ECN-203 - Assignment 3

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Batch - 04
CSE - BTech - II Year
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Q.1. (a) $y(t) = t^2 x(t-1)$

$y_1(t) = t^2 x_1(t-1)$

$y_2(t) = t^2 x_2(t-1)$

$a x_1(t) + b x_2(t) \Rightarrow y(t) = t^2 [a x_1(t-1) + b x_2(t-1)]$

$= a y_1(t) + b y_2(t)$

$\boxed{\Rightarrow \text{Linear}}$

$y_1(t) = t^2 x_1(t-1)$

$x_2(t) = x_1(t - t_0)$

$\Rightarrow y_2(t) = t^2 x_2(t-1)$

$= t^2 x_1(t - t_0 - 1)$

$y_1(t - t_0) = (t - t_0)^2 x_1(t - t_0 - 1)$

$\therefore y_2(t) \neq y_1(t - t_0) \Rightarrow \boxed{\text{Time variant}}$

(b) $y[n] = (x[n-2])^2$

$y_1[n] = (x_1[n-2])^2$

$y_2[n] = (x_2[n-2])^2$

$\Rightarrow a x_1[n] + b x_2[n] \rightarrow y[n] = (a x_1[n-2] + b x_2[n-2])^2$

$= a^2 (x_1[n-2])^2 + b^2 (x_2[n-2])^2$
$+ 2ab (x_1[n-2])(x_2[n-2])$

$\Rightarrow y[n] \neq y_1[n] + y_2[n]$

$\Rightarrow \boxed{\text{Non-linear}}$

$y_1[n] = (x_1[n-2])^2$

$x_2[n] = x_1[n-n_0]$

$y_2[n] = (x_2[n-2])^2$

$= (x_1[n-n_0-2])^2$

$\Rightarrow y_1[n-n_0] = (x_1[n-n_0-2])^2$

$\because y_1[n-n_0] = y_2[n]$

$\Rightarrow$ $\boxed{\text{Time invariant}}$

(c) $y[n] = x[n+1] - x[n-1]$

$y_1[n] = x_1[n+1] - x_1[n-1]$

$y_2[n] = x_2[n+1] - x_2[n-1]$

$ax_1[n] + bx_2[n] \Rightarrow y[n] = (ax_1[n+1] + bx_2[n+1]) - (ax_1[n-1] + bx_2[n-1])$

$\Rightarrow y[n] = a(x_1[n+1] - x_1[n-1]) + b(x_2[n+1] - x_2[n-1])$

$\Rightarrow y[n] = ay_1[n] + by_2[n]$

$\Rightarrow$ $\boxed{\text{Linear}}$

$y_1[n] = x_1[n+1] - x_1[n-1]$

$x_2[n] = x_1[n-n_0]$

$\Rightarrow y_2[n] = x_2[n+1] - x_2[n-1]$

$= x_1[n-n_0+1] - x_1[n-n_0-1]$

$\Rightarrow y_1[n-n_0] = x_1[n-n_0+1] - x_1[n-n_0-1]$

$\because y_1[n-n_0] = y_2[n]$

$\Rightarrow$ $\boxed{\text{Time invariant}}$

(d). $y(t) = \mathbb{E}\mathbb{V}(x(t))$

$$= \frac{x(t) + x(-t)}{2}$$

$$y_1(t) = \frac{x_1(t) + x_1(-t)}{2} \qquad y_2(t) = \frac{x_2(t) + x_2(-t)}{2}$$

$$ax_1(t) + bx_2(t) \Rightarrow y(t) = \frac{ax_1(t) + bx_2(t) + ax_1(-t) + bx_2(-t)}{2}$$

$$= \frac{a(x_1(t) + x_1(-t)) + b(x_2(t) + x_2(-t))}{2}$$

$$= ay_1(t) + by_2(t)$$

$\Rightarrow \because y(t) = ay_1(t) + by_2(t)$, $\boxed{\text{Linear}}$

$$y_1(t) = \frac{x_1(t) + x_1(-t)}{2}$$

$x_2(t) = x_1(t - t_0)$

$$\Rightarrow y_2(t) = \frac{x_2(t) + x_2(-t)}{2} = \frac{x_1(t - t_0) + x_1(-(t) - t_0)}{2}$$

$$y_1(t - t_0) = \frac{x_1(t - t_0) + x_1(-(t - t_0))}{2}$$

$\because y_1(t - t_0) \neq y_2(t)$

$\Rightarrow \boxed{\text{Time variant}}$

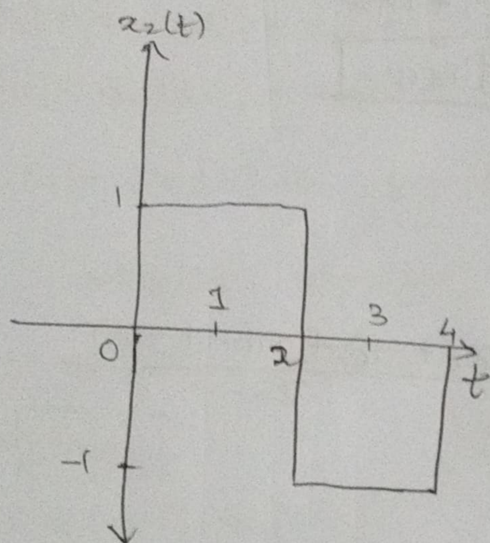
Q.2, $x_1(t) = \begin{cases} 1, & 0 \leq t \leq 2 \\ 0, & \text{otherwise} \end{cases}$ $y_1(t) = \begin{cases} 2t, & 0 \leq t \leq 1 \\ 4-2t, & 1 < t \leq 2 \\ 0, & \text{otherwise.} \end{cases}$

