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ASSIGNMENT - 2 , 19114018, Ayushman Tripathy

① ⓐ $x_1(t) = 2j\, e^{j12t}$

If T is the (any) period of signal

$\Rightarrow x_1(t+T) = x_1(t)$

$\Rightarrow 2j\, e^{j12(t+T)} = 2j\, e^{j12t}$

$\Rightarrow e^{j.12t + j12.T} = e^{j12t}$

$\Rightarrow e^{j12T} = 1 = e^{j(2n\pi)}$

$\Rightarrow T = \dfrac{2n\pi}{12} = \dfrac{n\pi}{6}$

$\therefore$ for $x_1(t)$

for fundamental period $n = 1 \Rightarrow \boxed{T_0 = \dfrac{\pi}{6}}$ fundament.

So, for a continuous signal $T = \dfrac{2\pi}{|\omega_0|}$, here $\boxed{T_0 = \dfrac{\pi}{6}}$

] PERIODIC

ⓑ $x_2[n] = e^{-j0.7n}$

Let N be the period

$\Rightarrow x_2[n+N] = x_2[n]$

$\Rightarrow e^{-0.7(n+N)} = e^{-0.7nj} \Rightarrow e^{j(-0.7N)} = 1 = e^{(-2m\pi)-j}$

$\Rightarrow N = \dfrac{2m\pi}{0.7} = \dfrac{20m\pi}{7} \Rightarrow \dfrac{N}{m} = \dfrac{20\pi}{7}$

But $\dfrac{N}{m}$ is a rational no, and RHS is irrational $\left(\dfrac{20\pi}{7}\right)$

{here $m \neq 0$} as $m=0$ is 0 period which is meaningless.

$\therefore$ signal is $\boxed{\text{NOT PERIODIC}}$

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(c) $x_3[n] = e^{3\pi j/5 (n+1/2)}$

Let N be the period

$\Rightarrow x_3[n+N] = x_3[n]$

$\Rightarrow e^{3\pi j/5 (n+N+1/2)} = e^{3\pi j/5 (n+1/2)}$

$\Rightarrow e^{3\pi j/5 N} = 1 = e^{j2m\pi}$

$\Rightarrow \frac{3\cancel{\pi}\cancel{j}}{5} N = 2m\cancel{\pi}\cancel{j}$

$\Rightarrow N = 2m \cdot \frac{5}{3} = \frac{10m}{3}$

But since N is an integer, $m = 3k \Rightarrow N = 10k$

$\therefore$ for fundamental period $k=1 \Rightarrow m=3 \Rightarrow \boxed{N=10}$

$\therefore$ PERIODIC with Fundamental Period $= 10$

(d) $x_4(t) = 5e^{j(2\pi t)}$

$x_4(t+T) = x_4(t)$

$\Rightarrow 5e^{j(2\pi(t+T))} = 5e^{j(2\pi t)}$

$\Rightarrow e^{j \cdot 2\pi T} = 1 = e^{j(2\pi m)}$

$\Rightarrow 2\pi T = 2\pi m$

$\Rightarrow \boxed{T = m}$ $\therefore$ for fundamental period, $m=1$

$\Rightarrow \boxed{T_0 = 1}$

$\therefore$ PERIODIC with Fundamental Period $= 1$

(2) (a) $x(t) = 2\cos(7t+3) + 3\sin(3t+4)$

During composition of two signals, their fundamental period of combined signal would be the LCM of periods of both signals.

We have, $\omega_1 = 7 \Rightarrow T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{7}$

$\left\{ \text{Proof: } 7(t+T)+3 = 7t+3+2\pi m \Rightarrow 7T = 2\pi \Rightarrow T = \frac{2\pi m}{7} \right.$

$\left. \Rightarrow F.P = 2\pi/7 \right\}$

Similarly, $\omega_2 = 3 \Rightarrow T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{3}$

$$\text{LCM of } T_1 \text{ \& } T_2 = \frac{\text{LCM of } (2\pi, 2\pi)}{\text{GCD of } (3, 7)}$$

$$= \frac{2\pi}{1} = 2\pi$$

$\left\{ \because \text{LCM of two fractions} = \frac{\text{LCM of Numerators}}{\text{GCD of Denominators}} \right\}$

$\therefore$ the $\boxed{\text{fundamental period of } x(t) \text{ is } 2\pi}$

PTO

(b) $x[n] = 1 + e^{j4\pi n/7} - e^{j2n/5}$

For signal 1, Fundamental Period

$\omega_{0,1} = \frac{4\pi}{7}$ $\quad N_{0,1} = m \cdot \frac{2\pi}{\omega_{0,1}}$, $N_{0,1} \in \mathbb{Z}$ {Integer}

$= m \cdot \frac{2\pi}{4\pi/7} = \frac{7m}{2}$

$\Rightarrow m = 2$

$\boxed{N_{0,1} = 7}$

For signal 2, Fundamental Period,

$\omega_{0,2} = \frac{2}{5}$ $\quad N_{0,2} = m \cdot \frac{2\pi}{\omega_{0,2}} = m \cdot \frac{2\pi}{2/5}$

