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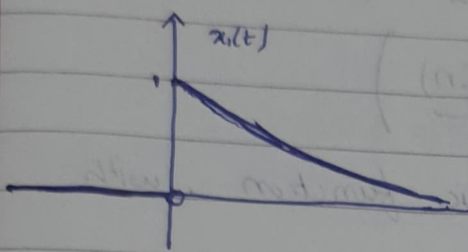
① (a) We know that for signal $x(t)$,

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

Given $x_1(t) = \begin{cases} e^{-2t} & ; t \geq 0 \\ 0 & ; \text{otherwise} \end{cases}$



$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x_1(t)|^2 dt$$

$$= \lim_{T\to\infty} \left(\int_{-T}^{0} |x_1(t)|^2 dt + \int_{0}^{T} |x_1(t)|^2 dt \right)$$

$$= \lim_{T\to\infty} \left(0 + \int_0^T (e^{-2t})^2 dt \right)$$

$$= \lim_{T\to\infty} \int_0^T e^{-4t} dt$$

$$= \lim_{T\to\infty} \frac{1}{-4} \left[e^{-4t} \right]_0^T = \lim_{T\to\infty} \frac{\left[e^{-4T} - e^{-4\cdot 0} \right]}{-4}$$

$$= \lim_{T\to\infty} \frac{\left[1 - e^{-4T} \right]}{4}$$

$$= \frac{1}{4} - 0 \quad \left\{ \text{as } \lim_{T\to\infty} e^{-4T} = 0 \right\}$$

$$\therefore E_\infty = \frac{1}{4}$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x_1(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \frac{\left[1 - e^{-4T}\right]}{4} = \lim_{T\to\infty} \frac{1}{8T} - \lim_{T\to\infty} \frac{e^{-4T}}{8T}$$

$$= \lim_{T\to\infty} \left(\frac{1}{8T} - \frac{1}{8Te^{4T}} \right) = 0 \quad \left\{ \lim_{T\to\infty} \frac{(1 - e^{-4T})}{8T} = \frac{0}{\infty} = 0 \right\}$$

$$\boxed{\therefore P_\infty = 0 \text{ and } E_\infty = 1/4}$$

1 (b) $x_2[n] = \cos\left[\frac{\pi}{4}n\right]$, $-\infty < n < \infty$

$$E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} |x_2[n]|^2$$

$$= \lim_{N\to\infty} \sum_{n=-N}^{N} \cos^2\left(\frac{\pi}{4}n\right)$$

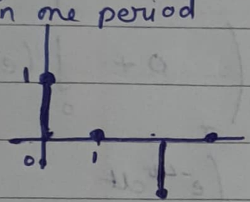
$$= \lim_{N\to\infty} \sum_{n=-N}^{N} \frac{1+\cos\left(2\cdot\frac{\pi}{4}n\right)}{2} \qquad \left\{\cos^2\theta = \frac{1+\cos 2\theta}{2}\right\}$$

$$= \lim_{N\to\infty}\left(\sum_{-N}^{N}\frac{1}{2} + \sum_{-N}^{N}\frac{1}{2}\cos\left(\frac{\pi}{2}n\right)\right)$$

$$= \lim_{N\to\infty}\left(\frac{2N+1}{2} + \sum_{-N}^{N}\frac{\cos\left(\frac{\pi}{2}n\right)}{2}\right)$$

$\cos\left(\frac{\pi}{2}n\right)$ is a periodic function, with a period of 4.

In one period



$\Sigma \cos\left(\frac{\pi}{2}n\right)$ in one period $= 0$ $\{\underbrace{1+0+-1+0}_{=0}\}$

$\therefore$ sum of it for all intervals would be 0

$\therefore \lim_{N\to\infty}\sum_{n=-N}^{N}\cos\left(\frac{\pi n}{2}\right) = 0$

$$\therefore E_\infty = \lim_{N\to\infty}\left(\frac{2N+1}{2}\right) + 0 = \lim_{N\to\infty} N + \frac{1}{2} = \infty$$

$\therefore$ it is an infinite energy signal.

$$P_\infty = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}|x[n]|^2$$

$$= \lim_{N\to\infty}\frac{1}{2N+1}\left[N+\frac{1}{2}\right]$$

$$= \lim_{N\to\infty}\frac{N+\frac{1}{2}}{2N+1} = \lim_{N\to\infty}\frac{N}{2N} = \frac{1}{2}$$

$$\therefore \boxed{P_\infty = \frac{1}{2} \text{ and } E_\infty = \infty}$$

The above two ques. are consistent with fact that E_∞ can only be finite if $P_\infty = 0$ & vice versa.

