ME581 Homework 1
Due: 4:15pm September 12, 2017

The following problems are to be documented, solved, and presented in a Jupyter notebook.

On-Campus students: Save the notebook as a single PDF, then print and return a hard copy in class.
Off-Campus students: Save the notebook as a single PDF, then upload and submit the PDF in Blackboard. The name of the file should be SURNAME-HW1.pdf.

Problem 1
Verify that the function $f(x) = x^3 + x^2 - 3x - 3$ has a zero on the interval $(1,2)$ by plotting the function. Next, perform the first five iterations of the bisection method and verify that each approximation satisfies the theoretical error bound of the bisection method, but that the actual errors do not steadily decrease. The exact location of the zero is $p = \sqrt{3}$.

Problem 2
Verify that the function $f(x) = x^6 - 3$ has a zero on the interval $(1,2)$. Next, perform the first five iterations of the bisection method and verify that each approximation satisfies the theoretical error bound of the bisection method.

Problem 3
Approximate $1/37$ to five significant decimal places by applying the bisection method to the equation $1/x - 37 = 0$. Include plots of the following:
a) Approximated Root vs Iteration Number
b) Absolute Error vs. Iteration Number.

Problem 4
The equation $x^7 = 3$ has a root on the interval $(1,2)$, namely $x = \sqrt[7]{3}$.
a) Perform five iterations of Newton's method.
b) For $n \geq 1$, compare $|p_n - p_{n-1}|$ with $|p_{n-1} - p|$ and $|p_n - p|$.
c) For $n \geq 1$, compute the ratio $|p_n - p|/|p_{n-1} - p|^2$ and show that this value approaches $|f''(p)/2f'(p)|$.

Problem 5
For each of the functions given below, use the Newton's method algorithm to approximate **all** real roots. Use an absolute tolerance of 10^{-6} as a stopping condition. For each of the roots plot the logarithm of the error e_n at each iteration n against the logarithm of the error at the previous iteration, and use this plot to calculate the order of convergence. Use these plots to comment on the convergence behavior of Newton's method.
a) $f(x) = e^x + x^2 - x - 4$
b) $f(x) = x^3 - x^2 - 10x + 7$
c) $f(x) = 1.05 - 1.04x + \ln(x)$

Problem 6
Consider the function $f(x) = (3x)(1 - \cos(\pi x))$, which has a root of multiplicity three at $x = 0$. Apply ten iterations of Newton's method to this equation with a starting value of $p_0 = 0.5$. Comment on the order of convergence.

Problem 7
The function $f(x) = \sin(x)$ has a zero on the interval $(3,4)$, namely $x = \pi$. Perform three iterations of Newton's method to approximate this zero, using $p_0 = 4$. Determine the absolute error in each of the computed approximations. What is the apparent order of convergence? What explanation can you provide for this behavior?

Problem 8
(a) Verify that the equation $x^4 - 18x^2 + 45 = 0$ has a root on the interval $(1,2)$. Next, perform three iterations of Newton's method, with $p_0 = 1$. Given that the exact value of the root is $x = \sqrt{3}$, compute the absolute error in the approximations just obtained. What is the apparent order of convergence? What explanation can you provide for this behavior?
(b) Verify that the equation $x^4 - 18x^2 + 45 = 0$ also has a root on the interval $(3,4)$. Perform five iterations of Newton's

method, and compute the absolute error in each approximation. The exact value of the root is $x = \sqrt{15}$. What is the apparent order of convergence in this case?

(c) What explanation can you provide for the different convergence behavior between parts (a) and (b)?

Problem 9

The function $f(x) = \sin(x)$ has a zero on the interval $(3,4)$, namely $x = \pi$. Perform five iterations of the secant method to approximate this zero, using $p_0 = 3$ and p_1=4. Determine the absolute error in each of the computer approximations. What is the apparent order of convergence? What explanation can you provide for this behavior?