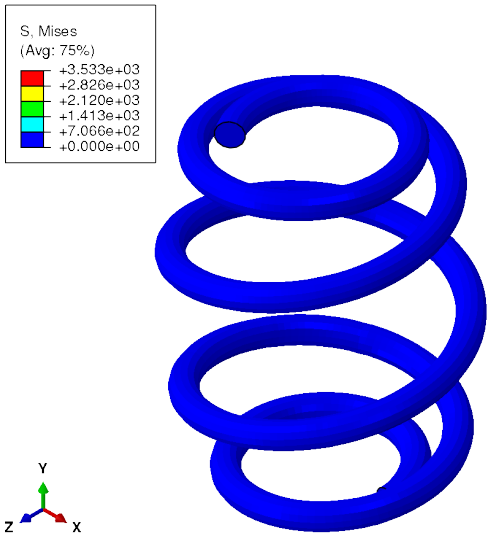
Influence of Geometry and Placement Configuration on Side Forces in Compression Springs



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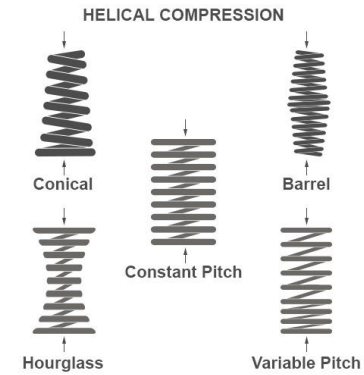
August 25, 2019

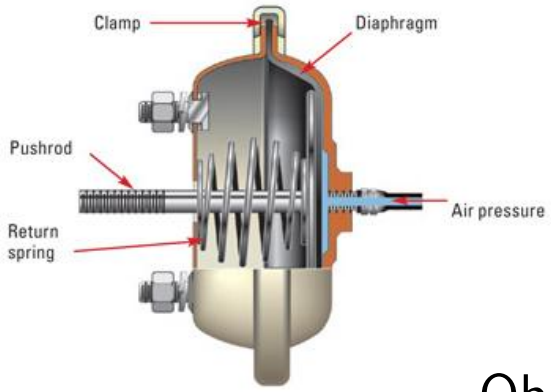
Outline

Introduction: Helical Springs & Application

Types



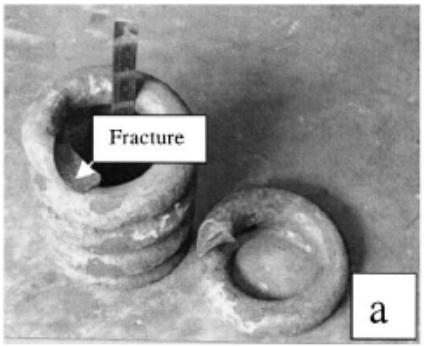
Application



Loads

- Axial Forces: Designed loading
- Side Forces:
  - Unwanted: Wear & tear
  - Fatigue failure

Failure

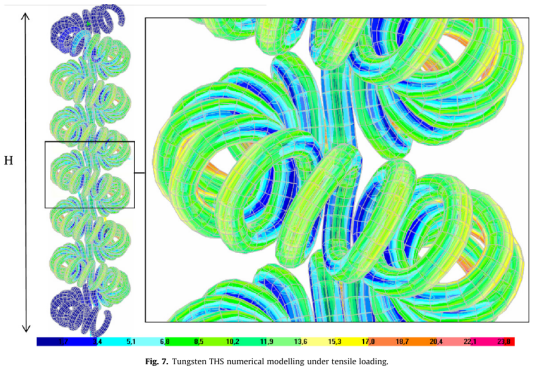


Objective

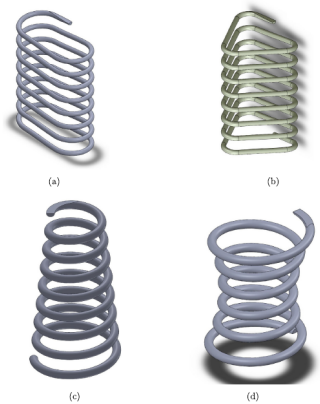
- Model side forces in barrel springs
- Identify cause of side force

Introduction: Existing Literature

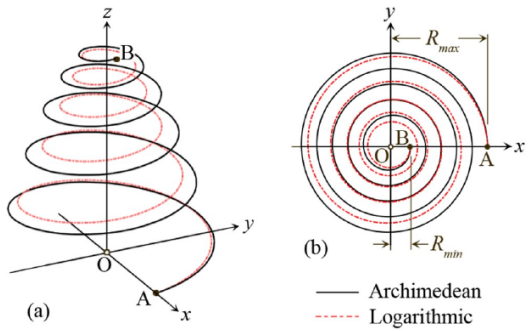
- Analytical studies: using castigliano's theorem, beam theory & verification: FEM, experiments



Kaoua et al 2011

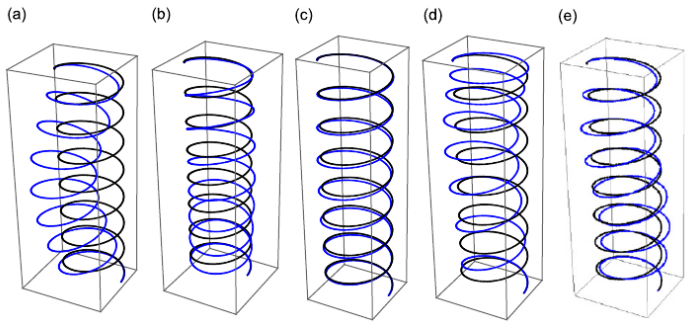


Chaudhury et al 2017

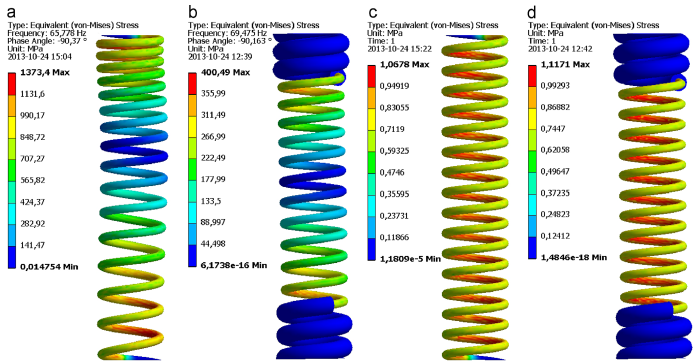


Aribas et al 2019

- Dynamic analysis: free vibration/ buckling - beam theory, wave equation, customized finite elements



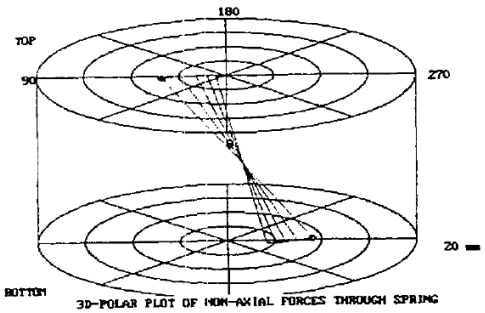
Yu et al 2011



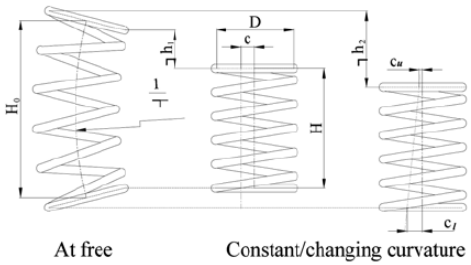
Michalczyk et al 2015

Introduction: Existing Literature

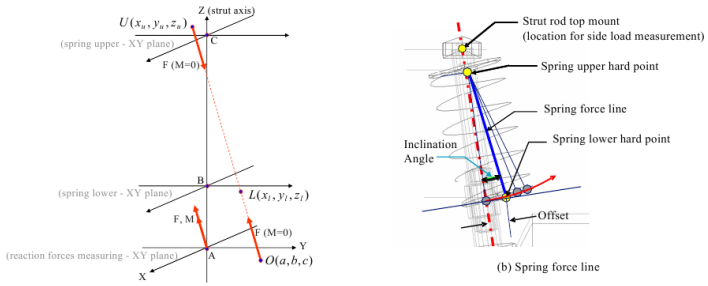
- Side-Force studies: Using force-centerline, FEM and reducing by tilting or changing shape of centerline



Hastey et al 1997

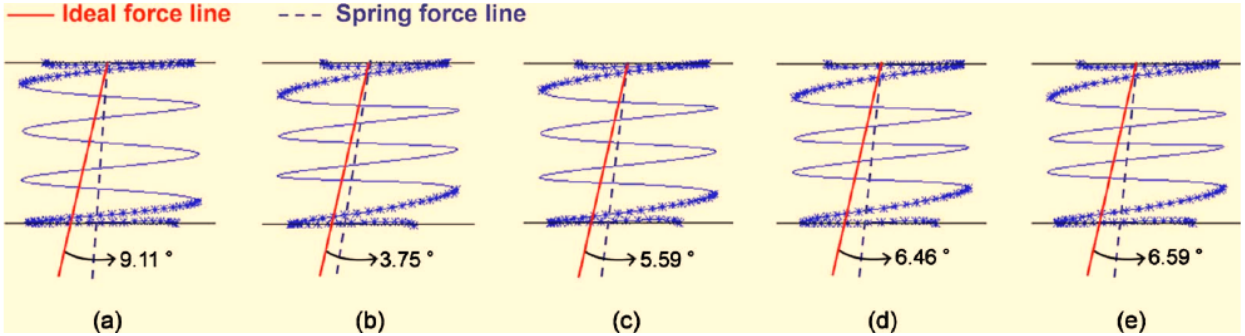


Liu et al 2008



Ryu et al 2010

- Optimization: Analytical, FEM models, meta-models, probabilistic formulation - SQP, GRG, GA

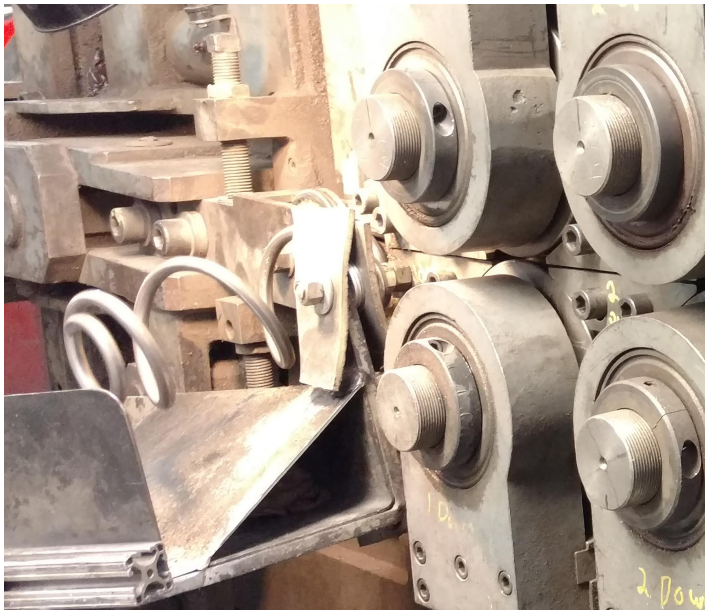


Choi et al 2010

Manufacturing Process

Steps:

1. Coiling: From circular wire
2. Heat Treatment: Two stage
   1. Heating and oil Quenching: Becomes hard & brittle
   2. Tempering: Increases ductility
3. Pre-setting: Induces residual stresses
4. Powder Coating: Increases hardness



Coiling

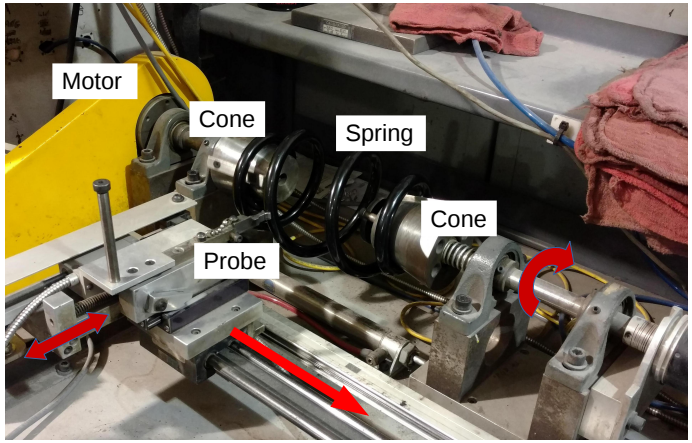
Before & after pre-setting

Final product

Spring Geometry and Measurements

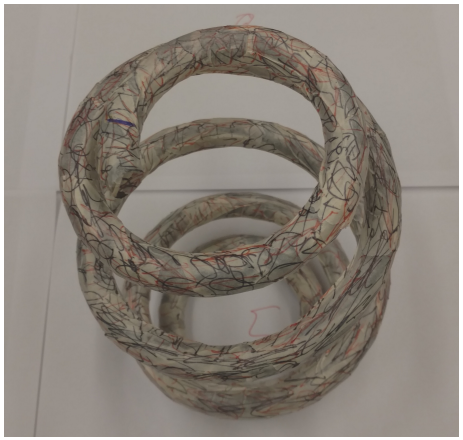
Measurement: using Profiler

- Measurement using moving probe
- Spring is rotated about central axis of cones
- Probe travels on outer surface of spring

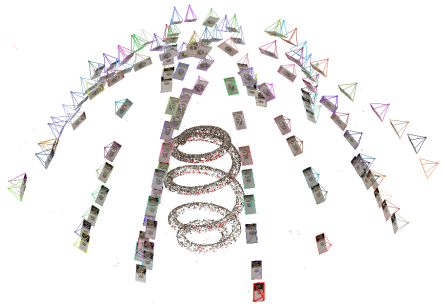


Measurement: using Optical Sensors

- Idea: generate point cloud and extract center-line
- Using: SFM or 3D-Scanners
- Difficulties:
  - Reflective, Feature-deviod surfaces
  - Structure occlusion
  - Incomplete point-cloud



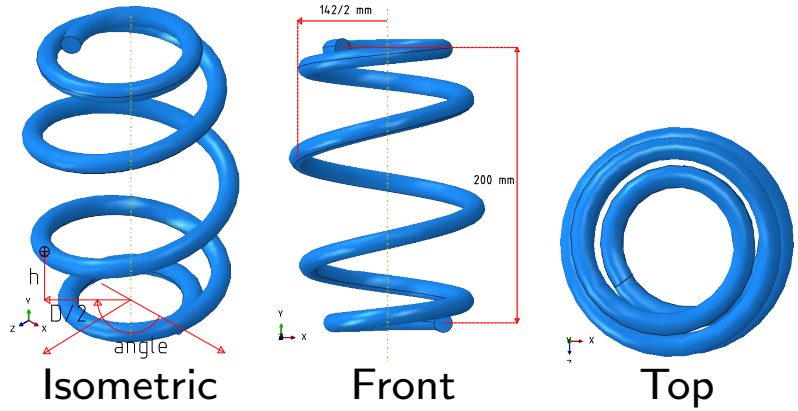
Masked spring



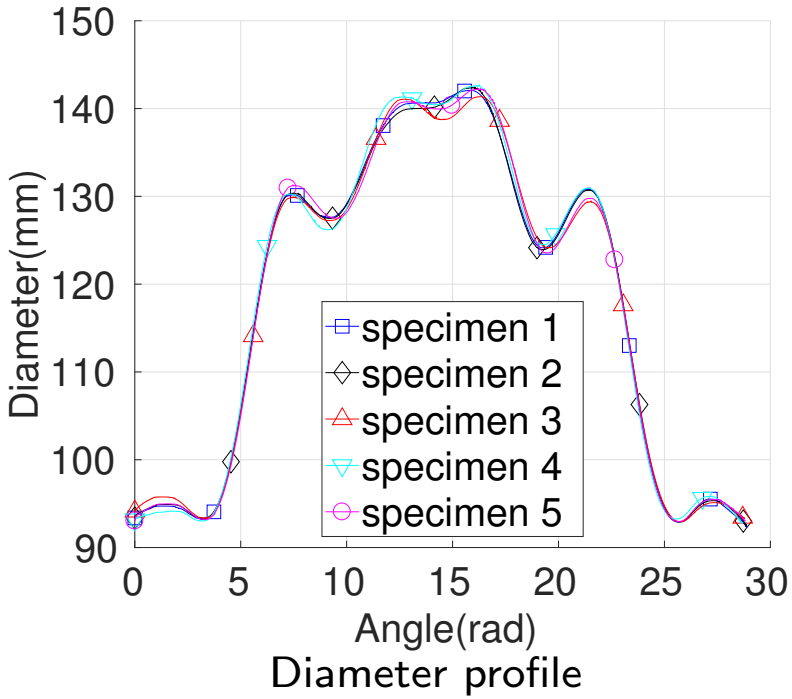
3D reconstruction

Spring Geometry and Measurements

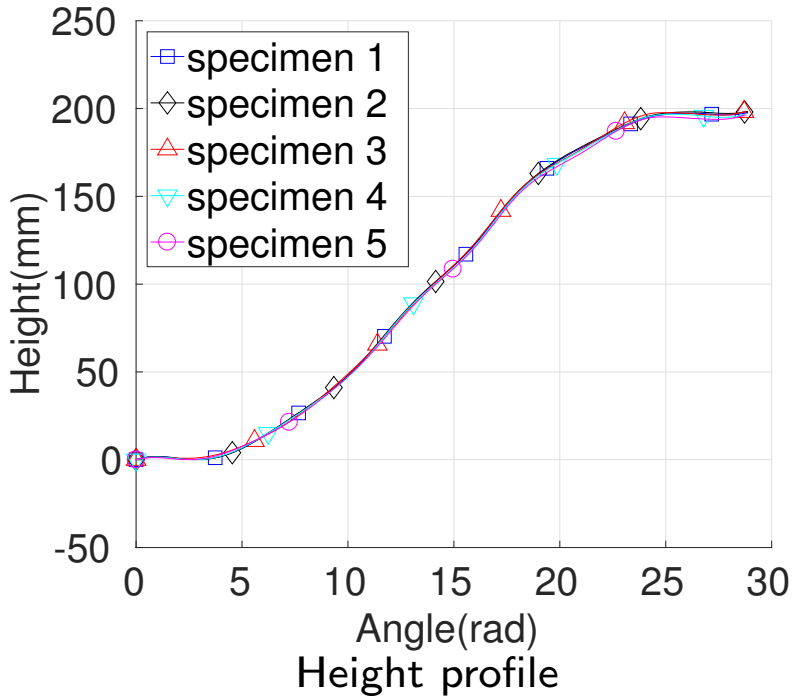
- Wire diameter: $d = 13.5$ mm
- Number of coils: 4.75
- Solid height: less than 38 mm
- Center line $\{\boldsymbol{x}_i\}_{i=1}^{N}$ coordinates: Follow diameter & height profiles.



Isometric Front Top



Diameter profile

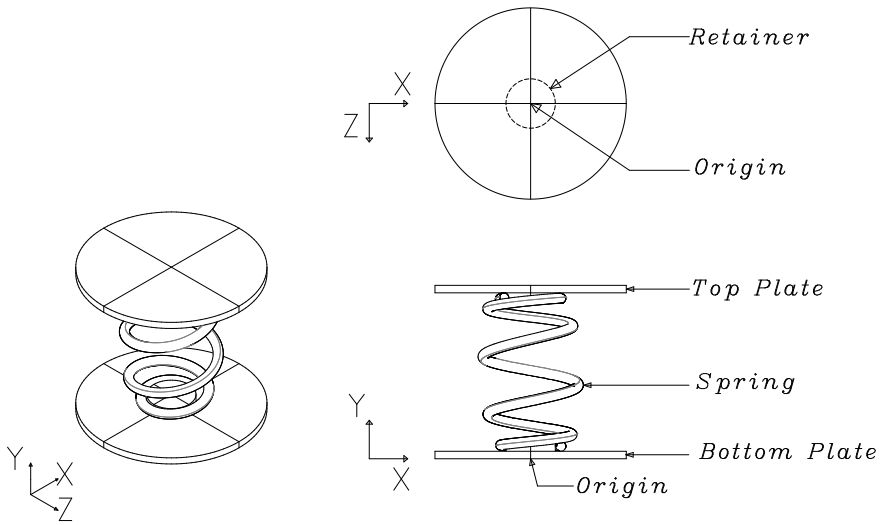


Height profile

Loading & Testing Procedure

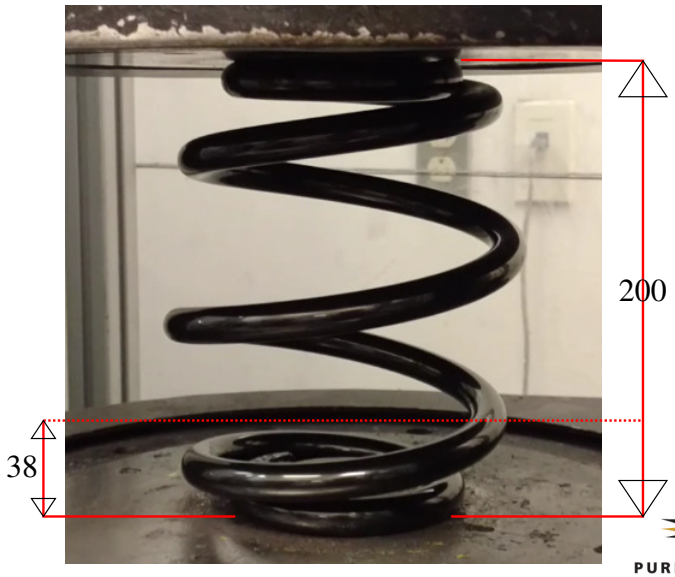
Setup

- Load Testing using UTM
- Bottom Plate is fixed
- Top plate applies the force
- Retainer: assists in placement

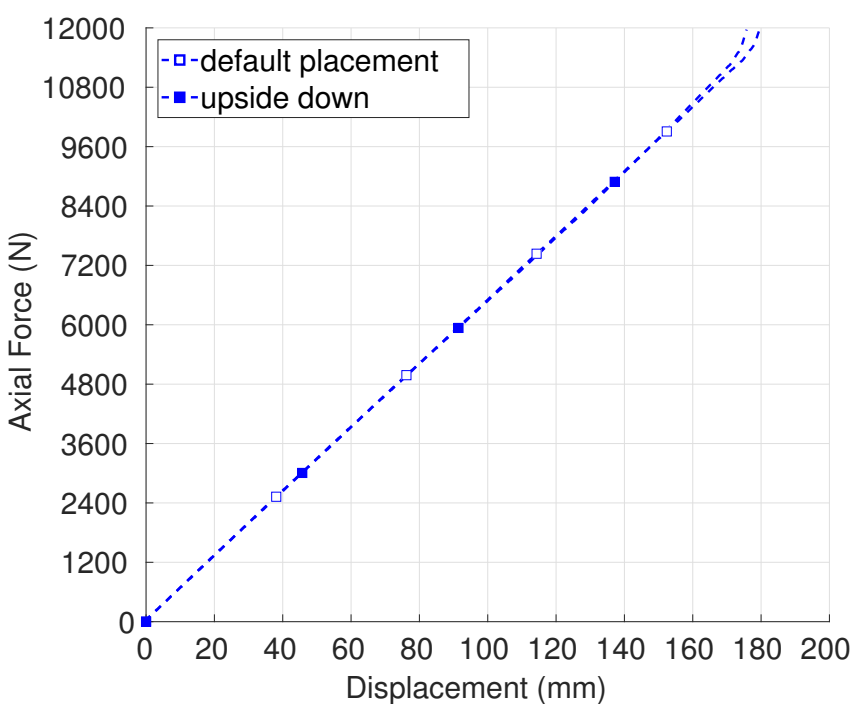


Loading

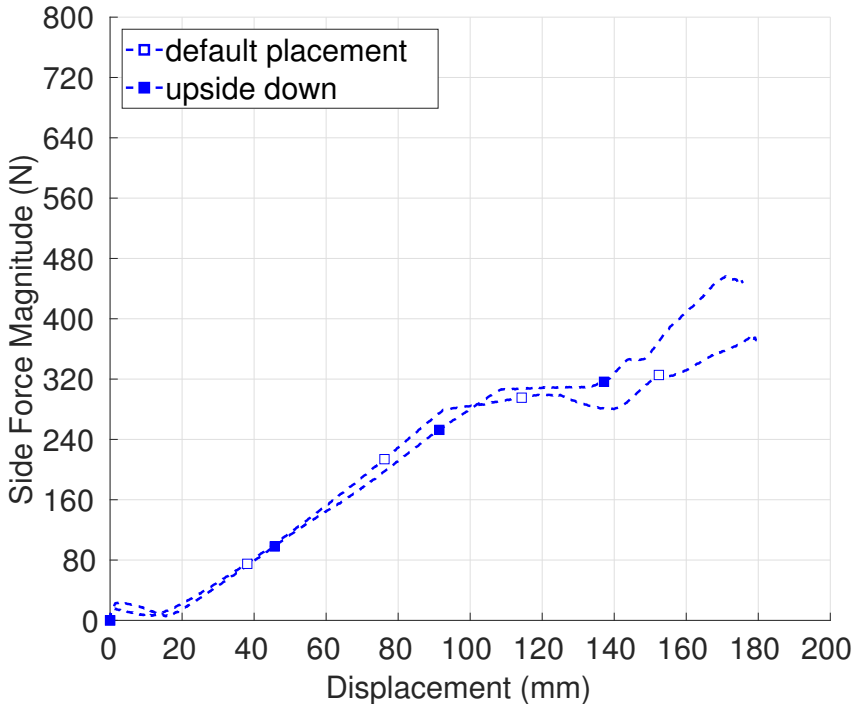
- Spring compressed up-to 38 mm
- Force response: recorded by load cells
- Testing done for several types of placements of spring



Loading & Testing Procedure: Specimen-1



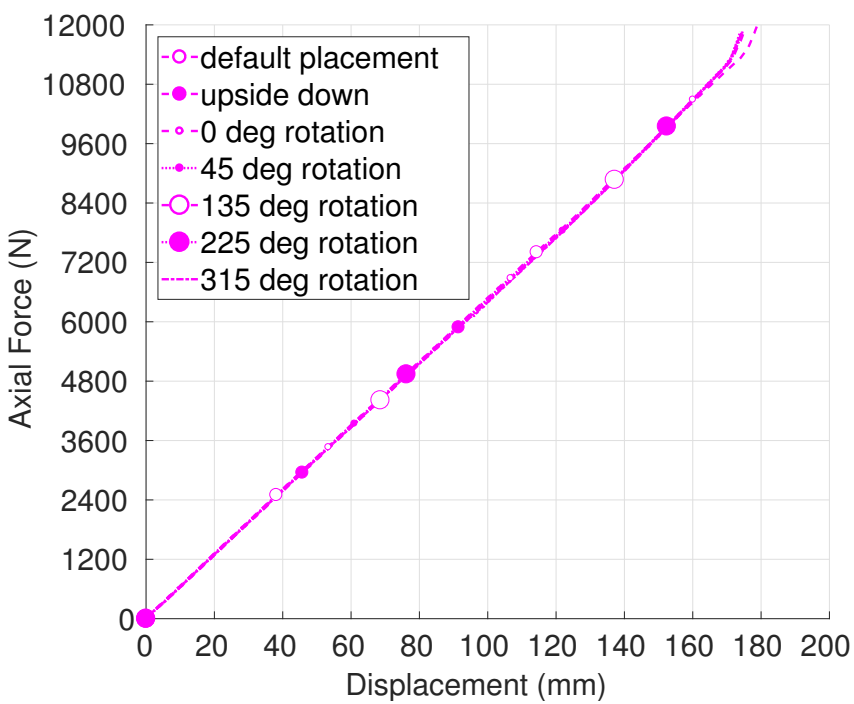
Axial Force



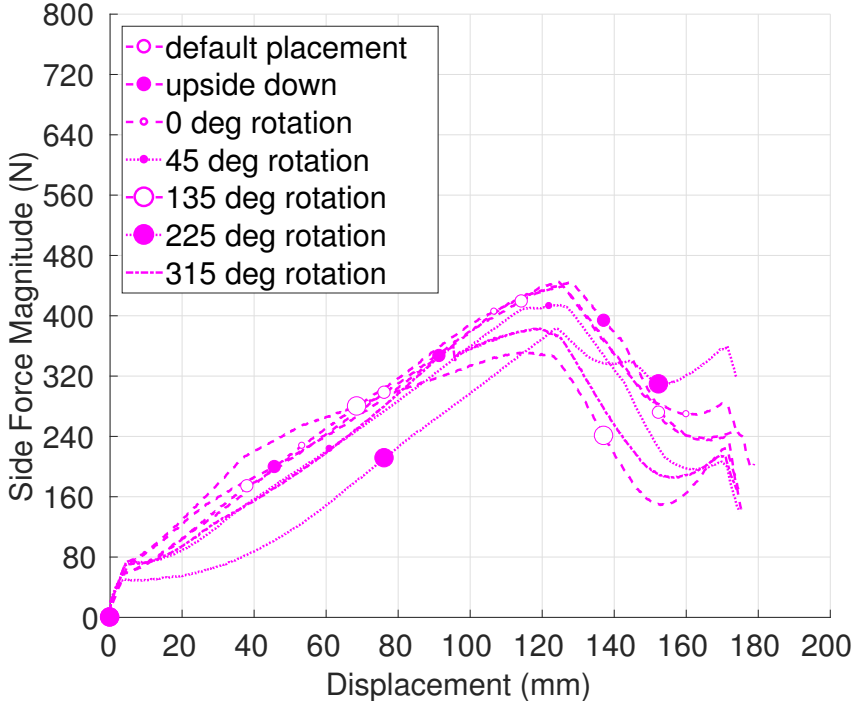
Side Force

Figure: Measured load response for specimen 1

Loading & Testing Procedure: Specimen-5



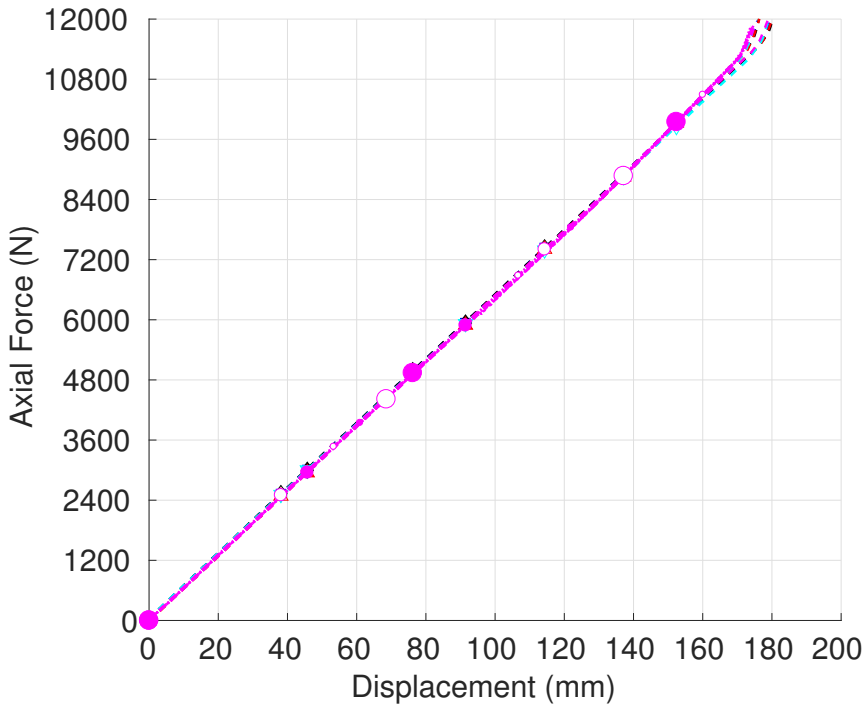
Axial Force



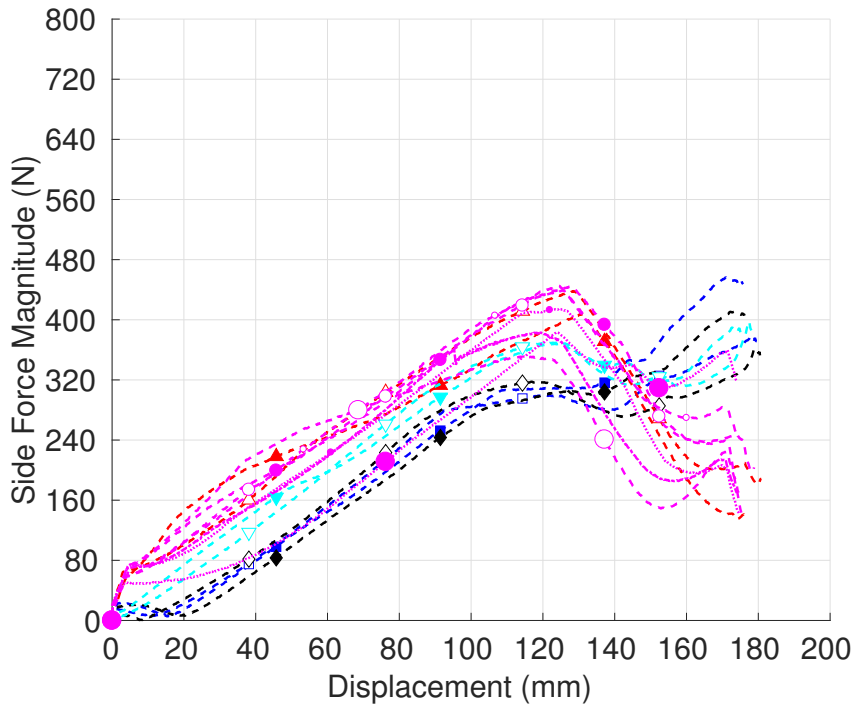
Side Force

Figure: Measured load response for specimen 5

Loading & Testing Procedure: All Specimens



Axial Force



Side Force

Figure: Measured load response for all specimens

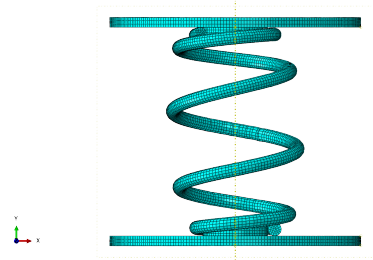
Response is different for:

