Assignment 3 19114018, Ayushman Tripathy

1 (a) $y(t) = t^2(x(t-1))$

Input $= x_1(t)$
$\Rightarrow y_1(t) = t^2 \, x_1(t-1)$

Input $= x_2(t)$
$\Rightarrow y_2(t) = t^2 \, x_2(t-1)$

If input $= a\,x_1(t) + b\,x_2(t) = x(t)$
$\{x(t-1) = a x_1(t-1) + b x_2(t-1)\}$

$\Rightarrow y(t) = t^2\left(a x_1(t-1) + b\,x_2(t-1)\right)$

$= a\,t^2 x_1(t-1) + b\,t^2 x_2(t-1)$

$y(t) = a\,y_1(t) + b\,y_2(t)$

$\Rightarrow a x_1(t) + b\,x_2(t) \rightarrow a\,y_1(t) + b\,y_2(t)$

$\therefore$ $\boxed{\text{Linear System}}$

$x_1(t) \rightarrow y_1(t) = t^2 \, x_1(t-1)$

Let $x_2(t) = x_1(t-t_0)$

If input $= x_2(t) \Rightarrow y_2(t) = t^2 \, x_2(t-1)$

$x_2(t) = x_1(t-t_0)$
$\Rightarrow x_2(t-1) = x_1(t-1-t_0)$
$\Rightarrow \boxed{x_2(t-1) = x_1(t-t_0-1)}$

$y_1(t) = t^2(x_1(t-1))$
$\Rightarrow y_1(t-t_0) = (t-t_0)^2 \, x_1(t-t_0-1)$
$\Rightarrow \boxed{y_1(t-t_0) = (t-t_0)^2 \, x_1(t-t_0-1)}$

$\Rightarrow y_2(t) = t^2 x_2(t-1) = t^2 \, x_1(t-t_0-1) \neq y_1(t-t_0)$

$\because x_1(t) \rightarrow y_1(t)$ but $x_1(t-t_0) \not\rightarrow y_1(t-t_0)$

$\therefore$ $\boxed{\text{Not Time Invariant}}$

$\therefore$ Linear but not time invariant

classmate

Date

Page

(b) $y[n] = (x[n-2])^2$

$x_1[n] \rightarrow y_1[n]$ | $x_2[n] \rightarrow y_2[n]$

$\Rightarrow y_1[n] = (x_1[n-2])^2$ | $\Rightarrow y_2[n] = (x_2[n-2])^2$

$\{ x[n-2] = a x_1[n-2] + b x_2[n-2] \}$

if input $= a x_1[n] + b x_2[n] = x[n]$

$\Rightarrow y[n] = (a x_1[n-2] + b x_2[n-2])^2$

$= a^2 (x_1[n-2])^2 + b^2 (x_2[n-2])^2 + 2ab\, x_1[n-2]\, x_2[n-2]$

$y[n] = a^2 y_1[n] + b^2 y_2[n] + 2ab \sqrt{y_1[n] \cdot y_2[n]}$

$\therefore a x_1[n] + b x_2[n] \rightarrow a^2 (y_1[n]) + b^2 (y_2[n]) + 2ab \sqrt{y_1[n]\, y_2[n]}$

$\neq a y_1[n] + b y_2[n]$

$\therefore$ $\boxed{\text{NOT LINEAR}}$

$x_1[n] \rightarrow y_1[n]$

$\Rightarrow y_1[n] = (x_1[n-2])^2$

$x_2[n] \rightarrow y_2[n] \Rightarrow y_2[n] = (x_2[n-2])^2$

If $x_2[n] = x_1[n-n_0] \Rightarrow y_2[n] = (x_1[n-n_0-2])^2$

$y_1[n] = (x_1[n-2])^2$

$\Rightarrow y_1[n-n_0] = (x_1[n-n_0-2])^2$

$\Rightarrow y_2[n] = y_1[n-n_0]$ when $x_2[n] = x_1[n-n_0]$

$\therefore$ $\boxed{\text{TIME INVARIANT}}$

$\therefore$ System is NOT LINEAR BUT IT IS TIME INVARIANT

(c) $y[n] = x[n+1] - x[n-1]$

$x_1[n] \rightarrow y_1[n] \Rightarrow y[n] = x_1[n+1] - x_1[n-1]$

$x_2[n] \rightarrow y_2[n] \Rightarrow y_2[n] = x_2[n+1] - x_2[n-1]$

if input $= a x_1[n] + b x_2[n] = x[n]$

$\{ x[n+1] = a x_1[n+1] + b x_2[n+1]$, Similarly for $x[n-1] \}$

$\Rightarrow y[n] = (\underbrace{a x_1[n+1] + b x_2[n+1]}_{x[n+1]}) - (\underbrace{a x_1[n-1] + b x_2[n-1]}_{x[n-1]})$

$= a(x_1[n+1] - x_1[n-1]) + b(x_2[n+1] - x_2[n-1])$

$\boxed{y[n] = a y_1[n] + b y_2[n]}$

$\Rightarrow a x_1[n] + b x_2[n] \longrightarrow a y_1[n] + b y_2[n]$

$\therefore$ $\boxed{\text{LINEAR SYSTEM}}$

$x_1[n] \rightarrow y_1[n] \Rightarrow y_1[n] = x_1[n+1] - x_1[n-1]$

if $x_2[n] = x_1[n-n_0] \Rightarrow \begin{cases} x_2[n+1] = x_1[n+1-n_0] = x_1[n-n_0+1] \\ x_2[n-1] = x_1[n-1-n_0] = x_1[n-n_0-1] \end{cases}$

$x_2[n] \rightarrow y_2[n] \Rightarrow y_2[n] = x_2[n+1] - x_2[n-1]$

$\Rightarrow y_2[n] = x_1[n-n_0+1] - x_1[n-n_0-1]$

$y_1[n-n_0] = x_1[n-n_0+1] - x_1[n-n_0-1]$

$\therefore y_2[n] = y_1[n-n_0]$

$\Rightarrow x_1[n] \rightarrow y_1[n] \Rightarrow x_1[n-n_0] \longrightarrow y_1[n-n_0]$

$\therefore$ $\boxed{\text{TIME INVARIANT}}$

$\therefore$ $\boxed{\text{Both Linear and Time Invariant}}$ $\Rightarrow$ LTI System

(d) $y(t) = EV\{x(t)\} = \frac{1}{2}[x(t) + x(-t)]$

$\therefore y(t) = \dfrac{x(t) + x(-t)}{2}$

$x_1(t) \rightarrow y_1(t) \Rightarrow y_1(t) = \dfrac{x_1(t) + x_1(-t)}{2}$

$x_2(t) \rightarrow y_2(t) \Rightarrow y_2(t) = \dfrac{x_2(t) + x_2(-t)}{2}$

if $x(t) = a x_1(t) + b x_2(t)$

$\Rightarrow x(-t) = a x_1(-t) + b x_2(-t)$

$\Rightarrow y(t) = \frac{1}{2}(x(t) + x(-t))$

$= \frac{1}{2}(a x_1(t) + b x_2(t) + a x_1(-t) + b x_2(-t))$

$= \frac{1}{2} a (x_1(t) + x_1(-t)) + \frac{b}{2}(x_2(t) + x_2(-t))$

$\Rightarrow y(t) = a\, y_1(t) + b\, y_2(t)$

$\therefore a x_1(t) + b x_2(t) \longrightarrow a y_1(t) + b y_2(t)$

$\therefore$ $\boxed{\text{LINEAR SYSTEM}}$

if $x_2(t) = x_1(t - t_0)$

$x_2(-t) = x_1(-t - t_0)$

$y_2(t) = \frac{1}{2}(x_2(t) + x_2(-t)) = \frac{1}{2}(x_1(t - t_0) + x_1(-t - t_0))$

