Number Systems Class 9 Extra Questions Maths Chapter 1

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Extra Questions for Class 9 Maths Chapter 1 Number Systems

Question-1

If a is a positive rational number and n is a positive integer greater than 1, prove that a^n is a rational number.

Solution:

We know that product of two rational number is always a rational number.
Hence if $\boldsymbol{a}$ is a rational number then

$a^2 = a \times a$ is a rational number,

$a^3 = a^2 \times a$ is a rational number.

$\therefore a^n = a^{n-1} \times a$ is a rational number.

Write the following rational numbers in decimal form:

(i) $\frac{42}{100}$ (ii) $\frac{327}{500}$ (iii) $3\frac{3}{8}$ (iv) $\frac{5}{6}$ (v) $\frac{1}{5}$ (vi) $\frac{1}{7}$ (vii) $\frac{2}{13}$ (viii) $\frac{11}{17}$

Solution:

(i) $\frac{42}{100} = 0.42$

(ii) $\frac{327}{500} = 0.654$

```
     0. 654
5 | 3.27
  | 30
  | 27
  | 25
  |   20
  |   20
  |    0
```

(iii) $3\frac{3}{8} = \frac{27}{8} = 3.375$

```
     3.375
8 | 27
  | 24
  |  30
  |  24
  |   60
  |   56
  |    40
  |    40
  |     0
```

(iv) $\frac{5}{6} = 0.833\ldots = 0.8\overline{333}$

```
     0.8333
6 | 50
  | 30
  |  20
  |  18
  |   20
  |   18
  |    20
  |    18
  |     2
```

(v) $\frac{1}{5} = 0.2$

$$
\begin{array}{r|l}
 & 0.2 \\
\hline
5 & 10 \\
 & \underline{10} \\
 & 0
\end{array}
$$

(vi) $\frac{1}{7} = 0.\overline{142857}$

$$
\begin{array}{r|l}
 & 0.\overline{142857} \\
\hline
7 & 10 \\
 & \underline{7} \\
 & 30 \\
 & \underline{28} \\
 & 20 \\
 & \underline{14} \\
 & 60 \\
 & \underline{56} \\
 & 40 \\
 & \underline{35} \\
 & 50 \\
 & \underline{49} \\
 & 1
\end{array}
$$

(vii) $\frac{2}{13} = 0.\overline{153846}$

$$
\begin{array}{r|l}
 & 0.\overline{153846} \\
\hline
13 & 20 \\
 & \underline{13} \\
 & 70 \\
 & \underline{65} \\
 & 50 \\
 & \underline{39} \\
 & 110 \\
 & \underline{104} \\
 & 60 \\
 & \underline{52} \\
 & 80 \\
 & \underline{78} \\
 & 2
\end{array}
$$

(viii) $\frac{11}{17} = 0.\overline{6470588235294117}$

$0.\overline{6470588235294117}$

17 | 110

102

80

68

120

119

100

85

150

136

40

34

60

51

90

85

50

34

160

153

70

68

20

17

30

17

130

119

11

Question-3

Give three rational numbers lying between $\frac{1}{3}$ and $\frac{1}{2}$.

Solution:

The rational number lying between is $\frac{1}{3}$ and $\frac{1}{2}$.

$$\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}\right) = \frac{1}{2}\left(\frac{1\times2+1\times3}{3\times2}\right) = \frac{1}{2}\left(\frac{2+3}{6}\right) = \frac{1}{2}\times\frac{5}{6} = \frac{5}{12}$$

Therefore, $\frac{1}{3} < \frac{5}{12} < \frac{1}{2}$.

Now, the rational number lying between $\frac{1}{3}$ and $\frac{5}{12}$ is

$$\frac{1}{2}\left(\frac{1}{3}+\frac{5}{12}\right) = \frac{1}{2}\left(\frac{4}{12}+\frac{5}{12}\right) = \frac{1}{2}\left(\frac{4+5}{12}\right) = \frac{1}{2}\times\frac{9}{12} = \frac{9}{24} = \frac{3}{8}$$

Therefore, $\frac{1}{3} < \frac{3}{8} < \frac{5}{12}$.

The rational number lying between $\frac{5}{12}$ and $\frac{1}{2}$. is

$$\frac{1}{2}\left(\frac{5}{12}+\frac{1}{2}\right) = \frac{1}{2}\left(\frac{5}{12}+\frac{6}{12}\right) = \frac{1}{2}\left(\frac{5+6}{12}\right) = \frac{1}{2}\times\frac{11}{12} = \frac{11}{24}$$

Therefore, $\frac{5}{12} < \frac{11}{24} < \frac{1}{2}$

Hence the three rational numbers lying between $\frac{1}{3}$ and $\frac{1}{2}$ are $\frac{3}{8}, \frac{5}{12}, \frac{11}{24}$.

Question-4

Insert a rational and an irrational number between 2 and 3.

Solution:

A rational number between 2 and 3 = $\frac{2+3}{2}$ = 2.5

An irrational number between 2 and 3 is $\sqrt{5}$.

Question-5

How many rational numbers and irrational numbers can be inserted between 2 and 3 ?

Solution:

There are infinite number of rational and irrational numbers between 2 and 3.

Question-6

Find three rational numbers lying between 0 and 0.1. Find twenty rational numbers between 0 and 0.1. Give a method to determine any number of rational numbers between 0 and 0.1.

Solution:

The three rational numbers lying between 0 and 0.1 are 0.01, 0.05, 0.09.

The twenty rational numbers between 0 and 0.1 are 0.001, 0.002, 0.003, 0.004, ... 0.011, 0.012, ... 0.099.

To determine any number of rational numbers between 0 and 0.1 insert the square root of its product.

i.e. The rational numbers between ***a*** and ***b*** is $\sqrt{a \times b}$

Question-7

Find three rational numbers lying between $-\frac{2}{5}, -\frac{1}{5}$.

Solution:

The rational number lying between $-\frac{2}{5}$ and $-\frac{1}{5}$

$$\frac{1}{2}\left(\frac{-2}{5}+\frac{-1}{5}\right)=\frac{1}{2}\left(\frac{-2-1}{5}\right)=\frac{1}{2}\times\frac{-3}{5}=\frac{-3}{10}$$

Therefore, $-\frac{2}{5} < -\frac{3}{10} < -\frac{1}{5}$

The rational number lying between $-\frac{2}{5}$ and $-\frac{3}{10}$

$$\frac{1}{2}\left(\frac{-2}{5}+\frac{-3}{10}\right)=\frac{1}{2}\left(\frac{-4}{10}+\frac{-3}{10}\right)=\frac{1}{2}\times\left(\frac{-4-3}{10}\right)=\frac{1}{2}\times\frac{-7}{10}=\frac{-7}{20}$$

Therefore, $-\frac{2}{5} < \frac{-7}{20} < -\frac{3}{10}$

The rational number lying between $-\frac{3}{10}$ and $-\frac{1}{5}$

$$\frac{1}{2}\left(\frac{-3}{10}+\frac{-1}{5}\right)=\frac{1}{2}\left(\frac{-3}{10}+\frac{-2}{10}\right)=\frac{1}{2}\times\left(\frac{-3-2}{10}\right)=\frac{1}{2}\times\frac{-5}{10}=\frac{-5}{20}=\frac{-1}{4}$$

Therefore, $-\frac{3}{10} < \frac{-1}{4}. < -\frac{1}{5}$

Therefore the three rational numbers are $\frac{-3}{10}, \frac{-7}{20}$ *and* $\frac{-1}{4}$.

Question-8

Complete the following:

(i) Every point on the number line corresponds to a ………………. number which may be either ………………….. or ……………………

(ii) The decimal form of an irrational number is neither ……………… nor ……………………

(iii) The decimal representation of the rational number $\frac{8}{27}$ is …………………

(iv) 0 is …………….. number. [Hint: a rational /an irrational]

Solution:

(i) Every point on the number line corresponds to a <u>real</u> number which may be either <u>rational</u> or <u>irrational.</u>

(ii) The decimal form of an irrational number is neither <u>recurring</u> nor <u>terminating.</u>

(iii) The decimal representation of the rational number $\frac{8}{27}$ is <u>0.296</u>

(iv) 0 is <u>a rational</u> number.

Question-9

Give an example for each, if two irrational numbers, whose

(i) difference is a rational number.
(ii) difference is a irrational number.
(iii) sum is a rational number.
(iv) sum is an irrational number.
(v) product is an irrational number,
(vi) product is an irrational number.
(vii) quotient is a rational number.
(viii) quotient is an irrational number.

Solution:

(i) $2+\sqrt{5}$, and $3+\sqrt{5}$
(ii) $10+\sqrt{3}$, $5-\sqrt{3}$
(iii) $3+\sqrt{2}$, $3-\sqrt{2}$
(iv) $5\sqrt{3}$, $3\sqrt{3}$
(v) $3-\sqrt{5}$, $5+\sqrt{3}$
(vi) $5\sqrt{8}$, $\sqrt{2}$
(vii) $2\sqrt{3}$, $\sqrt{3}$
(viii) $2\sqrt{10}$, $2\sqrt{2}$

Which of the following rational numbers have the terminating decimal representation?

(i) 3/5 (ii) 7/20 (iii) 2/13

(iv) 27/40 (v) 133/125 (vi) 23/7

[Making use of the result that a rational number *p/q* where *p* and *q* have no common factor(s) will have a terminating representation if and only if the prime factors of *q* are 2's or 5's or both.]

Solution:

(i) The prime factor of 5 is 5. Hence 3/5 has a terminating decimal representation.

(ii) $20 = 4 \times 5 = 2^2 \times 5$.

The prime factors of 20 are both 2's and 5's. Hence 7/20 has a terminating decimal.

(iii) The prime factor of 13 is 13. Hence 2/13 has non- terminating decimal.

(iv) $40 = 2^3 \times 5$.

The prime factors of 40 are both 2's and 5's. Hence 27/40 has a terminating decimal.

(v) $125 = 5^3$

The prime factor of 125 is 5's. Hence 133/125 has a terminating decimal.

(vi) The prime factor of 7 is 7. Hence 23/7 has a non-terminating decimal representation.

Question-11

Find the decimal representation of 1/7, 2/7. Deduce from the decimal representation of 1/7, without actual calculation, the decimal representation of 3/7, 4/7, 5/7 and 6/7.

Solution:

$$\begin{array}{r|l} & 0.\overline{142857} \\ \hline 7 & 10000000 \\ & \underline{07} \\ & 030 \\ & \underline{028} \\ & 0020 \\ & \underline{0014} \\ & 00060 \\ & \underline{00056} \\ & 000040 \\ & \underline{000035} \\ & 0000050 \\ & \underline{0000042} \\ & 0000008 \end{array}$$

The decimal representation of 1/7 $= 0.\overline{142857}$.

$$\begin{array}{r|l} & 0.\overline{285714} \\ \hline 7 & 2.000000 \\ & \underline{14} \\ & 060 \\ & \underline{056} \\ & 0040 \\ & \underline{0035} \\ & 00050 \\ & \underline{00049} \\ & 000010 \\ & \underline{000007} \\ & 0000030 \\ & \underline{0000028} \\ & 0000002 \end{array}$$

The decimal representation of 2/7 $= 0.\overline{285714}$

We observe that $2/7 = 2(1/7) = 2(0.\overline{142857}) = 0.\overline{285714}$

Therefore the decimal representation of

$3/7 = 3(1/7) = 3(0.\overline{142857}) = 0.\overline{428571}$

$4/7 = 4(1/7) = 4(0.\overline{142857}) = 0.\overline{571428}$

$5/7 = 5(1/7) = 5(0.\overline{142857}) = 0.\overline{714285}$

$6/7 = 6(1/7) = 6(0.\overline{142857}) = 0.\overline{857142}$

Question-12

Express 0.6666.........in the form of $\frac{p}{q}$

Solution:

Let $x = 0.6666..........$ ...(i)

$\therefore 10x = 6.\overline{6666}........$ ...(ii)

(ii) − (i), $9x = 6$ $\therefore x = \frac{6}{9} = \frac{2}{3}$

$\therefore 0.6666 = \frac{2}{3}$

Question-13

If *a* and *b* are two rational numbers, prove that *a + b*, *a - b*, *ab* are rational numbers. If *b* ≠ 0, show that *a/b* is also a rational number.

Solution:

Let ***a = p/q*** where ***q ≠ 0*** and ***b = r/s*** where ***s*** ≠ 0 be the rational numbers then

(i) $a + b = \frac{p}{q} + \frac{r}{s} = \frac{ps + rq}{qs}$

where ***q s*** ≠ 0, since ***q*** ≠ 0 and ***s*** ≠ 0.

Also ***p s + r q*** is an integer.

Hence ***a + b*** is a rational number.

(ii) $a - b = \frac{p}{q} - \frac{r}{s} = \frac{ps - qr}{qs}$

where ***q s*** ≠ 0, since ***q*** ≠ 0 and ***s*** ≠ 0.

Also ***p s - r q*** is an integer

Hence ***a - b*** is a rational number.

(iii) $ab = \frac{p}{q} \times \frac{r}{s} = \frac{pr}{qs}$

where ***q s*** ≠ 0, since ***q*** ≠ 0 and ***s*** ≠ 0.
Also ***pr*** is an integer.
Hence ***ab*** is a rational number.

(iv) Since ***b*** ≠ 0, we have ***r/s*** ≠ 0 thus ***r*** ≠ 0 and ***s*** ≠ 0.
$\frac{a}{b} = \frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \times \frac{s}{r} = \frac{ps}{qr}$

where ***q*** ≠ 0 and ***r*** ≠ 0.
Also ***ps*** is an integer
Hence ***a/b*** is a rational number.

Question-14

Express 0.272727.........in the form of $\frac{p}{q}$.

Solution:
Let x = 0.272727.......... ...(i) ∴ 100x = 27.2727..... ...(ii)

(ii) − (i), 99x = 27 ∴ $x = \frac{27}{99} = \frac{3}{11}$
∴ 0.272727.........= $\frac{3}{11}$

Question-15

You have seen that √2 is not a rational number. Show that 2 + √2 is not a rational number.

Solution:
Let 2 + √2 be a rational number say ***r***.
Then 2 + √2 = ***r***
√2 = ***r*** -2
But, √2 is an irrational number.
Therefore, ***r*** - 2 is also an irrational number.
=> ***r*** is an irrational number.
Hence our assumption ***r*** is a rational number is wrong.

Question-16

Express 3.7777.......... in the form of $\frac{p}{q}$.

Solution:

Let x = 3.7777.......... ...(i)

$\therefore 10x = 37.\overline{777......}$ ...(ii)

(ii) − (i), $9x = 34$ $\therefore x = \frac{34}{9}$ $\therefore$ 3.7777......... = $\frac{34}{9}$

Question-17

Prove that 3√3 is not a rational number.

Solution:

Let 3√3 be a rational number say *r* .

Then 3√3 = ***r***

√3 = (1/3) ***r***

(1/3) ***r*** is a rational number because product of two rational number is a rational number.

=> √3 is a rational number but √3 is not a rational number.

Therefore our assumption 3√3 is a rational number is wrong.

Question-18

Express 18.484848................in the form of $\frac{p}{q}$.

Solution:

Let x = 18.484848...(i) $\therefore 100x$ = 1848.4848..............................(ii)

(ii) − (i), $99x = 1830$ $\therefore$ $x = \frac{1830}{99} = \frac{610}{33}$ $\therefore$ 18.484848.........= $\frac{610}{33}$

Question-19

Show that $\sqrt[3]{6}$ is not a rational number.

Solution:

Let $\sqrt[3]{6}$ be a rational number, say $\frac{p}{q}$ where q ≠ 0.

Then $\sqrt[3]{6} = \frac{p}{q}$

Since $1^3 = 1$, and $2^3 = 8$, it follows that $1 < \frac{p}{q} < 2$

Then $q > 1$ because if $q = 1$ then $\frac{p}{q}$ will be an integer, and there is no integer between 1 and 2.

Now, $6 = \left(\frac{p}{q}\right)^3$

$6 = \frac{p^3}{q^3}$

$6q^2 = \frac{p^3}{q}$

q being an integer, $6q^2$ is an integer, and since $q > 1$ and q does not have a common factor with p and consequently with p^3. So, $\frac{p^3}{q}$ is a fraction, different from an integer.

Thus $6q^2 \neq \frac{p^3}{q}$.

This contradiction proves the result.

Question-20

Identify the following as rational or irrational numbers. Give the decimal representation of rational numbers.

(i) $\sqrt{4}$

(ii) 3 $\sqrt{18}$

(iii) $\sqrt{1.44}$

(iv) $\sqrt{\frac{9}{27}}$

(v) - $\sqrt{.64}$

(vi) $\sqrt{100}$

Solution:

(i) $\sqrt{4} = 2$ is rational.

(ii) $3\sqrt{18} = 3\sqrt{9\times 2} = 3 \times 3\sqrt{2} = 9\sqrt{2}$ is irrational.

(iii) $\sqrt{1.44} = \sqrt{\frac{144}{100}} = \frac{12}{10} = 1.2$ is rational.

(iv) $\sqrt{\frac{9}{27}} = \sqrt{\frac{9}{9\times 3}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$ is irrational.

(v) $-\sqrt{.64} = -0.8$ is rational.

(vi) $\sqrt{100} = 10$ is rational.

Question-21

Find two irrational numbers between 2 and 2.5.

Solution:

The two irrational numbers between 2 and 2.5 are 2.101001000100001….. and 2.201001000100001…..

Question-22

In the following equations, find which of the variables x, y, z etc. represent rational numbers and which represent irrational numbers:

(i) $x^2 = 5$
(ii) $y^2 = 9$
(iii) $z^2 = 0.04$
(iv) $u^2 = \frac{17}{4}$
(v) $v^2 = 3$
(vi) $w^3 = 27$
(vii) $t^2 = 0.4$

Solution:

(i) $x^2 = 5$
$\therefore x = \sqrt{5}$ is irrational.

(ii) $y^2 = 9$
$\therefore y = 3$ is rational.

(iii) $z^2 = 0.04$
$\therefore z = 0.2$ is rational.

(iv) $u^2 = \frac{17}{4}$
$\therefore u = \sqrt{\frac{17}{4}}$
$= \frac{\sqrt{17}}{\sqrt{4}} = \frac{\sqrt{17}}{2}$ is irrational.

(v) $v^2 = 3$
$\therefore v = \sqrt{3}$ is irrational.
(vi) $w^3 = 27$ $\therefore w = \sqrt[3]{27} = 3$ is rational.

(vii) $t^2 = 0.4$ $\therefore t = \sqrt{0.4} = \sqrt{\frac{4}{10}} = \frac{2}{\sqrt{10}}$ is irrational.

Question-23

Find two irrational numbers between 0.1 and 0.12.

Solution:

The two irrational numbers between 0.1 and 0.12 are 0.1010010001… and 0.1101001000100001……

Question-24

Give an example to show that the product of a rational number and an irrational number may be a rational number.

Solution:

A rational number 0 multiplied by an irrational number gives the rational number 0.

Question-25

Give two irrational number lying between $\sqrt{2}$ and $\sqrt{3}$.

Solution:

The two irrational number lying between $\sqrt{2}$ and $\sqrt{3}$ are $\sqrt{2.1}$ and $\sqrt{2.2}$

Question-26

State with reason which of the following are surds and which are not.