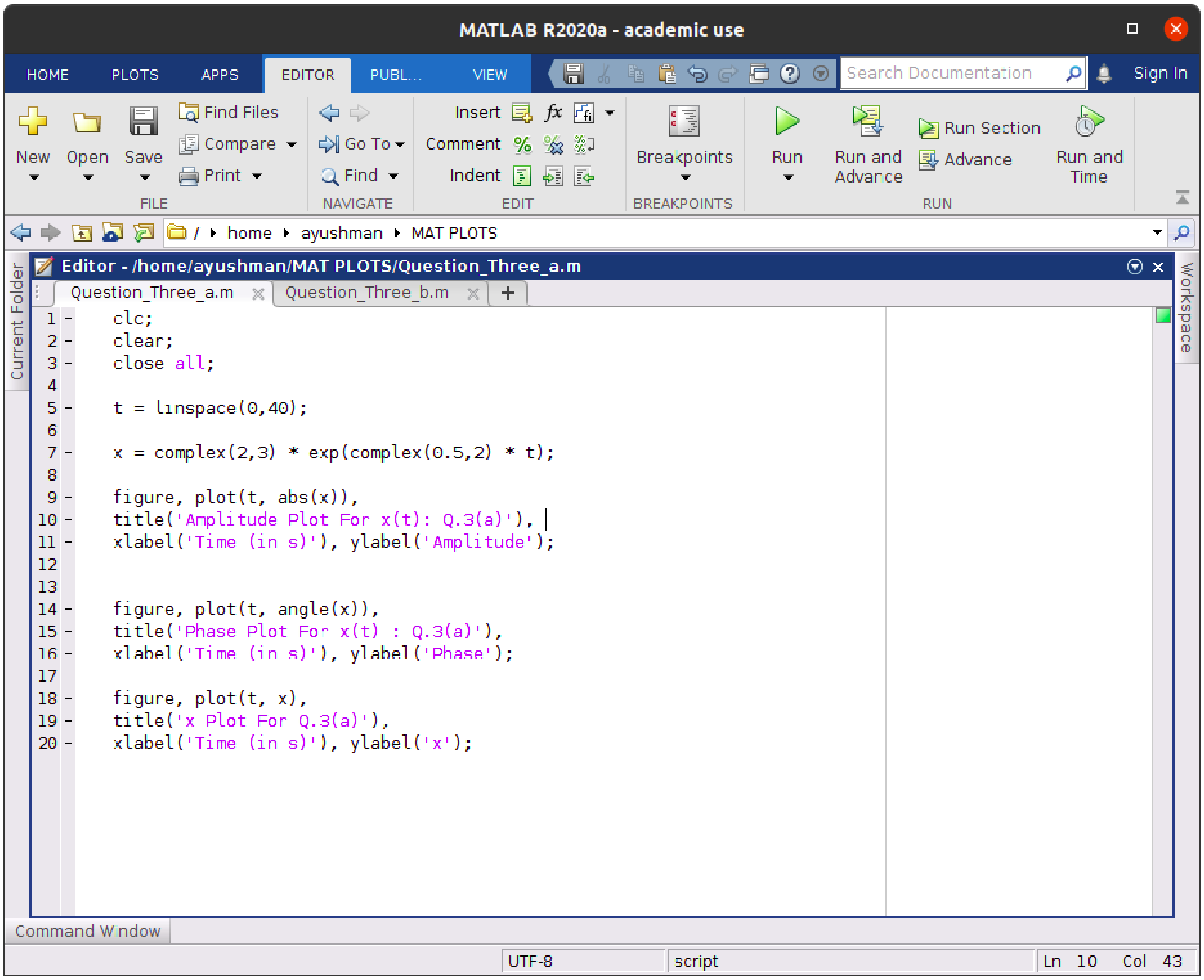
$= 5\pi m$

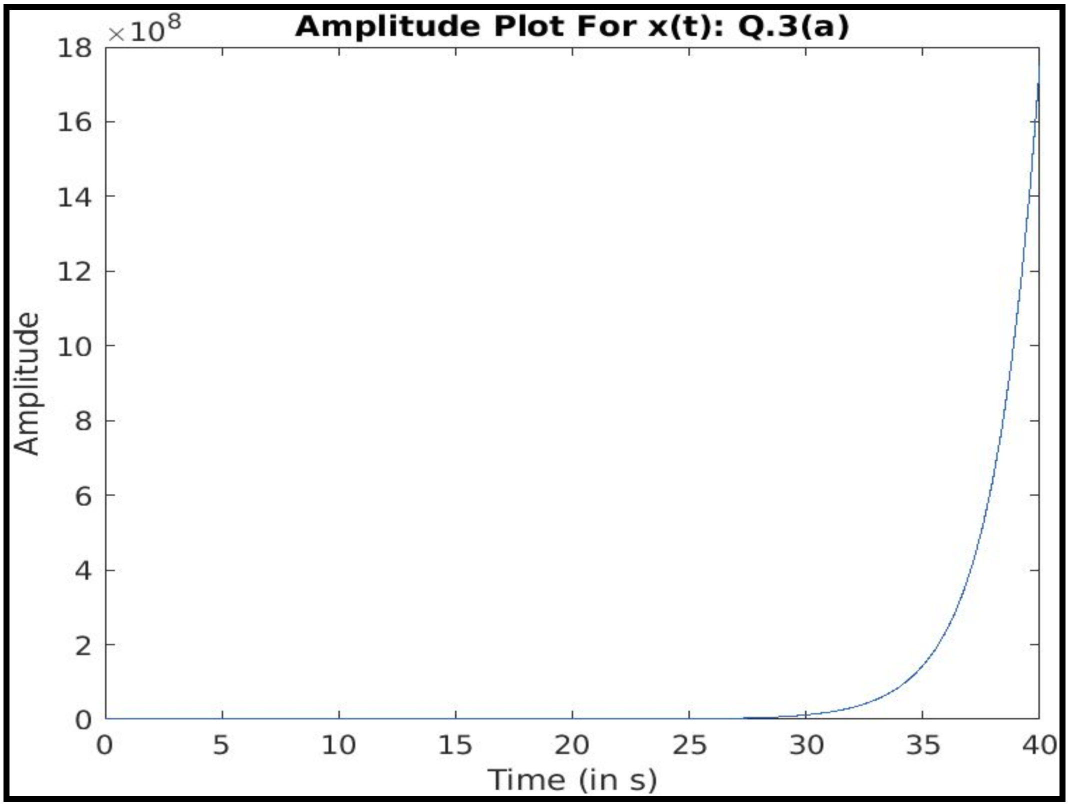
For no value of m, $5m\pi \in \mathbb{Z}$ {integers}