(a) $x_2(t) = \begin{cases} 1, & 0 \leq t \leq 2 \\ -1, & 2 < t \leq 4 \end{cases}$

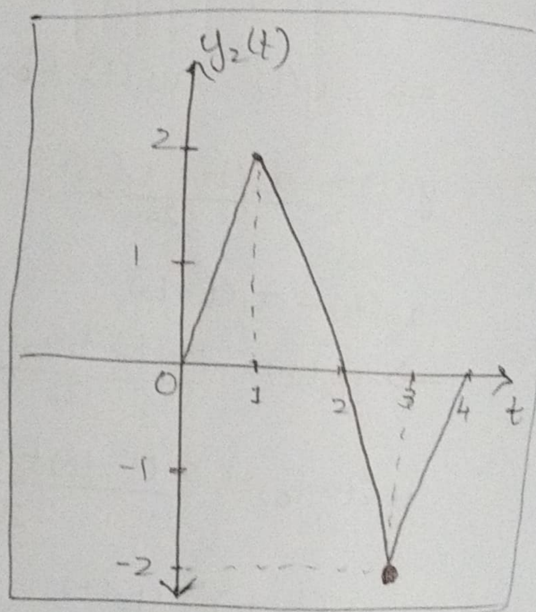
$\Rightarrow x_2(t) = x_1(t) - x_1(t-2)$

$\because$ We are operating on an LTI system,

$y_2(t) = y_1(t) - y_1(t-2)$



$\Longrightarrow$



*Note: $y_1(t-2) = \begin{cases} 2(t-2), & 0 < t-2 < 1 \\ 4-2(t-2), & 1 < t-2 < 2 \\ 0, & \text{otherwise} \end{cases}$

$\Rightarrow y_1(t-2) = \begin{cases} 2t-4, & 2 < t < 3 \\ 8-4t, & 3 < t < 4 \\ 0, & \text{otherwise} \end{cases}$

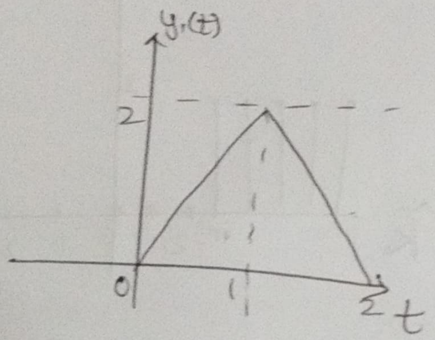
$\Rightarrow \boxed{y_2(t) = y_1(t) - y_1(t-2) = \begin{cases} 2t, & 0 < t < 1 \\ 4-2t, & 1 < t < 2 \\ 4-2t, & 2 < t < 3 \\ 4t-8, & 3 < t < 4 \\ 0, & \text{otherwise.} \end{cases}}$

(b) $x_3(t) = \begin{cases} 1, & -1<t<0 \\ 2, & 0<t<1 \\ 1, & 1<t<2 \\ 0, & \text{otherwise} \end{cases}$

$\Rightarrow x_3(t) = x_1(t) + x_1(t+1)$

$\because$ System is LTI system,

$\Rightarrow y_3(t) = y_1(t) + y_1(t+1)$



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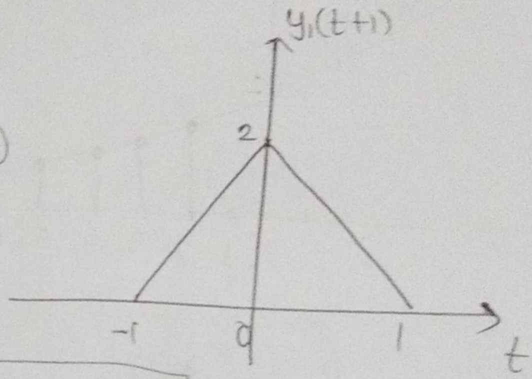
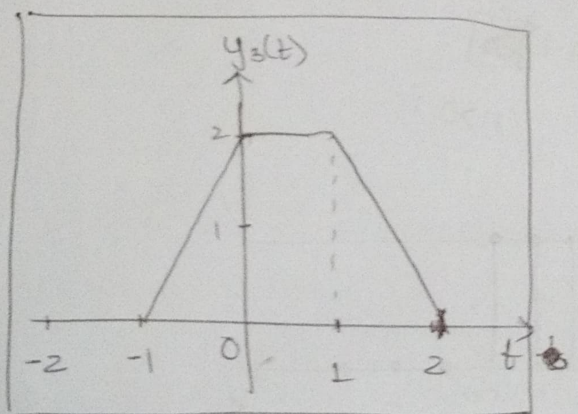


$\Rightarrow$



Note: $y_3(t) = y_1(t) + y_1(t+1)$

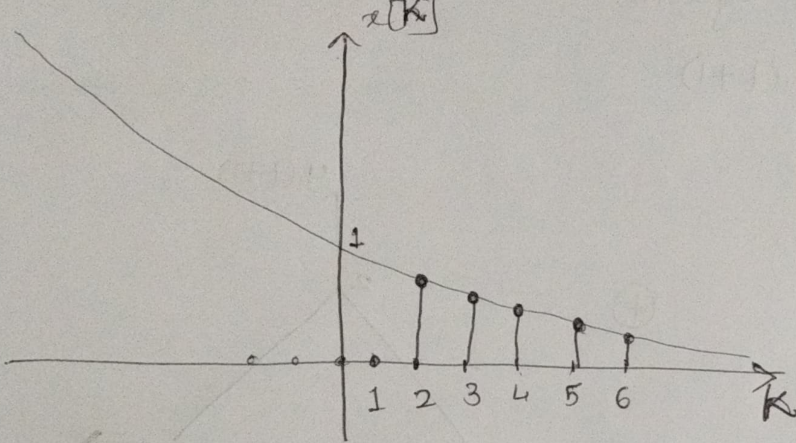
where $y_1(t+1) = \begin{cases} 2t+2, & 0<t+1<1 \\ 4-2(t+1), & 1<t+1<2 \\ 0, & \text{otherwise} \end{cases} = \begin{cases} 2t+2, & -1<t<0 \\ 2-2t, & 0<t<1 \\ 0, & \text{otherwise} \end{cases}$