$y_1(t) = \frac{1}{2}(x_1(t) + x_1(-t)) \Rightarrow$

$\Rightarrow y_1(t - t_0) = \frac{1}{2}(x_1(t - t_0) + x_1(-(t - t_0)))$

PT.O

$$\Rightarrow y_1(t-t_0) = \frac{1}{2}\left(x_1(t-t_0) + x_1(-t+t_0)\right)$$

$$y_2(t) = \frac{1}{2}\left(x_1(t-t_0) + x_1(-t-t_0)\right)$$

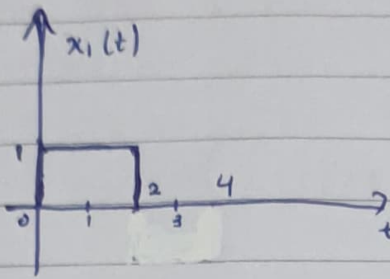
$$\therefore \; y_2(t) \neq y_1(t-t_0)$$

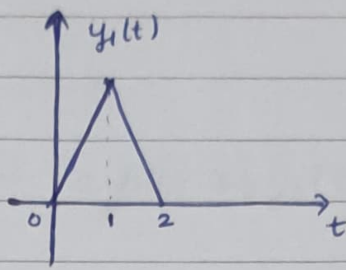
$$\Rightarrow \text{if } \; x_1(t) \rightarrow y_1(t), \quad x_1(t-t_0) \not\rightarrow y_1(t-t_0)$$

$$\therefore \; \boxed{\text{NOT TIME INVARIANT}}$$

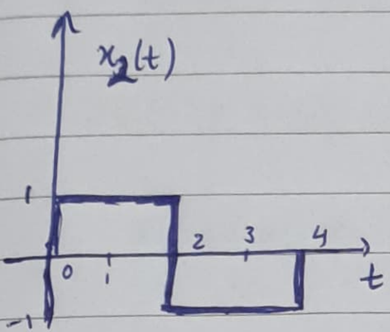
$$\boxed{\therefore \text{ System is Linear, but not Time Invariant}}$$

(2)



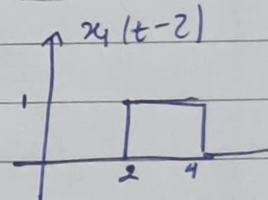


(a)

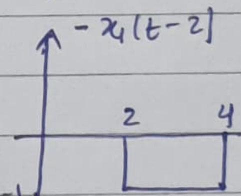


$x_2(t)$ can be split into two parts.
We will try to represent each one of them in terms of $x_1(t)$ and use properties of LTI to map the output

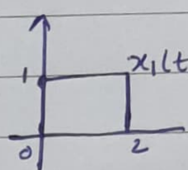
when $0 \le t \le 2$, $x_2(t) = x_1(t)$



If $2 < t \leq 4$ $\Rightarrow -x_1(t-2) = x_2(t)$



$\Rightarrow x_2(t) =$



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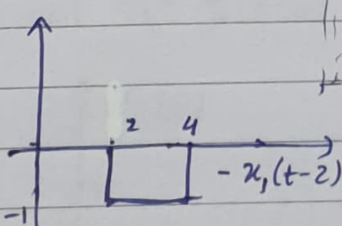


$= x_1(t) + - x_1(t-2)$

$$\boxed{x_2(t) = x_1(t) - x_1(t-2)}$$

$\Rightarrow y_2(t) = y_1(t) - y_1(t-2)$

By property of LTI system as both linear and time invariant

Proof

For LTI system

if $x_1(t) \to y_1(t)$

$x_0(t) \to y_0(t)$

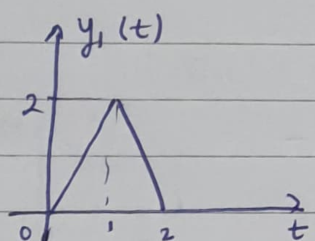
$\Rightarrow \quad a x_1(t) + b x_0(t) \longrightarrow a y_1(t) + b y_0(t)$ {Linear}

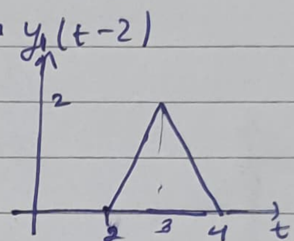
if $x_0(t) = x_1(t-t_0) \Rightarrow \quad y_0(t) = y_1(t-t_0)$ {Time invariant}

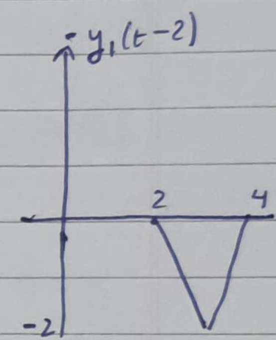
$\Rightarrow a x_1(t) + b x_1(t-t_0) \to a y_1(t) + b y_1(t-t_0)$

$\therefore \quad x_1(t) - x_1(t-2) \longrightarrow y_1(t) - y_1(t-2)$

$\therefore \quad y_2(t) = y_1(t) - y_1(t-2)$







$\Rightarrow \quad y_2(t) = y_1(t) + (-y_1(t-2))$

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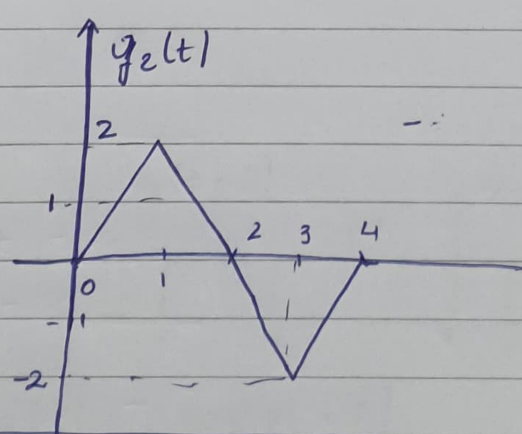
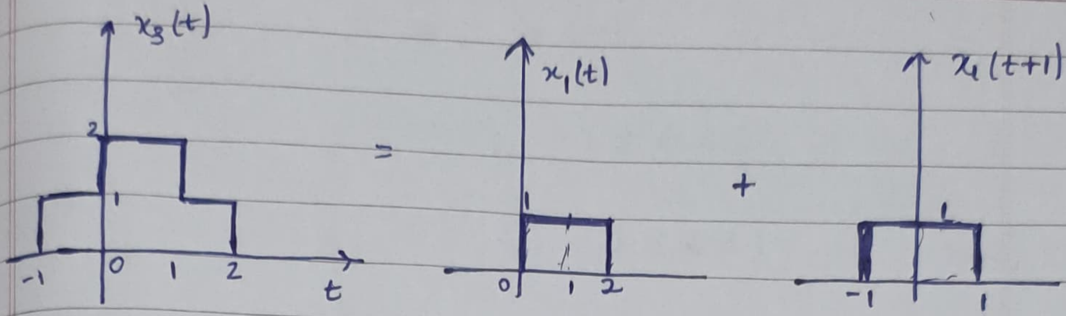


$$\boxed{\Rightarrow y_2(t) = \begin{cases} 0, & t < 0 \\ 2t, & 0 \le t \le 1 \\ 4-2t, & 1 < t \le 3 \\ -8+2t; & 3 < t \le 4 \\ 0, & t > 4 \end{cases}}$$

(b)



