Jitesh Jain (19114039) (02) Assignment - 2 ECN-203

Ans 1 (a) $x_1(t) = 2j\, e^{j12t}$ → periodic func^n with $\frac{\pi}{6}$ as its period

(b) $x_2[n] = e^{-0.7jn}$ → $\boxed{\text{not periodic}}$ as $\omega_0 = 0.7$ is not a multiple of 2π

(c) $x_3[n] = 3e^{\frac{j3\pi(n+1/2)}{5}}$ → periodic func^n with period $= \left(\frac{2\pi}{3\pi/5}\right)n = \left(\frac{10}{3}\right)n = \boxed{10}$; $n=3$

(d) $x_4(t) = 5e^{j2\pi t}$ → periodic func^n with period $= 1$

Ans.2 (a) $x(t) = \underbrace{2\cos(7t+3)}_{A} + \underbrace{3\sin(3t+4)}_{B}$

A's period $= \frac{2\pi}{7}$, B's period $= \frac{2\pi}{3}$

$\boxed{x(t) \text{ period} = 2\pi}$

~~Code is 1~~

Ans 2(b) $x[n] = 1 + \underbrace{e^{j\frac{4\pi n}{7}}}_{T_1} - \underbrace{e^{j\frac{2n}{5}}}_{T_2}$

$T_1 \rightarrow$ period $= 7$

$T_2 \rightarrow$ period $= \frac{2\pi}{2/5} = 10\pi \rightarrow$ Irrational

$\therefore$ $x[n]$ is not periodic

Ans.3 (a) $x(t) = (2+3j)\, e^{(0.5+2j)t}$

$$= \sqrt{13}\, e^{j\theta} \cdot e^{(0.5+2j)t}$$

$$= \sqrt{13}\, e^{0.5t} \cdot e^{(\theta+2t)j}$$

Amplitude $= \sqrt{13}\, e^{0.5t}$; $\tan\theta = \frac{3}{2}$

Phase $= \cos(\theta+2t)$

(b) $x(t) = (2+3j)\, e^{(0.5+2j)n}$

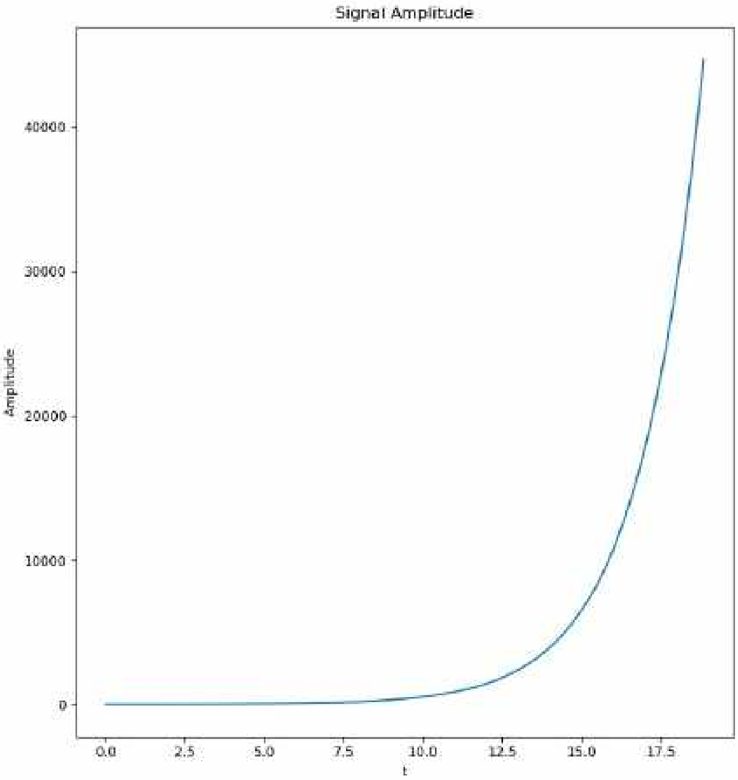
$$= \sqrt{13}\, e^{0.5n}\, e^{(\theta+2n)j}$$