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(2) (a) $x_1(n) = \begin{cases} 2 & ; \quad 0 \le n \le 6 \\ 0 & ; \quad \text{otherwise} \end{cases}$

$\boxed{OD\{x_1[n]\} = \frac{1}{2}[x_1[n] - x_1[-n]]}$

$\boxed{EV\{x_1(n)\} = \frac{1}{2}[x_1[n] + x_1[-n]]}$

(i) → if $0 < n \le 6$

$x_1[n] = 2; \quad -n \in [-6, 0) \Rightarrow x_1(-n) = 0$

$\therefore EV\{x_1(n)\}$ if $0 < n \le 6 = \frac{1}{2}[2+0] = 1$

$\therefore OD\{x_1(n)\}$ if $0 < n \le 6 = \frac{1}{2}[2-0] = 1$

(ii) if $n = 0$

$x_1[n] = x_1[0] = 2, \quad x_1[-n] = x_1[0] = 2$

$\therefore EV\{x_1(n)\} = \frac{1}{2}[2+2] = 2$

$OD\{x_1(n)\} = \frac{1}{2}[2-2] = 0$

(iii) if $-6 \le n < 0$

$\Rightarrow x_1[n] = 0, \quad -n \in (0, 6] \Rightarrow x_1[-n] = 2$

$\therefore EV\{x_1(n)\} = \frac{1}{2}[0+2] = 1$

$OD\{x_1(n)\} = \frac{1}{2}[0-2] = -1$

(iv) Otherwise i.e. $n < -6 \vee n > 6$

$\Rightarrow x_1[n] = 0, \quad -n \in (-\infty, -6) \cup (6, \infty)$

$\Rightarrow x_1[-n] = 0$

$\therefore OD\{x_1(n)\} = \frac{1}{2}[0-0] = 0$

$EV\{x_1(n)\} = \frac{1}{2}[0+0] = 0$

∴ EVEN DECOMPOSITION

$$Ev\{x_1(n)\} = \begin{cases} 0 & , n < -6 \\ 1 & , -6 \le n < 0 \\ 2 & , n = 0 \\ 1 & , 0 < n \le 6 \\ 0 & , n > 6 \end{cases}$$

$$OD\{x_1(n)\} = \begin{cases} 0 & , n < -6 \\ -1 & , -6 \le n < 0 \\ 0 & , n = 0 \\ 1 & , 0 < n \le 6 \\ 0 & , n > 6 \end{cases}$$

(b) $x_2(t) = \sin\left(\frac{t}{2}\right) \quad , -\infty < t < \infty$

$$Ev\{x_2(t)\} = \frac{1}{2}\left[x_2(t) + x_2(-t)\right]$$

$$= \frac{1}{2}\left[\sin\left(\frac{t}{2}\right) + \sin\left(\frac{-t}{2}\right)\right]$$

$$= \frac{1}{2}\left[\sin\left(\frac{t}{2}\right) - \sin\left(\frac{t}{2}\right)\right] \qquad \left\{ \sin(-\theta) = -\sin(\theta) \right\}$$

$$\boxed{\therefore Ev\{x_2(t)\} = 0}$$

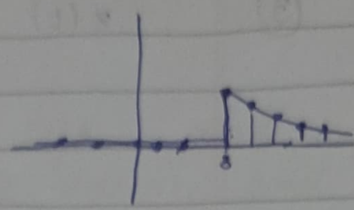
$$OD\{x_2(t)\} = \frac{1}{2}\left[x_2(t) - x_2(-t)\right]$$

$$= \frac{1}{2}\left[\sin\left(\frac{t}{2}\right) - \sin\left(-\frac{t}{2}\right)\right]$$

$$= \frac{1}{2}\left[2\sin\left(\frac{t}{2}\right)\right]$$

$$\therefore \boxed{OD\{x_2(t)\} = \sin\left(\frac{t}{2}\right)}$$

(c) $x_3(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & n \geq 3 \\ 0, & \text{otherwise} \end{cases}$



(i) if $n \geq 3$, $-n \leq -3$

$\therefore x_3(n) = \left(\frac{1}{2}\right)^n$, $x_3(-n) = 0$

$\therefore Ev\{x_3(n)\} = \frac{1}{2}\left[\left(\frac{1}{2}\right)^n + 0\right] = \left(\frac{1}{2}\right)^{n+1}$