$x_3(t) = x_1(t) + x_1(t+1)$

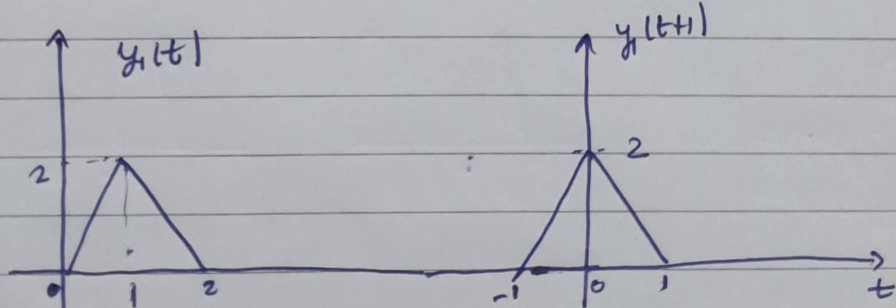
{ As, we can see that the overlap can be represented as the signal being doubled }

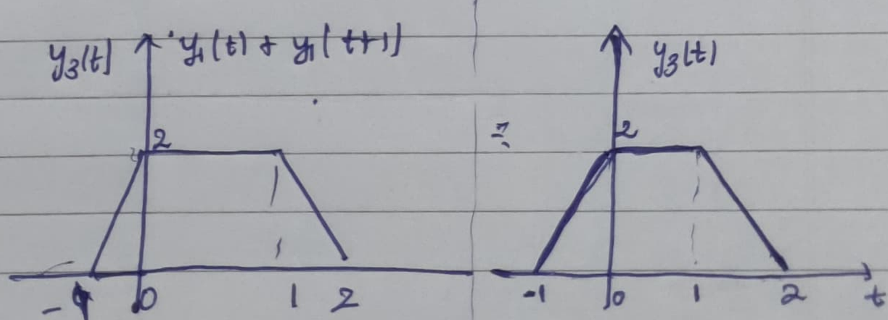
i.e. $x_3(t) = \begin{cases} 0, & t < -1,\ t > 1 \\ 1, & -1 \leq t < 0 \\ 2, & 0 \leq t < 1 \\ 1, & t \geq 1 \end{cases} = x_1(t) + x_1(t+1)$

Since the system is an LTI system

$\because x_3(t) = x_1(t) + x_1(t+1)$

$\Rightarrow \quad y_3(t) = y_1(t) + y_1(t+1)$





P.T.O

$$\Rightarrow \quad y_3(t) = \begin{cases} 0, & t < -1 \\ 2t+2; & -1 \leq t < 0 \\ 1 ; & 0 \leq t < 1 \\ 4-2t ; & t \geq 1 \end{cases}$$

- We didn't do time scaling and adding three signals as time scaling doesn't necessarily represent LTI. }

So, we took our system as time shifted addition as LTI sys. only works during time shifting and linear combinati.
{ It doesn't necessarily hold in case of time scaling }

3. Input $x[n] = \left(\frac{1}{2}\right)^n u[n-2]$

Unit impulse response: $h_0[n] = h[n] = u[n+2]$

{ $h[n]$ is output of LTI sys, when $\delta[n]$ is the input }

★ { Here we have assumed system to be LTI as other values of $h_k[n]$ have not been given to us. }

Thus,

$$h_k[n] = h_0[n-k]$$ { By time-invariance property of LTI systems }

If input is $\delta[n] \Rightarrow$ response/output $= h_0[n] = h[n]$

input is $\delta[n-k] \Rightarrow$ output $= h_k[n] = h_0[n-k]$

$= h[n-k]$

$\Rightarrow$ if input is $x[k]\,\delta[n-k] \Rightarrow$ response/output $= x[k]\,h[n-k]$

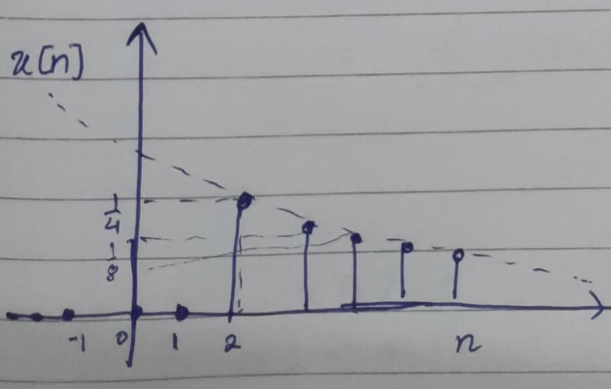
{By linearity of LTI}

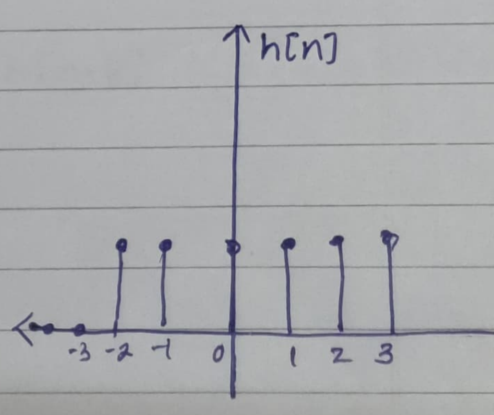
As $x[n]$ {input} $\equiv \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$

$\Rightarrow$ Output : $y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$

{ Also we know the result for LTI, $y[n] = x[n] * h[n]$ }

$u[n-2]\cdot\left(\frac{1}{2}\right)^n = x[n]$ $u[n+2] = h[n]$

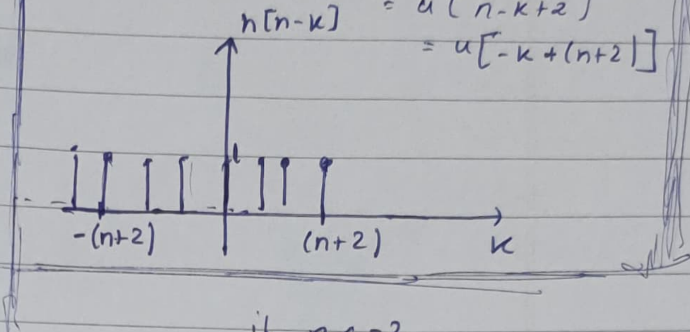




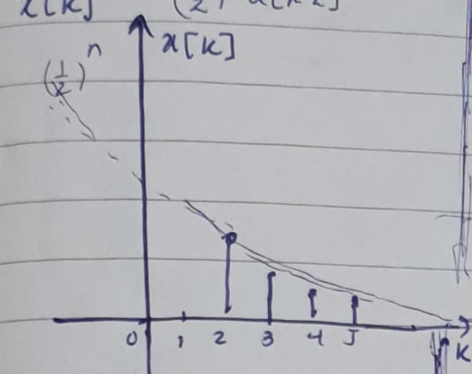
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] =$$

$n \geq -2$

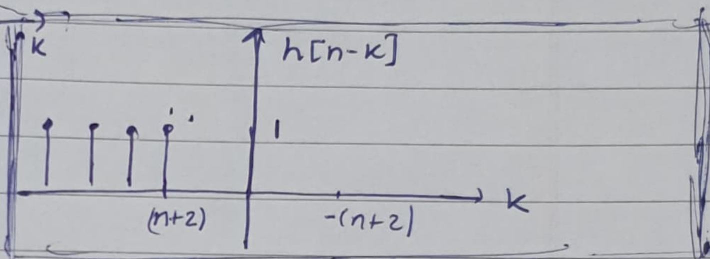
$h[n-k] = u[n-k+2]$
$= u[-k+(n+2)]$



$x[k] = \left(\frac{1}{2}\right)^k u[k-2]$



if $n < -2$



if $k \geq 2$
$x[k] = \left(\frac{1}{2}\right)^k$
else $x[k] = 0$

- if $n \leq -2 \Rightarrow y[n] = 0$ as they don't intersect positively
- $y[n]$: if $(n+2) < 2$ & $n \geq -2$ $\Rightarrow$ if $-2 \leq n < 0 \Rightarrow y[n] = 0$

