ME581 Homework 2

Due: 4:15pm September 26, 2017

The following problems are to be documented, solved, and presented in a Jupyter notebook.

On-Campus students: Save the notebook as a single PDF, then print and return a hard copy in class.

Off-Campus students: Save the notebook as a single PDF, then upload and submit the PDF in Blackboard. The name of the file should be SURNAME-HW2.pdf.

Problem 1

A system of equations $A\boldsymbol{x} = \boldsymbol{b}$ is given as

$$\begin{bmatrix} 2.01 & 1.99 \\ 1.99 & 2.01 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}.$$

The solution to this system of equations is

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

and one approximate solution is

$$\widetilde{\boldsymbol{x}} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

(i) Compute the error $\boldsymbol{e} = \widetilde{\boldsymbol{x}} - \boldsymbol{x}$.

(ii) Compute the residual $\boldsymbol{r} = A\widetilde{\boldsymbol{x}} - \boldsymbol{b}$.

(iii) Using the l_∞ norm, compute the relative error $\frac{\|\boldsymbol{e}\|_\infty}{\|\boldsymbol{x}\|_\infty}$.

(iv) Using the l_∞ norm, compute the condition number κ_∞.

(v) Using the l_∞ norm, compute the relative residual $\frac{\|\boldsymbol{r}\|_\infty}{\|\boldsymbol{b}\|_\infty}$.

(vi) Compute the product of the condition number and the relative residual.

(vii) Compare the relative error to the product of the condition number and the relative residual.

Problem 2

Let

$$A = \begin{bmatrix} 5.1 & 8.7 \\ 2.4 & 4.1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 9.48 \\ 4.48 \end{bmatrix}.$$

(i) Using the l_∞ norm, compute the condition number $\kappa_\infty(A)$.

(ii) Solve the system of equations $A\boldsymbol{x} = \boldsymbol{b}$ for $\boldsymbol{x}$.

(iii) Perturb the coefficient matrix A and the right-side vector $\boldsymbol{b}$ by

$$\delta A = \begin{bmatrix} -0.001 & 0 \\ 0.001 & 0 \end{bmatrix} \text{ and } \delta \boldsymbol{b} = \begin{bmatrix} 0.05 \\ -0.05 \end{bmatrix}$$

and solve the resulting perturbed system of equations $(A + \delta A)\widetilde{\boldsymbol{x}} = (\boldsymbol{b} + \delta \boldsymbol{b})$ for the approximate solution $\widetilde{\boldsymbol{x}}$.

(iv) Using the l_∞ norm, compute the actual value of the relative change in the solution, $\frac{\|\delta \boldsymbol{x}\|_\infty}{\|\boldsymbol{x}\|_\infty}$ for the perturbation in part (iii).

(v) Using the l_∞ norm, compute the theoretical upper bound of the relative change in the solution, $\frac{\|\delta \boldsymbol{x}\|_\infty}{\|\boldsymbol{x}\|_\infty}$ for the perturbation in part (iii).

(vi) For the perturbation in part (iii), compare the actual value of $\frac{\|\delta \boldsymbol{x}\|_\infty}{\|\boldsymbol{x}\|_\infty}$ to its theoretical upper bound.

(vii) Perturb the original coefficient matrix A and the original right-side vector $\boldsymbol{b}$ by

$$\delta A = \begin{bmatrix} 0.001 & -0.001 \\ -0.001 & 0.001 \end{bmatrix} \text{ and } \delta \boldsymbol{b} = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix}$$

and solve the resulting perturbed system of equations $(A + \delta A)\widetilde{\boldsymbol{x}} = (\boldsymbol{b} + \delta \boldsymbol{b})$ for $\widetilde{\boldsymbol{x}}$.

(viii) Using the l_∞ norm, compute the actual value of the relative change in the solution, $\frac{\|\delta \boldsymbol{x}\|_\infty}{\|\boldsymbol{x}\|_\infty}$ for the perturbation in part (vii).

(ix) Using the l_∞ norm, compute the theoretical upper bound of the relative change in the solution, $\frac{\|\delta \boldsymbol{x}\|_\infty}{\|\boldsymbol{x}\|_\infty}$ for the perturbation in part (vii).

(x) For the perturbation in part (vii), Compare the actual value of $\frac{\|\delta \boldsymbol{x}\|_\infty}{\|\boldsymbol{x}\|_\infty}$ to its theoretical upper bound.