$OD\{x_3(n)\} = \frac{1}{2}\left[\left(\frac{1}{2}\right)^n - 0\right] = \left(\frac{1}{2}\right)^{n+1}$

(ii) if $-3 < n < 3$, $-n \in (-3, 3)$

$\therefore x_3(n) = 0$ and $x_4(-n) = 0$

$\therefore Ev\{x_3(n)\} = \frac{1}{2}[0+0] = 0$

$OD\{x_3(n)\} = \frac{1}{2}[0-0] = 0$

(iii) if $n \leq -3$, $\Rightarrow -n \geq 3$

$x_3(n) = 0$, $x_3(-n) = \left(\frac{1}{2}\right)^{-n}$

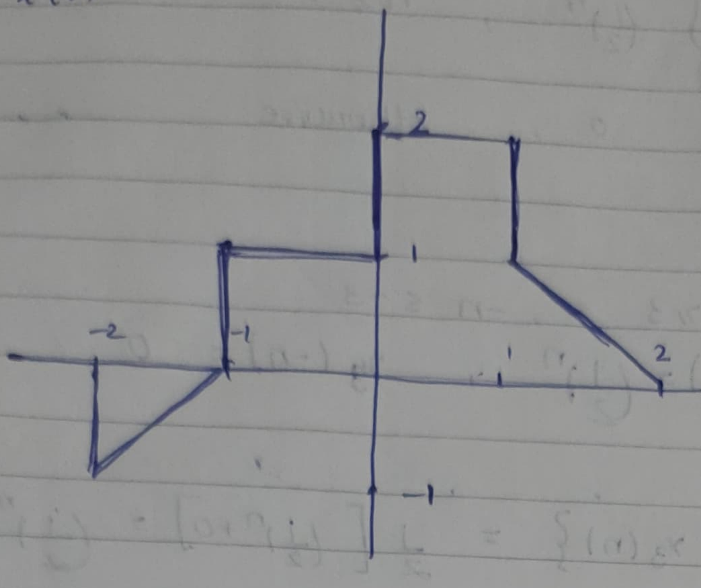
$Ev\{x_3(n)\} = \frac{1}{2}\left[0 + \left(\frac{1}{2}\right)^{-n}\right] = \frac{1}{2^{1-n}} = \left(\frac{1}{2}\right)^{1-n}$

$OD\{x_3(n)\} = \frac{1}{2}\left[0 - \left(\frac{1}{2}\right)^{-n}\right] = -\left(\frac{1}{2}\right)^{1-n}$

$$\boxed{Ev\{x_3(n)\} = \begin{cases} \left(\frac{1}{2}\right)^{1-n}, & n \leq -3 \\ 0, & -3 < n < 3 \\ \left(\frac{1}{2}\right)^{n+1}, & n \geq 3 \end{cases}}$$

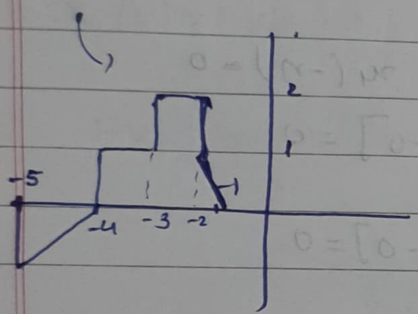
$$\boxed{OD\{x_3(n)\} = \begin{cases} -\left(\frac{1}{2}\right)^{1-n}, & n \leq -3 \\ 0, & -3 < n < 3 \\ \left(\frac{1}{2}\right)^{1+n}, & n \geq 3 \end{cases}}$$