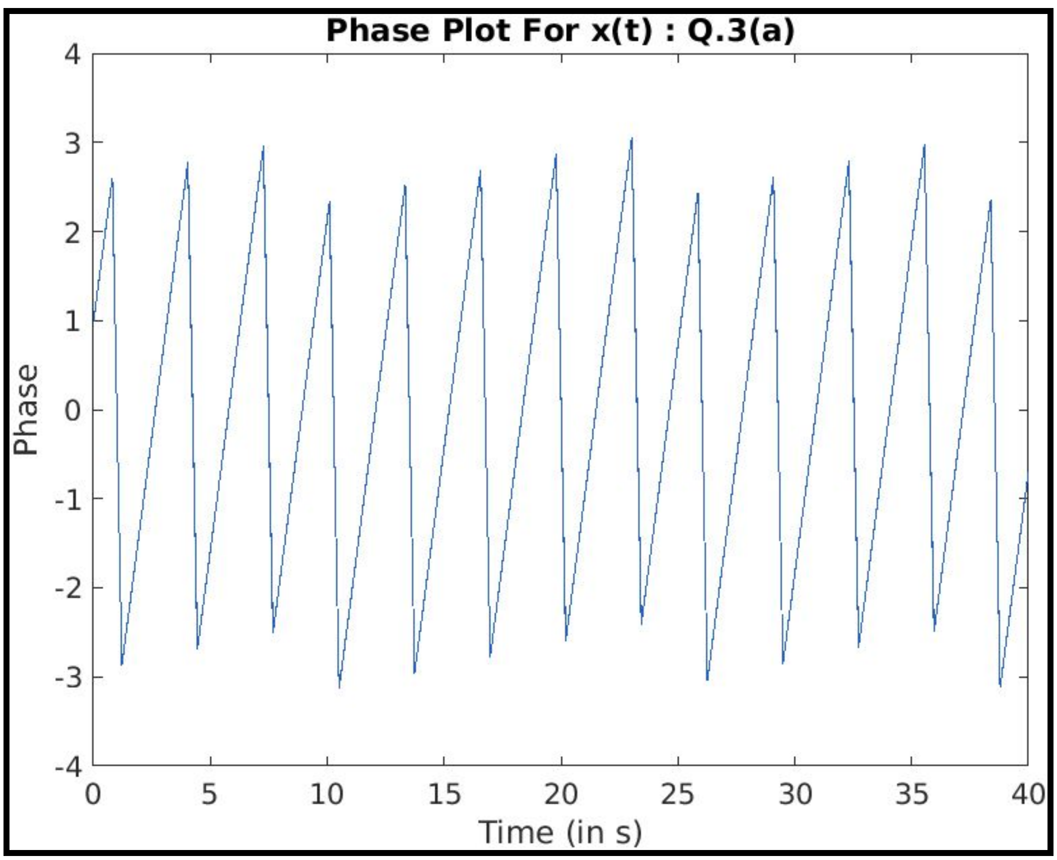
$\therefore$ signal 2 is not periodic

$\therefore$ The signal $\boxed{x[n] \text{ is NOT PERIODIC}}$

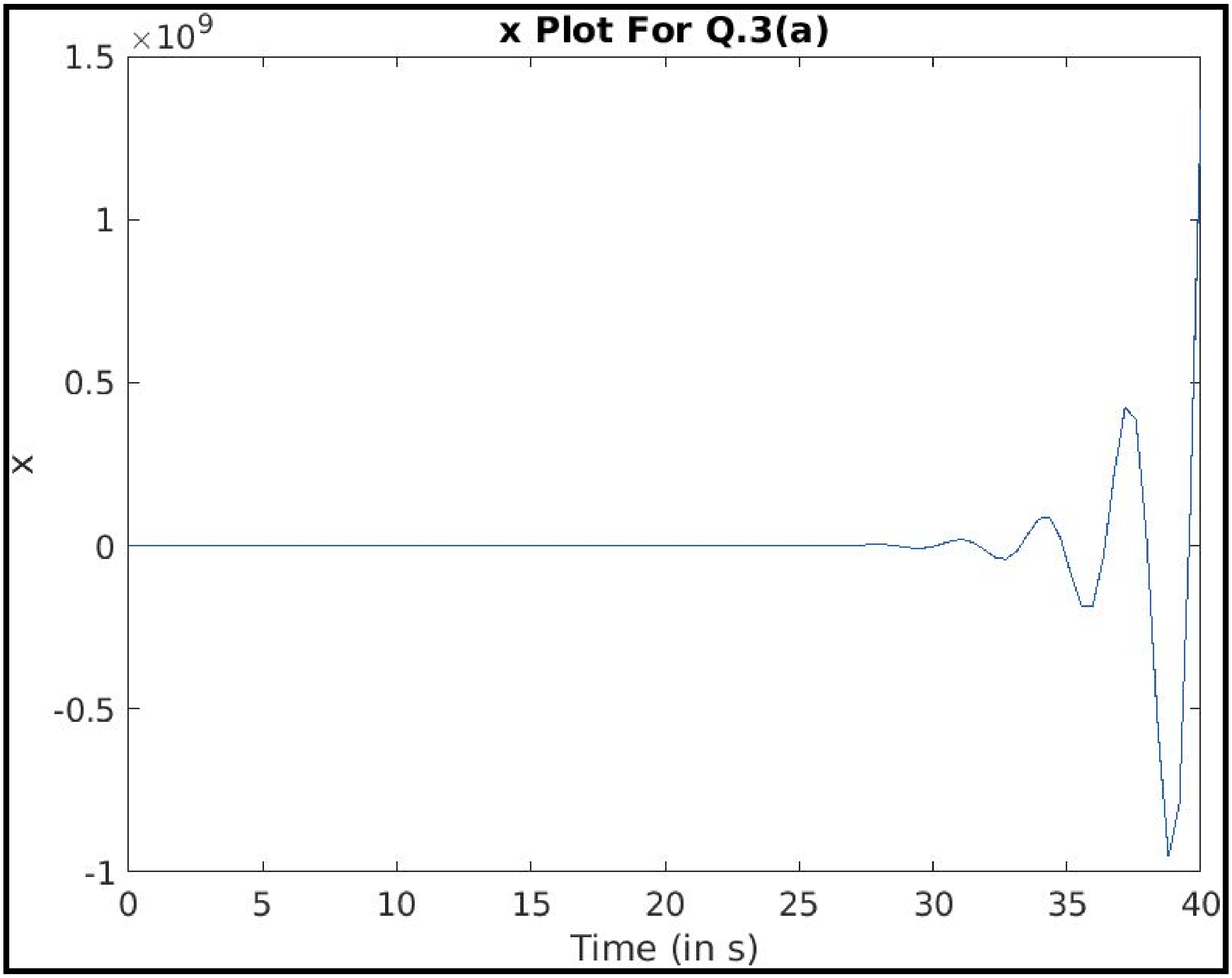