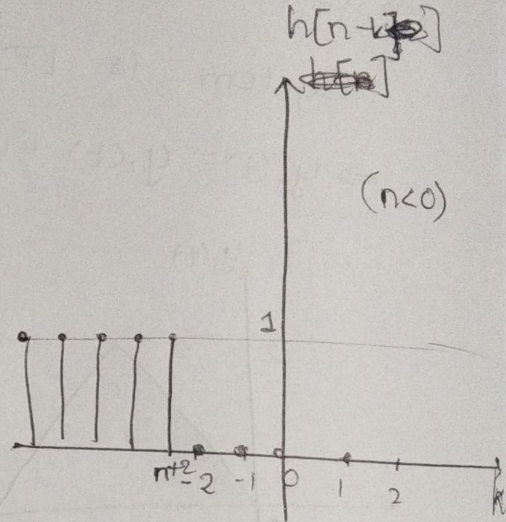
$\Rightarrow \boxed{y_3(t) = \begin{cases} 2t+2, & -1<t<0 \\ 2, & 0<t<1 \\ 4-2t, & 1<t<2 \\ 0, & \text{otherwise} \end{cases}}$

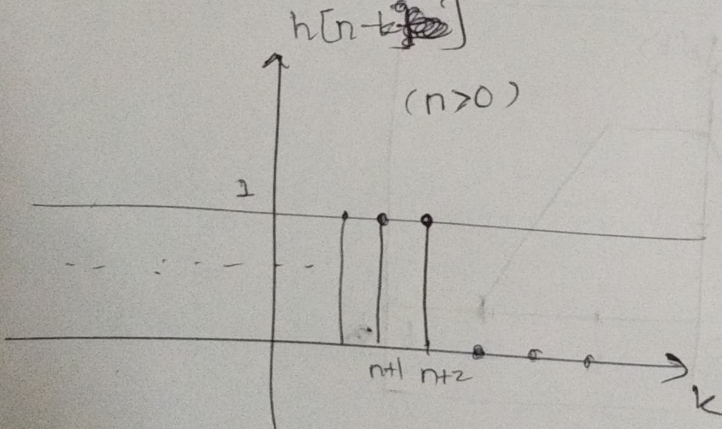
Q-3. $x[n] = \left(\frac{1}{2}\right)^n u[n-2]$; $h[n] = u[n+2]$

For an LTI system, $h_k[n] = h_0[n]$

$$\Rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n] * h[n]$$







Now, $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

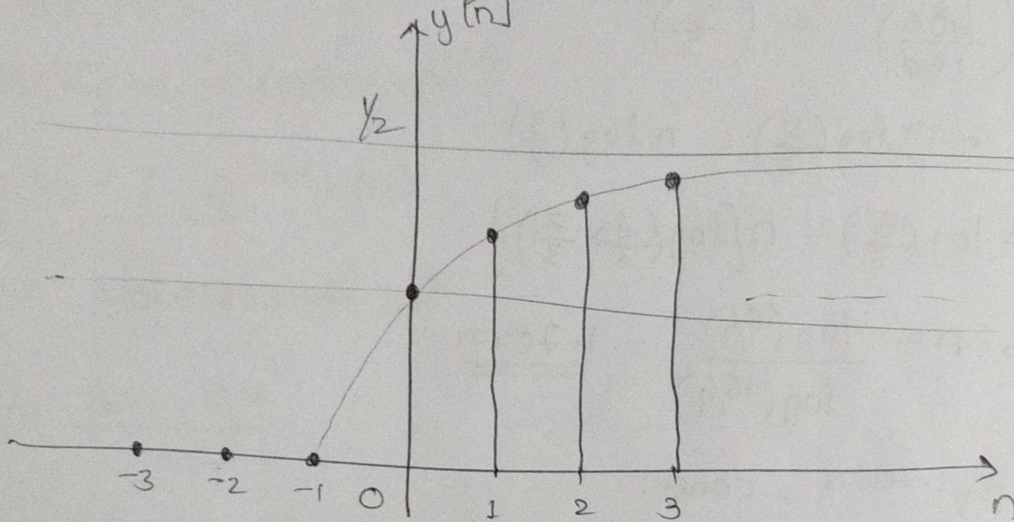
$$x[k]h[n-k] = \begin{cases} \left(\frac{1}{2}\right)^k, & \overset{0}{\cancel{2}} \leq k \leq n \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow y[n] = \begin{cases} 0, & n < \cancel{2}0 \\ \sum_{k=2}^{n} \left(\frac{1}{2}\right)^k, & n \geq \cancel{2}0 \end{cases}$$

$$y[n] = \begin{cases} 0, & n<0 \\ \left(\frac{1}{2}\right)^2\left[\frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}\right], & n\geq 0 \end{cases}$$

$$\Rightarrow y[n] = \begin{cases} 0, & n<0 \\ \frac{1}{2}-\left(\frac{1}{2}\right)^{n+2}, & n\geq 0 \end{cases}$$

$$\Rightarrow \boxed{y[n] = \left(\frac{1}{2}-\left(\frac{1}{2}\right)^{n+2}\right)u[n]}$$



