Biped Patrol

Task 3.3: Think & Answer

Team Id	2550
College	Maharaja Agrasen Institute of Technology
Team Leader Name	Shubham Aggarwal
e-mail	shubham.agg70@gmail.com
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Question No.	Max. Marks	Marks Scored
Q1	10	
Q2	20	
Q3	5	
Q4	5	
Q5	5	
Q6	10	
Q7	15	
Q8	8	
Q9	4	
Q10	8	
Q11	10	
Total	100	

Biped Patrol

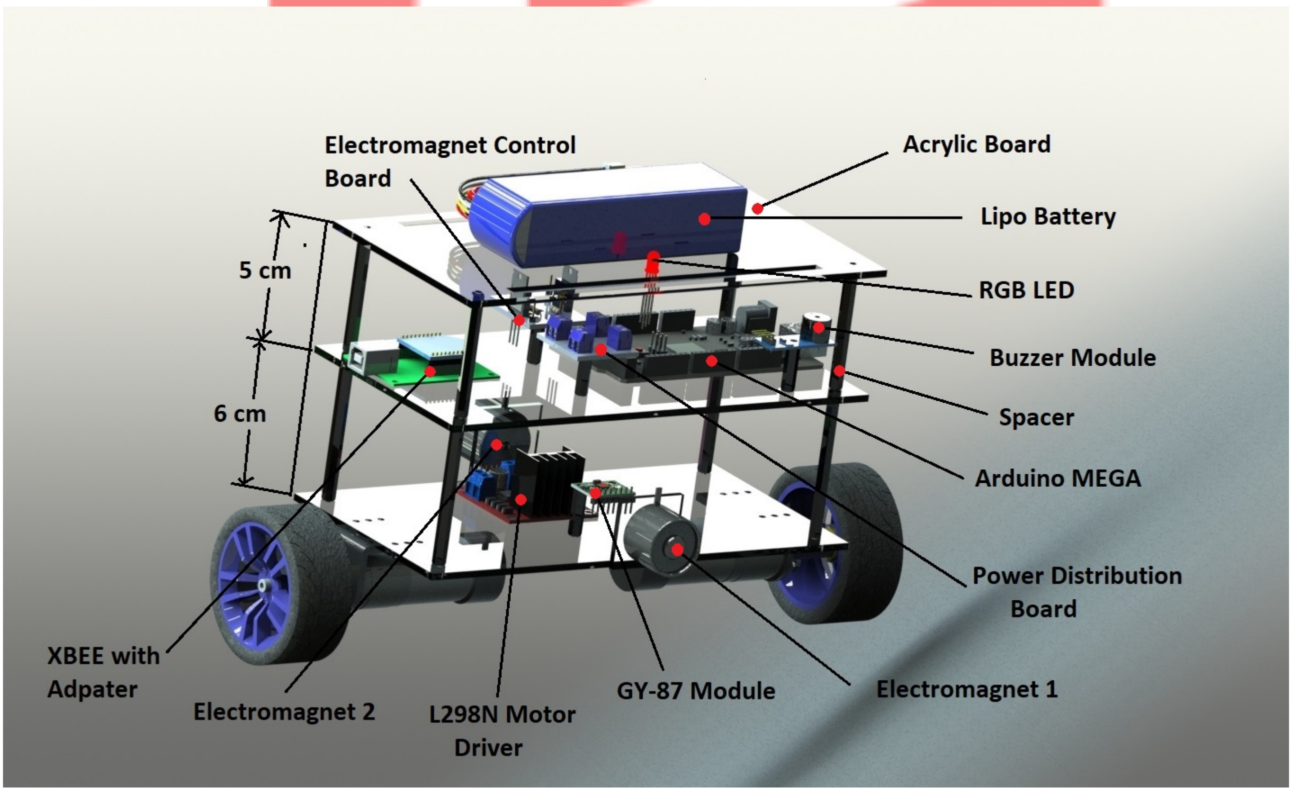
Task 3.3: Think & Answer

Instructions:

- There are no negative marks.
- Unnecessary explanation will lead to less marks even if answer is correct.
- If required, draw the image in a paper with proper explanation and add the snapshot in your corresponding answer.

Q 1. Describe hardware design for the Medbot, your team is constructing. Describe various parts with well labeled image. Give reasons for selection of design. [10]

A 1. Our MedBot will look something like this:



Our bot will have a stackable design as shown for the following reasons:

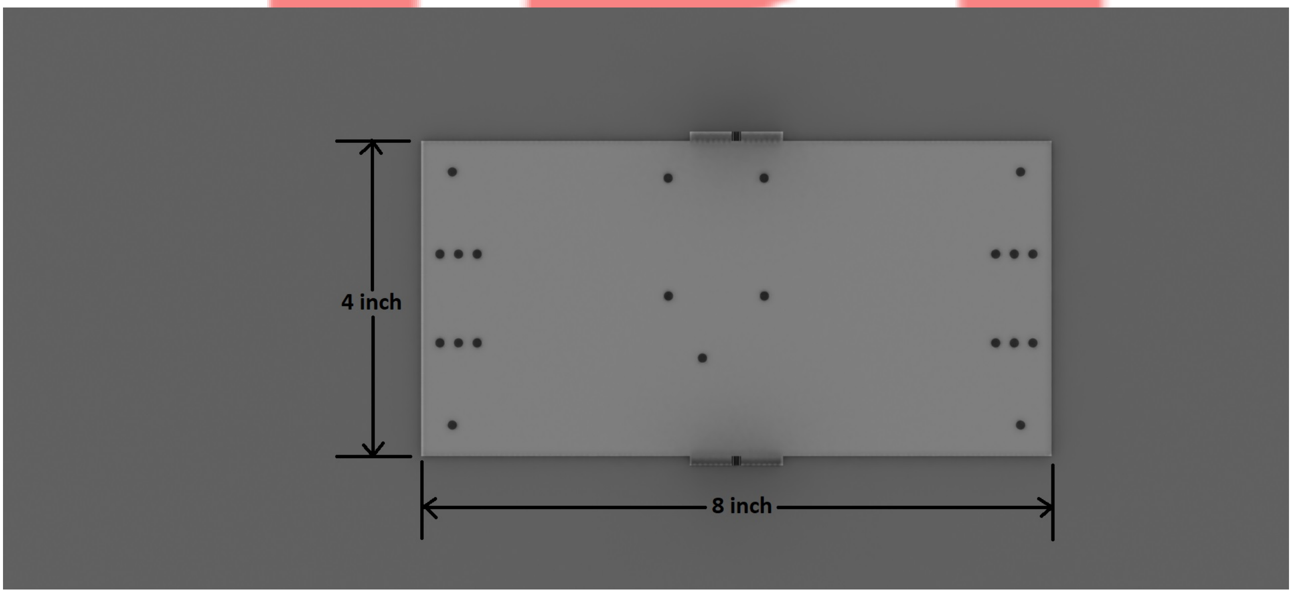
i) Stability and Response – The mass moment of inertia of the robot must be high so that it tilts more slowly when external torque is applied. The mass in stacked design is distributed as different tiers. Hence, the different tiers can be moved up and down in order to shift the center of gravity as required. The taller the robot, more stable it is. One can understand this point by trying to balance a short stick and a longer stick on his/her hand. One will find that it is easier to balance the long stick compared to the shorter stick for the same reason.