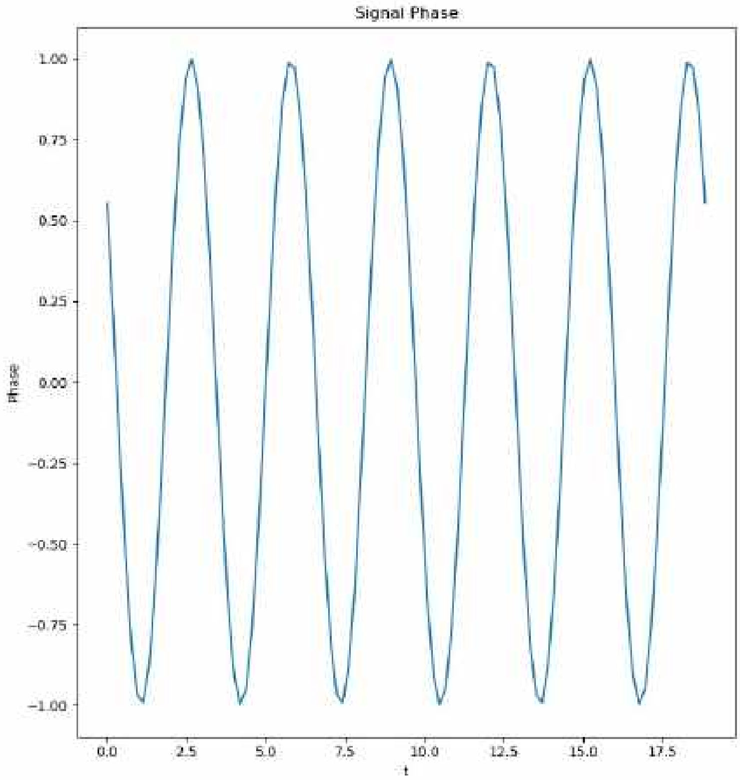
Amplitude $= \sqrt{13}\, e^{0.5n}$; $\tan\theta = \frac{3}{2}$