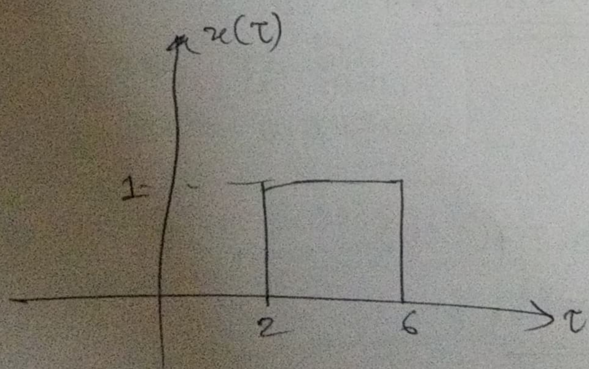
Q.4, $x(t) = u(t-2) - u(t-6)$ & $h(t) = e^{-2t}u(t)$

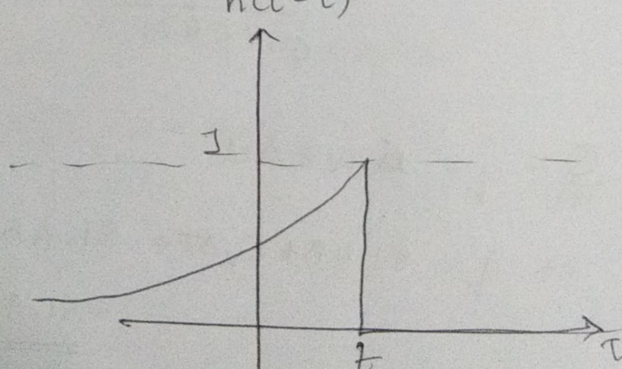
(a) $y(t) = x(t) * h(t)$

$$= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} [u(\tau-2) - u(\tau-6)]e^{-2(t-\tau)}u(t-\tau)d\tau$$

Complex $\Rightarrow$ Drawing $x(t)$ & $h(t-\tau)$





~~Q-1 $x(t) = u(t-2) - u(t-6)$ & $h(t) = e^{-2t}u(t)$~~

~~$\Rightarrow x(\tau)\,h(t-\tau)$~~

For $t<2$, $x(\tau)h(t-\tau)=0$

For $2 \le t \le 6$, $x(\tau)h(t-\tau) = \begin{cases} e^{-2(t-\tau)}, & 2 \le \tau \le t \\ 0, & \text{otherwise} \end{cases}$

For $t>6$, $x(\tau)h(t-\tau) = \begin{cases} e^{-2(t-\tau)}, & 2 \le \tau \le 6 \\ 0, & \text{otherwise.} \end{cases}$

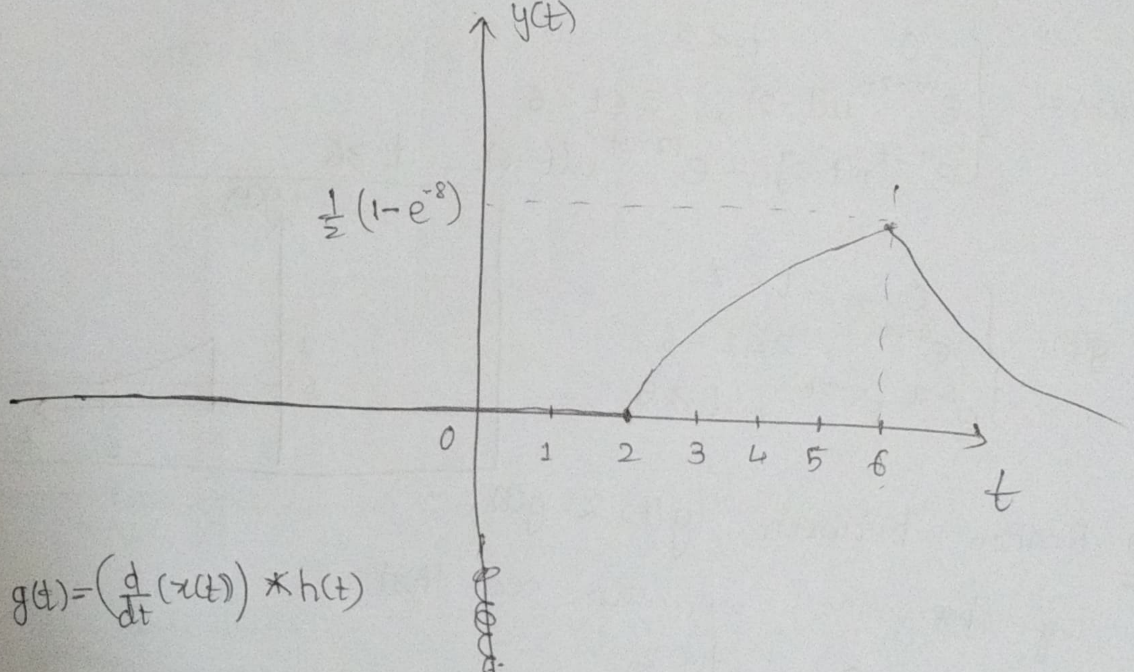
Now, $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$

$$= \begin{cases} 0, & t<2 \\ \int_2^t e^{2\tau-2t}\,d\tau, & 2 \le t < 6 \\ \int_2^6 e^{2\tau-2t}\,d\tau, & t>6 \end{cases}$$

$$= \begin{cases} 0, & t<2 \\ \frac{e^{-2t}}{2}\, e^{2\tau}\Big|_2^t & 2 \le t \le 6 \\ \frac{e^{-2t}}{2}\, e^{2\tau}\Big|_2^6, & t>6 \end{cases}$$