(i) $\sqrt{5} \times \sqrt{10}$ (ii) $\sqrt{8} \times \sqrt{6}$ (iii) $\sqrt{27} \times \sqrt{3}$ (iv) $\sqrt{16} \times \sqrt{4}$ (v) $5\sqrt{8} \times 2\sqrt{6}$

(vi) $\sqrt{125} \times \sqrt{5}$ (vii) $\sqrt{100} \times \sqrt{2}$ (viii) $6\sqrt{2} \times 9\sqrt{3}$ (ix) $\sqrt{120} \times \sqrt{45}$ (x) $\sqrt{15} \times$

Solution:

(i) $\sqrt{5} \times \sqrt{10} = \sqrt{5} \times \sqrt{5 \times 2} = \sqrt{5} \times \times = 5\sqrt{2}$ is a surd.

(ii) $\sqrt{8} \times \sqrt{6} = \sqrt{4 \times 2} \times \sqrt{3 \times 2} = 2 \times\times\times$

$= 4\sqrt{3}$ is a surd.

(iii) $\sqrt{27} \times \sqrt{3} = \sqrt{9 \times 3} \times \sqrt{3} = 3\sqrt{3} \times \sqrt{3} = 9$ is not a surd.

(iv) $\sqrt{16} \times \sqrt{4} = 4 \times 2 = 8$ is not a surd.

(v) $5\sqrt{8} \times 2\sqrt{6} = 5\sqrt{4 \times 2} \times 2\sqrt{3 \times 2} = 5 \times 2\sqrt{2} \times 2\sqrt{2} \times$

$= 5 \times 2 \times 2 \times 2 \times$

$= 40\sqrt{3}$ is a surd.

(vi) $\sqrt{125} \times = \sqrt{25 \times 5} \times = 5\sqrt{5} \times = 5 \times 5 = 25$ is not a surd.

(vii) $\sqrt{100} \times \sqrt{2} = 10\sqrt{2}$ is a surd.

(viii) $6\sqrt{2} \times 9\sqrt{3} = 54\sqrt{6}$ is a surd.

(ix) $\sqrt{120} \times \sqrt{45} = \sqrt{4 \times 30} \times$

$= 2\sqrt{6 \times 5} \times 3\sqrt{5}$

$= 2 \times\times\times 3 \times$

$= 30\sqrt{6}$ is a surd.

(x) $\sqrt{15} \times \sqrt{6} = \sqrt{5 \times 3} \times \sqrt{2 \times 3} = \sqrt{5} \times \sqrt{3} \times \sqrt{2} \times \sqrt{3}$

$= 3\sqrt{10}$ is a surd.

Question-27

Express the following recurring decimals into vulgar fractions :

(a) $0.\bar{6}$ (b) $0.\overline{16}$ (c) $0.\overline{234}$ (d) $0.12\overline{54}$

Solution:

(a) $0.\bar{6}$ = 0.666.... ....(1)

10 × $0.\bar{6}$ = 6.666... .... (2)

(2) - (1) ⇒ 9 × $0.\bar{6}$ = 6

∴ $0.\bar{6}$ = 6/9 = 2/3

(b) $0.\overline{16}$ = 0.161616..... ---(1)

100 × $0.\overline{16}$ = 16.1616..... -----(2)

Subtracting (1) from (2) , we get

99 ´ $0.\overline{16}$ = 16

∴ $0.\overline{16}$ = 16/99

(c) $0.\overline{234}$ = 0.234234..... ----(i)

1000 × $0.\overline{234}$ = 234.234234... -----(ii)

Subtracting (i) from (ii) , we get

999 × $0.\overline{234}$ = 234

∴ $0.\overline{234}$ = 234/999.

(d) $0.12\overline{54}$ = 0.125454.... -----(i)

∴ 100 × $0.12\overline{54}$ = 12.545454.... -----(ii)

And, 10000 ´ $0.12\overline{54}$ = 1254.5454... -----(iii)

Subtracting (ii) from (iii) , we get

9900 × $0.12\overline{54}$ = 1242

∴ $0.12\overline{54}$ = 1242/9900 = 69/550.

Question-28

Find the values of *a* and *b* if $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$

Solution:

$\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$ [Multiplying the numerator and the denominator by ($\sqrt{3}$ -1)]

$$= \frac{3-2\sqrt{3}+1}{3-1} = \frac{4-2\sqrt{3}}{2} = 2 - \sqrt{3}$$

It is given that $2 - \sqrt{3} = a + b\sqrt{3}$

$\therefore$ ***a*** = 2 and ***b*** = -1.

Question-29

Give two examples to show that the product of two irrational numbers may be a rational number.

Solution:

Take ***a*** = $(2+\sqrt{3})$ and ***b*** = $(2-\sqrt{3})$; ***a*** and ***b*** are irrational numbers, but their product = 4 - 3 = 1,
is a rational number.
Take ***c*** = $\sqrt{3}$ and ***d*** = $-\sqrt{3}$; ***c*** and ***d*** are irrational numbers, but their product = -3,
is a rational number.

Question-30

Find the values of *a* and *b* if $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$

Solution:

$\frac{3+\sqrt{2}}{3-\sqrt{2}} = \frac{3+\sqrt{2}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$ [Multiplying the numerator and the denominator by $(3 + \sqrt{2})$]

$= \frac{9+6\sqrt{2}+2}{9-2}$

$= \frac{11+6\sqrt{2}}{7}$

It is given that $\frac{11+6\sqrt{2}}{7}$ **= *a* + *b*** $\sqrt{2}$

$\therefore$ ***a*** = $\frac{11}{7}$ and ***b*** = $\frac{6}{7}$

Question-31

Find the value of $\sqrt{5}$ correct to two places of decimal.

Solution:

We know that $2^2 = 4 < 5 < 9 = 3^2$
Taking positive square roots we get
$2 < \sqrt{5} < 3$.

Next, $(2.2)^2 = 4.84 < 5 < 5.29 = (2.3)^2$
Taking positive square roots, we have
$2.2 < \sqrt{5} < 2.3$

Again, $(2.23)^2 = 4.9729 < 5 < 5.0176 = (2.24)^2$
Taking positive square roots, we obtain
$2.23 < \sqrt{5} < 2.24$
Hence the required approximation is 2.24 as $(2.24)^2$ is nearest to 5 than $(2.23)^2$.

Question-32

Find the values of a and b if $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$

Solution:

$\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$ [Multiplying the numerator and the denominator by 7 - 4 $\sqrt{3}$]

$$= \frac{35 - 20\sqrt{3} + 14\sqrt{3} - 24}{49 - 48}$$

$$= \frac{11 - 6\sqrt{3}}{1}$$

It is given that $\frac{11-6\sqrt{3}}{1} = a + b\sqrt{3}$

$\therefore$ $a = 11$ and $b = -6$.

Question-33

Prove that $\sqrt{3} - \sqrt{2}$ is irrational.

Solution:

Let $\sqrt{3} - \sqrt{2}$ be a rational number, say r

Then $\sqrt{3} - \sqrt{2} = r$

On squaring both sides we have

$$(\sqrt{3} - \sqrt{2})^2 = r^2$$
$$3 - 2\sqrt{6} + 2 = r^2$$
$$5 - 2\sqrt{6} = r^2$$
$$-2\sqrt{6} = r^2 - 5$$
$$\sqrt{6} = -(r^2 - 5)/2$$

Now $-(r^2 - 5)/2$ is a rational number and $\sqrt{6}$ is an irrational number. Since a rational number cannot be equal to an irrational number. Our assumption that $\sqrt{3} - \sqrt{2}$ is rational is wrong.

Question-34

Find the values of *a* and *b* if $\frac{5+\sqrt{6}}{5-\sqrt{6}} = a + b\sqrt{6}$

Solution:

$\frac{5+\sqrt{6}}{5-\sqrt{6}} = \frac{5+\sqrt{6}}{5-\sqrt{6}} \times \frac{5+\sqrt{6}}{5+\sqrt{6}}$ [Multiplying the numerator and the denominator by 5 + $\sqrt{6}$]

$$= \frac{25+10\sqrt{6}+6}{25-6}$$

$$= \frac{25+10\sqrt{6}+6}{25-6}$$

$$= \frac{31+10\sqrt{6}}{19}$$

It is given that $\frac{31+10\sqrt{6}}{19} = \boldsymbol{a + b}\sqrt{6}$

$\therefore$ $\boldsymbol{a} = \frac{31}{19}$ and $\boldsymbol{b} = \frac{10}{19}$

Question-35

Prove that $\sqrt{3} + \sqrt{5}$ is an irrational number.

Solution:

Let $\sqrt{3} + \sqrt{5}$ be a rational number, say $\boldsymbol{r}$

Then $\sqrt{3} + \sqrt{5} = \boldsymbol{r}$

On squaring both sides,

$(\sqrt{3} + \sqrt{5})^2 = \boldsymbol{r}^2$

$3 + 2\sqrt{15} + 5 = \boldsymbol{r}^2$

$8 + 2\sqrt{15} = \boldsymbol{r}^2$

$2\sqrt{15} = \boldsymbol{r}^2 - 8$

$\sqrt{15} = (\boldsymbol{r}^2 - 8)/2$

Now $(\boldsymbol{r}^2 - 8)/2$ is a rational number and $\sqrt{15}$ is an irrational number.

Since a rational number cannot be equal to an irrational number. Our assumption that

$\sqrt{3} + \sqrt{5}$ is rational is wrong.

Question-36

Find the values of *a* and *b* if $\frac{3+\sqrt{7}}{3-4\sqrt{7}} = a + b\sqrt{7}$

Solution:

$\frac{3+\sqrt{7}}{3-4\sqrt{7}} = \frac{3+\sqrt{7}}{3-4\sqrt{7}} \times \frac{3+4\sqrt{7}}{3+4\sqrt{7}}$ [Multiplying the numerator and the denominator by $3 + 4\sqrt{7}$]

$$= \frac{9 + 12\sqrt{7} + 3\sqrt{7} + 28}{9 - 112}$$

$$= \frac{15\sqrt{7} + 37}{-103}$$

It is given that $\frac{3+\sqrt{7}}{3-4\sqrt{7}} = a + b\sqrt{7}$

$$\frac{15\sqrt{7} + 37}{-103} = a + b\sqrt{7}$$

$$\therefore a = \frac{-15}{103} \text{ and } b = \frac{-37}{103}$$

Question-37

If $\frac{\sqrt{7}-1}{\sqrt{7}+1} - \frac{\sqrt{7}+1}{\sqrt{7}-1} = a + b\sqrt{7}$, find the values of *a* and *b*.

Solution:

$$\frac{\sqrt{7}-1}{\sqrt{7}+1} - \frac{\sqrt{7}+1}{\sqrt{7}-1} = \frac{(\sqrt{7}-1)^2 - (\sqrt{7}+1)^2}{(\sqrt{7}+1)(\sqrt{7}-1)}$$

$$= \frac{-4\sqrt{7}}{6}$$

$$= \frac{-2}{3}\sqrt{7}$$

It is given that $\frac{\sqrt{7}-1}{\sqrt{7}+1} - \frac{\sqrt{7}+1}{\sqrt{7}-1} = a + b\sqrt{7}$

$$\therefore \frac{-2}{3}\sqrt{7} = a + b\sqrt{7}$$

$$\therefore a = 0 \text{ and } b = \frac{-2}{3}$$

Question-38

Give two rational numbers lying between 0.232332333233332... and 0.212112111211111.....

Solution:
The two rational numbers are 0.222. and 0.221

Question-39

Simplify by rationalising the denominator: $\frac{1}{\sqrt{6}-\sqrt{5}}$

Solution:
$\frac{1}{\sqrt{6}-\sqrt{5}} = \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}$ [Rationalising the denominator]

$= \frac{\sqrt{6}+\sqrt{5}}{6-5} = \sqrt{6}+\sqrt{5}$

Question-40

Examine, whether the following numbers are rational or irrational:

(i) $(\sqrt{2}+2)^2$

(ii) $(2-\sqrt{2})x(2+\sqrt{2})$

(iii) $(\sqrt{2}+\sqrt{3})^2$

(iv) $\frac{6}{3\sqrt{2}}$

Solution:

(i) $(\sqrt{2}+2)^2 = (\sqrt{2})^2 + 2\sqrt{2}x2 + (2)^2 = 2 + 4\sqrt{2} + 4 = 6 + 4\sqrt{2}$.

$\therefore$ It is an irrational number.

(ii) $(2-\sqrt{2})x(2+\sqrt{2}) = (2)^2 - (\sqrt{2})^2 = 4 - 2 = 2$.

$\therefore$ It is a rational number.

(iii) $(\sqrt{2}+\sqrt{3})^2 = (\sqrt{2})^2 + 2\sqrt{2}x\sqrt{3} + (\sqrt{3})^2 = 2 + 2\sqrt{6} + 3 = 5 + 2\sqrt{6}$

$\therefore$ It is an irrational number.

(iv) $\frac{6}{3\sqrt{2}} = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\therefore$ It is an irrational number.

Question-41

Prove that

(a) $2 + \sqrt{3}$ is not a rational number and

(b) $\sqrt[3]{7}$ is not a rational number.

Solution:

If possible, let $2 + \sqrt{3} = \boldsymbol{a}$, where $\boldsymbol{a}$ is rational.

Then, $(2 + \sqrt{3})^2 = \boldsymbol{a}^2$

$7 + 4\sqrt{3} = \boldsymbol{a}^2$

$\sqrt{3} = \frac{a^2 - 7}{4}$ -------(i)

Now, $\boldsymbol{a}$ is rational $\Rightarrow \frac{a^2 - 7}{4}$ is rational.

$\sqrt{3}$ is rational [from (i)]

This is a contradiction.

Hence, $2 + \sqrt{3}$ is not a rational number.

(b) If possible, let $\sqrt[3]{7} = \boldsymbol{p/q}$, where $\boldsymbol{p}$ and $\boldsymbol{q}$ are integers, having no common factors and $\boldsymbol{q} \neq 0$.

Then, $(\sqrt[3]{7})^3 = (p/q)^3$

$\Rightarrow 7\boldsymbol{q}^3 = p^3$------(i)

$\Rightarrow \boldsymbol{p}^3$ is a multiple of 7

$\Rightarrow \boldsymbol{p}$ is multiple of 7.

Let $\boldsymbol{p}$ = 7m, where m is an integer.

Then, $\boldsymbol{p}^3 = 343\ m^3$ ------(ii)

$\Rightarrow 7\boldsymbol{q}^3 = 343\ m^3$ [from (i) and (ii)]

$\Rightarrow \boldsymbol{q}^3 = 49\ m^3$

$\Rightarrow \boldsymbol{q}^3$ is a multiple of 7.

$\Rightarrow \boldsymbol{q}$ is a multiple of 7.

Thus, $\boldsymbol{p}$ and $\boldsymbol{q}$ are both multiples of 7, or 7 is a factor of $\boldsymbol{p}$ and $\boldsymbol{q}$.

This contradicts our assumption that p and $\boldsymbol{q}$ have no common factors.

Hence $\sqrt[3]{7}$ is not a rational number.

Question-42

Simplify by rationalising the denominator: $\frac{4}{\sqrt{7}+\sqrt{3}}$

Solution:

$\frac{4}{\sqrt{7}+\sqrt{3}} = \frac{4}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}$ [Rationalising the denominator]

$= \frac{4(\sqrt{7}-\sqrt{3})}{7-3} = \frac{4(\sqrt{7}-\sqrt{3})}{4} = \sqrt{7}-\sqrt{3}$

Question-43

Simplify by rationalising the denominator: $\frac{30}{5\sqrt{3}-3\sqrt{5}}$

Solution:

$\frac{30}{5\sqrt{3}-3\sqrt{5}} = \frac{30}{5\sqrt{3}-3\sqrt{5}} \times \frac{5\sqrt{3}+3\sqrt{5}}{5\sqrt{3}+3\sqrt{5}}$ [Rationalisation the denominator]

$= \frac{30(5\sqrt{3}+3\sqrt{5})}{25\times 3-9\times 5}$

$= \frac{30(5\sqrt{3}+3\sqrt{5})}{75-45}$

$= \frac{30(5\sqrt{3}+3\sqrt{5})}{30}$

$= 5\sqrt{3}+3\sqrt{5}$

Question-44

Examine whether the following numbers are rational or irrational:
(i) $\left(3+\sqrt{2}\right)^2$,(ii) $(3-\sqrt{3})(3+\sqrt{3})$,(iii) $\frac{6}{2\sqrt{3}}$

Solution:

(i) $(3+\sqrt{2})^2 = 9+2+6\sqrt{2} = 11+6\sqrt{2}$, which is irrational.

(ii) $(3-\sqrt{3})(3+\sqrt{3}) = (3)^2-(\sqrt{3})^2 = 9-3 = 6$, which is rational.

(iii) $\frac{6}{2\sqrt{3}} = \frac{6}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\times\sqrt{3}}{6} = \sqrt{3}$, which is irrational.

Question-45

Simplify by rationalising the denominator: $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

Solution:

$\frac{6-4\sqrt{2}}{6+4\sqrt{2}} = \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}}$ [Rationalising the denominator]

$$= \frac{36-48\sqrt{2}+16\times 2}{36-16\times 2} = \frac{68-48\sqrt{2}}{36-32}$$

$$= \frac{68-48\sqrt{2}}{4}$$

$$= 17 - 12\sqrt{2}$$

Question-46

Find three irrational numbers between 2 and 2.5 .

Solution:

If a and b are any two distinct positive rational numbers such that ab is not a perfect square, then the irrational number $\sqrt{ab}$ lies between a and b.

$\therefore$ Irrational number between 2 and 2.5 is $\sqrt{2\times 2.5}$, i.e $\sqrt{5}$

Irrational number between 2 and $\sqrt{5}$ is $\sqrt{2\times\sqrt{5}}$ = $2^{(1/2)} \times 5^{(1/4)}$

Irrational number between $\sqrt{5}$ and 2.5 is $\sqrt{\sqrt{5}\times 2.5}$ =

Thus, the three irrational numbers between 2 and 2.5 are $\sqrt{5}$, $2^{(1/2)} \times 5^{(1/4)}$ and $(1/2) \times 5^{3/4} \times 2^{1/2}$.

Question-47

Simplify by rationalising the denominator: $\frac{\sqrt{7}+\sqrt{2}}{9+2\sqrt{14}}$

Solution:

$\frac{\sqrt{7}+\sqrt{2}}{9+2\sqrt{14}} = \frac{\sqrt{7}+\sqrt{2}}{9+2\sqrt{14}} \times \frac{9-2\sqrt{14}}{9-2\sqrt{14}}$ [Rationalising the denominator]

$$= \frac{9\sqrt{7}-14\sqrt{2}+9\sqrt{2}-4\sqrt{7}}{81-4\times 14}$$

$$= \frac{5(\sqrt{7}-\sqrt{2})}{81-56}$$

$$= \frac{5(\sqrt{7}-\sqrt{2})}{25}$$

$$= \frac{1}{5}(\sqrt{7}-\sqrt{2})$$

Question-48

Find two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$.

Solution:

Irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$ is $\sqrt{\sqrt{2}\times\sqrt{3}}$,i.e $\sqrt{\sqrt{6}} = 6^{(1/4)}$

Irrational numbers lying between $\sqrt{2}$ and $6^{(1/4)}$ is $\sqrt{\sqrt{2}\times 6^{\frac{1}{4}}} = 2^{(1/4)}\times 6^{(1/8)}$

Hence two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$ are $6^{(1/4)}$ and $2^{(1/4)}\times 6^{(1/8)}$.

