Homework 6

ME 581

Due: 4:15 PM November 28, 2017

The following problems are to be documented, solved, and presented in a Jupyter notebook.
On-Campus students: Save the notebook as a single PDF, then print and return a hard copy in class.
Off-Campus students: Save the notebook as a single PDF, then upload and submit the PDF in Blackboard. The name of the file should be SURNAME-HW4.pdf.

1. Consider the ODE $y' = -5y$, with initial condition $y(0)=1$.
    a. Is this ODE stable?
    b. Is Euler's method stable for this ODE using a step size h=0.5?
    c. Calculate the numerical approximate solution for t=0 to t=5 given by Euler's method. Use time step size of h = 0.5 and 0.1. Plot the analytical solution and numerical solution for each time step size.
    d. Is the backward Euler method stable for this ODE using a step size h=0.5?
    e. Calculate the numerical approximate solution for t=0 to t=5 given by the backward Euler method. Use a time step size, h = 0.5 and 0.1. Plot the analytical solution and numerical solution for each time step size.

2. Consider the initial value problem with a step size of $h = 0.08$:

$$y''+6y'+8.96y=0$$
$$y(0)=1$$
$$y'(0)=5$$

    a. Write the analytical solution.
    b. Perform numerical integration from t=0 to 8 using 4th order Runge-Kutta method.
    c. Perform numerical integration from t=0 to 8 using Euler's method.
    d. Perform numerical integration from t=0 to 8 using Taylor series method of order two.
    e. Plot analytical, Euler, Taylor and Runge-Kutta solutions in the same plot.

3. Consider the system of differential equations

$$\frac{dy_1}{dt} = -0.013y_1 - 1000y_1y_3$$
$$\frac{dy_2}{dt} = -2500y_2y_3$$
$$\frac{dy_3}{dt} = -0.013y_1 - 1000y_1y_3 - 2500y_2y_3$$

    Initial Conditions : $y_1(0) = 1, y_2(0) = 1, y_3(0) = 0$
    Approximate the solution from t=0 to 5 using the Runge-Kutta 4th order and Euler method with a step size of $h = 0.0001$.

4. Approximate the solution from t=0 to 5 using the 2nd order Runge-Kutta method (Heun method) for initial value problems given in (a), (b) and (c).

(a)

$$y' = -200y + 200\sin t + \cos t\,, \qquad y(0) = 1$$

(i) Determine the maximum allowable time step to maintain absolute stability
(ii) Compute the approximate solution using a step size, h= 0.008 and plot the solution with time
(iii) Compute the approximate solution using a step size, h=0.012 and plot the solution with time

(b)

$$u_1' = 9u_1 + 24u_2 + 5\,cos\,t - \frac{1}{3}sin\,t\,, \qquad u_1(0) = \frac{4}{3}$$

$$u_2' = -24u_1 - 51u_2 - 9\cos t + \frac{1}{3}\sin t\,, \qquad u_2(0) = \frac{2}{3}$$

(i) Determine the maximum allowable time step to maintain absolute stability
(ii) Compute the approximate solution using a step size, h=0.04 and plot the solution with time
(iii) Compute the approximate solution using a step size, h=0.06 and plot the solution with time

(c)

$$u_1' = -20u_1 - 19u_2 \qquad u_1(0) = 2$$

$$u_2' = -19u_1 - 20u_2 \qquad u_2(0) = 0$$

(i) Determine the maximum allowable time step to maintain absolute stability
(ii) Compute the approximate solution using a step size, h=0.04 and plot the solution with time
(iii) Compute the approximate solution using a step size, h=0.06 and plot the solution with time