Problem 3

Solve the augmented matrix

$$\left[\begin{array}{cccc|c} 3 & 1 & 4 & -1 & 7 \\ 2 & -2 & -1 & 2 & 1 \\ 5 & 7 & 14 & -9 & 21 \\ 1 & 3 & 2 & 4 & -4 \end{array}\right]$$

By means of

(i) Gaussian Elimination with Partial Pivoting.

(ii) Gaussian Elimination with Scaled Partial Pivoting.

Problem 4

(i) Solve the augmented matrix by means of Gaussian Elimination with Partial Pivoting in double precision:

$$\left[\begin{array}{ccccc|c} -9 & 11 & -21 & 63 & -252 & -356 \\ 70 & -69 & 141 & -421 & 1684 & 2385 \\ -575 & 575 & -1149 & 3451 & -13801 & -19551 \\ 3891 & -3891 & 7782 & -23345 & 93365 & 132274 \\ 1024 & -1024 & 2048 & -6144 & 24572 & 34812 \end{array}\right]$$

(ii) Using the l_∞ norm, estimate the condition number of the coefficient matrix based on your result. The exact solution for this problem is given by $\boldsymbol{x} = [1 \quad -1 \quad 1 \quad -1 \quad 1]^T$.

Problem 5

Determine the member and reaction forces within the plane truss shown in Figure 1 when the truss is subjected to each of the following loading configurations.

(a) 500-pound forces directed vertically downward at nodes#3 and #5, and a 1000-pound force directed vertically downward at node#4.

(b) A 500-pound force directed vertically downward at nodes#3, a 1000-pound force directed vertically downward at node#4, a 1500-pound force directed vertically downward at node#5.

(c) A 1500-pound force directed vertically downward at nodes#3, a 1000-pound force directed vertically downward at node#4, a 500-pound force directed vertically downward at node#5.

(d) 500-pound force acting at node#4, and a 1000-pound force acting at node #3, both forces acting horizontally to the right.

(e) 500-pound force acting at node#4, and a 1000-pound force acting at node #5, both forces acting horizontally to the left.

Solve the problem using your GE code with partial pivoting. Show the augmented matrix and the resulting forces for each case.

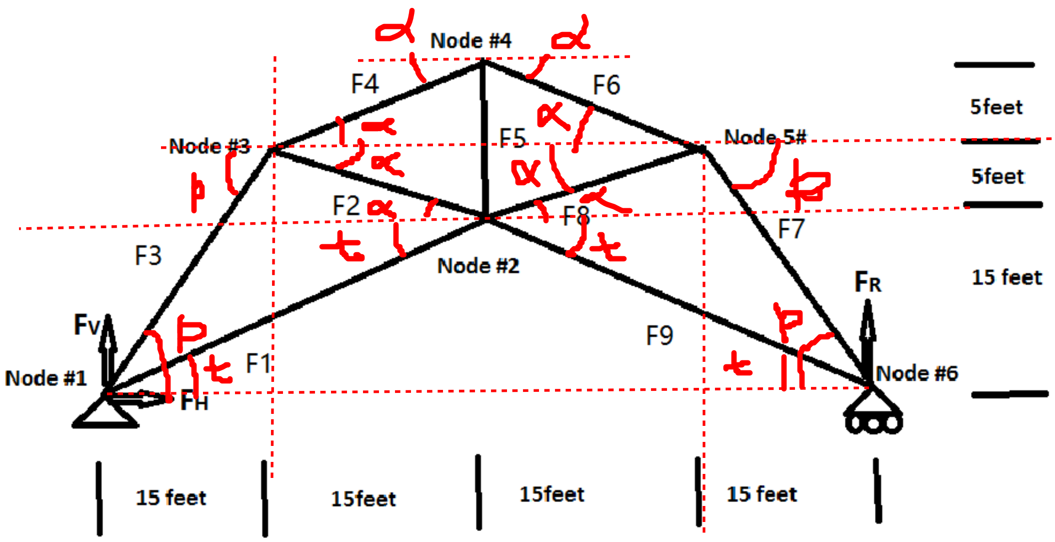

Figure 1

$$\alpha = \tan^{-1}(5/15)$$

$$\beta = \tan^{-1}(20/15)$$

$$t = \tan^{-1}(15/30)$$