	A	B	C	D	Availability
I	14	56	48	27	13
					+
II	82	35	21	81	19
					+
III	99	31	71	63	16
					=
Demand	7 +	14 +	21 +	6 =	$\boxed{48}$

Total demand = total supply ⇒ Balanced T.P.

Apply Northwest corner rule to get initial bfs

14	56	48	27	u_1	$x_{11} = 7$
7	6				$x_{12} = 6$
82	35	21	81	u_2	$x_{22} = 8$
	8	11			$x_{23} = 11$
99	31	71	63	u_3	$x_{33} = 10$
		10	6		$x_{34} = 6$
v_1	v_2	v_3	v_4		All other = 0

find u_i, v_j's

such that $u_i + v_j = c_{ij}$ for basic cells

Suppose $u_1 = 0 \Rightarrow v_1 + 0 = 14 \Rightarrow v_1 = 14$

⇒ Also $u_1 + v_2 = 56 \Rightarrow v_2 = 56$

$u_2 + v_2 = 35 \Rightarrow u_2 = 35 - 56 = -21$

$v_3 + u_2 = 21 \Rightarrow v_3 = 21 - (-21) = +42$

$v_3 + u_3 = 71 \Rightarrow u_3 = 71 - v_3 = \cancel{71+10=81} = 29$

$v_4 + u_3 = 63 \Rightarrow v_4 = 63 - u_3 = 63 - 29 = \cancel{18}\ 37$

for non-basic cells, calculate $k_{ij} = c_{ij} - u_i - v_j$

14 (7)	56 (6)	48 6	27 -10	$u_1 = 0$
82 → 89	35 (8) -	21 + (11)	81 65	$u_2 = -21$
99 → 56	31 + $\boxed{-54}$	71 - (10)	63 (6)	$u_3 = 29$
$v_1 = 14$	$v_2 = 56$	$v_3 = 42$	$v_4 = 37$	

k_{ij} for non-basic cells.

Most negative $k_{ij} = -54 \rightarrow$ make it basic cell (ie. ~~x_3~~ x_{32} enters basis)

Make a loop (there exist unique loop) using basic cell containing the cell x_{32}.

Put +, − sign starting from x_{32}, to each of basic cells in the loop. Select the minimum allocation in the −ve cells loop (ie 8) (x_{22} becomes non-basic) and add it to cells with + sign & subtract from cells with − sign. Recalculate u_i, v_j's & k_{ij}'s
(Currently x_{32} was zero, new $x_{32} = 0+8 = 8$)

14) 7	56) 6 −	48) 6	27) + −7	$u_1 = 0$
22) 75	35) 0	21) 19	81) 68	$u_2 = -21$
99) 56	31) 8 +	71) 2	63) − 6	$u_3 = 29$
$v_1 = 14$	$v_2 = 56$	$v_3 = 42$	$v_4 = 34$	

x_{14} enters basis with $x_{14} = 6$, Either of x_{12} or x_{34} leaves the basis. As cost associated with x_{12} is less than x_{34}, so we move x_{12} out of basis. Next table is given as :-

14) 7	56) 61	48) 13	27) 6	$u_1 = 0$
82) 82	35) 54	21) 19	81) 68	$u_2 = -14$
99) 49	31) 14	71) 2	63) 0	$u_3 = 36$
$v_1 = 14$	$v_2 = -5$	$v_3 = 35$	$v_4 = 27$	

As all $x_{ij} \geq 0$, so this is optimal table.

Optimal solution / assignment is

I → A : 7 ores

I → D ; 6 ores

II → C : 19 ores.

III → B : 14 ores

IV → C : 2 ores.

Total cost = $14 \times 7 + 27 \times 6 + \cancel{35 \times 54} + 21 \times 19$
$+ 31 \times 14 + 71 \times 2 = 1235$

2/

(8) 150	(-6)	(-5)	150	-3400
(-6) 50	(-6) 100	(-6)	150	
(-10)	(-8) 100	(-4) 50	150	
(9)	(-6)	(-4) 150	150	
200	200	200	600	

Total requirement = 200 + 200 + 200 = 600

Total demand = 150 + 150 + 150 + 150 = 600

∴ Problem is balanced. & b.f.s by northwest corner rule is given above. Calculate u_i, v_j, x_{ij}'s.

