Metaheuristic Optimization of Closed-Loop Supply Chain Networks Using Particle Swarm Optimization and Differential Evolution

Introduction

In the present competitive and environmentally conscious market, supply chains must balance cost reduction with sustainable practices. Traditional supply chains, focused mainly on forward logistics, often overlook the value recoverable from returned products. A closed-loop supply chain (CLSC) incorporating activities such as collection, screening, reworking, and remanufacturing. This not only reduces production costs but also conserves natural resources and minimizes environmental burden.

The CLSC model in this study consists of a primary manufacturer, retailer, customers, collection centre, and rework plant, with screening processes at both manufacturer and rework stages. Minor defectives are reworked to restore usability, while major defectives are segregated into remanufacturable or salvageable items. Given the complexity and nonlinear nature of such systems, metaheuristic approaches are highly effective. Particle Swarm Optimization (PSO) and Differential Evolution (DE) are applied to achieve cost minimization and ensure efficient resource utilization in the developed CLSC model.

Objectives

This study aims to develop and optimize a closed-loop supply chain model by minimizing costs through rework and remanufacturing. Particle Swarm Optimization (PSO) and Differential Evolution (DE) are applied to solve the cost minimization problem, with performance comparison ~~and sensitivity analysis~~ for effective decision-making.

Model Diagram

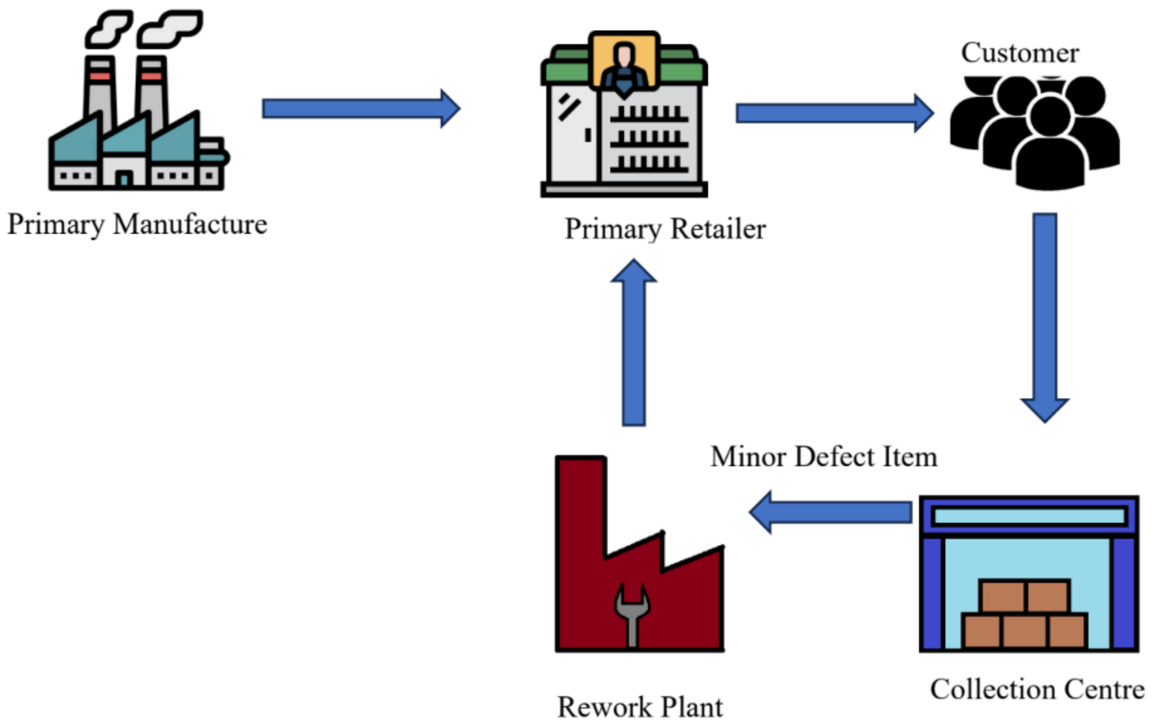


Fig. 1

Nomenclature

λ: Customer demand rate, r: rework rate, s_1: screened fraction retained, β: production rate, s_2: screened fraction retained from manufacture, μ: fraction of the customer's demand items collected by the collection centre, p: Fraction of collected items which are reworkable, T cycle length in month, C_{or} retailer order cost per cycle

Mathematical Model

Rework Plant

The inventory level for time $0 \leq t \leq T_1$

$$\frac{dI_r(t)}{dt} = rs_1 - \lambda$$

At $t = 0, I_m(0) = 0$

There is no rework from time $T_1 \leq t \leq T_2$.