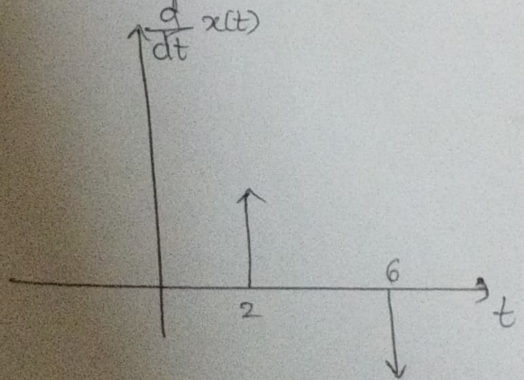
$$\Rightarrow y(t) = \begin{cases} 0 & , \ t<2 \\ \frac{e^{-2t}}{2}(e^{2t}-e^{4}) & , \ 2\le t\le 6 \\ \frac{e^{-2t}}{2}(e^{12}-e^{4}) & , \ t>6 \end{cases}$$

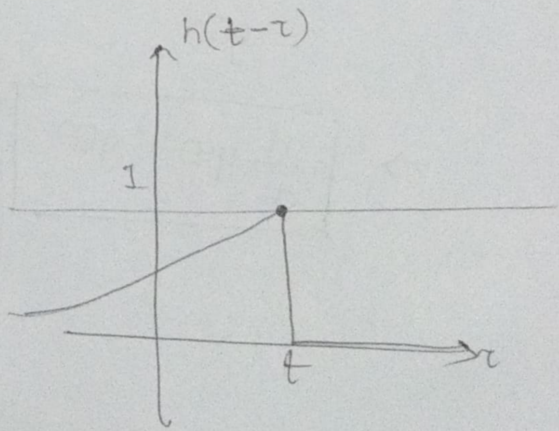
$$\Rightarrow y(t) = \begin{cases} 0 & , \ t<2 \\ \frac{1}{2}-\frac{e^{4}}{2}e^{-2t} & , \ 2\le t\le 6 \\ \left(\frac{e^{12}-e^{4}}{2}\right)e^{-2t} & , \ t>6 \end{cases}$$



(b) $g(t) = \left(\frac{d}{dt}(x(t))\right) * h(t)$

Now $\frac{d}{dt}(u(t)) = \delta(t)$





If $t<2 \Rightarrow x(\tau)h(t-\tau)=0$

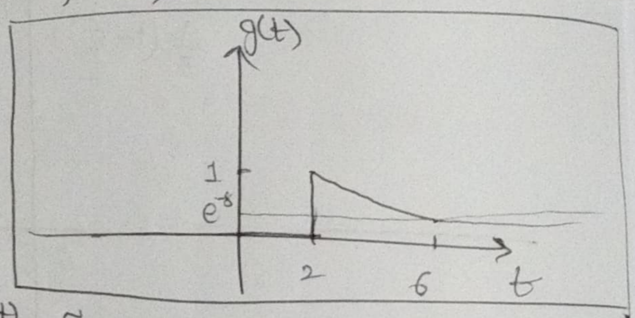
If $2\leq t < 6 \Rightarrow x(\tau)h(t-\tau)= \delta(\tau-2)\, e^{-2(t-\tau)} u(t-\tau)$

If $t>6 \Rightarrow x(\tau)h(t-\tau)= [\delta(\tau-2)-\delta(\tau-6)]\, e^{-2(t-\tau)}u(t-\tau)$

$$\Rightarrow g(t)= \begin{cases} 0, & t<2 \\ e^{-2(t-2)} u(t-2), & 2\leq t<6 \\ e^{-2(t-2)}u(t-2)\; e^{-2(t-6)}u(t-6) & t\geq 6 \end{cases}$$

$$\Rightarrow g(t)= \begin{cases} 0, & t<2 \\ e^{4-2t} u(t-2), & 2\leq t<6 \\ e^{4-2t}u(t-2) - e^{12-2t}u(t-6), & t\geq 6 \end{cases}$$

$$\Rightarrow g(t)= \begin{cases} 0, & t<2 \\ e^{4-2t}, & 2\leq t<6 \\ e^{4-2t}-e^{12-2t}, & t\geq 6 \end{cases}$$



(c) Relation between $y(t)$ & $g(t)$ ~

By observation, we can see that

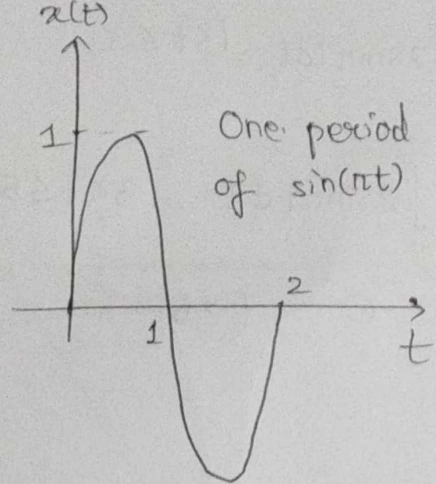
$$\frac{d}{dt} g(t)= \begin{cases} 0, & t<2 \\ e^{4}e^{-2t}, & 2\leq t<6 \\ (e^{4}-e^{12}).e^{-2t}, & t\geq 6 \end{cases}$$