{As, $x[k]$ is 0 while other $h[n-k]$ is non-zero} and vice versa

$\therefore$ $y[n] = 0$ if $n < 0$

if $n \geq 0 \Rightarrow (n+2) \geq 2$

$$\Rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$= \sum_{k=-\infty}^{1} x[k]\, h[n-k] + \sum_{k=2}^{n+2} x[k]\, h[n-k] + \sum_{k=n+3}^{\infty} x[k]\, h[n-k]$$

if $k \leq 1$, $x[k] = 0$
if $k > n+2 \Rightarrow h[n+2] = 0$

PTO

if $n \geq 0$

$$\Rightarrow \quad y[n] = \sum_{k=2}^{n+2} x[k]\,h[n-k]$$

$$= \sum_{k=2}^{n+2} \left(\tfrac{1}{2}\right)^{k} \cdot 1$$

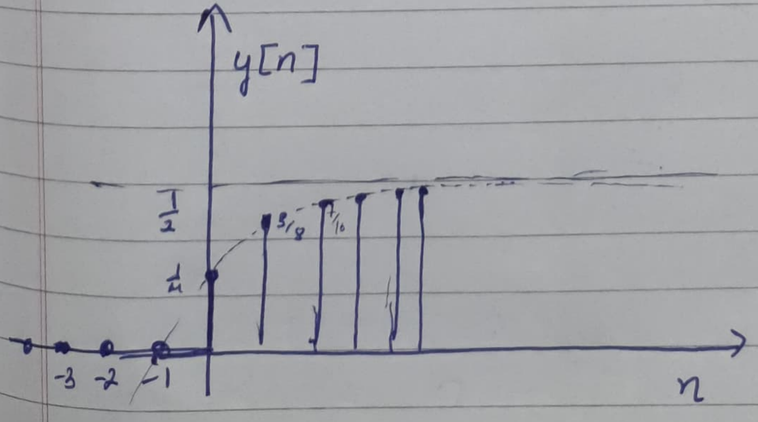
$$= \left(\tfrac{1}{2}\right)^{2} + \left(\tfrac{1}{2}\right)^{3} + \cdots + \left(\tfrac{1}{2}\right)^{n+2}$$

$$y[n] = \left(\tfrac{1}{2}\right)^{2} \frac{\left[1 - \left(\tfrac{1}{2}\right)^{n+1}\right]}{1 - \tfrac{1}{2}}$$

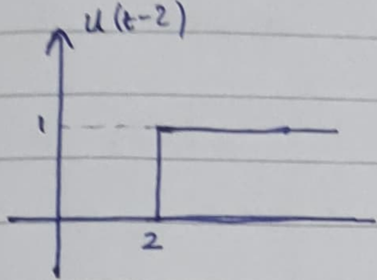
$$y[n] = \tfrac{1}{2}\left[1 - \left(\tfrac{1}{2}\right)^{n+1}\right] \quad , \text{ if } n \geq 0$$

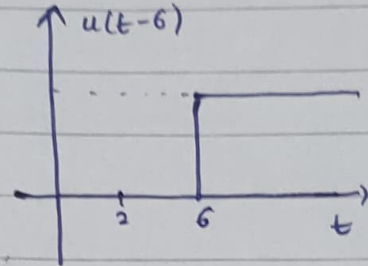
$$\therefore \quad y[n] = \begin{cases} 0 , & n < 0 \\ \tfrac{1}{2}\left(1 - \tfrac{1}{2^{n+1}}\right) , & n \geq 0 \end{cases}$$

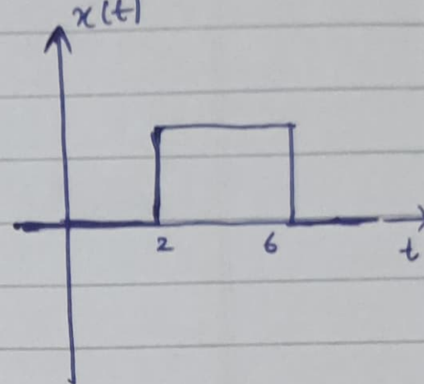
$$= \tfrac{1}{2} - \left(\tfrac{1}{2}\right)^{n+2}$$



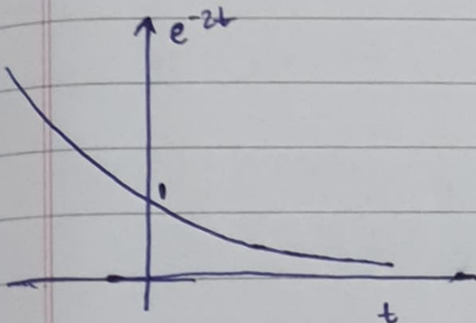
4 (a) $x(t) = u(t-2) - u(t-6)$

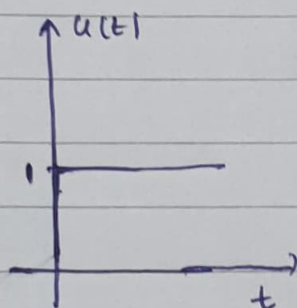


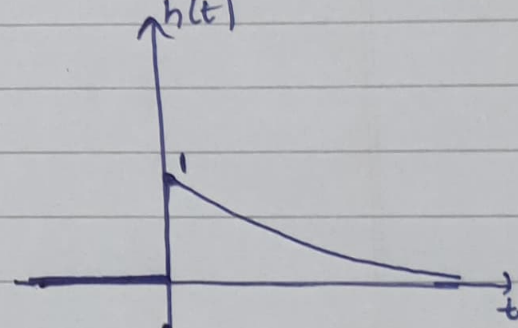




$h(t) = e^{-2t} u(t)$







$$y(t) = x(t) \star h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau) . h(t-\tau)\, d\tau$$

$\left\{ \begin{array}{l} \text{Here we assume system to be LTI, as no} \\ \text{information about } h_\tau(t) \text{ other than that has been} \\ \text{given, thus } h_\tau(t) = h(t-\tau) \Rightarrow x(\tau) h_\tau(t) = x(\tau) . h(t-\tau) \end{array} \right\}$

As we see from graph of $x(t)$ v/s t that $x(t)$ vanishes other than $2 \leq t \leq 6$

$$\therefore \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \equiv \int_{2}^{6} x(\tau)\, h(t-\tau)\, d\tau$$

In interval $2 \le \tau \le 6$ $\quad x(\tau) = 1$

$$\Rightarrow y(t) = \int_2^6 1 \cdot h(t-\tau)\, d\tau$$

$$= \int_2^6 h(t-\tau)\, d\tau$$

Let $t - \tau = k \Rightarrow dk = -d\tau$

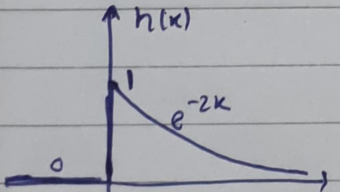
$\tau = 2 \Rightarrow k = t-2$

$\tau = 6 \Rightarrow k = t-6$

$$\Rightarrow y(t) = \int_{t-2}^{t-6} h(k)\,(-dk) = \int_{t-6}^{t-2} h(k)\, dk$$

$$\left\{ -\int_a^b f(x)\,dx = \int_b^a f(x)\,dx \right.$$

$$\boxed{y(t) = \int_{t-6}^{t-2} h(k)\,dk}$$



if $t - 2 < 0 \Rightarrow \boxed{t < 2}$

$\Rightarrow h(k) = 0 \Rightarrow \boxed{y(t) = 0}$

if $t - 6 \geq 0 \Rightarrow t \geq 0$

$\Rightarrow h(k) = e^{-2k}$

$$\Rightarrow y(t) = \int_{t-6}^{t-2} e^{-2k}\, dk$$

$$= \frac{-1}{2}\left[e^{-2k}\right]_{t-6}^{t-2}$$

$$\boxed{y(t) = \frac{1}{2}\left[e^{-2(t-6)} - e^{-2(t-2)}\right] \quad t \geq 6}$$