Customer demand is fulfilled by the reworked item

$$\frac{dI_r(t)}{dt} = -\lambda$$

$I_r(T_2) = 0$ and $I_r(T_1) = I_r(T_1)$

Thus $I_r(t) = \begin{cases} (rs_1 - \lambda)t & 0 \leq t \leq T_1 \\ \lambda(T_2 - t) & T_1 \leq t \leq T_2 \end{cases}$

Now from $I_r(T_1) = I_r(T_1)$ we get $T_1 = \frac{\lambda T_2}{rs_1}$.

All the collected units from the end customer of the FSC at the collection centre is a fraction μ of the FSC customer's demand, i.e., $\mu\lambda T$, which is utilized by the CLSC

p fraction of the collected units i.e., $\mu\lambda p$ are rework

The total item reworked is $p\mu\lambda$.

Thus, total rework cost $= C_r \int_0^{T_1} p\mu\lambda\, dt = C_r p\mu\lambda T_1$.

Now holding cost of the reworked item

$$= h_r \left[\int_0^{T_1}(rs_1 - \lambda)t\, dt + \int_{T_1}^{T_2} \lambda(T_2 - t)\, dt\right] = \frac{h_r}{2}\left[rs_1T_1^2 - 2\lambda T_1T_2 + \lambda T_2^2\right]$$

Primary manufacturer

$$\frac{dI_m(t)}{dt} = \begin{cases} \beta s_2 & T_1 \leq t \leq T_2 \\ \beta s_2 - \lambda & T_2 \leq t \leq T_3 \\ -\lambda & T_3 \leq t \leq T \end{cases}$$

$$I_m(T_1) = 0, I_m(T_2) = I_m(T_2), I_m(T_3) = I_m(T_3), I_m(T) = 0$$

$$I_m(t) = \begin{cases} \beta s_2(t - T_1) \quad T_1 \le t \le T_2 \\ (\beta s_2 - \lambda)t - \beta s_2 T_1 + \lambda T_2 \quad T_2 \le t \le T_3 \\ \lambda(T - t) \quad T_3 \le t \le T \end{cases}$$

$$T_3 = \frac{\lambda(T - T_2) + \beta s_2 T_1}{\beta s_2}$$

Setup Cost of manufacture: C_{sm}

Production cost= $C_p \int_{T_1}^{T_3} \beta dt = C_p\beta(T_3 - T_1)$

Holding cost of manufacture

$$= C_{hm}\left[\int_{T_1}^{T_2} I_m(t)\, dt + \int_{T_2}^{T_3} I_m(t)\, dt + \int_{T_3}^{T} I_m(t) dt\right]$$

$$= C_{hm}\left[\frac{\beta s_2}{2}\{(T_2 - T_1)^2 + T_3^2 - T_2^2 + 2T_1(T_2 - T_3)\} + \lambda\left\{(T_3 - T)^2 - \frac{(T_3 - T_2)^2}{2}\right\}\right]$$

Thus total cost of manufacturer is

$$C_{sm} + C_p\beta(T_3 - T_1) + C_{hm}\left[\frac{\beta s_2}{2}\{(T_2 - T_1)^2 + T_3^2 - T_2^2 + 2T_1(T_2 - T_3)\} + \lambda\left\{(T_3 - T)^2 - \frac{(T_3 - T_2)^2}{2}\right\}\right]$$

Primary Retailer:

$\frac{dI_{re}(t)}{dt} = -\lambda,\ 0 \le t \le T$ where $I_{re}(T) = 0$.

Now the equation will reduce as: $I_{re}(t) = \lambda(T - t), 0 \le t \le T$

Retailer order cost per cycle= C_{or}

Total inventory of the primary retailer $I_{re} = \int_0^T I_{re}(t)dt = \lambda\frac{T^2}{2}$.

Inventory holding cost per cycle: $\lambda C_{hr}\frac{T^2}{2}$

Total cost of the primary retailer: $C_{or} + \lambda C_{hr}\frac{T^2}{2}$

Collection centre:

Collection cost of the used item with screening cost= $C_c\mu\lambda T$

The holding cost at the collection centre is: $h_c \int_0^T \mu\lambda t dt = h_c\lambda\mu\frac{T^2}{2}$

Total cost of the collection centre $TC_c = C_c\mu\lambda T + h_c\lambda\mu\frac{T^2}{2}$

Decision variable: T, α where $T_2 = \alpha T$

Objective function: Minimize $C_r p\mu\lambda T_1 + \frac{h_r}{2}[rs_1 T_1^2 - 2\lambda T_1 T_2 + \lambda T_2^2] + C_{sm} + C_p\beta(T_3 - T_1) + C_{hm}\left[\frac{\beta s_2}{2}\{(T_2 - T_1)^2 + T_3^2 - T_2^2 + 2T_1(T_2 - T_3)\} + \lambda\left\{(T_3 - T)^2 - \frac{(T_3 - T_2)^2}{2}\right\}\right] + C_{or} + \lambda C_{hr}\frac{T^2}{2} + C_c\mu\lambda T + h_c\lambda\mu\frac{T^2}{2}$