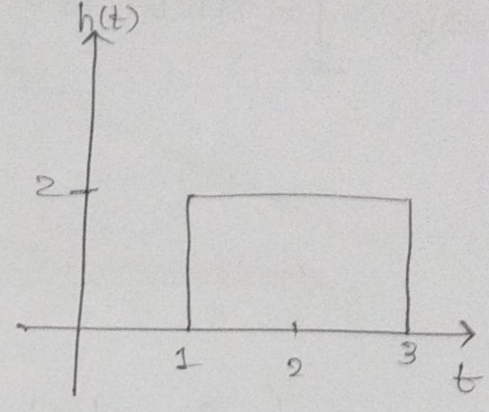
$= g(t)$

$$\Rightarrow \boxed{\frac{d}{dt} y(t) = g(t)}$$

Q5 - Plot y(t) for the following x(t) & h(t)

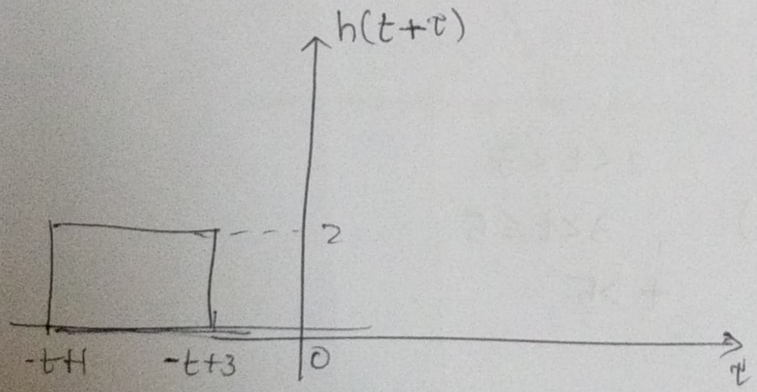
(a)

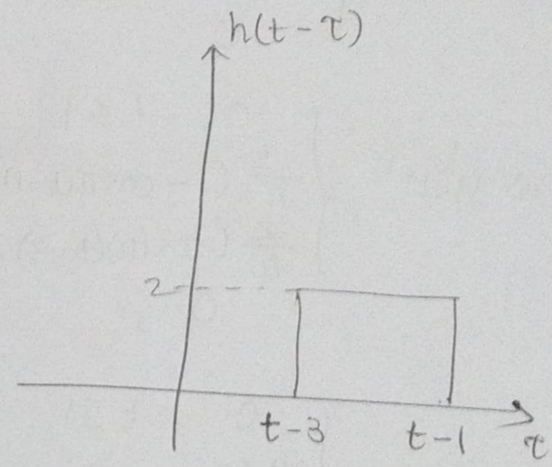




$y(t) = x(t) * h(t)$

Now,





~~Now,~~

~~$x(\tau)h(t-\tau) = \begin{cases} 0, & t<1 \\ & t-1\le 2 \end{cases}$~~

For $t \le 1$, $x(\tau)h(t-\tau) = 0$

For $0 < t-1 \le 2$, $x(\tau)h(t-\tau) = \begin{cases} 2\sin(\pi\tau), & 0 \le \tau \le t-1 \\ 0, & \text{otherwise} \end{cases}$

i.e. $1 < t \le 3$

For $2 < t-1 \le 4$, $x(\tau)h(t-\tau) = \begin{cases} 2\sin(\pi\tau), & t-3 \le \tau \le 2 \\ 0, & \text{otherwise} \end{cases}$