Question-49

Simplify by rationalising the denominator: $\frac{3}{5-\sqrt{3}}+\frac{2}{5+\sqrt{3}}$

Solution:

$\frac{3}{5-\sqrt{3}}+\frac{2}{5+\sqrt{3}} = \frac{3(5+\sqrt{3})+2(5-\sqrt{3})}{(5-\sqrt{3})(5+\sqrt{3})}$

$$= \frac{15+3\sqrt{3}+10-2\sqrt{3}}{25-3}$$

$$= \frac{25+\sqrt{3}}{22}$$

Question-50

Express $\frac{7}{64}$ as a decimal fraction.

Solution:

0.109375

$64\overline{)7.000}$

$\underline{64}$

600

$\underline{576}$

240

$\underline{192}$

480

$\underline{448}$

320

$\underline{320}$

0

Therefore $\frac{7}{64} = 0.109375$

Question-51

Simplify by rationalising the denominator: $\frac{2}{5+\sqrt{3}} - \frac{2}{5-\sqrt{3}}$

Solution:

$$\frac{2}{5+\sqrt{3}} - \frac{2}{5-\sqrt{3}} = \frac{2(5-\sqrt{3}) - 2(5+\sqrt{3})}{(5+\sqrt{3})(5-\sqrt{3})}$$

$$= \frac{10 - 2\sqrt{3} - 10 - 2\sqrt{3})}{25 - 3}$$

$$= \frac{-4\sqrt{3}}{22}$$

$$= \frac{-2\sqrt{3}}{11}$$

Question-52

Express $\frac{12}{125}$ as a decimal fraction.

Solution:

$$\begin{array}{r|l} & 0.096 \\ 125 & 12.000 \\ & \underline{1125} \\ & 750 \\ & \underline{750} \\ & 0 \end{array}$$

Therefore $\frac{12}{125} = 0.096$

Question-53

Express $\frac{1}{9}$ as a decimal fraction.

Solution:

$$\begin{array}{r|l} & 0.111 \\ 9 & 1.000 \\ & \underline{9} \\ & 10 \\ & \underline{9} \\ & 10 \\ & \underline{9} \\ & \underline{1} \end{array}$$

Therefore $\frac{1}{9} = 0.111$

Question-54

Simplify by rationalising the denominator: $\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}$

Solution:

$$\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}=\frac{(4+\sqrt{5})(4+\sqrt{5})+(4-\sqrt{5})(4-\sqrt{5})}{(4-\sqrt{5})(4+\sqrt{5})}$$

$$=\frac{2[4^2+(\sqrt{5})^2]}{16-5}$$

$$=\frac{2[16+5]}{11}$$

$$=\frac{2\times 21}{11}$$

$$=\frac{42}{11}$$

Question-55

Represent $0.\overline{57}$ in the form of $\frac{p}{q}$.

Solution:

$\frac{p}{q} = 0.\overline{57}$ ....(i)

$100\frac{p}{q} = 57.\overline{57}$ ....(ii)

(ii) – (i)

$99\frac{p}{q} = 57$

Therefore $\frac{p}{q}$ = 57/99

Question-56

Simplify by rationalising the denominator: $\frac{\sqrt{5}-2}{\sqrt{5}+2}-\frac{\sqrt{5}+2}{\sqrt{5}-2}$

Solution:

$$\frac{\sqrt{5}-2}{\sqrt{5}+2}-\frac{\sqrt{5}+2}{\sqrt{5}-2}=\frac{(\sqrt{5}-2)^2-(\sqrt{5}+2)^2}{(\sqrt{5}+2)(\sqrt{5}-2)}$$

$$=\frac{5-4\sqrt{5}+4-(5+4\sqrt{5}+4)}{5-4}$$

$$=\frac{5-4\sqrt{5}+4-5-4\sqrt{5}-4}{5-4}$$

$$=-8\sqrt{5}$$

Question-57

Simplify by rationalising the denominator: $\frac{4}{2+\sqrt{3}+\sqrt{7}}$

Solution:

$$\frac{4}{2+\sqrt{3}+\sqrt{7}}=\frac{4}{(2+\sqrt{3})+\sqrt{7}}\times\frac{(2+\sqrt{3})-\sqrt{7}}{(2+\sqrt{3})-\sqrt{7}}$$

$$=\frac{4[(2+\sqrt{3})-\sqrt{7}]}{(2+\sqrt{3})^2-(\sqrt{7})^2}$$

$$=\frac{8+4\sqrt{3}-4\sqrt{7}}{4+3+4\sqrt{3}-7}$$

$$=\frac{8+4\sqrt{3}-4\sqrt{7}}{4\sqrt{3}}$$

$$=\frac{\sqrt{3}(8+4\sqrt{3}-4\sqrt{7})}{\sqrt{3}(4\sqrt{3})}$$ [Rationalising the denominator]

$$=\frac{2\sqrt{3}+3-\sqrt{21}}{3}$$

$$=\frac{1}{3}\left(2\sqrt{3}+3-\sqrt{21}\right)$$

Question-58

Represent $0.2\overline{341}$ in the form of $\frac{p}{q}$.

Solution:

$\frac{p}{q} = 0.2\overline{341}$ ....(1)

$10000\frac{p}{q} = 2341.2\overline{341}$ ....(2)

(2)-(1) $\Rightarrow 9999\frac{p}{q} = 2341$

Therefore $\frac{p}{q} = 2341/9999$

Question-59

Simplify by rationalising the denominator: $\frac{1}{\sqrt{3}+\sqrt{2}-\sqrt{5}}$

Solution:

$$\frac{1}{\sqrt{3}+\sqrt{2}-\sqrt{5}} = \frac{1}{(\sqrt{3}+\sqrt{2})-\sqrt{5}} \times \frac{(\sqrt{3}+\sqrt{2})+\sqrt{5}}{(\sqrt{3}+\sqrt{2})+\sqrt{5}}$$

$$= \frac{\sqrt{3}+\sqrt{2}+\sqrt{5}}{(\sqrt{3}+\sqrt{2})^2-(\sqrt{5})^2}$$

$$= \frac{\sqrt{3}+\sqrt{2}+\sqrt{5}}{3+2\sqrt{6}+2-5}$$

$$= \frac{\sqrt{3}+\sqrt{2}+\sqrt{5}}{2\sqrt{6}}$$

$$= \frac{\sqrt{3}+\sqrt{2}+\sqrt{5}}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{\sqrt{6}(\sqrt{3}+\sqrt{2}+\sqrt{5})}{12}$$

$$= \frac{1}{12}(\sqrt{18}+\sqrt{12}+\sqrt{30})$$

$$= \frac{1}{12}(3\sqrt{2}+2\sqrt{3}+\sqrt{30})$$

Question-60

Which of the following is surds:

(i) $\sqrt{64}$

(ii) $\sqrt{45}$

(iii) $3\sqrt{12} \div 6\sqrt{27}$

Solution:

(i) $\sqrt{64} = \sqrt{2\times2\times2\times2\times2\times2} = 2\times2\times2 = 8$

$\therefore \sqrt{64}$ is not a surd

(ii) $\sqrt{45} = \sqrt{3\times3\times5} = 3\sqrt{5}$

$\therefore \sqrt{45}$ is a surd

(iii) $3\sqrt{12} \div 6\sqrt{27} = \dfrac{3\times2\sqrt{3}}{6\times3\sqrt{3}} = \dfrac{1}{3}$

$\therefore 3\sqrt{12} \div 6\sqrt{27}$ is not a surd

Question-61

Express as a pure surd:

(i) $5\sqrt{6}$ (ii) $2\sqrt[3]{4}$ (iii) $3\sqrt[4]{5}$

(iv) $10\sqrt{3}$ (v) $\frac{2}{3}\sqrt{32}$ (vi) $\frac{3}{4}\sqrt{8}$

Solution:

(i) $5\sqrt{6} = \sqrt{6\times5^2} = \sqrt{6\times25} = \sqrt{150}$

(ii) $2\sqrt[3]{4} = \sqrt[3]{4\times2^3} = \sqrt[3]{4\times8} = \sqrt[3]{32}$

(iii) $3\sqrt[4]{5} = \sqrt[4]{5\times3^4} = \sqrt[4]{5\times81} = \sqrt[4]{405}$

(iv) $10\sqrt{3} = \sqrt{3\times10^2} = \sqrt{3\times100} = \sqrt{300}$

(v) $\frac{2}{3}\sqrt{32} = \sqrt{32\times\left(\frac{2}{3}\right)^2} = \sqrt{32\times\frac{4}{9}} = \sqrt{\frac{128}{9}}$

(vi) $\frac{3}{4}\sqrt{8} = \sqrt{8\times\left(\frac{3}{4}\right)^2} = \sqrt{8\times\frac{9}{16}} = \sqrt{\frac{9}{2}}$

Question-62

Express as a mixed surd in its simplest form:

(i) $\sqrt{80}$ **(ii)** $\sqrt[3]{72}$ **(iii)** $\sqrt[5]{288}$

(iv) $\sqrt{1350}$ **(v)** $\sqrt[5]{320}$ **(vi)** $5\sqrt[3]{135}$

Solution:

(i) $\sqrt{80} = \sqrt{2\times2\times2\times2\times5} = \sqrt{2^2\times2^2\times5} = 2\text{x}2\text{x}\sqrt{5} = 4\sqrt{5}$

(ii) $\sqrt[3]{72} = \sqrt[3]{2\times2\times2\times3\times3} = \sqrt[3]{2^3\times3^2} = 2\sqrt[3]{9}$

(iii) $\sqrt[5]{288} = \sqrt[5]{2\times2\times2\times2\times2\times3\times3} = \sqrt[5]{2^5\times3^2} = 2\sqrt[5]{9}$

(iv) $\sqrt{1350} = \sqrt{2\times3\times3\times3\times5\times5} = \sqrt{2\times3^2\times3\times5^2} = 3\text{x}5\text{x}\sqrt{6} = 15\sqrt{6}$

(v) $\sqrt[5]{320} = \sqrt[5]{2\times2\times2\times2\times2\times2\times5} = \sqrt[5]{2^5\times2\times5} = 2\sqrt[5]{10}$

(vi) $5\sqrt[3]{135} = 5\sqrt[3]{3\times3\times3\times5} = 5\sqrt[3]{3^3\times5} = 5\text{X}3\sqrt[3]{5} = 15\sqrt[3]{5}.$

Question-63

Which is greater?

(i) $\sqrt{2}$ or $\sqrt[3]{3}$

(ii) $\sqrt{3}$ or $\sqrt[4]{10}$

(iii) $\sqrt[4]{5}$ or $\sqrt[3]{4}$

(iv) $\sqrt[3]{6}$ or $\sqrt[4]{8}$

(v) $\sqrt[8]{12}$ or $\sqrt[4]{6}$

(vi) $\sqrt[3]{3}$ or $\sqrt[4]{4}$

Solution:

(i) L.C.M. of 2 and 3 is 6.

Thus, $\sqrt{2} = \sqrt[6]{2^3} = \sqrt[6]{8}$

And $\sqrt[3]{3} = \sqrt[6]{3^2} = \sqrt[6]{9}$

Therefore $\sqrt[6]{9} > \sqrt[6]{8}$

Hence, $\sqrt[3]{3} > \sqrt{2}$

(ii) L.C.M. of 1 and 4 is 4.

Thus, $\sqrt{3} = \sqrt[4]{3^2} = \sqrt[4]{9}$

and $\sqrt[4]{10} = \sqrt[4]{10}$

Therefore $\sqrt[4]{10} > \sqrt[4]{9}$

Hence, $\sqrt[4]{10} > \sqrt{3}$

(iii) L.C.M. of 4 and 3 is 12.

Thus, $\sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$

and $\sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}$

Therefore $\sqrt[12]{256} > \sqrt[12]{125}$

Hence, $\sqrt[3]{4} > \sqrt[4]{5}$

(iv) L.C.M. of 3 and 4 is 12.

Thus, $\sqrt[3]{6} = \sqrt[12]{6^4} = \sqrt[12]{1296}$

and $\sqrt[4]{8} = \sqrt[12]{8^3} = \sqrt[12]{512}$

Therefore $\sqrt[12]{1296} > \sqrt[12]{512}$

Hence, $\sqrt[3]{6} > \sqrt[4]{8}$

(v) L.C.M. of 8 and 4 is 8.

Thus, $\sqrt[8]{12} = \sqrt[8]{12}$

and $\sqrt[4]{6} = \sqrt[8]{6^2} = \sqrt[8]{36}$

Therefore $\sqrt[8]{36} > \sqrt[8]{12}$

Hence, $\sqrt[4]{6} > \sqrt[8]{12}$

(vi) L.C.M. of 3 and 4 is 12

Thus, $\sqrt[3]{3} = \sqrt[12]{3^4} = \sqrt[12]{81}$

and $\sqrt[4]{4} = \sqrt[12]{4^3} = \sqrt[12]{64}$

Therefore $\sqrt[12]{81} > \sqrt[12]{64}$

Hence, $\sqrt[3]{3} > \sqrt[4]{4}$.

Question-64

Arrange in descending order of magnitude

(i) $\sqrt[3]{4}, \sqrt[4]{5}, \sqrt{3}$

(ii) $\sqrt[3]{2}$, $\sqrt[4]{3}$, $\sqrt[3]{4}$

(iii) $\sqrt[4]{10}$, $\sqrt[3]{6}$, $\sqrt{3}$

Solution:

(i) L.C.M. of 3, 4 and 2 is 12

Thus, $\sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}$

$\sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$

$\sqrt{3} = \sqrt[12]{3^6} = \sqrt[12]{729}$

Therefore $\sqrt[12]{729} > \sqrt[12]{256} > \sqrt[12]{125}$

Hence, $\sqrt{3} > \sqrt[3]{4} > \sqrt[4]{5}$

(ii) L.C.M of 3,4 and 3 is 12

Thus, $\sqrt[3]{2} = \sqrt[12]{2^4} = \sqrt[12]{16}$

$\sqrt[4]{3} = \sqrt[12]{3^3} = \sqrt[12]{27}$

$\sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}$

$\therefore \sqrt[12]{256} > \sqrt[12]{27} > \sqrt[12]{16}$

Hence, $\sqrt[3]{4} > \sqrt[4]{3} > \sqrt[3]{2}$

(iii) L.C.M. of 4,3 and 2 is 12

Thus, $\sqrt[4]{10} = \sqrt[12]{10^3} = \sqrt[12]{1000}$

$\sqrt[3]{6} = \sqrt[12]{6^4} = \sqrt[12]{1296}$

$\sqrt{3} = \sqrt[12]{3^6} = \sqrt[12]{729}$

Therefore $\sqrt[12]{1296} > \sqrt[12]{1000} > \sqrt[12]{729}$

Hence, $\sqrt[3]{6} > \sqrt[4]{10} > \sqrt{3}$.

Question-65

Simplify by combining similar terms:

$5\sqrt{2} + 20\sqrt{2}$

Solution:

$5\sqrt{2}+20\sqrt{2} = (5+20)\sqrt{2} = 25\sqrt{2}$

Question-66

Simplify by combining similar terms:

$2\sqrt{3} + \sqrt{27}$

Solution:

Reducing into simplest form

$\sqrt{27} = \sqrt{3^2 \times 3} = 3\sqrt{3}$

Therefore $2\sqrt{3} + \sqrt{27} = 2\sqrt{3} + 3\sqrt{3} = (2+3)\sqrt{3} = 5\sqrt{3}$.

Question-67

Simplify by combining similar terms:

$4\sqrt{3} - 3\sqrt{12} + 2\sqrt{75}$

Solution:

Reducing into simplest form

$\sqrt{12} = \sqrt{2^2 \times 3} = 2\sqrt{3}$

$\sqrt{75} = \sqrt{3 \times 5^2} = 5\sqrt{3}$

Therefore $4\sqrt{3} - 3\sqrt{12} + 2\sqrt{75} = 4\sqrt{3} - 3 \times 2\sqrt{3} + 2 \times 5\sqrt{3}$

$= 4\sqrt{3} - 6\sqrt{3} + 10\sqrt{3}$

$= (4-6+10)\sqrt{3}$

$= 8\sqrt{3}$.

Question-68

Simplify by combining similar terms:

$\sqrt{8} + \sqrt{32} - \sqrt{2}$

Solution:

Reducing into simplest form.

$\sqrt{8} = \sqrt{2^2 \times 2} = 2\sqrt{2}$

$\sqrt{32} = \sqrt{2^2 \times 2^2 \times 2} = 2 \times 2\sqrt{2} = 4\sqrt{2}$

$\therefore \sqrt{8} + \sqrt{32} - \sqrt{2} = 2\sqrt{2} + 4\sqrt{2} - \sqrt{2} = (2 + 4 - 1)\sqrt{2} = 5\sqrt{2}$.

