ECE 580 Spring 2020

FunWork #2

Due on February 19

1. We say that $g(k)$ is of the order-of-magnitude of $f(k)$ and write $g(k) = O(f(k))$ if for sufficiently large k and some positive constant K, we have

$$g(k) \leq K f(k).$$

What is the order-of-magnitude of

$$3k^3 + 2k^2 + 5?$$

2. The "big-O" notation can also be used when analyzing the accuracy of approximation formulas such as the Taylor series. In this case we write $g(h) = O(f(h))$ to mean

$$g(h) \leq K f(h) \quad \text{as} \quad h \to 0$$

(Note that the above is equivalent to the previous definition if we take $h = 1/k$.) We know that for $|h| < 1$,

$$\frac{1}{1-h} = 1 + h + h^2 + h^3 + \cdots$$

From this we can conclude that

$$\frac{1}{1-h} = 1 + h + O\left(h^2\right)$$

Find a constant K for the above order-of-magnitude estimation.

Hint: Consider the case when $|h| < 1/2$.

3. Many iterative optimization methods use a variable step size. The step size is determined by using a line search which involves locating the minimizer of a function of many variables in a specified direction $\boldsymbol{d}$. This involves the location of an interval in which the minimizer lies and then the interval is reduced. Once the uncertainty interval is determined, one can use the Golden Section search or the Fibonacci method to reduce the uncertainty interval.

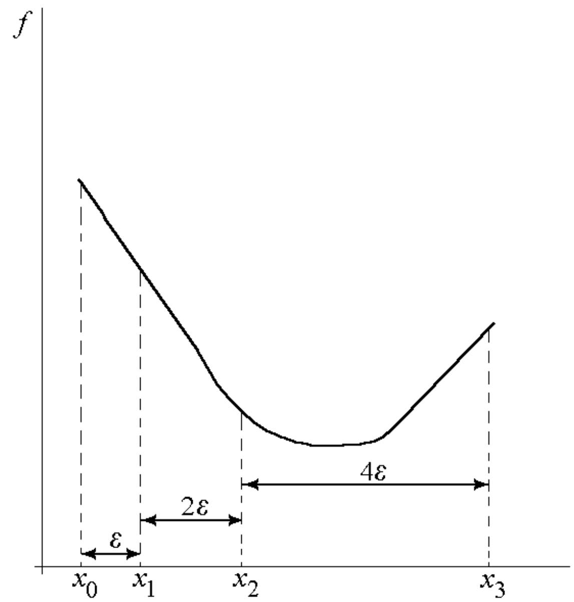

Figure 1: An illustration of the process of bracketing a minimizer.

We now describe a method that can be used to determine an interval containing the minimizer. We begin by evaluating the given function, say f, at an initial point $\boldsymbol{x}^{(0)}$. The next step is to evaluate the function at a second point which is a distance ε from $\boldsymbol{x}^{(0)}$, where ε is the chosen parameter, that is, we evaluate f at $\boldsymbol{x}^{(0)} + \varepsilon \boldsymbol{d}$. We then continue to evaluate f at new points, successively doubling the distance between the points. The process stops when the function increases between two consecutive evaluations. In the one-dimensional example in Figure 1, the function f increases between x_2 and x_3 and therefore the minimizer is bracketed in the interval $[x_1, x_3]$.

Consider the function

$$f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^\top \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \boldsymbol{x}$$

with the initial guess $\boldsymbol{x}^{(0)} = \begin{bmatrix} 0.8 & -0.25 \end{bmatrix}^\top$. Assume that the direction of travel is the negative gradient of f. Take $\varepsilon = 0.1$. Bracket the minimizer, that is, use the above described method to find an initial uncertainty region.

4. Apply the Golden Section search to the previous problem to reduce the uncertainty region width to 0.05. Organize the results of your computations in table format similar to that of Exercise **7.2** on page 126.

5. Repeat Problem **4** using the Fibonacci search method.

6. Repeat Problem **4** using the Newton method.

7. For the function

$$f(x_1, x_2) = (x_2 - x_1)^4 + 12x_1x_2 - x_1 + x_2 - 3,$$

(a) use MATLAB's commands `meshgrid` and `mesh` to generate its 3D plot. The range of x_1 and x_2 is the same and it should be equal to $[-1.2, 1.2]$. Set the `box` on.

(b) use the command `contour` to generate 10 contours. Use the same range for x_1 and x_2 as in (a).

8. Minimize the above function using the method of the gradient descent when $\alpha = 0.05$ and locate these points on the level sets of f. Connect the successive points with lines or lines with arrows to show clearly the progression of the optimization process. Use two staring points,

$$\boldsymbol{x}^{(0)} = \begin{bmatrix} 0.55 \\ 0.7 \end{bmatrix} \quad \text{and} \quad \boldsymbol{x}^{(0)} = \begin{bmatrix} -0.9 \\ -0.5 \end{bmatrix}.$$

Obtain the sequence of points using the steepest descent method and locate these points on the level sets of f.

9. Minimize the above function using Newton's method. Locate the points on the level sets of f.