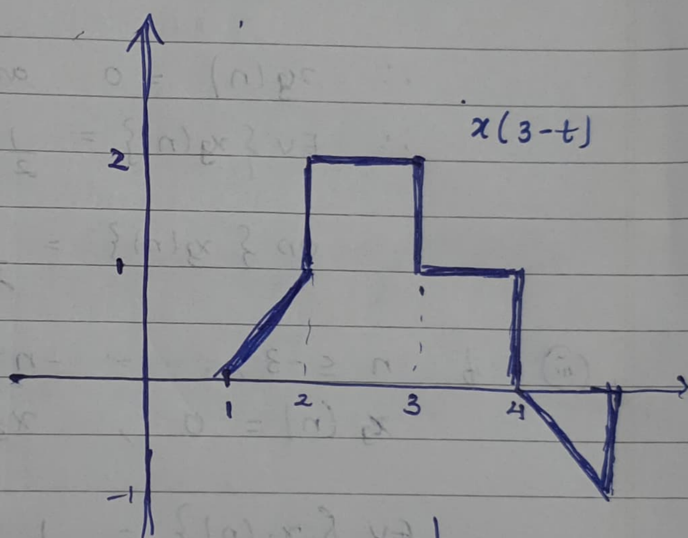
(3) $x(t)$



(a) $x(3-t) = x(-t+3)$

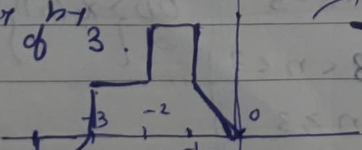
First we shift by 3 to left and then reflect about 'y' axis'

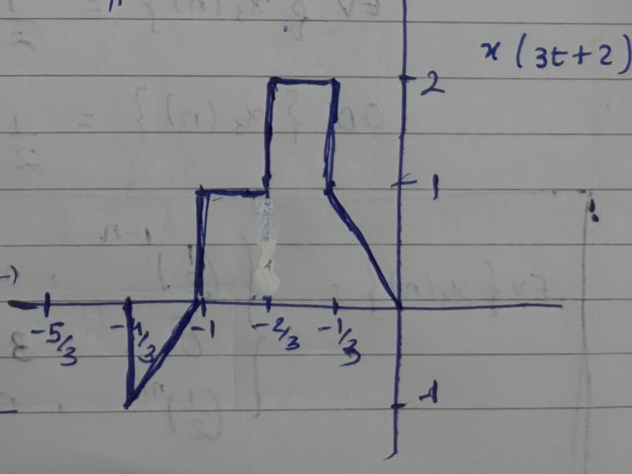




(b) $x(3t+2)$

First we shift it by 2 to left & then compress it linearly by a factor of 3.