classmate
Date
Page

If $t-6 < 0 \leq t-2$

$\Rightarrow$ if $2 \leq t < 6$

$$\Rightarrow \int_{t-6}^{t-2} h(\kappa)\, d\kappa = \int_{t-6}^{0} 0\, d\kappa + \int_{0}^{t-2} e^{-2\kappa}\, d\kappa$$

$$= -\frac{1}{2}\left[e^{-2\kappa}\right]_0^{t-2} = \frac{1}{2}\left[1 - e^{-2(t-2)}\right]$$

$$\therefore y(t) = \begin{cases} 0 \ ; & t < 2 \\ \dfrac{1 - e^{-2(t-2)}}{2} \ ; & 2 \leq t < 6 \\ \dfrac{e^{-2(t-6)} - e^{-2(t-2)}}{2} \ ; & t \geq 6 \end{cases}$$

$$= \frac{e^{-2t}}{2}\left(e^{12} - e^{4}\right) , \quad t \geq 6$$

classmate

Date

Page

(b) $x(t) = u(t-2) - u(t-6)$

$$\frac{d}{dt}(x(t)) = \frac{d}{dt}(u(t-2)) - \frac{d}{dt}(u(t-6))$$

$$= \delta(t-2) - \delta(t-6) \quad \left\{ \text{By definition } \delta(t) = \frac{d}{dt}(u(t)) \right.$$

$$\frac{d}{dt}(x(t)) \star h(t) = g(t)$$

$$g(t) = (\delta(t-2) - \delta(t-6)) \star h(t)$$

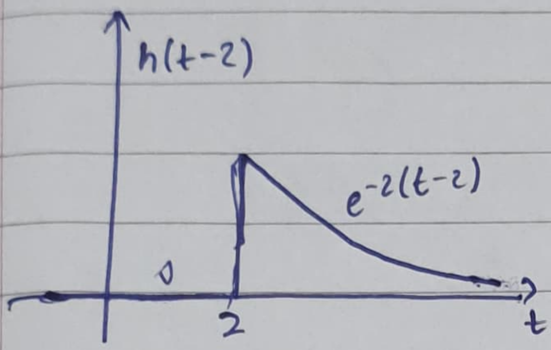
By distributive property

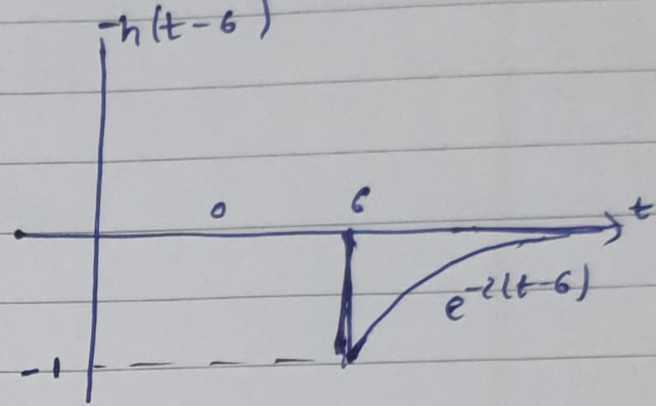
$$g(t) = \delta(t-2) \star h(t) - \delta(t-6) \star h(t)$$

$$= h(t-2) - h(t-6)$$

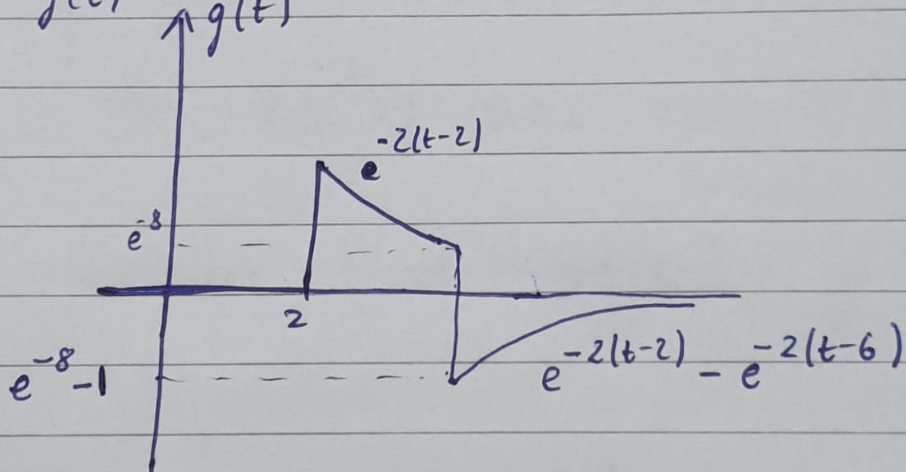
{ As for any arbitrary signal $x(t)$, the convulation $\delta(t-t_0) \star x(t) = x(t-t_0)$ }

$$g(t) = h(t-2) - h(t-6)$$





$h(t-2) - h(t-6)$
$= g(t)$



$$g(t) = \begin{cases} 0, & t < 2 \\ e^{-2(t-2)}, & 2 \le t < 6 \\ e^{-2(t-2)} - e^{-2(t-6)}, & t \geq 6 \end{cases}$$

$$\equiv e^{-2t}(e^{4} - e^{12})$$

$$\equiv -(e^{12} - e^{4})\, e^{-2t}$$

classmate

Date

Page

(c) $y(t) = \begin{cases} 0 \ ; & t < 2 \\ \dfrac{1 - e^{-2(t-2)}}{2} \ ; & 2 \leq t < 6 \\ (e^{12} - e^{4}) \dfrac{e^{-2t}}{2} \ ; & t \geq 6 \end{cases}$

$g(t) = \begin{cases} 0 , & t < 2 \\ e^{-2(t-2)} \ ; & 2 \leq t < 6 \\ -(e^{12} - e^{4}) e^{-2t} \ ; & t \geq 6 \end{cases}$

We can see that $\dfrac{d(y(t))}{dt} = g(t)$ except at $t = 2$ & 6 where it is not defined

$\dfrac{d(y(t))}{dt} = \begin{cases} 0 \ , & t < 2 \\ 0 - \frac{1}{2} \cdot (-2)\, e^{-2(t-2)} = e^{-2(t-2)} \ ; & 2 < t < 6 \\ (e^{12} - e^{4}) \cdot \frac{1}{2} (-2)\, e^{-2t} = -(e^{12} - e^{4}) e^{-2t} \ ; & t > 6 \end{cases}$

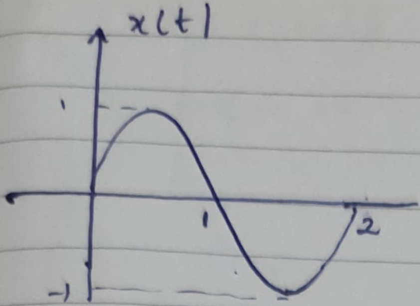
$\therefore$ $\boxed{g(t) = \dfrac{d(y(t))}{dt}}$ except at $t = 2$ and $t = 6$

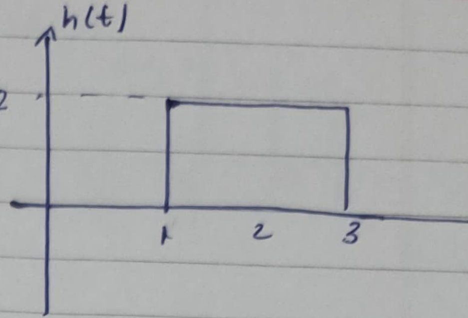
{as not defined there}

{Here we have not taken the points where both functions are discontinuous at $t = 2$ and $t = 6$}

as not defined at these pts.