ii) Construction – The stacked design is more feasible to construct. Various components can be easily mounted and removed, without having to worry much about the CG shift as the tiers can easily be shifted later as required.

iii) Scalability - Another major advantage is that, the stacked design is scalable. For example, if we want the robot to carry some weight, this can be easily done in the stacked design by adding a new tier. The design can be constantly updated and new features can be incorporated with ease without messing with the already mounted components.

Components have been divided in three different tiers as shown. These tiers will be made using acrylic sheet as it is lightweight and strong. Due to its lightweight properties it will be easier to balance the bot as the torque required by the motor will be less.

DIMENSIONS OF EACH LEVEL:



PLACEMENTS OF COMPONENTS

i) GY-87 - IMU module has been placed at the bottom most tier just above the wheel axle. The angular rate measured by the gyroscope is not affected by the placement of the sensor, as it will vary by the same amount at all points on the robot. However, as the distance from the ground increases, linear acceleration and vibrations, measured at higher points, increases. This could lead to false readings on the accelerometer. Therefore, the sensor is placed at the bottom where these disturbances are minimum.

ii) Lipo Battery - As it is one of the most heaviest component among all and we want our center of mass to be as high as possible, we have placed it at the very top.

iii) Electromagnet - It will be at the bottom most tier. It will be at a height of around 5 cm from the ground because supply items will also be at a height of 5 cm. One electromagnet is at the front and the second electromagnet will be at the back.

iv) Motor, Wheels & L298N - They will be placed at the bottom most tier.

v) RGB LED: It will be placed at the top, so that it is easily visible.

vi) Rest of the components - All other components are placed at 2nd Level because these components have to be interfaced with Arduino and closer they are to Arduino, easier it is to do the wiring. Also, this way, it helps us to keep the center of mass high.

Q 2. In Task 1.2, you were asked to model different systems such as Simple Pulley, Complex Pulley, Inverted Pendulum with and without input and stabilizing the unstable equilibrium point using Pole Placement and LQR control techniques. There you had to choose the states; Derive the equations (usually non-linear), find equilibrium points and then linearize around the equilibrium points. You were asked to find out the linear system represented in the form

$$\dot{X}(t) = AX(t) + BU(t) \tag{1}$$

Where $X(t)$ is a vector of all the state,i.e., $X(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$, and $U(t)$ is the vector of input to the system, i.e. $U(t) = [u_1(t), u_2(t), \ldots, u_m(t)]^T$. A is the State Matrix & B is the Input Matrix.

In this question, you have to choose the states for the Medbot you are going to design. Model the system by finding out the equations governing the dynamics of the system using Euler-Lagrange Mechanics. Linearize the system via Jacobians around the equilibrium points representing your physical model in the form given in equation 1.
Note: You may choose symbolic representation such as M_w for Mass of wheel, etc. [20]

A 2. We will be designing a stackable design as shown in answer of question 1. All the components will be distributed on 3 levels.

Components on Level 1: Motor, Motor Driver, GY-87 module, electromagnets and Wheels.

Components on Level 2: Arduino Mega, XBEE with Apdapter, Power Distribution Board, Electromagnet control board and Buzzer.

Components on Level 3: Lipo Battery and RGB LED.

Let:

M_w = Mass of one Wheel

M_{md} = Mass of Motor Driver

M_m = Mass of one Motor

M_{pdb} = Mass of Power Distribution Board

M_{emcb} = Mass of Electromagnet Control Board

M_{ard} = Mass of Arduino Mega

M_{bat} = Mass of Lipo Battery

M_{imu} = Mass of GY-87 module

M_{XB} = Mass of XBEE with Adapter

M_{em} = Mass of one Electromagnet

M_{bz} = Mass of Buzzer

R_w = Radius of Wheels

Lenght of a Level = l

Width of a level = w

$\implies$ Total Mass at Level 1 = M_1 = 2*M_m + M_{md} + 2*M_{em} + M_{imu} + 2*M_w

Total Mass at Level 2 = M_2 = M_{ard} + M_{XB} + M_{pdb} + M_{emcb} + M_{bz}

Total Mass at Level 3 = M_3 = M_{bat}

Diatnce of Level 1 from wheel axle = L_1 = 0 cm

Diatnce of Level 2 from wheel axle = L_2 cm

Diatnce of Level 3 from wheel axle = L_3 cm

System can be divided into two sub-systems as:

System 1: For Pitch

System 2: For Roll

In total there will be 6 state variables which are

x - Horizontal position

$\dot{x}$ - Horizontal velocity

θ - pitch angle

$\dot{\theta}$ - angular velocity of pitch

δ - roll angle

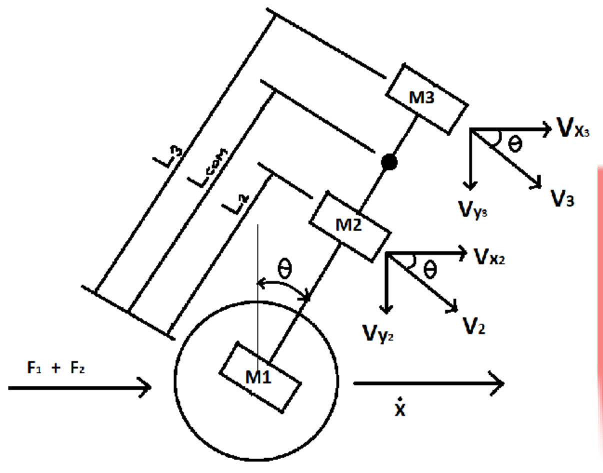
$\dot{\delta}$ - angular velocity of roll

And 2 inputs which are

F_1 - Force applied by motor 1

F_2 - Force applied by motor 2.