Question 3-(a). Code for the plot



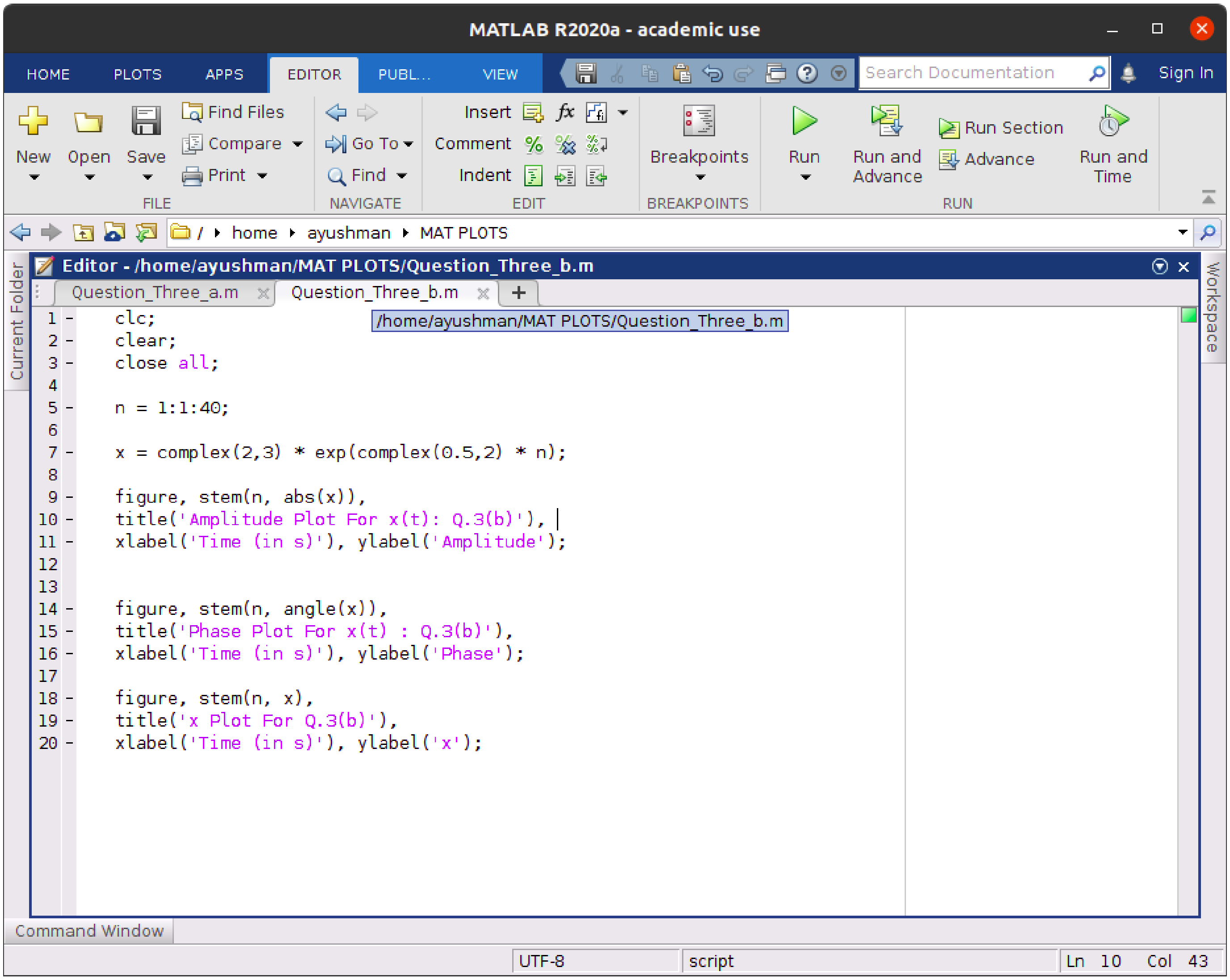

Amplitude Plot For x(t): Q.3(a)
$\times 10^8$
18
16
14
12
10
8
6
4
2
0
Amplitude
0
5
10
15
20
25
30
35
40
Time (in s)