5 (a)

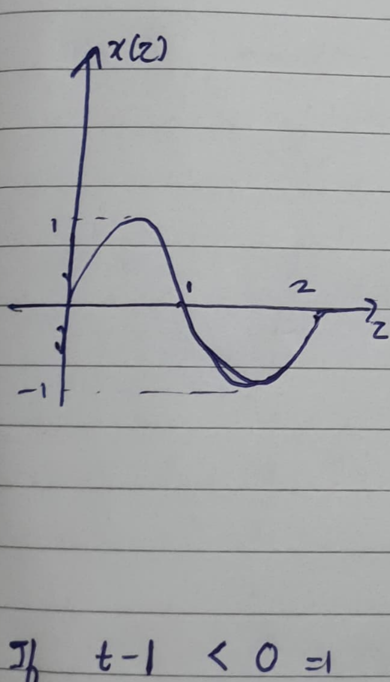


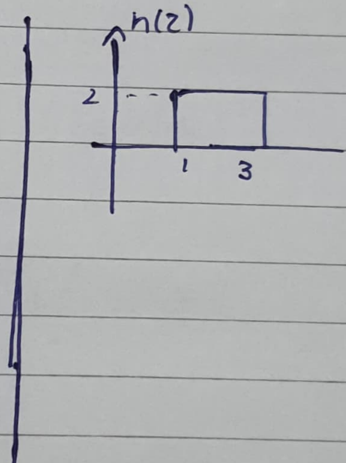


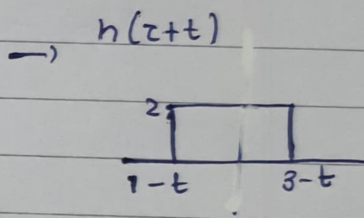
Assuming system to be LTI, we have

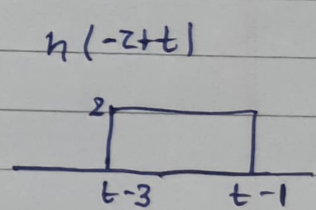
$$y(t) = x(t) \star h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

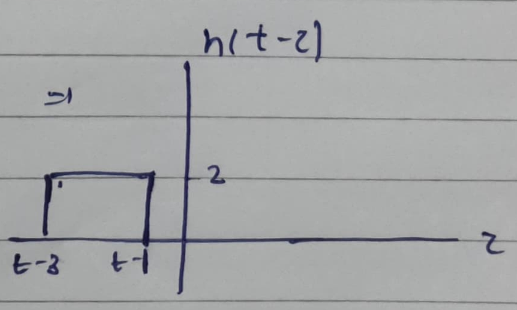




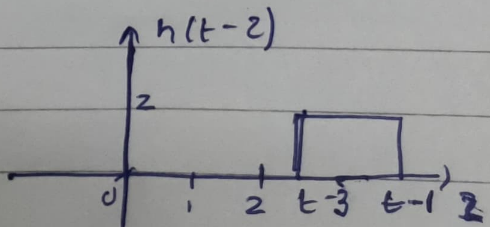




If $t-1 < 0 \Rightarrow t < 1 \Rightarrow$

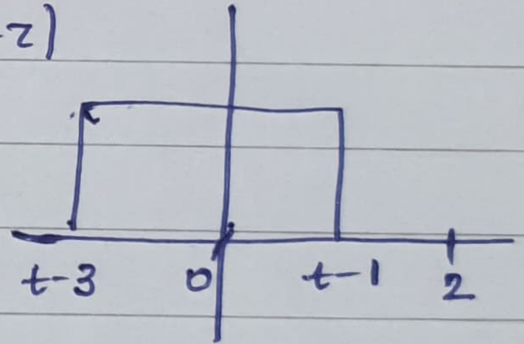


If $t-3 > 2 \Rightarrow t > 5 \Rightarrow$



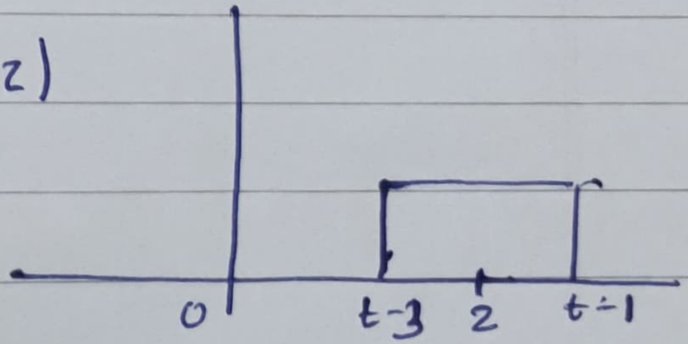
If $t \geq 1$ & $t \leq 3$ ⇒ $h(t-\tau)$

$3 < t \leq 1$



If $t \geq 3$ and $t \leq 5$ $h(t-\tau)$

⇒ $3 \leq t \leq 5$



$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

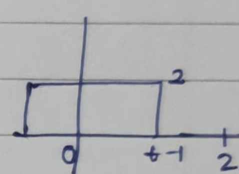
$$= \int_{0}^{2} x(\tau)\, h(t-\tau)\, d\tau \qquad \left\{ \text{As } x(\tau) \text{ is non-zero only for } 0 \le \tau \le 2 \right\}$$

If $t < 1$ or $t > 5$

$h(t-\tau) = 0 \quad \forall \quad 0 \le \tau \le 2$

$\therefore \int_0^2 x(\tau)\, h(t-\tau) = 0 \quad \Rightarrow y(t) = 0$

If $1 \le t < 3$



$$\Rightarrow h(t-\tau) = \begin{cases} 2, & 0 \le \tau \le t-1 \\ 0, & t-1 \le \tau < 2 \end{cases}$$

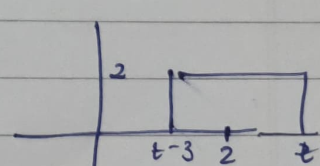
$x(\tau) = \sin(\pi\tau)$

$$\Rightarrow \int_0^2 h(t-\tau)\, x(\tau)\, d\tau$$

$$= \int_0^{t-1} \sin(\pi\tau)\,(2)\, d\tau = \left[\frac{-2\cos(\pi\tau)}{\pi}\right]_0^{t-1} = 2\left(\frac{1 - \cos(\pi(t-1))}{\pi}\right)$$

$$= \frac{1 - \cos(\pi(t-1))}{\pi}$$

If $3 \le t \le 5$



$$h(t-\tau) = \begin{cases} 0, & 0 \le \tau < t-3 \\ 2, & t-3 \le \tau \le 2 \end{cases}$$

$$\Rightarrow \int_0^2 x(\tau)\, h(t-\tau)\, d\tau = \int_{t-3}^{2} x(\tau)\, h(t-\tau)\, d\tau = \int_{t-3}^{2} \sin(\pi\tau) \cdot 2\, d\tau$$

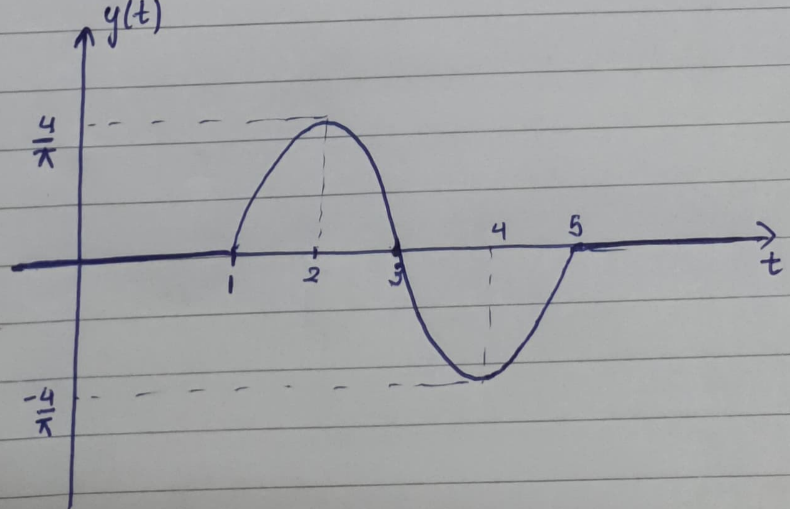
$$= 2\left[\frac{-\cos(\pi\tau)}{\pi}\right]_{t-3}^{2}$$

$$= 2\left(\frac{\cos(\pi(t-3)) - \cos(2\pi)}{\pi}\right) = 2\left(\frac{\cos(\pi(t-3)) - 1}{\pi}\right)$$