For system 1:



SYSTEM 1: For Pitch

$v_{x_3} = L_3\dot{\theta}cos(\theta), v_{y_3} = L_3\dot{\theta}sin(\theta)$

$v_{x_2} = L_2\dot{\theta}cos(\theta), v_{y_2} = L_2\dot{\theta}sin(\theta)$

Translational K.E of system 1 $= T.K.E_1$

$T.K.E_1 = \frac{M_1\dot{x}^2}{2} + \frac{M_2(\dot{x}+L_2\dot{\theta}cos\theta)^2}{2} + \frac{M_3(\dot{x}+L_3\dot{\theta}cos\theta)^2}{2} + \frac{M_2(L_2\dot{\theta}sin\theta)^2}{2} + \frac{M_3(L_3\dot{\theta}sin\theta)^2}{2}$

$= \frac{\dot{x}^2(M_1+M_2+M_3)}{2} + \frac{(M_2L_2^2+M_3L_3^2)\dot{\theta}^2}{2} + \dot{x}\dot{\theta}cos\theta(L_2 + L_3)(M_2 + M_3)$

Rotational K.E of system 1 $= R.K.E_1$

Distance of Center of Mass from axle of Wheel $= L_{com}$

$L_{com} = \frac{M_1L_1+M_2L_2+M_3L_3}{M_1+M_2+M_3}$

Let the moment of inertia about the axis passing through the center of mass parallel to the ground be I_{com}

$\implies I_{com} = M_1(L_1 - L_{com})^2 + M_2(L_2 - L_{com})^2 + M_3(L_3 - L_{com})^2$

Let the moment of inertia about the axis passing through the axle of wheels parallel to the ground be I

Then, by Parallel Axis Theorem:

$I = I_{com} + (M_1 + M_2 + M_3)L_{com}^2 + 2M_wR_w^2$

Last term is added to consider moment of inertia due to rotation of wheels. Wheels are assumed to be like a ring as most of its mass is concentrated on the outside.

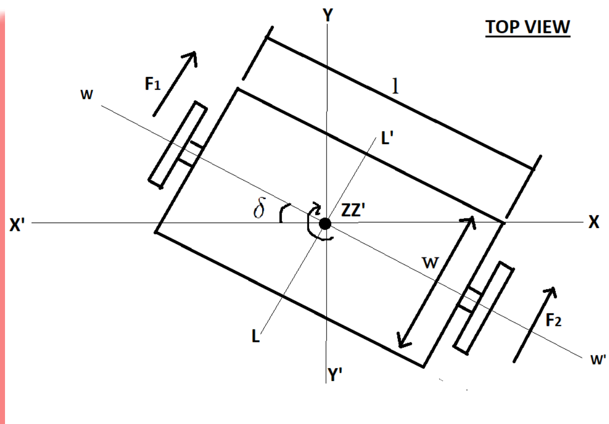
$\implies R.K.E_1 = \frac{I\dot{\theta}^2}{2}$

Total K.E of system 1 $= K.E_1 = T.K.E_1 + R.K.E_1$

$\implies K.E_1 = \frac{\dot{x}^2(M_1+M_2+M_3)}{2} + \frac{(M_2L_2^2+M_3L_3^2)\dot{\theta}^2}{2} + \dot{x}\dot{\theta}cos\theta(L_2+L_3)(M_2+M_3) + \frac{I\dot{\theta}^2}{2}$

Potential Energy of system 1 $= P.E_1 = M_3gL_3cos(\theta) + M_2gL_2cos(\theta)$

For system 2:



SYSTEM 2: For Roll

We are assuming that at each level, mass is uniformly distributed and each level is a rectangle of length l and width w.

Moment of Inertia about the axis WW' at level 1 $= I_{WW'} = \frac{M_1w^2}{12} + 2M_wR_w^2$

Moment of Inertia about the axis LL' at level 1 $= I_{LL'} = \frac{M_1l^2}{12} + 2M_w(l/2)^2$

Let the moment of inertia about the perpendicular to the plane and passing through the center of level 1, 2 and 3 be I_{z1}, I_{z2} and I_{z3} respectively.

Then, by Perpendicular Axis Theorem:

$I_{z1} = I_{WW'} + I_{LL'}$

Similarly for Level 2 and 3

$I_{z2} = \frac{M_2w^2}{12} + \frac{M_2l^2}{12}$

$I_{z3} = \frac{M_3w^2}{12} + \frac{M_3l^2}{12}$

$\implies$ Total moment of inertia about the z-axis $= I_{zz'} = I_{z1} + I_{z2} + I_{z3}$

$I_{zz'} = \frac{M_1w^2}{12} + 2M_wR_w^2 + \frac{M_1l^2}{12} + 2M_w(l/2)^2 + \frac{M_2w^2}{12} + \frac{M_2l^2}{12} + \frac{M_3w^2}{12} + \frac{M_3l^2}{12}$

Rotational K.E of system 2 $= R.K.E_2$

$\implies R.K.E_2 = \frac{I_{zz'}\dot{\delta}^2}{2}$

Translational K.E of system 2 = $T.K.E_2 = 0$

P.E of system 2 = $P.E_2 = 0$ (as Total P.E was considered in system 1)

$K.E_2 = R.K.E_2 + T.K.E_2 = \frac{I_{zz'}\dot{\delta}^2}{2}$

Total K.E of the complete system = K.E = $K.E_1 + K.E_2$

$\implies$ K.E = $\frac{\dot{x}^2(M_1+M_2+M_3)}{2} + \frac{(M_2L_2^2+M_3L_3^2)\dot{\theta}^2}{2} + \dot{x}\dot{\theta}cos\theta(L_2+L_3)(M_2+M_3) + \frac{I\dot{\theta}^2}{2} + \frac{I_{zz'}\dot{\delta}^2}{2}$

Total P.E of the complete system = P.E = $P.E_1 + P.E_2$

$\implies$ P.E = $M_3gL_3cos(\theta) + M_2gL_2cos(\theta)$

Lagrangian = L = K.E - P.E

$\implies$ L = $\frac{\dot{x}^2(M_1+M_2+M_3)}{2} + \frac{(M_2L_2^2+M_3L_3^2)\dot{\theta}^2}{2} + \dot{x}\dot{\theta}cos\theta(L_2+L_3)(M_2+M_3) + \frac{I\dot{\theta}^2}{2} + \frac{I_{zz'}\dot{\delta}^2}{2}$ - $M_3gL_3cos(\theta) - M_2gL_2cos(\theta)$

By Euler-Lagrange Method:

$\frac{d}{dt}(\frac{\partial L}{\partial \dot{x}}) - \frac{\partial L}{\partial x} = F_1 + F_2$

After simplifying the above eqn. we get:

$$\ddot{x}(M_1+M_2+M_3) + \ddot{\theta}(L_2+L_3)(M_2+M_3)cos\theta - \dot{\theta}^2sin\theta(L_2+L_3)(M_2+M_3) = F_1 + F_2 \quad ....(1)$$