Phase $= \cos(\theta+2n)$

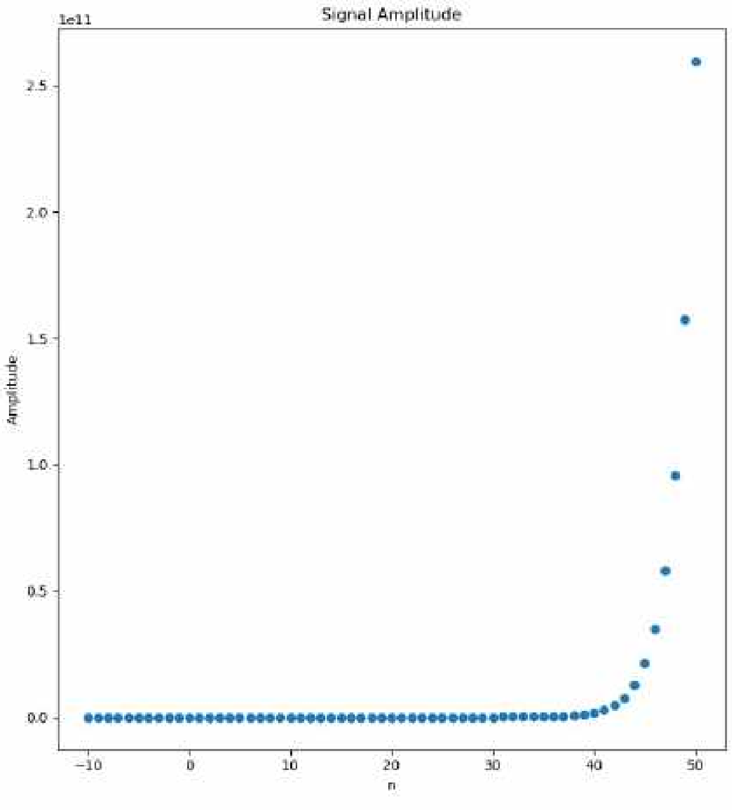
Code in Python

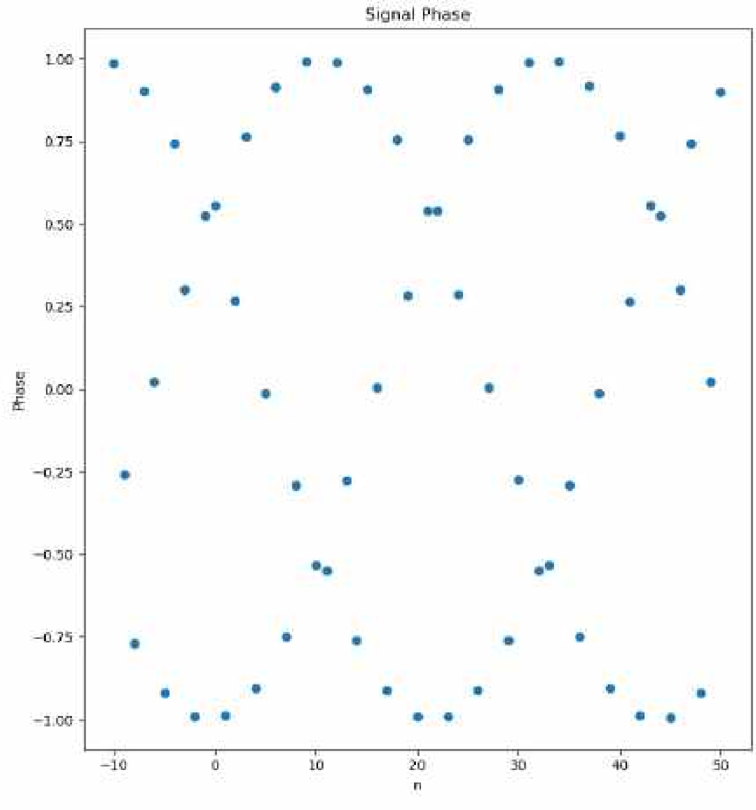
```python
## Solution 3 Part -> (a)

import numpy as np
import matplotlib.pyplot as plt

## Amplitude Plot

x = np.linspace(0,6*np.pi,num=100)
fig,ax = plt.subplots(nrows=1,ncols=2,squeeze=False,figsize=(40,40))
ax[0,0].plot(x,np.sqrt(13) * np.exp(0.5 * x))
ax[0,0].set(ylabel='Amplitude')
ax[0,0].set_title("Signal Amplitude")
ax[0,1].plot(x, np.cos(2 * x + np.arctan(1.5)))
ax[0,1].set(ylabel='Phase')
ax[0,1].set_title("Signal Phase")

for axs in ax.flat[0:2]:
    axs.set(xlabel='t')

plt.show()
```


Signal Amplitude
Amplitude
t
0
10000
20000
30000
40000
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5


Signal Phase
Phase
t
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5

```python
## Solution 3 Part -> (b)

import numpy as np
import matplotlib.pyplot as plt

## Amplitude Plot

x = np.linspace(-10,50,num=61)
fig,ax = plt.subplots(nrows=1,ncols=2,squeeze=False,figsize=(40,40))
ax[0,0].plot(x,np.sqrt(13) * np.exp(0.5 * x),'o')
ax[0,0].set(ylabel='Amplitude')
ax[0,0].set_title("Signal Amplitude")
ax[0,1].plot(x, np.cos(2 * x + np.arctan(1.5)),'o')
ax[0,1].set(ylabel='Phase')
ax[0,1].set_title("Signal Phase")

for axs in ax.flat[0:2]:
    axs.set(xlabel='n')

plt.show()
```


Signal Amplitude
1e11
2.5
2.0
1.5
1.0
0.5
0.0
Amplitude
−10
0
10
20
30
40
50
n


Signal Phase
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Phase
−10
0
10
20
30
40
50
n

Ans. 4 $x[n] = 1 - \sum_{k=-2}^{\infty} \delta[n-1-k]$

Now, $u[n] = \sum_{k=0}^{\infty} \delta[n-k]$

$\Rightarrow u[n-1] = \sum_{k=0}^{\infty} \delta[n-k-1]$

$u[n+1] = \sum_{k=0}^{\infty} \delta[n-1-(k-2)]$

$\Rightarrow u[n+1] = \cancel{\sum_{k'=-2}^{\infty} \delta[n-1-k]} \sum_{k'=-2}^{\infty} \delta[n-1-k'] \; ; \; k' = k-2$

$\therefore \boxed{x[n] = 1 - u[n+1]} = u[-2-n]$

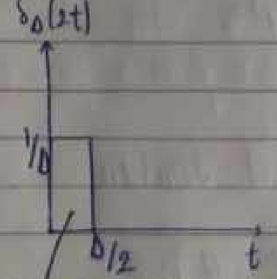
Ans. 5 Given: $\delta(t)$ ... $\lim$ ... $(1/\Delta)\, dt$; $0 \le t \le \Delta$