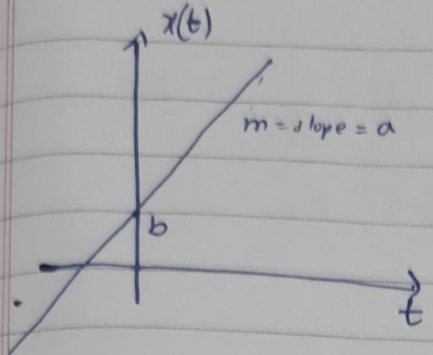
$$\Rightarrow y(t) = \begin{cases} 0, & t < 1 \\ \dfrac{2(1-\cos(\pi(t-1)))}{\pi} = \dfrac{2}{\pi}(1-\cos(\pi t-\pi)) = \dfrac{2}{\pi}(1+\cos(\pi t)) & 1 \le t < 3 \\ \dfrac{2}{\pi}(\cos(\pi(t-3))-1) = \dfrac{2}{\pi}(\cos(\pi t - 3\pi) - 1) = -\dfrac{2}{\pi}(\cos(\pi t)+1) & 3 \le t \le 5 \\ 0, & t > 5 \end{cases}$$

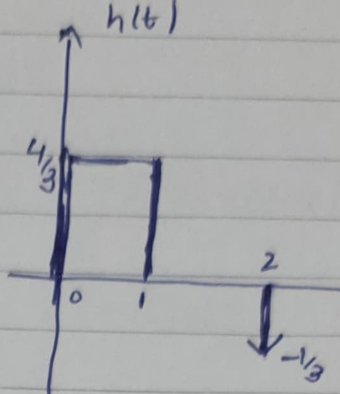
$$\Rightarrow y(t) = \begin{cases} 0 \; ; & t < 1 \\ \dfrac{2}{\pi}(1+\cos(\pi t)) \; ; & 1 \le t < 3 \\ -\dfrac{2}{\pi}(1+\cos(\pi t)) \; ; & 3 \le t \le 5 \\ 0 \; , & t > 5 \end{cases}$$

Plot (a)



(6)





$x(t) = at + b$

$$h(t) = \begin{cases} 0, & t < 0 \\ 4/3, & 0 \le t \le 1 \\ 0, & 1 < t < 2 \\ -\frac{1}{3}\delta(t-2); & t = 2 \\ 0; & t > 2 \end{cases}$$

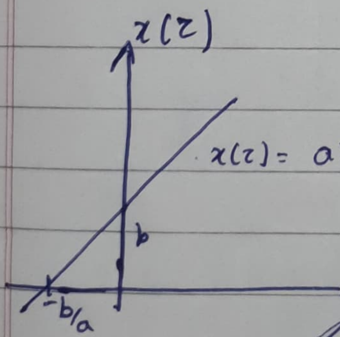
$y(t) = x(t) \star h(t)$

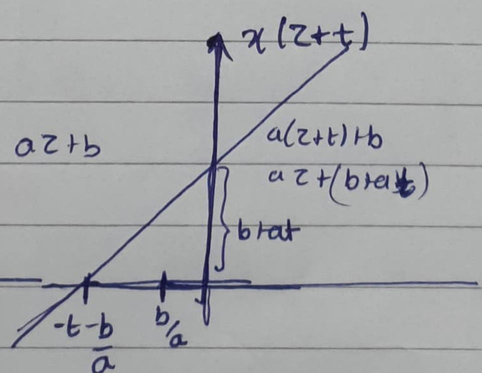
{considering LTI sys}

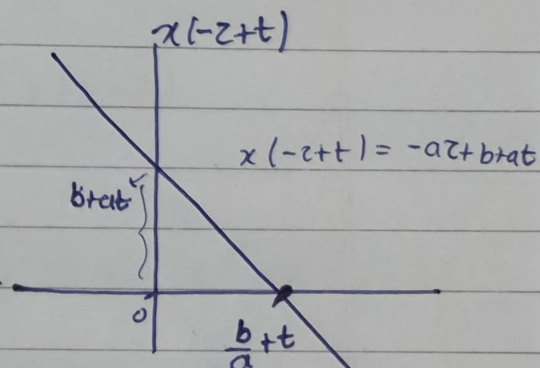
$\Rightarrow y(t) = h(t) \star x(t)$ {By Commutative Property}

we are using this as $h(t)$ is only defined for a short interval...

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$







{t is not necessarily positive but it won't affect calculation, As, we will take cases accordingly}

$\therefore x(-\tau+t)$

$= -a\tau + b + at$

classmate

Date

Page

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{0} h(\tau)\, x(t-\tau)\, d\tau + \int_{0}^{1} h(\tau)\, x(t-\tau)\, d\tau$$

$$+ \int_{1}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= 0 + \int_0^1 \frac{4}{3}(-a\tau + b + at)\, d\tau + \underbrace{-\frac{1}{3} x(t-\tau)}_{\text{at } \tau = 2} + 0$$

{ as

{ As we can write $h(t) = h_1(t) + h_2(t)$

where $h_2(t) = -\frac{1}{3}\delta(t-2)$

$$x(t) * h(t) = x(t) * h_1(t) + x(t) * -\frac{1}{3}\delta(t-2)$$

$$= \underbrace{x(t) * h_1(t)}_{\int_0^1 \frac{4}{3}(-a\tau+b+at)\, d\tau} - \frac{1}{3} x(t-2) \quad \}$$

$$= \frac{4}{3} \int_0^1 (-a\tau + b + at)\, d\tau - \frac{1}{3} x(t-2)$$

$$= \frac{4}{3}\left[-\frac{a}{2}\tau^2 + (b+at)\tau\right]_0^1 - \frac{1}{3}(a(t-2) + b)$$

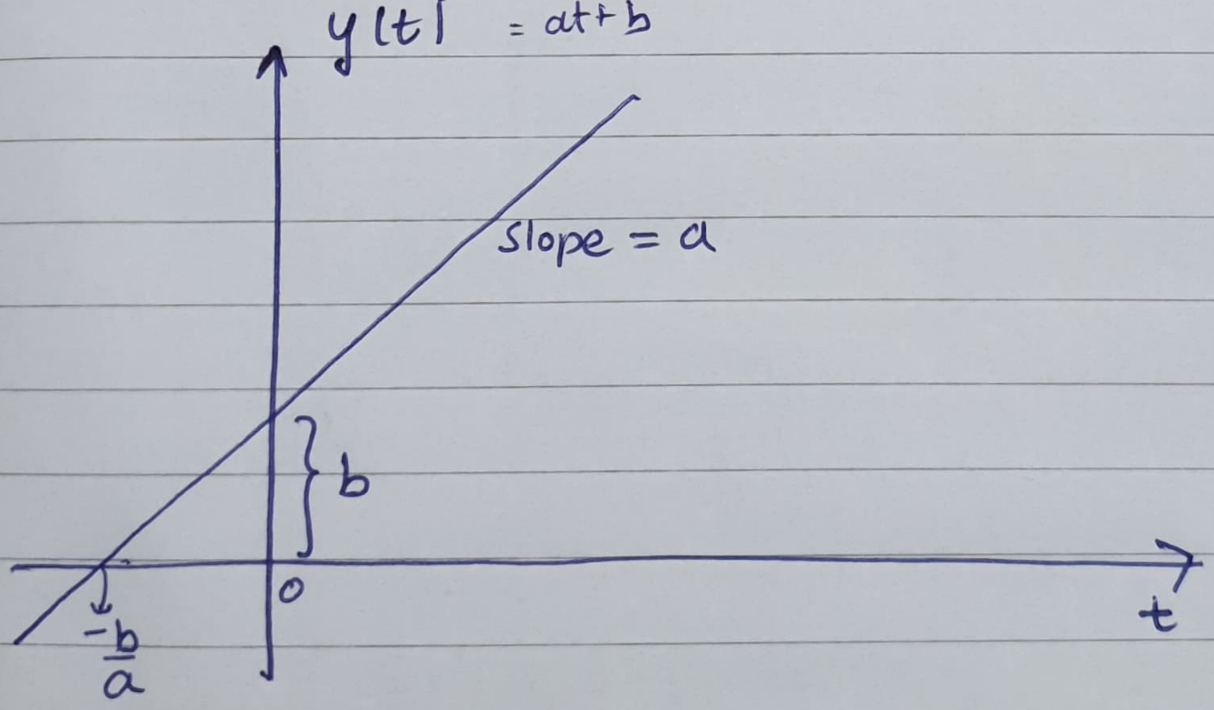
$$= -\frac{2}{3}a + \frac{4}{3}(b+at) - \frac{1}{3}a(t-2) - \frac{b}{3}$$