Subjected to constrain:

$$\beta s_2 - \lambda \geq 0$$

$$T - T_2 > 0$$

$$T_1 = \frac{\lambda T_2}{rs_1}$$

$$T_3 = \frac{\lambda(T - T_2) + \beta s_2 T_1}{\beta s_2} = \frac{\lambda(T - T_2)}{\beta s_2} + \frac{\lambda T_2}{rs_1}$$

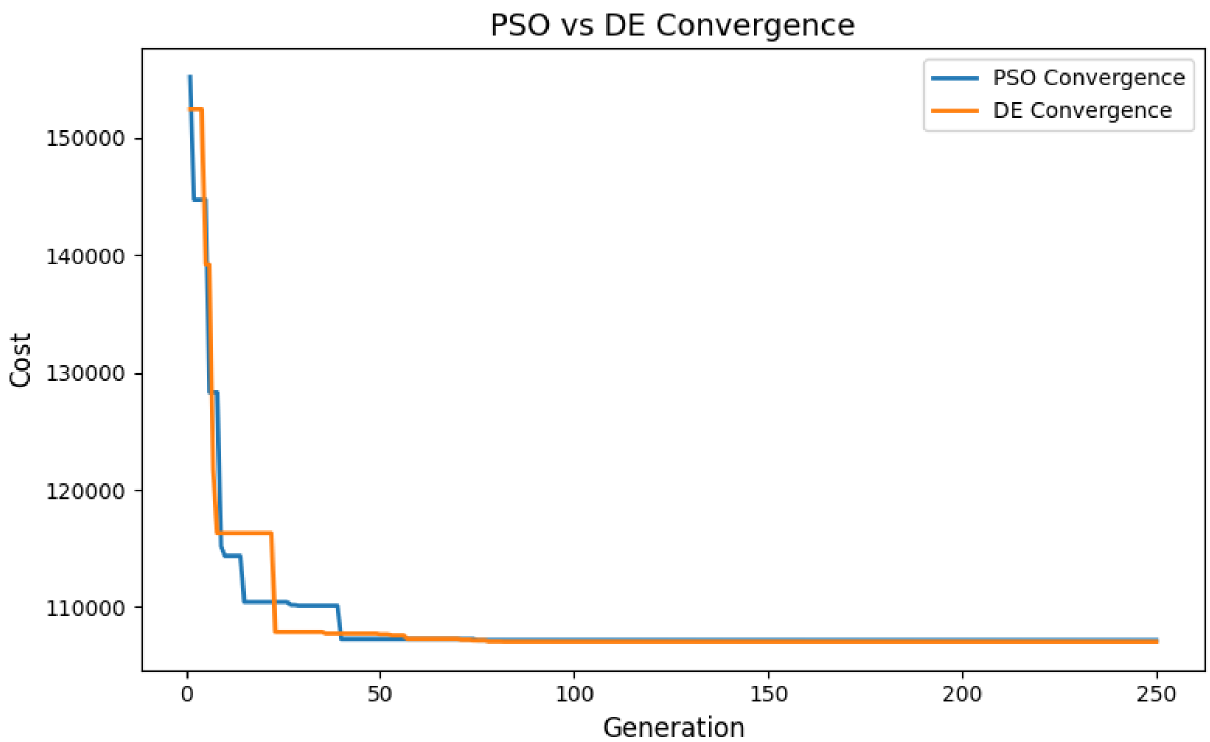
Numerical Analysis

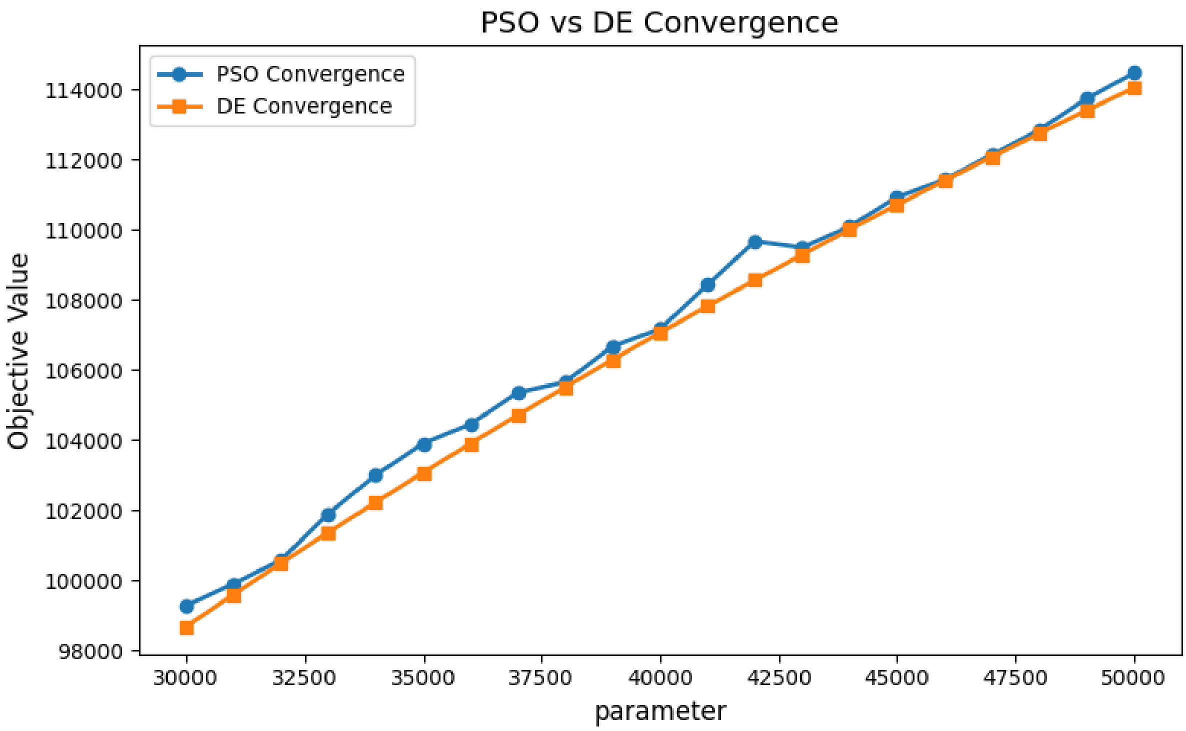
$s_1 = s_2 = 0.995, \lambda = 20000\ unit/month, \beta = 40000, \mu = 0.3, p = 0.4,$

$h_r = Rs\ 3\ per\frac{unit}{month},\ r = 30000, C_{sm} = 12000\ per\ cycle,$

$C_p = 12\ per\ unit, C_{hm} = 3.5\ per\ unit\ per\ month\ \ C_{or} = 10000\ per\ cycle,$

$C_{hr} = 3\ per\ unit\ per\ month, C_c = 7\ per\ unit, h_c = 2\ per\ month$





	PSO	DE
T	0.5000000006165148	0.5000000002697224
α	0.6027477201919046	0.603621728811301
Optimal Cost	107162.12182970179	107058.46275020837

Conclusion

The closed-loop supply chain problem is successfully solved using PSO and DE. Both algorithms achieved near-optimal solutions with comparable parameter values. PSO produced a cost of $\mathbf{107162.12}$, while DE achieved a slightly lower cost of $\mathbf{107058.46}$**,** demonstrating DE's superior performance in this case. Future work may focus on tackling uncertainties in demand, return rates, and defect levels, which significantly affect supply chain performance. Advanced learning-based methods, such as **Deep Q-Networks (DQN)**, can be explored to dynamically handle uncertainties and support real-time decision-making. Additionally, hybrid approaches combining metaheuristics with reinforcement learning may further enhance scalability, robustness, and adaptability of CLSC models.

