ECE661 HW1

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```
In [1]: import numpy as np
```

1) What are all the points in the representational space $\Re^3$ that are the homogenous coordinatesof the origin in the physical space $\Re^2$?

Origin in physical space $\Re^2$ is $(0,0)'$, it's homogeneous coordinates will be $(0,0,1)'$. Also, we know that any point $\vec{x}$ in representational space is equivalent to $k * \vec{x}\ \forall k \neq 0$. Therefore all the points $(0,0,k)'\ \forall k \neq 0$ will represent the origin in the representational space.

2) Are all the points at infinity in the physical plane $\Re^2$ the same? Justify your answer.

Any point at infinity in the physical space can be represented as an Ideal point $(a,b,0)$ in the representational space. While the point is located at infinity, it certainly has a direction associated to it, $a\vec{i} + b\vec{j}$,and thus, all points at infinity are **not** the same as they can have different directions.

3) Argue that the matrix rank of a degenerate conic can never exceed 2.

Any degenerate Conic C can be written as $C = \vec{l}\vec{m}^T + \vec{m}\vec{l}^T$.

Now $\vec{l}$ can be written as $\begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$ and $\vec{m} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$

$$\vec{l}\vec{m}^T = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} * \begin{bmatrix} m_1 & m_2 & m_3 \end{bmatrix} = \begin{bmatrix} l_1m_1 & l_1m_2 & l_1m_3 \\ l_2m_1 & l_2m_2 & l_2m_3 \\ l_3m_1 & l_3m_2 & l_3m_3 \end{bmatrix}$$

We can note that the second column of $\vec{l}\vec{m}^T$ can be written as $\frac{m_2}{m_1} *$ first column.
Similarly, The third column of $\vec{l}\vec{m}^T$ can be written as $\frac{m_3}{m_1} *$first column.
Therefore the column space of $\vec{l}\vec{m}^T$ is of dim = 1 $\Rightarrow \vec{l}\vec{m}^T$ is of Rank 1
Similarly, $\vec{m}\vec{l}^T$ is also Rank=1
Now using triangle inequality, $Rank(A + B) \leq Rank(A) + Rank(B)$
We get: $Rank(C) = Rank(\vec{l}\vec{m}^T + \vec{m}.\vec{l}^T) \leq Rank(\vec{l}\vec{m}^T) + Rank(\vec{m}.\vec{l}^T) = 1 + 1 = 2$
$\Rightarrow Rank(C) \leq 2$

4) Derive in just 3 steps the intersection of two lines l_1 and l_2 with l_1 passing through the points (0,0)and (2,6), and with l_2 passing through the points (-6,8) and (-3,2). How many steps would take you if the second line passed through (-10,-3) and (10,3)?

In [2]:
```
x1=[0,0,1];x2=[2,6,1]
l1=np.cross(x1,x2) # step 1
x3 = [-6,8,1];x4=[-3,2,1]
l2 =np.cross(x3,x4) #step 2
x5 = np.cross(l1,l2) #step 3
print('point of intersection of l1 and l2 is '+str(x5))
x5 = x5/x5[-1]; x5 = x5[:-1]
print('point of intersection of l1 and l2 in 2D space is '+str(x5))
```

```
point of intersection of l1 and l2 is [ 24  72 -30]
point of intersection of l1 and l2 in 2D space is [-0.8  -2.4]
```

now second line is changed

In [3]:
```
x3 = [-10,-3,1];x4=[10,3,1]
l2 =np.cross(x3,x4) #step 1
x5 = np.cross(l1,l2) #step 2
print('new point of intersection of l1 and l2 is '+str(x5))
```

```
new point of intersection of l1 and l2 is [   0    0 -108]
```

we were able to find the new point of intersection in 2 steps. However analytically we can note that the points (-10,-3) and (10,3) are symmetric about the origin, and line joining these two points will pass through (0,0) which is one of the point on line l_1. Therefore, analytically we can deduce the new point of intersection in just one step (for this case only).

5) Consider that there are two lines. The first line is passing though points (0,0) and (2,-2). The second line is passing through poiints (-3,0) and (0,-3). Find the intersection between these two lines. Comment on your answer.

In [4]:
```
x1 = [0,0,1]; x2=[2,-2,1]
l1 = np.cross(x1,x2)
x3=[-3,0,1];x4=[0,-3,1]
l2 = np.cross(x3,x4)
x5 = np.cross(l1,l2) #intersection
print('point of intersection of l1 and l2 is '+str(x5))
```

```
point of intersection of l1 and l2 is [ 18 -18   0]
```

The above point is an Ideal Point which means the lines l_1 and l_2 were parallel lines, let us check that:

In [5]:
```
print('l1 is' + str(l1))
print('l2 is '+ str(l2))
print('slope of l1 is '+ str(l1[1]/l1[0])+ ' and slope of l2 is '\
      +str(l2[1]/l2[0])) #slope is b/a
```

```
l1 is[2 2 0]
l2 is [3 3 9]
slope of l1 is 1.0 and slope of l2 is 1.0
```

As the computed slope of the two lines l_1 and l_2 is identical, therefore they are indeed parallel

6) As you know, when a point x is on a conic, the tangent to the conic at that point is given by $l = Cx$. That raises the question of what the Cx corresponds to when x is, say, outside of a conic. As you'll see later in class, when x is outside the conic, Cx is the line that joins the two points of contact if you draw tangents to C from the point x. This line is referred to as the polar line. Now consider for our conic a circle of radius 1 that is centered at the coordinates (5,5) and let x be the origin of $\Re^2$ physical plane. Where does the polar line intersect the x and y axes in this case?

```
In [11]: x = np.array([0,0,1]);
         #equation of circle is (x-5)**2 +(y-5)**2=1
         a=1.0; c=1.0; d=-10.0;e=-10.0; f=49.0; b=0.0;
         C=np.array([[a,b/2,d/2],[b/2,c,e/2],[d/2,e/2,f]])
         L=np.dot(C,x)
         print('The polar line is '+str(L))
```

```
The polar line is [-5. -5. 49.]
```

```
In [12]: xpt=np.cross(L,np.array([0,1,0]))
         xpt = xpt/xpt[-1]
         print('intersection of polar line with x axes is at' +str(xpt))
         ypt=np.cross(L,np.array([1,0,0]))
         ypt = ypt/ypt[-1]
         print('intersection of polar line with y axes is at' +str(ypt))
```

```
intersection of polar line with x axes is at[ 9.8 -0.   1. ]
intersection of polar line with y axes is at[-0.   9.8  1. ]
```

```
In [13]: print('In the physical space the points will be')
         print('x axes intersection at '+ str(xpt[:-1]) \
               +'and y axes intersection at '+str(ypt[:-1]))
```

```
In the physical space the points will be
x axes intersection at [ 9.8 -0. ]and y axes intersection at [-0.   9.8]
```

7) Find the intersection of two lines whose equations are given by x=1 and y=1

```
In [9]: l1=[1,0,-1];l2=[0,1,-1]
        x=np.cross(l1,l2)
        print('The point of intersection of the two lines is at '+str(x))
```

```
The point of intersection of the two lines is at [1 1 1]
```

```
In [10]: x2d = x/x[-1]; x2d = x2d[:-1]
         print('The intersection point in the physical space is '+str(x2d))
```

```
The intersection point in the physical space is [1. 1.]
```