- Spring profiles
- Spring placements

Continuum Model

Geometry

- Circular cross-section swept along the center-line
- Solid parts for plates and retainers



Type of Elements

- Spring Part:
  - Bending-suited element: C3D8I (Quadratic behavior)
- Plate and Retainer Part:
  - Linear element: C3D8

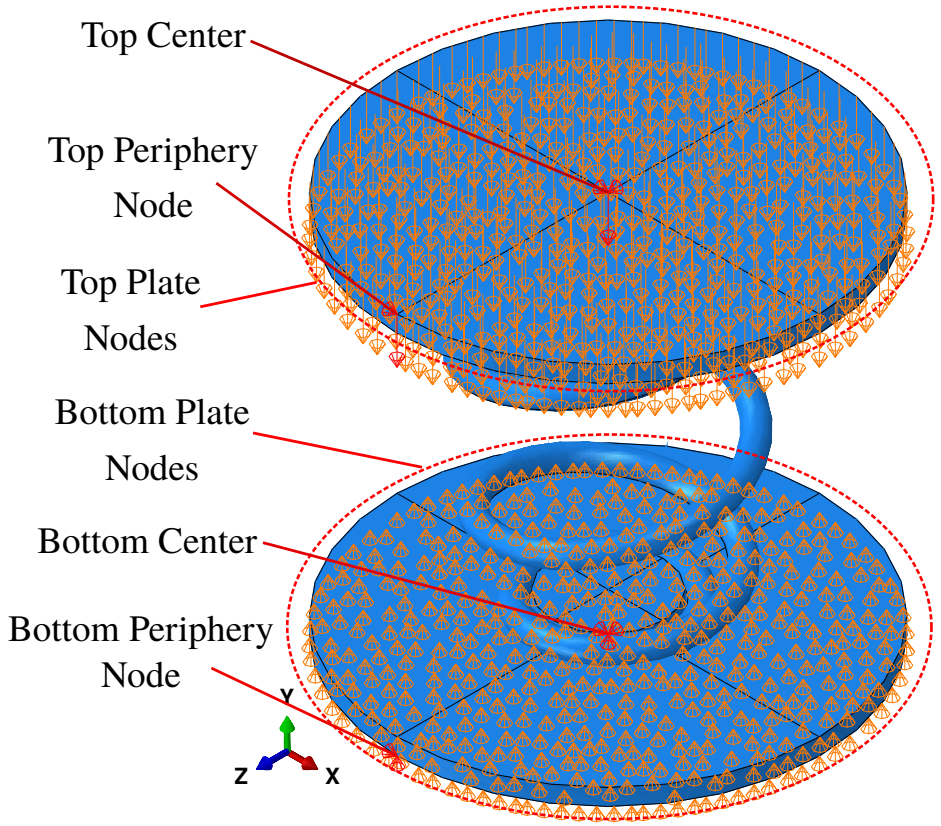
Material properties

- Linear elastic Hooke's model
- Loading plate and retainer: $E = 207$ GPa and $\nu = 0.3$
- Spring : $E = 250$ GPa and $\nu = 0.3$

Boundary Conditions

Boundary Conditions

- Bottom center: all fixed
- Top center: specified displacement in Y ($\alpha = (H + d - 38.1)$)
- Bottom plate nodes: fixed in Y, free in X & Z
- Top plate nodes: specified displacement(α) in Y , free in X & Z
- Bottom periphery node: fixed in Y & X
- Top periphery node: specified displacement(α) in Y, fixed in X



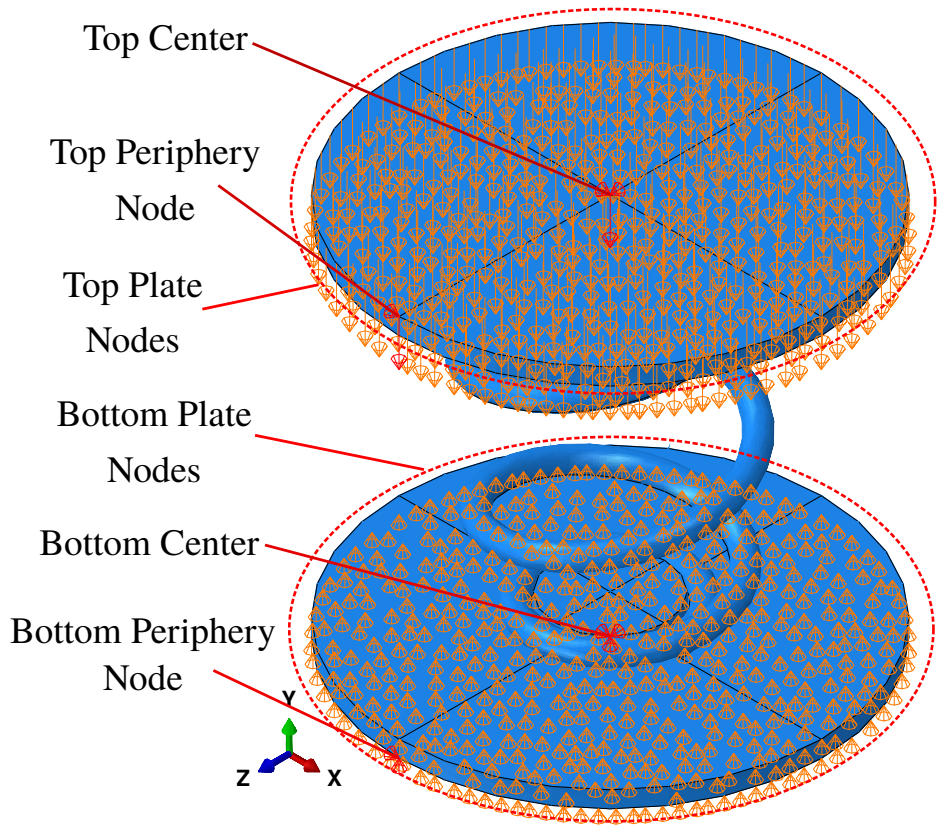
Analysis & Post-Processing

Analysis details

- Static with geometric non-linearity
- Frictional contact modeled with $\mu = 0.7$
- Automatic stabilization with volume proportional damping

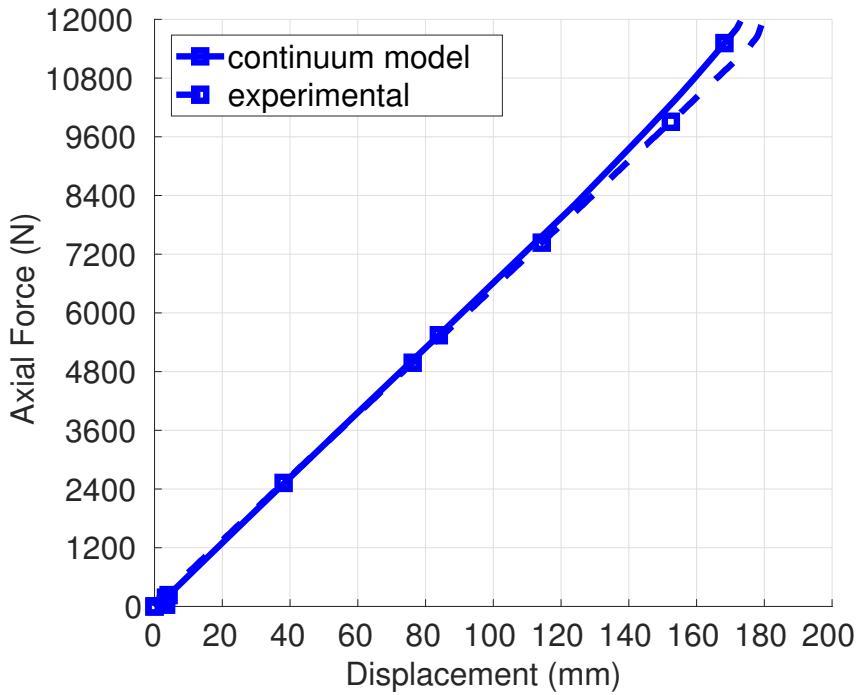
Post-Processing to extract:

- Displacement: displacement of the top center node.
- Axial-force: Sum of axial force for all nodes on bottom face of the bottom plate.
- Side-force: Sum of reaction force at the bottom center and bottom periphery node.

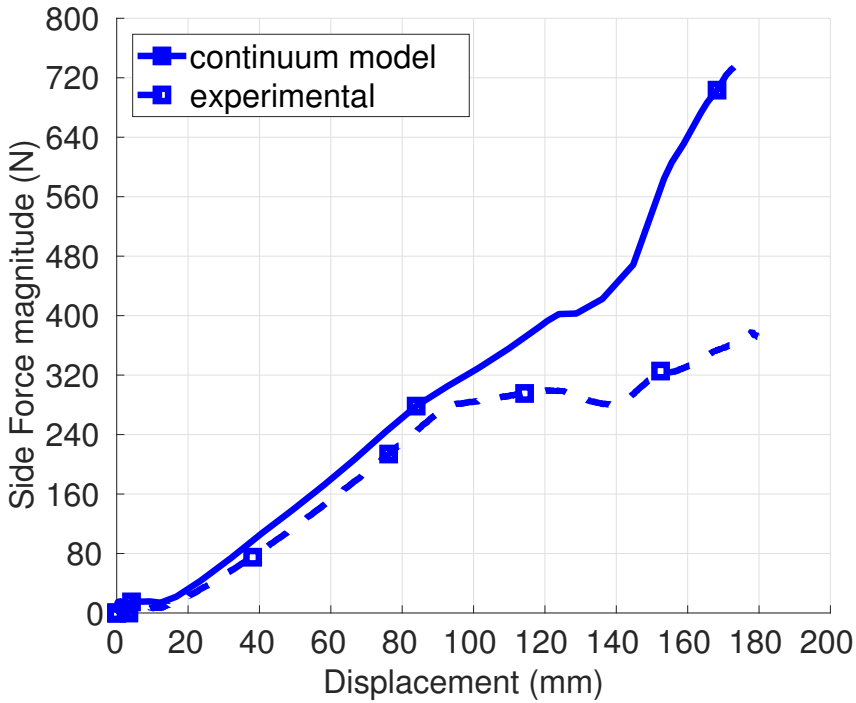


Model Results

- Axial force matches experimental data
- Side-force response stiffer



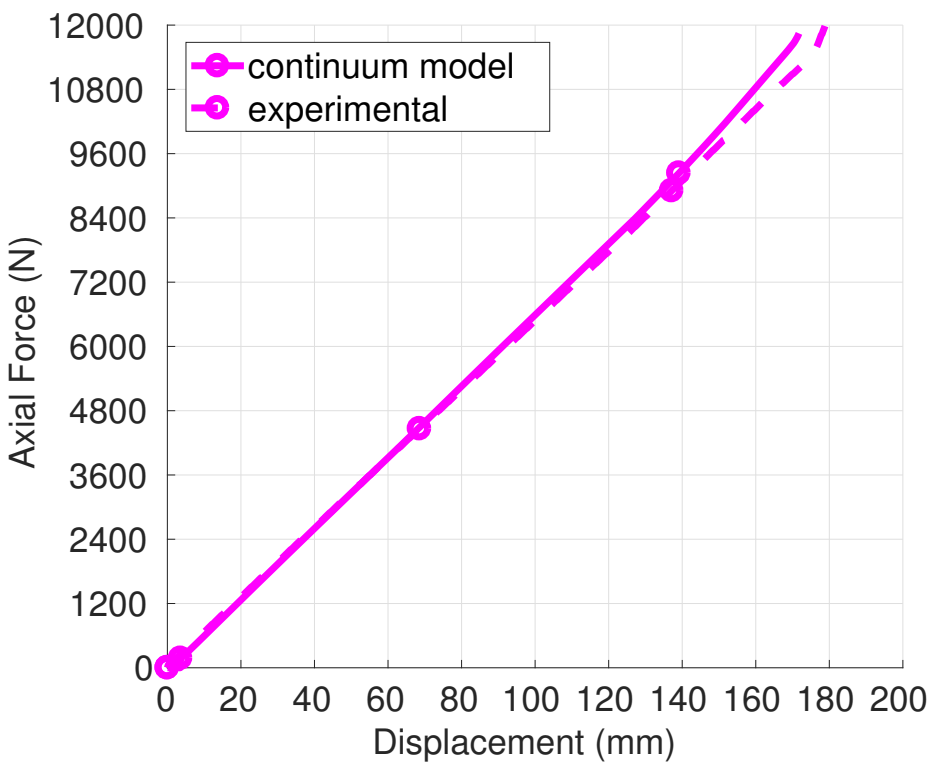
Axial force



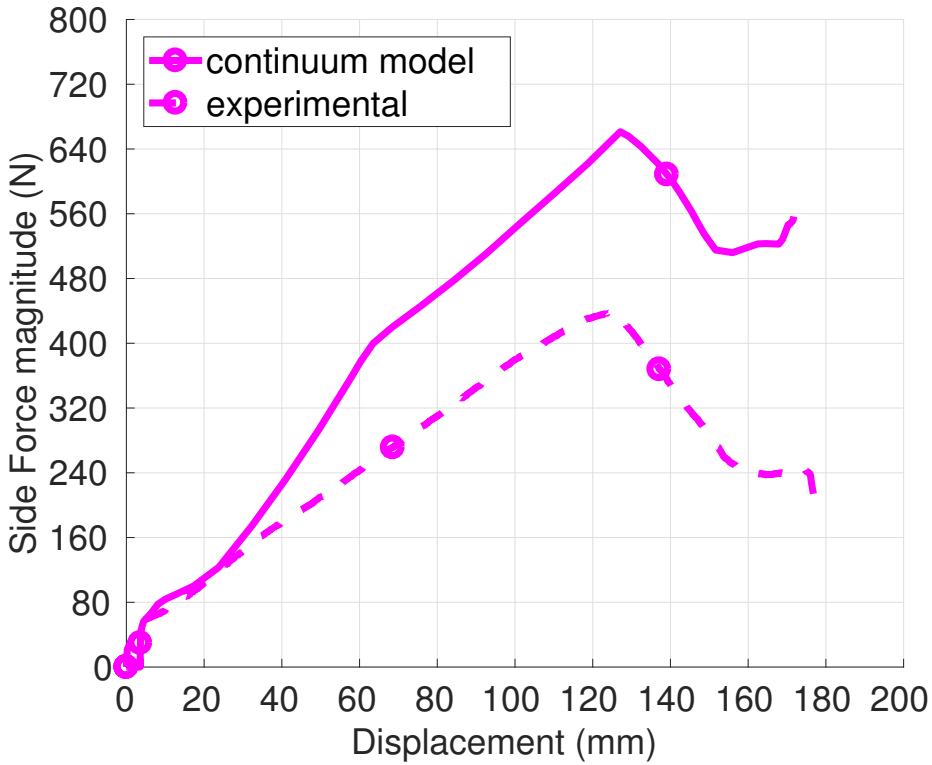
Side force

Figure: Results for Continuum Model: Specimen-1

Model Results



Axial force



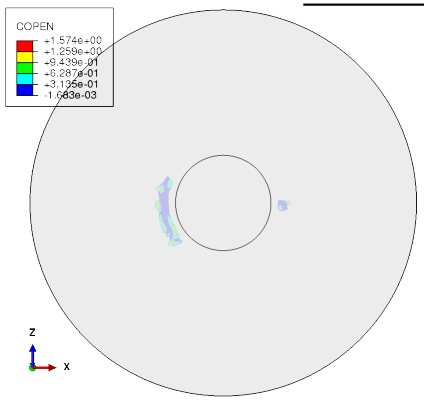
Side force

Figure: Results for Continuum Model: Specimen-5

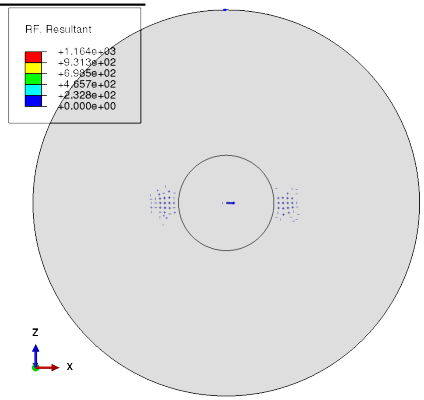
- Validation: Model results match experiments

Model Results: Specimen 1

Bottom Plate

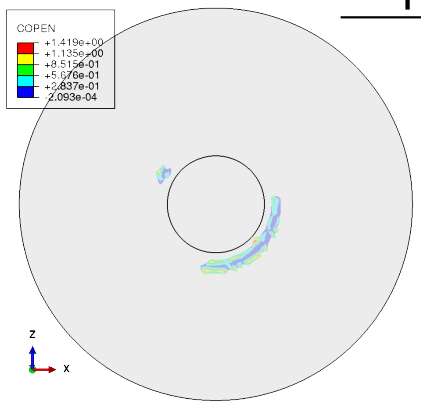


Contact region

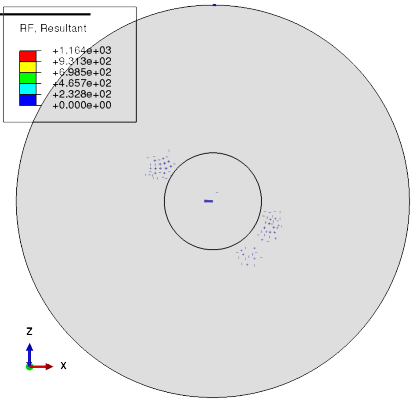


Reaction force

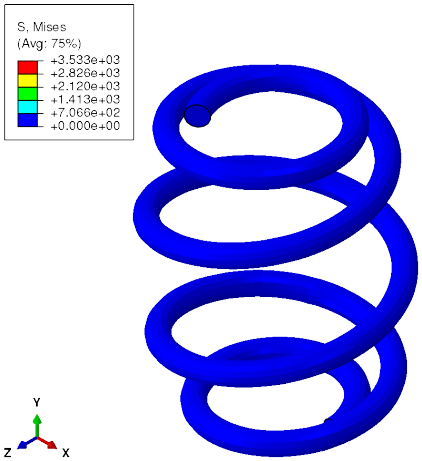
Top Plate



Contact region



Reaction force



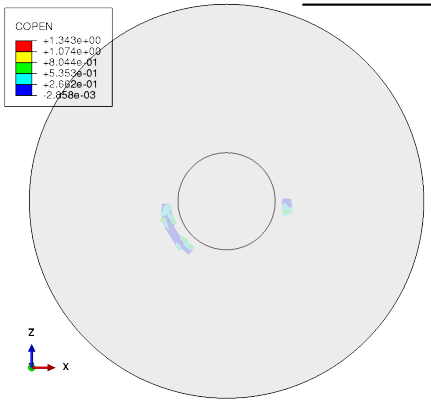
Deformation shape of spring

Observations

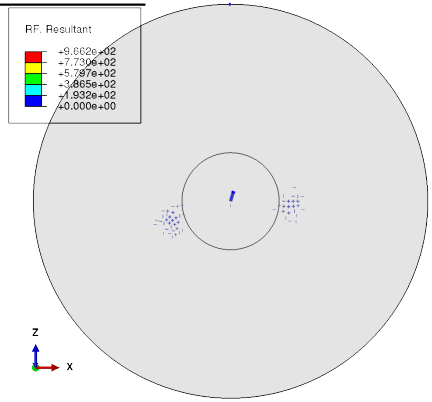
- Contact regions change
- Reaction forces are concentrated: friction force higher
- Regions of concentration change during loading: frictional forces will also change

Model Results: Specimen 5

Bottom Plate

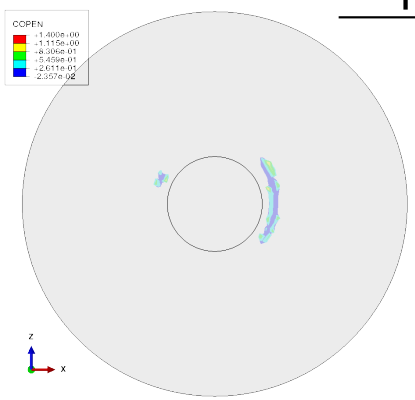


Contact region

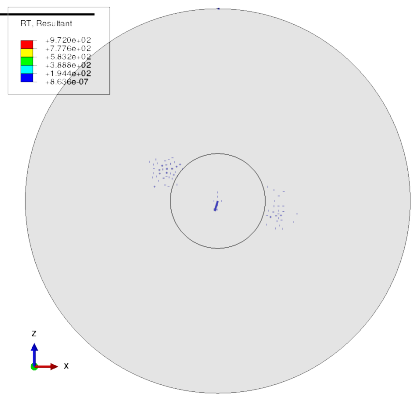


Reaction force

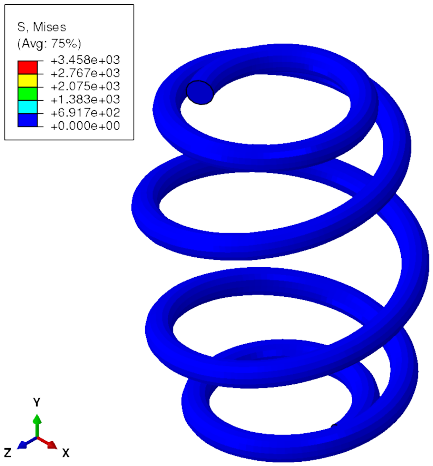
Top Plate



Contact region



Reaction force



Deformation shape of spring

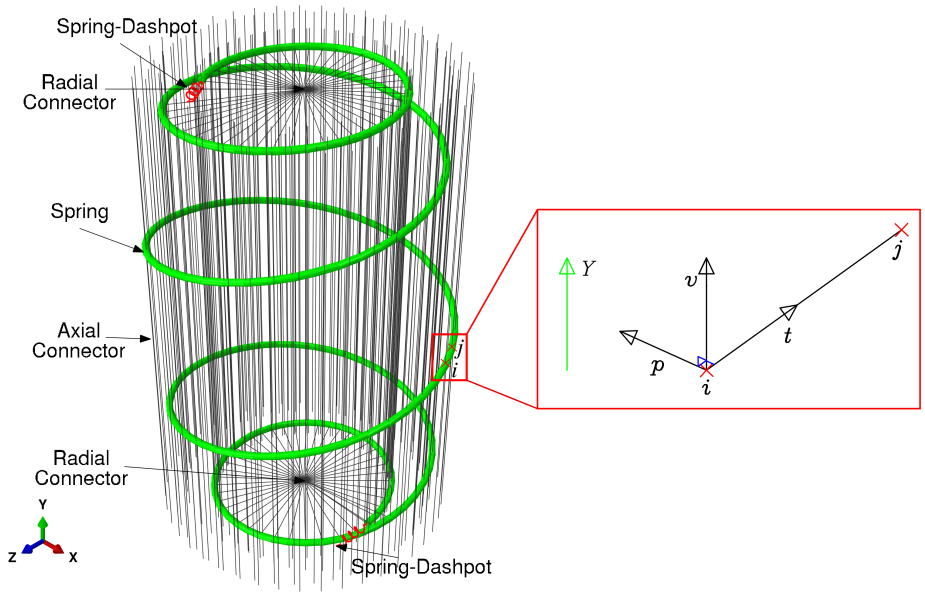
Observations

- Regions of concentration: different position than Specimen 1

Reduced-order Model using Beam Elements and Connectors

Geometry

- Spring part: wire from center-line
- No parts for plates and retainers
- Axial connectors: Normal contact with plates
- Radial connectors:
  - Frictional contact on plate
  - Normal contact with retainer

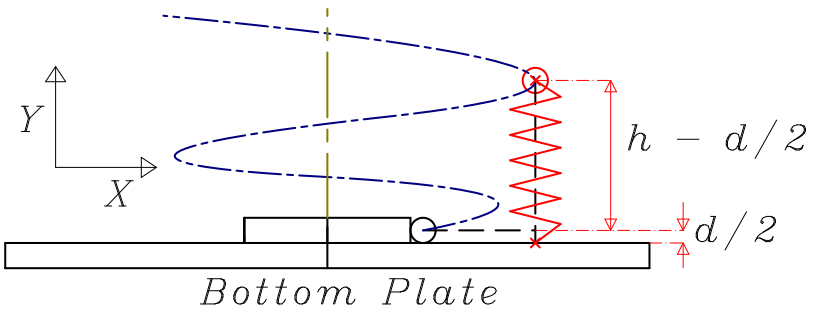


Type of Elements

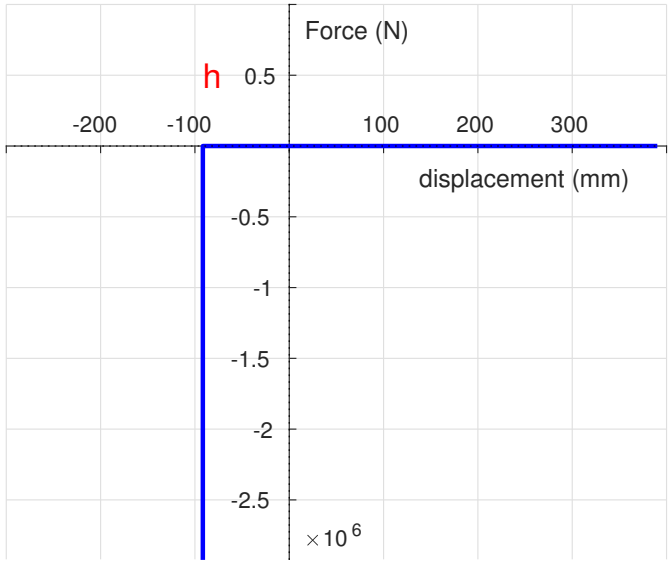
- Spring element: Timoshenko beam element (B32) - 200 elements
  - Orientation: $\boldsymbol{p} = \boldsymbol{t} \times \boldsymbol{v}$
  - With circular cross-section
- Spring dashpots: at end faces to restrict rigid body motion
- Connectors: Slot connectors with non-linear behavior

Geometry & Type of elements: Connector Stiffness

- Non-linear behavior to mimic normal contact



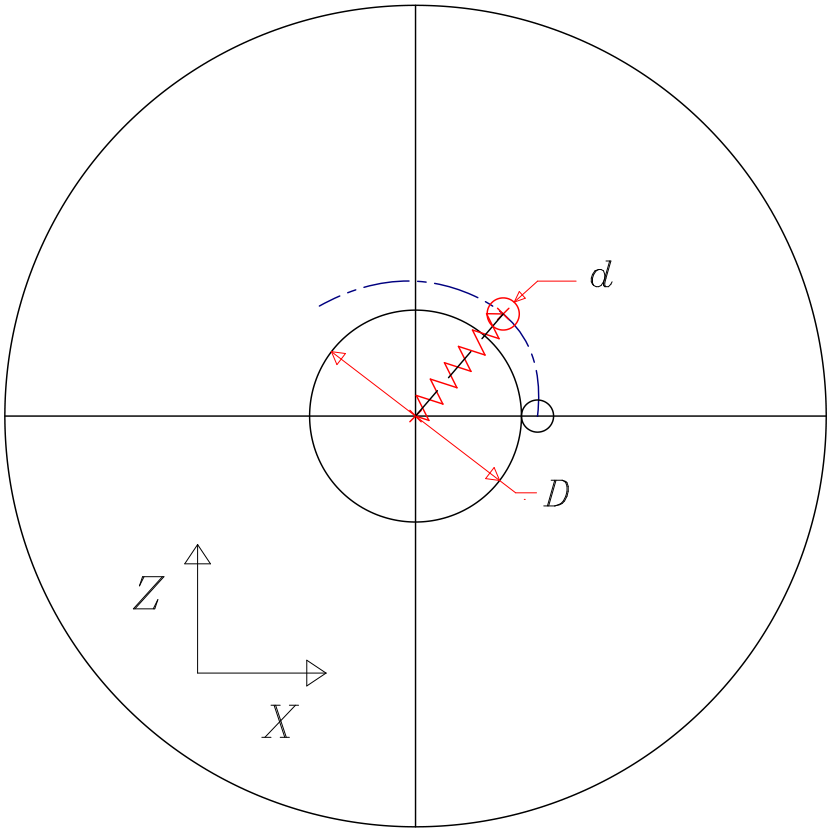
axial connector



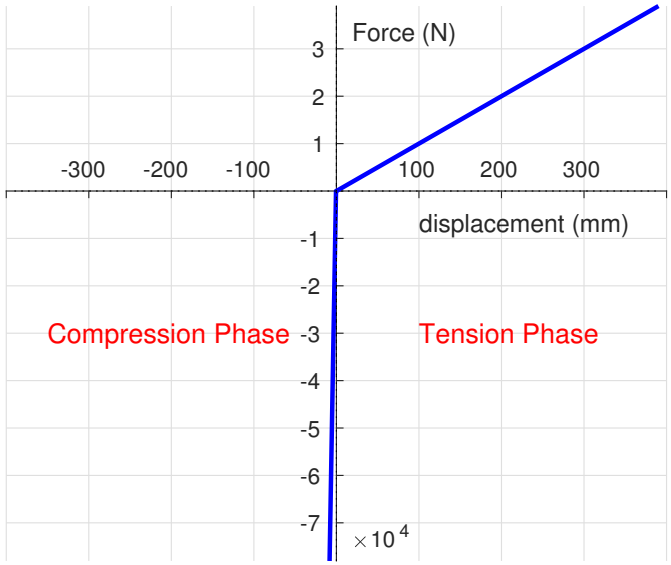
axial connector response (K_a)

Geometry & Type of elements: Connector Stiffness

- Non-linear behavior to mimic normal and frictional contact



radial connector



radial connector response (K_c,K_t)