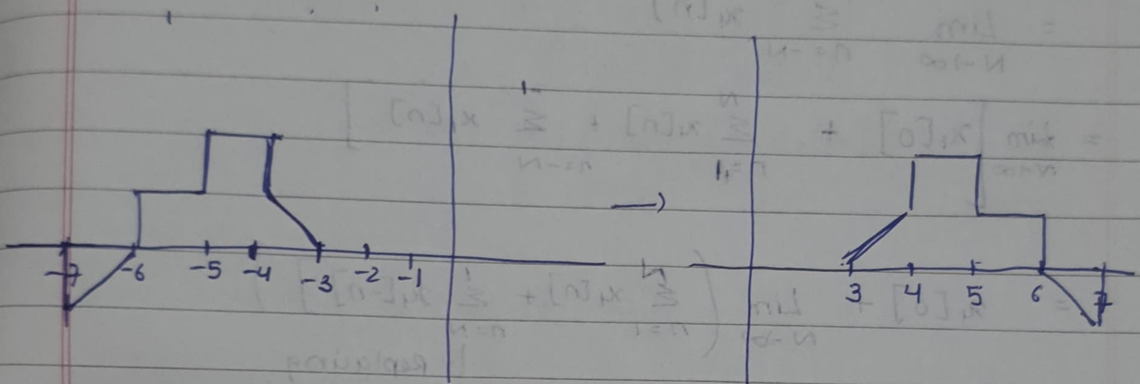


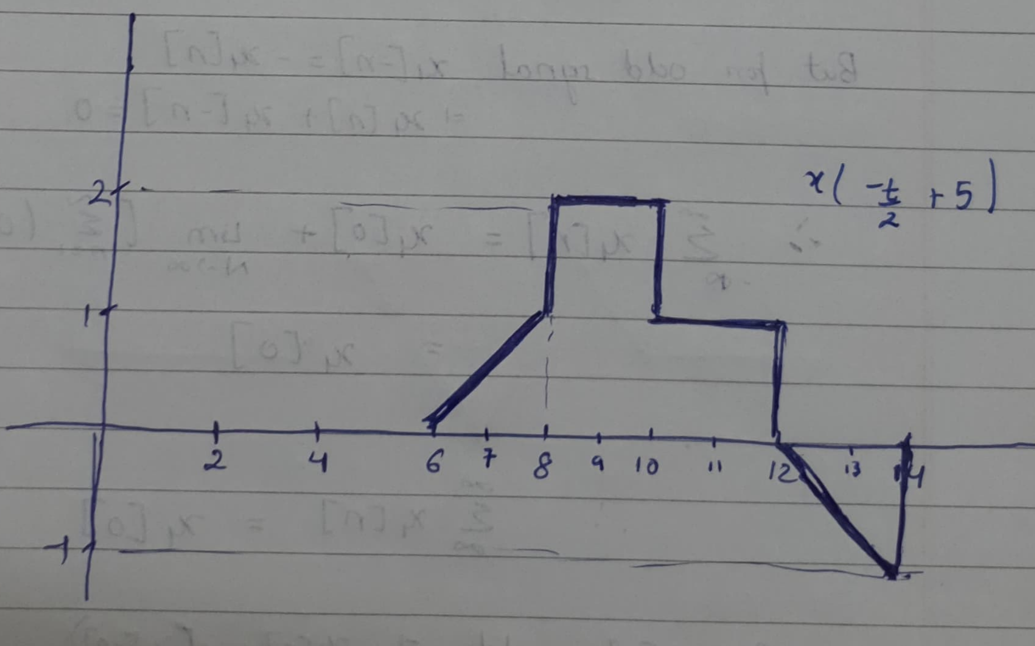


① $x\left(5-\frac{t}{2}\right) = x\left(-\frac{t}{2}+5\right)$

First we ① shift by 5 to left and then ② stretch it by a factor of 2 linearly and ③ then reverse it about 'y' axis.

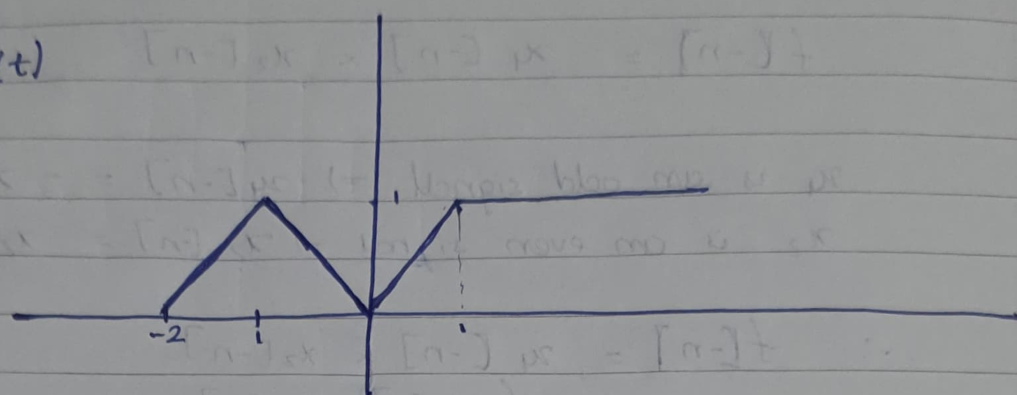
{Steps ② and ③ can be interchanged in orders}



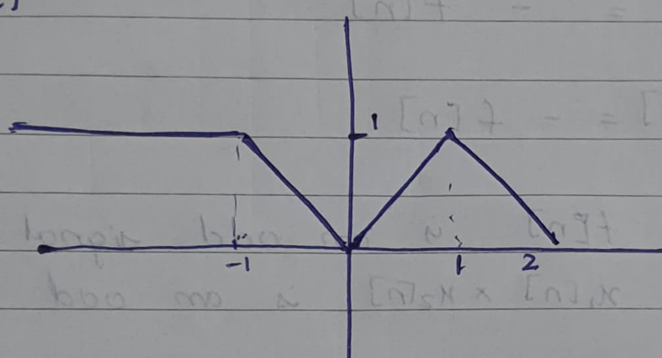


(4) (a)