i.e. $3 < t \le 5$

For $t-1 \ge 4$, $x(\tau)h(t-\tau) = 0$

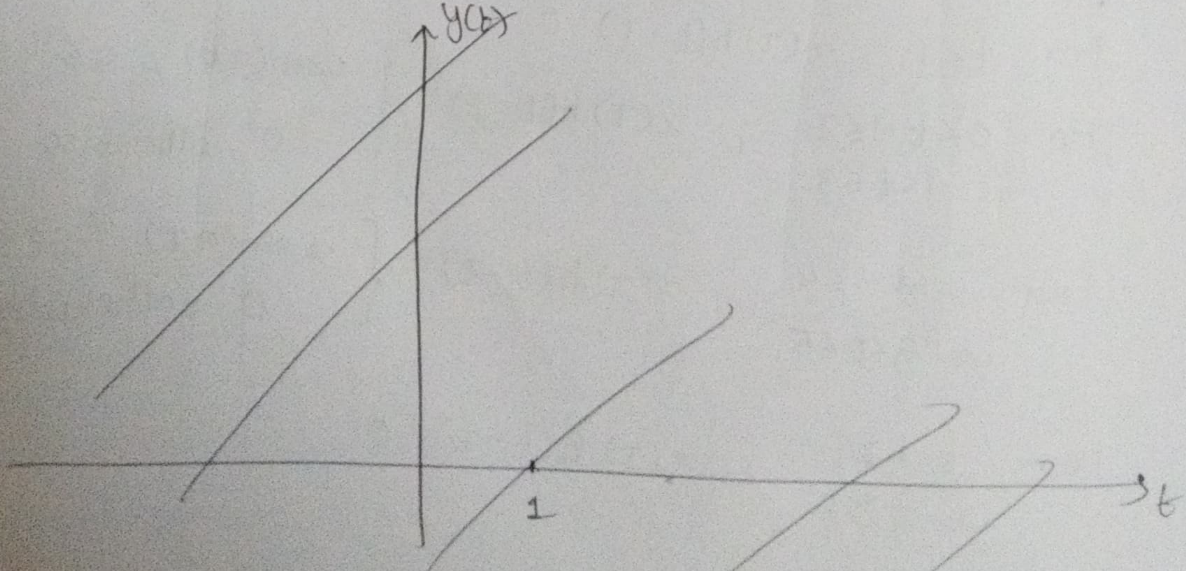
i.e. $t \ge 5$

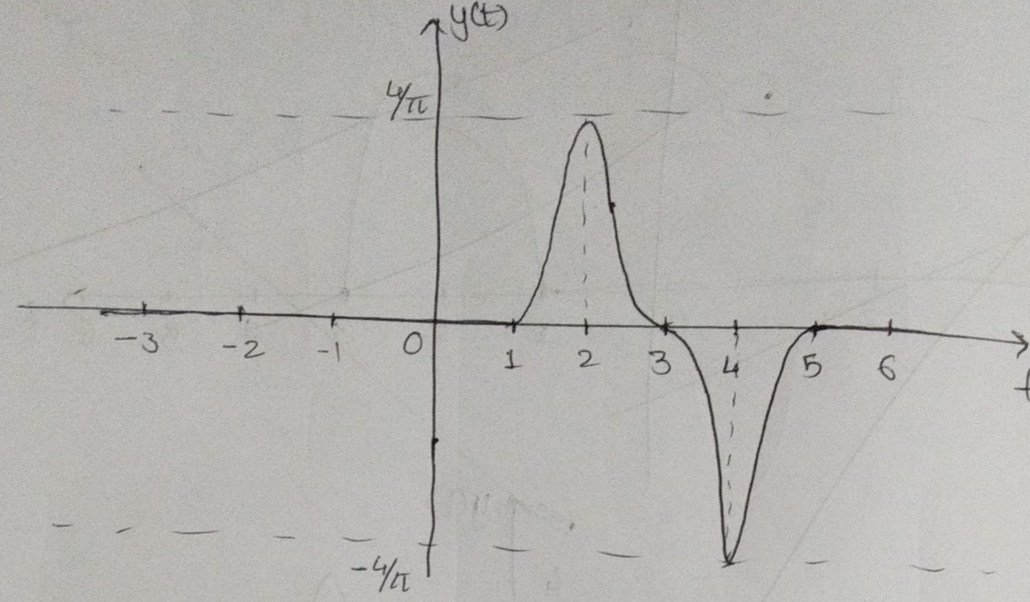
$$\Rightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \begin{cases} 0, & t \le 1 \\ \int_{0}^{t-1} 2\sin\pi\tau\, d\tau, & 1 < t \le 3 \\ \int_{t-3}^{2} 2\sin\pi\tau\, d\tau, & 3 < t \le 5 \\ 0, & t > 5 \end{cases}$$

$$\Rightarrow y(t) = \begin{cases} 0, & t \le 1 \\ -\frac{2}{\pi} \cos\pi\tau \Big|_{0}^{t-1}, & 1 < t \le 3 \\ -\frac{2}{\pi} \cos\pi\tau \Big|_{t-3}^{2}, & 3 < t \le 5 \\ 0, & t > 5 \end{cases}$$

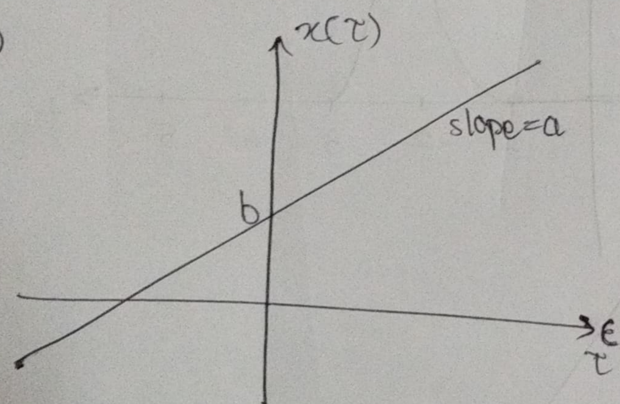
$$\Rightarrow y(t) = \begin{cases} 0, & t \le 1 \\ \frac{2}{\pi}(1 - \cos\pi(t-1)), & 1 < t \le 3 \\ \frac{2}{\pi}(\cos(\pi)(t-3) - 1), & 3 < t \le 5 \\ 0, & t > 5 \end{cases}$$

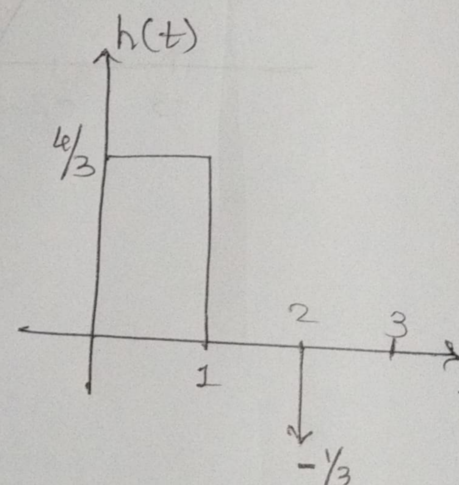
$$\Rightarrow y(t) = \begin{cases} 0, & t \le 1 \\ \frac{2}{\pi}(1 + \cos\pi t), & 1 < t \le 3 \\ \frac{2}{\pi}(-\cos\pi t - 1), & 3 < t \le 5 \\ 0, & t > 5 \end{cases}$$