Material properties

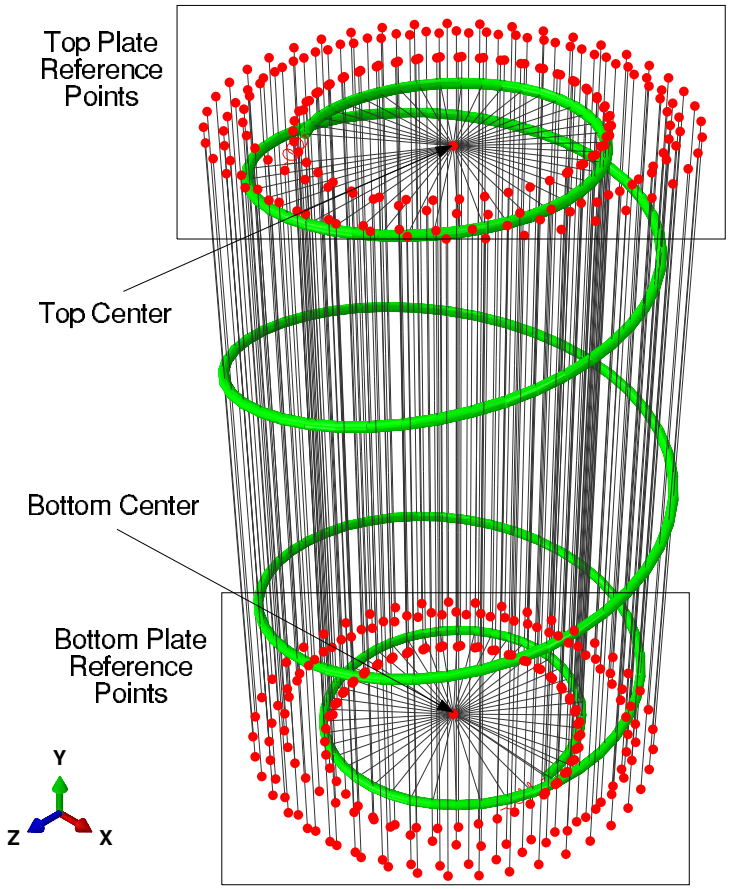
Material properties (from calibration)

- Linear elastic Hooke's material model
- Barrel spring material: $E = 225$ GPa and $\nu = 0.3$
- Axial connectors stiffness: $K_a = 10^6$ N/mm
- Radial connectors stiffness:
  - Compression: $K_c = 10^4$ N/mm
  - Tension: $K_t = 100$ N/mm
- Spring dashpots stiffness: $K = 1$ N/mm

Boundary Conditions

Boundary Conditions:

- Bottom center: all fixed
- Top center: specified displacement ($\alpha = (H + d - 38.1)$)
- Bottom plate ref pts: fixed in Y, free in X & Z
- Top plate ref pts: specified displacement(α) in Y, free X & Z
- Bottom spring-dashpot: fixed in Z, free in X & Y
- Top spring-dashpot: specified displacement (α) in Y, fixed in X & Z



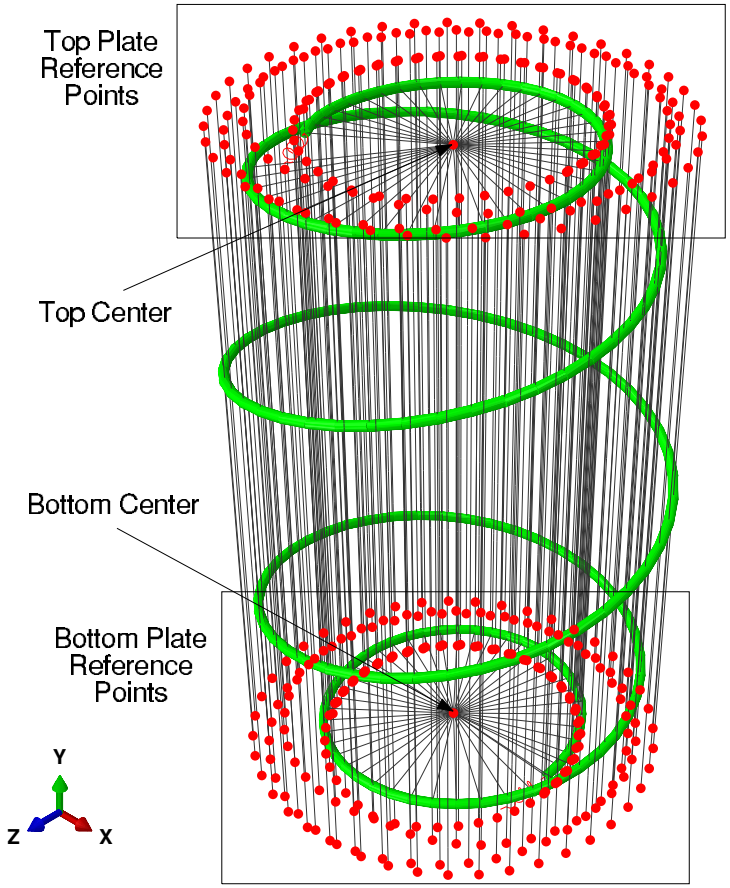
Analysis & Post-Processing

Analysis details

- Static analysis with geometric non-linearity
- Automatic stabilization with volume proportional damping

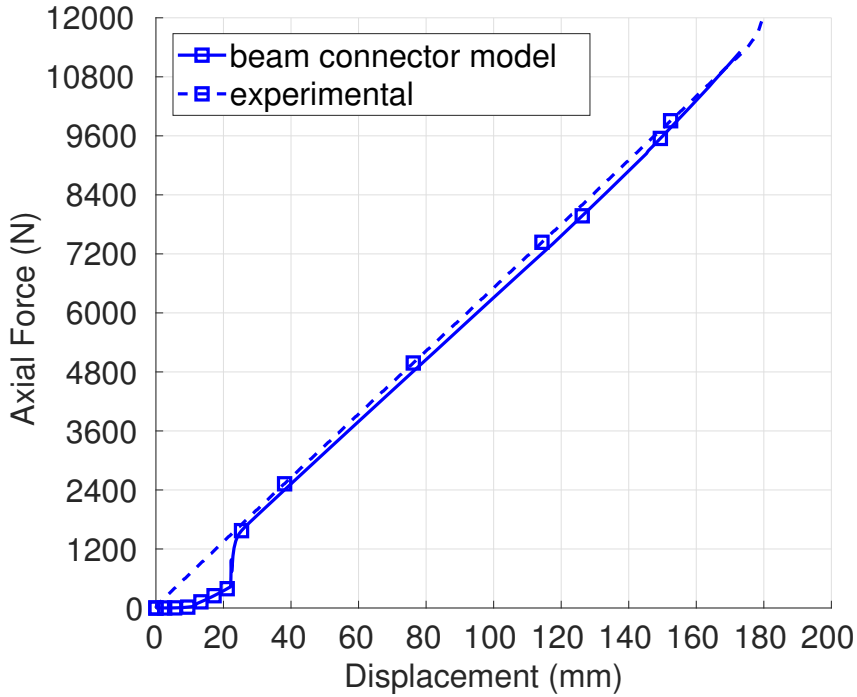
Post-Processing to extract:

- Displacement: Top center displacement.
- Axial-force: Sum of axial force for all bottom pts.
- Side-force: Sum of reaction force at the bottom center.

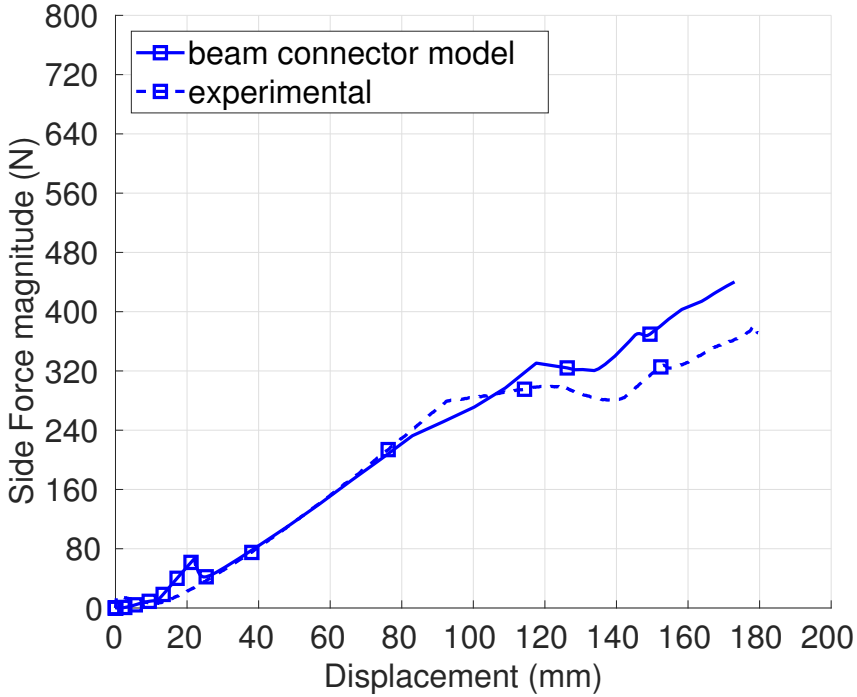


Model Results

- Axial force matches experimental data
- Side force also matches closely



Axial force



Side force

Figure: Results for Beam-Connector Model: Specimen-1

Model Results

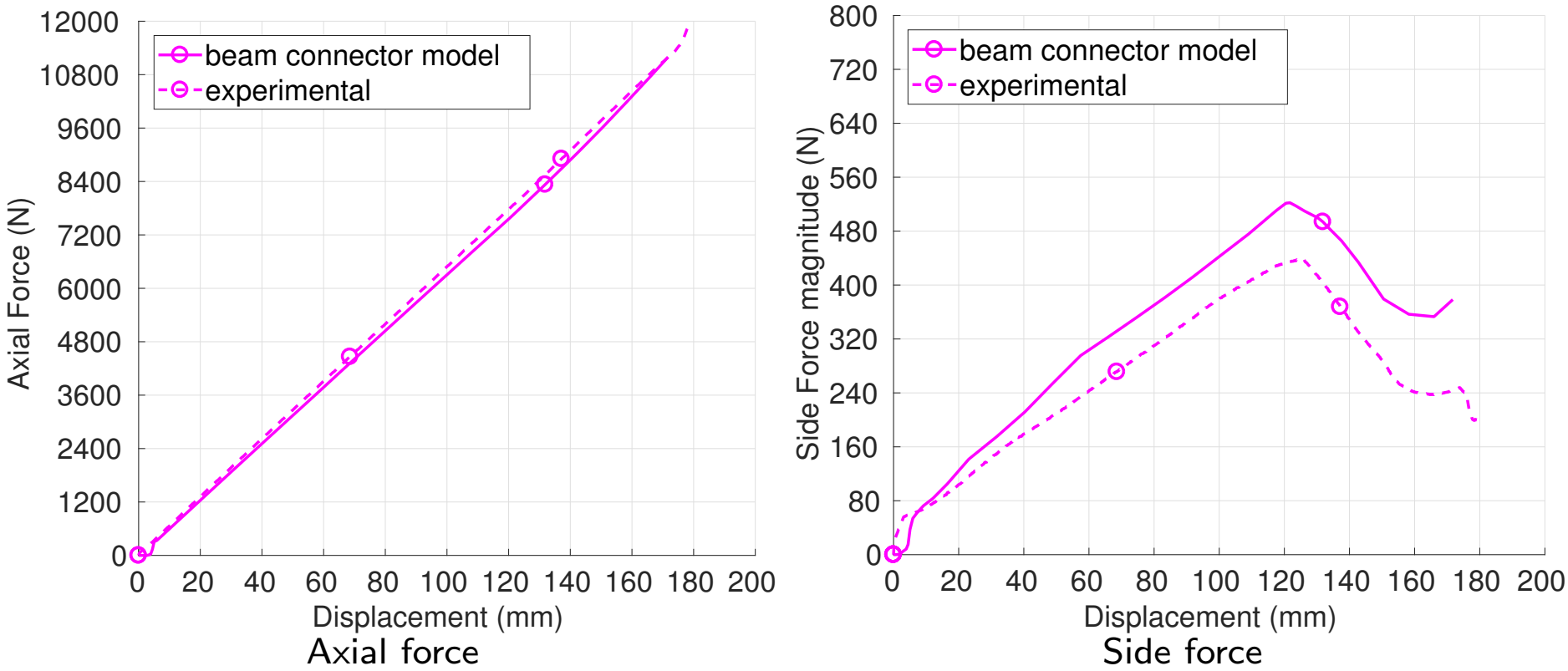
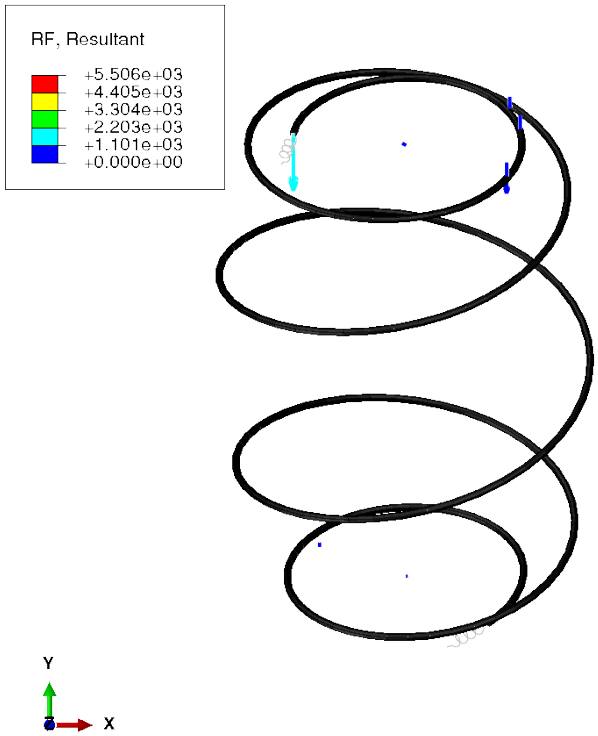


Figure: Results for Continuum Model: Specimen-5

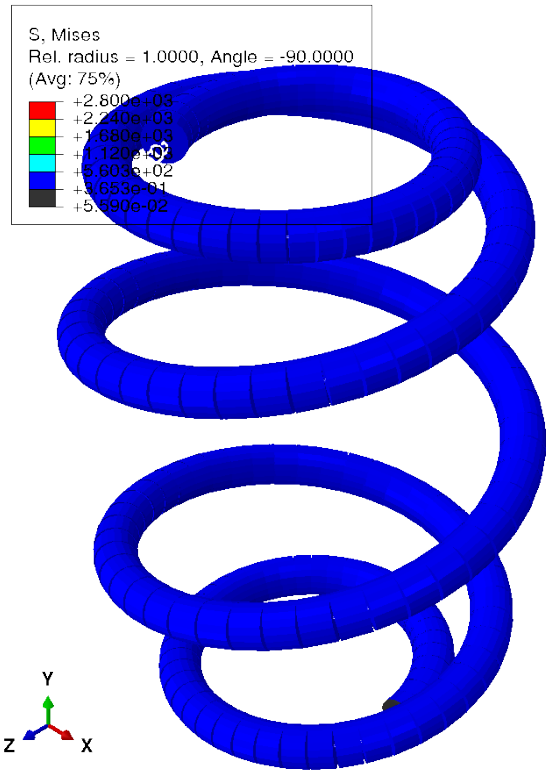
- Validation: Model results match experiments

Model Results: Specimen 1

Reaction forces



Von-Mises stress

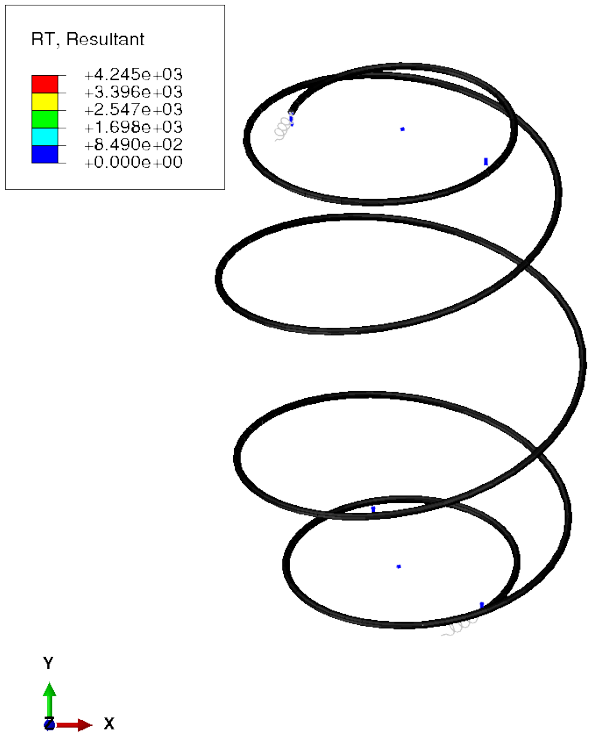


Observations

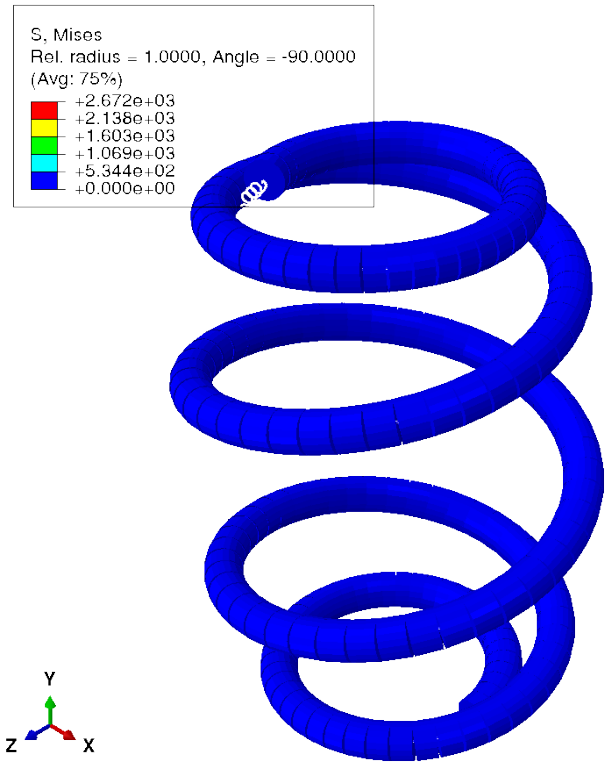
- Reaction forces concentrated at certain regions of concentration
- Regions localized at 2 or 3 locations: Spring supported by 2 or 3 points only

Model Results: Specimen 5

Reaction forces



Von-Mises stress



Parameter Variation Analysis

Observations

- Response varies for different placement configurations
- Response varies for different profiles

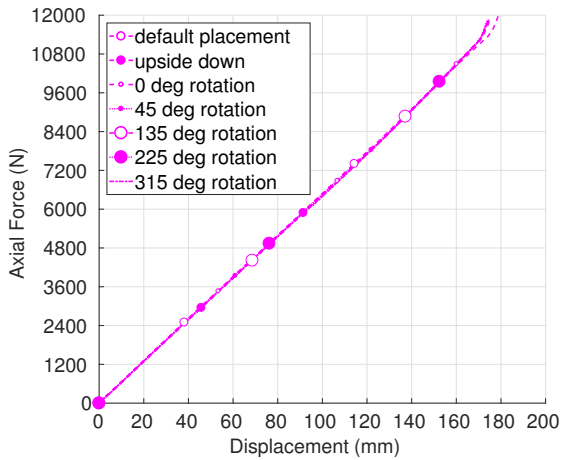
Source of Variations

- Placement: Clearances - User error
- Geometry: Manufacturing Tolerances - Different profiles

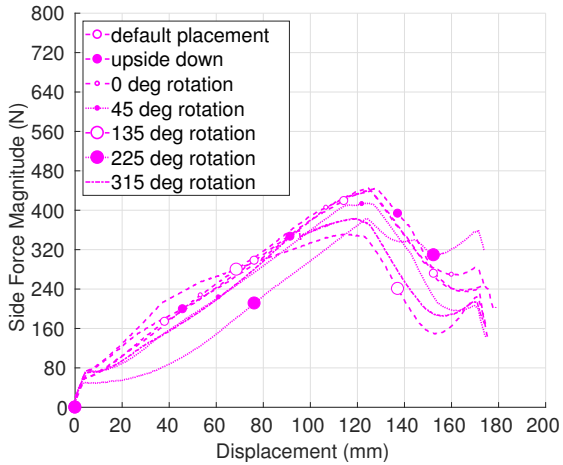
Parametric variation analysis - to study the effect on side-forces

Placement Variations

- What: Model allowable placements (Translation & Rotation)
- Why: User places spring for testing
- How: estimate allowable variation and observe effect



Axial Force

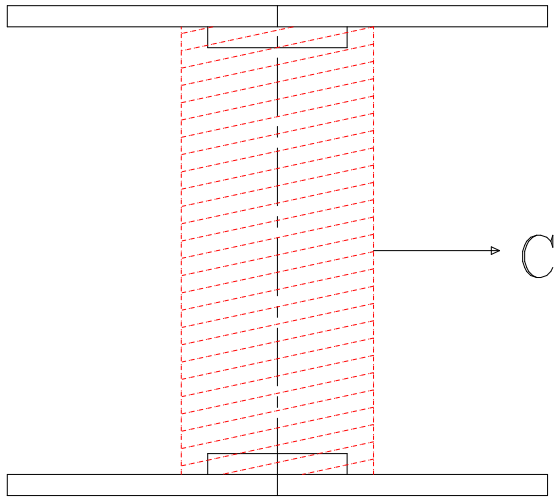


Side Force

Estimating allowable variation

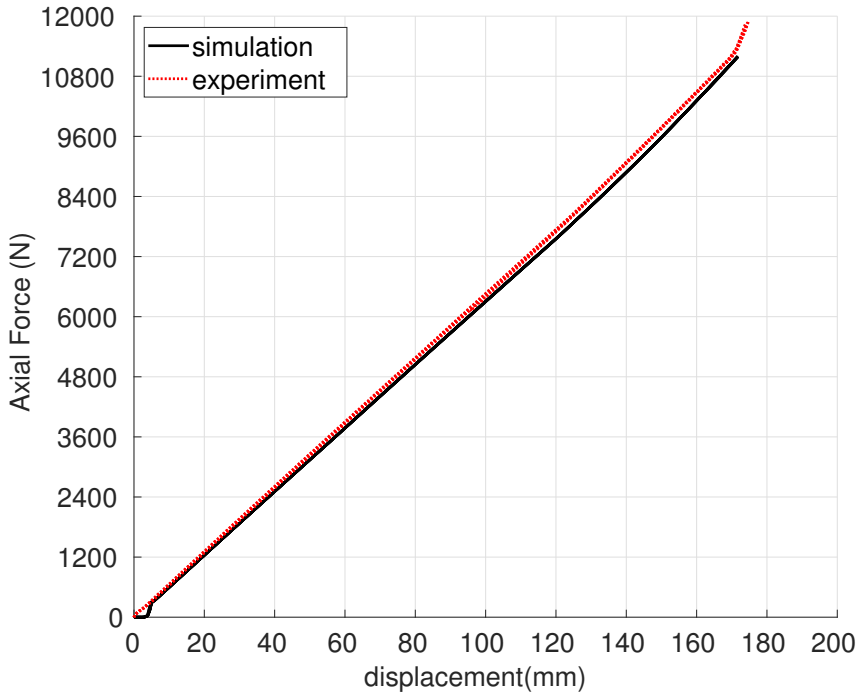
$$p^* = \underset{p}{\operatorname{argmax}}\, p$$

st: $\boldsymbol{z}_i(p) = \mathcal{A}(p)\boldsymbol{x}_i \notin \mathcal{C} \quad \forall \quad i = \{1, 2, ..., N\}$

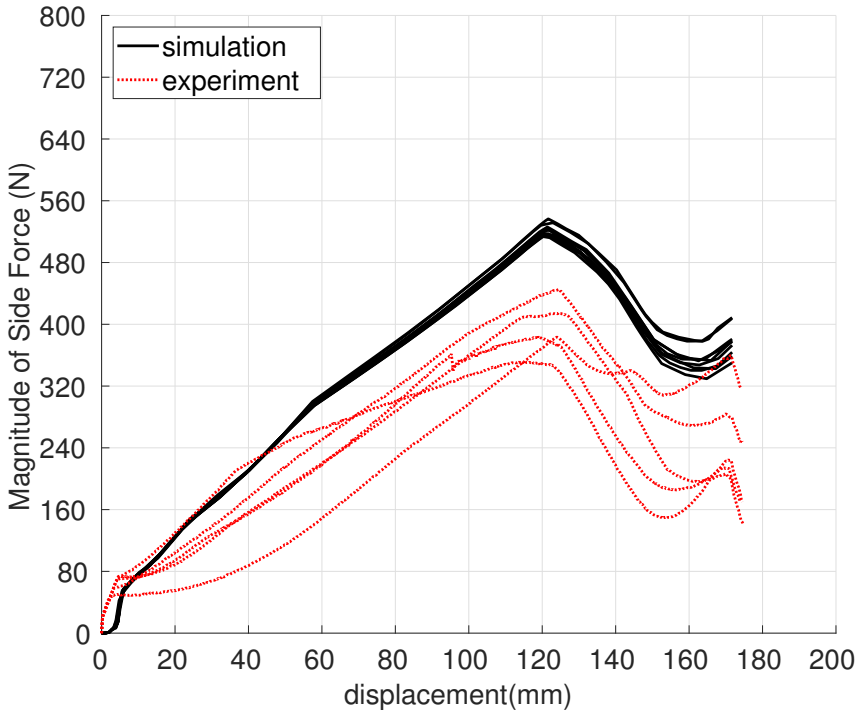


- $\mathcal{A}(p)$: Translation or Rotation operator
- Solve by enumeration

Translation (x & z) - unaffected

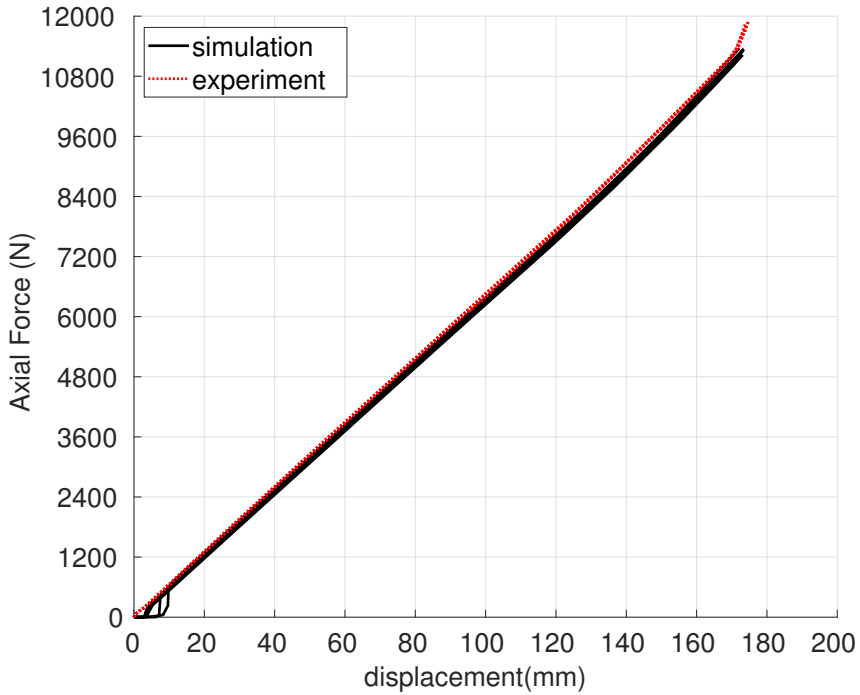


axial force

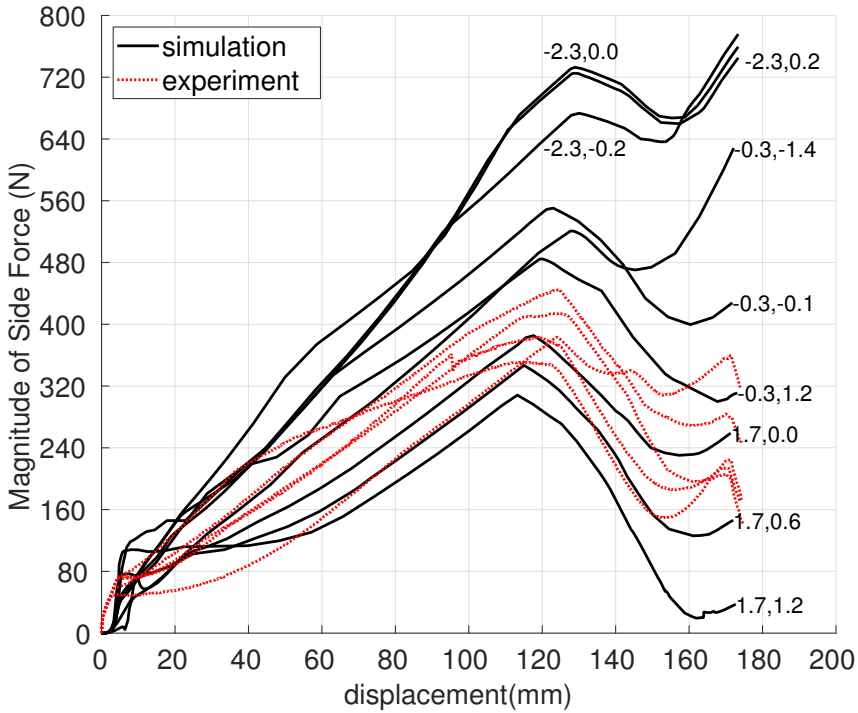


side force

Rotation (x & z) - Specimen 5



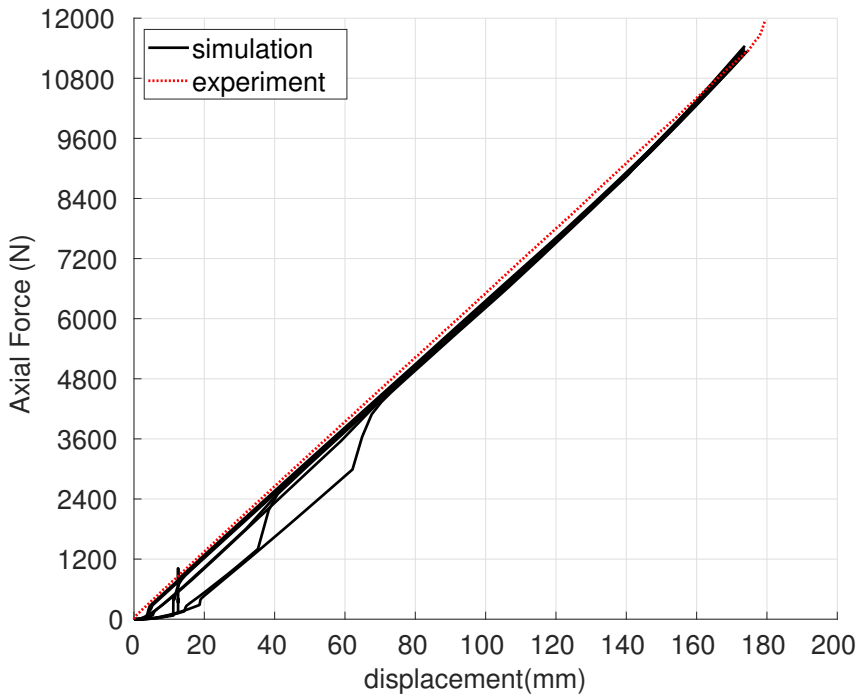
axial force



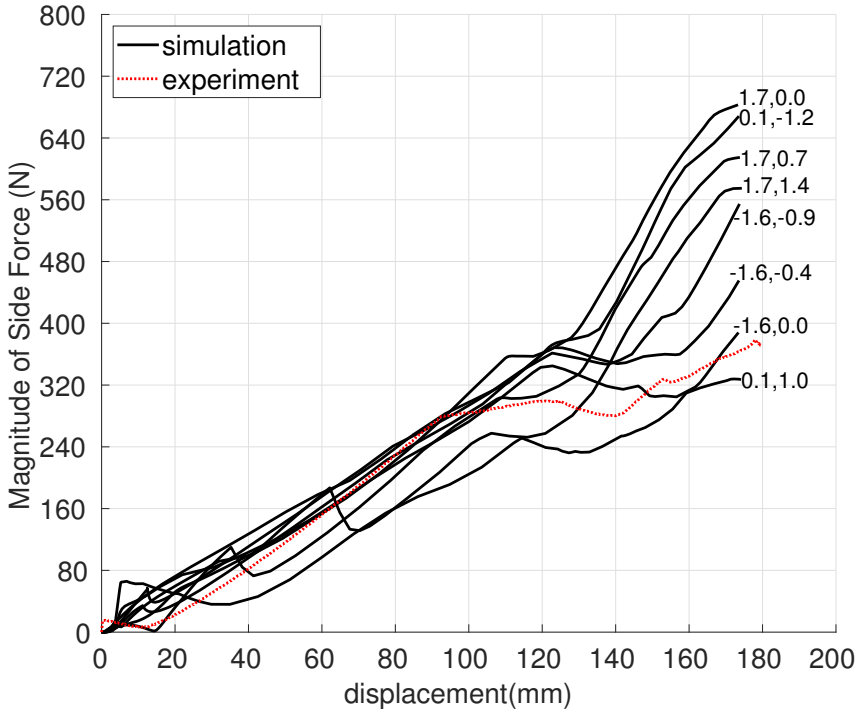
side force

- Rotation Variation: is critical

Rotation (x & z) - Specimen 1



axial force



side force

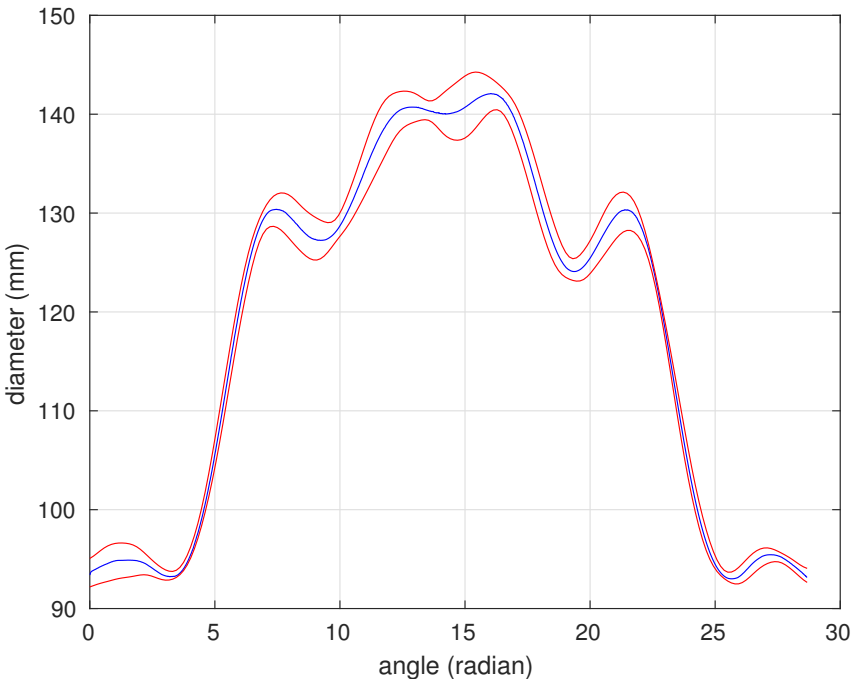
- Inconsistent trend accross specimens: Geometry also affects