Question-69

Simplify by combining similar terms:

$$\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$$

Solution:

Reducing into simplest form

$\sqrt{45} = \sqrt{3^2 \times 5} = 3\sqrt{5}$

$\sqrt{20} = \sqrt{2^2 \times 5} = 2\sqrt{5}$

$\therefore \sqrt{45} - 3\sqrt{20} + 4\sqrt{5} = 3\sqrt{5} - 3 \times 2\sqrt{5} + 4\sqrt{5}$

$= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5}$

$= \sqrt{5}$

Question-70

Simplify by combining similar terms:

$$4\sqrt{12} - \sqrt{50} - 7\sqrt{48}$$

Solution:

Reducing into simplest form

$\sqrt{12} = \sqrt{2^2 \times 3} = 2\sqrt{3}$

$\sqrt{50} = \sqrt{2 \times 5^2} = 5\sqrt{2}$

$\sqrt{48} = \sqrt{2^2 \times 2^2 \times 3} = 2x2\sqrt{3} = 4\sqrt{3}$

$\therefore 4\sqrt{12} - \sqrt{50} - 7\sqrt{48} = 4x\ 2\sqrt{3} - 5\sqrt{2} - 7 \times 4\sqrt{3}$

$= 8\sqrt{3} - 5\sqrt{2} - 28\sqrt{3}$

$= (8-28)\sqrt{3} - 5\sqrt{2}$

$= -20\sqrt{3} - 5\sqrt{2}$

Question-71

Simplify by combining similar terms:

$$2\sqrt[3]{4} + 7\sqrt[3]{32} - \sqrt[3]{500}$$

Solution:

Reducing into simplest form

$\sqrt[3]{32} = \sqrt[3]{2^3 \times 2^2} = 2\sqrt[3]{4}$

$\sqrt[3]{500} = \sqrt[3]{5^3 \times 2^2} = 5\sqrt[3]{4}$

$\therefore\ 2\sqrt[3]{4} + 7\sqrt[3]{32} - \sqrt[3]{500} = 2\sqrt[3]{4} + 7 \text{x} 2\sqrt[3]{4} - 5\sqrt[3]{4}$

$= 2\sqrt[3]{4} + 14\sqrt[3]{4} - 5\sqrt[3]{4}$

$= (2+14-5)\sqrt[3]{4}$

$= 11\sqrt[3]{4}$

Question-72

Simplify by combining similar terms:

$$2\sqrt[3]{40} + 3\sqrt[3]{625} - 4\sqrt[3]{320}$$

Solution:

Reducing into simplest form

$\sqrt[3]{40} = \sqrt[3]{2^3 \times 5} = 2\sqrt[3]{5}$

$\sqrt[3]{625} = \sqrt[3]{5^3 \times 5} = 5\sqrt[3]{5}$

$\sqrt[3]{320} = \sqrt[3]{2^3 \times 2^3 \times 5} = 2x2\sqrt[3]{5} = 4\sqrt[3]{5}$

$\therefore 2\sqrt[3]{40} + 3\sqrt[3]{625} - 4\sqrt[3]{320} = 2\sqrt[3]{5} + 3x5\sqrt[3]{5} - 4x4\sqrt[3]{5}$

$= 4\sqrt[3]{5} + 15\sqrt[3]{5} - 16\sqrt[3]{5}$

$= (4 + 15 - 16)\sqrt[3]{5}$

$= 3\sqrt[3]{5}$

Question-73

Simplify by combining similar terms:

$$3\sqrt{147} - \frac{7}{3}\sqrt{\frac{1}{3}} + 7\sqrt{\frac{1}{3}}$$

Solution:

Reducing into simplest form

$$\sqrt{147} = \sqrt{3 \times 7^2} = 7\sqrt{3}$$

$$\sqrt{\frac{1}{3}} = \sqrt{\frac{3}{9}} = \frac{\sqrt{3}}{3}$$

$$\therefore 3\sqrt{147} - \frac{7}{3}\sqrt{\frac{1}{3}} + 7\sqrt{\frac{1}{3}} = 3 \times 7\sqrt{3} - \frac{7}{3} \times \frac{\sqrt{3}}{3} + 7 \times \frac{\sqrt{3}}{3}$$

$$= 21\sqrt{3} - \frac{7}{9}\sqrt{3} + \frac{7}{3}\sqrt{3}$$

$$= \left\{21 - \frac{7}{9} + \frac{7}{3}\right\}\sqrt{3}$$

$$= \left\{\frac{189 - 7 + 21}{9}\right\}\sqrt{3}$$

$$= \frac{203}{9}\sqrt{3}.$$

Question-74

Simplify and express the result in its simplest form:

$$\sqrt{14} \times \sqrt{21}$$

Solution:

$$\sqrt{14} \times \sqrt{21} = \sqrt{14 \times 21} = \sqrt{2 \times 7 \times 3 \times 7} = \sqrt{2 \times 3 \times 7^2}$$

$$= 7\sqrt{6}.$$

Question-75

Simplify and express the result in its simplest form:

$$\sqrt{15} \times \sqrt{7}$$

Solution:

$$\sqrt{15} \times \sqrt{7} = \sqrt{15 \times 7} = \sqrt{105}.$$

Question-76

Simplify and express the result in its simplest form:

$\sqrt[3]{4} \times \sqrt[3]{22}$

Solution:

$\sqrt[3]{4} \text{ X } \sqrt[3]{22} = \sqrt[3]{4 \times 22} = \sqrt[3]{2^2 \times 2 \times 11} = \sqrt[3]{2^3 \times 11} = 2\sqrt[3]{11}$.

Question-77

Simplify and express the result in its simplest form:

$4\sqrt{12} \times 7\sqrt{6}$

Solution:

$4\sqrt{12} \text{ X } 7\sqrt{6} = 28\sqrt{12 \times 6} = 28\sqrt{6^2 \times 2} = 28 \text{x} 6\sqrt{2}$

$= 168\sqrt{2}$.

Question-78

Simplify and express the result in its simplest form:

$\sqrt[3]{2} \times \sqrt{5}$

Solution:

L.C.M. of 3, 2 is 6

$\sqrt[3]{2} = \sqrt[6]{2^2} = \sqrt[6]{4}$

$\sqrt{5} = \sqrt[6]{5^3} = \sqrt[6]{125}$

$\sqrt[3]{2} \text{ X } \sqrt{5} = \sqrt[6]{4} \text{ X } \sqrt[6]{125} = \sqrt[6]{4 \times 125} = \sqrt[6]{500}$

Question-79

Simplify and express the result in its simplest form:

$\sqrt[3]{2} \times \sqrt[4]{3}$

Solution:

L.C.M. of 3, 4 is 12

$\sqrt[3]{2} = \sqrt[12]{2^4} = \sqrt[12]{16}$

$\sqrt[4]{3} = \sqrt[12]{3^3} = \sqrt[12]{27}$

$\sqrt[3]{2} \times \sqrt[4]{3} = \sqrt[12]{16} \times \sqrt[12]{27} = \sqrt[12]{16 \times 27} = \sqrt[12]{432}$

Question-80

Simplify and express the result in its simplest form:

$4\sqrt{28} \div 3\sqrt{7}$

Solution:

$4\sqrt{28} \div 3\sqrt{7} = \frac{4\sqrt{28}}{3\sqrt{7}}$

$\frac{4\sqrt{28}}{3\sqrt{7}} = \frac{4}{3}\sqrt{4} = \frac{4}{3} \times 2 = \frac{8}{3}$.

Question-81

Simplify and express the result in its simplest form:

(1) $\sqrt[6]{12} \div (\sqrt{3} \times \sqrt[3]{2})$

(2) $\sqrt{2} \times \sqrt[3]{3} \times \sqrt[4]{4}$

Solution:

1) L.C.M of 2 and 3 is 6

$\sqrt{3} = \sqrt[6]{3^3} = \sqrt[6]{27}$

$\sqrt[3]{2} = \sqrt[6]{2^2} = \sqrt[6]{4}$

$\sqrt{3} \times \sqrt[3]{2} = \sqrt[6]{27} \times \sqrt[6]{4} = \sqrt[6]{27 \times 4} = \sqrt[6]{108}$

$\sqrt[6]{12} \div (\sqrt{3} \times \sqrt[3]{2}) = \sqrt[6]{12} \div \sqrt[6]{108} = \sqrt[6]{\frac{12}{108}} = \sqrt[6]{\frac{1}{9}} = \sqrt[3]{\frac{1}{3}}$

2) L.C.M. of 2,3 and 4 is 12

$\sqrt{2} = \sqrt[12]{2^6}$

$\sqrt[3]{3} = \sqrt[12]{3^4}$

$\sqrt[4]{4} = \sqrt[12]{4^3} = \sqrt[12]{(2^2)^3} = \sqrt[12]{2^6}$

$\sqrt{2} \times \sqrt[3]{3} \times \sqrt[4]{4} = \sqrt[12]{2^6} \times \sqrt[12]{3^4} \times \sqrt[12]{2^6} = \sqrt[12]{2^6 \times 3^4 \times 2^6}$

$= \sqrt[12]{2^{12} \times 3^4} = 2\sqrt[12]{3^4} = 2\sqrt[3\times4]{3^4} = 2\sqrt[3]{3}$

Question-82

Write the simplest rationalising factor of

(i) $2\sqrt{2}$ (ii) $\sqrt{10}$ (iii) $\sqrt{75}$ (iv) $2\sqrt[3]{5}$ (v) $\sqrt[3]{36}$

Solution:

(i) $2\sqrt{2}$ x $\sqrt{2}$ = 2 x $\sqrt{2\times 2}$ = 4

$\therefore$ $\sqrt{2}$ is the simplest rationalising factor of $2\sqrt{2}$

(ii) $\sqrt{10}$ x $\sqrt{10}$ = $\sqrt{10\times 10}$ = $\sqrt{10^2}$ = 10

$\therefore$ $\sqrt{10}$ is the simplest rationalising factor of $\sqrt{10}$

(iii) $\sqrt{75} = \sqrt{3\times 5\times 5} = \sqrt{3\times 5^2} = 5\sqrt{3}$

$5\sqrt{3}$ x $\sqrt{3}$ = $5\sqrt{3\times 3}$ = 5x3 = 15

$\therefore$ $\sqrt{3}$ is the simplest rationalising factor of $\sqrt{75}$

(iv) $2\sqrt[3]{5}$ x $\sqrt[3]{5^2}$ = $2\sqrt[3]{5\times 5^2}$ = $2\sqrt[3]{5^3}$

= 2x5=10

$\therefore$ $\sqrt[3]{25}$ is the simplest rationalising factor of $2\sqrt[3]{5}$

(v) $\sqrt[3]{36} = \sqrt[3]{6^2}$

Now $\sqrt[3]{6^2}$ x $\sqrt[3]{6}$ = $\sqrt[3]{6^3}$ = 6

$\therefore$ $\sqrt[3]{6}$ is the simplest rationalising factor of $\sqrt[3]{36}$.

Question-83

Express with a rational denominator the following surds:

(i) $\frac{2}{\sqrt{5}}$ **(ii)** $\frac{2}{3\sqrt{3}}$ **(iii)** $\frac{1}{\sqrt{12}}$ **(iv)** $\frac{\sqrt{2}}{\sqrt{5}}$ **(v)** $\frac{2\sqrt{7}}{\sqrt{11}}$ **(vi)** $\frac{3\sqrt[3]{5}}{\sqrt[3]{9}}$

Solution:

(i) $\frac{2}{\sqrt{5}}$

The simplest Rational factor of $\sqrt{5}$ is $\sqrt{5}$ itself.

$\therefore \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

(ii) $\frac{2}{3\sqrt{3}}$

The simplest R.F of $\sqrt{3}$ is $\sqrt{3}$ itself.

$\therefore \frac{2}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3\times 3} = \frac{2\sqrt{3}}{9}$

(iii) $\frac{1}{\sqrt{12}}$

$\sqrt{12} = \sqrt{2\times 2\times 3} = 2\sqrt{3}$

The simplest R.F of $\sqrt{3}$ is $\sqrt{3}$ itself.

$\therefore \ \frac{1}{\sqrt{12}} = \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{6}$

(iv) $\frac{\sqrt{2}}{\sqrt{5}}$

The simplest R.F of $\sqrt{5}$ is $\sqrt{5}$ itself.

$\therefore \ \frac{\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$

(v) The simplest R.F of $\sqrt{11}$ is $\sqrt{11}$ itself.

$\frac{2\sqrt{7}}{\sqrt{11}} \times \frac{\sqrt{11}}{\sqrt{11}} = \frac{2\sqrt{77}}{11}$

(vi) The simplest R.F of $\sqrt[3]{9}$ is $\sqrt[3]{3}$ itself as $\sqrt[3]{9} = \sqrt[3]{3^2}$

$\therefore \ \frac{3\sqrt[3]{5}}{\sqrt[3]{9}} = \frac{3\sqrt[3]{5}}{\sqrt[3]{9}} \times \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{3\sqrt[3]{5 \times 3}}{\sqrt[3]{3^3}} = \frac{3\sqrt[3]{15}}{3} = \sqrt[3]{15}$

Question-84

(a) Which is greater $\sqrt{3}$ or $\sqrt[3]{5}$?

(b) Arrange in ascending order of magnitude $\sqrt[4]{10}$, $\sqrt[3]{6}$, $\sqrt{3}$.

Solution:

(a) L.C.M of 2 and 3 is 6

Thus $\sqrt{3} = \sqrt[6]{3^3} = \sqrt[6]{27}$

$\sqrt[3]{5} = \sqrt[6]{5^2} = \sqrt[6]{25}$

Now $\sqrt[6]{27} > \sqrt[6]{25}$

$\therefore \sqrt{3} > \sqrt[3]{5}$

(b) L.C.M of 4,3, and 2 is 12

Thus, $\sqrt[4]{10} = \sqrt[12]{10^3} = \sqrt[12]{1000}$

$\sqrt[3]{6} = \sqrt[12]{6^4} = \sqrt[12]{1296}$

$\sqrt{3} = \sqrt[12]{3^6} = \sqrt[12]{729}$

Now, $\sqrt[12]{729} < \sqrt[12]{1000} < \sqrt[12]{1296}$

$\therefore \sqrt{3} < \sqrt[4]{10} < \sqrt[3]{6}$.

Find the value to three places of decimals, of each of the following. It is given that $\sqrt{2}$ = 1.414, $\sqrt{3}$ = 1.732, $\sqrt{10}$ = 3.162 and $\sqrt{5}$ = 2.236 (approx.) (i) $\frac{1}{\sqrt{2}}$ (ii) $\frac{1}{\sqrt{3}}$ (iii) $\frac{1}{\sqrt{10}}$ (iv) $\frac{\sqrt{2}+1}{\sqrt{5}}$ (v) $\frac{2-\sqrt{3}}{\sqrt{3}}$ (vi) $\frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}}$

Solution:

(i) $\frac{1}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1.414}{2} = 0.707$$

(ii) $\frac{1}{\sqrt{3}}$

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} X \frac{\sqrt{3}}{\sqrt{3}} = \frac{1.732}{3} = 0.5773 = 0.577$$

(iii) $\frac{1}{\sqrt{10}}$

$$\frac{1}{\sqrt{10}} = \frac{1}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10} = \frac{3.162}{10} = 0.3162$$

(iv) $\frac{\sqrt{2}+1}{\sqrt{5}}$

$$\frac{\sqrt{2}+1}{\sqrt{5}} = \frac{\sqrt{2}+1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}+\sqrt{5}}{5} = \frac{3.162+2.236}{5} = \frac{5.398}{5} = 1.0796 = 1.080$$

(v) $\frac{2-\sqrt{3}}{\sqrt{3}}$

$$\frac{2-\sqrt{3}}{\sqrt{3}} = \frac{2-\sqrt{3}}{\sqrt{3}} X \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}-3}{3} = \frac{2(1.732)-3}{3} = \frac{3.464-3}{3} = \frac{0.46}{3} = 0.1546 = 0.155$$

(vi) $\frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}}$ $\frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}} = \frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}} X \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{20}-\sqrt{10}}{2}$

$$= \frac{2\sqrt{5}-\sqrt{10}}{2} = \frac{2(2.236)-(3.162)}{2} = \frac{4.472-3.162}{2} = \frac{1.310}{2} = 0.655$$

Question-86

If both of *a* and *b* are rational numbers, find the values of a and b in each of the following equalities:

(i) $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a+b\sqrt{3}$ (ii) $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$ (iii) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$

(iv) $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a-b\sqrt{6}$

Solution:

(i) $\sqrt{3}$ -1 is the conjugate surd of $\sqrt{3}+1$

\ $\frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = a+b\sqrt{3}$

$$\frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-1^2} = a+b\sqrt{3}$$

$$\frac{(\sqrt{3})^2-2\sqrt{3}+1}{3-1} = a+b\sqrt{3}$$

$$\frac{3-2\sqrt{3}+1}{2} = a+b\sqrt{3}$$

$2-\sqrt{3} = \boldsymbol{a+b}\sqrt{3}$

Hence, $\boldsymbol{a} = 2$, $\boldsymbol{b} = -1$

(ii) $3+\sqrt{2}$ is the conjugate of $3-\sqrt{2}$

\ $\frac{3+\sqrt{2}}{3-\sqrt{2}} = \frac{3+\sqrt{2}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{(3+\sqrt{2})^2}{(3)^2-(\sqrt{2})^2}$

$= \frac{(3)^2+2(3)(\sqrt{2})+(\sqrt{2})^2}{9-2}$

$= \frac{9+6\sqrt{2}+2}{7} = \frac{11+6\sqrt{2}}{7} = \frac{11}{7}+\frac{6}{7}\sqrt{2}$

On comparing both sides

$\frac{11}{7}+\frac{6}{7}\sqrt{2} = a+b\sqrt{2}$

We have ***a*** = $\frac{11}{7}$ and ***b*** = $\frac{6}{7}$

(iii) $7-4\sqrt{3}$ is the conjugate of $7+4\sqrt{3}$

$\therefore \frac{5+2\sqrt{3}}{7+4\sqrt{3}} = \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{(5+2\sqrt{3})\times(7-4\sqrt{3})}{(7)^2-(4\sqrt{3})^2}$

$= \frac{5(7-4\sqrt{3})+2\sqrt{3}(7-4\sqrt{3})}{49-48}$

$= \frac{35-20\sqrt{3}+14\sqrt{3}-8\text{x}3}{1}$

$= 35-20\sqrt{3}+14\sqrt{3}-24 = 11-6\sqrt{3}$

On comparing both sides

$11-6\sqrt{3} = a+b\sqrt{3}$

We have, ***a*** = 11, ***b*** = -6.

(iv) $3\sqrt{2}+2\sqrt{3}$ is the conjugate of the denominator $3\sqrt{2} - 2\sqrt{3}$

$$\therefore \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = \frac{\left(\sqrt{2}+\sqrt{3}\right)\times\left(3\sqrt{2}+2\sqrt{3}\right)}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2}$$

$$= \frac{\sqrt{2}\left(3\sqrt{2}+2\sqrt{3}\right)+\sqrt{3}\left(3\sqrt{2}+2\sqrt{3}\right)}{18-12}$$

$$= \frac{6+2\sqrt{6}+3\sqrt{6}+6}{6} = \frac{12+5\sqrt{6}}{6} = \frac{12}{6}+\frac{5}{6}\sqrt{6}$$

$$= 2 + \frac{5}{6}\sqrt{6}$$

On comparing both sides

$2 + \frac{5}{6}\sqrt{6} = a-b\sqrt{6}$

we have, ***a*** = 2 and ***b*** = $-\frac{5}{6}$

Question-87

Simplify each of the following by rationalizing the denominator:

(i) $\frac{5+\sqrt{6}}{5-\sqrt{6}}$ (ii) $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$ (iii) $\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}-3\sqrt{3}}$

Solution:

(i) \ $\frac{5+\sqrt{6}}{5-\sqrt{6}} = \frac{5+\sqrt{6}}{5-\sqrt{6}} \times \frac{5+\sqrt{6}}{5+\sqrt{6}} = \frac{(5+\sqrt{6})^2}{(5)^2-(\sqrt{6})^2}$

$5+\sqrt{6}$ is the rationalizing factor of $5-\sqrt{6}$

$$= \frac{(5)^2+2(5)\sqrt{6}+(\sqrt{6})^2}{25-6} = \frac{25+10\sqrt{6}+6}{19} = \frac{31+10\sqrt{6}}{19}$$

(ii) $\sqrt{7}-\sqrt{5}$ is the rationalizing factor of $\sqrt{7}+\sqrt{5}$

\ $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{(\sqrt{7}-\sqrt{5})^2}{(\sqrt{7})^2-(\sqrt{5})^2}$

$$= \frac{(\sqrt{7})^2-2(\sqrt{7})(\sqrt{5})+(\sqrt{5})^2}{7-5} = \frac{(7-2\sqrt{35}+5)}{2} = \frac{12-2\sqrt{35}}{2} = 6-\sqrt{35}.$$

(iii) $2\sqrt{2}+3\sqrt{3}$ is the rationalizing factor of $2\sqrt{2}-3\sqrt{3}$

$$\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}-3\sqrt{3}}$$
$$= \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}-3\sqrt{3}} \times \frac{2\sqrt{2}+3\sqrt{3}}{2\sqrt{2}+3\sqrt{3}}$$
$$= \frac{(2\sqrt{3}-\sqrt{5})\times(2\sqrt{2}+3\sqrt{3})}{(2\sqrt{2})^2-(3\sqrt{3})^2}$$

$$= \frac{2\sqrt{3}(2\sqrt{2}+3\sqrt{3})-\sqrt{5}(2\sqrt{2}+3\sqrt{3})}{8-27}$$
$$= \frac{4\sqrt{6}+18-2\sqrt{10}-3\sqrt{15}}{-19}$$

$$= \frac{-18-4\sqrt{6}+2\sqrt{10}+3\sqrt{15}}{+19}$$

Rationalize the denominator of

(i) $\frac{1}{3+\sqrt{5}-2\sqrt{2}}$ **(ii)** $\frac{1}{\sqrt{3}-\sqrt{2}-1}$ **(iii)** $\frac{1}{\sqrt{6}+\sqrt{5}-\sqrt{11}}$

Solution:

(i) The denominator is a trinomial surd.

