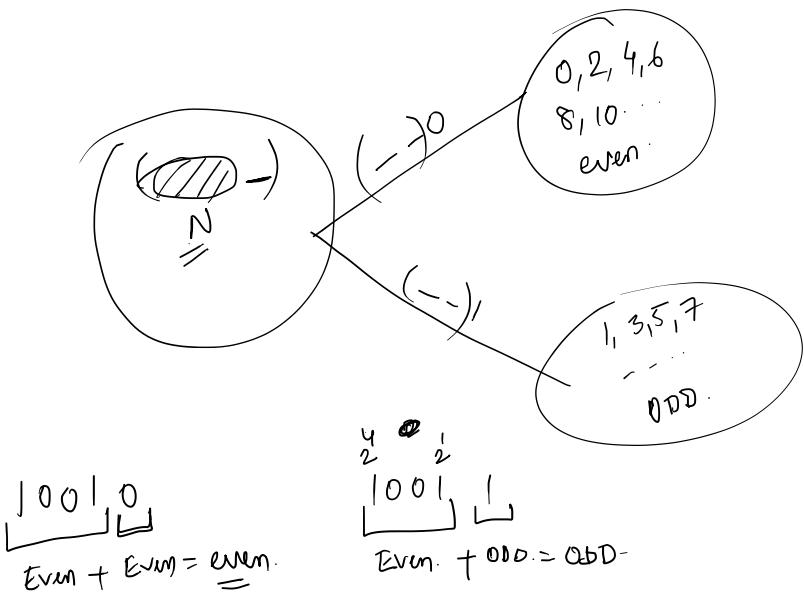
$$\begin{array}{r} & \cancel{0} \\ \& \cancel{0} \\ \hline 0 & 0 & 0 & 1 \end{array}$$

$$A = 6 (0110)$$

$$\begin{array}{r} & \cancel{0} \\ \& \cancel{0} \\ \hline 0 & 0 & 0 & 0 \end{array}$$

\Rightarrow if last bit is set bit in A

\Rightarrow ——— unset bit in A.



$$(ii) A \& 0 = 0 \quad 0101$$

$$\begin{array}{r} & \cancel{0} \\ \& \cancel{0} \\ \hline 0 & 0 & 0 & 0 \end{array}$$

$$(iii) A \& A = A \quad 0101$$

$$\begin{array}{r} & \cancel{0} \\ \& \cancel{0} \\ \hline 0 & 1 & 0 & 1 \end{array}$$

$$(iv) A \mid 0 = A \quad 0101$$

$$\begin{array}{r} 1 \quad \cancel{0} \\ \hline 0 & 1 & 0 & 1 \end{array} \Rightarrow A$$

$$(v) A \& A = A \quad 0101$$

$$\begin{array}{r} & \cancel{0} \\ \& \cancel{0} \\ \hline 0 & 1 & 0 & 1 \end{array}$$

$$(vi) A \wedge 0 = A \quad \text{✓}$$

$$\begin{array}{r} 0101 \\ \times 0000 \\ \hline 0101 \end{array}$$

$$\textcircled{vii} \quad A \wedge A = \underline{\underline{A}}$$

$$\begin{array}{r} 0101 \\ \times 0101 \\ \hline 0000 \end{array}$$

Commutative Prop

$$(a \& b) = (b \& a) \quad \checkmark$$

$$(a | b) = (b | a) \quad \checkmark$$

$$(a \wedge b) = (b \wedge a) \quad \checkmark$$

↗ Associativity

$$\boxed{a \& b \& c} = \boxed{(c * b) * a} ?$$

$$\begin{aligned} &= \boxed{(b * (c)) * a} \\ &= \boxed{b * a \& c} \\ &= \boxed{(a * b) * c} \end{aligned}$$

$$\boxed{a \wedge b \wedge c} = \boxed{(c \wedge a) \wedge b}$$

$$\begin{aligned} &\stackrel{P}{=} \boxed{(a \wedge c) \wedge b} \\ &\stackrel{P}{=} \boxed{a \wedge b \wedge c} \end{aligned}$$

eg

$$\boxed{a \wedge b \wedge c \wedge b \wedge a} = ?$$

$$\begin{aligned} &\stackrel{P}{=} \boxed{a \wedge a \wedge b \wedge b \wedge c} = ? \\ &\stackrel{P}{=} \boxed{0 \wedge 0 \wedge c} \end{aligned}$$

$$\boxed{0 \wedge c} = \textcircled{C} \quad \times$$

eg

$$1 \wedge 1 \wedge 3 \wedge 5 \wedge 2 \wedge 5 \wedge 3$$

$$0 \neq \textcircled{2}$$

$$\begin{array}{r} 1 \wedge 1 \wedge 3 \wedge 3 \wedge 5 \wedge 5 \wedge 2 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 0 \wedge 0 \wedge 0 \wedge 2 \\ \hline 0 \wedge 0 \wedge 2 \\ \hline 0 \wedge 2 \\ \hline \textcircled{2} \end{array}$$

Given an int arr of size N .

st. all ele. appear twice except for '1' ele. which appears once.