Profile Variations

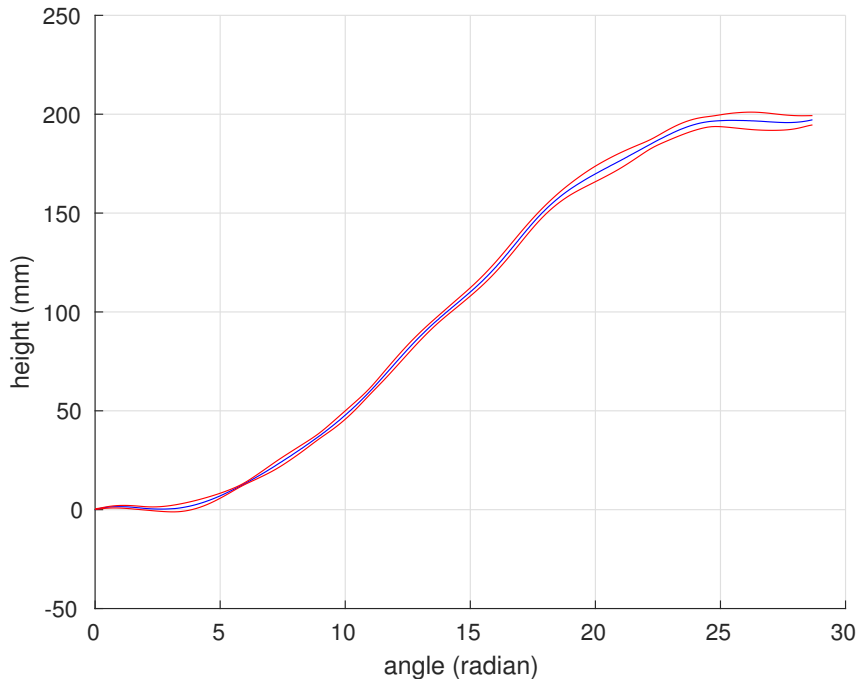
- Why: Manufacturing Tolerances - Different profiles - Different side force
- Variations about which spring? - Reference Spring
- How much variation? - Estimate from specimens
- How to carry out realistic variations? - Approximate the profile
  - Spline approximation with 13 points - unrealistic results
  - Linear approximation with 200 points - realistic results
- How many parameters? - Identify from specimens
  - Regions of profile variations
- Variations on regions: realistic?
  - Smooth-out variations accross regions

Reference Spring & Bounds for variation

- Reference spring: average profile of 5 specimens
- Bounds for variation: Max. absolute difference of average spring with all specimens



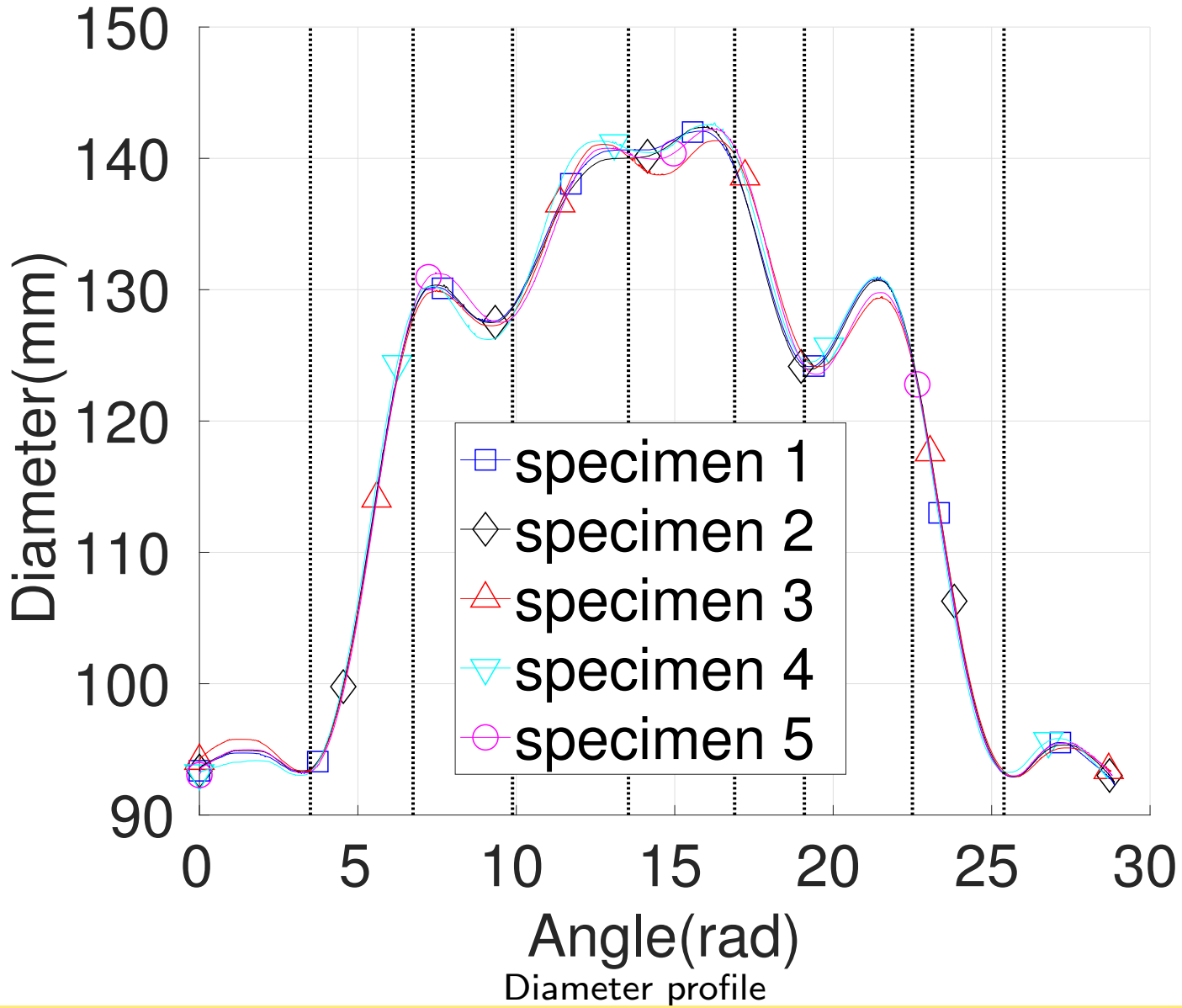
Diameter bounds



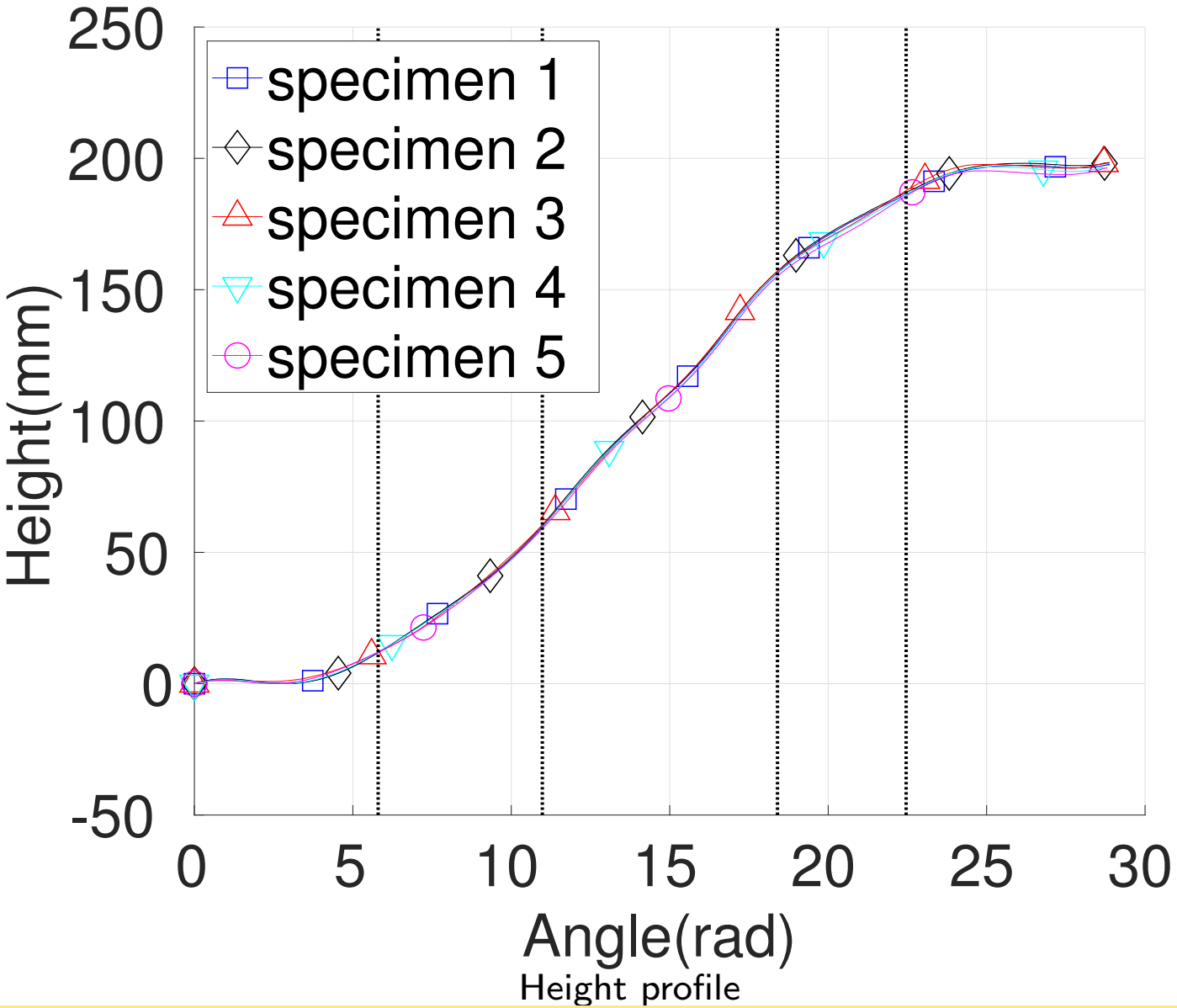
Height bounds

Figure: 2x bounds

Identifying parameters to vary



Identifying parameters to vary



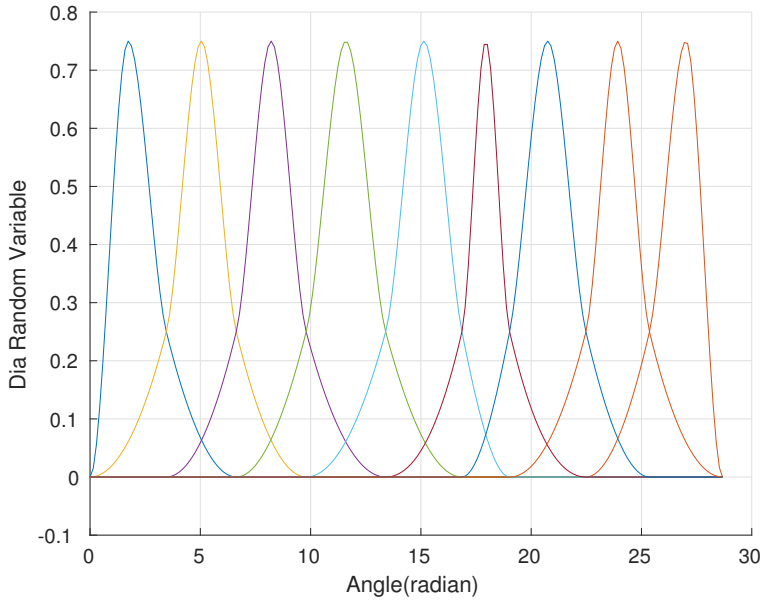
Approximating the Profile & Realistic variations

Approximating the profile

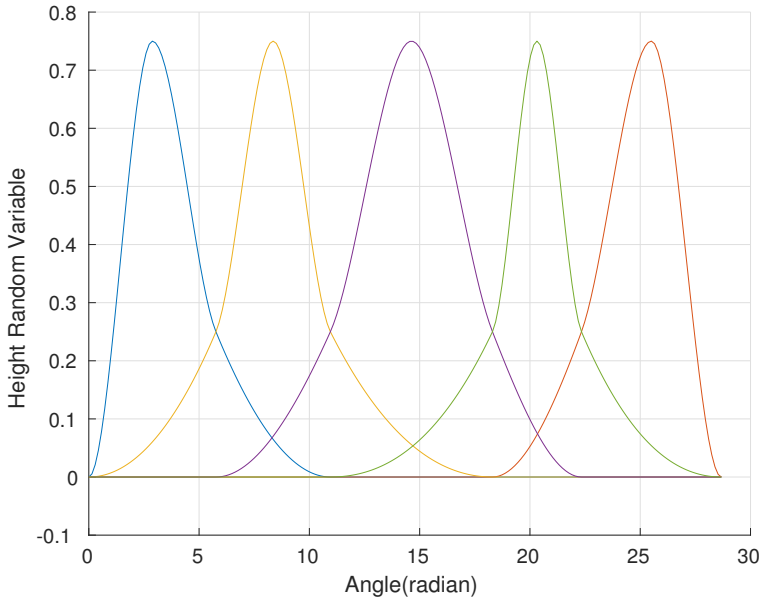
- Beam Model: 200 Nodes - defined by diameter & height profile
- Variations to the 200 Nodes - random variable for regions

Realistic Variations

- Manufacturing Variations- smooth variations at boundary
- Simulated Variations: smooth variations using wt. functions
- Using PW cubic wt. functions



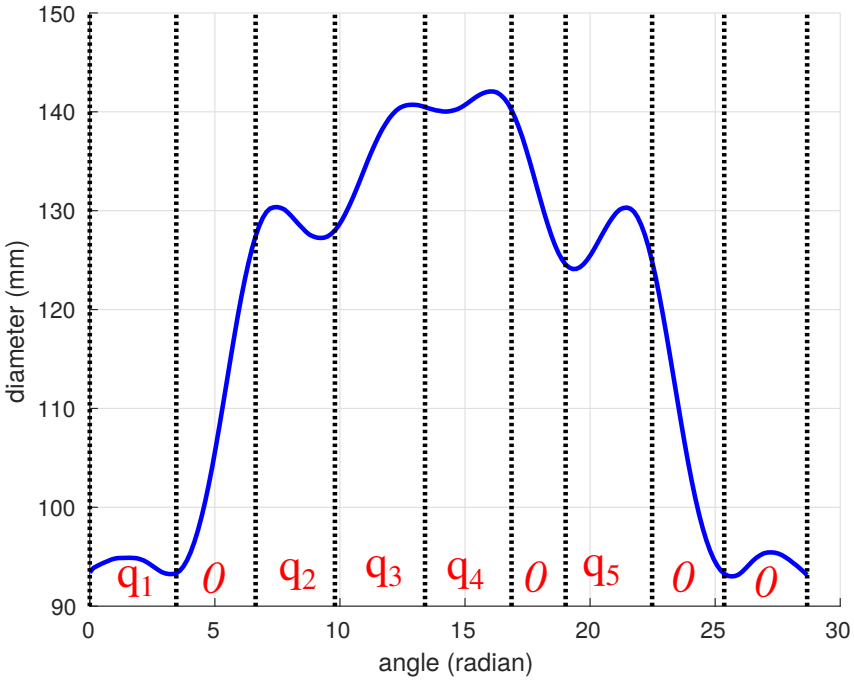
Wt functions over Diameter regions



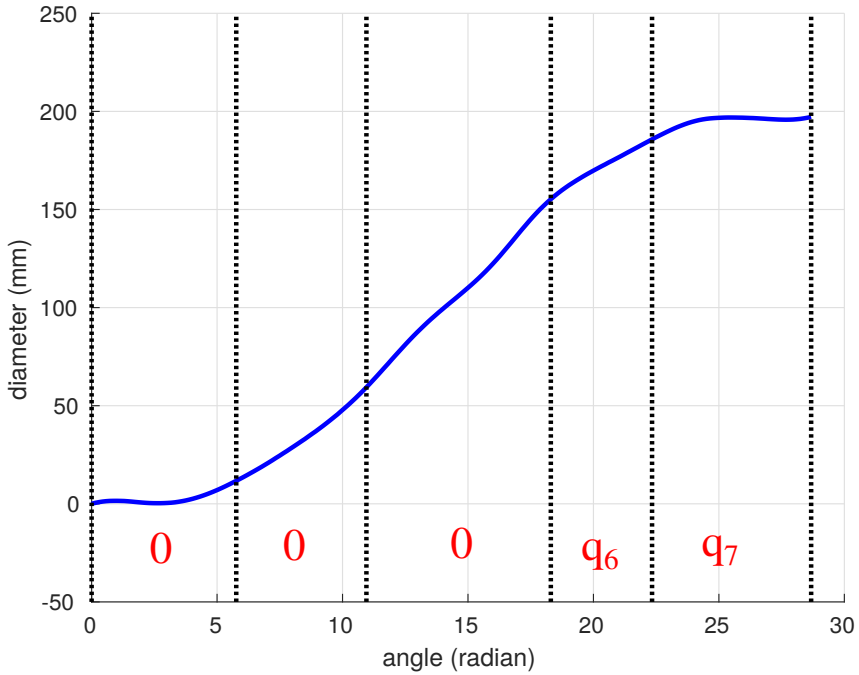
Wt functions over Height regions

Experiment-1: Diameter & Height Variation - 15 simulations

Parameters varied:

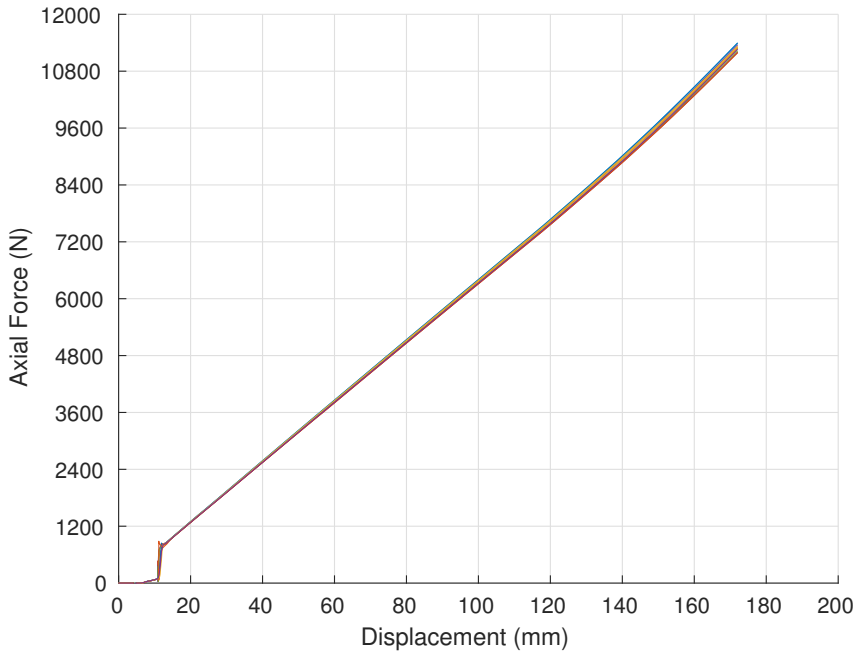


Random variables for diameter regions

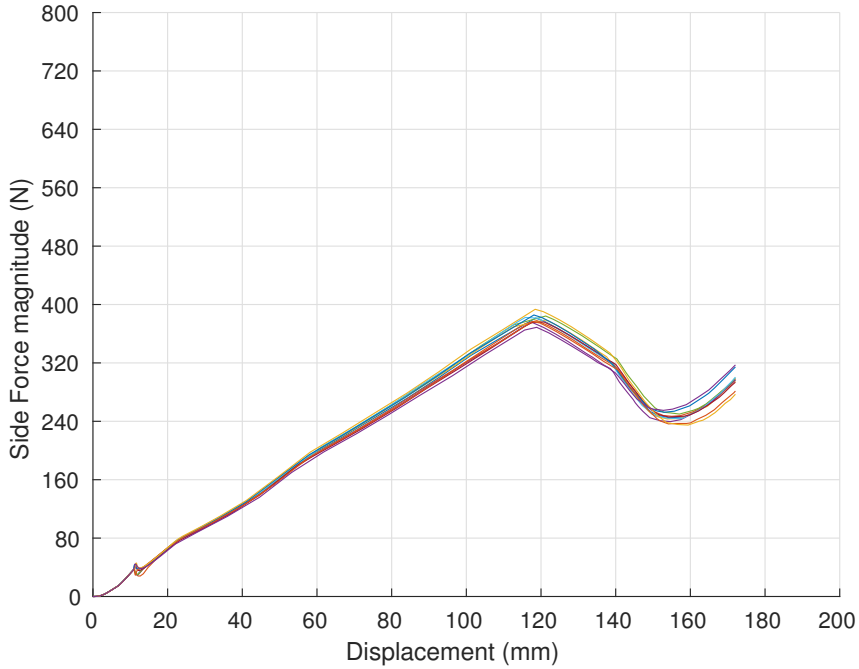


Random variables for height regions

Diameter variation results - no effect

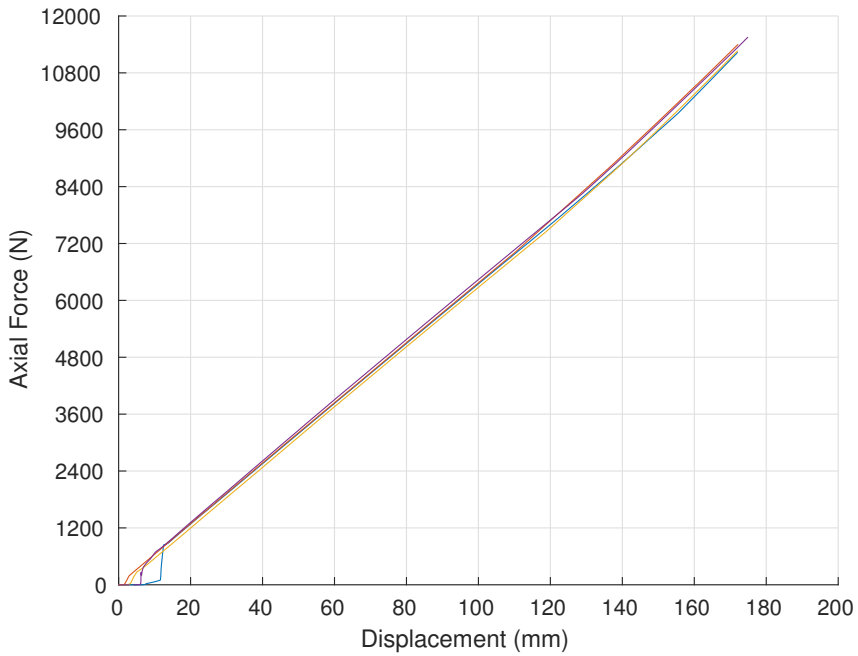


Axial Force

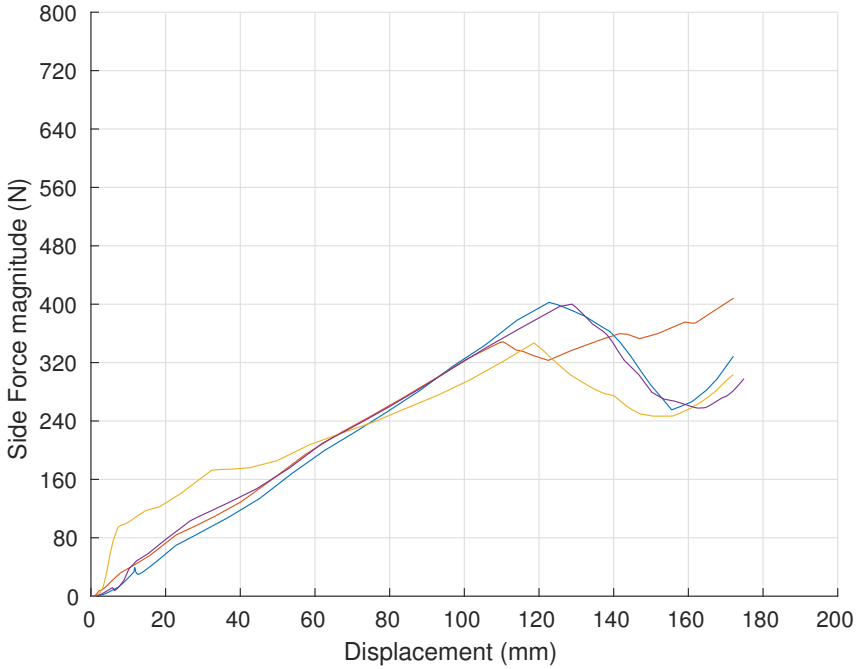


Side Force Magnitude

Height variation results - affects



Axial Force

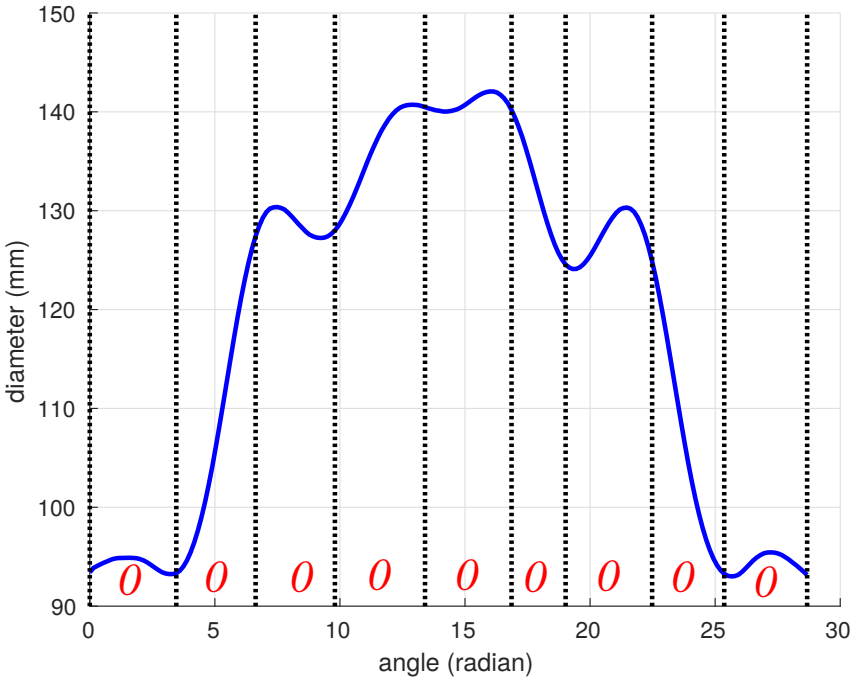


Side Force Magnitude

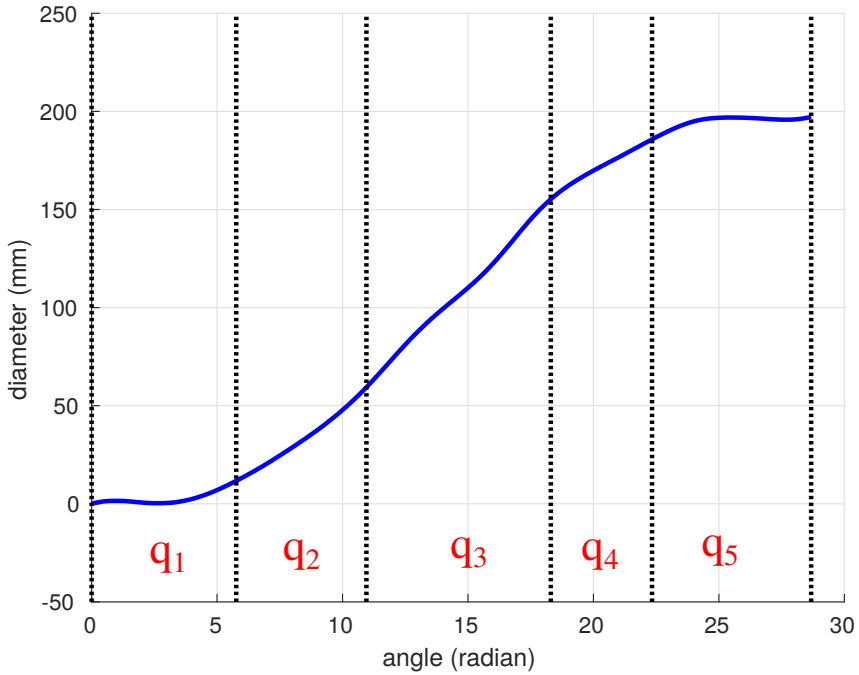
- Height variations: critical
- Diameter variation: no effect