(-8) 150	(-6) 2	(-5) -1	0
(-6) 50	(-6) 100	(-6) $\boxed{-4}$	2
(-10) -2	(-8) 100	(-4) 50	0
(-8) 0	(-6) 2	(-4) 150	0
-8	-8	-4	

(-8) 150	(-6) 2	(-5) 2	0
(-6) 50	(-6) 50	(-6) 50	2
(-10) -2	(-8) 150	(-4) 4	0
(-8) $\boxed{-4}$	(-6) -2	(-4) 150	4
-8	-8	-8	

5

			(A)	
	-8 · 150	-6 · -2	-5 · -1	0
	-6 · 4	-6 · (-) 50	-6 · (+) 100	-2
	-10 · 2	-8 · 150	-4 · 4	-4
	-8 · 50	-6 · (+) $\boxed{-2}$	-4 · (-) 100	0
	-8	-4	-4	

→ Alternate Solution

-8 · 150	-6 · $\boxed{-4}$	-5 · -1	0
-6 · 4	-6 · -2	-6 · 150	-2
-10 · 4	-8 · 150	-4 · 6	-6
-8 · 50	-6 · 50	-4 · 50	0
-8	-2	-4	

-8 · 100	-6 · 50	-5 · $\boxed{-1}$	0
-6 · 4	-6 · 2	-6 · 150	-2
-10 · 0	-8 · 150	-4 · 2	-2
-8 · 100	-6 · 0	-4 · 50	0
-8	-6	-4	

-8 · 50	-6 · 50	-5 · 50	0
-6 · 3	-6 · 1	-6 · 150	-1
-10 · 0	-8 · 150	-4 · 3	-2
-8 · 150	-6 · 0	-4 · 1	0
-8	-6	-5	

∴ Total 400 + 300 + 250 + 900 + 1200 + 1200

= 4250

Alternate solution : Proceed from (A)

(-8) (-) 150	(-6) $\boxed{-2}$	(-5) -1
(-6) 4	(-6) 50 (-)	(-6) (+) 100
(-10) 2	(-8) 150	(-4) 4
(-8) (+) 50	(-6) -2	(-4) (-) 100

(-8) (-) 100	(-6) 50	(-5) (+) $\boxed{-1}$	0
(-6) 4	(-6) 2	(-6) 150	-2
(-10) 0	(-8) 150	(-4) 2	-2
(-8) (+) 100	(-6) 0	(-4) (-) 50	0
-8	-6	-4	

(-8) 50	(-6) 50	(-5) 50	0
(-6) 3	(-6) 1	(-6) 150	-1
(-10) 0	(-8) 150	(-4) 5	-2
(-8) 150	(-6) 0	(-4) 1	0
-8	-6	-5	

∴ Solution is same as previous one.

$B_1 \rightarrow T_1$: 50 | $B_2 \rightarrow T_3$: 50

$B_1 \rightarrow T_2$: 50 | $B_3 \rightarrow T_2$: 150

$B_1 \rightarrow T_3$: 50 | $B_4 \rightarrow T_1$: 150.

3 Total requirement = 4+4+6+8+8 = 30

Total capacities = 8+12+14 = 34 ⇒ Problem is unbalanced.

Add dummy furnance M_6 with requirement 4 quintals. and associate a high cost M with F_1M_6, F_2M_6 & F_3M_6. and find initial b.f.s

M_1	M_2	M_3	M_4	M_5	M_6	
4) 4	2) 4	3) 0	2)	6)	M)	8
5)	4)	5) 6	2) 6	1)	M)	12
6)	5)	4)	7) 2	3) 8	M) 4	14
4	4	6	8	8	4	

find u_i, v_j & k_{ij}'s.

						u_i↓
4) 4	2) 4	3) 0	2) 2	6) 10	M) 7	0
5) −1	4) 0	5) 6 −	2) + 6	1) 3	M) 5	2
6) −5	5) −4	4) + −6	7) − 2	3) 8	M) 4	7
v_j → 4	2	3	0	−4	M−7	