Similarly,

$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}}) - \frac{\partial L}{\partial \theta} = 0$

$$\therefore \ddot{\theta}(M_2L_2^2+M_3L_3^2+I) + \ddot{x}(L_2+L_3)(M_2+M_3)cos\theta - 2\dot{x}(L_2+L_3)(M_2+M_3)\dot{\theta}sin\theta - gsin\theta(M_2L_2+M_3L_3) = 0 \quad ....(2)$$

$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\delta}}) - \frac{\partial L}{\partial \delta} = F_1 + F_2$

$$\implies I_{zz'}\ddot{\delta} = F_1 + F_2 \quad ....(3)$$

$$\implies \ddot{\delta} = \frac{F_1+F_2}{I_{zz'}} \quad ....(4)$$

Dividing eq(1) by $(L_2+L_3)(M_2+M_3)cos\theta$ and eq(2) by $(M_2L_2^2+M_3L_3^2+I)$

$$\implies \frac{\ddot{x}(M_1+M_2+M_3)}{(L_2+L_3)(M_2+M_3)cos\theta} + \ddot{\theta} - \frac{\dot{\theta}^2sin\theta(L_2+L_3)(M_2+M_3)}{(L_2+L_3)(M_2+M_3)cos\theta} = \frac{F_1+F_2}{(L_2+L_3)(M_2+M_3)cos\theta} \quad ....(5)$$

$$\implies \ddot{\theta} + \frac{\ddot{x}(L_2+L_3)(M_2+M_3)cos\theta}{(M_2L_2^2+M_3L_3^2+I)} - \frac{2\dot{x}(L_2+L_3)(M_2+M_3)\dot{\theta}sin\theta}{(M_2L_2^2+M_3L_3^2+I)} - \frac{gsin\theta(M_2L_2+M_3L_3)}{(M_2L_2^2+M_3L_3^2+I)} = 0 \quad ...(6)$$

Subtracting eq(5) from eq(6) and solving for $\ddot{x}$ we get:

$$\ddot{x} = \frac{(I+M_2L_2^2+M_3L_3^2)(F_1+F_2+\dot{\theta}^2sin\theta(L_2M_2+L_2M_3+L_3M_2+L_3M_3)) - (L_2M_2+L_2M_3+L_3M_2+L_3M_3)(g(L_2M_2+L_3M_3)+2\dot{\theta}\dot{x}(L_2M_2+L_2M_3+L_3M_2+L_3M_3))sin\theta cos\theta}{(I+M_2L_2^2+M_3L_3^2)(M_1+M_2+M_3) - (L_2M_2+L_2M_3+L_3M_2+L_3M_3)^2cos^2\theta}$$

Dividing eq(1) by $(M_1 + M_2 + M_3)$ and eq(2) by $(L_2 + L_3)(M_2 + M_3)cos\theta$

$$\implies \ddot{x} + \frac{\ddot{\theta}(L_2+L_3)(M_2+M_3)cos\theta}{(M_1+M_2+M_3)} - \frac{\dot{\theta}^2 sin\theta(L_2+L_3)(M_2+M_3)}{(M_1+M_2+M_3)} = \frac{F_1+F_2}{(M_1+M_2+M_3)} \qquad \text{......................(7)}$$

$$\implies \frac{\ddot{\theta}(I+M_2L_2^2+M_3L_3^2)}{(L_2+L_3)(M_2+M_3)cos\theta} + \ddot{x} - \frac{2\dot{x}(L_2+L_3)\dot{\theta}sin\theta}{(L_2+L_3)(M_2+M_3)cos\theta} - \frac{gsin\theta(M_2L_2+M_3L_3)}{(L_2+L_3)(M_2+M_3)cos\theta} = 0 \qquad \text{...(8)}$$

Subtracting eq(8) from eq(7) and solving for $\ddot{\theta}$ we get:

$$\ddot{\theta} = \frac{(M_1+M_2+M_3)(g(L_2M_2+L_3M_3)+2\dot{\theta}\dot{x}(L_2M_2+L_2M_3+L_3M_2+L_3M_3))sin\theta-(L_2M_2+L_2M_3+L_3M_2+L_3M_3)(F_1+F_2+\dot{\theta}^2sin\theta(L_2M_2+L_2M_3+L_3M_2+L_3M_3))cos\theta}{(I+M_2L_2^2+M_3L_3^2)(M_1+M_2+M_3)-(L_2M_2+L_2M_3+L_3M_2+L_3M_3)^2cos^2\theta}$$

$$\text{....................(9)}$$

By putting $\dot{x} = 0$, $\ddot{x} = 0$, $\dot{\theta} = 0$, $\ddot{\theta} = 0$, $\dot{\delta} = 0$ and $\ddot{\delta} = 0$ we get 1 unstable equilibrium point:

$$(x, \dot{x}, \theta, \dot{\theta}, \delta, \dot{\delta}) = (\text{x}, 0, 0, 0, \delta, 0)$$

Equilibrium point is independent of x and δ

Linearizing the Jacobian of the system around the above equilibrium point:

$$\text{A} = \begin{bmatrix} \frac{\partial\dot{x}}{\partial x} & \frac{\partial\dot{x}}{\partial\dot{x}} & \frac{\partial\dot{x}}{\partial\theta} & \frac{\partial\dot{x}}{\partial\dot{\theta}} & \frac{\partial\dot{x}}{\partial\delta} & \frac{\partial\dot{x}}{\partial\dot{\delta}} \\ \frac{\partial\ddot{x}}{\partial x} & \frac{\partial\ddot{x}}{\partial\dot{x}} & \frac{\partial\ddot{x}}{\partial\theta} & \frac{\partial\ddot{x}}{\partial\dot{\theta}} & \frac{\partial\ddot{x}}{\partial\delta} & \frac{\partial\ddot{x}}{\partial\dot{\delta}} \\ \frac{\partial\dot{\theta}}{\partial x} & \frac{\partial\dot{\theta}}{\partial\dot{x}} & \frac{\partial\dot{\theta}}{\partial\theta} & \frac{\partial\dot{\theta}}{\partial\dot{\theta}} & \frac{\partial\dot{\theta}}{\partial\delta} & \frac{\partial\dot{\theta}}{\partial\dot{\delta}} \\ \frac{\partial\ddot{\theta}}{\partial x} & \frac{\partial\ddot{\theta}}{\partial\dot{x}} & \frac{\partial\ddot{\theta}}{\partial\theta} & \frac{\partial\ddot{\theta}}{\partial\dot{\theta}} & \frac{\partial\ddot{\theta}}{\partial\delta} & \frac{\partial\ddot{\theta}}{\partial\dot{\delta}} \\ \frac{\partial\dot{\delta}}{\partial x} & \frac{\partial\dot{\delta}}{\partial\dot{x}} & \frac{\partial\dot{\delta}}{\partial\theta} & \frac{\partial\dot{\delta}}{\partial\dot{\theta}} & \frac{\partial\dot{\delta}}{\partial\delta} & \frac{\partial\dot{\delta}}{\partial\dot{\delta}} \\ \frac{\partial\ddot{\delta}}{\partial x} & \frac{\partial\ddot{\delta}}{\partial\dot{x}} & \frac{\partial\ddot{\delta}}{\partial\theta} & \frac{\partial\ddot{\delta}}{\partial\dot{\theta}} & \frac{\partial\ddot{\delta}}{\partial\delta} & \frac{\partial\ddot{\delta}}{\partial\dot{\delta}} \end{bmatrix}$$