Experiment-2: Height variations only - 156 simulations

Parametes varied



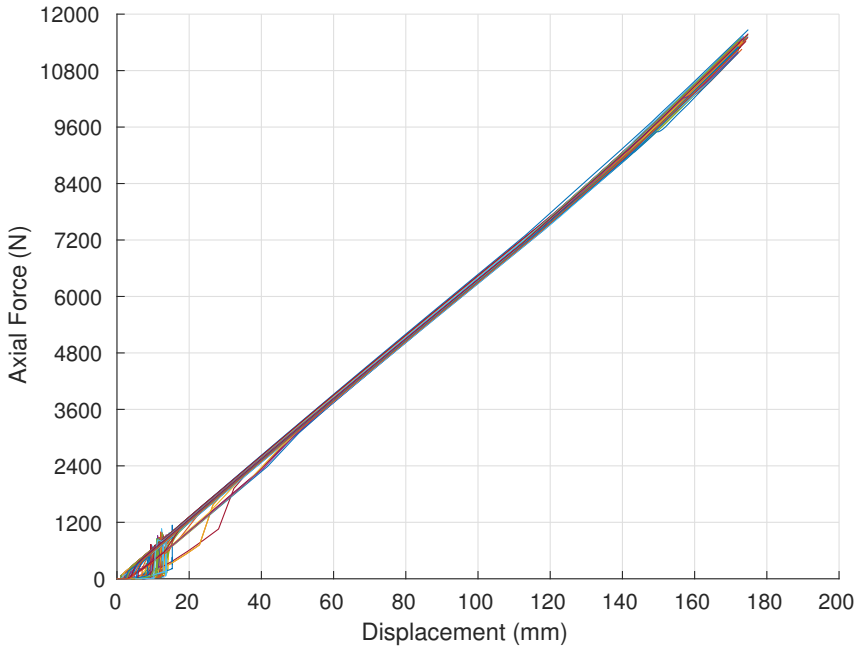
Random variables for diameter regions



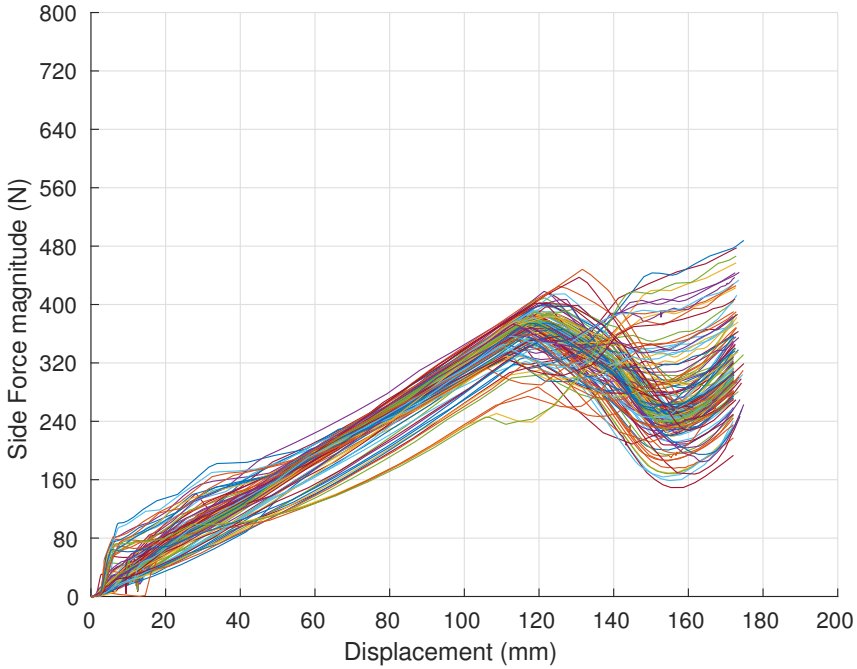
Random variables for height regions

Design of Experiments: Experiment 2

Results



Axial Force



Side Force

- Big spread of side-forces
- How to analyse results? - Sensitivity analysis using L1-Regularized Least Squares

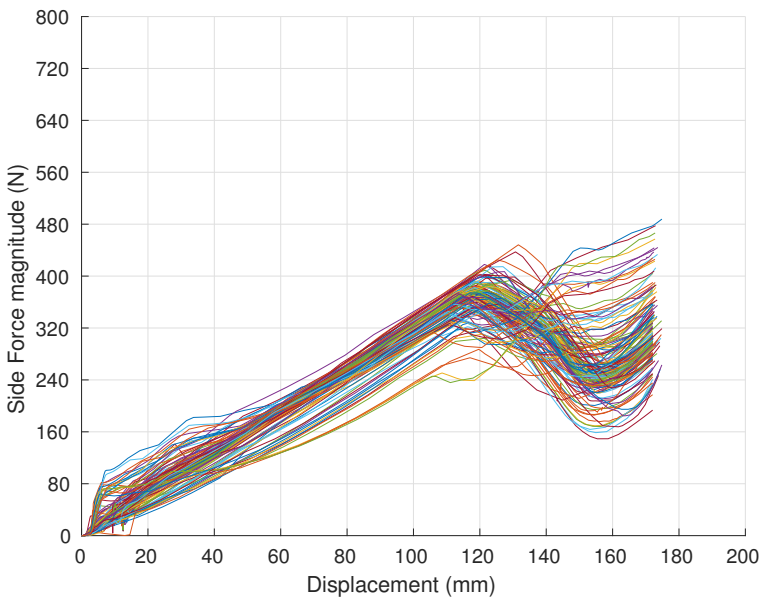
Analyzing results

Assume a linear response model

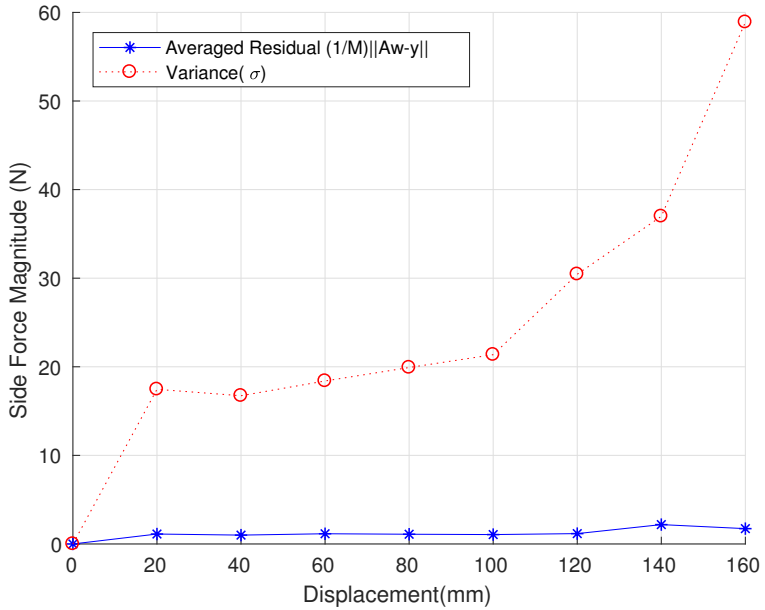
- Displacement fixed - force response y
- Response function - linear function of variations $\boldsymbol{q}$ with weigths $\boldsymbol{w}$
- Solve for weights using 156 simulation data- using Linear Regression

$$\boldsymbol{w}^* = \underset{\boldsymbol{w}}{\operatorname{argmin}} \frac{1}{M} \|A\boldsymbol{w} - \boldsymbol{y}\|_2^2$$

Linear response model- verification



Side Force



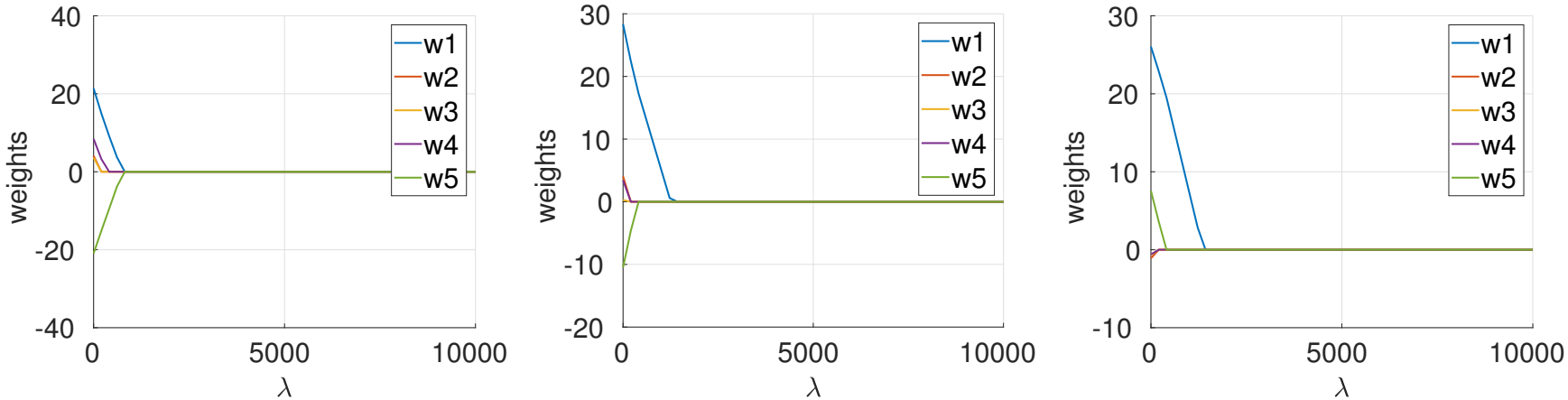
Verification

Analyzing results

Sensitivity Analysis: L1-Regularized Least Squares (LASSO)

- L1 norm promotes sparsity
- λ - acts as control knob
- Sensitivity: parameter which activates first

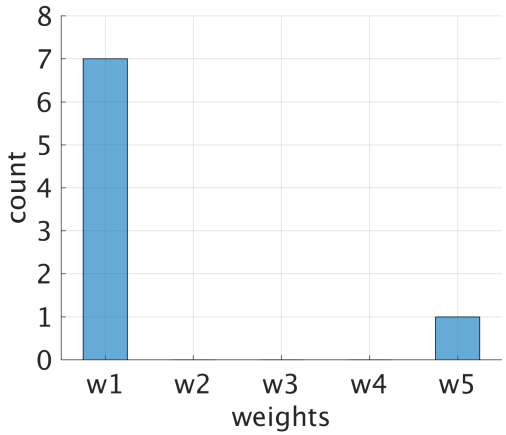
$$\boldsymbol{w}^* = \underset{\boldsymbol{w}}{\operatorname{argmin}} \frac{1}{M} \|A\boldsymbol{w} - \boldsymbol{y}\|_2^2 + \lambda \|\boldsymbol{w}\|_1$$



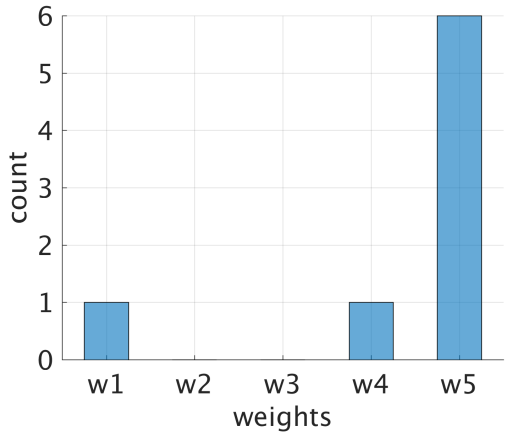
displacement = 20 mm displacement = 40 mm displacement = 60 mm

Figure: Regression results for 2x max variations

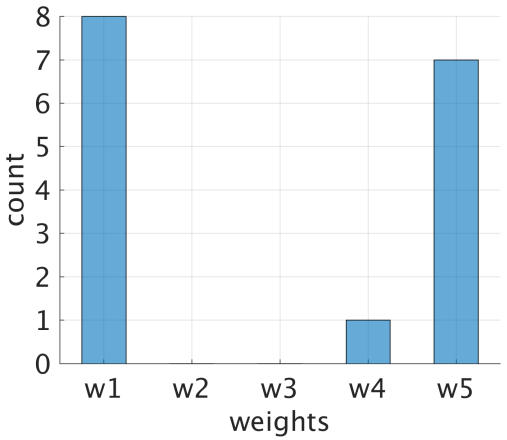
Analyzing results



Top factor



Second factor



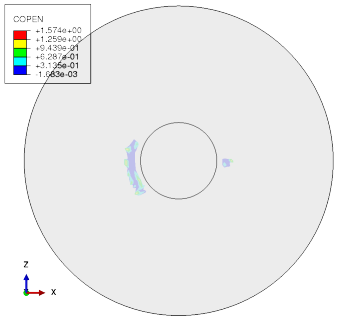
Top two factors

Figure: Histogram for top, second and top two factors for 2x max variations

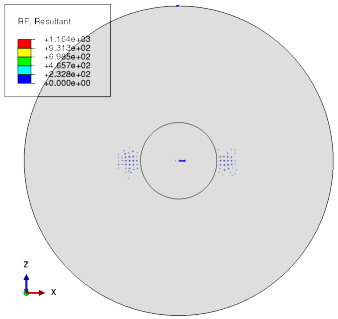
- Height variations at ends: most sensitive
- Large displacement: w_4 also becomes important

Insights

Bottom Plate: Specimen 1

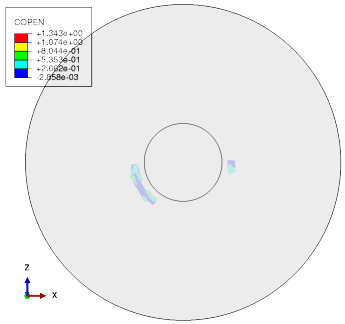


Contact region

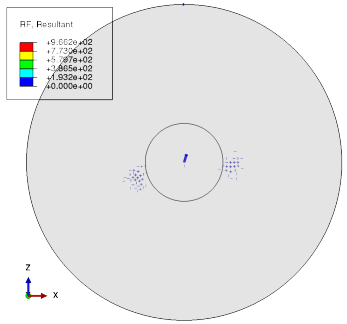


Reaction force

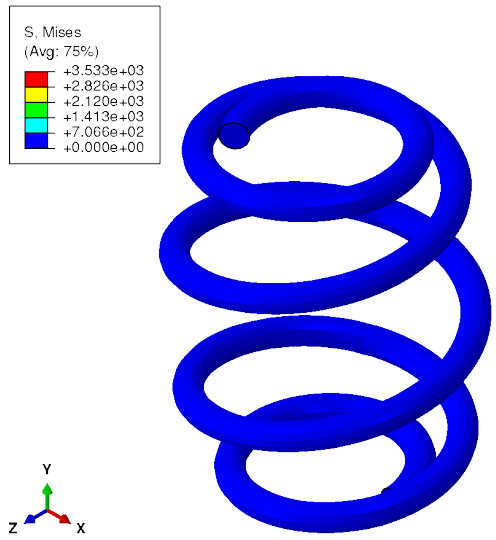
Bottom Plate: Specimen 5



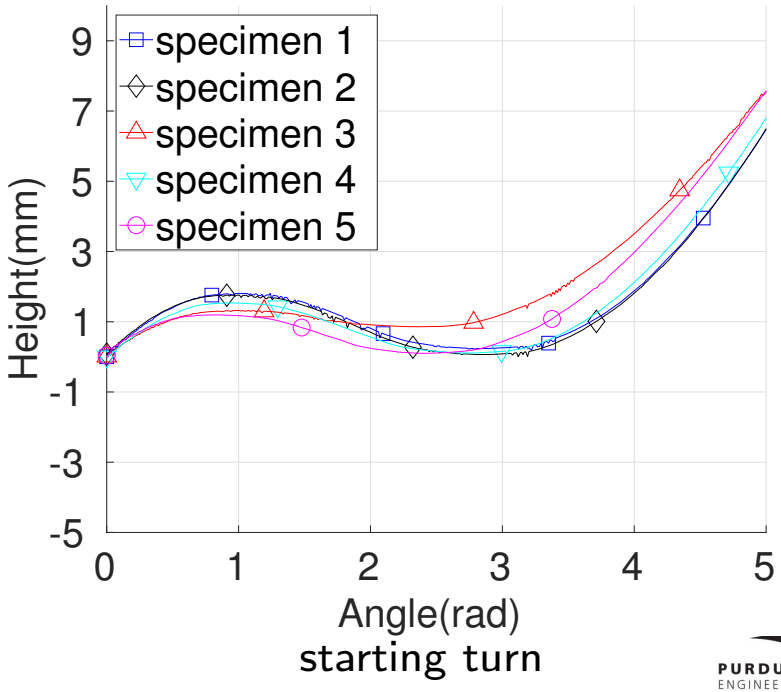
Contact region



Reaction force



Deformation shape of spring



Summary & Conclusions

Summary

- Two models generated
  - High-fidelity Model using Continuum Elements and Contact
  - Reduced-order Model using Beam Elements and Connectors
- Model results:
  - Side-force acts over localized regions
  - Regions change during loading - Non-Linearity
- Parameter variation
  - Placement variation:
    - Translation: no effect
    - Rotation: affects side-force
  - Profile variations:
    - Diameter variations: no effect
    - Height variations: affects side-force
  - Sensitivity analysis
    - Height variation at ends most sensitive to side-force

Methods to reduce side-force

- Tight manufacturing tolerance for height profile
- Place at an optimal rotation angle

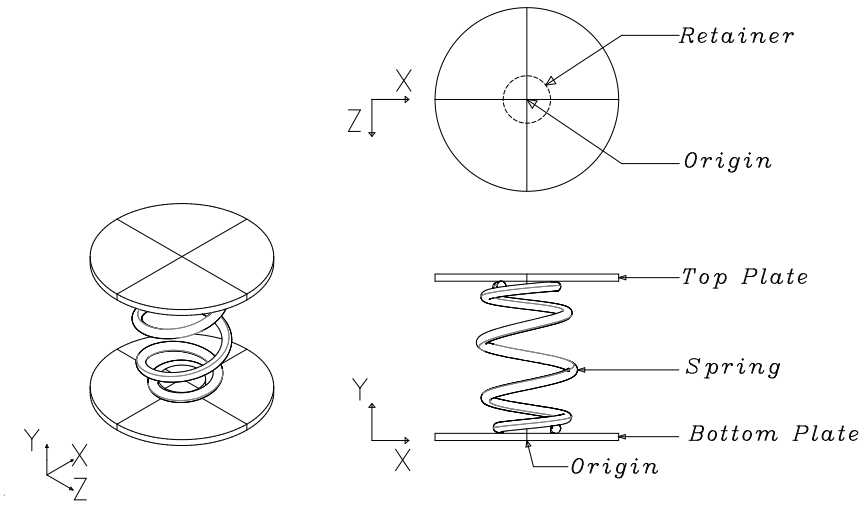
Questions?

backup

Coordinate System & Notation

Coordinate System

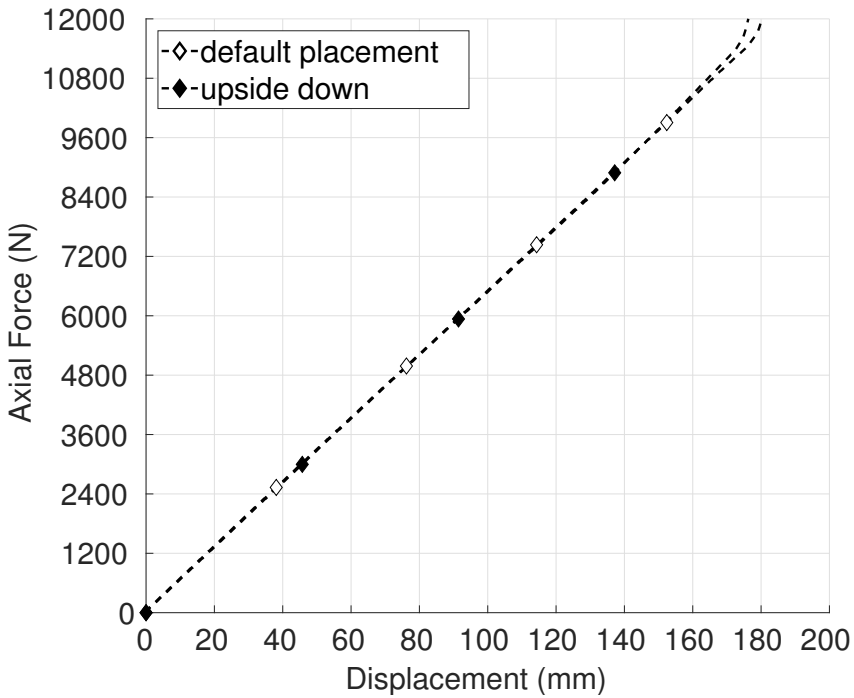
- Origin: Bottom center of bottom plate
- Y axis: Normal from bottom to top plate
- X axis: Starting face in XY plane and outward normal along Z axis



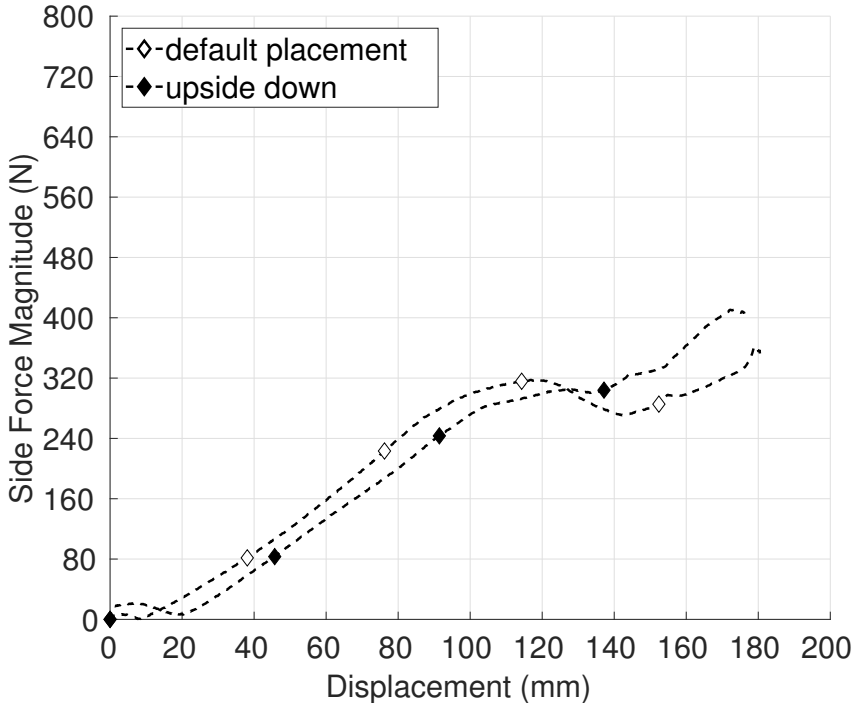
Notation

- Position vector: $\boldsymbol{x} = x_1\boldsymbol{e}_1 + x_2\boldsymbol{e}_2 + x_3\boldsymbol{e}_3$
- Projection tensor (for XZ projection): $\boldsymbol{P} = \boldsymbol{I} - \boldsymbol{e}_2 \otimes \boldsymbol{e}_2$
- Center line of the spring: Collection of N points $\{\boldsymbol{x}_i\}_{i=1}^{N}$

Loading & Testing Procedure: Specimen-2



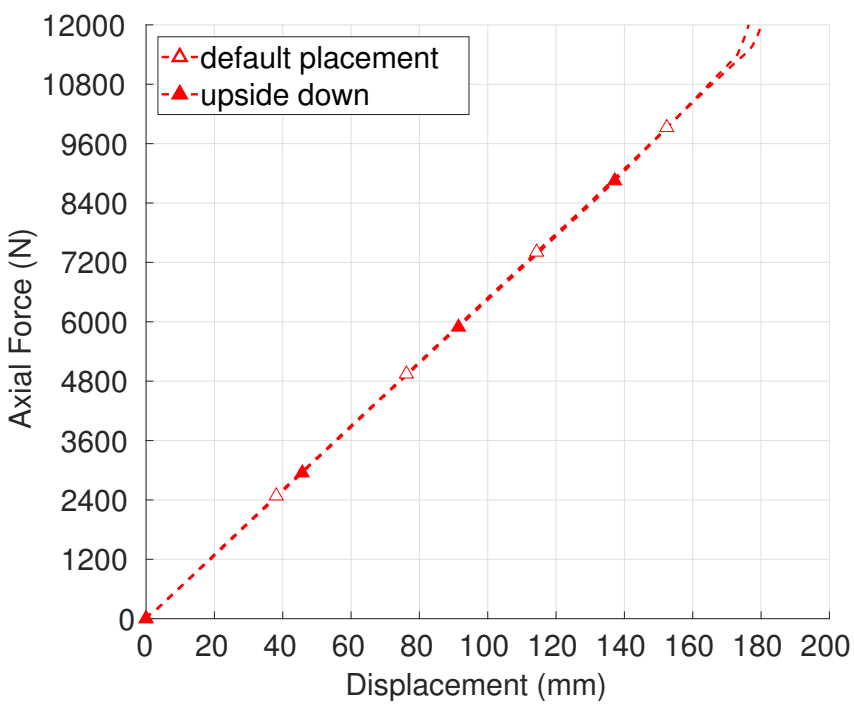
Axial Force



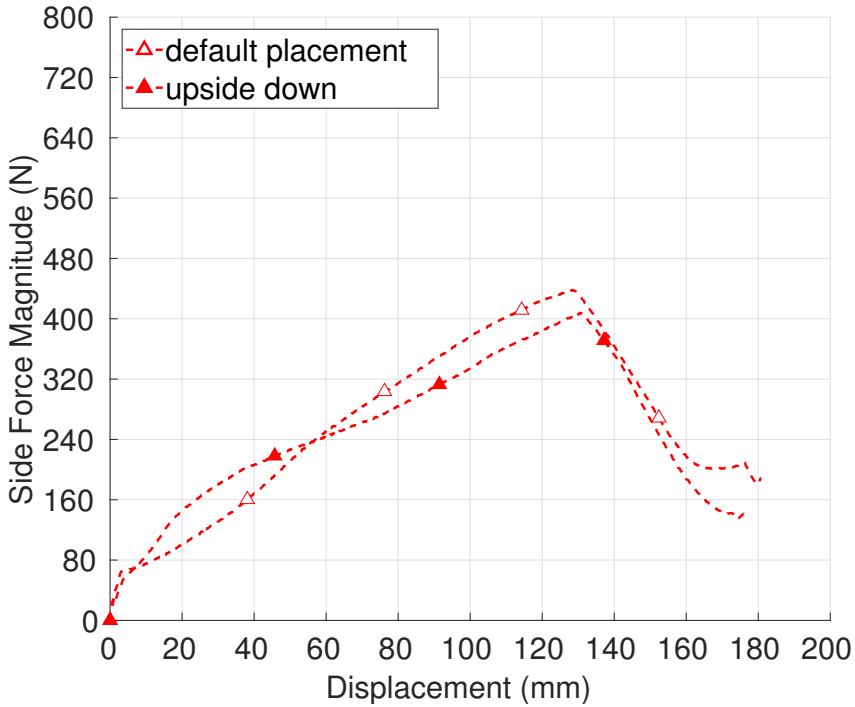
Side Force

Figure: Measured load response for specimen 2

Loading & Testing Procedure: Specimen-3



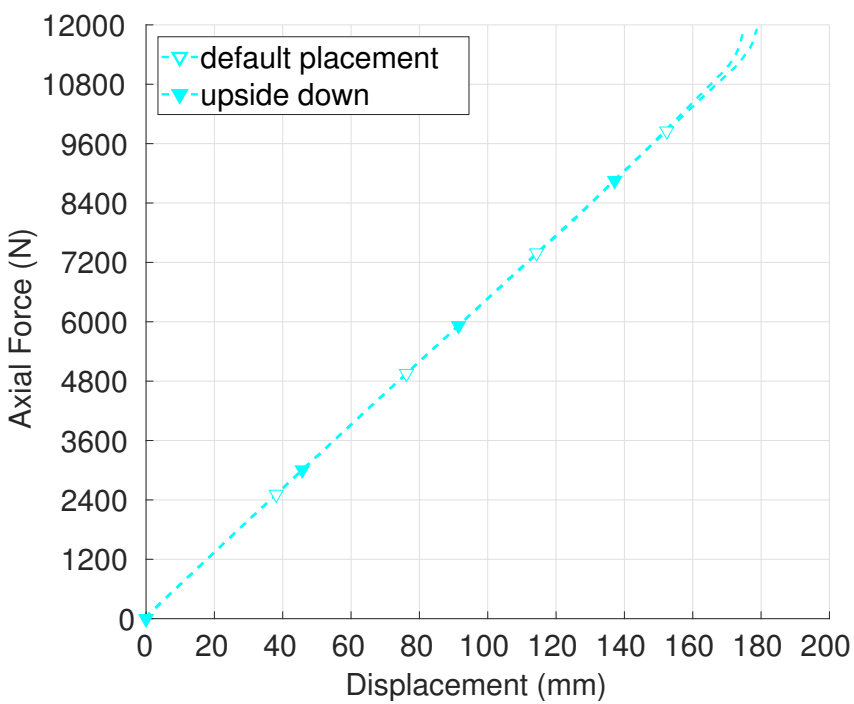
Axial Force



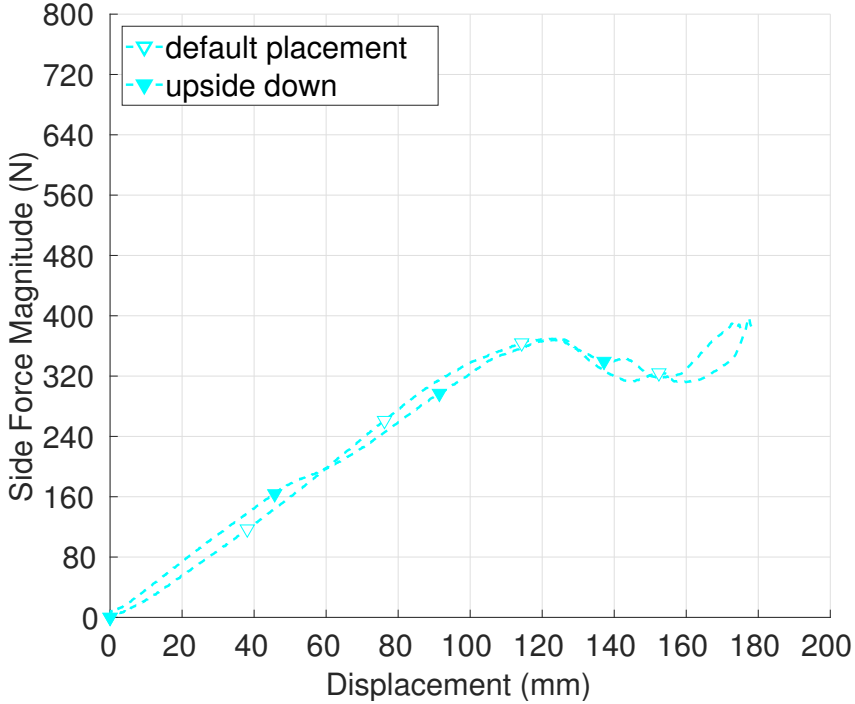
Side Force

Figure: Measured load response for specimen 3

Loading & Testing Procedure: Specimen-4



Axial Force



Side Force

Figure: Measured load response for specimen 4

Mechanical Response of Barrel Springs

- Bending dominated loading

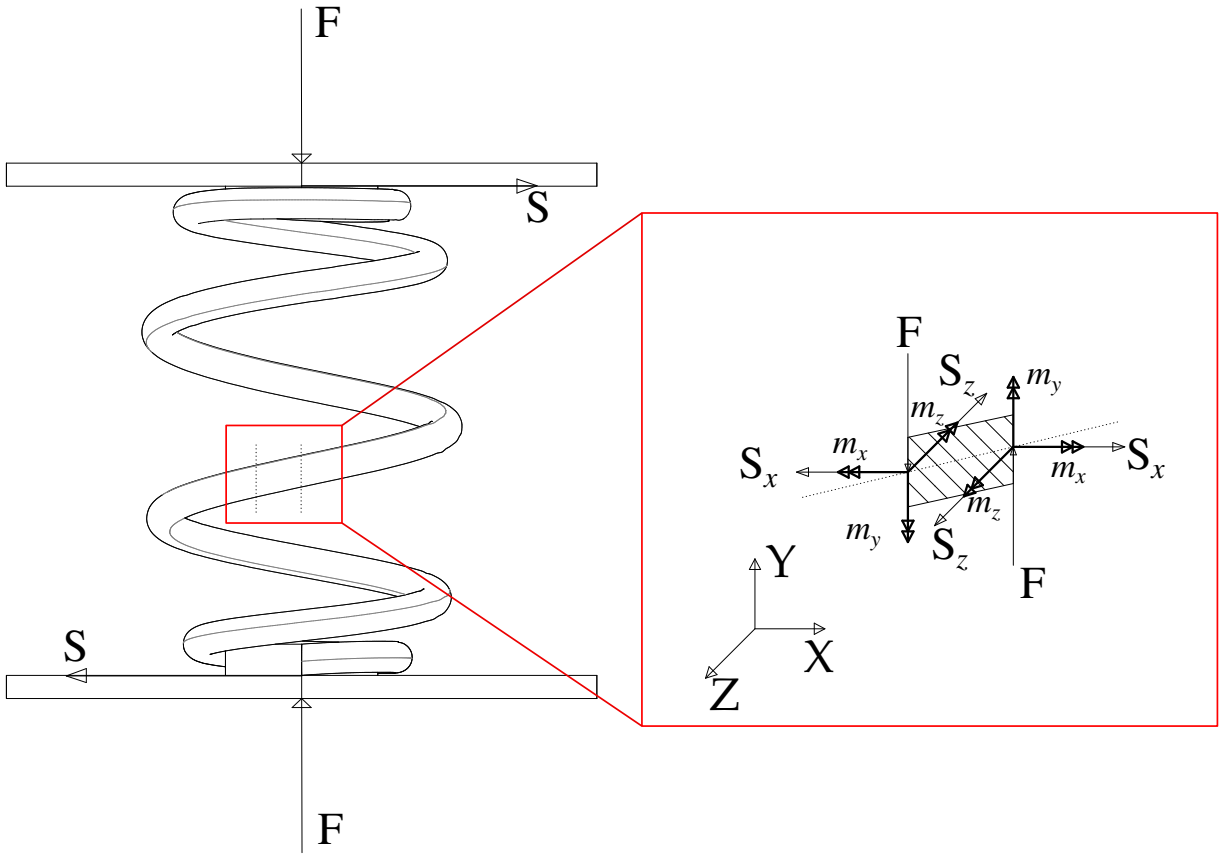


Figure: Free body diagram of an element (in inset) of spring

High-fidelity Model using Continuum Elements and Contact

Advantages

- Exact Physical model without approximations
- Using 3D continuum elements with contact formulation
- Can visualize contact behavior: reason why non-linear behavior os side-force