4) 0 −	2) 4	3) + 0	2) 2	6) 4	M) 1	0
5) + −1	4) 0	5) − 4	2) 8	1) −3	M) −1	2
6) 1	5) 2	4) 2	7) 6	3) 8	M) 4	1
4	2	3	0	2	M−1	

Break Tie : enter x_{21}

4) (0)	2) (4)	3) (4)	2) 1	6) 4	M) 1	0
5) (4)	4) 1	5) (8)	2) (8)	1) -2	M) 0	1
6) 1	5) 2	4) (2)	7) 5	3) (8)	M) (4)	1
4	2	3	1	2	M-1	

4) (4)	2) (4)	3) (0)	2) -1	6) 4	M) 1	0
5) 2	4) 3	5) 3	2) (8)	1) (4)	M) 2	-1
6) 1	5) 2	4) (6)	7) 3	3) (4)	M) (4)	1
4	2	3	3	2	M-1	

4) (4)	2) (4)	3) 1	2) (0)	6) 5	M) 2	0
5) 1	4) 2	5) 3	2) (8)	1) (4)	M) 2	0
6) 0	5) 1	4) (6)	7) 3	3) (4)	M) (4)	2
4	2	~~3~~ 2	2	1	M-2	

$\therefore Z = 4\times4 + 2\times4 + \overset{2\times0}{\cancel{3\times3}} + 2\times8 + 1\times4 + 4\times6 + 3\times4 = 80$

Also, the third furnance will run ~~6~~ 4 quintals below it max. capacity.

optimal assignment:

$F_1 \to M_1 : 4$

$F_1 \to M_2 : 4$

$F_2 \to M_4 : 8$

$F_2 \to M_5 : 4$

$F_3 \to M_3 : 6$

$F_3 \to M_5 : 4$

	D_1	D_2	D_3	D_4	D_5	u_i ↓
O_1	20) 30	19) 10	14) 2	21) 7	16) −M+22	0
O_2	15) −6	20) 30	13) 30	19) 4	16) −M+21	1
O_3	18) −8	15) −10	18) 20	20) 40	M) 10	6
O_4	M) M−26	M) M−25	M) M−18	M) M−20	M) 50	6
v_j →	20	19	12	14	M−6	

To Balance the T.P add origin O_4 with availability 50, and high cost M and find initial bfs.

	20) 30	19) 10	14) 2	21) 7	16) 1	0
	15) −6	20) 30	13) 20	19) 4	16) 10	1
	18) −8	15) −10	18) 30	20) 40	M) M−21	6
	M) −5	M) −4	M) 3	M) 1	M) 50	M−15
	20	19	12	14	15	

	20) 0	19) 40	14) 14	21) 19	16) 13	0
	15) −18	20) −12	13) 50	19) 4	16) 10	13
	18) 30	15) −22	18) 0	20) 40	M) M−21	18
	M) −17	M) −16	M) 3	M) 1	M) 50	M−3
	20	19	0	2	3	

20) 30	19) 10	14) -8	21) -3	16) -9	0
15) 4	20) 10	13) 50	19) 4	15) 10	-9
18) 2	15) 30	18) 0	20) 40	M) M-21	-4
M) 5	M) 6	M) 3	M) 1	M) 50	M-25
20	19	22	24	25	

20) 30	19) 10	14) 1	21) -3	16) 0	0
15) -5	20) 1	13) 50	19) -5	16) 10	0
18) 2	15) 30	18)	20) 40	M) M-12	-4
M) -4	M) -3	M) 3	M) -8	M) 50	M-16
20	19	13	24	16	

20) 30	19) 8	14) 1	21) 5	16) 10	0
15) -5	20) 9	13) 50	19) 3	16) 10	0
18) -6	15) 40	18) 1	20) 30	M) M-20	4
M) -4	M) 5	M) 3	M) 10	M) 40	M-16
20	11	13	16	16	

20) 6	19) 8	14) 1	21) 5	16) 40	0
15) 1	20) 9	13) 50	19) 5	16) 10	0
18) 30	15) 40	18) 1	20) 0	M) M-20	4
M) 2	M) 5	M) 3	M) 40	M) 10	M-16
14	11	13	16	16	

$Z = 16\times40 + 13\times50 + 16\times10 + 18\times30 + 15\times40 + 20\times0$

$= 2590.$

Optimal Assi

1→5 : 40

2→3 : 50

2→5 : 10

3→1 : 30

3→2 : 40

D_4 will receive 40 units less than requirement & D_5 will receive 10 units than its requirement.