$$\text{B} = \begin{bmatrix} \frac{\partial\dot{x}}{\partial F_1} & \frac{\partial\dot{x}}{\partial F_2} \\ \frac{\partial\ddot{x}}{\partial F_1} & \frac{\partial\ddot{x}}{\partial F_2} \\ \frac{\partial\dot{\theta}}{\partial F_1} & \frac{\partial\dot{\theta}}{\partial F_2} \\ \frac{\partial\ddot{\theta}}{\partial F_1} & \frac{\partial\ddot{\theta}}{\partial F_2} \\ \frac{\partial\dot{\delta}}{\partial F_1} & \frac{\partial\dot{\delta}}{\partial F_2} \\ \frac{\partial\ddot{\delta}}{\partial F_1} & \frac{\partial\ddot{\delta}}{\partial F_2} \end{bmatrix}$$

Calculating above partial derivatives, we get:

Let D $= (I + M_2L_2^2 + M_3L_3^2)(M_1 + M_2 + M_3) - (L_2M_2 + L_2M_3 + L_3M_2 + L_3M_3)^2cos^2\theta$

$$\frac{\partial \ddot{x}}{\partial \dot{x}} = \frac{-(L_2M_2+L_2M_3+L_3M_2+L_3M_3)(2\dot{\theta}(L_2M_2+L_2M_3+L_3M_2+L_3M_3)sin\theta cos\theta)}{D}$$

$$\frac{\partial \ddot{x}}{\partial \dot{\theta}} = \frac{(I+M_2L_2^2+M_3L_3^2)(2\dot{\theta}sin\theta(L_2M_2+L_2M_3+L_3M_2+L_3M_3))-(L_2M_2+L_2M_3+L_3M_2+L_3M_3)(2\dot{x}(L_2M_2+L_2M_3+L_3M_2+L_3M_3))sin\theta cos\theta}{D}$$

$$\frac{\partial \ddot{x}}{\partial \theta} = \frac{(I+M_2L_2^2+M_3L_3^2)(\dot{\theta}^2cos\theta(L_2M_2+L_2M_3+L_3M_2+L_3M_3))-(L_2M_2+L_2M_3+L_3M_2+L_3M_3)(g(L_2M_2+L_3M_3)+2\dot{\theta}\dot{x}(L_2M_2+L_2M_3+L_3M_2+L_3M_3))cos2\theta}{D} -$$

$$\frac{2(I+M_2L_2^2+M_3L_3^2)(F_1+F_2+\dot{\theta}^2sin\theta(L_2M_2+L_2M_3+L_3M_2+L_3M_3))(L_2M_2+L_2M_3+L_3M_2+L_3M_3)^2sin\theta cos\theta}{D^2} +$$

$$\frac{2(L_2M_2+L_2M_3+L_3M_2+L_3M_3)(g(L_2M_2+L_3M_3)+2\dot{\theta}\dot{x}(L_2M_2+L_2M_3+L_3M_2+L_3M_3))(L_2M_2+L_2M_3+L_3M_2+L_3M_3)^2sin\theta cos\theta}{D^2}$$

$$\frac{\partial \ddot{\theta}}{\partial \dot{x}} = \frac{(M_1+M_2+M_3)(2\dot{\theta}(L_2M_2+L_2M_3+L_3M_2+L_3M_3))sin\theta}{D}$$

$$\frac{\partial \ddot{\theta}}{\partial \dot{\theta}} = \frac{(M_1+M_2+M_3)(2\dot{x}(L_2M_2+L_2M_3+L_3M_2+L_3M_3))sin\theta-(L_2M_2+L_2M_3+L_3M_2+L_3M_3)(2\dot{\theta}sin\theta(L_2M_2+L_2M_3+L_3M_2+L_3M_3))cos\theta}{D}$$

$$\frac{\partial \ddot{\theta}}{\partial \theta} = \frac{(M_1+M_2+M_3)(g(L_2M_2+L_3M_3)+2\dot{\theta}\dot{x}(L_2M_2+L_2M_3+L_3M_2+L_3M_3))cos\theta-(L_2M_2+L_2M_3+L_3M_2+L_3M_3)^2\dot{\theta}^2cos^2\theta}{D} +$$

$$\frac{(L_2M_2+L_2M_3+L_3M_2+L_3M_3)(F_1+F_2\dot{\theta}^2sin\theta(L_2M_2+L_2M_3+L_3M_2+L_3M_3))sin\theta}{D} -$$

$$\frac{2[(M_1+M_2+M_3)(g(L_2M_2+L_3M_3)+2\dot{\theta}\dot{x}(L_2M_2+L_2M_3+L_3M_2+L_3M_3))sin\theta(L_2M_2+L_2M_3+L_3M_2+L_3M_3)^2sin\theta cos\theta}{D^2} +$$

$$\frac{2(L_2M_2+L_2M_3+L_3M_2+L_3M_3)(F_1+F_2+\dot{\theta}^2sin\theta(L_2M_2+L_2M_3+L_3M_2+L_3M_3))cos\theta](L_2M_2+L_2M_3+L_3M_2+L_3M_3)^2sin\theta cos\theta}{D^2}$$

$$\frac{\partial \ddot{x}}{\partial F_1} = \frac{(I+M_2L_2^2+M_3L_3^2)}{D}$$

$$\frac{\partial \ddot{x}}{\partial F_2} = \frac{(I+M_2L_2^2+M_3L_3^2)}{D}$$

$$\frac{\partial \ddot{\theta}}{\partial F_1} = \frac{-(L_2M_2+L_2M_3+L_3M_2+L_3M_3)}{D}$$

$$\frac{\partial \ddot{\theta}}{\partial F_2} = \frac{-(L_2M_2+L_2M_3+L_3M_2+L_3M_3)}{D}$$

$$\frac{\partial \ddot{\delta}}{\partial F_1} = \frac{1}{I_{zz'}}$$

$$\frac{\partial \ddot{\delta}}{\partial F_2} = \frac{1}{I_{zz'}}$$

Rest of the partial derivatives becomes 0 or 1 when equilibrium point is substituted.