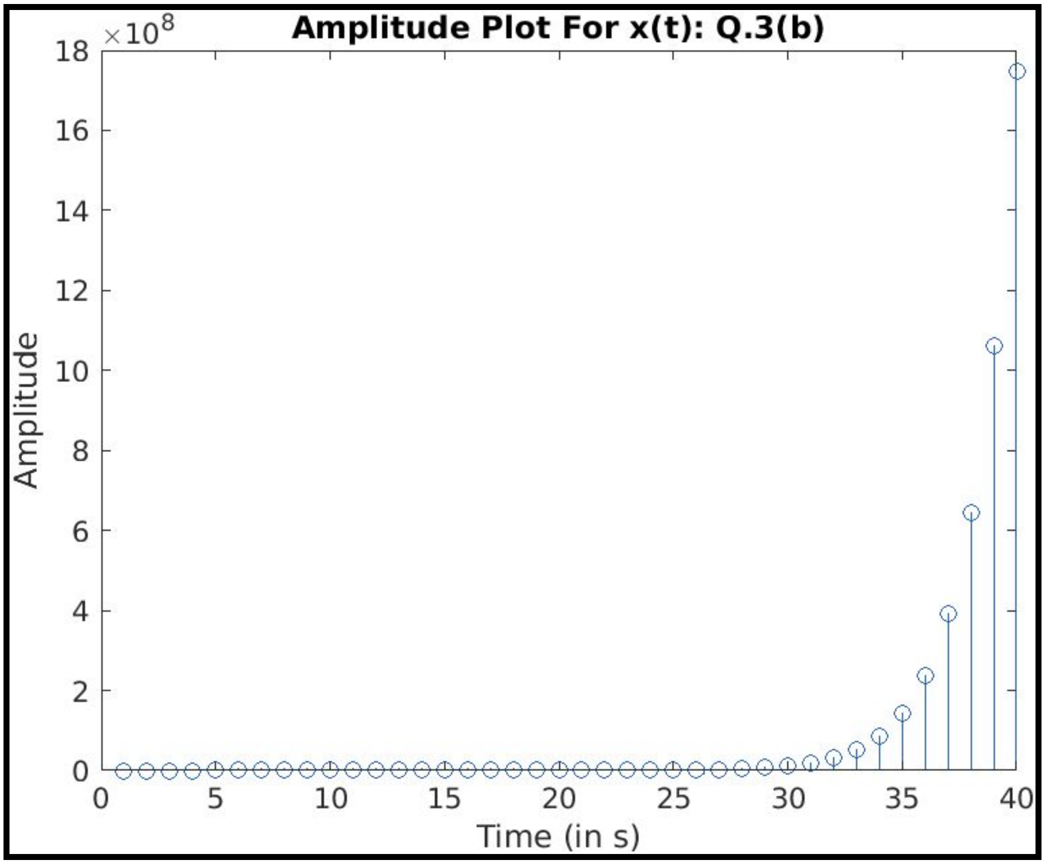

Phase Plot For x(t) : Q.3(a)
4
3
2
1
0
-1
-2
-3
-4
Phase
0
5
10
15
20
25
30
35
40
Time (in s)

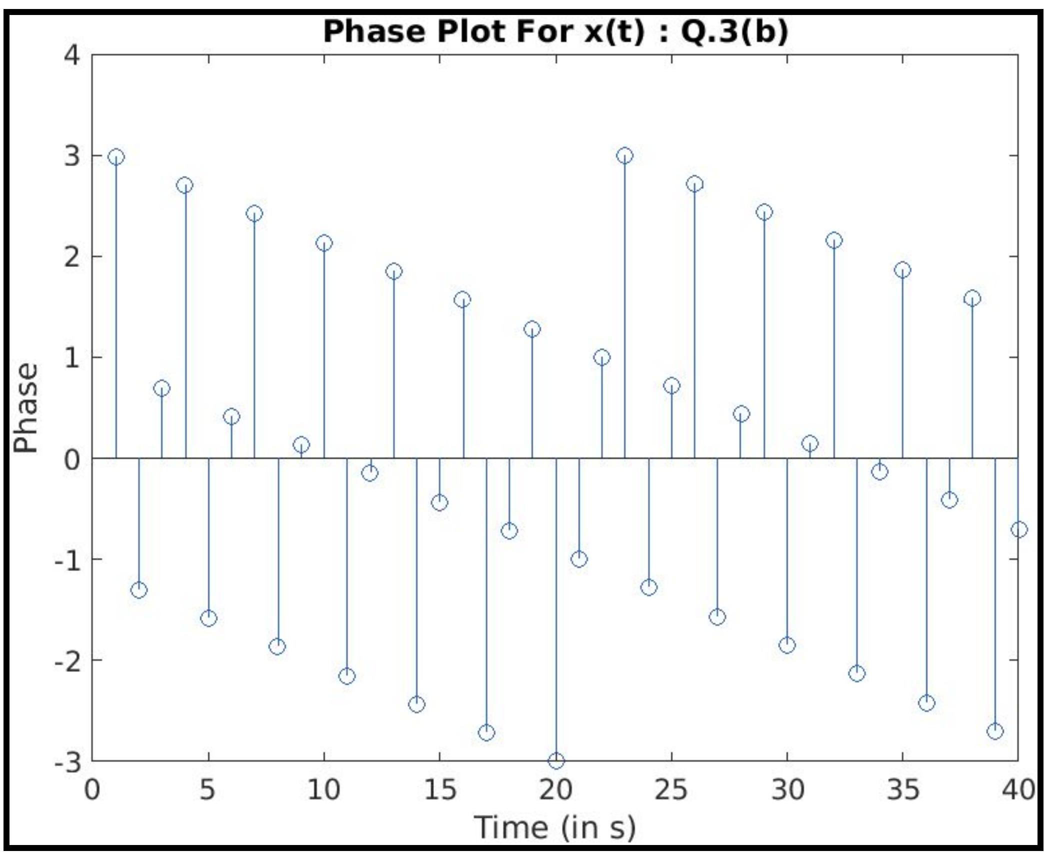
I have also plotted the signal for the question to get an extra perspective into how it would look like



