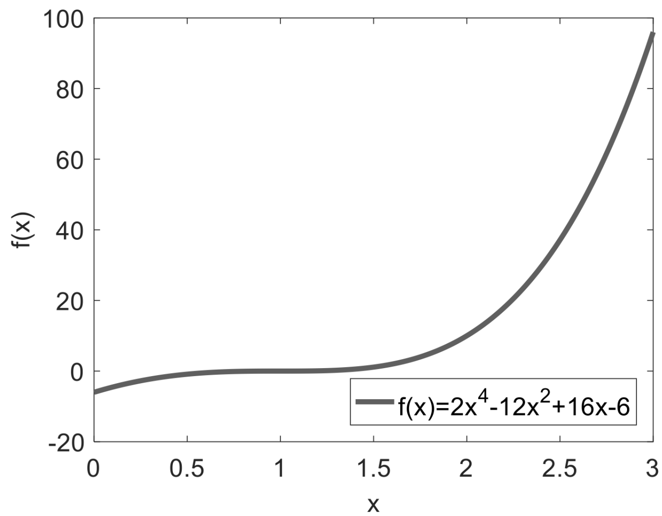
ME 581
Midterm Exam
Tuesday October 17 2017 3:00PM EST to
Wednesday October 18 2017 3:00PM EST

Problem 1 (60 pts)	
Problem 2 (40 pts)	
Total (100 pts)	

1) Time limit: 24 hours. Start Time: 3 :00 P.M.EST on October 17, 2017 End Time: 3.00 P.M.EST on October 18, 2017.
2) It will be solved using Jupyter notebooks and submitted as a single pdf file in Blackboard.
3) Notes, books and homework are allowed during the exam.
4) Laptops and calculators are allowed.
5) You can use all the functions that you wrote for previous homework assignments in your exam, so make sure that they are working properly.
6) You do not need a proctor.
7) The midterm exam should be done on your own and should not be discussed with any other person.

Problem 1 (60 points)

The function $f(x) = 2x^4 - 12x^2 + 16x - 6$ is plotted below. The function has a root at $x = 1$.



(a) Find an approximation of the root by performing 10 iterations of Newton's method, beginning with an initial guess of $x_0 = 0$. Print and plot the value of x_i versus the iteration number i.

(b) Plot the absolute error $e_i = |x_{i+1} - x_i|$ versus the iteration number i.

(c) Determine the order of convergence and explain if you expected this order.

(d) Perform 10 iterations using the bisection method starting in an interval [0,3]. Compare the order of convergence with the one in Newton's method.

(e) A further root exists at $x = -3$. Perform 10 iterations using Newton's method, beginning with an initial guess of $x_0 = -4$. Print and plot the value of x_i versus the iteration number i.

(f) What order of convergence do you find for this root? Why?

Problem 2 (40 points)

The matrix R summarizes the correlations among speeds for the 100m, 200m, 400m, 800m, 1500m, 5000m, 10000m, and marathon events extracted from Olympic track records.

$$R = \begin{pmatrix} 1.00 & 0.91 & 0.82 & 0.70 & 0.69 & 0.60 & 0.70 & 0.50 \\ 0.91 & 1.00 & 0.85 & 0.80 & 0.77 & 0.60 & 0.69 & 0.60 \\ 0.82 & 0.85 & 1.00 & 0.90 & 0.83 & 0.77 & 0.78 & 0.67 \\ 0.70 & 0.80 & 0.90 & 1.00 & 0.97 & 0.85 & 0.87 & 0.79 \\ 0.69 & 0.77 & 0.83 & 0.97 & 1.00 & 0.92 & 0.95 & 0.80 \\ 0.60 & 0.60 & 0.77 & 0.85 & 0.92 & 1.00 & 0.97 & 0.92 \\ 0.70 & 0.69 & 0.78 & 0.87 & 0.95 & 0.97 & 1.00 & 0.94 \\ 0.50 & 0.60 & 0.67 & 0.79 & 0.80 & 0.92 & 0.94 & 1.00 \end{pmatrix}$$

The events in the matrix are provided in the order 100m, 200m, 400m, 800m, 1500m, 5000m, 10000m, and marathon. The eigenvector associated with the largest eigenvalue of R is called the principal component. Further, the percentage of variation accounted for by the principal component is given by the ratio of the corresponding eigenvalue and the number of variables (8 in this case).

(a) Perform 10 iterations using the power method to calculate the largest eigenvalue of R and the principal component, beginning with an initial guess of $x_0 = [1,1,1,1,1,1,1,1]^T$. Normalize the eigenvector using the L_2 norm.

(b) Calculate the percentage of variation accounted for by the principal component.