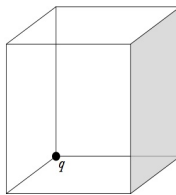


Department of Physics
Electrodynamics and Optics (PH005)

Tutorial 1 (July 30 , 2019)

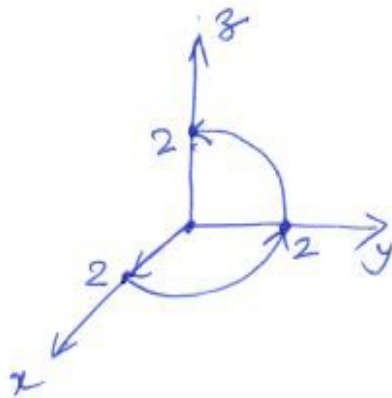
- (1) For an observer moving with velocity $v_1 = 20$ km/h in the positive x-direction, the wave equation is $y = A \sin k(x - vt)$ and for another observer moving along the same direction with velocity $v_2 = 40$ km/h, the equation is $y' = A \sin k(x' + vt)$. Calculate the actual velocity of the wave.
- (2) Find the gradients of following functions:
a) $f(x, y, z) = x^2 + y^3 + z^4$
b) $f(x, y, z) = x^2 y^3 z^4$
c) $f(x, y, z) = e^x \sin(y) \ln(z)$
(Problem: 1.11 Griffith)
- (3) Prove that $\nabla \cdot (\nabla \times \mathbf{V}) = 0$. Check it for $f(x) = xy\hat{i} + 2yz\hat{j} + 3zx\hat{k}$
b) Prove that $\nabla \times (\nabla f) = 0$.
- (4) Which one can not be an electrostatic field?
(a) $\mathbf{E}_1 = (y\hat{i} + z^3\hat{j} + x^2\hat{k})$
(b) $\mathbf{E}_2 = (x^2\hat{i} + 2yz\hat{j} + y^2\hat{k})$
- (5) Calculate the flux of \mathbf{E} through shaded region due to charge q at the corner of the cube shown in following figure



(Problem: 3.2 Griffith)

- (6) Using Gauss's law, find the electric field inside a uniformly charged sphere at radial distance r_1 from origin ($r_1 < R$). R is radius of the sphere. Volume charge density of the sphere is ρ . Also find electric field outside the sphere at radial distance r_2 ($r_2 > R$). Plot $|\mathbf{E}|$ as a function of r .
- (7) Is curl of a vector field is always orthogonal to the vector field. Justify your argument with example.

- (8) Calculate the divergence and curl of the following vector fields;
- (a) $\mathbf{A}(\mathbf{r}) = -\frac{y}{x^2+y^2}\hat{i} + \frac{x}{x^2+y^2}\hat{j}$
- (b) $\mathbf{A}(\mathbf{r}) = -\frac{x}{x^2+y^2}\hat{i} + \frac{y}{x^2+y^2}\hat{j}$
- (b) Let C be a circle of unit radius in the x - y plane, enclosing the unit surface $a=S$ having a vector field $\mathbf{A} = y\hat{i}$ Calculate the line integral of \mathbf{A} around C and (ii) the flux of $\nabla \times \mathbf{A}$ through S .
- (9) Calculate the gradient and Laplacian of the function $T = r(\cos\theta + \sin\theta\cos\phi)$. Check the Laplacian by converting to Cartesian coordinates. Test the gradient theorem for this function using the path shown in following figure



(Problem: 1.39 Griffith)

- 10 A vector field is defined by $\mathbf{A}(x, y) = xy^2\hat{i} + (x+y)\hat{j}$ and surface S is surrounded by the curves $y = x^2$ and $y = x$ Calculate $\iint_S (\nabla \times \mathbf{A}) \cdot d\mathbf{s}$ also repeat the calculation for surface bound by the curves $y = x^2$, $y = -x^2$ and $x = 1$.