Disadvantages

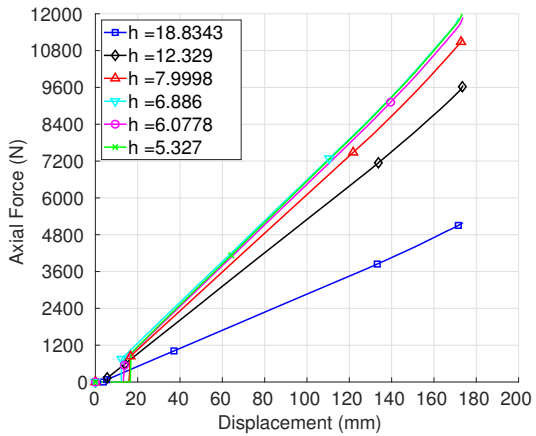
- Contact formulation can require very small time-steps: high computational cost and high memory cost
- Contact problems are sometimes difficult to converge

Model Assumptions

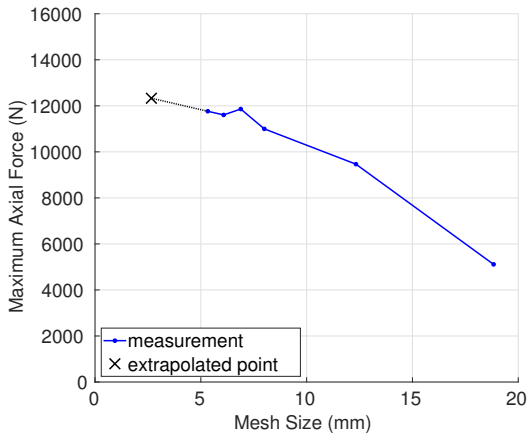
- Cross-section or wire assumed as strictly circular: can be non-circular due to forming process
- Material Properties assumed as constant for the whole volume: differential heating can lead to variation of matrial property
- Residual stresses not modelled: pre-setting results in residual stresses

Model Convergence

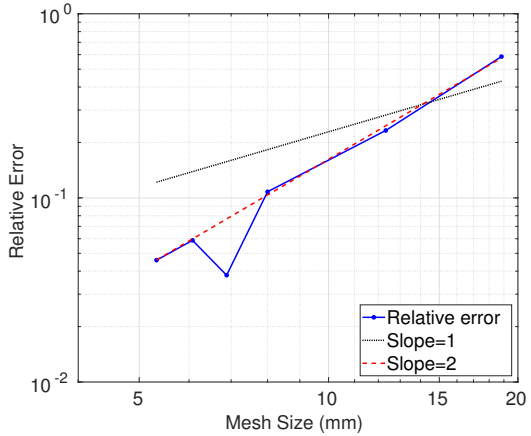
- Relative error of maximum axial force converges with a quadratic rate
- Mesh sizes smaller than and equal to 6.886 have relative errors less than 10%
- Mesh size of 6.886 mm chosen: computationally cheaper than finest mesh



(a) Axial force



(b) plot of max. axial force



(c) Log-Log plot

Figure: Convergence of Solid Model *(a)* Axial response of the model converging to the same line as mesh size decreases *(b)* Measured value of maximum axial force for different mesh sizes along with the extrapolated point (at half the edge length of finest mesh) *(c)* Relative error (using extrapolated point as the assumed true solution) converges quadratically

Analysis & Post-Processing

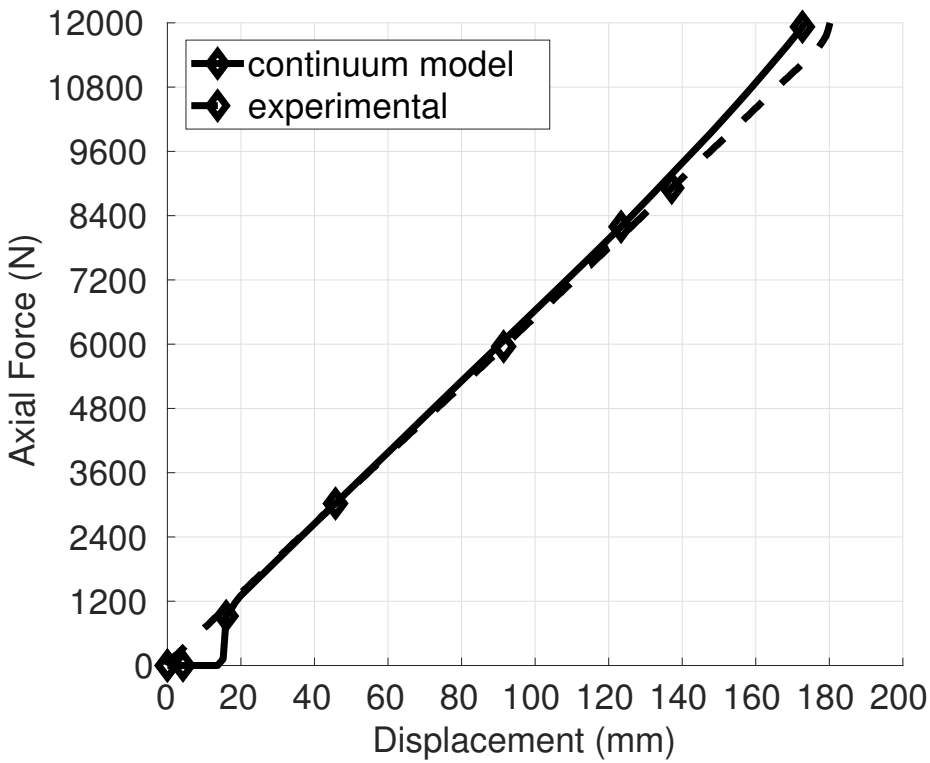
Analysis details

- Static with geometric non-linearity
- Frictional contact modeled with $\mu = 0.7$ (Penalty formulation for contact: induces numerical softness (relatively easier to converge).
- Automatic stabilization with volume proportional damping used: to resolve initial large forces due to contact
  - Viscous forces of the form $\boldsymbol{F}_v = c\boldsymbol{M}^*\boldsymbol{v}$ are added to the global equilibrium equations $\boldsymbol{P} - \boldsymbol{I} - \boldsymbol{F}_v = \boldsymbol{0}$
  - Where $\boldsymbol{M}^*$ is an artificial mass matrix calculated with unit density, c is damping factor.
  - $\boldsymbol{v} = \Delta\boldsymbol{u}/\Delta t$ is the vector of nodal velocities, Δt is the increment of time, $\boldsymbol{P}$ is the external load and $\boldsymbol{I}$ is the internal forces developed due to loading.

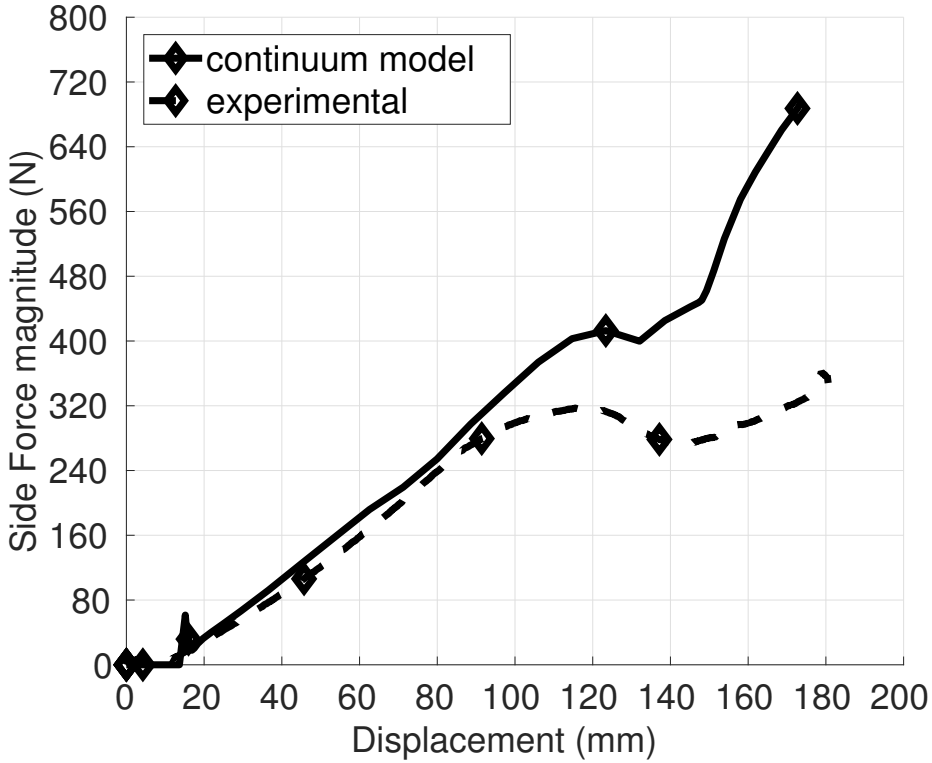
Post-Processing to extract:

- Side-force: Vector sum of reaction force at the bottom center and bottom periphery node.
- Displacement: displacement of the top center node.
- Axial-force: Sum of axial force (along Y direction) for all the nodes on the bottom face of the bottom plate.

Model Results



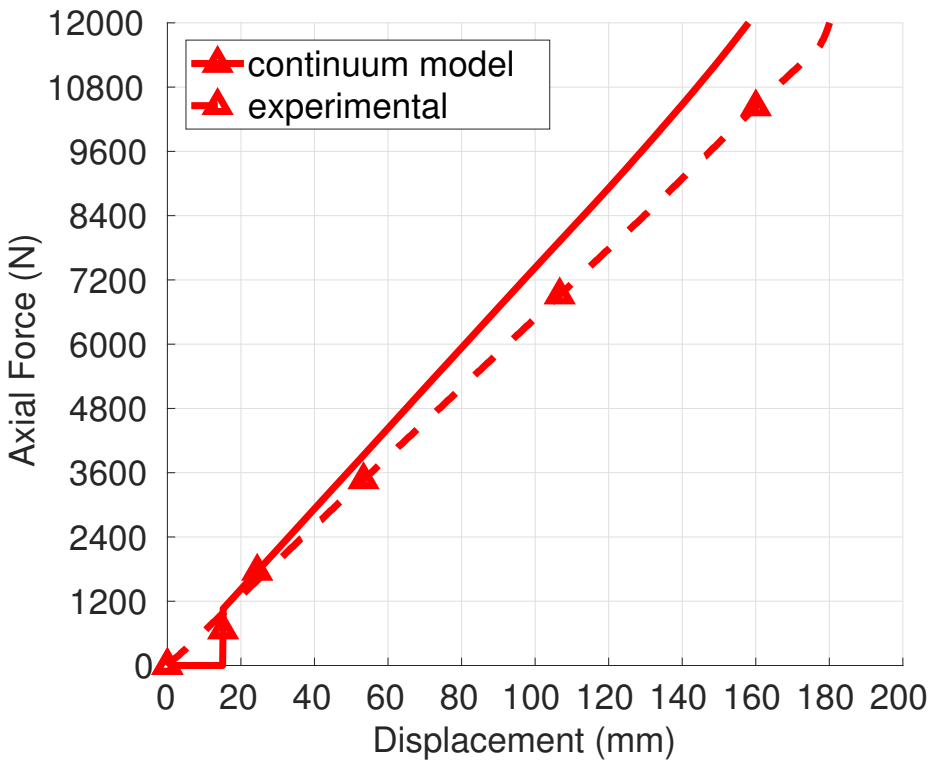
(a) Axial force



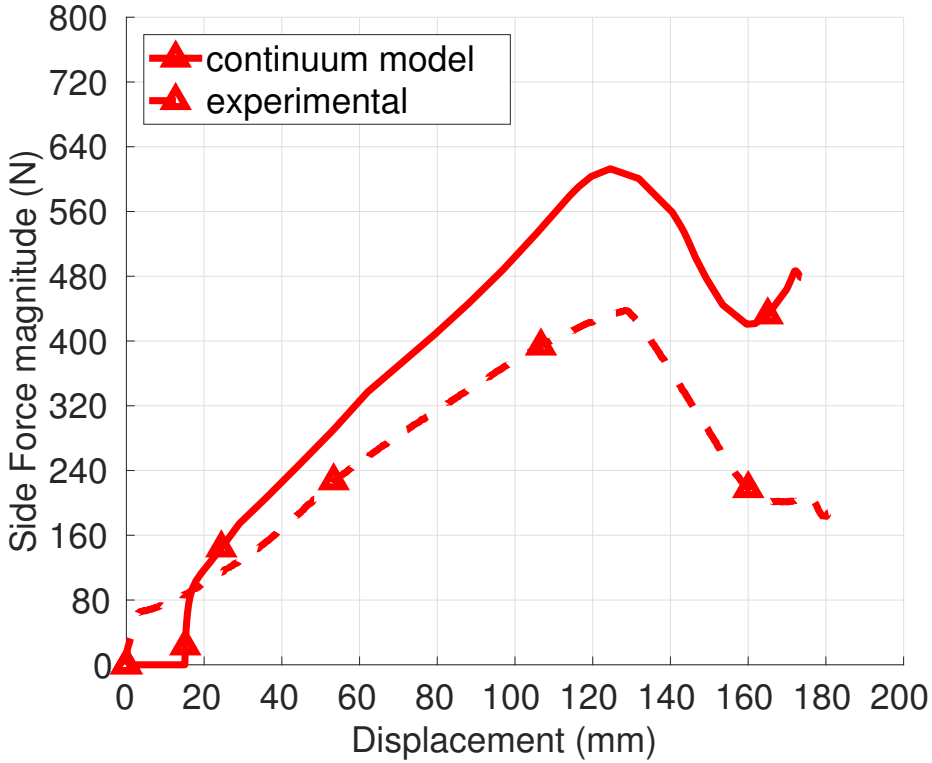
(b) Side force

Figure: Results for Continuum Model: Specimen-2

Model Results



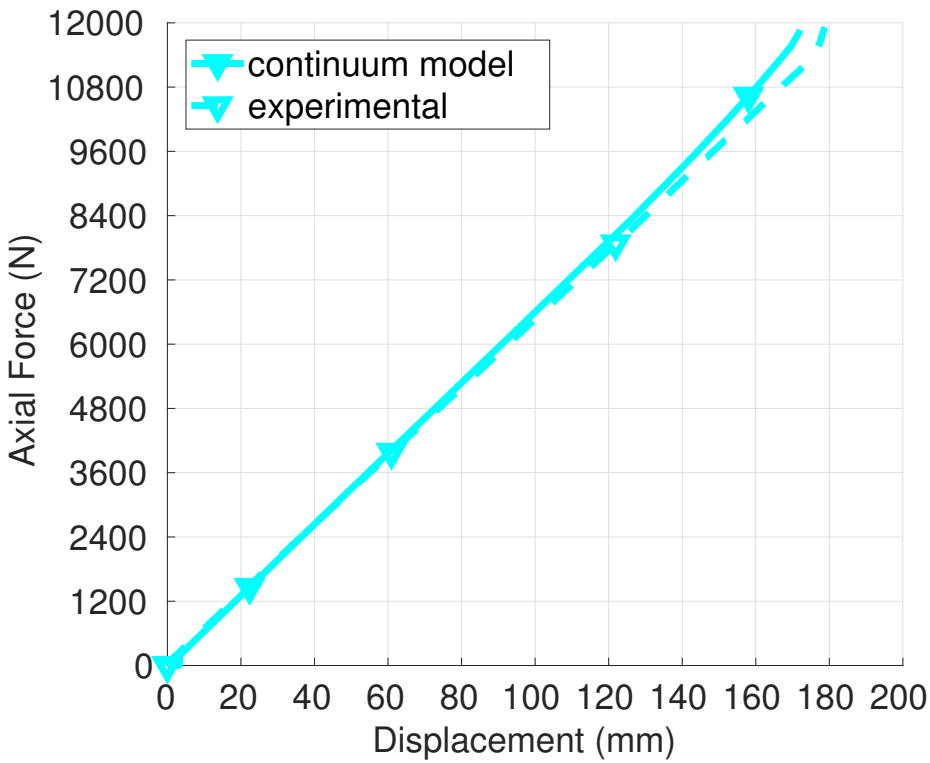
(a) Axial force



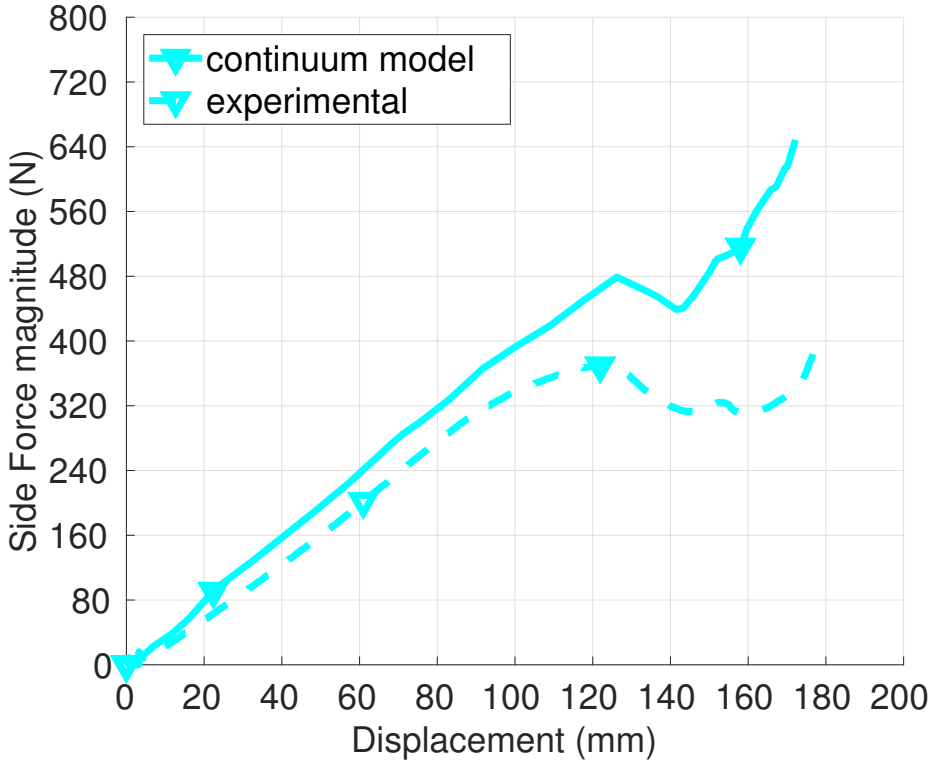
(b) Side force

Figure: Results for Continuum Model: Specimen-3

Model Results



(a) Axial force



(b) Side force

Figure: Results for Continuum Model: Specimen-4

Reduced-order Model using Beam Elements and Connectors

Advantages

- Computationaly cheaper than Continuum Model
- Use Axial and Radial coonectors with non-linear spring behavior to mimic contact and friction
- Use of beam elements to model the spring reduces the total number of nodes

Disadvantages

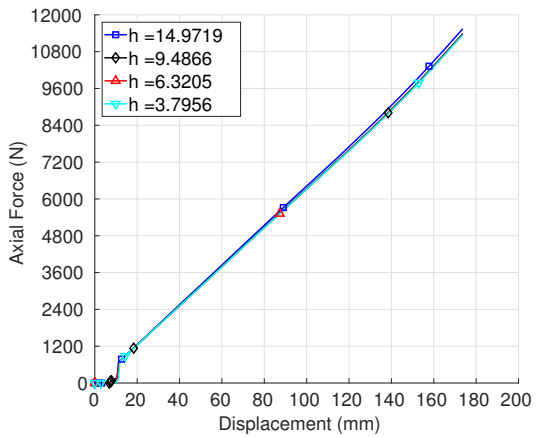
- Contact not simulated: cannot visualize contact regions and forces
- Approximate model: does not model the "exact" physics

Model Assumptions (In addition to assumptions stated in continuum model)

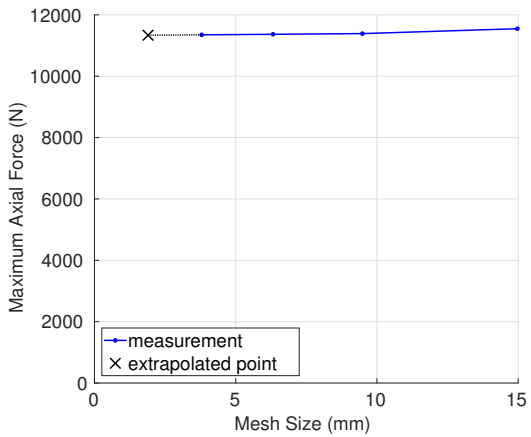
- Self-contact between the beams is not modeled
- Only the first and last full turn of the spring is assumed to contribute to the side-forces

Model Convergence

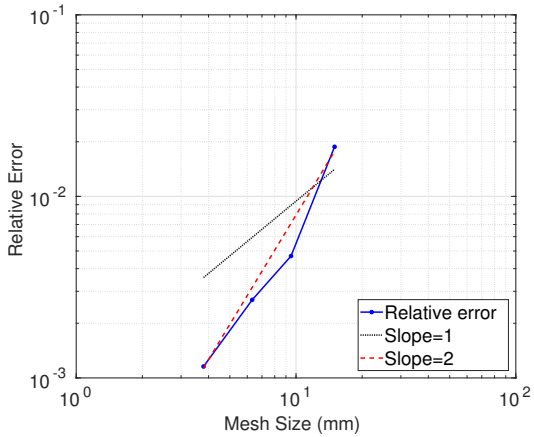
- Relative error of maximum axial force converges with a quadratic rate
- Mesh sizes smaller than 14.97 mm have relative errors less than 1%
- Mesh size of 9.48mm chosen (200 nodes): computationally cheaper than finest mesh and has a relative error of 0.5%



(a) axial force



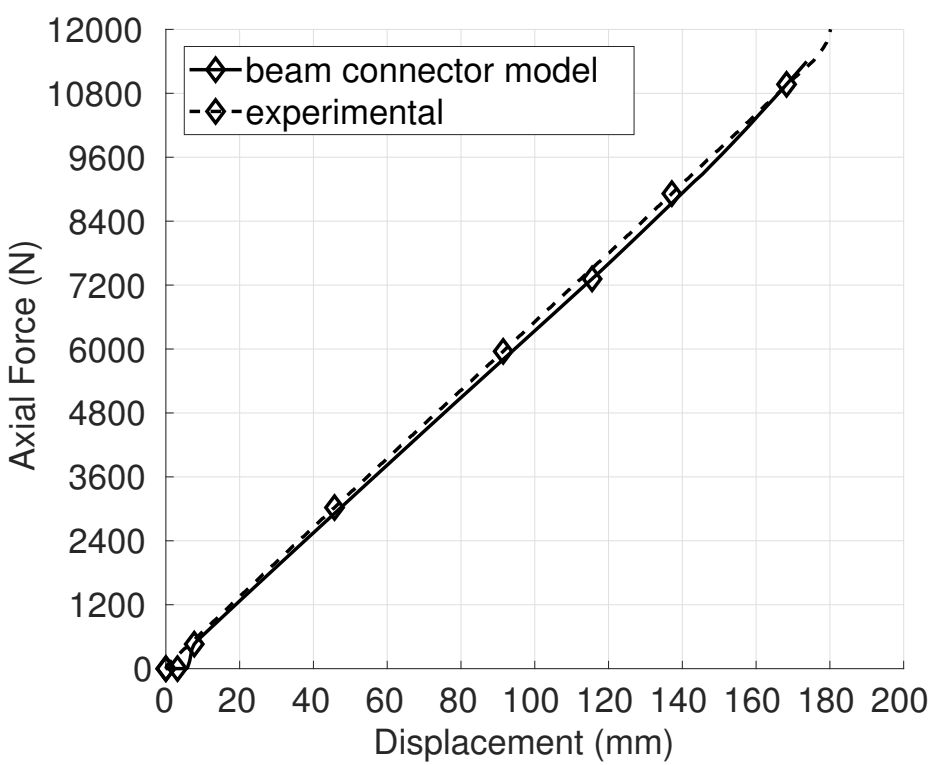
(b) convergence plot



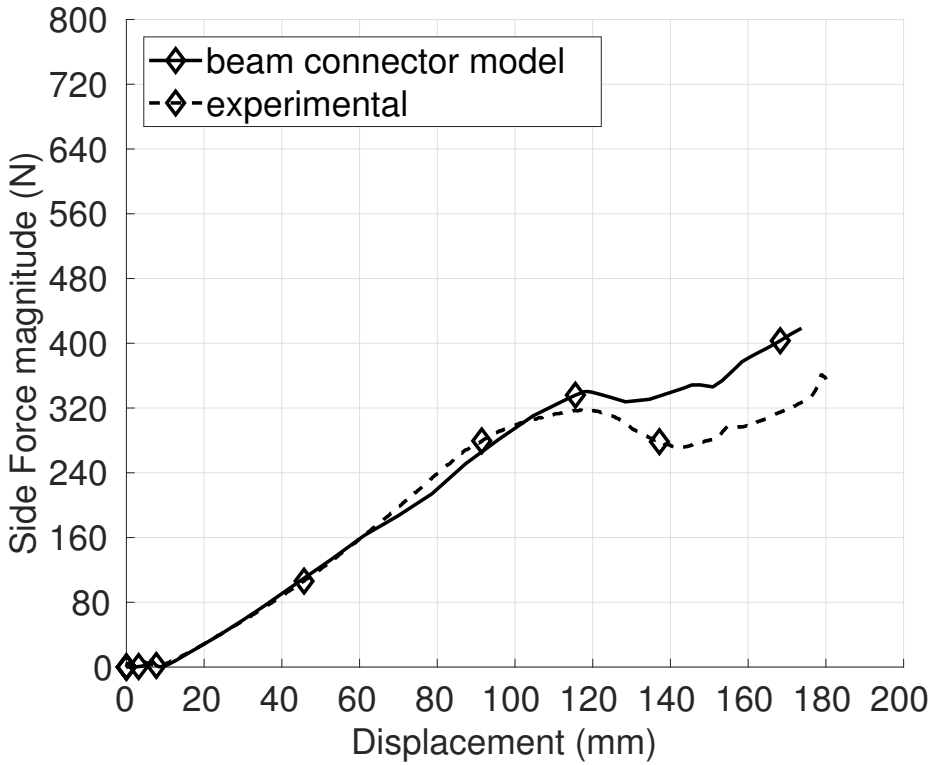
(c) Log-Log plot

Figure: Convergence of Beam Model *(a)* Axial response converges to the same sloped line *(b)* Maximum axial force for different mesh sizes along with the extrapolated point (at half the edge length of finest mesh) *(c)* Relative error (using extrapolated point as the assumed true solution) converges quadratically