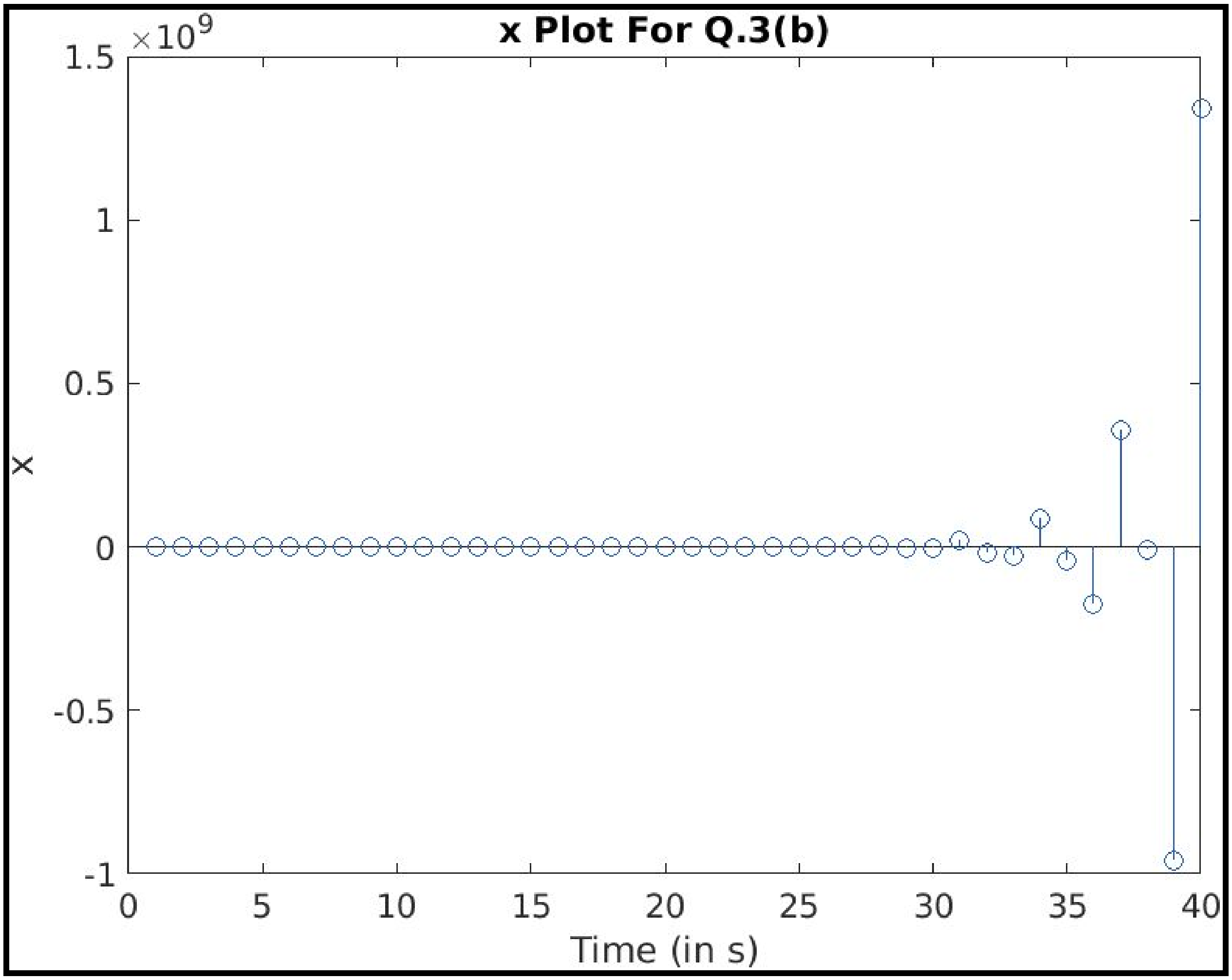
Question. 3(b). Code for the plot




Amplitude Plot For x(t): Q.3(b)
×10^8
18
16
14
12
10
8
6
4
2
0
Amplitude
0
5
10
15
20
25
30
35
40
Time (in s)


Phase Plot For x(t) : Q.3(b)
4
3
2
1
0
-1
-2
-3
Phase
0
5
10
15
20
25
30
35
40
Time (in s)

I have also plotted the x[n]



(4) $x[n] = 1 - \sum_{k=-2}^{\infty} \delta[n-1-k]$

Let $m = n-1-k$

if $k = -2 \Rightarrow m = n-1+2 = n+1$
if $k \to \infty \Rightarrow m = n-1-k \to -\infty$

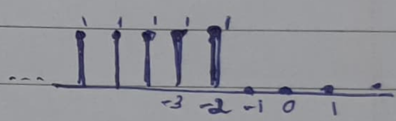
$\Rightarrow x[n] = 1 - \sum_{m=n+1}^{-\infty} \delta[m]$

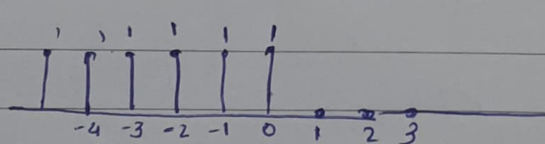
$x[n] = 1 - \sum_{m=-\infty}^{n+1} \delta[m]$

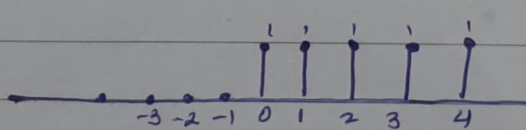
$u[n] = \sum_{m=-\infty}^{n} \delta[m] \Rightarrow \sum_{m=-\infty}^{n+1} \delta[m] = u[n+1]$

$\Rightarrow x[n] = 1 - u[n+1]$ [ALITER IN NEXT PAGE]

$x[n] = \begin{cases} 0 & , n+1 \geq 0 \quad \{as \ u[n+1] = 1\} \\ 1 & , n+1 < 0 \quad \{as \ u[n+1] = 0\} \end{cases}$

$x[n] =$ 

$x[n-2] =$ 

$x[-n-2] =$  $= u[n]$

$x[-n-2] = u[n]$ $\quad -n-2 = k \Rightarrow n = -(2+k)$

$x[k] = u[-(2+k)]$

$\Rightarrow \boxed{x[k] = u[-k-2]}$ $\quad \therefore \boxed{x[n] = u[-n-2]}$

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Aliter: for Q4.

we got

$$x[n] = 1 - u[n+1]$$

$$x[n] = \begin{cases} 0 & n+1 \geq 0 > -1 \\ 1 & n+1 < 0 \quad \Rightarrow n+1 \leq -1 \end{cases}$$

