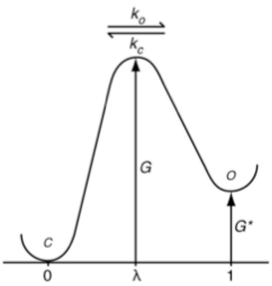
Class Test 1 (spring problem solution)



$$\Delta G_S = \int_{d_c}^{d_o} s\,\xi\, d\xi = s\left(\frac{d_o^2}{2} - \frac{d_c^2}{2}\right)$$

$$k_c = (\text{const})\, e^{-(G-G^{*}-(1-\lambda)\Delta G_S)/RT}$$

$$k_o = (\text{const})\, e^{-(G+\lambda \Delta G_S)/RT}$$

T is the total concentration of channels

$$\frac{dO}{dt} = -k_c O + k_o C = -k_c O + k_o(T-O) = k_o T - (k_o + k_c)O$$

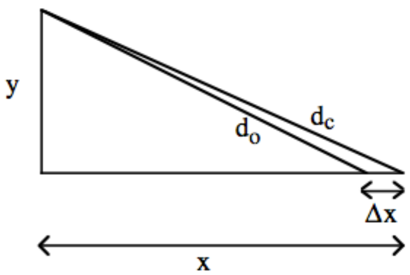
$$O = T\,\frac{k_o}{k_o + k_c}$$

so that

$$\text{prob. open} = \frac{O}{T} = \frac{(\text{const})\, e^{-(G+\lambda\Delta G_S)/RT}}{(\text{const})\, e^{-(G+\lambda\Delta G_S)/RT} + (\text{const})\, e^{-(G-G^{*}-(1-\lambda)\Delta G_S)/RT}}$$

Hence=>

$$\frac{O}{T} = \frac{1}{1+e^{(G^{*}+\Delta G_S)/RT}} = \frac{1}{1+e^{(G^{*}+s(d_o^2/2-d_c^2/2)/RT}}$$



$$d_c = \sqrt{x^2+y^2}$$

$$\begin{aligned} d_o &= \sqrt{(x-\Delta x)^2 + y^2} \\ &= \sqrt{x^2+y^2-2x\Delta x+\Delta x^2} \\ &= \sqrt{x^2+y^2}\sqrt{1-\frac{2x\Delta x-\Delta x^2}{x^2+y^2}} \\ &\approx \sqrt{x^2+y^2}\left(1-\frac{x\Delta x}{x^2+y^2}\right) = d_c\left(1-\frac{x\Delta x}{d_c^2}\right) = d_c - \frac{x\Delta x}{d_c} \end{aligned}$$

$$s\left(d_o^2/2 - d_c^2/2\right) = \frac{s}{2}(d_o+d_c)(d_o-d_c) \approx -s\, d_c\, x\Delta x / d_c = -s\, x\Delta x$$

Now system is in steady state at t=0^-

$O(0^-)$

$$\frac{O}{T} = \frac{1}{1+e^{(\Delta G^{*}-s\,x\Delta x)/RT}}$$

Cilia are now moved apart to x_1, thus

$$\frac{dO}{dt} = k_o(x_1 T - \left[k_o(x_1) + k_c(x_1)\right]O$$

$$\begin{aligned} \left.\frac{dO}{dt}\right|_{t=0^+} &= k_o(x_1)T - \left[k_o(x_1)+k_c(x_1)\right]T\,\frac{k_o(x_0)}{k_o(x_0)+k_c(x_0)} \\ &= \frac{k_o(x_1)k_c(x_0) - k_o(x_0)k_c(x_1)}{k_o(x_0)+k_c(x_0)}T \end{aligned}$$