$$= b + at$$

$$\Rightarrow \boxed{y(t) = at + b}$$

P.T.O

plot for (b)



(c) As the input to the LTI system here is periodic with fundamental period = 2
$\Rightarrow$ Output would also be periodic
{As time invariant}

Proof. $y(t) = x(t) * h(t) = h(t) * x(t)$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

If T_0 is periodic of $x(t)$

$$\Rightarrow y(t+T_0) = \int_{-\infty}^{\infty} h(\tau)\, x(t+T_0-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau \quad \{\text{Periodicity of } x\}$$

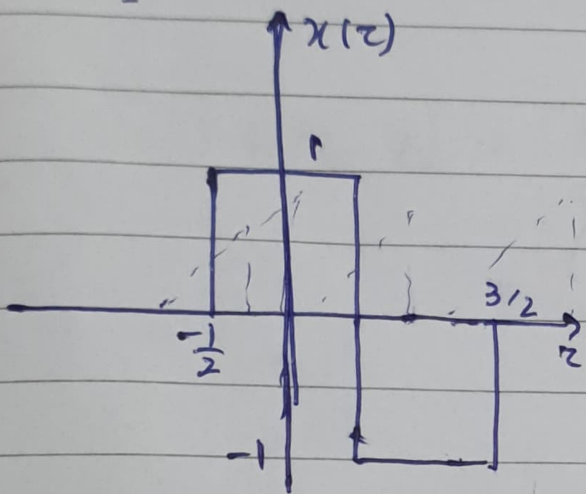
$$= y(t) \qquad y(t) \text{ has a period} = T_0$$

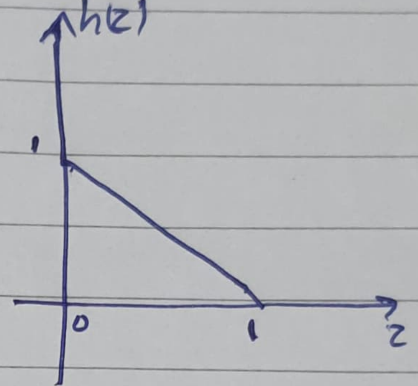
$\therefore$ We need to calculate for only one period of 'x' $\{-\frac{1}{2}$ to $\frac{3}{2}\}$ and then output will repeat

5(c) $x(\tau)$

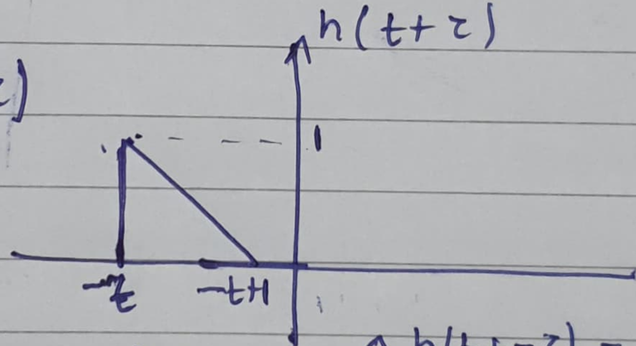
$=$

This will repeat $\{y(t)\}$ with period $= 2$

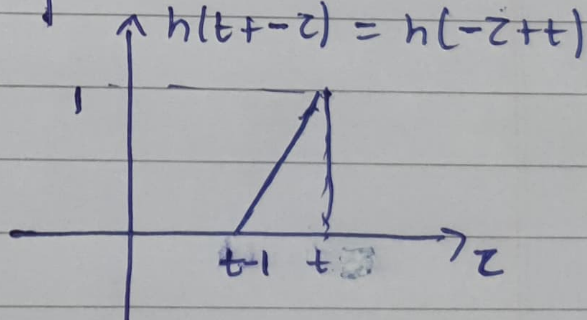




$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)$$



{Here 't' maybe negative but output can be handled accordingly by constraints mathematically}



$$h(t-\tau) = \begin{cases} \tau-(t-1) \\ = 1+\tau-t, & t-1 \le \tau \le t \\ 0, & \text{otherwise} \end{cases}$$

For, $-\frac{1}{2} < t < \frac{1}{2}$

$$x(\tau)\,h(t-\tau) = \begin{cases} -(\tau-t+1) & ; \quad t-1 < \tau < -1/2 \\ 1+t-\tau & ; \quad -1/2 < \tau < t \end{cases}$$

For $\frac{1}{2} < t < \frac{3}{2}$

$$x(\tau)\,h(t-\tau) = \begin{cases} 1+t-\tau & ; \quad t-1 < \tau < 1/2 \\ -(1+t-\tau) & ; \quad \frac{1}{2} < \tau < t \end{cases}$$

Here we took only $t \in [-\frac{1}{2}, \frac{3}{2}]$ as we had only taken $x(t)$ for that interval.

$$y_1(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

$$y(t) = \begin{cases} \displaystyle\int_{t-1}^{-1/2} -(\tau-t+1)\,d\tau + \int_{-1/2}^{t} (\tau-t+1)\,d\tau & ; \quad -\frac{1}{2} < t < \frac{1}{2} \\ \displaystyle\int_{t-1}^{1/2} (\tau-t+1)\,d\tau + \int_{\frac{1}{2}}^{t} -(\tau-t+1)\,d\tau & ; \quad \frac{1}{2} < t < \frac{3}{2} \end{cases}$$

$$= \begin{cases} -\left(\frac{1}{2}\left(\frac{1}{4}-(t-1)^2\right) - t\left(-\frac{1}{2}-t+1\right) + \left(-\frac{1}{2}+1-t\right)\right) + \left[\frac{1}{2}\left(t^2-\frac{1}{4}\right) - t\left(t+\frac{1}{2}\right) + t + \frac{1}{2}\right] & \text{for } -1/2 \le t \le 1/2 \\ \left[\frac{1}{2}\left[\frac{1}{4}-(t-1)^2\right] - t\left(\frac{1}{2}-t+1\right) + \frac{1}{2}-t+1\right] - \left[\frac{1}{2}\left(t^2-\frac{1}{4}\right) - t\left(t-\frac{1}{2}\right) + t - \frac{1}{2}\right] & \text{for } \frac{1}{2} \le t < \frac{3}{2} \end{cases}$$

$$\therefore \quad y_1(t) = \begin{cases} \frac{1}{4} + t - t^2 \; ; & -\frac{1}{2} < t < \frac{1}{2} \\ t^2 - 3t + \frac{7}{4} \; ; & \frac{1}{2} < t < \frac{3}{2} \end{cases}$$

Plot for (c)