Homework 7
ME 581
Due: 4:15 PM December 7, 2017

The following problems are to be documented, solved, and presented in a Jupyter notebook.
On-Campus students: Save the notebook as a single PDF, then print and return a hard copy in class.
Off-Campus students: Save the notebook as a single PDF, then upload and submit the PDF in Blackboard. The name of the file should be SURNAME-HW7.pdf.

For problems 1 to 3

Approximate the solution of the following boundary value problems using the finite central difference method. Plot the approximate solution.

Problem 1

$$u'' = -(x+1)u' + 2u + (1-x^2)e^{-x}\,, \qquad u(0) = -1\,, u(1) = 0$$

Problem 2

$$u'' + 3u' = x^2 + \sin x\ ,\ \ u(-5) = 10,\ \ u(13.2) = 23$$

Problem 3

$$\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{du}{d\rho}\right) = -1,\ \ u(1) = 0, \qquad u(2) = -\frac{1}{2}$$

Problem 4

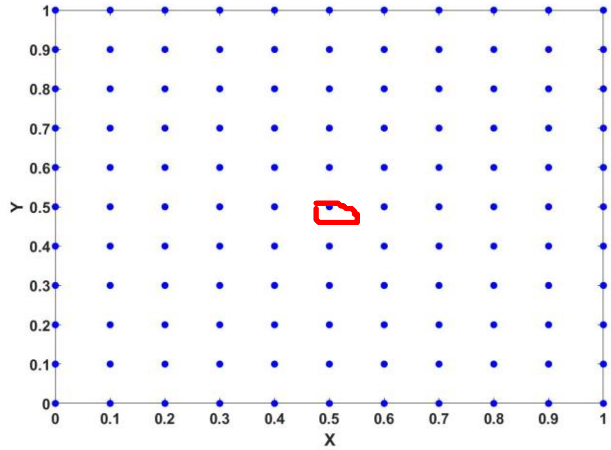
Consider solving finite difference solution of the given Poisson equation.

$$u_{xx} + u_{yy} = x + y$$

on the unit square using the mesh shown below ($\Delta x = 0.1$ and $\Delta y = 0.1$), subject to the boundary conditions,

$$u(0, y) = 0$$
$$u(1, y) = 1$$
$$u(x, 0) = 0$$
$$u(x, 1) = 1$$

Use a second-order accurate, centered finite-difference scheme to compute the approximate solution. Show the contours of final solution u on x-y plot.



Problem 5

An aquifer is located between two rivers, and fluctuations in the water table are monitored at two wells located 1100 meters apart. During a flood, the rise in the water table as measured at both wells was found to be

$$r(t) = \begin{cases} \frac{5}{3}t\,, & t \le 3 \\ 5e^{-(t-3)/5}, & t > 3 \end{cases}$$

where r is measured in meters, and t is measured in days. The change in the water table, $h(x,t)$, as a result of the flood is modeled by the following partial differential equation

$$\frac{\partial h}{\partial t} = \propto \frac{\partial^2 h}{\partial x^2}, \quad h(x,0) = 0, h(0,t) = h(1100,t) = r(t)$$

The hydraulic diffusivity of the soil has been experimentally determined to be

$$\propto = 0.0059\frac{m^2}{s} = 509.76\frac{m^2}{day}$$

(a) Determine $h(x,t)$ at the peak of the flood , t=3.
(b) Plot $h(x,t)$ for t=10 , t= 15 , and t= 20.

.