Group 1

Directions: This is an open book, open class notes exam. You may not use internet resources or other materials besides the class notes and textbook. You may use a calculator. You have 75 minutes to work the exam. Write your numbered solutions to the five problems on paper, scan your solutions to a pdf file, and send the scanned solutions to me via email (mrb@ecn.purdue.edu) or Purdue Filelocker. (See "Exam 1 Instructions(Group 1).pdf", sent via email, for complete details.)

---

1. **(20 pts.)** A standard die is tossed twice and the number of dots facing up in each toss are counted and noted in the order of occurance.
   (a) Find the sample space $\mathcal{S}$ for this experiment.
   (b) Find a reasonable event space for this experiment.
   (c) Find the set A corresponding to the event "the number of dots in the first toss is greater than or equal to the number of dots in the second toss."
   (d) Find the set B corresponding to the event "the number of dots in the first toss is 6."
   (e) Find $A \cap \overline{B}$ and describe it in words.
   (f) Let C be the set corresponding to the event "the number of dots on the two dice differs by 2." Find $A \cap C$.
   (g) What is the value of $P(A|B)$? Justify your answer.

---

2. **(20 pts.)** This problem involves two short problems involving the axioms of probability.
   (a) Let $(\mathcal{S}, \mathcal{F}, P)$ be a probability space and let $M \in \mathcal{F}$ be an event with nonzero probability. Show that $P(\cdot|M)$ is a valid probability measure satisfying the axioms of probability.
   (b) Let Let $(\mathcal{S}, \mathcal{F}, P)$ be a probability space having a finite sample space with $|\mathcal{S}| = n$, event space $\mathcal{F}$ equal to the power set of $\mathcal{S}$, and probability measure P given by the classical probability measure. Show that P satisfies the axioms of probability.

---

3. **(20 pts.)** Consider a probability space $(\mathcal{S}, \mathcal{F}, P)$ with events $A, B, C \in \mathcal{F}$ and probabilities $P(A) = 1/2$, $P(B) = 1/3$, $P(C) = 1/4$, $P(A \cap B) = 1/6$, $P(B \cap C) = 1/12$, $P(A \cap C) = 1/6$, and $P(A \cap B \cap C) = 1/36$.
   (a) Find the probability $P(A \cup B \cup C)$.
   (b) Find the conditional probability $P(A|C)$.
   (c) Find the conditional probability $P(A|\overline{C})$.
   (d) Find the conditional probability $P(B|\overline{A} \cup \overline{C})$.
   (e) Are events B and C statistically independent? Justify your answer.

Continued on next page.

4. **(20 pts.)** Suppose I flip a fair coin until heads occurs. The pmf of the number of flips until a heads occurs is a geometic pmf of the form

$$p(k) = \frac{1}{2}\left(\frac{1}{2}\right)^{k-1} \qquad k = 1, 2, 3, \ldots .$$

Next suppose I independently roll a fair die until a "1" occurs. The pmf of the number of rolls until a "1" occurs is a geometic pmf of the form

$$q(n) = \frac{1}{6}\left(\frac{5}{6}\right)^{n-1} \qquad n = 1, 2, 3, \ldots .$$

Now suppose I form the the joint experiment made up of these two independent experiments. What is the probability that the number of coin flips until a heads occurs is equal to the number of die rolls until a "1" occurs?

5. **(20 pts.)** A court is investigating the possible occurence of an unlikely event T. The reliability of two independent witnesses named Art and Bob is known to the court: Art tells the truth with probability α and Bob tells the truth with probability β, and there is no collusion in their answers (i.e., they answer independently.) Let A and B be the events that Art and Bob assert (respectively) that T occured, and the probability that T has occured (without considering Art's and Bob's testimony) is $P(T) = \tau$. What is the probability that T occured given that both Art and Bob declare that T occured?