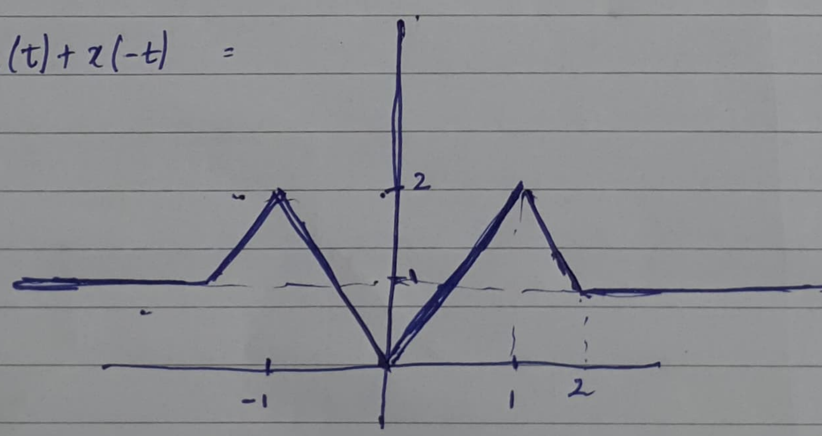
$x(t)$



$x(-t)$

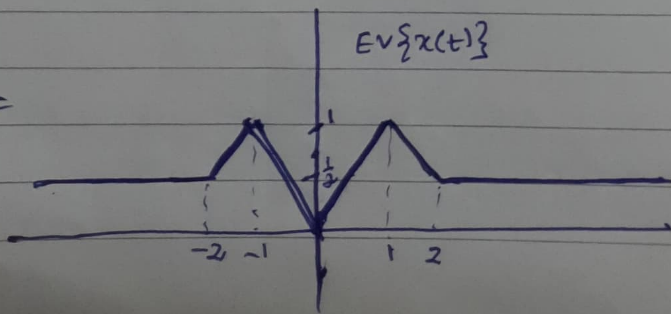


$x(t) + x(-t) =$



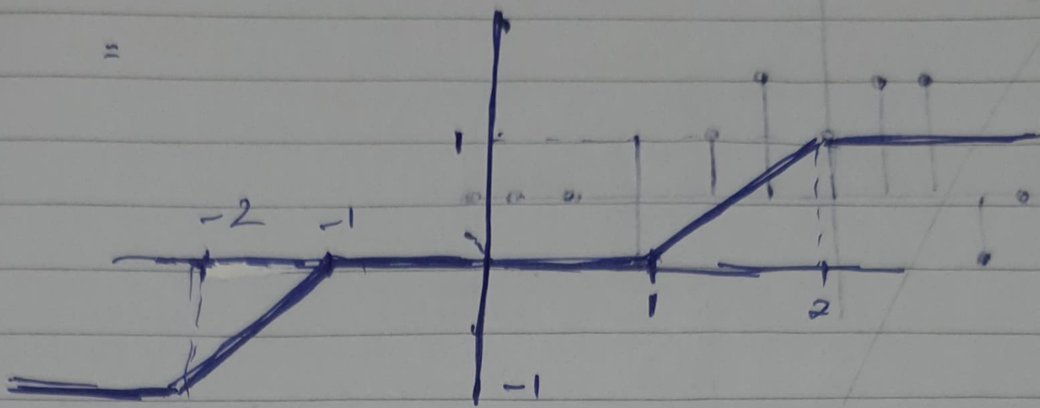
$$EV\{x(t)\} = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$=$

