AAE550: HW0

Rahul Deshmukh PUID: 0030004932

August 26, 2018

I FUNCTIONS OF ONE VARIABLE

1) Write a MATLAB function file called func_one.m that accepts a single input variable ‘x’, evaluates the function (1), and returns an output variable ‘f’. MATLAB, by default, uses radians when evaluating the sine function.

$$f(x) = e^{-0.25x} cos(x) \tag{1}$$

```
function f = func_one(x)
% 1.1
% evaluates the function e^(-0.25*x)*cos(x)
% x in radians vector or single valued
f = exp(-0.25*x).*cos(x);
```

2) Write a MATLAB script that plots the function $f(x)$ over the domain $0 \leq x \leq 3\pi$ using the plot command and func_one.m. Be sure to label all axes and give an appropriate title; add labels either via commands in your script or via the menus in the MATLAB figure window.

```
%1.2
% plot func_one over x = (0,3*pi)
x = 0:0.1:3*pi;
y = func_one(x);
figure(1);
plot(x,y,'-ro'); title('1.2 plot of function one');
xlabel('x');ylabel('y');
```

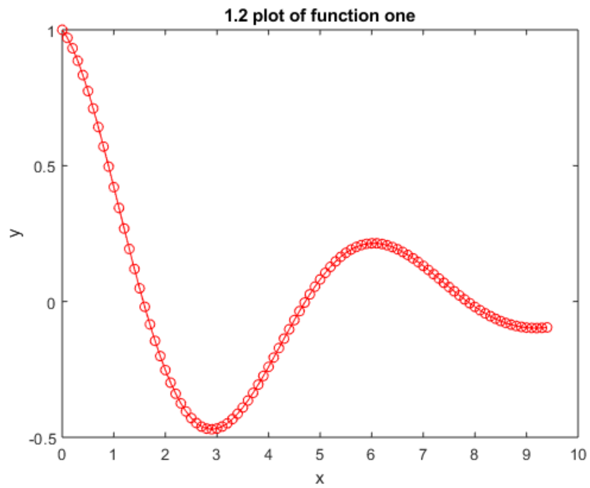


Figure 1: plot of single variable function

3) Create a formula in Excel that computes the value of $f(x)$ in one cell that uses a value of x from another cell as input. Excel also uses radians when evaluating the sine function.

i	x	fx		start	0	
	F1+A3*(F2-F1)/F3	EXP(-0.25*B3)*COS(B3)		end	9.424778	3*pi'
0	0	1		num_pts	20	
1	0.471238898	0.791984662				
2	0.942477796	0.464398126				
3	1.413716694	0.109860128				
4	1.884955592	-0.192897194				
5	2.35619449	-0.39234167				
6	2.827433388	-0.469052706				
7	3.298672286	-0.432983288				
8	3.769911184	-0.315242482				
9	4.241150082	-0.15724243				
10	4.71238898	-5.65769E-17				
11	5.183627878	0.1242343				
12	5.654866776	0.196783321				
13	6.126105675	0.213543548				
14	6.597344573	0.182771611				
15	7.068583471	0.120787865				
16	7.539822369	0.046919843				
17	8.011061267	-0.021112639				
18	8.482300165	-0.070512262				
19	8.953539063	-0.095008559				
20	9.424777961	-0.094780225				

Figure 2: Excel table for f(x)

4) Repeat question 2) using the chart tool in Excel. You should create a table of x and $f(x)$ values that can be used by the chart tool. Create the plot as an "XY scatter" plot, and connect the points you computed in the table with an interpolating curve.

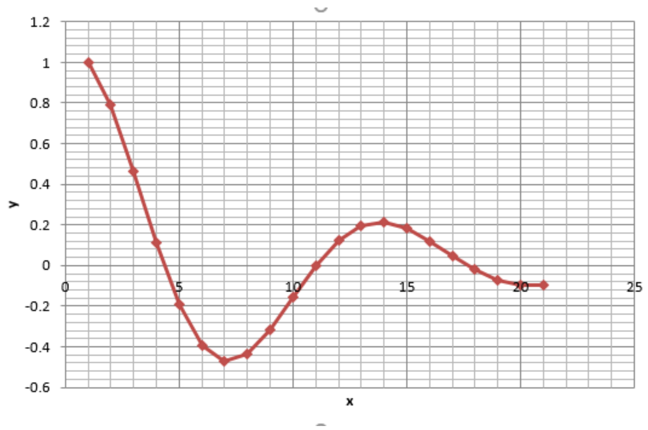


Figure 3: plot of single variable function using excel

II FUNCTIONS OF TWO VARIABLE

1) Write a MATLAB function file called func_two.m that accepts a vector ‘x’ that contains values of x_1 and x_2, evaluates the function (2), and returns an output variable ‘f’.

