Class Test 2 (small signal analysis)

Eqns with the change

$$I_{ext}^r + i_{ext} = C\frac{d\left(V^r + v\right)}{dt} + \bar{g}_K\left(n^r + \eta\right)^4\left(V^r + v - E_K\right) + \bar{g}_L\left(V^r + v - E_L\right)$$

$$\frac{d\left(n^r + \eta\right)}{dt} = \left(\alpha^r + k_\alpha v\right)\left(1 - n^r - \eta\right) - \left(\beta^r + k_\beta v\right)\left(n^r + \eta\right)$$

$$\left(n^r + \eta\right)^4 \approx n^{r4} + 4n^{r3}\eta \quad \text{ignoring terms in } \eta^2, \eta^3, \text{and } \eta^4$$

$$I_{ext}^r = \bar{g}_K n^{r4}\left(V^r - E_K\right) + \bar{g}_L\left(V^r - E_L\right)$$

$$0 = \alpha^r\left(1 - n^r\right) - \beta^r n^r$$

$$i_{ext} \approx C\frac{dv}{dt} + \bar{g}_K n^{r4} v + 4\bar{g}_k n^{r3}\eta\left(V^r - E_K\right) + \bar{g}_L v$$

$$\frac{d\eta}{dt} \approx -\alpha^r\eta + k_\alpha\left(1 - n^r\right)v - \beta^r\eta - k_\beta n^r v$$

$$\begin{bmatrix}\dot{v}\\ \dot{\eta}\end{bmatrix} = \begin{bmatrix}-\left(\bar{g}_K n^{r4} + \bar{g}_L\right)/C & -4\bar{g}_K n^{r3}\left(V^r - E_K\right)/C\\ k_\alpha\left(1 - n^r\right) - k_\beta n^r & -\left(\alpha^r + \beta^r\right)\end{bmatrix}\begin{bmatrix}v\\ \eta\end{bmatrix} + \begin{bmatrix}i_{ext}/C\\ 0\end{bmatrix}$$

Now taking Laplace transform

$$\mathbf{i}_{\text{ext}}(s) = \left[Cs + \bar{g}_K n^{r4} + \bar{g}_L\right]\mathbf{v}(s) + 4\bar{g}_k n^{r3}\left(V^r - E_K\right)\mathbf{h}(s)$$

$$0 \approx \left[k_\alpha\left(1 - n^r\right) - k_\beta n^r\right]\mathbf{v}(s) - \left[s + \alpha^r + \beta^r\right]\mathbf{h}(s)$$

$$\mathbf{i}_{\text{ext}}(s) = \left\{Cs + \bar{g}_K n^{r4} + \bar{g}_L + \frac{4\bar{g}_k n^{r3}\left(V^r - E_K\right)\left[k_\alpha\left(1 - n^r\right) - k_\beta n^r\right]}{s + \alpha^r + \beta^r}\right\}\mathbf{v}(s)$$

[after eliminating h(s) above]. Now the RLC circuit currents in Laplace domain written as:

$$I = I_C + I_{R_1} + I_{LR_0}$$

$$\mathbf{I} = sC\,\mathbf{V} + \frac{1}{R_1}\mathbf{V} + \frac{\mathbf{V}}{sL + R_0}$$

$$\frac{1}{R_1} = \bar{g}_K n^{r4} + \bar{g}_L$$

So we get the values of L R_1 and R_0, C is same.

$$L = \frac{1}{4\bar{g}_K n^{r3}\left(V^R - E_K\right)\left[k_\alpha\left(1 - n^r\right) - k_\beta n^r\right]}$$

$$R_0 = \left(\alpha^r + \beta^r\right)L$$