We proceed as with a binomial surd by grouping two of the terms. Thus,

$$\{(3+\sqrt{5})-2\sqrt{2}\}\times\{(3+\sqrt{5})+2\sqrt{2}\}$$
$$=(3+\sqrt{5})^2-(2\sqrt{2})^2$$
$$=3^2+2\times3\times\sqrt{5}+(\sqrt{5})^2-(2\sqrt{2})^2$$
$$= 9 + 6\sqrt{5} + 5 - 8$$
$$= 6 + 6\sqrt{5}$$
$$= 6(1+\sqrt{5})$$

\ $\frac{1}{3+\sqrt{5}-2\sqrt{2}}=\frac{1}{3+\sqrt{5}-2\sqrt{2}}\times\frac{3+\sqrt{5}+2\sqrt{2}}{3+\sqrt{5}+2\sqrt{2}}$

$$=\frac{3+\sqrt{5}+2\sqrt{2}}{\{(3+\sqrt{5})-2\sqrt{2}\}\{(3+\sqrt{5})+2\sqrt{2}\}}$$
$$=\frac{3+\sqrt{5}+2\sqrt{2}}{6(1+\sqrt{5})}\times\frac{1-\sqrt{5}}{1-\sqrt{5}}$$

$$=\frac{(3+\sqrt{5}+2\sqrt{2})\times(1-\sqrt{5})}{6\{(1)^2-(\sqrt{5})^2\}}$$
$$=\frac{3-3\sqrt{5}+\sqrt{5}-5+2\sqrt{2}-2\sqrt{10}}{6(1-5)}$$

$$=\frac{-2-2\sqrt{5}+2\sqrt{2}-2\sqrt{10}}{6(-4)}$$

$$=\frac{-2(1+\sqrt{5}-\sqrt{2}+\sqrt{10})}{-24}$$

$$=\frac{1+\sqrt{5}-\sqrt{2}+\sqrt{10}}{12}$$

(ii) The denominator is a trinomial surd.

We proceed as with a binomial surd by grouping two of the terms.

Thus, $\{\sqrt{3}-(\sqrt{2}+1)\}\{\sqrt{3}+(\sqrt{2}+1)\} = (\sqrt{3})^2 - (\sqrt{2}+1)^2$

$$= 3 - (2 + 2\sqrt{2} + 1) = 3 - 3 - 2\sqrt{2} = -2\sqrt{2}$$

$$\frac{1}{\sqrt{3}-\sqrt{2}-1} = \frac{\sqrt{3}+\sqrt{2}+1}{\{\sqrt{3}-(\sqrt{2}+1)\}\{\sqrt{3}+(\sqrt{2}+1)\}}$$

$$= \frac{\sqrt{3}+\sqrt{2}+1}{-2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{6}+2+\sqrt{2}}{-2\times 2}$$
$$= \frac{2+\sqrt{2}+\sqrt{6}}{-4}$$

$$= \frac{-1}{4}(2+\sqrt{2}+\sqrt{6})$$

(iii) The denominator is a trinomial surd.

We proceed as with a trinomial surd, by grouping two of the terms.

Thus, $\{(6+\sqrt{5})-\sqrt{11}\}\{(6+\sqrt{5})+\sqrt{11}\}$

$$= (\sqrt{6}+\sqrt{5})^2 - (\sqrt{11})^2$$
$$= (\sqrt{6})^2 + 2\sqrt{6}\sqrt{5} + (\sqrt{5})^2 - 11$$

$$= 6 + 2\sqrt{30} + 5 - 11$$
$$= 2\sqrt{30}$$

\ $\frac{1}{\sqrt{6}-\sqrt{5}-\sqrt{11}} = \frac{\sqrt{6}-\sqrt{5}+\sqrt{11}}{\{\sqrt{6}+\sqrt{5}-\sqrt{11}\}\{\sqrt{6}+\sqrt{5}+\sqrt{11})\}}$

$$= \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{2\sqrt{30}}$$
$$= \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{2\sqrt{30}} \times \frac{\sqrt{30}}{\sqrt{30}}$$
$$= \frac{\sqrt{6\times 30}+\sqrt{5\times 30}+\sqrt{11\times 30}}{2\times 30}$$

$$= \frac{6\sqrt{5}+5\sqrt{6}+\sqrt{330}}{60}$$

Question-89

Simplify each of the following:

(i) $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}+\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}$ (ii) $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

Solution:

(i) Let us rationalise the denominator of each term:

$$\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}=\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}=\frac{\sqrt{6\times 2}-\sqrt{6\times 3}}{\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}=\frac{2\sqrt{3}-3\sqrt{2}}{2-3}=\frac{2\sqrt{3}-3\sqrt{2}}{-1}=-2\sqrt{3}+3\sqrt{2}$$

$$\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}=\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}\times\frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}}=\frac{3\sqrt{2\times 6}-3\sqrt{2\times 3}}{\left(\sqrt{6}\right)^2-\left(\sqrt{3}\right)^2}=\frac{6\sqrt{3}-3\sqrt{6}}{6-3}$$

$$=\frac{3\left(2\sqrt{3}-\sqrt{6}\right)}{3}$$

$$=2\sqrt{3}-\sqrt{6}$$

$$\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}=\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}\times\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}=\frac{4\sqrt{3\times 6}-4\sqrt{3\times 2}}{\left(\sqrt{6}\right)^2-\left(\sqrt{2}\right)^2}=\frac{12\sqrt{2}-4\sqrt{6}}{6-2}=\frac{4\left(3\sqrt{2}-\sqrt{6}\right)}{4}=3\sqrt{2}-\sqrt{6}$$

\ $$\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}+\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}=-2\sqrt{3}+3\sqrt{2}+2\sqrt{3}-\sqrt{6}-3\sqrt{2}+\sqrt{6}=0$$

(ii) Let us rationalize the denominator of each term:

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}=\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}\times\frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}}=\frac{7\sqrt{3\times 10}-7\sqrt{3\times 3}}{\left(\sqrt{10}\right)^2-\left(\sqrt{3}\right)^2}=\frac{7\sqrt{30}-21}{10-3}=\frac{7\left(\sqrt{30}-3\right)}{7}=\sqrt{30}-3$$

$$\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}=\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}\times\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}=\frac{2\sqrt{5\times 6}-2\sqrt{5\times 5}}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2}=\frac{2\sqrt{30}-10}{6-5}=\frac{2\sqrt{30}-10}{1}=2\sqrt{30}-10$$

$$\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}=\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}\times\frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}}=\frac{3\sqrt{2\times 15}-9\sqrt{2\times 2}}{\left(\sqrt{15}\right)^2-\left(3\sqrt{2}\right)^2}=\frac{3\sqrt{30}-18}{15-18}=\frac{3\left(\sqrt{30}-6\right)}{-3}=-\sqrt{30}+6$$

\ $$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}=\sqrt{30}-3-\left(2\sqrt{30}-10\right)-\left(-\sqrt{30}+6\right)=\sqrt{30}-3-2\sqrt{30}+10+\sqrt{30}-6=1$$

Question-90

Taking $\sqrt{2}=1.414, \sqrt{3}=1.732, \sqrt{5}=2.236$ and $\sqrt{6}=2.449$ (approx.). Find the value to three places of decimals of each of the following: (i) $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$ (ii) $\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}+\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Solution:

(i) $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}} = \frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})+(1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$

$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{2\times 5}-\sqrt{2\times 3}+\sqrt{5}+\sqrt{3}-\sqrt{2\times 5}-\sqrt{2\times 3}}{(\sqrt{5})^2-(\sqrt{3})^2}$

$= \frac{2\sqrt{5}+\sqrt{10}-\sqrt{6}-\sqrt{10}-\sqrt{6}}{5-3}$

$= \frac{2\sqrt{5}-2\sqrt{6}}{2}$

$= \frac{2}{2}(\sqrt{5}-\sqrt{6})$

= (2.236 - 2.449)

= (-0.213)

= - 0.213

(ii) $\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}=\frac{(2+\sqrt{3})(2+\sqrt{3})+(2-\sqrt{3})(2-\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$

$$\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}=\frac{(2+\sqrt{3})(2+\sqrt{3})+(2-\sqrt{3})(2-\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$$

$$= -\frac{(2+\sqrt{3})^2+(2-\sqrt{3})^2}{(2)^2-(\sqrt{3})^2}$$

$$= \frac{(2)^2+2(2)(\sqrt{3})+(\sqrt{3})^2+(2)^2-2(2)(\sqrt{3})+(\sqrt{3})^2}{4-3}$$

$$= \frac{4+4\sqrt{3}+3+4-4\sqrt{3}+3}{1}$$

= 14.268

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}=\frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}=\frac{(\sqrt{3})^2-2\sqrt{3}+1}{3-1}$$

$$\frac{3-2\sqrt{3}+1}{2}=\frac{4-2\sqrt{3}}{2}=2-\sqrt{3}$$

$$\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}+\frac{\sqrt{3}-1}{\sqrt{3}+1}=14+2-\sqrt{3}=16-\sqrt{3}$$

= 16 - 1.732

= 14.268.

Question-91

Simplify (a) $\sqrt[3]{2}$ **+** $\sqrt[3]{16}$ **-** $\sqrt[3]{54}$ **(b)** $\sqrt[3]{32}\times\sqrt[3]{250}$

Solution:

(a) $\sqrt[3]{16}$ **=** $\sqrt[3]{2\times2\times2\times2}$ **=** $2\sqrt[3]{2}$

$\sqrt[3]{54}$ **=** $\sqrt[3]{2\times3\times3\times3}$ **=** $3\sqrt[3]{2}$

$\therefore$ $\sqrt[3]{2}$ **+** $\sqrt[3]{16}$ **-** $\sqrt[3]{54}$ **=** $\sqrt[3]{2}$ **+** $2\sqrt[3]{2}$ **-** $3\sqrt[3]{2}$ **=** (1+2-3) $\sqrt[3]{2}$ = 0

(b) $\sqrt[3]{32}$ = $\sqrt[3]{2\times2\times2\times2}$ = $2\sqrt[3]{4}$

$\sqrt[3]{250}$ = $\sqrt[3]{2\times5\times5\times5}$ = $5\sqrt[3]{2}$

$\therefore$ $\sqrt[3]{32}\times\sqrt[3]{250}$ **=** $2\sqrt[3]{4}$ X $5\sqrt[3]{2}$ = $10\sqrt[3]{4\times2}$ = 10 x 2 =20

Question-92

Given that $\sqrt{3}$ = 1.7321, find correct to 3 places of decimals, the value of

$\sqrt{192}-\frac{1}{2}\sqrt{48}-\sqrt{75}$

Solution:

$\sqrt{192} = \sqrt{2\times2\times2\times2\times2\times2\times3} = 2 \times 2 \times 2\sqrt{3} = 8\sqrt{3}$

$\sqrt{48} = \sqrt{2\times2\times2\times2\times3} = 2 \times 2\sqrt{3} = 4\sqrt{3}$

$\sqrt{75} = \sqrt{3\times5\times5} = 5\sqrt{3}$

$\sqrt{192}-\frac{1}{2}\sqrt{48}-\sqrt{75} = 8\sqrt{3}-\frac{1}{2} \times 4\sqrt{3}-5\sqrt{3} = 8\sqrt{3}-2\sqrt{3}-5\sqrt{3}$

$= (8-2-5)\sqrt{3} = \sqrt{3} = 1.732.$

Question-93

Simplify by rationalizing the denominator :

(i) $\frac{4}{\sqrt[3]{16}}$ (ii) $\frac{2\sqrt[3]{3}}{4\sqrt[3]{5}}$

Solution:

(i) $\frac{4}{\sqrt[3]{16}}=\frac{4}{2\sqrt[3]{2}}=\frac{2}{\sqrt[3]{2}}\times\frac{\sqrt[3]{4}}{\sqrt[3]{4}}=\frac{2\sqrt[3]{4}}{\sqrt[3]{2^3}}=\frac{2\sqrt[3]{4}}{2}=\sqrt[3]{4}$

(ii) $\frac{2\times\sqrt[3]{3}}{4\sqrt[3]{5}}=\frac{2\sqrt[3]{3}}{4\sqrt[3]{5}}\times\frac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}}=\frac{\sqrt[3]{3}\times\sqrt[3]{5^2}}{2\sqrt[3]{5^3}}=\frac{\sqrt[3]{75}}{2\times5}=\frac{1}{10}\sqrt[3]{75}$

Question-94

If $\sqrt{5}$ = 2.236 (approx.), evaluate $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$ correct to three places of decimals.

Solution:

$3-2\sqrt{5}$ is a rationalising factor of $3+2\sqrt{5}$

$$\frac{3-\sqrt{5}}{3+2\sqrt{5}} = \frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$
$$= \frac{9-6\sqrt{5}-3\sqrt{5}+10}{9-20}$$
$$= \frac{19-9\sqrt{5}}{-11}$$
$$= -\frac{19+9(2.236)}{11}$$
$$= -\frac{19+20.124}{11}$$
$$= \frac{1.124}{11}$$
$$= 0.102$$

Question-95

Simplify : (i) $3\times\sqrt[5]{-96}$ (ii) $12\times\sqrt[5]{\frac{-1}{256}}$

Solution:

$$\text{(i)}\,3\times\sqrt[5]{-96} = 3\times\sqrt[5]{(-32)\times 3}$$
$$= 3\times\sqrt[5]{(-2)^5}\times\sqrt[5]{3}$$
$$= 3\times(-2)\times\sqrt[5]{3} = -6\sqrt[5]{3}$$

$$\text{(ii)}\,12\times\sqrt[5]{\frac{-1}{256}} = 12\times\frac{-1}{\sqrt[5]{4^4}}$$
$$= \frac{-12}{\sqrt[5]{4^4}}\times\frac{\sqrt[5]{4}}{\sqrt[5]{4}}$$
$$= -3\sqrt[5]{4}$$

Question-96

Simplify : (i) $\sqrt[4]{\frac{81}{8}}$ (ii) $\sqrt{\frac{125}{63}}$

Solution:

(i) $\sqrt[4]{\dfrac{81}{8}} = \dfrac{\sqrt[4]{81}}{\sqrt[4]{8}} = \dfrac{\sqrt[4]{3^4}}{\sqrt[4]{8}} \times \dfrac{\sqrt[4]{2}}{\sqrt[4]{2}} = \dfrac{3 \times \sqrt[4]{2}}{\sqrt[4]{16}}$

$= \dfrac{3}{2} \times \sqrt[4]{2}$

(ii) $\sqrt{\dfrac{125}{63}} = \dfrac{\sqrt{25 \times 5}}{\sqrt{9 \times 7}} - \dfrac{\sqrt{25} \times \sqrt{5}}{\sqrt{9} \times \sqrt{7}} - \dfrac{5 \times \sqrt{5}}{3 \times \sqrt{7}}$

$= \dfrac{5}{3} \times \dfrac{\sqrt{5}}{\sqrt{7}} \times \dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{5\sqrt{35}}{21} = \dfrac{5}{21} \times \sqrt{35}$

Question-97

Express each of the following as a mixed surd in the simplest form:

(i) $2 \times \sqrt[3]{270}$ (ii) $5 \times \sqrt[6]{1458}$

Solution:

(i) $2 \times \sqrt[3]{270} = 2 \times \sqrt[3]{3^3 \times 10}$

$= 2 \times 3 \times \sqrt[3]{10}$

$= 6\sqrt[3]{10}$

(ii) $5 \times \sqrt[6]{1458} = 5 \times \sqrt[6]{3^6 \times 2}$

$= 5 \times 3 \times \sqrt[6]{2}$

$= 15\sqrt[6]{2}$

Question-98

Express each of the following as a mixed surd in the simplest form:

(i) $\sqrt[4]{\frac{16}{27}}$ (ii) $5\sqrt[3]{135}$

Solution:

(i) $\sqrt[4]{\frac{16}{27}} = \frac{\sqrt[4]{16}}{\sqrt[4]{27}} = \frac{\sqrt[4]{2^4}}{\sqrt[4]{3^3}} \times \frac{\sqrt[4]{3}}{\sqrt[4]{3}} = \frac{2\times\sqrt[4]{3}}{\sqrt[4]{3^4}} = \frac{2}{3}\sqrt[4]{3}$

(ii) $5\sqrt[3]{135} = 5\sqrt[3]{3^3\times 5} = 5\times\sqrt[3]{3^3}\times\sqrt[3]{5}$

$= 5\times 3\times\sqrt[3]{5} = 15\times\sqrt[3]{5}$

Question-99

Arrange in descending order of magnitude:

$\sqrt[3]{2}$, $\sqrt[6]{3}$ and $\sqrt[9]{4}$.

Solution:

The given surds are of order 3,6 and 9 respectively.

The L.C.M. of 3, 6 and 9 is 18.

Reducing each surd to a surd of order 18,

$$\sqrt[3]{2} = 2^{\frac{1}{3}} = 2^{\left(\frac{1}{3}\right)\times\left(\frac{6}{6}\right)} = (2^6)^{\frac{1}{18}} = (64)^{\frac{1}{18}} = \sqrt[18]{64}$$

$$\sqrt[6]{3} = 3^{\frac{1}{6}} = 3^{\left(\frac{1}{6}\right)\times\left(\frac{3}{3}\right)} = (3^3)^{\frac{1}{18}} = (27)^{\frac{1}{18}} = \sqrt[18]{27}$$

$$\sqrt[9]{4} = 4^{\frac{1}{9}} = 4^{\left(\frac{1}{9}\right)\times\left(\frac{2}{2}\right)} = (4^2)^{\frac{1}{18}} = (16)^{\frac{1}{18}} = \sqrt[18]{16}$$

Hence , $\sqrt[18]{64} > \sqrt[18]{27} > \sqrt[18]{16}$

$$\sqrt[3]{2} > \sqrt[6]{3} > \sqrt[9]{4}.$$

Question-100

Simplify : $\sqrt[3]{a^4b} + \sqrt[3]{a\,b^4}$

Solution:

Reducing to the simplest form, we get

$\sqrt[3]{a^4b} = \sqrt[3]{a^3 \times ab} = a.\sqrt[3]{a\,b}$

$\sqrt[3]{a\,b^4} = \sqrt[3]{b^3 \times ab} = b.\sqrt[3]{a\,b}$

$\sqrt[3]{a^4b} + \sqrt[3]{a\,b^4} = a\sqrt[3]{a\,b} + b\sqrt[3]{a\,b}$

$= (a+b)\sqrt[3]{a\,b}$

Question-101

Simplify :

$\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

Solution:

$\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

$\sqrt[4]{81} = \sqrt[4]{3 \times 3 \times 3 \times 3} = 3$;

$\sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6} = 6$;

$\sqrt[5]{32} = \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2} = 2$;

$\sqrt{225} = \sqrt{15 \times 15} = 15$;

$\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225} = (3 - 8 \times 6 + 15 \times 2 + 15)$

$= (3 - 48 + 30 + 15) = 0$

Question-102

Simplify:

$$(3\sqrt{5} - 5\sqrt{2})(4\sqrt{5} + 3\sqrt{2})$$

Solution:

$$\left(3\sqrt{5}-5\sqrt{2}\right)\left(4\sqrt{5}+3\sqrt{2}\right)$$

$$=\left[12\times\left(\sqrt{5}\right)^2+9\sqrt{5}\times\sqrt{2}-20\sqrt{2}\times\sqrt{5}-15\left(\sqrt{5}\right)^2\right]$$

$$=\left(12\times5+9\sqrt{10}-20\sqrt{10}-15\times2\right)$$

$$=(60-30)+(9-20)\sqrt{10}=\left(30-11\sqrt{10}\right)$$

Question-103

Simplify:

$$(\sqrt[6]{12} \div \sqrt{3}\ \sqrt[3]{2})$$

Solution:

L.C.M of 6,2 and 3 is 6.

$$(\sqrt[6]{12} \div \sqrt{3}\ \sqrt[3]{2}) = \frac{\sqrt[6]{12}}{\sqrt[6]{3^3}\times\sqrt[6]{2^2}}=\sqrt[6]{\frac{12}{3^3\times2^2}}=\sqrt[6]{\frac{1}{3^2}}=\sqrt[6]{\frac{1}{9}}$$

Question-104

Find the rationalising factor of $\sqrt[5]{a^2b^3c^4}$

Solution:

$\sqrt[5]{a^2b^3c^4} = \sqrt[5]{a^2} \times \sqrt[5]{b^3} \times \sqrt[5]{c^4} = a^{\frac{2}{5}}\, b^{\frac{3}{5}}\, c^{\frac{4}{5}}$

$\left(a^{\frac{2}{5}}\, b^{\frac{3}{5}}\, c^{\frac{4}{5}}\right) \times \left(a^{\frac{3}{5}}\, b^{\frac{2}{5}}\, c^{\frac{1}{5}}\right) = abc$

Hence the required R.F = = $\sqrt[5]{a^3b^2c}$

Question-105

Find the rationalising factor of $\sqrt[5]{160}$

Solution:

$\sqrt[5]{160} = \sqrt[5]{2^5 \times 5} = \sqrt[5]{2^5} \times \sqrt[5]{5} = 2´ \sqrt[5]{5} = 2 \times 5^{(1/5)}$

$2 \times 5^{(1/5)} \times 5^{(4/5)} = 2 \times 5 = 10$

Hence the required R.F is $5^{(1/5)} = (5^4)^{1/5} = \sqrt[5]{625}$

Question-106

Convert $\sqrt[4]{3}$ and $\sqrt[6]{2}$ into surds of the same but smallest order.