$$f(x) = x_1^3 + 2x_2^2 - 2x_1 - 3x_2 \tag{2}$$

The `class02func.m` file provides an example of a MATLAB function file that computes a value for a function of two-variables.

```
function f = func_two(x)
%evaluates a function of two variables
% x is a vector
x1 = x(1);
x2= x(2);
f = x1^3+2*x2^2-2*x1-3*x2;
```

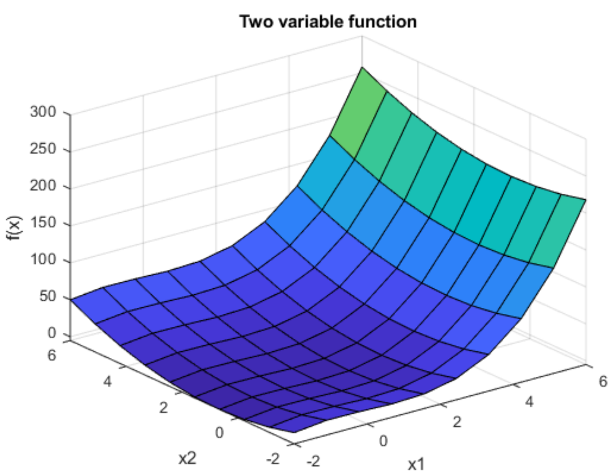
2) Using the surf command and your func_two.m function, write a script file that plots the function over the following region: $2 \leq x_i \leq 6$ Also, using the `contour` and `clabel` commands, plot a series of contours with labels of the function values. Again, be sure to label all axes and give an appropriate title.

The `contour_problem.m` file provides an example that includes a labeled contour plot of a function of two variables, where the function is evaluated using the `class02func` function in the `class02func.m` file. The `contour_problem.m` file also plots two additional functions on the same figure; for this part, only the contours of $f(x)$ in equation (2) should appear in the plot.

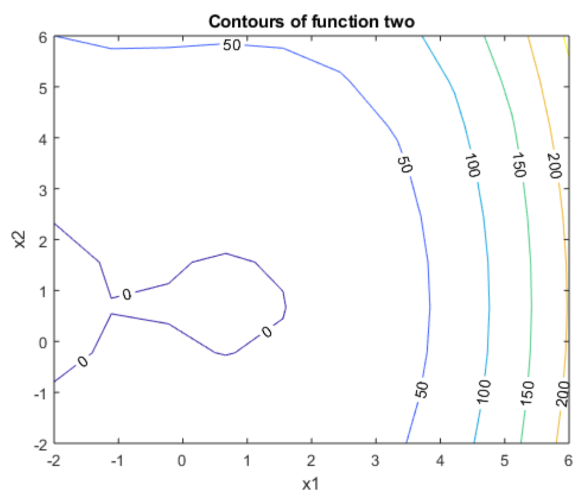
```
%plot func_two over [-2 6]x[-2 6]
clc; clear all; close all;
a = -2;b = 6; N =10;
x = linspace(a,b,N);
[X,Y]=meshgrid(x,x);
figure(1);
Z =zeros(N);
```

```
g1=zeros(N);
g2 = zeros(N);
for i=1:N
    for j = 1:N
        Z(i,j)= func_two([X(i,j);Y(i,j)]);
        temp=cons_three([X(i,j);Y(i,j)]);
        g1(i,j)=temp(1);
        g2(i,j)=temp(2);
    end
end
figure(1);
surf(X,Y,Z);title('Two variable function');
xlabel('x1');ylabel('x2');zlabel('f(x)');

figure(2);% for contour plots
[c,h]=contour(X,Y,Z);
clabel(c,h); xlabel('x1');ylabel('x2');
title('Contours of function two');
```