References

1. Guide Jr, V. Daniel R., and Luk N. Van Wassenhove. "OR FORUM—The evolution of closed-loop supply chain research." *Operations research* 57.1 (2009): 10-18.
2. Maheshwari, Sumit, et al. "Sustainable inventory model for a three-layer supply chain using optimal waste management." *International Journal of System Assurance Engineering and Management* 14.Suppl 1 (2023): 216-235.

3. Kausar, Amrina, Ahmad Hasan, and Chandra K. Jaggi. "Sustainable inventory management for a closed-loop supply chain with learning effect and carbon emission under the multi-shipment policy." *International Journal of System Assurance Engineering and Management* 14.5 (2023): 1738-1755.
4. Suhandi, Victor, and Ping-Shun Chen. "Closed-loop supply chain inventory model in the pharmaceutical industry toward a circular economy." *Journal of Cleaner Production* 383 (2023): 135474.
5. Fleischmann, Moritz, et al. "Quantitative models for reverse logistics: A review." *European journal of operational research* 103.1 (1997): 1-17.
6. Nemtajela, Ndivhuwo, and Charles Mbohwa. "Relationship between inventory management and uncertain demand for fast moving consumer goods organisations." *Procedia Manufacturing* 8 (2017): 699-706.
7. Khalili-Fard, Alireza, et al. "Multi-objective optimization of closed-loop supply chains to achieve sustainable development goals in uncertain environments." *Engineering Applications of Artificial Intelligence* 133 (2024): 108052.
8. Mnih, Volodymyr, et al. "Playing atari with deep reinforcement learning." *arXiv preprint arXiv:1312.5602* (2013).