(5) Problem is balanced.

Add 2 basic cells by hidden trial method with 0 assignment. To avoid cycling we may let the assignment be ε instead of 0.

(4) 2	(5) 30	(2) 1	0
(4) 40	(1) −6	(3) ε	2
(3) ε	(6) 20	(2) 0	1
(2) −6	(3) −8	(7) 60	6
2	5	1	

(4) −6	(5) 30	(2) −7	0
(4) 20	(1) 2	(3) 20	−6
(3) 20	(6) 8	(2) 0	−7
(2) −6	(3) 20	(7) 40	−2
10	5	9	

(4) 1	(5) 7	(2) 30	0
(4) 20	(1) 2	(3) 20	1
(3) 20	(6) 8	(2) 0	0
(2) −6	(3) 50	(7) 10	5
3	−2	2	

(4) 1	(5) 1	(2) 30	0
(4) 10	(1) −4	(3) 30	1
(3) 20	(6) 2	(2) 0	0
(2) 10	(3) 50	(7) 6	−1
3	4	2	

(4) 4	(5) 5	(2) 30	0
(4) 3	(1) 10	(3) 30	1
(3) 20	(6) 3	(2) −3	3
(2) 20	(3) 40	(7) 3	2
0	0	2	

(4) 4	(5) 5	(2) 30	0
(4) 3	(1) 30	(3) 10	1
(3) 5	(6) 6	(2) 20	0
(2) 40	(3) 20	(7) 3	2
0	0	2	

∴ $Z = 2 \times 30 + 1 \times 30 + 3 \times 10 + 2 \times 20 + 2 \times 40 + 3 \times 20 = 300$

6

Step 1 : Subtract minimum of each row from respective row to get (Row reduction)

12

$$\begin{bmatrix} 4 & 2 & 3 & 6 & 0 \\ 0 & 1 & 5 & 4 & 3 \\ 3 & 0 & 4 & 1 & 3 \\ 2 & 7 & 0 & 1 & 2 \\ 3 & 1 & 4 & 6 & 0 \end{bmatrix}$$

Step 2 : Subtract minimum of each column from respective column. (Column reduction)

$$\begin{bmatrix} 4 & 2 & 3 & 5 & 0 \\ 0 & 1 & 5 & 3 & 3 \\ 3 & 0 & 4 & 0 & 3 \\ 2 & 7 & 0 & 0 & 2 \\ 3 & 1 & 4 & 5 & 0 \end{bmatrix}$$

~~Step 3 Cover all the zeros with minimum no. of lines.~~

$$\begin{bmatrix} 4 & 2 & 3 & 5 & 0 \\ 0 & 1 & 5 & 3 & 3 \\ 3 & 0 & 4 & 0 & 3 \\ 2 & 7 & 0 & 0 & 2 \\ 3 & 1 & 4 & 5 & 2 \end{bmatrix}$$

Step 3 : Assign zeroes.

$$\begin{bmatrix} 4 & 2 & 3 & 5 & \boxed{0} \\ \boxed{0} & 1 & 5 & 3 & 3 \\ 3 & \boxed{0} & 4 & 0 & 3 \\ 2 & 7 & \boxed{0} & 0 & 2 \\ 3 & 1 & 4 & 5 & 0 \end{bmatrix} \begin{matrix} \checkmark \\ \\ \\ \\ \checkmark \end{matrix}$$

$\downarrow$ has no assignments.