(a) Surface Plot



(b) Contour Plot

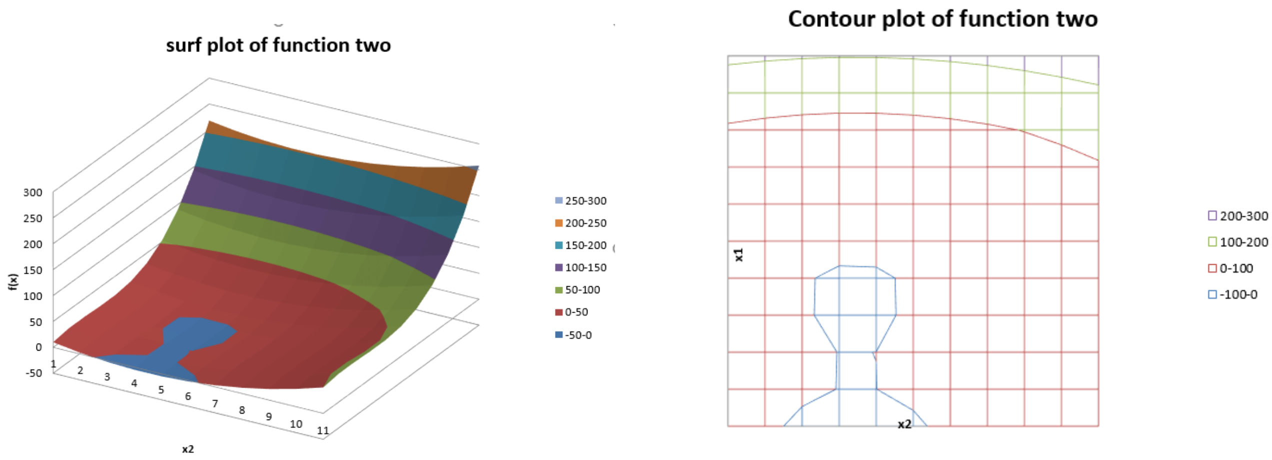
3) Repeat question 1) by creating a formula in Excel that computes the value of $f(x)$ in one cell given a value of x_1 and x_2 from two other cells as input.

C4 =$B4^3+2*C$3^2-2*$B4-3*C$3

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		start	-2	end	6	N	10						
2								x2					
3	i	x1	-2	-1.2	-0.4	0.4	1.2	2	2.8	3.6	4.4	5.2	6
4	0	-2	10	2.48	-2.48	-4.88	-4.72	-2	3.28	11.12	21.52	34.48	50
5	1	-1.2	14.672	7.152	2.192	-0.208	-0.048	2.672	7.952	15.792	26.192	39.152	54.672
6	2	-0.4	14.736	7.216	2.256	-0.144	0.016	2.736	8.016	15.856	26.256	39.216	54.736
7	3	0.4	13.264	5.744	0.784	-1.616	-1.456	1.264	6.544	14.384	24.784	37.744	53.264
8	4	1.2	13.328	5.808	0.848	-1.552	-1.392	1.328	6.608	14.448	24.848	37.808	53.328
9	5	2	18	10.48	5.52	3.12	3.28	6	11.28	19.12	29.52	42.48	58
10	6	2.8	30.352	22.832	17.872	15.472	15.632	18.352	23.632	31.472	41.872	54.832	70.352
11	7	3.6	53.456	45.936	40.976	38.576	38.736	41.456	46.736	54.576	64.976	77.936	93.456
12	8	4.4	90.384	82.864	77.904	75.504	75.664	78.384	83.664	91.504	101.904	114.864	130.384
13	9	5.2	144.208	136.688	131.728	129.328	129.488	132.208	137.488	145.328	155.728	168.688	184.208
14	10	6	218	210.48	205.52	203.12	203.28	206	211.28	219.12	229.52	242.48	258

Figure 4: Excel table for f(x)

4) Repeat question 2) using the chart tool in Excel. You should now generate a series of evenly spaced x_1 values in a row and a series of evenly spaced x_2 values in a column so that this defines the outside of a region in your spreadsheet. Then, write the function so that the value of a cell inside this defined region is a function of the x_1 value in the row at the top of the region and of the x_2 value in the column at the left of the region. Create the plot as a"surface" plot. Then, make a copy of the graph and change the chart type to show this as a wireframe contour plot.
The **contour_example.xls** and **contour_example.xlsx** files contain a worksheet that computes values of a two-variable function to generate a surface plot and as a contour plot or wireframe contour plot. Because of how Excel creates the surface plot, the data defining x_1 and x_2 must be available in evenly spaced increments.