{as discrete}

$$\Rightarrow x[n] = \begin{cases} 0 & n+1 > -1 \\ 1 & n+1 \leq -1 \end{cases}$$

$$\Rightarrow x[n] = \begin{cases} 0, & n > -2 \\ 1 & n \leq -2 \end{cases}$$

$$\Rightarrow x[n] = \begin{cases} 0 & -n-2 < 0 \\ 1 & -(n+2) \geq 0 \end{cases}$$

This is of the form of $u[-n-2]$

$$\therefore \boxed{x[n] = u[-n-2]}$$

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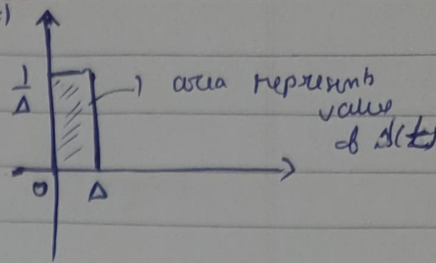
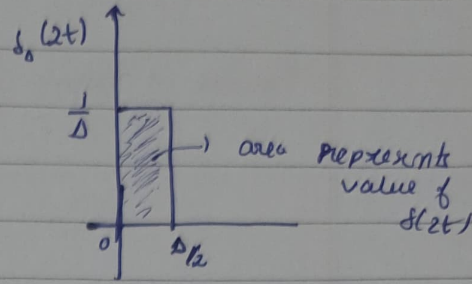
(5) $\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$



$\delta(2t) = \lim_{\Delta \to 0} \delta_\Delta(2t)$



$\int_{-\infty}^{\infty} \delta(t) = \int_{0}^{\Delta} \delta(t)dt = 1$

By time scaling

Taking $\int_{-\infty}^{\infty} \delta(2t)\, dt$. Let $2t = k$

$\Rightarrow t = \frac{k}{2}$

$= \int_{-\infty}^{\infty} \delta(k) \frac{dk}{2}$

$\Rightarrow dt = \frac{dk}{2}$

$= \frac{1}{2} \int_{-\infty}^{\infty} \delta(k) dk$

This integral represents the value of $\delta(2t)$

$= \frac{1}{2}$

$\Rightarrow \delta(2t) = \frac{1}{2} = \frac{1}{2}\delta(t)$

$\therefore \boxed{\delta(2t) = \frac{1}{2}\delta(t)}$

ALITER:

$\Rightarrow$ Area under the curve, by using time scaling property

For $\delta(t) =$ area1 $= \frac{1}{\Delta} \cdot \Delta = 1$

For $\delta(2t) =$ area 2 $= \frac{1}{\Delta} \cdot \frac{\Delta}{2} = \frac{1}{2}$ {By time scaling we get $\frac{\Delta}{2}$}

$\therefore \boxed{\delta(2t) = \frac{1}{2}\delta(t)}$

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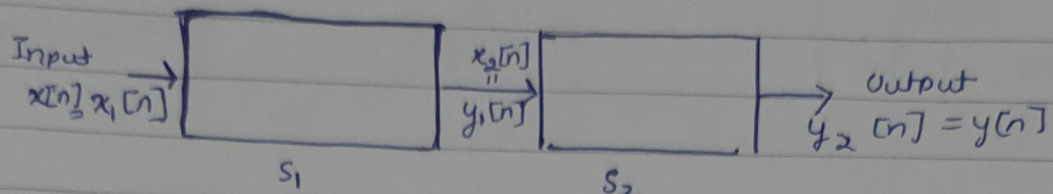
(6)

(a)



$\{ x_2[n] = y_1[n] \}$

Output of first sys. is input to second

$$y_1[n] = 2x_1[n] + 4x_1[n-1]$$

$$x_2[n] = y_1[n]$$

$$y_2[n] = x_2[n-2] + 0.5\, x_2[n-3]$$
$$= y_1[n-2] + 0.5\, y_1[n-3]$$

$$= \left(2x_1[n-2] + 4x_1[(n-2)-1]\right) + 0.5\left(2x_1[n-3] + 4x_1[(n-3)-1]\right)$$

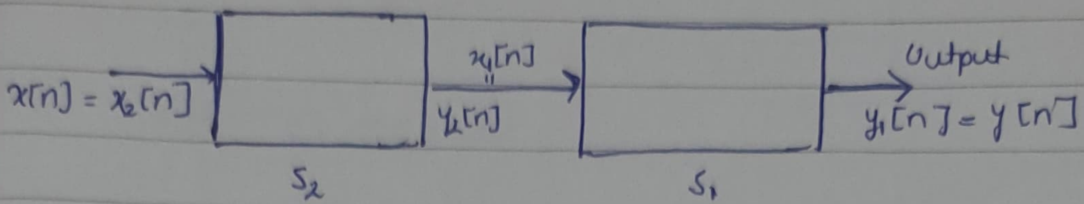
$$= 2x_1[n-2] + 4x_1[n-3] + (0.5\times2)x_1[n-3] + (0.5\times4)x_1[n-4]$$

$$y_2[n] = 2x_1[n-2] + 5x_1[n-3] + 2x_1[n-4]$$

$$y_2[n] = y[n]$$
$$x_1[n] = x[n]$$

$$\Rightarrow \boxed{y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]}$$

(6)



$y_2[n] = x_2[n-2] + 0.5\, x_2[n-3]$

$x_1[n] = y_2[n]$

$\Rightarrow x_1[n] = x_2[n-2] + 0.5\, x_2[n-3]$

$y_1[n] = 2x_1[n] + 4\, x_1[n-1]$

$= 2\, (x_2[n-2] + 0.5\, x_2[n-3])$

$+ 4\, (x_2[(n-1)-2] + 0.5\, x_2[(n-1)-3])$

$= 2\, x_2[n-2] + (2 \times 0.5)\, x_2[n-3] + 4\, x_2[n-3]$

$+ (4 \times 0.5)\, x_2[x-4]$

$y_1[n] = 2\, x_2[n-2] + 5\, x_2[n-3] + 2\, x_2[n-4]$

But $y_1[n] = y[n]$ and $x_2[n] = x[n]$

$\Rightarrow \boxed{y[n] = 2\, x[n-2] + 5x[n-3] + 2x[n-4]}$

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(7) $y(t) = x(\sin(t))$

(a) NOT MEMORYLESS

Say $t = -\pi \Rightarrow y(-\pi) = x(\sin(-\pi)) = x(0)$

$\Rightarrow$ output at t $\{-\pi, \text{here}\}$ is dependent on other time inputs $\{0, \text{here}\}$