Mark Tick as follows :

3.1. Tick all unassigned rows. (ie, row 5)

3.2. Tick all columns corresponding to zeroes in rows ticked in step 3.1 (ie column 5)

3.3 ~~Pick a tick~~ in column of step 3.2, put a tick to corresponding assigned ~~rows~~ zeroes rows. (ie row 1).

Now strike all unticked rows and ticked columns.

Step 4
Now $\theta =$ min of unstriked values $= 1$

- Add θ to intersection of strikes
- Subtract it from unstriked entries
- keep the entries left as they are.

Then assign zeroes.

$$\begin{bmatrix} 3 & 1 & 2 & 4 & \boxed{0} \\ \boxed{0} & 1 & 5 & 3 & 4 \\ 3 & \cancel{\boxed{0}} & 4 & \boxed{0} & 4 \\ 2 & 7 & \boxed{0} & 0 & 3 \\ 2 & \boxed{0} & 3 & 4 & 0 \end{bmatrix}$$

Assign 1 to 5
2 to 1
3 to 4
4 to 3
5 to 2.

& Min cost $= 1 + 2 + 2 + 1 + 2 = 8$

7. To convert max to min problem, subtract all elements from the highest element (ie 42) to get

$$\begin{bmatrix} 6 & 7 & 14 & 21 \\ 12 & 17 & 22 & 27 \\ 12 & 17 & 22 & 27 \\ 18 & 22 & 26 & 30 \end{bmatrix}$$

Step 1. Row Reduction

$$\begin{bmatrix} 0 & 7 & 14 & 21 \\ 0 & 5 & 10 & 15 \\ 0 & 5 & 10 & 15 \\ 0 & 4 & 8 & 12 \end{bmatrix}$$

Step 2 Column reduction

$$\begin{bmatrix} 0 & 3 & 6 & 9 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 3 assign zeroes.

$$\begin{bmatrix} \boxed{0} & 3 & 6 & 9 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & \boxed{0} & 0 & 0 \end{bmatrix} \begin{matrix} \checkmark \\ \checkmark \\ \checkmark \\ \\ \end{matrix}$$

$\checkmark$

$\theta = 1$

Step 4
Add/subtract θ as needed

$$\begin{bmatrix} 0 & 2 & 5 & 8 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Again assign zeroes.

$$\begin{bmatrix} \boxed{0} & 2 & 5 & 8 \\ 0 & \boxed{0} & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 0 & \boxed{0} & 0 \end{bmatrix} \quad \theta = 1$$

Step 5
Add/subtract θ as needed. & assign zeroes

$$\begin{bmatrix} \boxed{0} & 2 & 4 & 7 \\ 0 & \boxed{0} & 0 & 1 \\ 0 & 0 & \boxed{0} & 1 \\ 2 & 1 & 0 & \boxed{0} \end{bmatrix} \text{ or } \begin{bmatrix} \boxed{0} & 2 & 4 & 7 \\ 0 & 0 & \boxed{0} & 1 \\ 0 & \boxed{0} & 0 & 1 \\ 2 & 1 & 0 & \boxed{0} \end{bmatrix}$$

∴ optimal assignment: A → I, B → II, C → III, D → IV
or A → I, B → III, C → II, D → IV

$Z = 42 + 20 + 25 + 12 = 99$

(b) – D cannot be assigned to ~~A~~ IV

$$\begin{bmatrix} 0 & 7 & 14 & 21 \\ 12 & 17 & 22 & 27 \\ 12 & 17 & 22 & 27 \\ 18 & 22 & 26 & \infty \end{bmatrix}$$

Step 1
Row reduction

$$\begin{bmatrix} 0 & 7 & 14 & 21 \\ 0 & 5 & 10 & 15 \\ 0 & 5 & 10 & 15 \\ 0 & 4 & 8 & \infty \end{bmatrix} \Rightarrow \text{Step 2} \begin{bmatrix} \boxed{0} & 3 & 6 & 6 \\ 0 & 1 & 2 & \boxed{0} \\ 0 & 1 & 2 & 0 \\ 0 & \boxed{0} & 0 & \infty \end{bmatrix}$$