Substituting equilibrium point in above partial derivatives, we get:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{(L_2M_2 + L_2M_3 + L_3M_2 + L_3M_3)(g(L_2M_2 + L_3M_3))}{D} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{(M_1 + M_2 + M_3)(g(L_2M_2 + L_3M_3))}{D} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ \frac{(I + M_2L_2^2 + M_3L_3^2)}{D} & \frac{(I + M_2L_2^2 + M_3L_3^2)}{D} \\ 0 & 0 \\ \frac{-(L_2M_2 + L_2M_3 + L_3M_2 + L_3M_3)}{D} & \frac{-(L_2M_2 + L_2M_3 + L_3M_2 + L_3M_3)}{D} \\ 0 & 0 \\ \frac{1}{I_{zz'}} & \frac{1}{I_{zz'}} \end{bmatrix}$$

Where $D = (I + M_2L_2^2 + M_3L_3^2)(M_1 + M_2 + M_3) - (L_2M_2 + L_2M_3 + L_3M_2 + L_3M_3)^2 cos^2\theta$

We know that:

$\dot{X}(t) = AX(t) + BU(t)$

$$\implies \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \\ \dot{\delta} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{(L_2M_2 + L_2M_3 + L_3M_2 + L_3M_3)(g(L_2M_2 + L_3M_3))}{D} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{(M_1 + M_2 + M_3)(g(L_2M_2 + L_3M_3))}{D} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \\ \delta \\ \dot{\delta} \end{bmatrix} +$$

$$\begin{bmatrix} 0 & 0 \\ \frac{(I + M_2L_2^2 + M_3L_3^2)}{D} & \frac{(I + M_2L_2^2 + M_3L_3^2)}{D} \\ 0 & 0 \\ \frac{-(L_2M_2 + L_2M_3 + L_3M_2 + L_3M_3)}{D} & \frac{-(L_2M_2 + L_2M_3 + L_3M_2 + L_3M_3)}{D} \\ 0 & 0 \\ \frac{1}{I_{zz'}} & \frac{1}{I_{zz'}} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

Q 3. Equation 1 represents a continuous-time system. The equivalent discrete time system is represented as:

$$X(k+1) = A_dX(k) + B_dU(k) \tag{2}$$

Where $X(k)$ is a measure of the states at k_{th} sampling instant,i.e., $X(k) = [x_1(k), x_2(k), \ldots, x_n(k)]^T$, and $U(k)$ is the vector of input to the system at k_{th} sampling instant, i.e. $U(k) = [u_1(k), u_2(k), \ldots, u_m(k)]^T$. A_d is the Discrete State Matrix & B_d is the Discrete Input Matrix.

What should be the position of eigen values of A_d for system to be stable.
Hint: In frequency domain, continuous-time system is represented with Laplace transform and discrete-time system is represented with Z transform. [5]

A 3. For the system to be stable, it should return back to the equilibrium point when disturbed. Since the eq(2) holds true only when the system is around the vicinity of the equilibrium point.

Therefore:

For all the eigenvectors X(k),

X(k+1) = $\lambda X(k)$ when system is in the vicinity of equilibrium point.

Where λ is the eigenvalue of A_d.

So when the system is in the vicinity of the equilibrium point, X(k) will tend towards origin or 0 in z-plane as k tends to infinity iff $|\lambda| < 1$. Otherwise it will diverge away from it. Here origin is the equilibrium point about which the system was linearized.

Therefore the magnitude of eigen values of A_d should be less than 1. This means that in the z-plane, all eigenvalues must lie in the circle with the radius = 1 for the system to be stable.

It is marginally stable if modulus of eigen values is equal to 1. This means that system will always remain in the vicinity of the equilibrium point but will neither tend towards it nor tend away from it.

Q 4. Will LQR control always works? If No, then why not? and if Yes, Justify your answer.
Hint: Take a look at definition of Controllable System. What is controllability? [5]

A 4. Controllability: is the ability of a control system to bring the system to any possible state from any initial state in finite time by application of a suitable input. What this means is that there always exists an input which when applied to the system can drive the system to the desired state in finite time. Contrallabilty can be determined by calculating the rank of contrallabilty matrix(R) which is defined as:

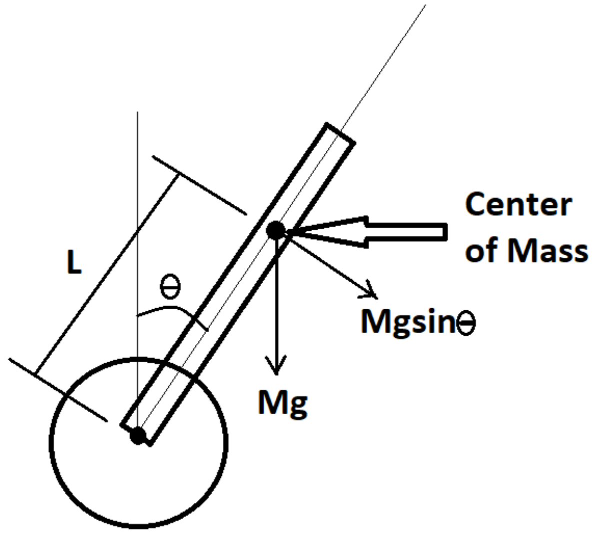
$$\text{R} = [\text{B AB } A^2B \text{ ..... } A^{n-1}B], \text{ where n is no. of state variables}$$

A system is controllable iff the rank(R) = n

If rank(R) is not equal to n, it means that the inputs we can give to the system cannot have effect on every state variables. Those state variables which are not in our control will make it impossible for us to control the system as we desire. Therefore we can say that LQR control do not always work.

Q 5. For balancing robot on two wheel i.e. as inverted pendulum, the center of mass should be made high or low? Justify your answer. [5]

A 5. Consider the following FBD:



Moment of Inertia of the Pendulum = I = ML^2 (where M is the total mass of Pendulum)

Torque on Pendulum = τ = MgLsinθ

Let the angular acceleration of the pendulum be α

Then,

$$\tau = I\alpha$$

As the distance L increases from the axis of rotation, both torque and moment of inertia increases but since I is quadratically dependent on L, it increaes at higher rate compared to torque. Therefore, for the above equation to hold true, angular acceleration decreases. This means the bot will tilt more slowly when under external torque Which is a good thing as it will give us more time to balance the pendulum before it gets out of control. This increases the stability of the bot.

Therefore center of mass should be **high**.

Q 6. Why do we require filter? Do we require both the gyroscope and the accelerometer for measuring the tilt angle of the robot? Why? [10]

A 6. Both accelerometer and gyroscope are prone to noise or disturbance.