Solution:

Since $\sqrt[4]{3} = 3^{1/4}$ and $\sqrt[6]{2} = 2^{1/6}$, it follows that the given surds are of order 4 and 6 respectively. The L.C.M. of 4 and 6 is 12.So,we shall convert each one of the given surds into a surd of order 12.

$\sqrt[4]{3} = 3^{1/4} = 3^{(1/4)\,(3/3)} = 3^{(3/12)} = (3^3)^{1/12} = (27)^{1/12} = \sqrt[12]{27}$

$\sqrt[6]{2} = 2^{1/6} = 2^{(1/6)\,(2/2)} = 2^{(2/12)} = (2^2)^{1/12} = (4)^{1/12} = \sqrt[12]{4}$

Question-107

If $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$, evaluate $\frac{\sqrt{10} - \sqrt{5}}{\sqrt{2}}$

Solution:

The rationalising factor of $\sqrt{2}$ is $\sqrt{2}$.

$$\frac{\sqrt{10} - \sqrt{5}}{\sqrt{2}} = \frac{\sqrt{10} - \sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10} \times \sqrt{2} - \sqrt{5} \times \sqrt{2}}{2} - \frac{\sqrt{20} - \sqrt{10}}{2}$$

$$= \frac{\sqrt{4 \times 5} - \sqrt{10}}{2} - \frac{2\sqrt{5} - \sqrt{10}}{2}$$

$$= \sqrt{5} - (1/2)\sqrt{10} = 2.236 - (1/2)(3.162)$$

$$= (2.236 - 1.581) = 0.655$$

Question-108

If $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a - b\sqrt{6}$, find the values of *a* and *b*.

Solution:

$$\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$$

$$= \frac{(\sqrt{2}+\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})}$$

$$= \frac{6+2\sqrt{6}+3\sqrt{6}+6}{(18-12)} - \frac{12+5\sqrt{6}}{6}$$

$$= \quad 2+\frac{5}{6}\sqrt{6} = a - b\sqrt{6}$$

$$\therefore a = 2,\ b = -\frac{5}{6}$$

Question-109

Express $\frac{\sqrt{2}}{(\sqrt{2}+\sqrt{3}-\sqrt{5})}$ with a rational denominator.

Solution:

$$\frac{\sqrt{2}}{(\sqrt{2}+\sqrt{3}-\sqrt{5})} = \frac{\sqrt{2}}{(\sqrt{2}+\sqrt{3})-\sqrt{5}} \times \frac{(\sqrt{2}+\sqrt{3})+\sqrt{5}}{(\sqrt{2}+\sqrt{3})+\sqrt{5}}$$

$$= \sqrt{2} \times \frac{(\sqrt{2}+\sqrt{3})+\sqrt{5}}{[(\sqrt{2}+\sqrt{3})^2-5]} = \frac{(2+\sqrt{6}+\sqrt{10})}{2\sqrt{6}}$$

$$= \frac{(2+\sqrt{6}+\sqrt{10})}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{(2+\sqrt{6}+\sqrt{10})\times\sqrt{6}}{12}$$

$$= \frac{2\sqrt{6}+6+\sqrt{60}}{12} = \frac{2\sqrt{6}+6+2\sqrt{15}}{12} = \frac{\sqrt{6}+3+\sqrt{15}}{6}$$

Question-110

Find the value of $\frac{1}{(\sqrt{3}-\sqrt{2}-1)}$ correct to three places of decimal, it being given that $\sqrt{2}$ = 1.4142.

Solution:

$$\frac{1}{(\sqrt{3}-\sqrt{2}-1)} = \frac{1}{(\sqrt{3}-1)-\sqrt{2}} \times \frac{(\sqrt{3}-1)+\sqrt{2}}{(\sqrt{3}-1)+\sqrt{2}} = \frac{(\sqrt{3}-1)+\sqrt{2}}{(\sqrt{3}-1)^2-2}$$

$$= \frac{(\sqrt{3}-1)+\sqrt{2}}{2-2\sqrt{3}} = \frac{(\sqrt{3}-1)+\sqrt{2}}{2(1-\sqrt{3})}$$

$$= \frac{(\sqrt{3}-1)+\sqrt{2}}{2(1-\sqrt{3})} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{2+\sqrt{2}+\sqrt{6}}{2(1-3)} = \frac{2+\sqrt{2}+\sqrt{6}}{-4}$$

$$= \frac{2+1.4142+2.4495}{-4} = \frac{5.8637}{-4} = -1.466$$

Q.111: Find five rational numbers between 1 and 2.

Solution:

We have to find five rational numbers between 1 and 2.

So, let us write the numbers with denominator 5 + 1 = 6

Thus, 6/6 = 1, 12/6 = 2

From this, we can write the five rational numbers between 6/6 and 12/6 as:

7/6, 8/6, 9/6, 10/6, 11/6

Q.112: Find five rational numbers between 3/5 and 4/5.

Solution:

We have to find five rational numbers between 3/5 and 4/5.

So, let us write the given numbers by multiplying with 6/6, (here 6 = 5 + 1)

Now,

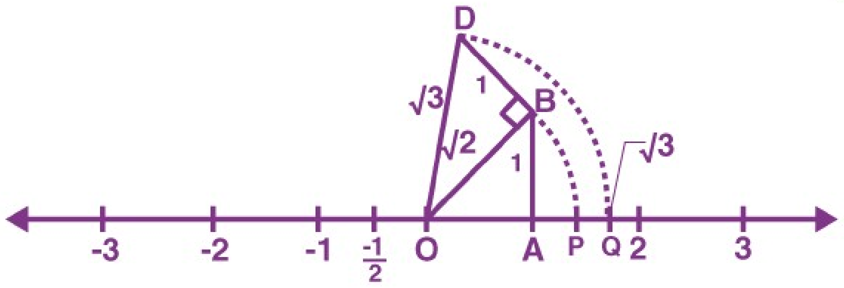
3/5 = (3/5) × (6/6) = 18/30

4/5 = (4/5) × (6/6) = 24/30

Thus, the required five rational numbers will be: 19/30, 20/30, 21/30, 22/30, 23/30

Q.113: Locate √3 on the number line.

Solution:



Construct BD of unit length perpendicular to OB (here, OA = AB = 1 unit) as shown in the figure.

By Pythagoras theorem,

OD = √(2 + 1) = √3

Taking O as the centre and OD as radius, draw an arc which intersects the number line at the point Q using a compass.

Therefore, Q corresponds to the value of √3 on the number line.

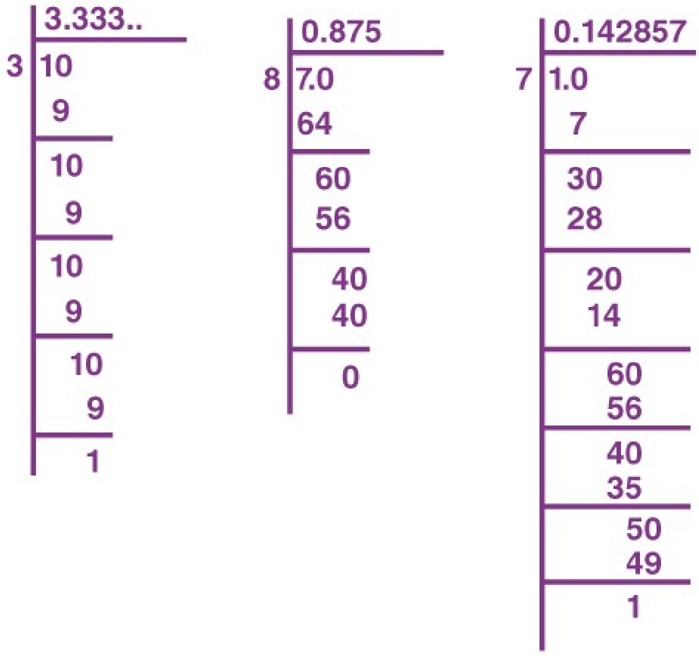
Q.114: Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Solution:

No, since the square root of a positive integer 16 is equal to 4. Here, 4 is a rational number.

Q.115: Find the decimal expansions of 10/3, 7/8 and 1/7.

Solution:



Therefore, 10/3 = 3.3333...

7/8 = 0.875

1/7 = 0.1428571...

Q.116:Show that 0.3333...=$0.\bar{3}$ can be expressed in the form p/q, where p and q are integers and q ≠0.

Solution:

Let x = 0.3333....

Multiply with 10,

10x = 3.3333...

Now, 3.3333… = 3 + x (as we assumed x = 0.3333…)

Thus, 10x = 3 + x

10x – x = 3

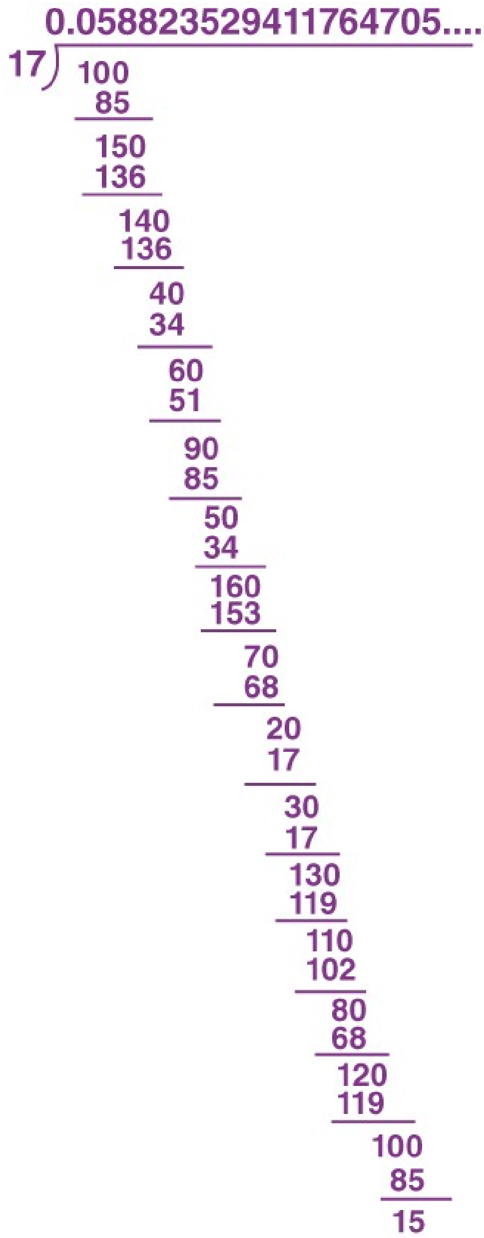
9x = 3

x = 1/3

Therefore, 0.3333… = 1/3. Here, 1/3 is in the form of p/q and q ≠ 0.

Q.117: What can the maximum number of digits be in the repeating block of digits in the decimal expansion of 1/17? Perform the division to check your answer.

Solution:



Thus, 1/17 = 0.0588235294117647....

Therefore, 1/17 has 16 digits in the repeating block of digits in the decimal expansion.

Q.118: Find three different irrational numbers between the rational numbers 5/7 and 9/11.

Solution:

The given two rational numbers are 5/7 and 9/11.

5/7 = 0.714285714.....

9/11 = 0.81818181……

Hence, the three irrational numbers between 5/7 and 9/11 can be:

0.720720072000…

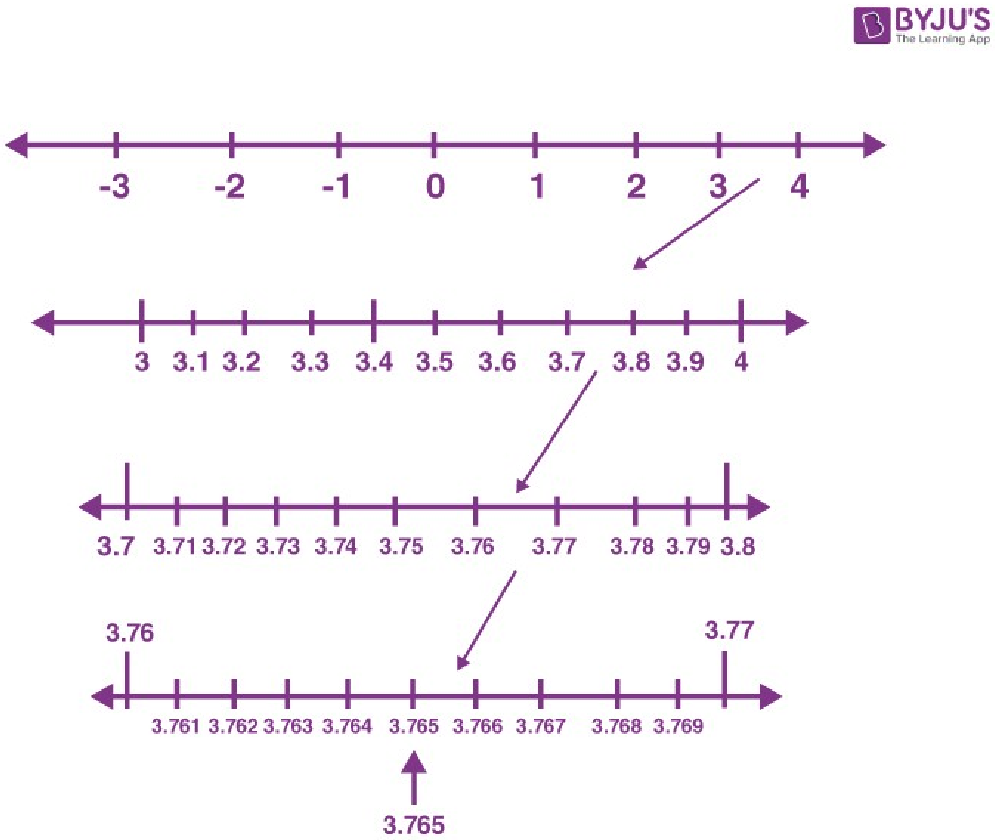
0.730730073000…

0.808008000…

Q.119: Visualise 3.765 on the number line, using successive magnification.

Solution:

Visualisation of 3.765 on the number line, using successive magnification is given below:



Q.120: Add 2√2+ 5√3 and √2 – 3√3.

Solution:

(2√2 + 5√3) + (√2 – 3√3)

= 2√2 + 5√3 + √2 – 3√3

$= (2 + 1)\sqrt{2} + (5 - 3)\sqrt{3}$

$= 3\sqrt{2} + 2\sqrt{3}$

Q.121: Simplify: $(\sqrt{3}+\sqrt{7})(\sqrt{3}-\sqrt{7})$.

Solution:

$(\sqrt{3} + \sqrt{7})(\sqrt{3} - \sqrt{7})$

Using the identity $(a + b)(a - b) = a^2 - b^2$,

$(\sqrt{3} + \sqrt{7})(\sqrt{3} - \sqrt{7}) = (\sqrt{3})^2 - (\sqrt{7})^2$

$= 3 - 7$

$= -4$

Q.122: Rationalise the denominator of $1/[7+3\sqrt{3}]$.

Solution:

$1/(7 + 3\sqrt{3})$

By rationalizing the denominator,

$= [1/(7 + 3\sqrt{3})]\ [(7 - 3\sqrt{3})/(7 - 3\sqrt{3})]$

$= (7 - 3\sqrt{3})/[(7)^2 - (3\sqrt{3})^2]$

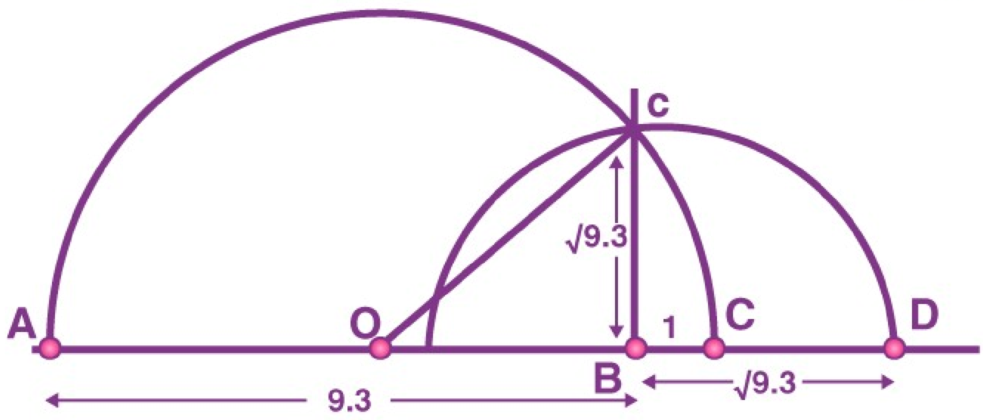
$= (7 - 3\sqrt{3})/(49 - 27)$

$= (7 - 3\sqrt{3})/22$

Q.123: Represent $\sqrt{(9.3)}$ on the number line.

Solution:

Representation of $\sqrt{9.3}$ on the number line is given below:



Q.124: Simplify:

(i) $7^{2/3}.7^{1/5}$
(ii) $10^{1/2}/10^{1/4}$

Solution:

(i) $7^{2/3}.7^{1/5}$

Bases are equal, so add the powers.

$7^{(2/3\ +\ 1/5)}$

$= 7^{(10\ +\ 3)/15}$

$= 7^{13/15}$

(ii) $10^{1/2}/10^{1/4}$

Bases are equal, so subtract the powers.