(a) Surface plot (b) Contour plot

Figure 5: Excel Plots

III GRAPHICAL SOLUTION OF AN OPTIMIZATION PROBLEM

It is difficult to superpose multiple contour plots in Excel; however, MATLAB makes this comparatively easy. For this part of the homework, you use the file func_two.m that you created in part II above.

1) A simple optimization problem with two variables can be expressed as:

$$\text{Minimize} \qquad f(x) = x_1^3 + 2x_2^2 - 2x_1 - 3x_2 \tag{3}$$

$$\text{Subject to:} \qquad x_1 + 3x_2 \geq 6 \tag{4}$$

$$x_1 \geq 1$$

Convert the constraints into functions $g_i(x) \leq 0$

The Converted Constraints are:

$$6 - x_1 - 3x_2 \leq 0$$
$$1 - x_1 \leq 0$$

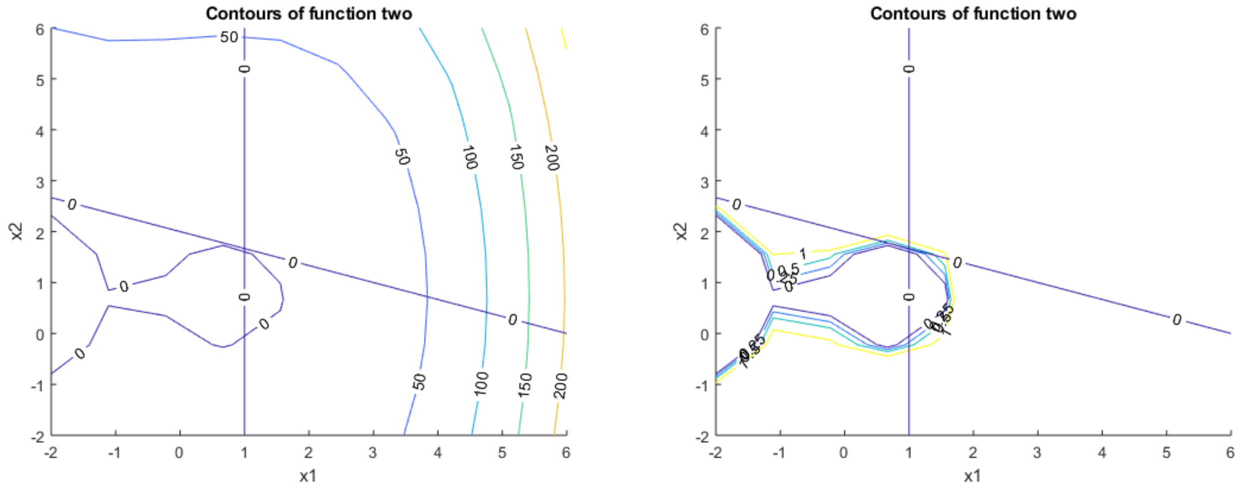
2) Write a MATLAB function file `cons_three.m` that computes the values of $g_1(x)$ and $g_2(x)$ and returns these in a single MATLAB vector g.
The `contour_problem.m` file plots the example function along with two constraint functions on the same figure. The `class02cons.m` file computes these two example constraint functions.

```
function g = cons_three(x)
%input x is a col vector of size=2
%evaluates the two convex constraints g1 and g2
%o/p g is a vector with [g1(x) g2(x)]
g =zeros(length(x),1);
g(1)=6-x(1)-3*x(2);
g(2)= 1-x(1);
```

