Question 2.1

Tuesday, April 21, 2020 6:00 PM

Given: X & Y are i.i.d.

& $Y_K \sim p(y)$, $W_K \sim \mathcal{N}(0, \sigma^2 I)$ & $x \in \mathbb{R}^p$

$$p_\theta(x|y) = p_\theta(\{x_0 \cdots x_{K-1}\} \mid \{y_0 \cdots y_{K-1}\})$$

$$= \prod_{K=0}^{K-1} p_\theta(x_K \mid y_K)$$

$W_K \sim \mathcal{N}(0, \sigma^2 I)$

$\Rightarrow W_K = X_K - f_\theta(Y_K) \sim \mathcal{N}(0, \sigma^2 I)$

$\Rightarrow X_K \sim \mathcal{N}(f_\theta(Y_K), \sigma^2 I)$

$$= \prod_{K=0}^{K-1} \frac{1}{(2\pi\sigma^2)^{p/2}} \exp\left\{ \frac{-1}{2\sigma^2} \| x_K - f_\theta(y_K) \|^2 \right\}$$

$$\boxed{p_\theta(x|y) = \frac{1}{(2\pi\sigma^2)^{Kp/2}} \cdot \exp\left\{ \frac{-1}{2\sigma^2} \sum_{K=0}^{K-1} \| x_K - f_\theta(y_K) \|^2 \right\}}$$