Ans 5 $\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$

$$\delta_\Delta(t) = \frac{d}{dt} u_\Delta(t) = \begin{cases} 1/\Delta, & 0 \le t \le \Delta \\ 0, & \text{otherwise} \end{cases}$$

$$\delta(2t) = \lim_{\Delta \to 0} \delta_\Delta(2t) = \lim_{\Delta \to 0} \begin{cases} 1/\Delta & 0 \le t \le \Delta/2 \\ 0, & \text{otherwise} \end{cases}$$

$\Rightarrow \delta_\Delta(2t) =$



Rectangle with area $\boxed{\frac{1}{2}}$

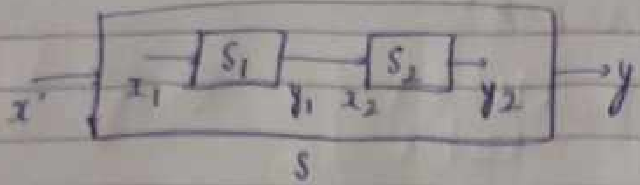
$\Rightarrow \delta(2t) =$

$= \begin{cases} \text{Pulse with } 1/2 \text{ unit area}, & t=0 \\ 0, & \text{otherwise} \end{cases}$

Arrow at $t=0$ indicates that area of pulse is concentrated at $t=0$ & $(1/2)$ represents area of pulse.

$\boxed{\text{So, } \delta(2t) \text{ represents a pulse with area } 1/2 \text{ at } t=0 \text{ \& otherwise the area} = 0.}$

Ans. 6



Given: $y_1[n] = 2x_1[x] + 4x_1[x-1]$

$y_2[n] = x_2[n-2] + 0.5\, x_2[n-3]$

(a) $x_2[n] = y_1[n] = 2x_1[n] + 4x_1[n-1]$ | $x[n] = x_1[n]$

$\therefore\ y[n] = y_2[n] = x_2[n-2] + 0.5x_2[n-3]$ |

$\Rightarrow y[n] = 2x_1[n-2] + 4x_1[n-3] + 0.5[2x_1[n-3] + 4x[n-4]]$

$\Rightarrow \boxed{y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]}$

(b) $x_1[n] = y_2[n] = x_2[n-2] + 0.5x_2[n-3]$

$\therefore\ y[n] = y_1[n] = 2x_1[n] + 4x_1[n-1]$

$\Rightarrow y[n] = 2x[n-2] + x[n-3] + 4x[n-3] + 2x[n-4]$

$\Rightarrow \boxed{y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]}$

$\therefore$ Input-Output relatⁿ is unconditional to the sequence of S_1-S_2

Ans 7 (a) $y(t) = x(\sin t)$

Now, $\sin(t) \to$ periodic with period 2π

As, ~~$y(t+(2\pi))$~~ $y(t + n(2\pi))$ depends on $x(t)$ $0 \le t \le 2\pi$, $n \in Z^+$

$\therefore$, $\boxed{\text{System is not memoryless}}$

(b) $\boxed{\text{The system is not causal.}}$

As, for $\forall\ t = t' - (2\pi)n \quad ; \quad n \in Z^+$,
output anticipates a future response at $t = t'$

(c) let $x(t) = 0 \quad \forall t$
then, $y(t) = 0 \quad \forall t$
let $x(t) = \delta(t-2)$
$y(t) = \delta(\sin(t-2)) = 0$

$\to$ System is not invertible as $y(t) = 0$ can't tell us nature of $x(t)$.

Ans 8 (a) $y(t) = x(t-4)$

$\to \boxed{x(t) = y(t+4)}$

(b) $y[n] = n\,x[n]$

$\to$ let, $x[n] = 0 \quad \forall n$
$\to y[n] = 0 \quad \forall n$

let $x[n] = \delta[n]$
$\to y[n] = 0 \quad \forall n$

Now, as different input signals correspond to same output

$\boxed{\text{System is non-invertible}}$

Ans. 8 (c) $y[n] = \left\{ \begin{array}{ll} x[n+1] & , \quad n \geq 0 \\ x[n] & , \quad n \leq -1 \end{array} \right\}$

The info^n about $x = 0$ is lost

$\therefore$ System is $\boxed{\text{non-invertible}}$