Find the unique ele.

e.g. $\text{arr}[] = \{ 6, 2, 1, 6, 2 \}$ op \Rightarrow ①

$$\overbrace{[1, 3, 5]}^{\Rightarrow q} \Rightarrow q$$

e.g. $\text{arr}[] = \{ 2, 3, 5, 6, 3, 6, 2 \}$ op \Rightarrow ⑤ ✗

$$\overbrace{[3, 1, 5]}^{\Rightarrow q} = q$$

M-1 $\text{ans} = 0$

for ($i=0; i < n; i++$)

$$TC = O(N)$$

$$\text{ans} = \text{ans} \wedge \text{arr}[i]$$

$$SC = O(1)$$

at ans;

M-2 (Brute Force)

for ($i=0; i < n; i++$) { — N }

$$\text{curEle} = \text{arr}[i]$$

count = 0;

for ($j=0; j < n; j++$) { }

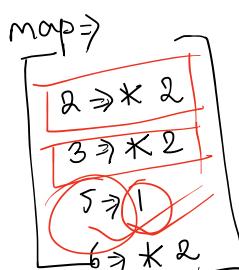
$$\begin{cases} \Rightarrow O(N) \\ TC = O(N^2) \\ SC = O(1) \end{cases}$$

if ($\text{arr}[j] == \text{curEle}$)

count++

if ($\text{count} == 1$)
 at $\text{arr}[i] //$ at curEle

M-3



$\{ 2, 3, 5, 6, 3, 6, 2 \}$

$$TC = O(N) + O(N) \Rightarrow O(N)$$

$$SC = O(N/2) \approx O(N)$$

for $i \leftarrow 0 \text{ to } N-1$

if $\text{arr}[i]$ in map

map[$\text{arr}[i]$] ++

else

map[$\text{arr}[i]$] = 1

$\underline{\underline{O(N)}}$

for k, v in map

if ($v \geq 1$)

rt key \times

$\underline{\underline{O(N)}}$

$\cancel{M=4}$ → [Searching]

$\cancel{\text{arr[sorted]}}$

$(1,1,1,2,2,\cancel{3},4,4,7,7,9,9)$ $\times \rightarrow$

$\cancel{O(N \log n)} \times$

Left shift ($<<$)

int \Rightarrow 4B \Rightarrow 32 bit

8 bit \times

$$a = 25 \quad (00011001)_2$$

$$(25 \ll 1) \Rightarrow \begin{array}{cccccccccc} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ \times & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ \hline (0 & 0 & 1 & 1 & 0 & 0 & 1 & 0)_2 \end{array}$$

32 16 2 1 0 1 1 0

101
110
~~101~~

$$a = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ & & & & 2^4 & & 2^0 \\ & & & & 0 & 1 & 2^1 \\ & & & & 0 & 0 & 1 \\ & & & & 0 & 1 & 0 \end{pmatrix}_2 \Rightarrow (9)_{16}$$

$$a \ll 1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}_2 \Rightarrow (18)_{10}$$

$$N = (a_1 \cdot 2^x + a_2 \cdot 2^y)$$

$$\begin{aligned} N \ll 1 &= a_1 \cdot 2^{x+1} + a_2 \cdot 2^{y+1} \\ &= 2 \cdot (a_1 \cdot 2^x) + 2 \cdot (a_2 \cdot 2^y) \end{aligned}$$

$$\begin{aligned} &= 2 \left[\underbrace{a_1 \cdot 2^x + a_2 \cdot 2^y}_{\textcircled{N}} \right] \\ &= 2 \textcircled{N} \times \end{aligned}$$

$$1 \ll 0 \quad 1 = (0000 \ 0001)$$

$$1 \ll 1 \Rightarrow 2 \quad (0000 \ 0010) = 2$$

$$1 \ll 2 \Rightarrow 4 \quad (0000 \ 0100) = 4$$

$$\boxed{1 \ll 3} \Rightarrow 8 \quad (0000 \ 1000) = 8 \times$$

$$\begin{aligned} N \ll 2 &\Rightarrow a_1 \cdot 2^{x+2} + a_2 \cdot 2^{y+2} \\ &= 2^2 [a_1 \cdot 2^x + a_2 \cdot 2^y] \end{aligned}$$

$$\begin{aligned} N \ll 3 &\Rightarrow a_1 \cdot 2^{x+3} + a_2 \cdot 2^{y+3} \\ &= 2^3 [a_1 \cdot 2^x + a_2 \cdot 2^y] \end{aligned}$$

$$\Rightarrow 8 \textcircled{N} \times$$

Right shift \Rightarrow

$$18 \gg 1 \quad \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \Rightarrow (18)_{10}$$

$$\quad \quad \quad \downarrow \quad \quad \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \Rightarrow (9)_{10}$$

$$27 \gg 1 \quad \begin{pmatrix} 0 & 0 & 0 & 1 & 1011 \end{pmatrix} \Rightarrow (27)_{10}$$

$$\quad \quad \quad \downarrow \quad \quad \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 1101 \end{pmatrix} \Rightarrow (13) \quad \boxed{27} = \boxed{13} \times$$

$\therefore 10 \dots \log 17$