Accelerometer is a electromechanical device which measures acceleration forces and therefore it also measures the forces acting on it other than the gravitational force which is the only force we want to measure. Therefore readings are not accurate and contains error. Error is highest when it is moving due to acceleration of the body itself and lowest when it as at rest.

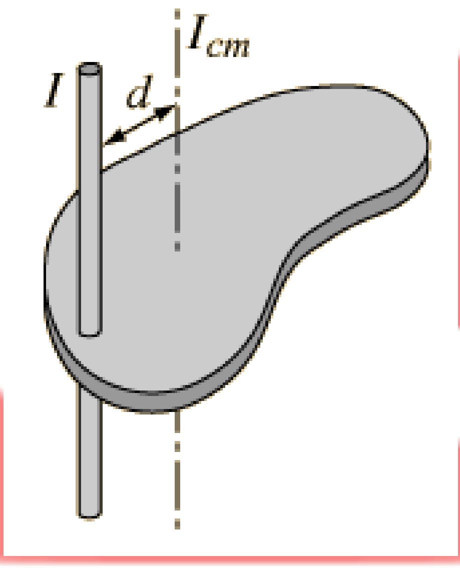
In case of gyroscope which measures the angular velocity have a small error in its each reading.

This error is negligible. But when we integrate these readings to calculate pitch and roll, these errors are also integrated and now we connot neglect this error. Therefore with time the readings of gyroscope drifts away from the original value.

Due to the above mentioned reasons we require filters to remove these errors from the readings of accelerometer and gyroscope.

We require both accelerometer and gyroscope to calculate tilt angle of the bot because filters never completely remove the noise but suppresses it. Therefore the gyroscope readings will still drift with time and accelerometer readings will still have high frequency noise. We can say that the accelerometer data is useful for long term as it do not drift with time while the gyro data is useful for short term. Therefore to measure the tilt angle with higher accuracy we need both accelerometer and gyroscope. We will use Complimentary Filter to combine both their values so that that the strength of one sensor will be used to overcome the weakness of the other sensor.

Q 7. What is Perpendicular and Parallel axis theorem for calculation of Moment of Inertia? Do you require this theorem for modelling the Medbot?Explain Mathematically. [15]

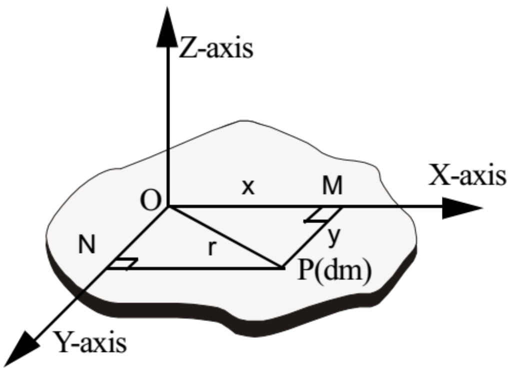


A 7.

Parallel axis theorem: states that for two parallel axis, one passing through the centre of mass of an object and the other displaced by some distance d as shown in above figure, the object's moment of inertia about the displaced axis is given by:

$$I = I_{cm} + Md^2$$

where M is the mass of object and I_{cm} is moment of inertia measured about the axis passing through the object's center of mass.



Perpendicular axis theorem: states that the moment of inertia of any plane lamina about an axis normal to the plane of the lamina is equal to the sum of the moments of inertia about any two mutually perpendicular axis passing through the given axis lying in the plane of the lamina. Therefore from the above figure:

$$I_z = I_x + I_y$$

where,

I_z is moment of inertia about Z axis

I_x is moment of inertia about X axis

I_y is moment of inertia about Y axis

Their use:

1)PARALLEL AXIS THEOREM

Consider the figure of system 1 given in answer 2.

We used parallel axis theorem to determine the moment of inertia about the axis passing through the center of both the wheels parallel to ground so that we can calculate rotational K.E of system 1.

Distance of Center of Mass from axle of Wheel = L_{com}

$L_{com} = \frac{M_1L_1+M_2L_2+M_3L_3}{M_1+M_2+M_3}$

Let the moment of inertia about the axis passing through the center of mass and parallel to the axis passing through the center of wheels be I_{com}

$\implies I_{com} = M_1(L_1 - L_{com})^2 + M_2(L_2 - L_{com})^2 + M_3(L_3 - L_{com})^2$

Let the moment of inertia about the axis passing through the center of wheels parallel to the ground be I

Then, by Parallel Axis Theorem:

$I = I_{com} + (M_1 + M_2 + M_3)L_{com}^2 + M_w R_w^2$

Last term is added to consider the moment of inertia due to rotation of wheels

$\implies R.K.E_1 = \frac{I\dot{\theta}^2}{2}$

2)PERPENDICULAR AXIS THEOREM

Consider the figure of system 2 given in answer 2.

We used perpendicular axis theorem to determine the moment of inertia about the axis(ZZ') passing through the center of mass and perpendicular to the axis passing through the center of mass of both wheels, so that we can calculate rotational K.E of system 2.

Since perpendicular axis theorem can only be applied to plane lamina, we are assuming that each level is a rectangle with its mass uniformly distributed.

This time mass M_1 does not include mass of wheels. Wheels are considered as ring as most of their mass is concentrated at the outside.

Moment of Inertia about the axis WW' at level 1 = $I_{WW'} = \frac{M_1 w^2}{12} + 2M_w R_w^2$

Moment of Inertia about the axis LL' at level 1 = $I_{LL'} = \frac{M_1 l^2}{12} + 2M_w (l/2)^2$

Let the moment of inertia about the perpendicular to the plane and passing through the center of level 1, 2 and 3 be I_{z1}, I_{z2} and I_{z3} respectively.

Then, by Perpendicular Axis Theorem:

$I_{z1} = I_{WW'} + I_{LL'}$

Similarly for Level 2 and 3

$I_{z2} = \frac{M_2 w^2}{12} + \frac{M_2 l^2}{12}$

$I_{z3} = \frac{M_3 w^2}{12} + \frac{M_3 l^2}{12}$

$\implies$ Total moment of inertia about the z-axis = $I_{zz'} = I_{z1} + I_{z2} + I_{z3}$

$I_{zz'} = \frac{M_1 w^2}{12} + 2M_w R_w^2 + \frac{M_1 l^2}{12} + 2M_w (l/2)^2 + \frac{M_2 w^2}{12} + \frac{M_2 l^2}{12} + \frac{M_3 w^2}{12} + \frac{M_3 l^2}{12}$

$\implies R.K.E_2 = \frac{I_{zz'}\dot{\delta}^2}{2}$

Q 8. What will happen in the following situations:

(a) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit falls inside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]

(b) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit falls outside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]

(c) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit and the Medbot both fall inside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]

(d) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit and the Medbot both fall outside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]

A 8. a) No there will be no penalty imposed and no points will be awarded. No the FAK will not be repositioned.

b) No there will be no penalty imposed but 20 points will be awarded for successful pick up. No the FAK will not be repositioned.

c) Yes Penalty of 50 points will be imposed for fall of bot. No points will be awarded. Yes the FAK will be repositoned.

d) Yes Penalty of 50 points will be imposed for fall of bot. 20 points will be awarded for successful pickup. Yes the FAK will be repositoned.