$\theta = 1$ ⇒ Add/subtract θ & assign zeroes.

$$\Rightarrow \begin{bmatrix} \boxed{0} & 2 & 5 & 6 \\ 0 & \boxed{0} & 1 & 0 \\ 0 & 0 & 1 & \boxed{0} \\ 1 & 0 & \boxed{0} & \infty \end{bmatrix} \text{ or } \begin{bmatrix} \boxed{0} & 2 & 5 & 6 \\ 0 & 0 & 1 & \boxed{0} \\ 0 & \boxed{0} & 1 & 0 \\ 1 & 0 & \boxed{0} & \infty \end{bmatrix}$$

A → I
B → II
C → IV
D → III

$Z = 42 + 25 + 15 + 16$
$= 98$

A → I
B → IV
C → II
D → III

$Z = 42 + 15 + 25 + 16$
$= 98$

Scanned with CamScanner

8 Subtract from highest element.

$$\begin{bmatrix} 9 & 3 & 1 & 12 & 1 \\ 1 & 17 & 12 & 20 & 5 \\ 0 & 14 & 8 & 11 & 4 \\ 19 & 3 & 0 & 5 & 5 \\ 12 & 8 & 1 & 10 & 2 \end{bmatrix}$$

Row reduction

$$\begin{bmatrix} 8 & 2 & 0 & 11 & 0 \\ 0 & 16 & 11 & 19 & 4 \\ 0 & 14 & 8 & 11 & 4 \\ 19 & 3 & 0 & 5 & 5 \\ 11 & 7 & 0 & 9 & 1 \end{bmatrix} \Rightarrow$$

column reduction

$$\begin{bmatrix} 8 & 0 & 0 & 6 & 0 \\ 0 & 14 & 11 & 14 & 4 \\ 0 & 12 & 8 & 6 & 4 \\ 19 & 1 & 0 & 0 & 5 \\ 11 & 5 & 0 & 4 & 1 \end{bmatrix}$$

Assign zeroes

$$\begin{bmatrix} 8 & \boxed{0} & 0 & 6 & 0 \\ 0 & 14 & 11 & 14 & 4 \\ \boxed{0} & 12 & 8 & 6 & 4 \\ 19 & 1 & 0 & \boxed{0} & 5 \\ 11 & 5 & \boxed{0} & 4 & 1 \end{bmatrix} \begin{matrix} \\ \checkmark \\ \checkmark \\ \\ \\ \end{matrix}$$

$\theta = 4$

Add/subtract θ as required & assign zeroes.

$$\begin{bmatrix} 12 & \boxed{0} & 0 & 6 & 0 \\ \boxed{0} & 10 & 7 & 10 & 0 \\ 0 & 8 & 4 & 2 & \boxed{0} \\ 23 & 1 & 0 & \boxed{0} & 5 \\ 15 & 5 & \boxed{0} & 4 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 12 & \boxed{0} & 0 & 6 & 0 \\ 0 & 10 & 7 & 10 & \boxed{0} \\ \boxed{0} & 8 & 4 & 2 & 0 \\ 23 & 1 & 0 & \boxed{0} & 5 \\ 15 & 5 & \boxed{0} & 4 & 1 \end{bmatrix}$$

Max profit $= 38 + 40 + 37 + 36 + 40 = 191$

b) As ~~III~~ fourth job is not assigned to II machine, so it does affect the solution.

7 To balance the problem, we add dummy subordinate IV

$$\begin{bmatrix} 9 & 26 & 15 & 0 \\ 13 & 27 & 6 & 0 \\ 35 & 20 & 15 & 0 \\ 18 & 30 & 20 & 0 \end{bmatrix} \overset{\text{Row/Col. Reduction}}{\sim} \begin{bmatrix} \boxed{0} & 6 & 9 & 0 \\ 4 & 7 & \boxed{0} & 0 \\ 26 & \boxed{0} & 9 & 0 \\ 9 & 10 & 14 & \boxed{0} \end{bmatrix}$$

$\therefore$ Assign I → A, II → C, III → B & D remains unassigned.

$Z = 9 + 6 + 20 = 35.$