Say $t = \pi/6$ $\quad y(\pi/6) = x(\sin(\frac{\pi}{6})) = x(\frac{1}{2})$

$\therefore$ output at $\frac{\pi}{6}$ is dependent on input at $\frac{1}{2}$

$\therefore$ it isn't memoryless

(b) NOT CAUSAL

Say $t = -\pi \Rightarrow y(-\pi) = x(\sin(-\pi)) = x(0)$

$\therefore$ output at $-\pi$ is dependent on a future input 0.

$\therefore$ it is not causal, as output at a time t is dependent on future input

7 c $y(t) = x(\sin(t))$ $\boxed{\text{NOT-INVERTIBLE}}$

For a ~~signal~~ system to be invertible, the input signal should always be uniquely mapped to the values of the output signal.
{ie. there exists a unique input signal for the output signal}

Taking the domain as R,

$\because$ $\sin(t) \in [-1, 1]$, $x(t)$ can be uniquely mapped to $y(t)$ in $[-1, 1]$

in $[-1, 1]$ inverse would have been $x(t) = y(\sin^{-1}(t))$

But the domain is 'R', as it hasn't been mentioned

$\therefore$ for $t < -1$ and for $t > 1$, there exists no unique mapping for $x(t)$ for these values

$\therefore$ it is $\boxed{\text{NOT - INVERTIBLE}}$

★ Any input signal with same values in range $[-1, 1]$ would satisfy this $\therefore$

Example: Let $y(t) = 0$ in $[-1, 1]$

Eg. Let $x(t) = \delta(t-2)$ be a solution

as $\sin(t) \in [-1, 1]$, $t-2 \in [-3, -1] \Rightarrow \delta(t-2) = 0$ always for given mapping

Similarly if we take $\delta(t-3)$
$t-3 \in [-4, -2]$, $\Rightarrow \{\delta(t-3) = 0$ always in $[-1, 1]\}$

$\therefore$ Both $\delta(t-2)$ & $\delta(t-3)$ would be solutions to the system

$\therefore$ NON-INVERTIBLE

(8) (a) $y(t) = x(t-4)$

Let $t - 4 = k$

$\Rightarrow t = k + 4$

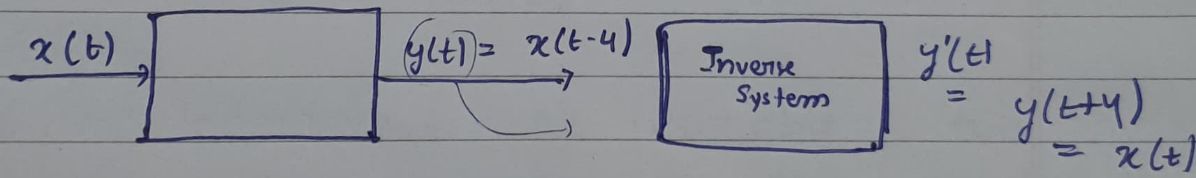
$\Rightarrow \boxed{x(k) = y(k+4)}$

$\Rightarrow \boxed{x(t) = y(t+4)}$

Since there exists a unique mapping from $x(t)$ to $y(t)$ and vice-versa and Distinct inputs would have corresponding outputs, $\therefore$ invertible

$\boxed{\text{INVERTIBLE}}$

Inverse - system



$\therefore$ $Z(t) = y(t+4)$ is the inverse system of

$y(t) = x(t-4)$

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(b) $y[n] = n\,x[n]$

At $n=0$, $y[n] = 0 \cdot x[0] = 0$.

Thus, whatever be the value of $x[0]$, $y[0]$ would always be 0

Hence we can always create a function

$$x[n] = \begin{cases} \frac{1}{n}\,y[n] & , n \neq 0 \\ c & , n = 0 \end{cases}$$

Here, 'c' can be variable, thus infinitely many answers {functions} to this are possible.

Eg. if $\delta[n]$ satisfies the required values for $x[n]$, $2\delta[n]$ or $e\,\delta[n]$ would also satisfy

$\therefore$ no unique mapping would be present

$\therefore$ $\boxed{\text{NON - INVERTIBLE}}$

(c) $y[n] = \begin{cases} x[n+1], & n \geq 0 \\ x[n], & n \leq -1 \end{cases}$

We tabulate all values of $y[n]$

$y[0] = x[1]$ $\qquad$ $y[-1] = x[-1]$
$y[1] = x[2]$ $\qquad$ $y[-2] = x[-2]$
$\vdots$ $\qquad\qquad\qquad$ $\vdots$

We see that no value of $y[n]$ is dependent on $x[0]$ for output

Thus we could always have functions satisfying at all other points, but having different values at $n = 0$.

$\therefore$ $\boxed{\text{NON-INVERTIBLE}}$

Let $x[n] = \delta[n]$ satisfy the function

$$\therefore \quad y[n] = \begin{cases} \delta[n+1], & n \geq 0 & = \delta[1], \delta[2] \cdots = 0 \\ \delta[n] & n \leq -1 & = 0 \end{cases}$$

$$= \begin{cases} 0, & n \geq 0 \\ 0 & n \leq -1 \end{cases}$$

Now instead of $\delta[n]$ if we take $2\,\delta[n]$ or $c\,\delta[n]$ all will satisfy, as they only differ in values at $n = 0$.