$= 10^{\ (1/2\ -\ 1/4)}$

$= 10^{1/4}$

Q.125: What is the product of a rational and an irrational number?
a) Always an integer
b) Always a rational number
c) Always an irrational number
d) Sometimes rational and sometimes irrational

Correct answer: Option (c)

Explanation:

The product of a rational and an irrational number is always an irrational number.

For example, 2 is a rational number and √3 is irrational. Thus, 2√3 is always an irrational number.

Q.126: What is the value of $(256)^{0.16} \times (256)^{0.09}$?

a) 4

b) 16

c) 64

d) 256.25

Correct answer: Option (a)

$(256)^{0.16} \times (256)^{0.09} = (256)^{(0.16 + 0.09)}$

$= (256)^{0.25}$

$= (256)^{(25/100)}$

$= (256)^{(1/4)}$

$= (4^4)^{(1/4)}$

$= 4^{4(1/4)}$

$= 4$

127. Every whole number is a natural number write true or false.

128. If $x = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, find the value of $x^2 + y^2 + xy$.

129. If $x = \dfrac{2-\sqrt{5}}{2+\sqrt{5}}$ and $y = \dfrac{2+\sqrt{5}}{2-\sqrt{5}}$, find the value of $x^2 - y^2$.

130. Determine rational numbers p and q if

$$\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = p - 7\sqrt{5}\,q.$$

131. Simplify: $\dfrac{6}{2\sqrt{3}-\sqrt{6}} + \dfrac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} - \dfrac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$.

132. Simplify: $\dfrac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} + \dfrac{2\sqrt{3}}{\sqrt{6}+2} - \dfrac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$.

133. Show that: $\dfrac{1}{3-\sqrt{8}} - \dfrac{1}{\sqrt{8}-\sqrt{7}} + \dfrac{1}{\sqrt{7}-\sqrt{6}} - \dfrac{1}{\sqrt{6}-\sqrt{5}} + \dfrac{1}{\sqrt{5}-2} = 5$

134. If: $x = \dfrac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}-\sqrt{p-q}}$, then find the value of $qx^2 - 2px + q$.

135. Show that: $\frac{x^{-1}+y^{-1}}{x^{-1}}+\frac{x^{-1}-y^{-1}}{x^{-1}}=\frac{x^2+y^2}{xy}$

136. If 2a = 3b = 6c then show that $c=\frac{ab}{a+b}$.

137. If $x=2+3\sqrt{2}$, then find the value of $\left(x+\frac{14}{x}\right)$.

Answers

1. False	2. 99	3. $-144\sqrt{5}$	4. $p=0,\ q=-\frac{1}{11}$
5. 0	6. 0	8. 0	11. $6\sqrt{2}$

Question 138. Simplify: (√5 + √2)2.

Solution: Here, (√5 + √22 = (√52 + 2√5√2 + (√2)2

= 5 + 2√10 + 2 = 7 + 2√10

Question 139. Identify a rational number among the following numbers : 2 + √2, 2√2, 0 and π

Solution: O is a rational number.

Question 140. Express 1.8181… in the form pq where p and q are integers and q ≠ 0.

Solution: Let x =1.8181… …(i)

100x = 181.8181… …(ii) [multiplying eqn. (i) by 100]

99x = 180 [subtracting (i) from (ii)]

x = 18099

Hence, 1.8181… = 18099 = 2011

Question 141. Simplify : √45 – 3√20 + 4√5

Solution: √45 – 3√20 + 4√5 = 3√5 – 6√5 + 4√5 = √5.

Question 142. Evaluate : (√5 + √22 + (√8 – √5)2

Solution: (√5 + √2)2 + (√8 – √52 = 5 + 2 + 2√10 + 8 + 5 – 2√40

= 20 + 2√10 – 4√10 = 20 – 2√10

Question 143.

Express $23.\overline{43}$ in pq form, where p, q are integers and q ≠ 0.
Solution:
Let x = $23.\overline{43}$
or x = 23.4343… ….(i)
100x = 2343.4343… …(ii) [Multiplying eqn. (i) by 100] 99x = 2320 [Subtracting (i) from (ii)
⇒ x = 2320

Question 144.

Let ‘a’ be a non-zero rational number and ‘b’ be an irrational number. Is ‘ab’ necessarily an irrational ? Justify your answer with example.
Solution:
Yes, ‘ab’ is necessarily an irrational.
For example, let a = 2 (a rational number) and b = √2 (an irrational number)
If possible let ab = 2√2 is a rational number.
Now, aba = 22√2 = √2 is a rational number.
[∵ The quotient of two non-zero rational number is a rational] But this contradicts the fact that √2 is an irrational number.
Thus, our supposition is wrong.
Hence, ab is an irrational number.

Question 145. Express 1.32 + 0.35 as a fraction in the simplest form.

Solution: Let . x = 1.32 = 1.3222…..(i)

Multiplying eq. (i) by 10, we have

10x = 13.222…

Again, multiplying eq. (i) by 100, we have

100x = 132.222… …(iii)

Subtracting eq. (ii) from (iii), we have

100x – 10x = (132.222…) – (13.222…)

90x = 119

⇒ x = 11990

Again, y = 0.35 = 0.353535……

Multiply (iv) by 100, we have …(iv)

100y = 35.353535… (v)

Subtracting (iv) from (u), we have

100y – y = (35.353535…) – (0.353535…)

99y = 35

y = 3599

Question 146. Find 'x', if $2^{x-7} \times 5^{x-4} = 1250$.

Solution: We have $2^{x-7} \times 5^{x-4} = 1250$

⇒ $2^{x-7} \times 5^{x-4}$ = 2 5 × 5 × 5 × 5

⇒ $2^{x-7} \times 5^{x-4}$ = 21 × 54

Equating the powers of 2 and 5 from both sides, we have

⇒ x – 7 = 1 and x – 4 = 4

⇒ x = 8 and x = 8

Hence, x = 8 is the required value.

Question 147: Is zero a rational number? Can you write it in the form p/q, where p and q are integers and q ≠ 0?
Solution:

Yes, zero is a rational number.

It can be written in p/q form, provided that q ≠ 0.

For example, 0/1 or 0/3 or 0/4 etc.

Question 148: Find five rational numbers between 1 and 2.
Solution:

We know that one rational number between two numbers m and n = (m+n)/2

To find: 5 rational numbers between 1 and 2

Step 1: Rational number between 1 and 2

= (1+2)/2

= 3/2

Step 2: Rational number between 1 and 3/2

= (1+3/2)/2

= 5/4

Step 3: Rational number between 1 and 5/4

= (1+5/4)/2

= 9/8

Step 4: Rational number between 3/2 and 2

= 1/2 [(3/2) + 2)]

= 7/4

Step 5: Rational number between 7/4 and 2

= 1/2 [7/4 + 2]

= 15/8

Arrange all the results: 1 < 9/8 < 5/4 < 3/2 < 7/4 < 15/8 < 2

Therefore required integers are, 9/8, 5/4, 3/2, 7/4, 15/8

Question 149: Find six rational numbers between 3 and 4.

Solution:

Steps to find n rational numbers between any two numbers:

Step 1: Multiply and divide both the numbers by n+1.

In this example, we have to find 6 rational numbers between 3 and 4. Here n = 6

Multiply 3 and 4 by 7

3 x 7/7 = 21/7 and

4 x 7/7 = 28/7

Step 2: Choose 6 numbers between 21/7 and 28/7

3 = 21/7 < 22/7 < 23/7 < 24/7 < 25/7 < 26/7 < 27/7 < 28/7 = 4

Therefore, 6 rational numbers between 3 and 4 are

22/7, 23/7, 24/7, 25/7, 26/7, 27/7

Question 150: Find five rational numbers between 3/5 and 4/5.

Solution:

Steps to find n rational numbers between any two numbers:

Step 1: Multiply and divide both the numbers by n+1.

In this example, we have to find 5 rational numbers between 3/5 and 4/5. Here n = 5

Multiply 3/5 and 4/5 by 6

3/5 x 6/6 = 18/30 and

4/5 x 6/6 = 24/30

Step 2: Choose 5 numbers between 18/30 and 24/30

$3/5 = 18/30 < 19/30 < 20/30 < 21/30 < 22/30 < 23/30 < 24/30 = 4/5$

Therefore, 5 rational numbers between 3/5 and 4/5 are

19/30, 20/30, 21/30, 22/30, 23/30

Question 151: Are the following statements true or false? Give reasons for your answer.

(i) Every whole number is a natural number.

(ii) Every integer is a rational number.

(iii) Every rational number is an integer.

(iv) Every natural number is a whole number,

(v) Every integer is a whole number.

(vi) Every rational number is a whole number.

Solution:

(i) False.

Reason: As 0 is not a natural number.

(ii) True.

(iii) False.

Reason: Numbers such as 1/2, 3/2, and 5/3 are rational numbers but not integers.

(iv) True.

(v) *False.*

Reason: Negative numbers are not whole numbers.

(vi) *False.*

Reason: Proper fractions are not whole numbers.

Exercise 1.2

Question 152: Express the following rational numbers as decimals.
(i) 42/100 (ii) 327/500 (iii) 15/4

Solution:

(i) By long division method

100) $\overline{42}$ (0.42

400

$\overline{200}$

200

$\overline{0}$

Therefore, $\frac{42}{100}$ = 0.42

(ii) By long division method

500) $\overline{327.000}$ (0.654

3000

$\overline{2700}$

2500

$\overline{2000}$

2000

$\overline{0}$

Therefore, $\frac{327}{500}$ = 0.654

(iii) By long division method

4) $\overline{15.00}$ (3.75

12

$\overline{30}$

28

$\overline{20}$

20

$\overline{0}$

Therefore, $\frac{15}{4}$ = 3.75

Question 153: Express the following rational numbers as decimals.
(i) 2/3 (ii) -4/9 (iii) -2/15 (iv) -22/13 (v) 437/999 (vi) 33/26
Solution:

(i) Divide 2/3 using long division:

$$
\begin{array}{r|l}
 & \underline{0.66666} \\
3 & 2.00000 \\
 & \underline{0} \\
 & 20 \\
 & \underline{18} \\
 & \;\;20 \\
 & \;\;\underline{18} \\
 & \;\;\;\;20 \\
 & \;\;\;\;\underline{18} \\
 & \;\;\;\;\;\;20 \\
 & \;\;\;\;\;\;\underline{18} \\
 & \;\;\;\;\;\;\;\;20 \\
 & \;\;\;\;\;\;\;\;\underline{18} \\
 & \;\;\;\;\;\;\;\;\;\;2
\end{array}
$$

$$\frac{2}{3} = 0.666\ldots = 0.\bar{6}$$

(ii) Divide using long division: -4/9

$9)\ \overline{4.000}\ (\ 0.444$

3600

$\overline{4000}$

3600

$\overline{4000}$

3600

$\overline{400}$

$$\frac{-4}{9} = -\,0.4444\ldots = -0.\bar{4}$$

(iii) Divide using long division: -2/15

$$
\begin{array}{r}
0.133 \\
15\overline{)2.0000} \\
\underline{15} \\
50 \\
\underline{45} \\
50 \\
\underline{45} \\
6
\end{array}
$$

$$\frac{-2}{15} = -0.133 = -0.1\bar{3}$$

(iv) *Divide using long division: -22/13*

$$
\begin{array}{l}
1.69230769 \\
13)\ \overline{22.000} \\
\underline{13} \\
90 \\
\underline{78} \\
120 \\
\underline{117} \\
30 \\
\underline{26} \\
40 \\
\underline{39} \\
100 \\
\underline{91} \\
90 \\
\underline{78} \\
120 \\
\underline{117} \\
3
\end{array}
$$

$$\frac{-22}{13} = -1.6923076923\ldots = -1.\overline{692307}$$

(v) *Divide using long division: 437/999*

```
999) 437.0000 ( 0.43743
     3996
     ----
     3740
     2997
     ----
     7430
     6993
     ----
     4370
     3996
     ----
     3740
     2997
     ----
      743
```

$$\frac{437}{999} = 0.43743... = 0.\overline{437}$$

(vi) *Divide using long division: 33/26*

```
    1.2692307692
26)33.000000000
   26
   --
    70
    52
    ---
    180
    156
    ---
     240
     234
     ---
       60
       52
       --
        80
        78
        ---
        200
        182
        ---
         180
         156
         ---
          24
```

$$\frac{33}{26} = 1.269230769... = 1.\overline{2692307}$$

Question 154: Look at several examples of rational numbers in the form p/q (q ≠ 0), where p and q are integers with no common factors other than 1 and having terminating decimal representations. Can you guess what property q must satisfy?

Solution:

The decimal representation will be terminating if the denominators have factors 2 or 5, or both. Therefore, p/q is a terminating decimal when the prime factorization of q must have only powers of 2 or 5 or both.

Exercise 1.3

Question 155: Express each of the following decimals in the form p/q:

(i) 0.39

(ii) 0.750

(iii) 2.15

(iv) 7.010

(v) 9.90

(vi) 1.0001

Solution:

(i)

0.39 = 39/100

(ii)

0.750 = 750/1000 = 3/4

(iii)

2.15 = 215/100 = 43/20

(iv)

7.010 = 7010/1000 = 701/100

(v)

9.90 = 990/100 = 99/10

(vi)

1.0001 = 10001/10000

Question 156: Express each of the following decimals in the form p/q:

(i) $0.\overline{4}$ (ii) $0.\overline{37}$

(iii) $0.\overline{54}$ (iv) $0.\overline{621}$

(v) $125.\overline{3}$ (vi) $4.\overline{7}$

(vii) $0.4\overline{7}$

Solution:

(i) Let $x = 0.\overline{4}$

or $x = 0.\overline{4} = 0.444$ …. (1)

Multiplying both sides by 10

$10x = 4.444$ …..(2)

Subtract (1) by (2), and we get

$10x - x = 4.444\ldots - 0.444\ldots$

$9x = 4$

$x = 4/9$

=> $0.\overline{4} = 4.9$

(ii) Let $x = 0.3737..$ …. (1)

Multiplying both sides by 100

$100x = 37.37\ldots$ …..(2)

Subtract (1) from (2), and we get

$100x - x = 37.37\ldots - 0.3737\ldots$

$100x - x = 37$

$99x = 37$

$x = 37/99$

(iii) Let $x = 0.5454\ldots$ (1)

Multiplying both sides by 100

$100x = 54.5454\ldots.$ (2)

Subtract (1) from (2), and we get

$100x - x = 54.5454.... - 0.5454....$

$99x = 54$

$x = 54/99$

(iv) Let $x = 0.621621...$ (1)

Multiplying both sides by 1000

$1000x = 621.621621....$ (2)

Subtract (1) from (2), and we get

$1000x - x = 621.621621.... - 0.621621....$

$999x = 621$

$x = 621/999$

or $x = 23/37$

(v) Let $x = 125.3333....$ (1)

Multiplying both sides by 10

$10x = 1253.3333....$ (2)

Subtract (1) from (2), and we get

$10x - x = 1253.3333.... - 125.3333....$

$9x = 1128$

or $x = 1128/9$

or $x = 376/3$

(vi) Let $x = 4.7777....$ (1)

Multiplying both sides by 10

$10x = 47.7777....$ (2)

Subtract (1) from (2), and we get

$10x - x = 47.7777.... - 4.7777....$

$9x = 43$

$x = 43/9$

(vii) Let $x = 0.47777....$

Multiplying both sides by 10

$10x = 4.7777....$ ...(1)

Multiplying both sides by 100

$100x = 47.7777....$ (2)

Subtract (1) from (2), and we get

$100x - 10x = 47.7777.... - 4.7777...$

$90x = 43$

$x = 43/90$

Exercise 1.4

Question 157: Define an irrational number.

Solution:

A number which cannot be expressed in the form of p/q, where p and q are integers and $q \neq 0$. It is a non-terminating or non-repeating decimal.

Question 158: Explain how irrational numbers differ from rational numbers.

Solution:

An irrational number is a real number which can be written as a decimal but not as a fraction i.e. it cannot be expressed as a ratio of integers.

It cannot be expressed as terminating or repeating decimals.

For example, $\sqrt{2}$ is an irrational number

A rational number is a real number which can be written as a fraction, and as a decimal i.e. it can be expressed as a ratio of integers.

It can be expressed as a terminating or repeating decimal.

For example, 0.10 and 5/3 are rational numbers

Question 159: Examine whether the following numbers are rational or irrational:

(i) $\sqrt{7}$ (ii) $\sqrt{4}$ (iii) $2 + \sqrt{3}$ (iv) $\sqrt{3} + \sqrt{2}$

(v) $\sqrt{3} + \sqrt{5}$ (vi) $(\sqrt{2} - 2)^2$ (vii) $(2 - \sqrt{2})(2 + \sqrt{2})$

(viii) $(\sqrt{3} + \sqrt{2})^2$ (ix) $\sqrt{5} - 2$ (x) $\sqrt{23}$

(xi) $\sqrt{225}$ (xii) 0.3796 (xiii) 7.478478……

(xiv) 1.101001000100001……

Solution:

(i) $\sqrt{7}$

Not a perfect square root, so it is an irrational number.

(ii) $\sqrt{4}$

A perfect square root of 2.

We can express 2 in the form of 2/1, so it is a rational number.

(iii) $2 + \sqrt{3}$

Here, 2 is a rational number, but $\sqrt{3}$ is an irrational number.

Therefore, the sum of a rational and irrational number is an irrational number.

(iv) $\sqrt{3} + \sqrt{2}$

$\sqrt{3}$ is not a perfect square, thus an irrational number.

$\sqrt{2}$ is not a perfect square, thus an irrational number.

Therefore, the sum of $\sqrt{2}$ and $\sqrt{3}$ gives an irrational number.

(v) $\sqrt{3} + \sqrt{5}$

$\sqrt{3}$ is not a perfect square, and hence, it is an irrational number

Similarly, $\sqrt{5}$ is not a perfect square, and it is an irrational number.

Since the sum of two irrational numbers is an irrational number, $\sqrt{3} + \sqrt{5}$ is an irrational number.

(vi) $(\sqrt{2} - 2)^2$

$(\sqrt{2} - 2)^2 = 2 + 4 - 4\sqrt{2}$

$= 6 - 4\sqrt{2}$

Here, 6 is a rational number but $4\sqrt{2}$ is an irrational number.

Since the sum of a rational and an irrational number is an irrational number, $(\sqrt{2} - 2)2$ is an irrational number.

(vii) $(2 - \sqrt{2})(2 + \sqrt{2})$

We can write the given expression as;

$(2 - \sqrt{2})(2 + \sqrt{2}) = ((2)^2 - (\sqrt{2})^2)$

[Since, $(a + b)(a - b) = a^2 - b^2$]
$= 4 - 2 = 2$ or $2/1$

Since 2 is a rational number, $(2 - \sqrt{2})(2 + \sqrt{2})$ is a rational number.

(viii) $(\sqrt{3} + \sqrt{2})^2$

We can write the given expression as;

$(\sqrt{3} + \sqrt{2})^2 = (\sqrt{3})^2 + (\sqrt{2})^2 + 2\sqrt{3} \times \sqrt{2}$

$= 3 + 2 + 2\sqrt{6}$

$= 5 + 2\sqrt{6}$

[using identity, $(a+b)^2 = a^2 + 2ab + b^2$]
Since the sum of a rational number and an irrational number is an irrational number, $(\sqrt{3} + \sqrt{2})^2$ is an irrational number.