Model Results



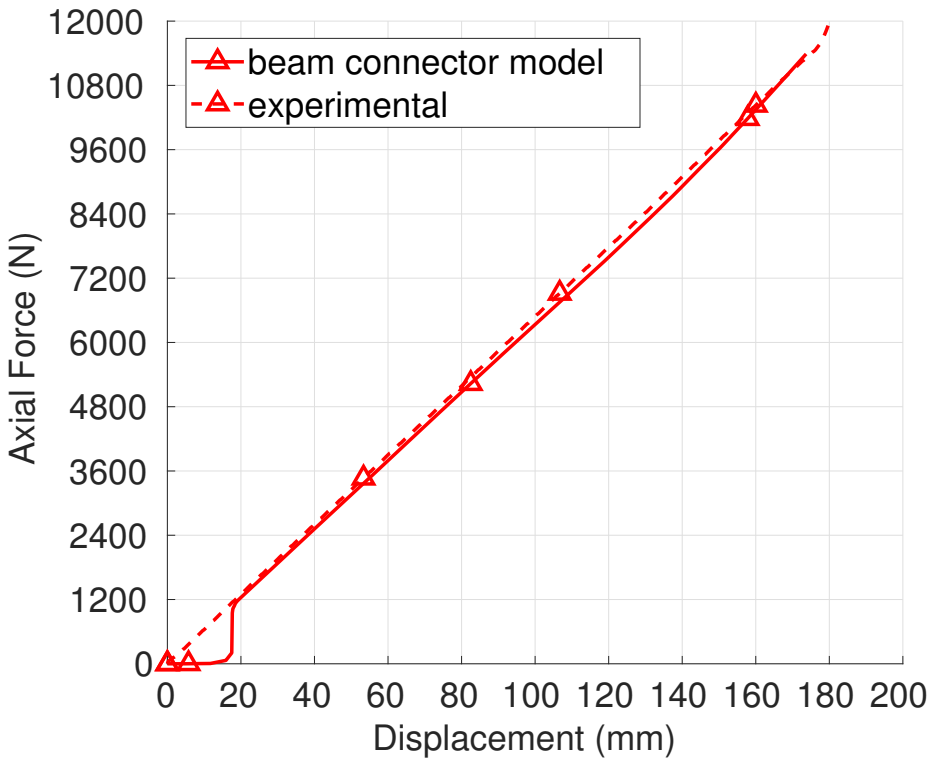
Axial force



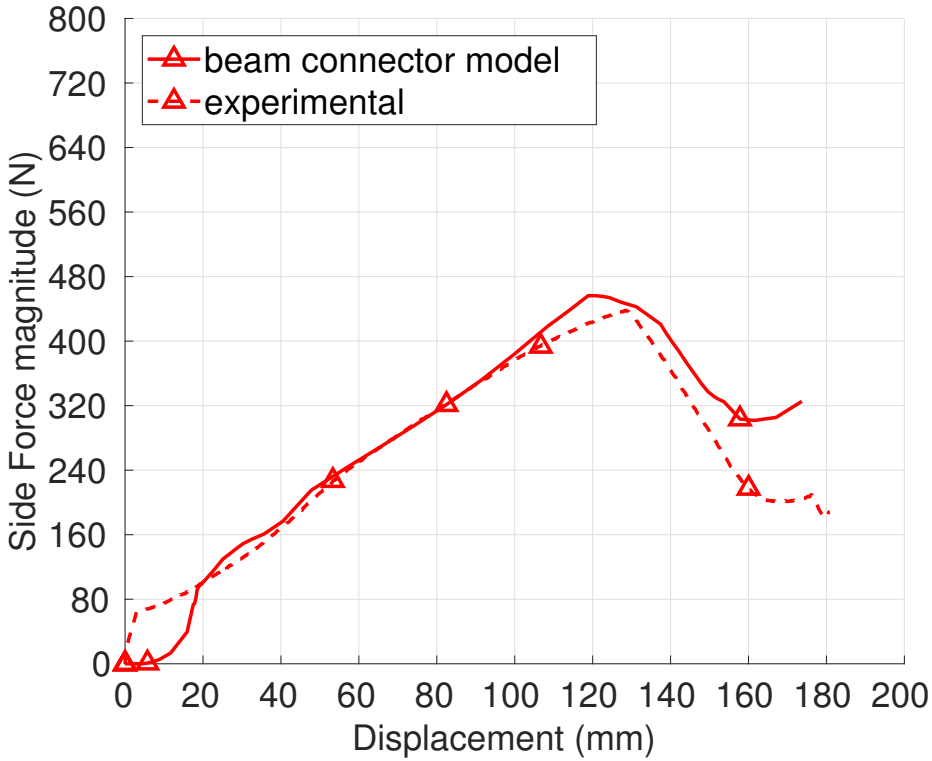
Side force

Figure: Results for Continuum Model: Specimen-2

Model Results



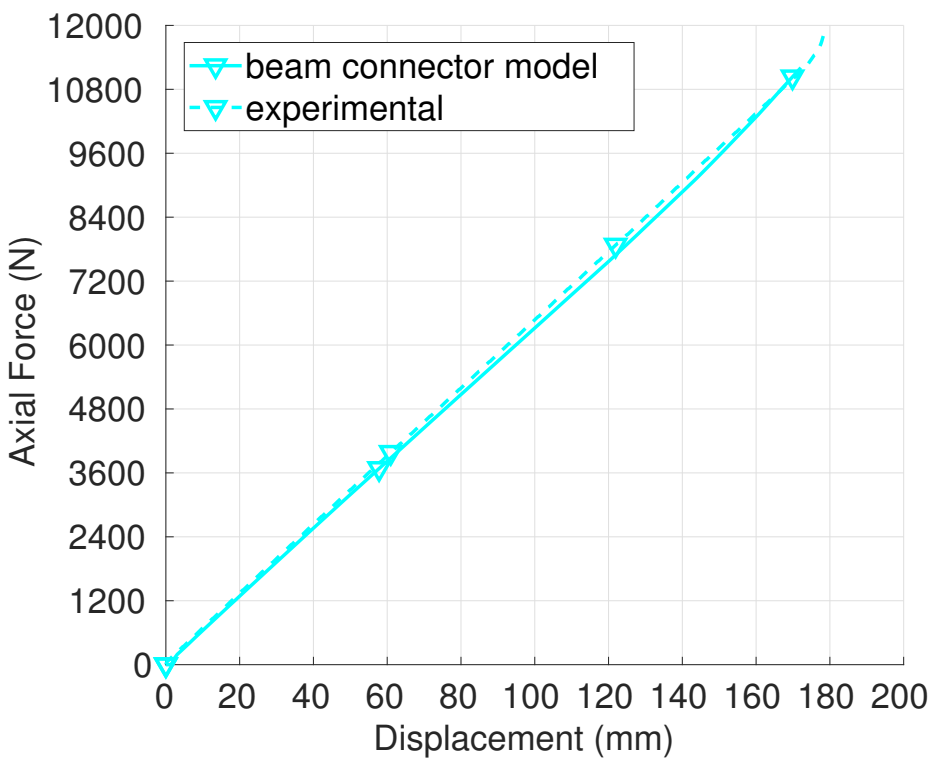
Axial force



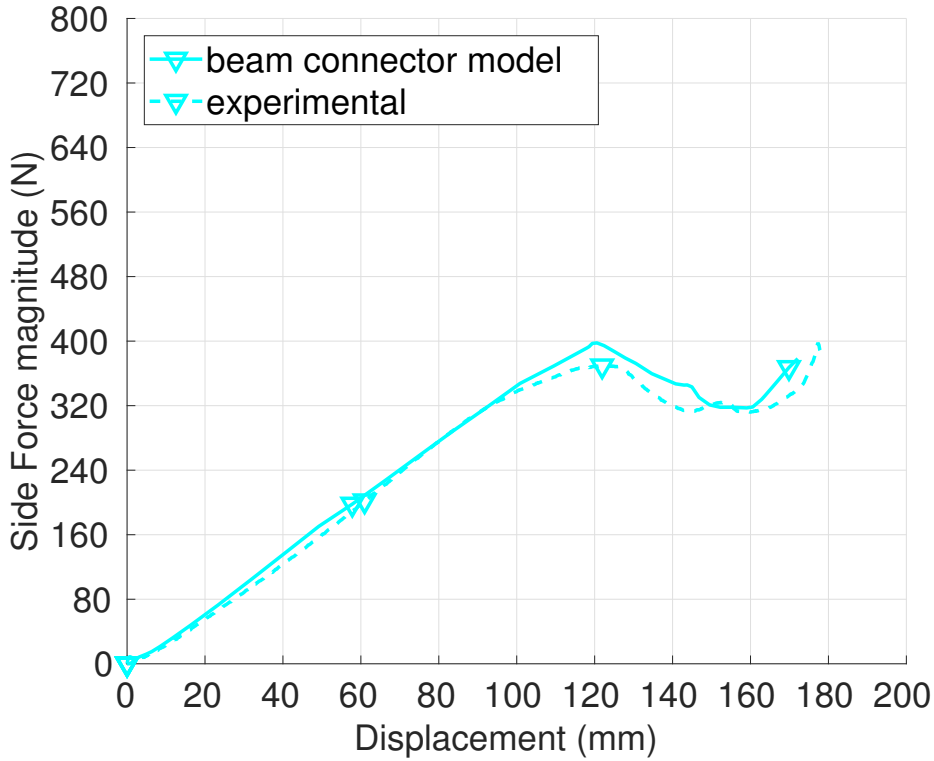
Side force

Figure: Results for Continuum Model: Specimen-3

Model Results



Axial force



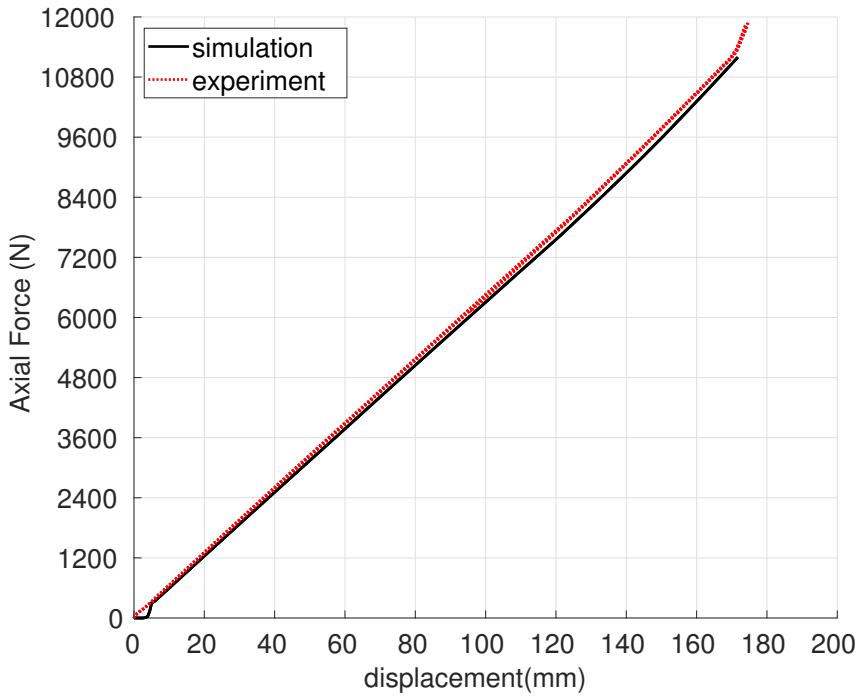
Side force

Figure: Results for Continuum Model: Specimen-4

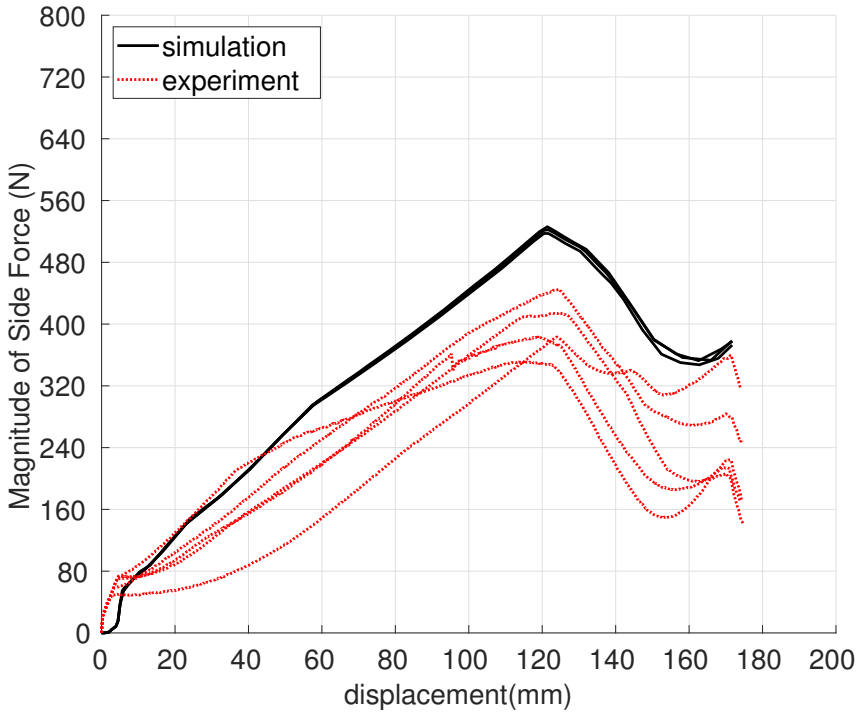
Design of Experiments: Experiment 1

Experiment 1: Rotation & Translation (x & z) - 81 simulations

Translation (x) - unaffected



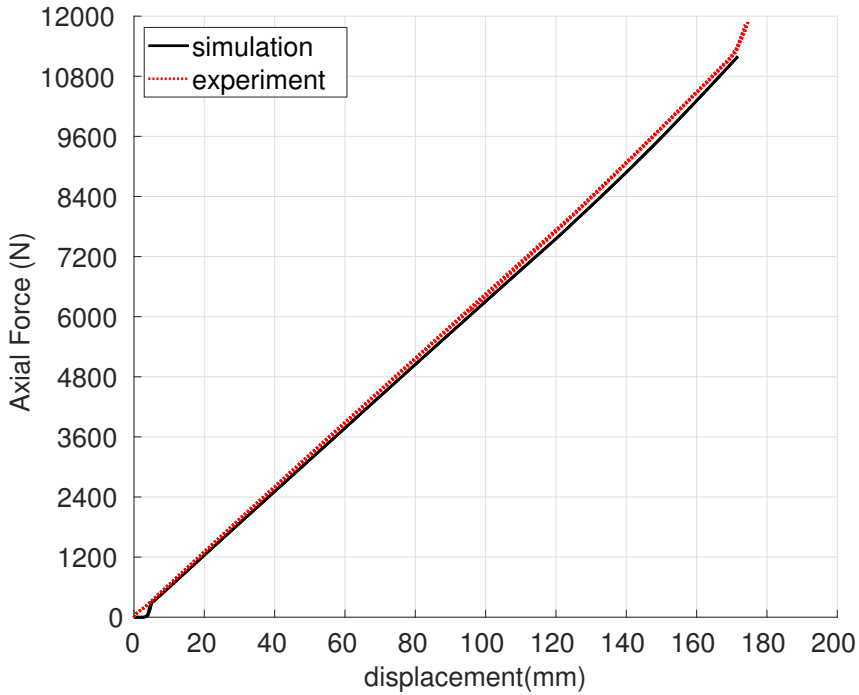
axial force



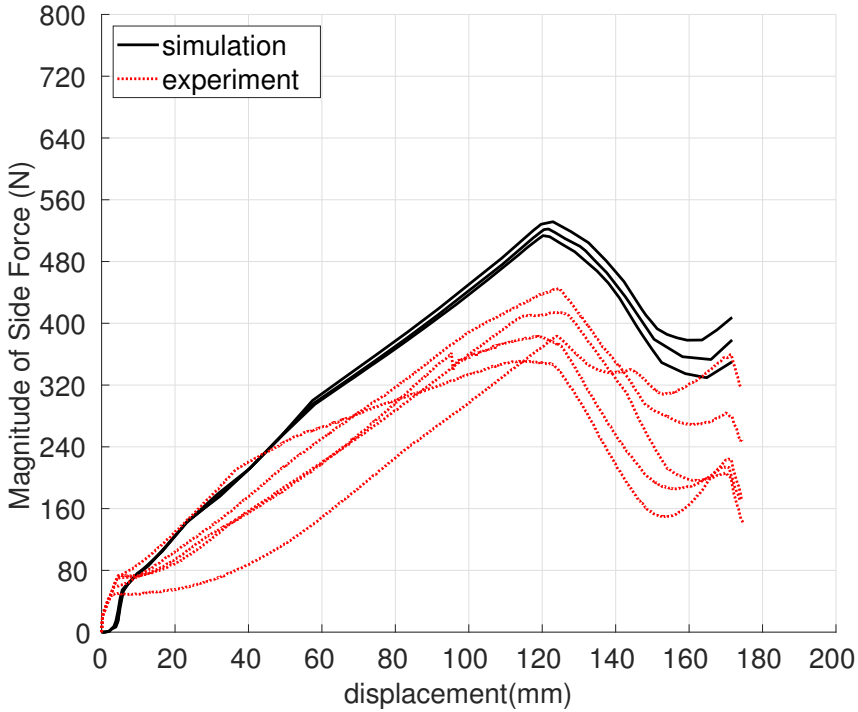
side force

Design of Experiments: Experiment 1

Translation (z) - unaffected



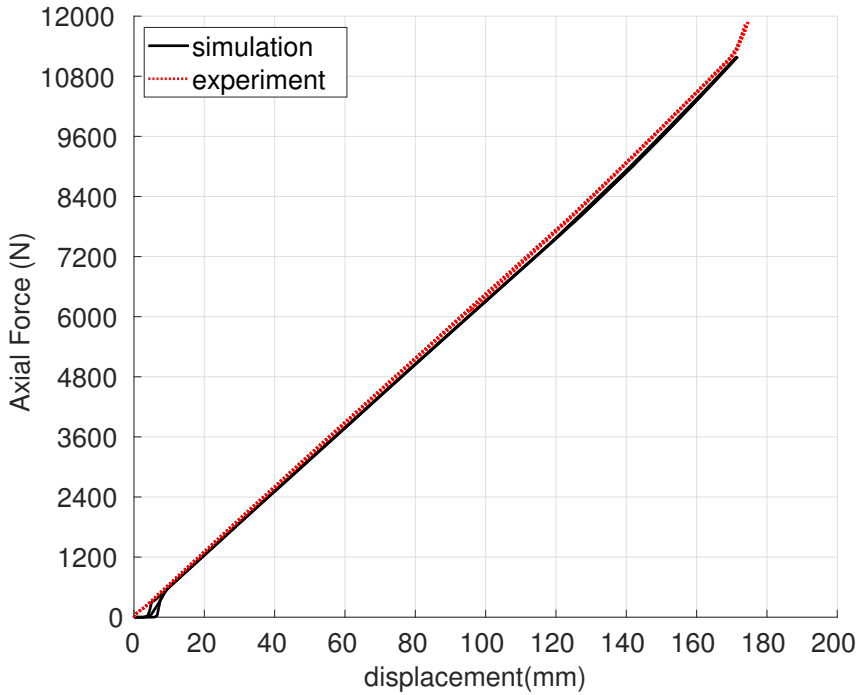
axial force



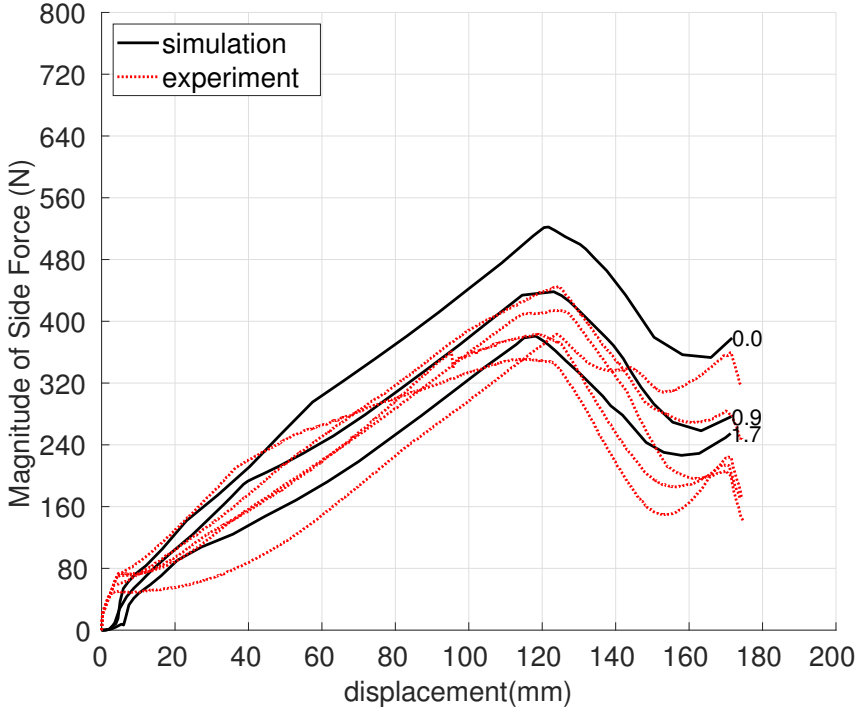
side force

Design of Experiments: Experiment 1

Rotation (x) - gets affected



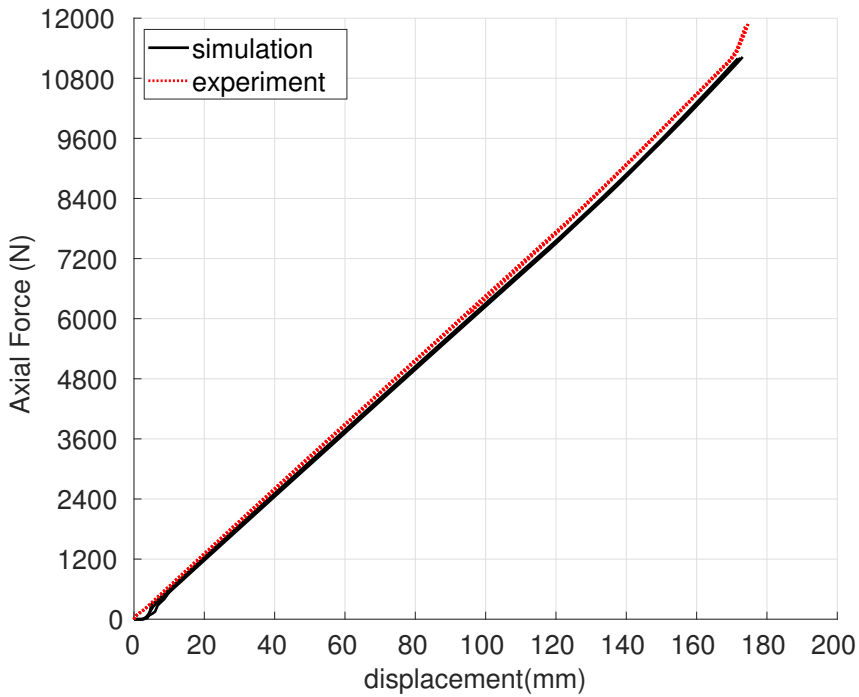
axial force



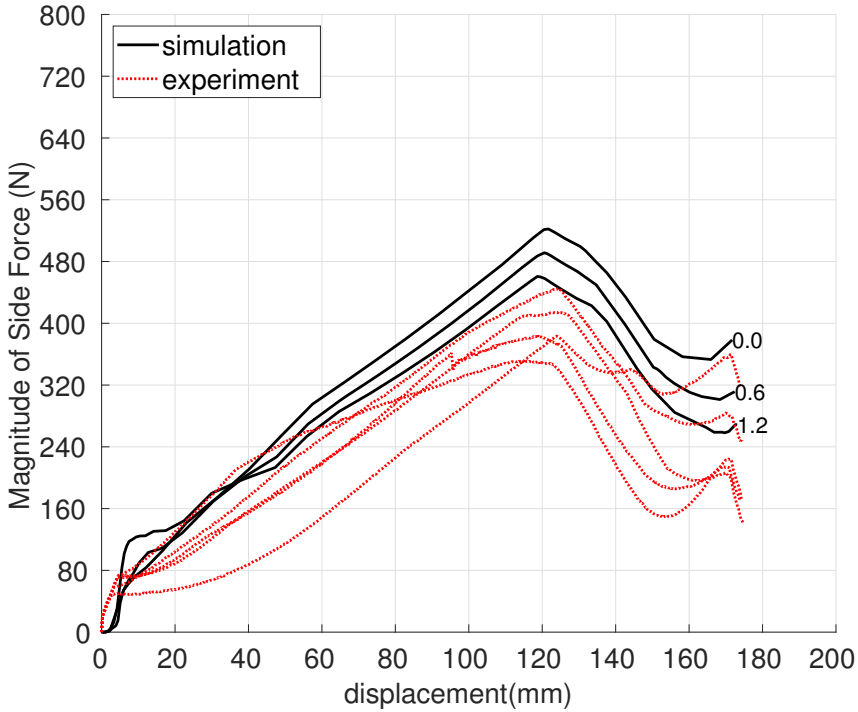
side force

Design of Experiments: Experiment 1

Rotation (z) - gets affected



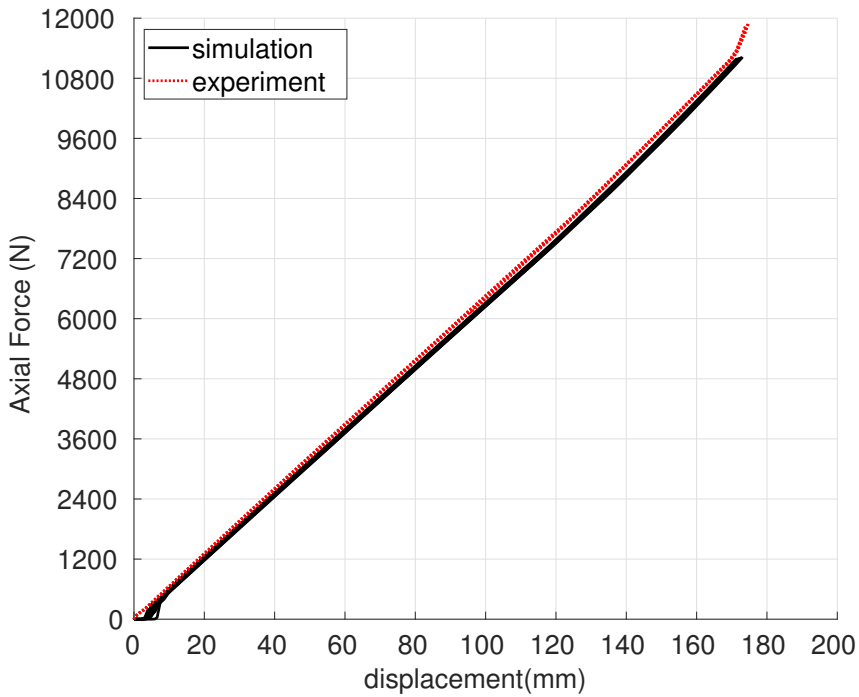
axial force



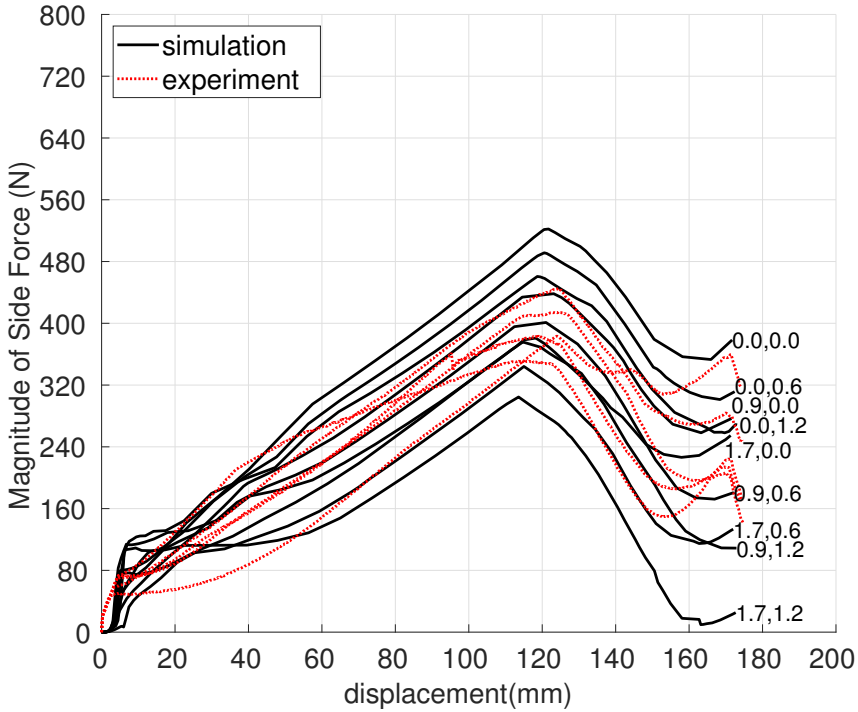
side force

Design of Experiments: Experiment 1

Rotation (x & z) - gets affected



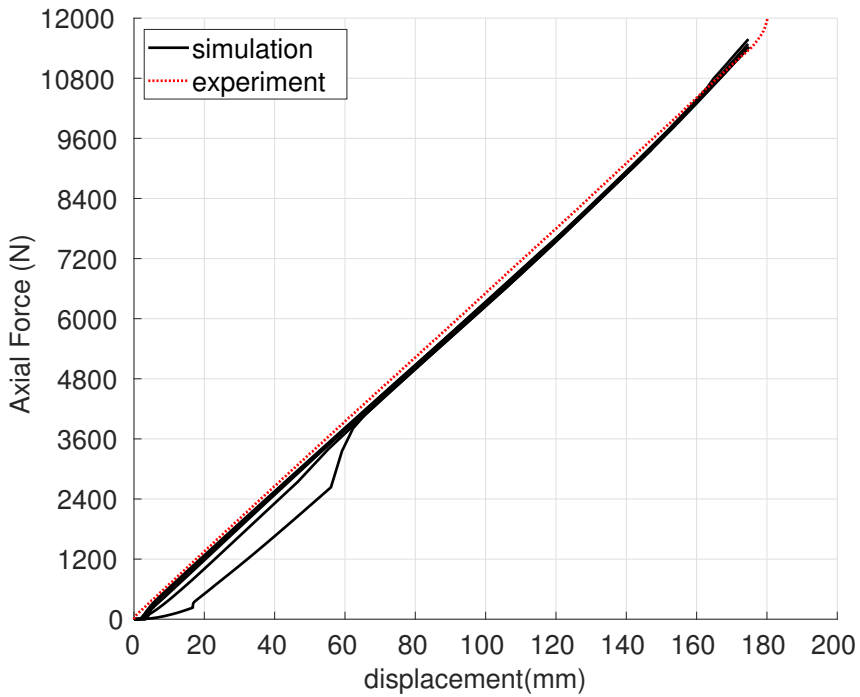
axial force



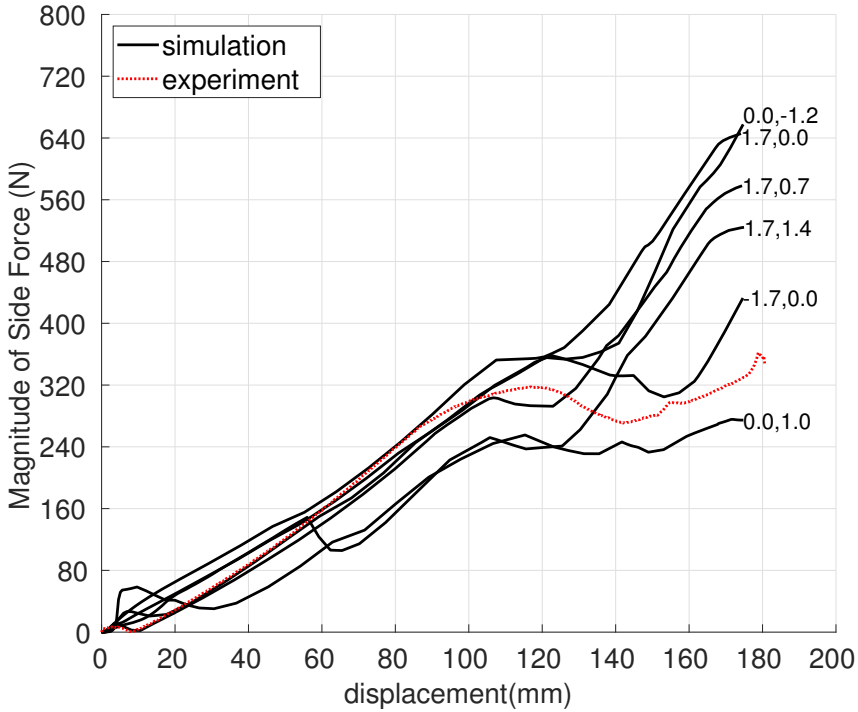
side force

- Rotation Variation: is critical

Rotation (x & z) - Specimen 2

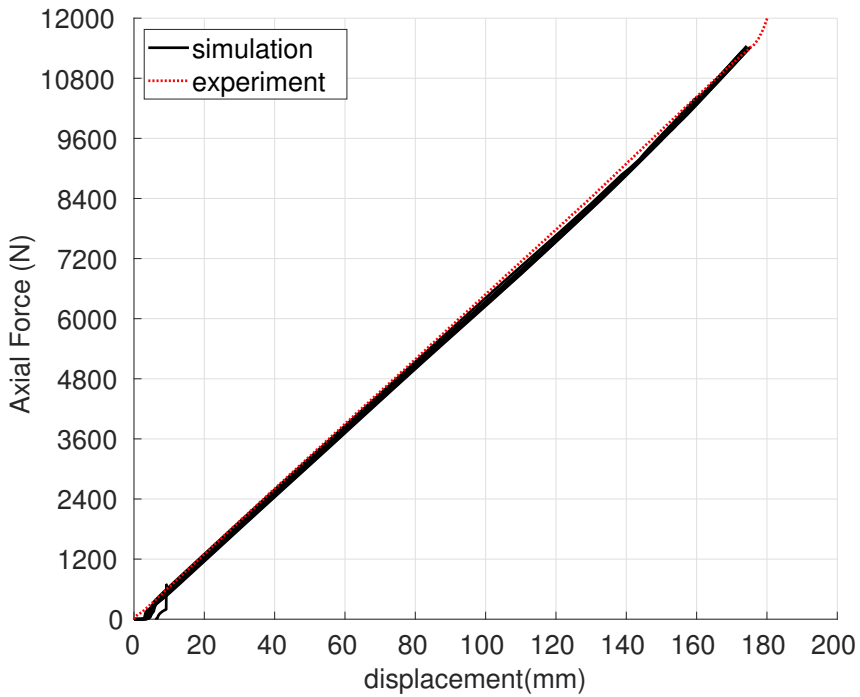


axial force

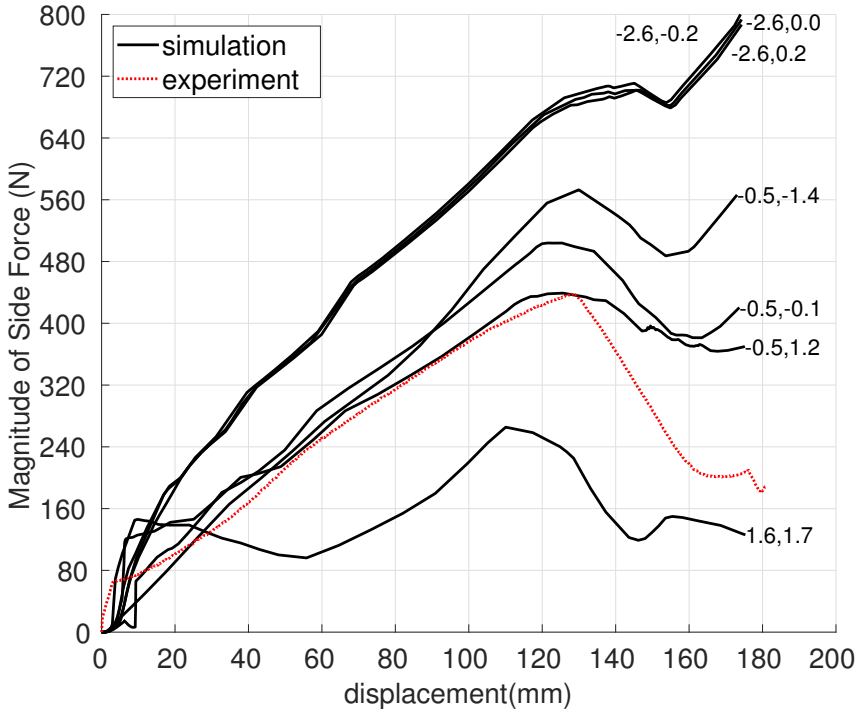


side force

Rotation (x & z) - Specimen 3



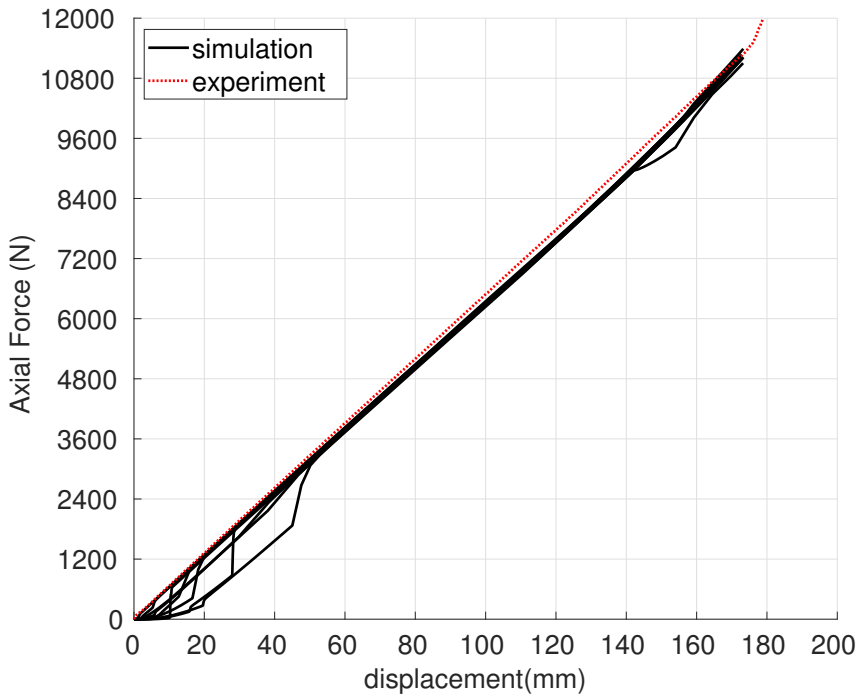
axial force



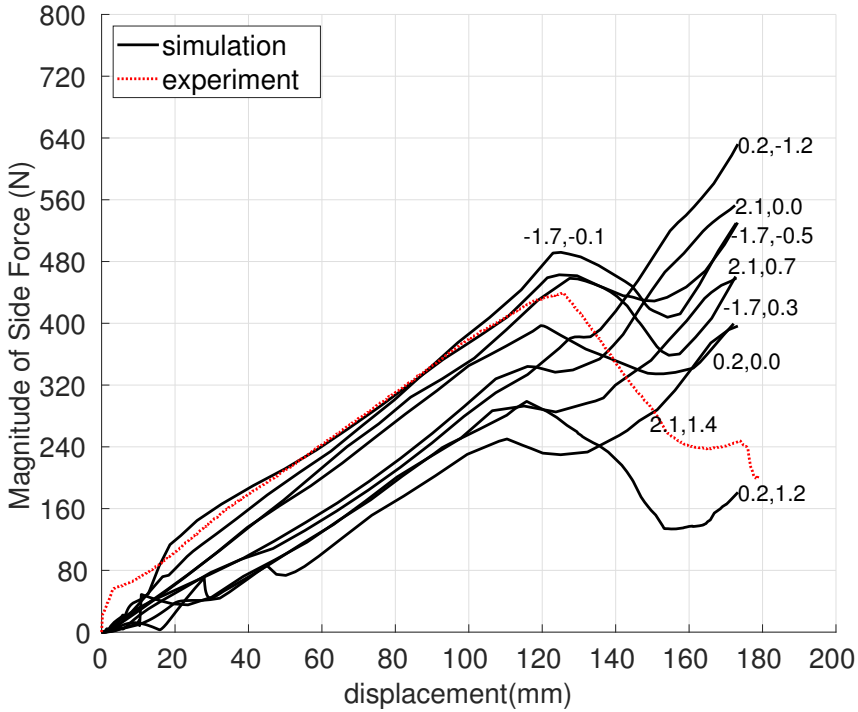
side force

Design of Experiments: Experiment 2

Rotation (x & z) - Specimen 4



axial force

