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Assignment

Q1

Sol Normal distribution

$$PMF(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

here $\mu = \theta_1$, $\sigma^2 = \theta_2$

$$\therefore f(x_i|\theta_1,\theta_2) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

Likelihood function

$$L(\theta_1,\theta_2) = \prod_{i=1}^{n} f(x_i|\theta_1,\theta_2)$$

$$L(\theta_1,\theta_2) = \prod_{i=1}^{n} \cancel{f} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\left(\frac{x_i-\theta_1}{2\theta_2}\right)^2}$$

$$L(\theta_1,\theta_2) = \prod_{i=1}^{n}(\theta)^{-\frac{1}{2}} \prod_{i=1}^{n}(2\pi)^{-1/2} \prod_{i=1}^{n} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

$$L(\theta_1, \theta_2) = (\theta_2)^{-\frac{n}{2}} (2\pi)^{-\frac{n}{2}} e^{-\frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2}$$

taking log both sides

$$\ln L(\theta_1,\theta_2) = \ln\left[(\theta_2)^{-\frac{n}{2}} (2\pi)^{-\frac{n}{2}} e^{-\frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2}\right]$$

$$-\frac{n}{2}\ln\theta_2 - \frac{n}{2}\ln 2\pi - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2 \quad -①$$

differentiating w.r.t θ_1

$$\frac{\partial l(\theta_1,\theta_2)}{\partial\theta_1} = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)$$

$$\frac{\partial l(\theta_1,\theta_2)}{\partial\theta_1} = 0$$

$$\therefore \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i-\theta_1) = 0$$

$$\Rightarrow \theta_1 = \sum_{i=1}^{n}\frac{x_i}{n}$$

$$\therefore \boxed{\theta_{1_{MLE}} = \overline{x}_n}$$

differentiating eq ① w.r.t θ_2

$$\frac{\partial l(\theta_1,\theta_2)}{\partial \theta_2} = \frac{-n}{2\theta_2} - \frac{1}{2(\theta_2)^2}\sum_{i=1}^{n}(x_i-\theta_1)^2 = 0$$

$$\Rightarrow \theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

$$\Rightarrow \boxed{\theta_{2_{MLE}} = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x}_n)^2}$$

Q2.

Sol. Binomial distribution

$$PMF(x_i) = {}^{m}C_{x_i}\, p^{x_i}\, q^{m-x_i}$$

$n = m$, $p = \theta$

$$\therefore P(x_i|m,\theta) = {}^{m}C_{x_i}\, \theta^{x_i}(1-\theta)^{m-x_i}$$

Likelihood function

$$L(P) = \prod_{i=1}^{n}(P(x_i|m,\theta))$$

$$= \prod_{i=1}^{n}\left({}^{m}C_{x_i}\, \theta^{x_i}(1-\theta)^{m-x_i}\right)$$

$$= \prod_{i=1}^{n}{}^{m}C_{x_i}\; \theta^{\sum_{i=1}^{n}x_i}\;(1-\theta)^{nm-\sum_{i=1}^{n}x_i}$$

taking log.

$$\ln L(P) = \ln\left(\prod_{i=1}^{n} {}^{m}C_{x_i} \; \theta^{\sum_{i=1}^{n} x_i} (1-\theta)^{nm - \sum_{i=1}^{n} x_i}\right)$$

$$= \ln\left(\prod_{i=1}^{n} {}^{m}C_{x_i}\right) + \ln(\theta) \cdot \sum_{i=1}^{n} x_i + \ln(1-\theta) \cdot nm - \sum_{i=1}^{n} x_i$$

differentiating wrt θ

$$\frac{\partial L(P)}{\partial \theta} = \frac{1}{\theta} \sum_{i=1}^{n} x_i + \left(\frac{-1}{1-\theta}\right) nm - \sum_{i=1}^{n} x_i$$

$$= \frac{1}{\theta} \sum_{i=1}^{n} x_i - \frac{1}{1-\theta}\left(nm - \sum_{i=1}^{n} x_i\right) = 0$$

$$\Rightarrow \frac{1-\theta}{\theta} = \frac{nm - \sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} x_i}$$

$$\therefore \theta = \frac{\sum_{i=1}^{n} x_i}{nm}$$

$$\Rightarrow \boxed{\theta_{MLE} = \frac{\bar{x}_n}{m}}$$