ECE 580 Spring 2020

FunWork #1

Due on February 05

INSTRUCTIONS: The assignment must be typed. Clearly identify the steps you have taken to solve each problem. Your grade depends on the completeness and clarity of your work as well as the resulting answer.

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1. Prove by contraposition, see page 5 in the textbook, the following statement:

For any integer x, that is, for any $x \in \mathbb{Z}$, if $7x + 9$ is even, then x is odd.

2. Prove by contradiction, see page 5 in the textbook, the following statement:

For any $x \in [0, \pi/2]$, we have $\sin x + \cos x \geq 1$.

3. Prove by induction, see page 5 in the textbook, the following statement:

For any natural number n, that is, for any $n \in \mathbb{N}$,

$$\sum_{i=1}^{n} = \frac{n(n+1)}{2}$$

4. Consider the following systems of equations,

(a)

$$\left.\begin{array}{rcl} x_1 + x_2 + 2x_3 + x_4 & = & 1 \\ x_1 + 2x_2 + 4x_3 - 2x_4 & = & 0 \end{array}\right\}$$

(b)

$$\left.\begin{array}{rcl} 2x_1 + x_2 + 2x_3 + x_4 & = & 0 \\ 6x_1 + 3x_2 + 6x_3 + 3x_4 & = & 1 \end{array}\right\}$$

Use Theorem 2.1, on page 17, to check if the system has a solution. If there is a solution, use the method of the proof of Theorem 2.2, on page 18, to find a general solution to the system.

5. Determine the condition under which the system of linear equations,

$$\left.\begin{array}{rcl} cx_1 + ax_3 & = & b \\ cx_2 + bx_3 & = & a \\ bx_1 + ax_2 & = & c \end{array}\right\}$$

has a unique solution. Then find the solution.

6. Compute

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}^{57}.$$

7. Let

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}, \quad f_1(x) = x^2 - 2x + 5, \quad \text{and} \quad f_2(x) = 7x + 5.$$

Find $5f_1(\boldsymbol{A}) - 3f_2(\boldsymbol{A})$.

8. Use the method of Section 3.2 to find the basis in which the matrix

$$\boldsymbol{A} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 2 & 5 & -2 & 1 \\ -1 & 1 & 0 & 3 \end{bmatrix}$$

is diagonal.

9. For the function

$$f = f(x_1, x_2) = x_1^2 + x_2^2 + 4x_1x_2 + \frac{2}{3}x_2^3 - 2x_2 + 7,$$

(a) find points that satisfy the first-order necessary conditions for the extremum;

(b) which point is a strict local minimizer? Justify your answer.

10. Represent the following quadratic forms,

(a) $f(x_1, x_2, x_3, x_4) = 7x_1^2 + x_3^2 - 2x_1x_3 + x_1x_4$;

(b) $f(x_1, x_2, x_3) = x_2^2 - 3x_1x_2$;

(c) $f(x_1, x_2, x_3) = 2x_1^2 - 5x_2^2 + 2x_1x_2$,

as

$$f = \frac{1}{2}\boldsymbol{x}^\top \boldsymbol{Q}\boldsymbol{x},$$

where $\boldsymbol{Q} = \boldsymbol{Q}^\top$.