ME 581
Homework 5
(Due: Tuesday November 14, 2017)

The following problems are to be documented, solved, and presented in a Jupyter notebook.
On-Campus students: Save the notebook as a single PDF, then print and return a hard copy in class.
Off-Campus students: Save the notebook as a single PDF, then upload and submit the PDF in Blackboard. The name of the file should be SURNAME-HW5.pdf.

1) Using either of the Trapezoidal, Simpson or the midpoint rule evaluate the following integrals. State the reason for your choice. Compare with the analytical solution. Will it give you the exact result? If not, why?

a) $\int_0^5 (5x^2 + 3)dx$

b) $\int_1^4 (2x + 1)dx$

c) $\int_1^2 (e^x)dx$

2) Using 2-point Gauss quadrature rule evaluate the following integrals.

a) $\int_1^6 (3x^4 + 5x^2)dx$. Did you expect to find the exact solution? Assume no rounding error.

b) $\int_1^4 (e^{x-5})dx$. Did you expect to find the exact solution? Assume no rounding error.

c) Generalize the change of variables you used in the previous integrals to compute $\int_a^b f(x)dx$.

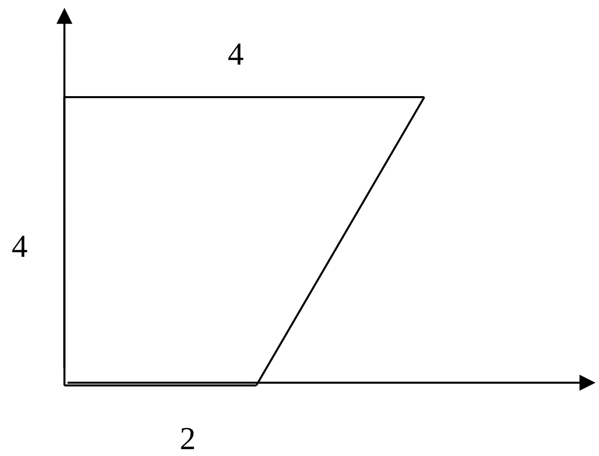
3) Newton-Cotes quadrature rules are derived by fixing the nodes and then determining the corresponding weights by the method of undetermined coefficients, so the degree is maximized for the given nodes. The opposite approach could also be taken, with the weights fixed and the nodes to be determined. In a Hermite quadrature rule, for example, all the weights are taken to have the same value, *w*.
Use the method of undetermined coefficients to determine the nodes and weight for a 3 point Hermite quadrature on the interval [-1,1]. Hermite polynomial basis is $\{1, 2t, 4t^2-2, 8t^3-12t\}$

4) Use two point Gauss quadrature to compute

a) $\int_0^6 \int_0^2 (-(x+3)^2 + y^2)dxdy$. Did you expect to find the exact solution? Assume no rounding error.

b) $\int_0^6 \int_0^2 e^{-(x^2+y^2)}dxdy$. Did you expect to find the exact solution? Assume no rounding error.

5) Map the integral $\iint_A xy\,dxdy$ from the quadrilateral region shown to the standard region [-1,1]x[-1,1] and then evaluate it analytically. Print the Jacobian



6) Map the quadrilateral region to the standard region and evaluate the following integrals using two point Gauss quadrature rule.

(a) $\iint_A (x+3)y\,dxdy$ over the region in problem 5.

(b) $\int_A e^{xy}dxdy$

over the following region: