ME581 Homework 3

Due: 4:15pm October 12, 2017

The following problems are to be documented, solved, and presented in a Jupyter notebook.

On-Campus students: Save the notebook as a single PDF, then print and return a hard copy in class.

Off-Campus students: Save the notebook as a single PDF, then upload and submit the PDF in Blackboard. The name of the file should be SURNAME-HW3.pdf.

Problem 1

Use the Gauss-Seidel method to solve the following system of equations. Take $\boldsymbol{x}^{(0)} = \mathbf{0}$, and terminate iteration when $\left\|\boldsymbol{x}^{(k+1)} - \boldsymbol{x}^{(k)}\right\|_\infty$ falls below 5×10^{-6}. Print the final iteration of the solution. Record the number of iterations required to achieve convergence.

$$
\begin{array}{rrrrrcr}
-14x_1 & +\ 6x_2 & & & & = & -\ 8 \\
-9x_1 & +27x_2 & +3x_3 & & & = & -18 \\
 & 2x_2 & +6x_3 & -\ 2x_4 & & = & 6 \\
 & & -3x_3 & +30x_4 & -12x_5 & = & 21 \\
 & & & -\ 2x_4 & +\ 3x_5 & = & 1
\end{array}
$$

Problem 2

Each of the following matrices has a unique dominant eigenvalue of multiplicity one but does not possess a complete set of linearly independent eigenvectors.

For each matrix:

Using the initial vector provided, apply 30 iterations of the power method to determine the dominant eigenvalue and an associated eigenvector. Print the final values of the estimated vector, eigenvalue, and convergence $\left|\lambda^{(n)} - \lambda^{(n-1)}\right|$. Plot all iterations of the eigenvalue and convergence. Explain the behavior of the eigenvalue and eigenvector sequences.

if we change the initial guess we may obtain a different eigen value

$$
\text{(a) } A = \begin{bmatrix} 2 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}
$$

$$
\text{(b) } A = \begin{bmatrix} 15 & -10 & 10 & 5 \\ 28 & -18 & 18 & 14 \\ 9 & -5 & 6 & 12 \\ -3 & 2 & -1 & 2 \end{bmatrix}, x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}
$$

Problem 3

Each of the following matrices has a unique dominant eigenvalue of multiplicity greater than one, and the eigenvalue does possess a complete set of linearly independent eigenvectors.

For each matrix:
Using each of the initial vectors provided, apply 30 iterations of the power method to determine the dominant eigenvalue and an associated eigenvector. Demonstrate that the eigenvalue sequence converges to the same value but the eigenvector sequence converges to different vectors. Print the final values of the estimated vector, eigenvalue, and convergence $|\lambda^{(n)} - \lambda^{(n-1)}|$. Plot all iterations of the eigenvalue and convergence. Explain the behavior of the eigenvalue and eigenvector sequences.

$$\text{(a) } A = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 5 & -1 & 5 \end{bmatrix}, x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, x_0 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

$$\text{(b) } A = \begin{bmatrix} 5 & -2 & 3 & -1 \\ -1 & 6 & -3 & 1 \\ -1 & 2 & 1 & 1 \\ 1 & -2 & 3 & 3 \end{bmatrix}, x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, x_0 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

Problem 4

Each of the following matrices has a unique dominant eigenvalue of multiplicity greater than one, but the eigenvalue does not possess a complete set of linearly independent eigenvectors.

For each matrix:
Using each of the initial vectors provided, apply 30 iterations of the power method to determine the dominant eigenvalue and an associated eigenvector. For each initial vector, print the final values of the estimated vector, eigenvalue, and convergence $|\lambda^{(n)} - \lambda^{(n-1)}|$ and plot all iterations of the eigenvalue and convergence. Explain the behavior of the eigenvalue and eigenvector sequences.

$$\text{(a) } A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 0 & 2 & 0 & -3 \\ 0 & 0 & 2 & -5 \end{bmatrix}, x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, x_0 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\text{(b) } A = \begin{bmatrix} 2 & 3 & 4 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, x_0 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$