ME 581 Fall 2017
Final Exam (Take Home)
Friday, December 8 2017 7:00PM EST to
Thursday, December 14 2017 7:00PM EST

Problem 1 (50 pts)	
Problem 2 (50 pts)	
Total (100 pts)	

1) Time limit: Start Time: 7 :00 P.M.EST on December 8, 2017   End Time: 7.00 P.M.EST on December 14, 2017.
2) It will be solved using Jupyter notebooks and submitted as a single pdf file in Blackboard.
3) Notes, books and homework are allowed during the exam.
4) Laptops and calculators are allowed.
5) You can use all the functions that you wrote for previous homework assignments in your exam, so make sure that they are working properly.
6) You do not need a proctor.
7) The final exam should be done on your own and should not be discussed with any other person.

Problem 1:

A given geothermal heat pump uses the heat naturally stored in the ground to heat cold water pumped through underground pipes during the winter (see Figure 1.1).

The time-dependent temperature distribution $\mathrm{T}(\mathrm{y},\mathrm{t})$ of the water pumped through a single pipe (see Figure 1.2) satisfies the equation

$$\frac{\partial \mathrm{T}}{\partial \mathrm{t}} + \mathrm{v}\frac{\partial \mathrm{T}}{\partial \mathrm{y}} = \frac{4\mathrm{h}}{\rho c_p \mathrm{d}}\left(\mathrm{T_g} - \mathrm{T}\right)$$

where $\mathrm{v} = 1\,\mathrm{m/s}$ is the velocity of the water pumped through the pipe, $\mathrm{h} = 990\,\mathrm{W/(m^2 \cdot {}^\circ C)}$ is the heat transfer coefficient, $\rho = 1000\,\mathrm{kg/m^3}$ is the density of the water, $c_p = 4180\,\mathrm{J/(kg \cdot {}^\circ C)}$ is the heat capacity of the water, and $\mathrm{d} = 70\mathrm{mm}$ is the diameter of the pipe.

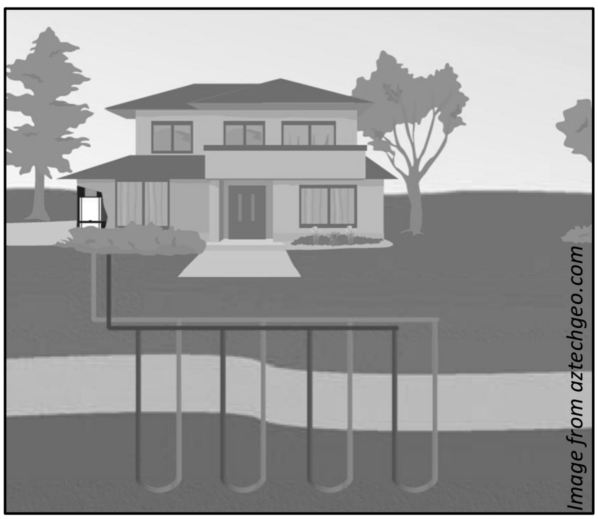


Figure 1.1: A geothermal heat pump system.

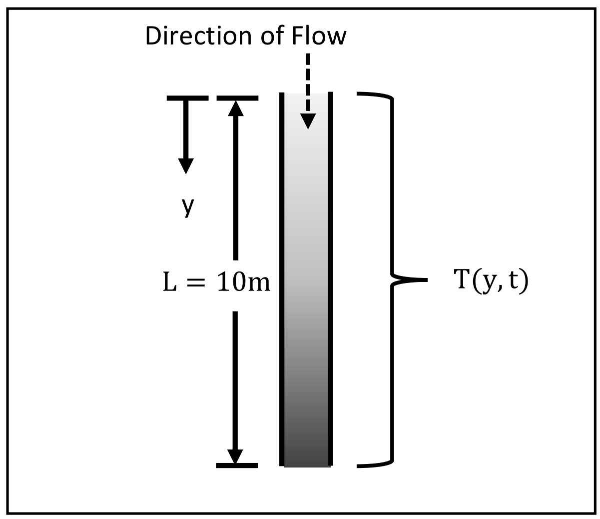


Figure 1.2: An illustration of the temperature distribution in a single pipe.

Consider a single vertical pipe in this system. The initial temperature distribution of the water along the pipe is taken as identical to the temperature profile of the ground,

$$\mathrm{T}(\mathrm{y},0) = \mathrm{T_g} = \left(3 + 10e^{-\frac{1}{2y}}\right)\,{}^\circ\mathrm{C}$$

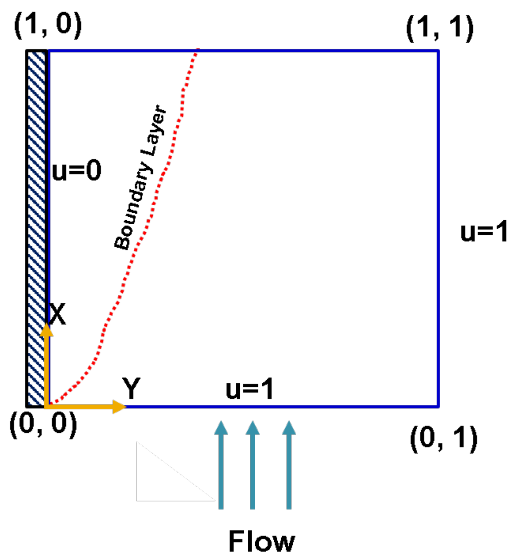
where y is in meters. As the heat pump is turned on and the water begins to flow, the temperature of the water flowing into the intake is constant at $\mathrm{T}(0,\mathrm{t}) = 3^\circ\mathrm{C}$.

Apply an explicit first-order finite difference method with a spatial discretization of $\Delta \mathrm{y} = 0.05\mathrm{m}$ and a temporal discretization of $\Delta \mathrm{t} = 0.005\mathrm{s}$ to approximate the temperature

T(y, t) of the water in the pipe for the first 10 seconds of operation. Graph the temperature profile T(y, t) at 1-second intervals from t = 0s to t = 10s.

Problem 2

Solve the given convection-diffusion equation using finite differencing methods over the domain $0 \leq x \leq 1$ and $0 \leq y \leq 1$ as shown in Figure 1.



$$C\frac{\partial u}{\partial x} = D\frac{\partial^2 u}{\partial y^2}, \text{ where C=1 } m/s \text{ and D} = 0.05\ m^2/s \quad (2.1)$$

subject to the boundary conditions,

$$Inflow\ boundary\ condition: \ u(x = 0, y) = 1\ m/s$$
$$No\ slip\ boundary\ condition\ at\ the\ wall: u(x, y = 0) = 0$$
$$Far\ away\ from\ the\ plate\ wall: \ u(x, y = \ 1) = 1 \text{ m/s}$$

(a) (40 points) Solve using finite difference with $\Delta X = \Delta Y = 0.01$. Show the approximate solution in a contour plot of u(x,y).

(b) (10 points) Find the exact solution to the above equation using separation of variables method. Compare (plot) the approximate solution and the exact solution u(x,y) at x=0.1 m, 0.5 m and 0.8 m as a function of y.

HINT: See that equation 2.1 is the "heat equation" if you replace x with t.