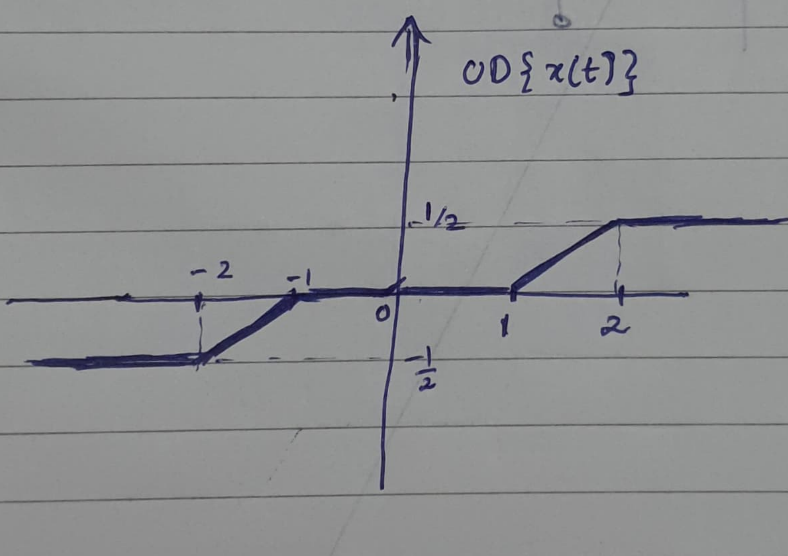


$x(t) - x(-t)$

$=$

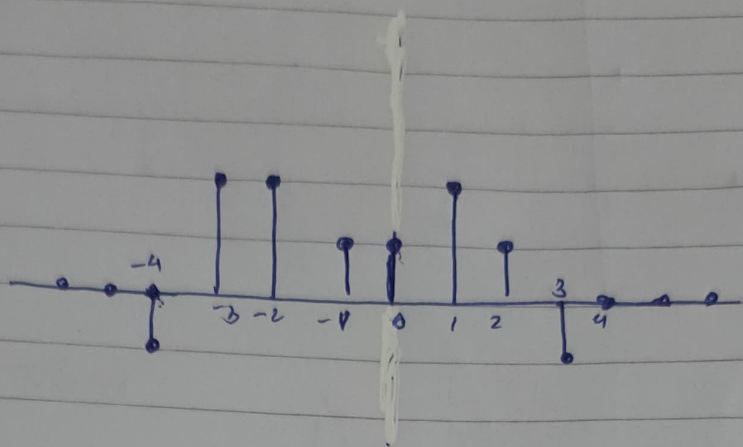


$OD\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$

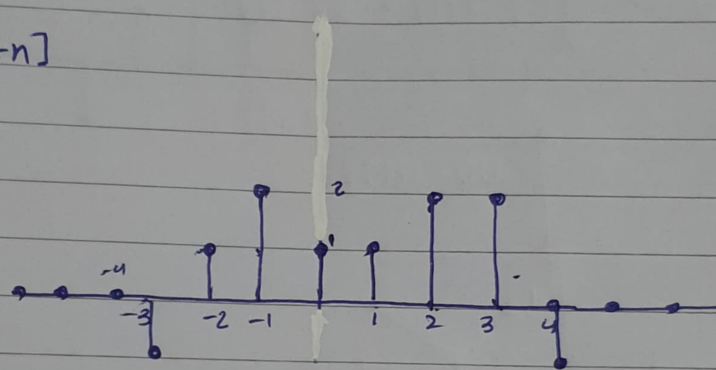


4 (b)