3) Use your script from part II that creates contour plots of $f(x)$ over the region: $-2 \leq x_i \leq 6$. Add commands to this file that add the contours of $g_1(x) \leq 0$ and $g_2(x) \leq 0$. Be sure to use the "hold on" command to allow this; see the `contour_problem.m` file for assistance. Generate this contour plot. Identify the optimum design x^* from this graph.

```
%plot func_two over [-2 6]x[-2 6]
clc; clear all; close all;
a = -2;b = 6; N =10;
x = linspace(a,b,N);
[X,Y]=meshgrid(x,x);
figure(1);
Z =zeros(N);
g1=zeros(N);
g2 = zeros(N);
for i=1:N
    for j = 1:N
        Z(i,j)= func_two([X(i,j);Y(i,j)]);
        temp=cons_three([X(i,j);Y(i,j)]);
        g1(i,j)=temp(1);
        g2(i,j)=temp(2);
    end
end
figure(1);
surf(X,Y,Z);title('Two variable function');
xlabel('x1');ylabel('x2');zlabel('f(x)');

figure(2);% for contour plots
hold on;
[c,h]=contour(X,Y,Z);
%[c,h]=contour(X,Y,Z,[0,0.25,0.5,1]);
clabel(c,h); xlabel('x1');ylabel('x2');
title('Contours of function two');
[cg1,hg1]=contour(X,Y,g1,[0 0]);clabel(cg1,hg1);
[cg2,hg2]=contour(X,Y,g2,[0 0]);clabel(cg2,hg2);
```



(a) Contour plot

(b) Zoomed

Figure 6: Contour plot of function and plot of constraints

From figure(6) we can estimate x^* is probably at $(1, 1.667)$ which is the intersection of the two constraint lines.

IV USING MATLAB'S OPTIMIZATION TOOLBOX GRAPHICAL USER INTERFACE

1) Converted the constraints to:

$$6 - x_1 - 3x_2 \leq 0$$
$$1 - x_1 \leq 0$$

2) Screen shot of configured optimtool GUI:

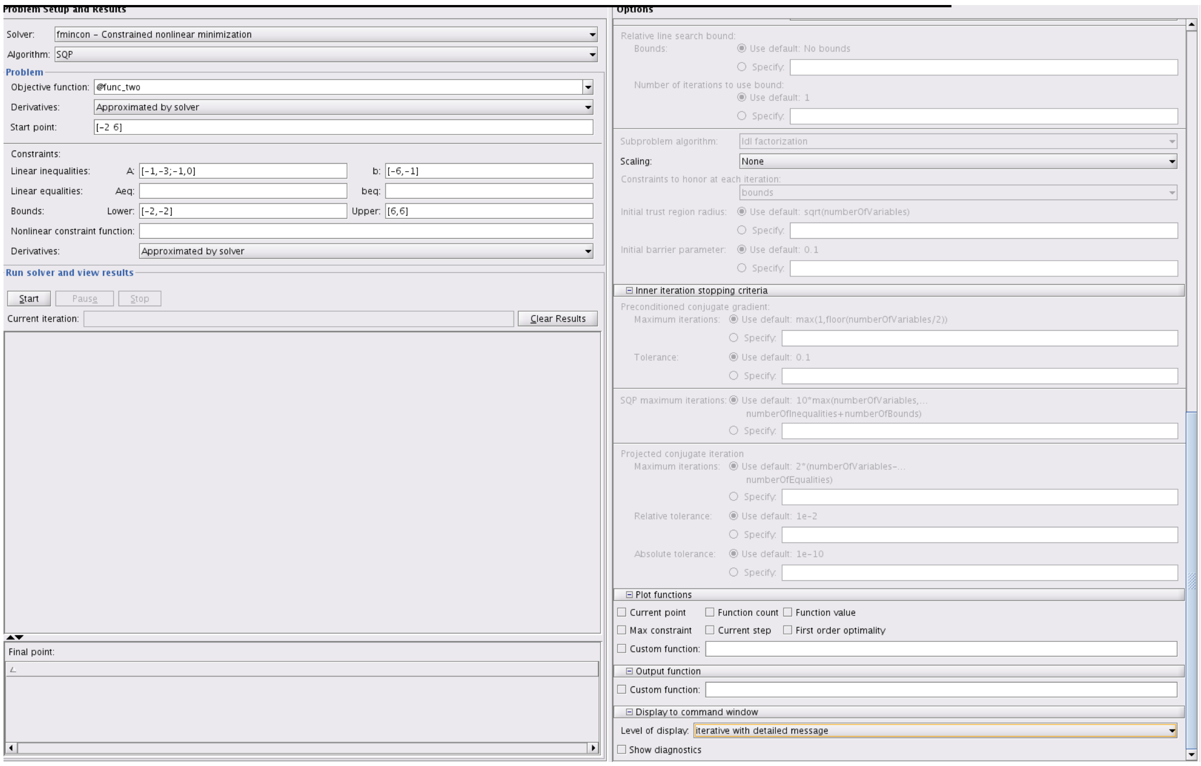


Figure 7: optimtool setup Screenshot

and screenshot of results window:

