Question 2.2

Tuesday, April 21, 2020 6:00 PM

MLE estimate $(\hat{\theta})$ for θ:

$$\hat{\theta} = \arg\min_{\theta} \left\{ -\log\left(p_\theta(x,y)\right) \right\}$$

$$= \arg\min \left\{ -\log\left(p_\theta(x|y)\right) - \cancel{\log\left(p(y)\right)} \right\}$$

Constant

$$= \arg\min_{\theta} \left\{ -\log\left(\frac{1}{(2\pi\sigma^2)^{Kp/2}} \cdot \exp\left\{ \frac{-1}{2\sigma^2} \sum_{k=0}^{K-1} \| x_k - f_\theta(y_k) \|^2 \right\} \right) \right\}$$

$$= \arg\min_{\theta} \left\{ + \frac{Kp}{2} \cancel{\log(2\pi\sigma^2)} + \cancel{\frac{1}{2\sigma^2}} \sum_{k=0}^{K-1} \| x_k - f_\theta(y_k) \|^2 \right\}$$

Constant Constant

$$\hat{\theta} = \arg\min_{\theta} \left\{ \sum_{k=0}^{K-1} \| x_k - f_\theta(y_k) \|^2 \right\}$$

FONC: $$\nabla_\theta \sum_{k=0}^{K-1} \| x_k - f_\theta(y_k) \|^2 \Big|_{\theta=\hat{\theta}} = 0$$

$$\Rightarrow \sum_{k=0}^{K-1} -2 \left(x_k - f_\theta(y_k)\right)^T \cdot \nabla_\theta f_\theta(y_k) \Big|_{\theta=\hat{\theta}} = 0$$

Solution to the above equation gives the estimate $\hat{\theta}$