(ix) $\sqrt{5} - 2$

$\sqrt{5}$ is an irrational number, whereas 2 is a rational number.

The difference of an irrational number and a rational number is an irrational number.

Therefore, $\sqrt{5} - 2$ is an irrational number.

(x) $\sqrt{23}$

Since, $\sqrt{23} = 4.795831352331\ldots$

As the decimal expansion of this number is non-terminating and non-recurring, it is an irrational number.

(xi) $\sqrt{225}$

$\sqrt{225} = 15$ or $15/1$

$\sqrt{225}$ is a rational number as it can be represented in the form of p/q, and q is not equal to zero.

(xii) 0.3796

As the decimal expansion of the given number is terminating, it is a rational number.

(xiii) 7.478478……

As the decimal expansion of this number is a non-terminating recurring decimal, it is a rational number.

(xiv) 1.101001000100001……

As the decimal expansion of the given number is non-terminating and non-recurring, it is an irrational number.

Question 160: Identify the following as rational or irrational numbers. Give the decimal representation of rational numbers:

(i) $\sqrt{4}$ **(ii)** $3\sqrt{18}$ **(iii)** $\sqrt{1.44}$

(iv) $\sqrt{\frac{9}{27}}$ **(v)** $-\sqrt{64}$ **(vi)** $\sqrt{100}$

Solution:

(i) $\sqrt{4}$

$\sqrt{4} = 2$, which can be written in the form of a/b. Therefore, it is a rational number.

Its decimal representation is 2.0.

(ii) $3\sqrt{18}$

$3\sqrt{18} = 9\sqrt{2}$

Since the product of a rational and an irrational number is an irrational number.

Therefore, $3\sqrt{18}$ is an irrational number.

Or $3 \times \sqrt{18}$ is an irrational number.

(iii) $\sqrt{1.44}$

$\sqrt{1.44} = 1.2$

Since every terminating decimal is a rational number, $\sqrt{1.44}$ is a rational number.

And its decimal representation is 1.2.

(iv) √9/27

√9/27 = 1/√3

Since the quotient of a rational and an irrational number is irrational numbers, √9/27 is an irrational number.

(v) – √64

– √64 = – 8 or – 8/1

Therefore, – √64 is a rational number.

Its decimal representation is –8.0.

(vi) √100

√100 = 10

Since 10 can be expressed in the form of a/b, such as 10/1, √100 is a rational number.

And its decimal representation is 10.0.

Question 161: In the following equation, find which variables x, y, z etc. represent rational or irrational numbers:

(i) $x^2 = 5$

(ii) $y^2 = 9$

(iii) $z^2 = 0.04$

(iv) $u^2 = 17/4$

(v) $v^2 = 3$

(vi) $w^2 = 27$

(vii) $t^2 = 0.4$

Solution:

(i) $x_2 = 5$

Taking square root on both sides,

x = √5

√5 is not a perfect square root, so it is an irrational number.

(ii) $y_2 = 9$

$y^2 = 9$

or $y = 3$

3 can be expressed in the form of a/b, such as 3/1, so it is a rational number.

(iii) $z^2 = 0.04$

$z^2 = 0.04$

Taking square root on both sides, we get

$z = 0.2$

0.2 can be expressed in the form of a/b, such as 2/10, so it is a rational number.

(iv) $u^2 = 17/4$

Taking square root on both sides, we get

$u = \sqrt{17}/2$

Since the quotient of an irrational and a rational number is irrational, u is an Irrational number.

(v) $v^2 = 3$

Taking square root on both sides, we get

$v = \sqrt{3}$

Since $\sqrt{3}$ is not a perfect square root, so v is an irrational number.

(vi) $w^2 = 27$

Taking square root on both sides, we get

$w = 3\sqrt{3}$

Since the product of a rational and irrational is an irrational number, w is an irrational number.

(vii) $t^2 = 0.4$

Taking square root on both sides, we get

$t = \sqrt{(4/10)}$

$t = 2/\sqrt{10}$

Since the quotient of a rational and an irrational number is an irrational number, t is an irrational number.

Exercise 1.5

Question 162: Complete the following sentences:

(i) Every point on the number line corresponds to a …… number which may be either …… or …….

(ii) The decimal form of an irrational number is neither ….. nor ……

(iii) The decimal representation of a rational number is either ….. or …..

(iv) Every real number is either … number or … number.

Solution:

(i) Every point on the number line corresponds to a **real** number which may be either **rational** or **irrational**.

(ii) The decimal form of an irrational number is neither **terminating** nor **repeating**.

(iii) The decimal representation of a rational number is either **terminating** or **non-terminating recurring**.

(iv) Every real number is either a **rational** number or **an irrational** number.

Question 2: Represent √6, √7, √8 on the number line.

Solution:

Find the equivalent values of √6, √7, √8

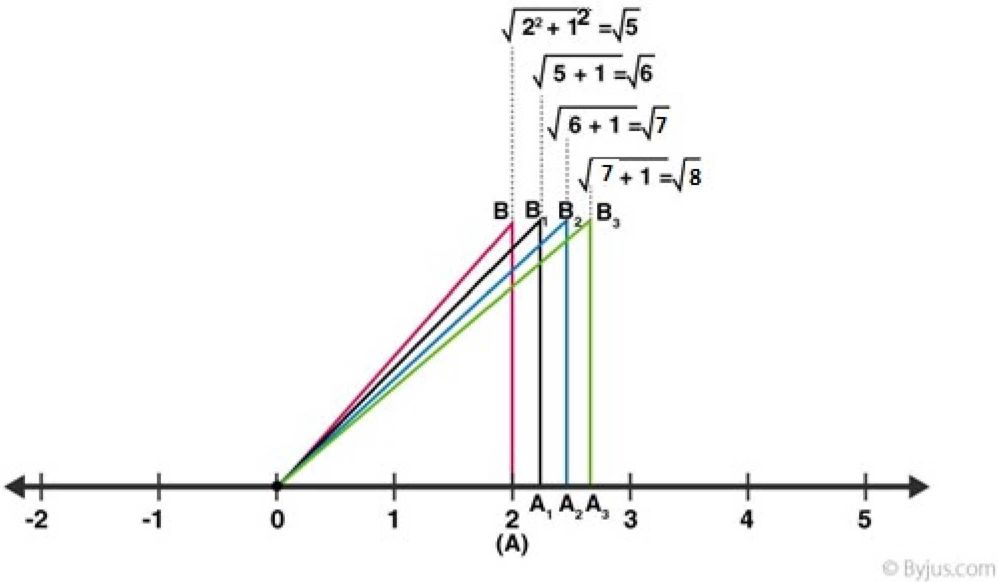
√6 = 2.449

√7 = 2.645

√8 = 2.828

We can see that all the given numbers lie between 2 and 3.

Draw on the number line:



Question 163: Represent √3.5, √9.4, √10.5 and on the real number line.

Solution:

Represent √3.5 on the number line

Step 1: Draw a line segment AB = 3.5 units

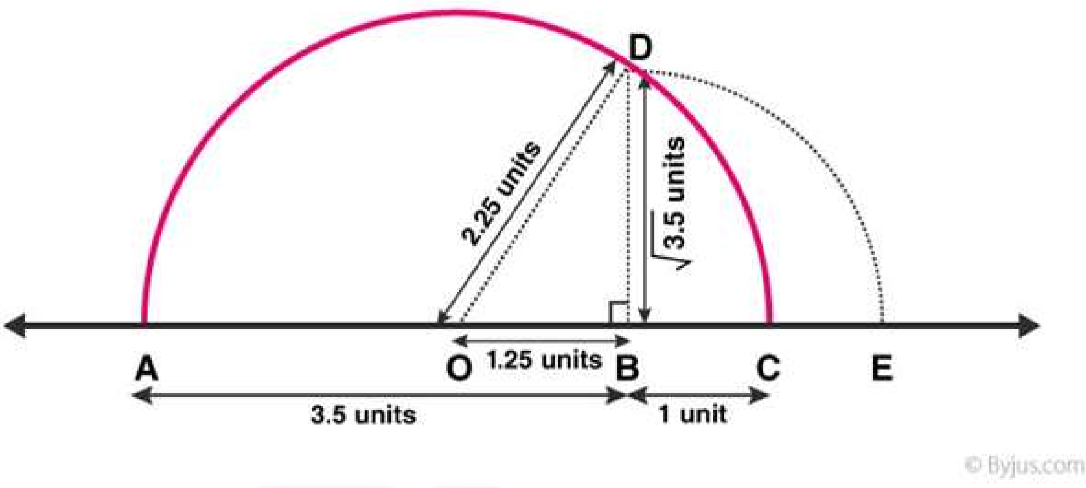
Step 2: Produce B till point C, such that BC = 1 unit

Step 3: Find the mid-point of AC, say O.

Step 4: Taking O as the centre, draw a semi circle, passing through A and C.

Step 5: Draw a line passing through B perpendicular to OB, and cut a semicircle at D.

Step 6: Consider B as a centre and BD as the radius draw an arc cutting OC produced at E.



Now, from the right triangle OBD,

$BD_2 = OD_2 - OB_2$

$= OC_2 - (OC - BC)_2$

(As, OD = OC)

$BD_2 = 2OC \times BC - (BC)_2$

$= 2 \times 2.25 \times 1 - 1$

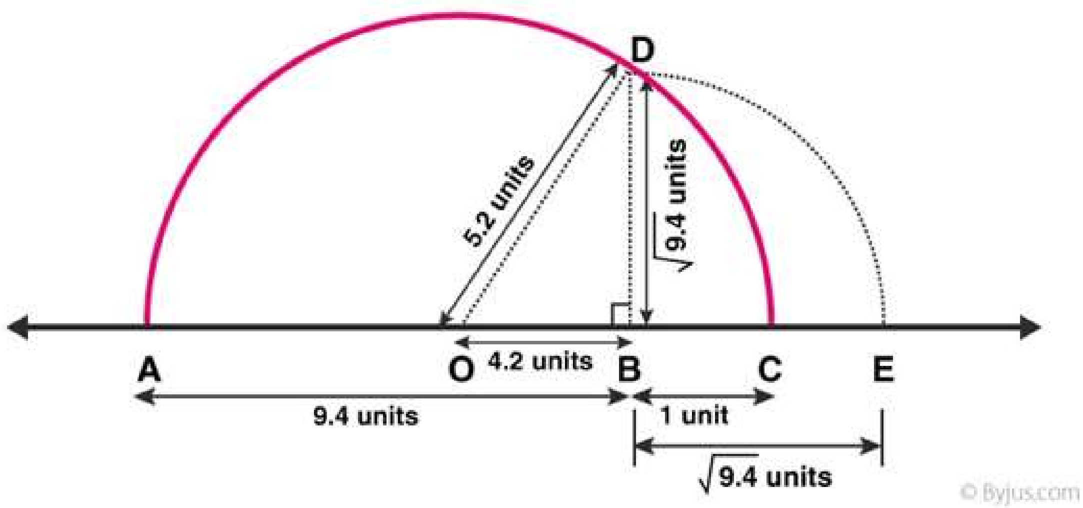
$= 3.5$

=> $BD = \sqrt{3.5}$

Represent √9.4 on the number line

Step 1: Draw a line segment AB = 9.4 units

Follow steps 2 to 6 mentioned above.



$BD_2 = 2OC \times BC - (BC)_2$

$= 2 \times 5.2 \times 1 - 1$

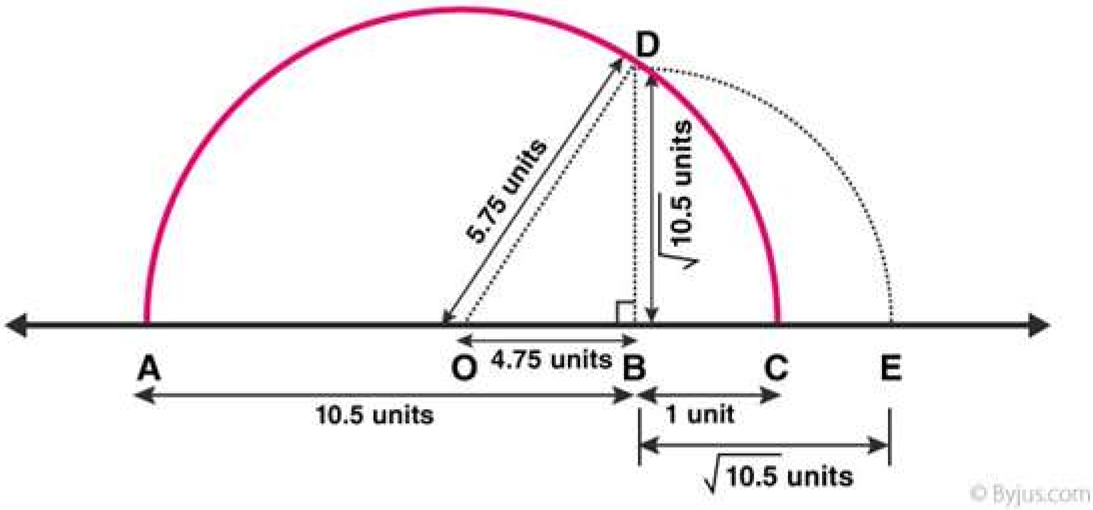
$= 9.4$

=> $BD = \sqrt{9.4}$

Represent √10.5 on the number line

Step 1: Draw a line segment AB = 10.5 units

Following the steps 2 to 6 mentioned above, we get



$BD^2 = 2OC \times BC - (BC)^2$

$= 2 \times 5.75 \times 1 - 1$

$= 10.5$

$\Rightarrow BD = \sqrt{10.5}$

Question 164: Find whether the following statements are true or false:

(i) Every real number is either rational or irrational.

(ii) π is an irrational number.

(iii) Irrational numbers cannot be represented by points on the number line.

Solution:

(i) True.

(ii) True.

(ii) False.

Exercise 1.6

Question 165: Visualise 2.665 on the number line using successive magnification.

Solution:

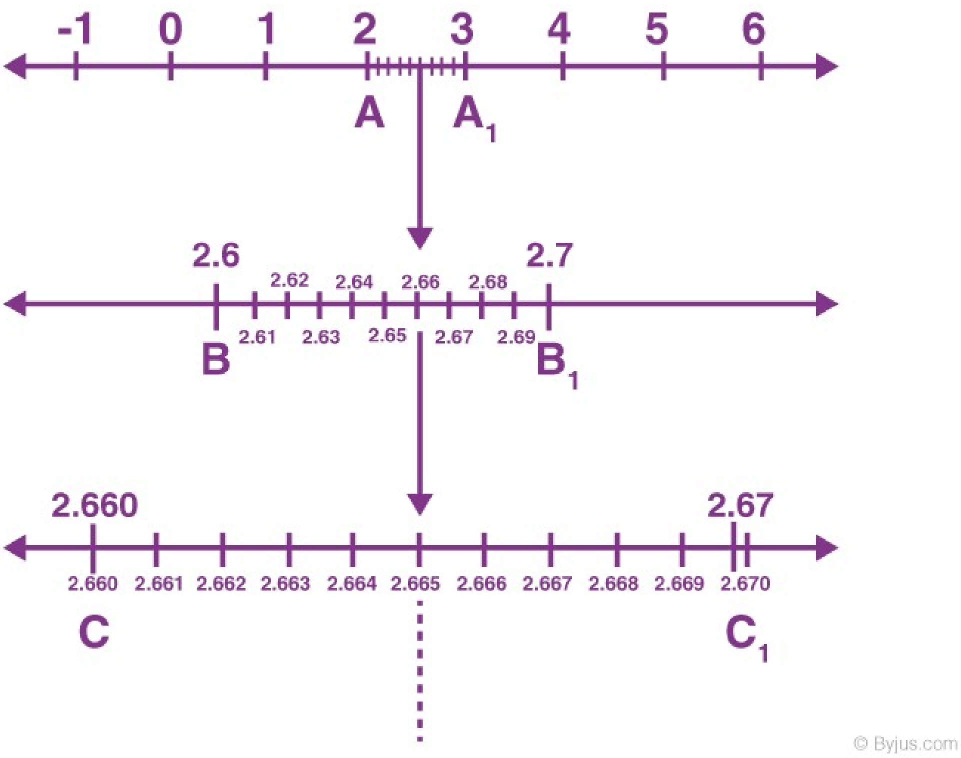
2.665 lies between 2 and 3 on the number line.

Divide the selected segment into 10 equal parts and mark each point of division as 2.1, 2.2, ….,2.9, 2.10

2.665 lies between 2.6 and 2.7

Divide the line segment between 2.6 and 2.7 into 10 equal parts, such as 2.661, 2.662, and so on.

Here we can see that the 5th point will represent 2.665.



Question 166: Visualise the representation of $5.3\overline{7}$ on the number line up to 5 decimal places, that is up to 5.37777.

Solution:

Clearly, $5.3\overline{7}$ is located between 5 and 6.

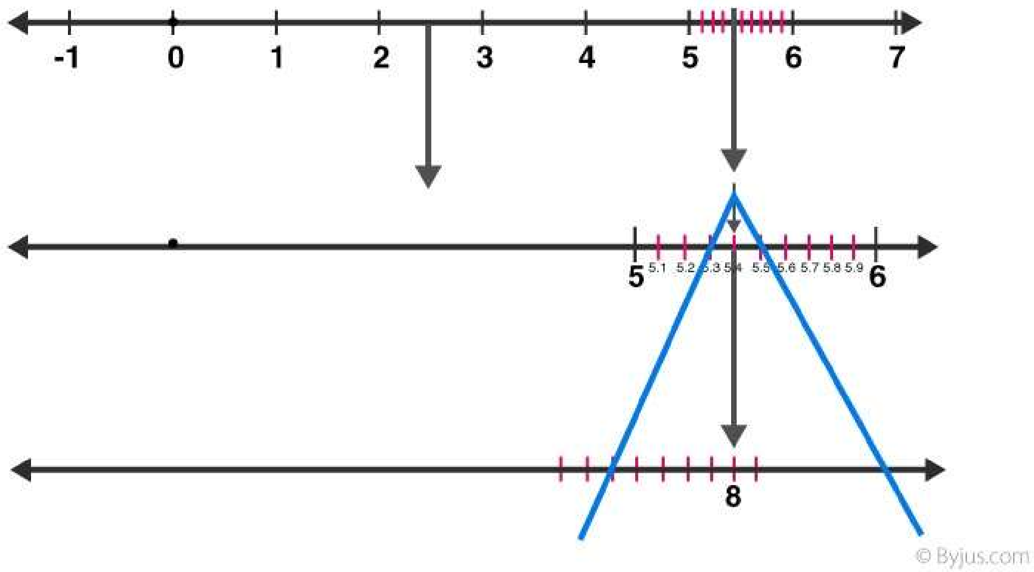
Again by successive magnification, and successively decrease $5.3\overline{7}$ located between 5.3 and 5.4.

For more clarity, divide the 5.3 and 5.4 portions of the number line into 10 equal parts, and we can see $5.3\overline{7}$ lies between 5.37 and 5.38.

To visualize $5.3\overline{7}$ more accurately, divide the line segment between 5.37 and 5.38 into ten equal parts.

$5.3\overline{7}$ lies between 5.377 and 5.378.

Again divide the above portion between 5.377 and 5.378 into 10 equal parts, which shows $5.3\overline{7}$ is located closer to 5.3778 than to 5.3777



-1
0
1
2
3
4
5
6
7
5
5.1
5.2
5.3
5.4
5.5
5.6
5.7
5.8
5.9
6
8
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