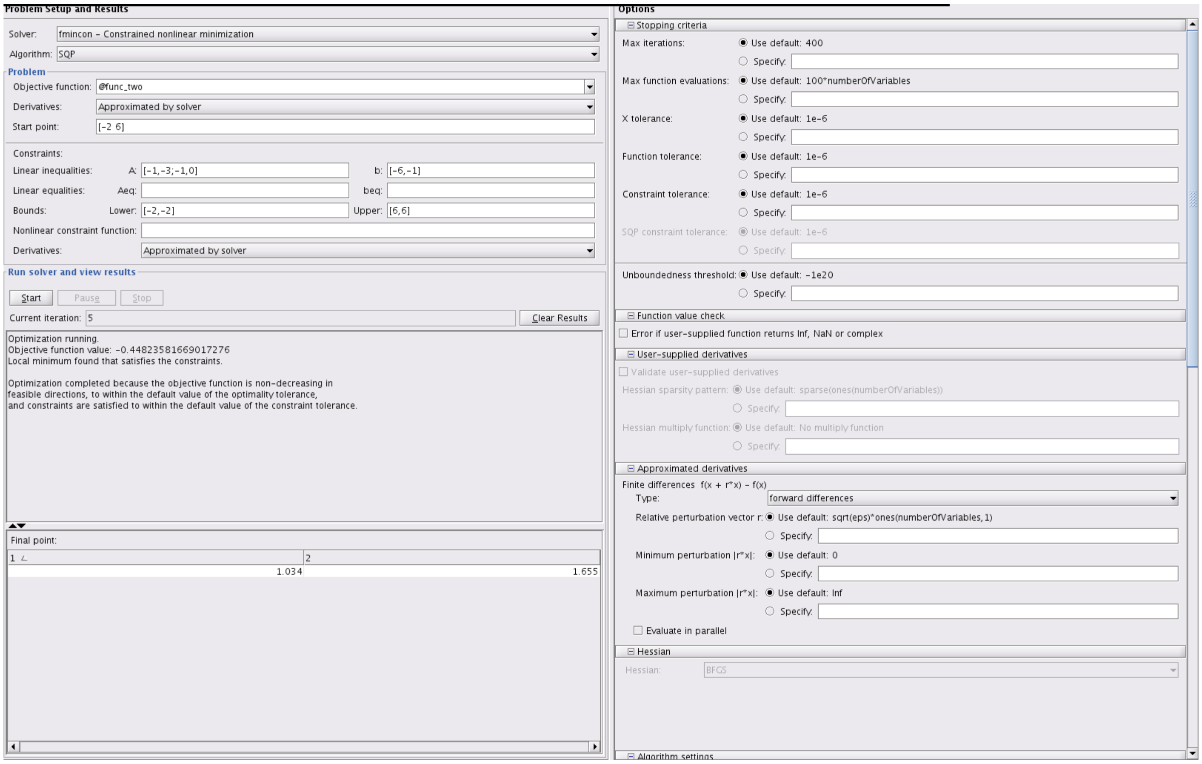


Figure 8: Results window

3) The optimum solution to the minimization problem given by the optimtool is at $x_1 = 1.034$, $x_2 = 1.655$ with the value of the objective function at -0.44823581669017276

   On comparing the above solution with the visual result, $x = (1, 1.667)$ with objective function $= -0.4444$, the value of the objective function obtained using the optimtool is smaller than the visually identified result.Therefore, we have obtained a better result using the optimtool.

4) The values of the parameters are as follows:

   i) "exitflag" $= 1$; which indicates that local minima was found
   ii) iterations $= 5$ were carried out
   iii) gradient $= [1.20713496199558; 3.62140499432140]$;