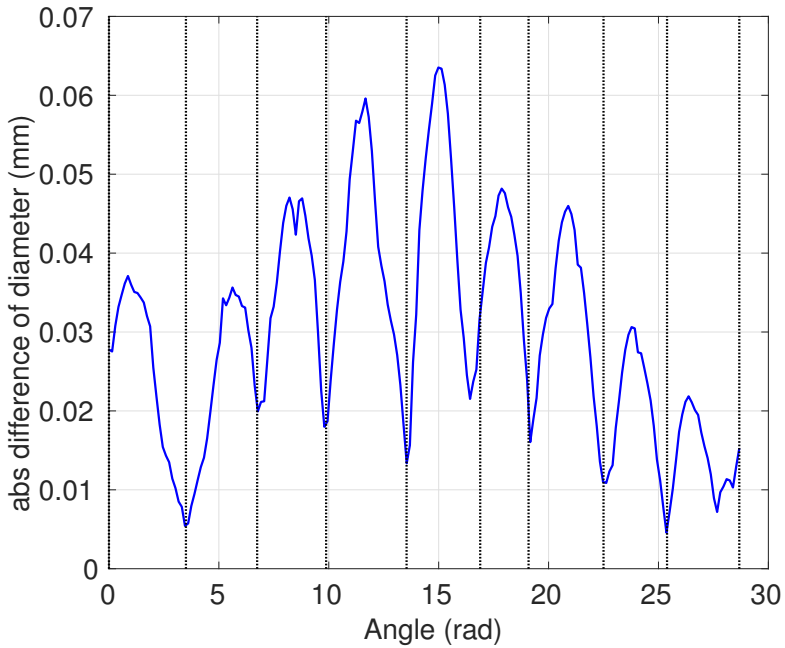


side force

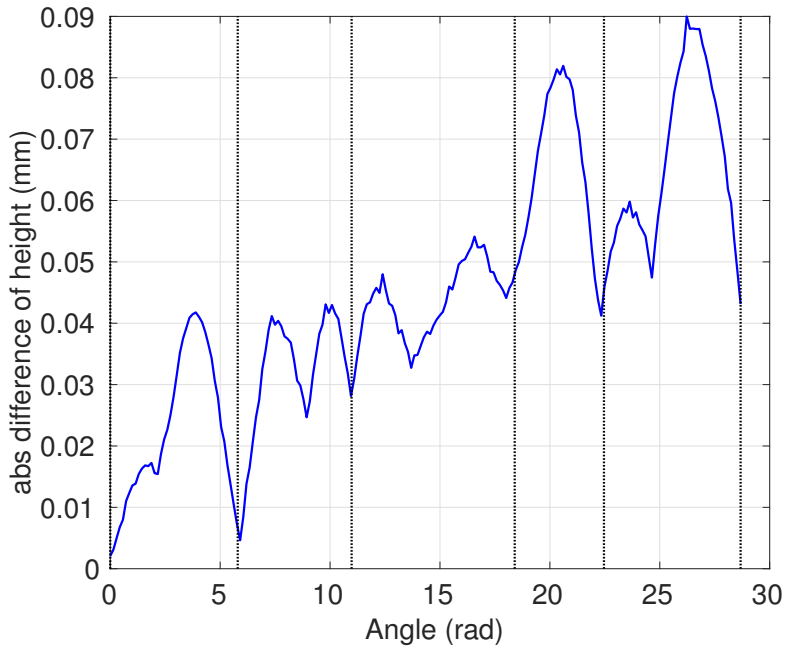
- Inconsistent trend accross specimens: Geometry also affects

Identifying parameters to vary

- Identifying regions of variations from max. absolute difference of average profile with all specimens



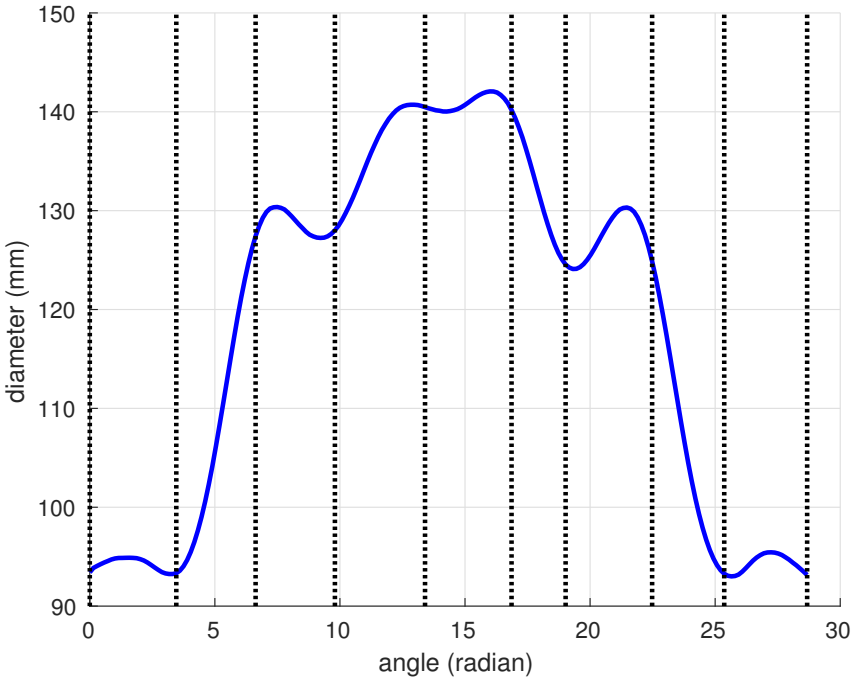
(a) Diameter regions



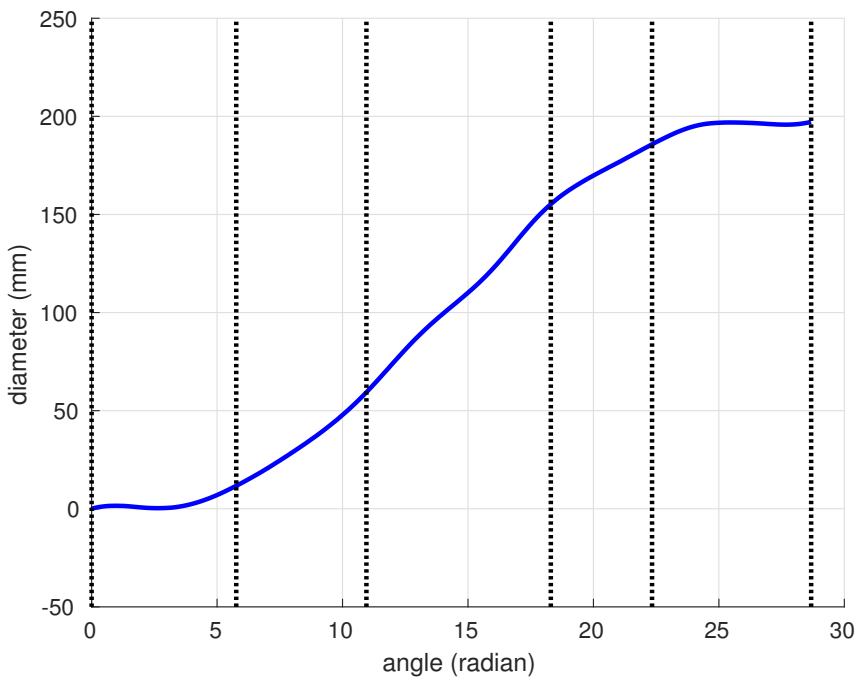
(b) Height regions

Identifying parameters to vary

- Identify regions of variations: max. absolute difference of average profile with all specimens
- One random variable for each region

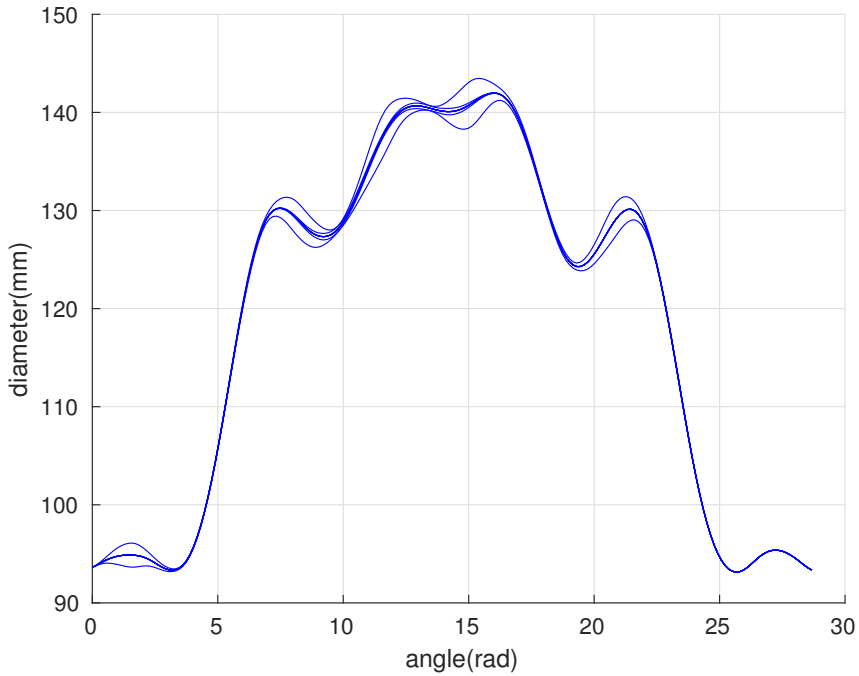


Diameter regions

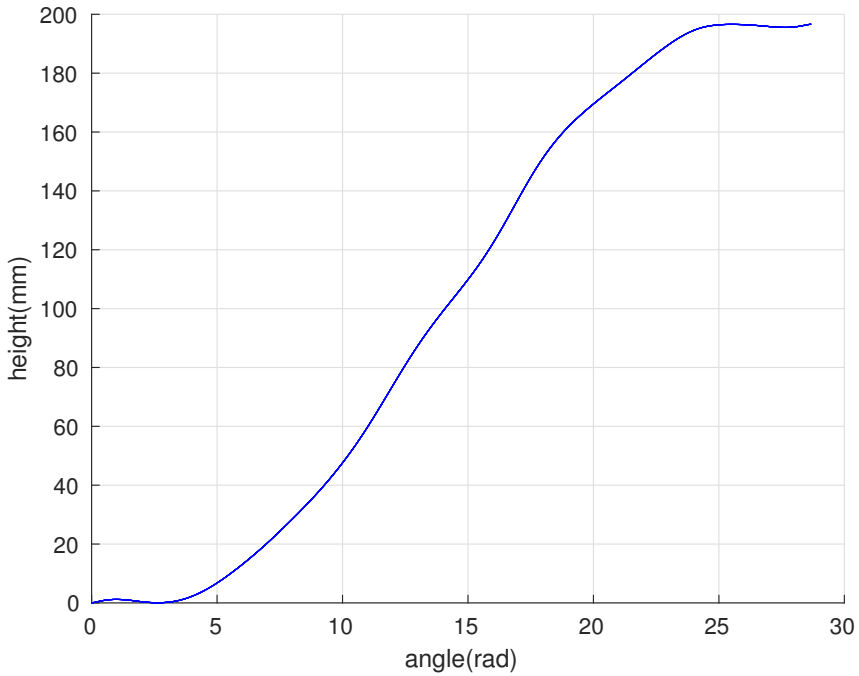


Height regions

Diameter variation profiles

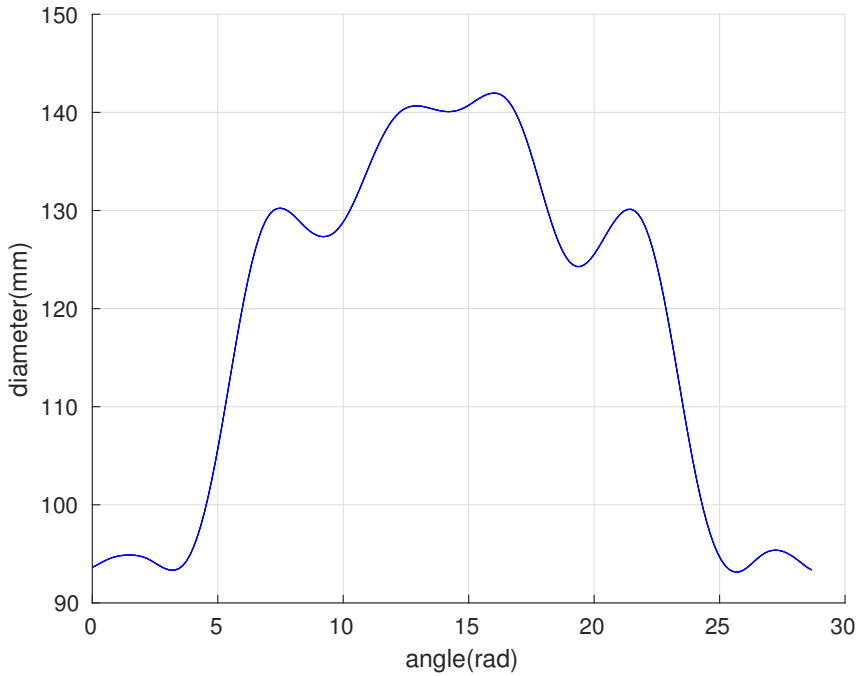


Diameter profiles

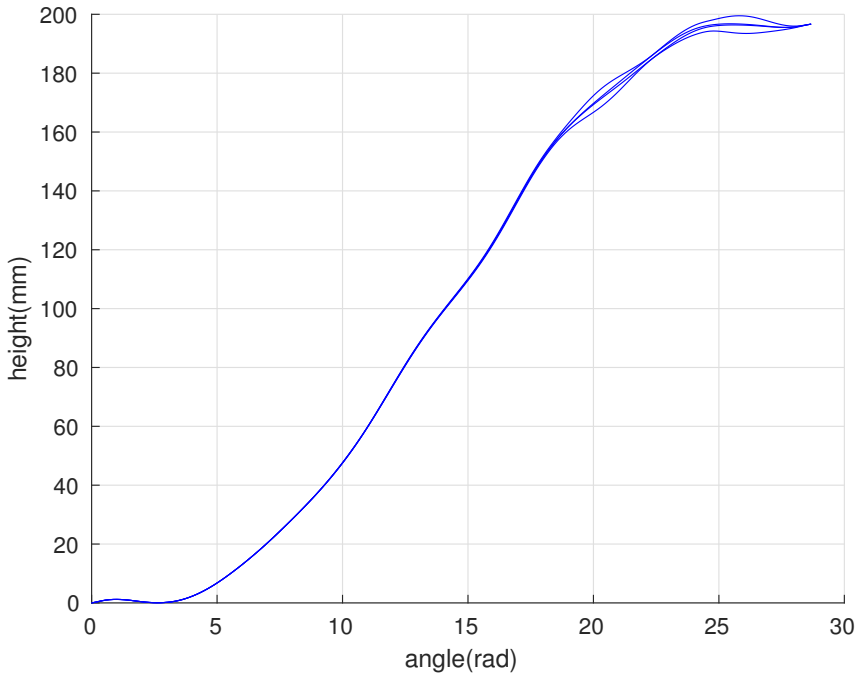


Height profiles

Height variation profiles



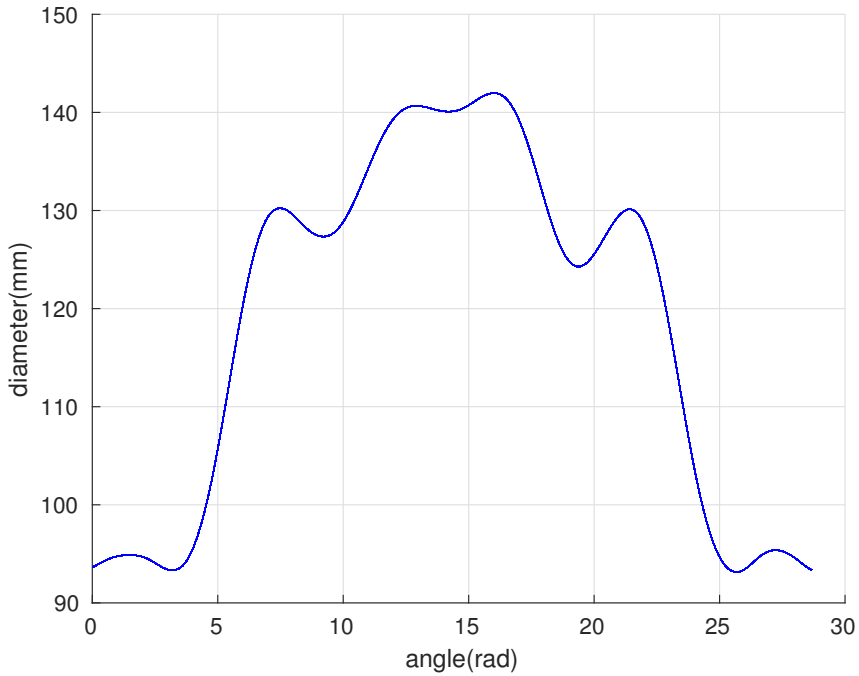
Diameter profiles



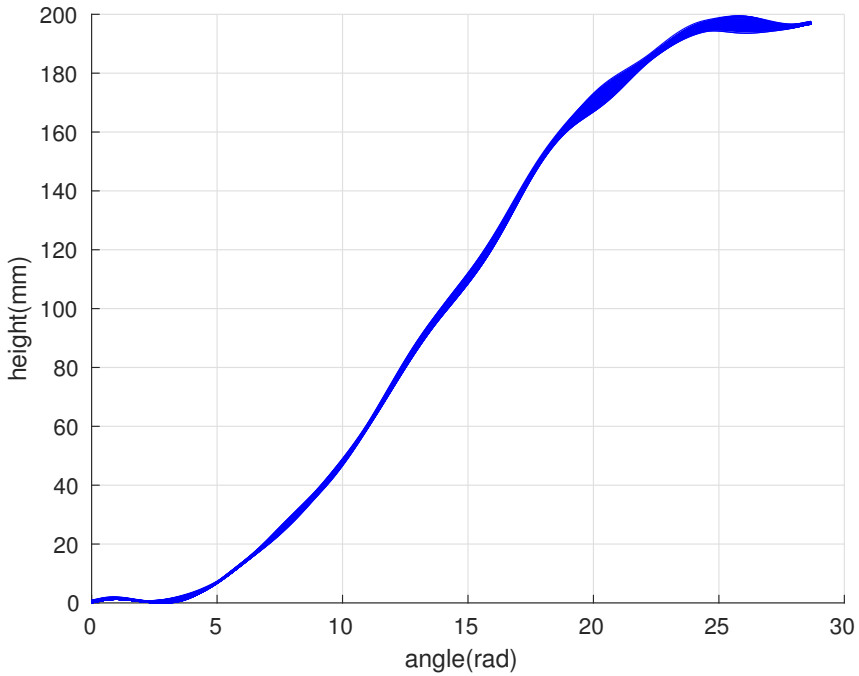
Height profiles

Design of Experiments: Experiment 2

All profiles

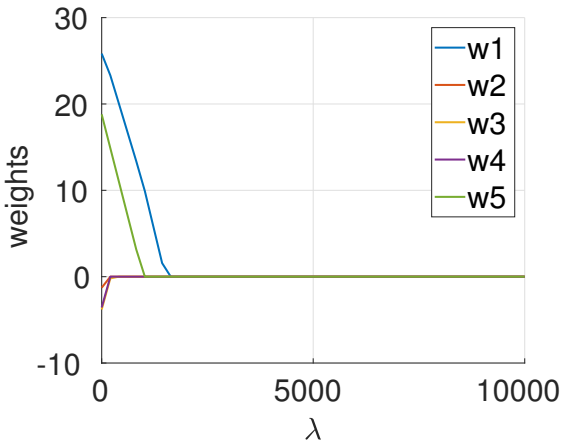


Diameter profiles

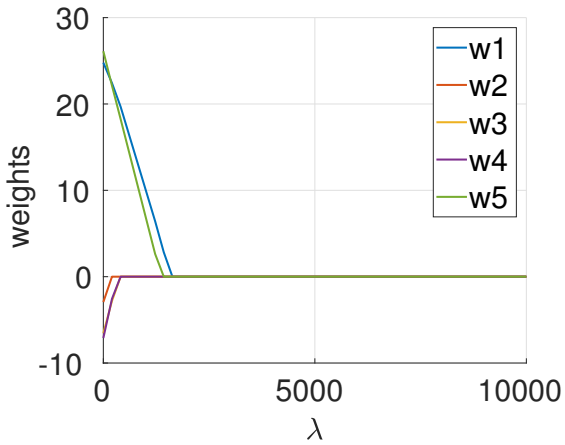


Height profiles

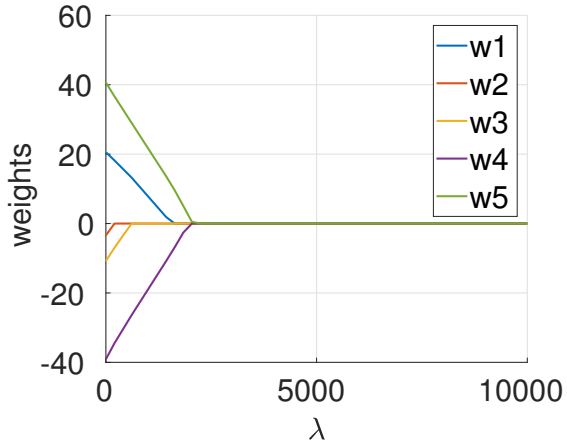
Analyzing results



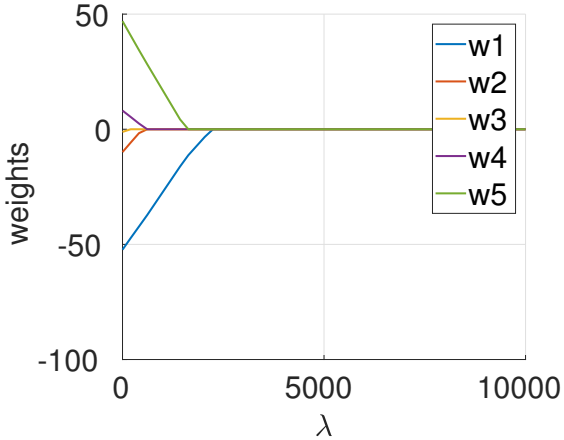
displacement = 80 mm



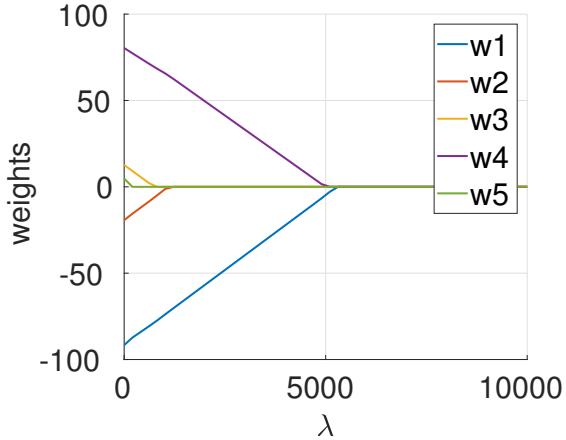
displacement = 100 mm



displacement = 120 mm



displacement = 140 mm



displacement = 160 mm

Figure: Regression results for 2x max variations

Analyzing results

Observations

- The closer any point on spring to the plate the earlier it makes contact and dictates the region of force concentration
- Regions of concentration for specimen 1: 180°apart on bottom plate
- Regions of concentration for specimen 5: 150°apart on bottom plate

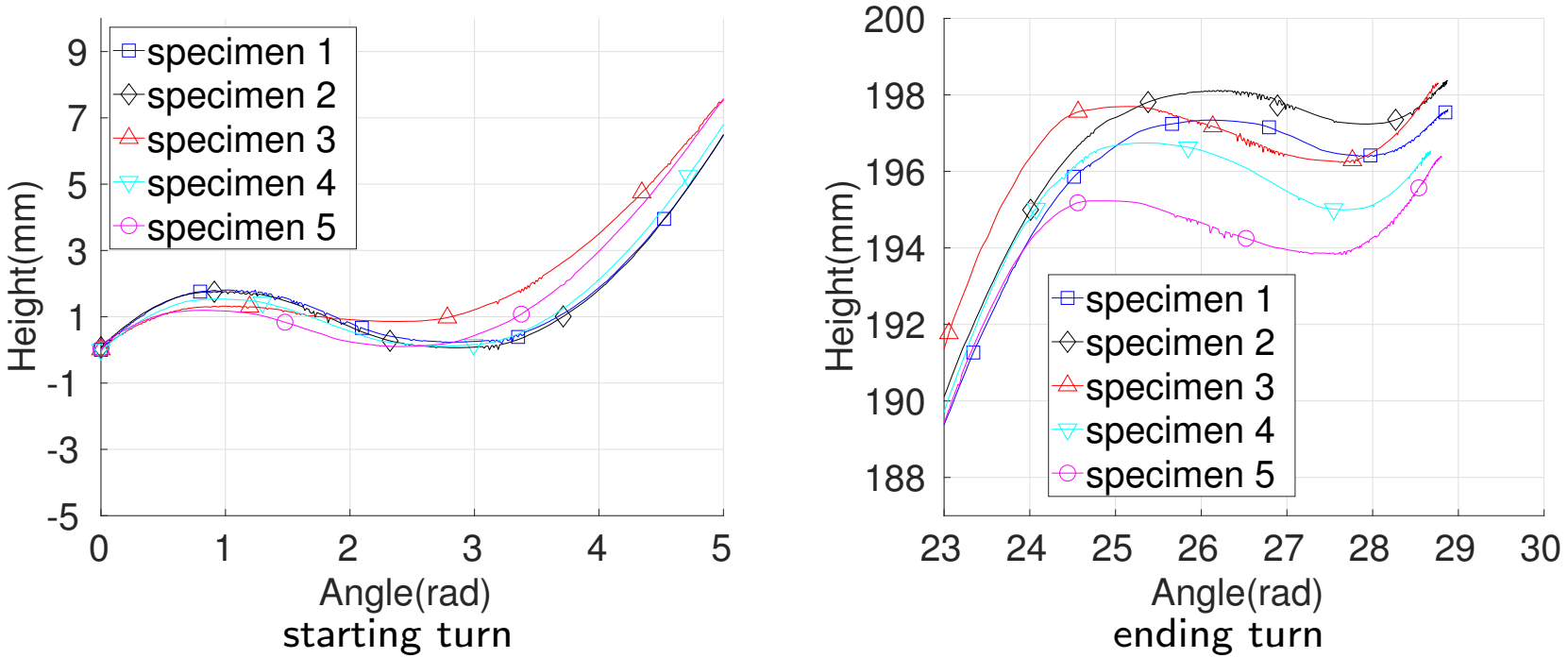


Figure: Zoomed-in plots for height profiles of all specimens at two ends