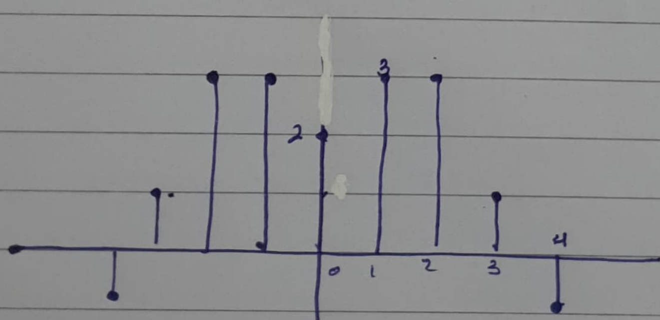
$x[n]$



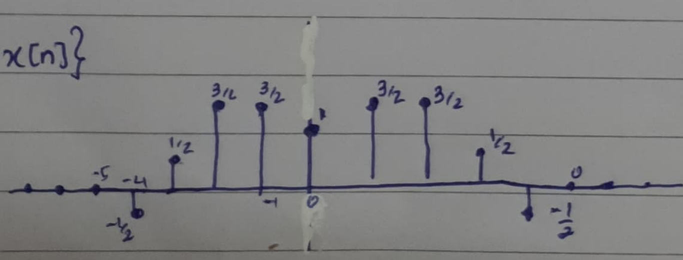
$x[-n]$



$x[n] + x[n]$

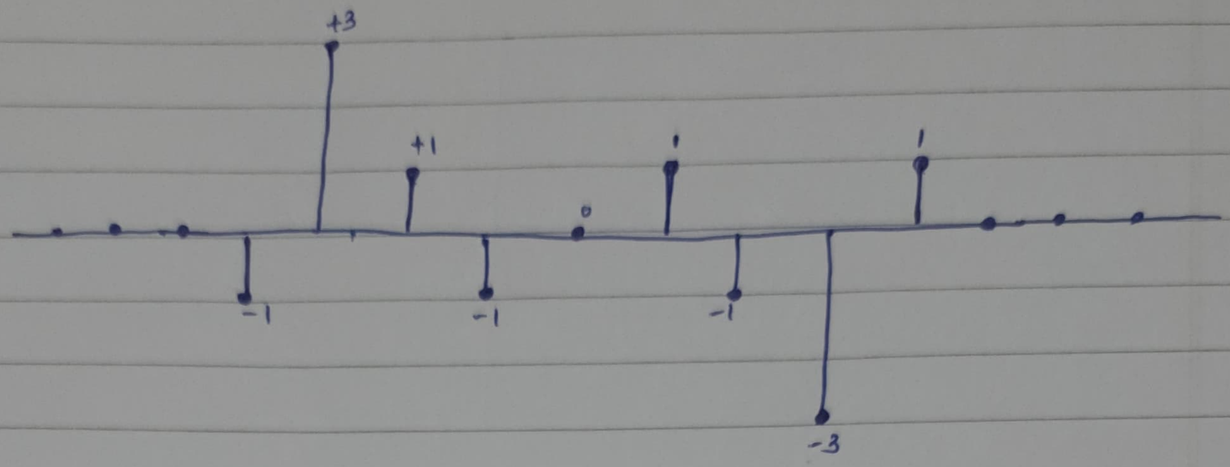


$EV\{x[n]\}$



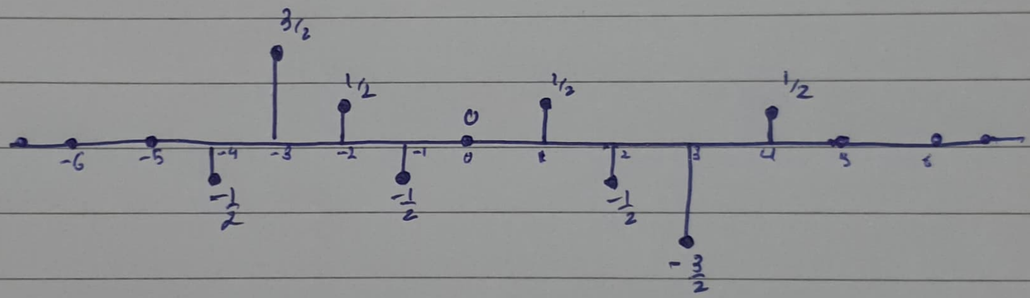
$$\left\{ \frac{x[n] + x[-n]}{2} \right\}$$

$x[n] - x[-n]$



+3
+1
0
1
1
-1
-1
-1
-3

$OD\{x[n]\}$



3/2
1/2
0
1/2
1/2
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
$-\frac{1}{2}$
$-\frac{1}{2}$
$-\frac{1}{2}$
$-\frac{3}{2}$

(5) Given $x_1[n]$ is an odd signal,
$x_2[n]$ is an even signal.

(a) $\sum_{-\infty}^{\infty} x_1[n]$

$= \lim_{N\to\infty} \sum_{n=-N}^{N} x_1[n]$

$= \lim_{N\to\infty} \sum_{n=-N}^{N} x_1[n]$

$= \lim_{N\to\infty} \left[x_1[0] + \sum_{n=1}^{N} x_1[n] + \sum_{n=-N}^{-1} x_1[n] \right]$

$= x_1[0] + \lim_{N\to\infty} \left(\sum_{n=1}^{N} x_1[n] + \sum_{n=N}^{1} x_1[-n] \right)$

↳ Replacing n by $-n$'

$= x_1[0] + \lim_{N\to\infty} \sum_{n=1}^{N} (x_1[n] + x_1[-n])$

But for odd signal $x_1[-n] = -x_1[n]$
$\Rightarrow x_1[n] + x_1[-n] = 0$

$\therefore \sum_{-\infty}^{\infty} x_1[n] = x_1[0] + \lim_{N\to\infty} \left[\sum_{n=1}^{N} (0) \right]$

$= x_1[0]$

$\therefore \sum_{-\infty}^{\infty} x_1[n] = x_1[0]$

But $x_1[n]$ is odd $\Rightarrow x_1[n] = -(x_1[-n])$
$\Rightarrow x_1[0] = -(x_1[0]) \Rightarrow 2x_1[0] = 0 \Rightarrow x_1[0] = 0$

$$\boxed{\therefore \sum_{-\infty}^{\infty} x_1[n] = x_1[0] = 0}$$

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(b) Let $f[n] = x_1[n] \times x_2[n]$

$f[-n] = x_1[-n] \times x_2[-n]$

x_1 is an odd signal $\Rightarrow x_1[-n] = -x_1[n]$
x_2 is an even signal $\Rightarrow x_2[-n] = x_2[n]$

$$\therefore f[-n] = x_1[-n] \times x_2[-n]$$
$$= (-x_1[n]) \times x_2[n]$$
$$= -(x_1[n] \times x_2[n])$$
$$= -f[n]$$

$\therefore f[-n] = -f[n]$.

$\therefore f[n]$ is an odd signal
$\therefore x_1[n] \times x_2[n]$ is an odd signal.