Q 9. What will be the points awarded if Medbot picks only one of the item from the medical store and repeatedly moves back and forth around the gravel pathway or the bridge for the entire run. [4]

A 9. Case 1: When it moves back and forth around Gravel Pathway

20 points for successful pickup + 50 points for traversing Gravel Pathway + 100 points for no penalty = Total of **170** points.

Case 2: When it moves back and forth around Bridge

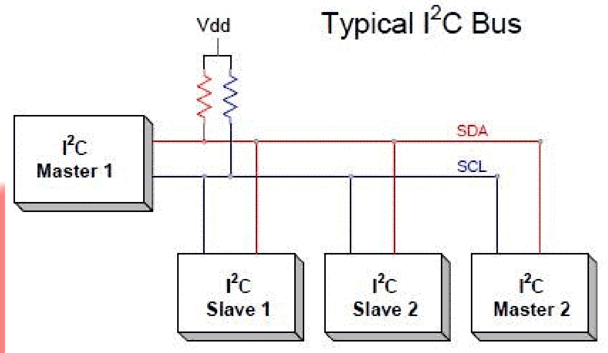
20 points for successful pickup + 70 points for traversing Bridge + 100 points for no penalty = Total of **190** points.

Q 10. What are the different communication protocols you'll be using? Name the hardware interfaced related to each of the communication protocols. Explain how these communication protocols works and what are the differences between them. [8]

A 10. We will be using 3 communication protocols namely:

1. UART: stands for Universal Asynchronous Receiver-Transmitter. Transmitter UART recives data from data bus in parallel form. After that, it adds a start bit, a parity bit, and a stop bit, creating the data packet. Next, the data packet is output serially, bit by bit at the Tx pin. The receiving UART reads the data packet bit by bit at its Rx pin. The receiving UART then converts the data back into parallel form and removes the start bit, parity bit, and stop bits. Finally, the receiving UART transfers the data packet in parallel to the data bus on the receiving end.

2. I2C(Inter-Integrated Communication): In I2C all the slaves and the master(s) are connected to the same data bus as shown.



Whenever master wants to send data to a slave, the SDA line switches from a high voltage level to a low voltage level before the SCL line switches from high to low to initiate start condition. Then it sends the address of the slave on data bus to each slave. Then each slave compares this address to its own. If it matches, it sends a low voltage ACK bit back to the master otherwise do nothing. After the master detects the ACK bit from the slave, the first data frame of 8 bits is sent bit by bit with the most significant bit first. Each data frame is immediately followed by an ACK(low)/NACK(high) bit to verify that the frame has been received successfully. The ACK bit must be received by either the master or the slave (depending on who is sending the data) before the next data frame can be sent.

After all of the data frames have been sent, the master can send a stop condition to the slave to halt the transmission. The stop condition is a voltage transition from low to high on the SDA line after a low to high transition on the SCL line, with the SCL line remaining high.

3. Zigbee/IEEE 802.15.4: In this type of communication there is one and only one coordinator which is responsible for initializing, maintaining and controlling the network. Connected to coordinator are called routers which are responsible for transmitting data from coordinator to end devices which are connected to router. This is similar to I2C because data transmission happens in frames which constitutes of different bytes such as start byte, Frame type, Source address, Source Network Address, Digital Channel Mask, Analog Channel Mask, Digital Data, Analog Data and Checksum. First the coordinator broadcasts the destination address to every router/end device. Routers/end devices compares the destination address to their own address. If a match is found, then the router/end device sends an acknowledgement back to the coordinator. After that the coordinator sends the data to the destination.

	UART	I2C	Zigbee/IEEE 802.15.4
Complexity	Simple	Easy to chain many devices	High
Speed	Lowest	Highest	Medium
Max No. of Devices	Upto 2	Upto 127	Upto 65536
No. of wires	2	2	Wireless
Duplex	Full	Half	Full
Range	15-30m	Few meters	Upto 1.6km
Number of masters and slaves	No multiple slaves and masters	Multiple slaves and masters	Only 1 Coordinator and multiple router and end devices

HARDWARES INTERFACED:

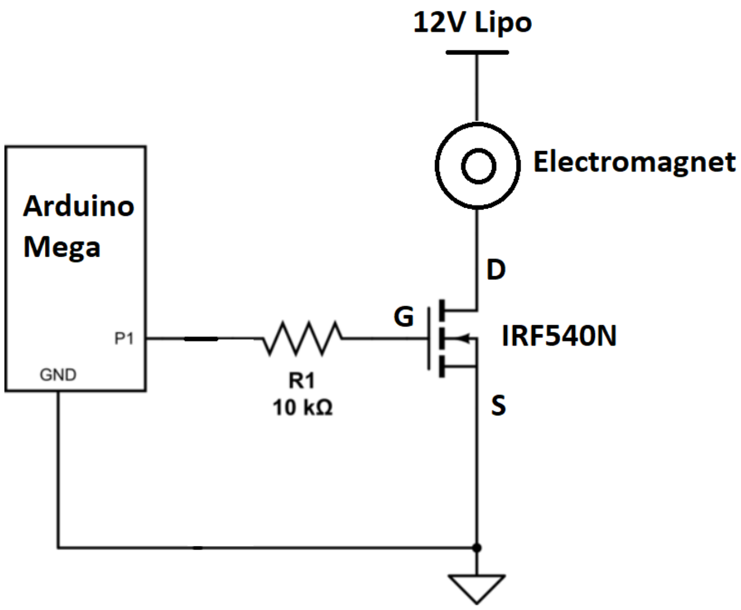
UART: Arduino and XBEE, Arduino and PC

I2C: GY-87 and Arduino

Zigbee/IEEE 802.15.4: Transmitter XBEE and Reciever XBEE

Q 11. Why do we require IRF540N? Provide circuit diagram for interfacing IRF540N with the microcontroller. [5+5]

A 11. We require IRF540N to control the electromagnet which will be used to pick and drop the supply items. It is an N-Channel enhancement type mosfet and will be used as an electronic switch to turn ON and OFF the electromagnet. It will be used in saturation region. To use it as a switch we connect it as shown below:



Here R1 is used to limit gate current to avoid damage to the arduino pin.