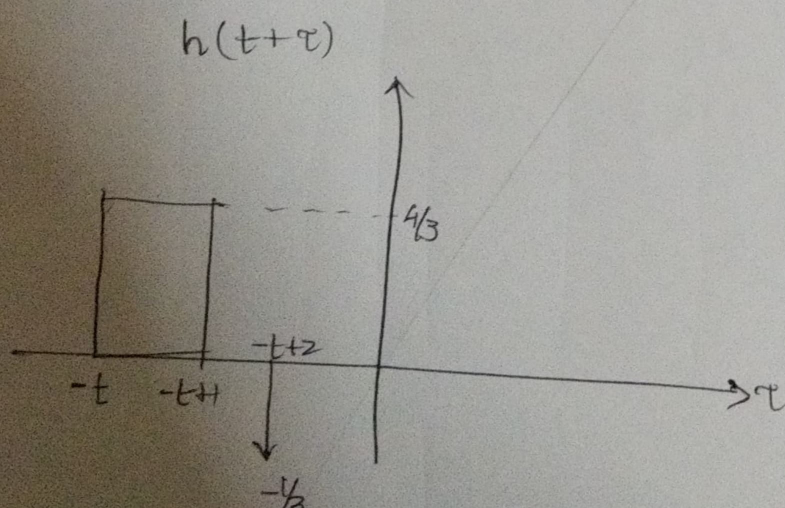


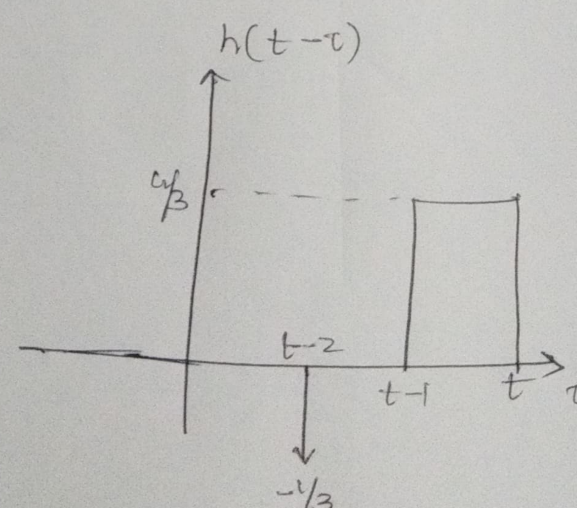

y(t)
4/π
−4/π
−3
−2
−1
0
1
2
3
4
5
6
t

(b)


x(τ)
slope=a
b
τ


h(t)
4/3
1
2
3
t
−1/3


h(t+τ)
4/3
−t
−t+1
−t+2
τ
−1/3


h(t−τ)
4/3
t−2
t−1
t
τ
−1/3

19

$$h(t-\tau) = \begin{cases} \cancel{\delta(t-2)} \\ \end{cases}$$

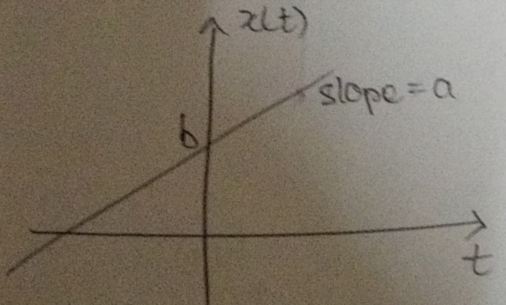
$$h(t-\tau) = \begin{cases} -\frac{1}{3}\delta(\tau-(t-2)) & , \tau = t-2 \\ \frac{4}{3} & , t-1 < \tau < t \\ 0 & , \text{otherwise} \end{cases}$$

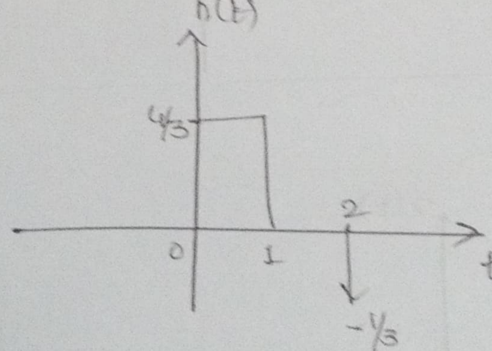
$x(\tau) = a\tau + b$

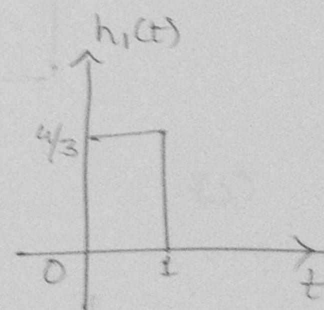
$$\Rightarrow x(\tau)h(t-\tau) = \begin{cases} -\frac{1}{3}\delta(\tau-(t-2)).(a\tau+b) & , \tau = t-2 \\ \frac{4}{3}(a\tau+b) & , t-1 < \tau < t \\ 0 & , \text{otherwise.} \end{cases}$$

This approach gets complicated.

Now, $y(t) = x(t) * h(t)$





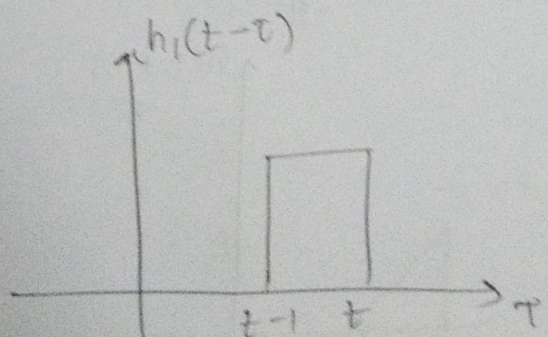


$x(t) = ax + b$

$$h_1(t) = \begin{cases} 4/3, & 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$h(t) = \delta(t-2) + h_1(t)$

$$\Rightarrow \boxed{y(t) = x(t) * h(t) = x(t) * h_1(t) - \frac{1}{3}x(t-2)}$$



$$x(t) * h_1(t) = \int_{-\infty}^{\infty} x(\tau)h_1(t-\tau)\,d\tau$$

$$= \int_{t-1}^{t} \frac{4}{3}(a\tau+b)\,d\tau$$

$$= \frac{4}{3}\left[\frac{a}{2}(t^2-(t-1)^2) + b\right]$$

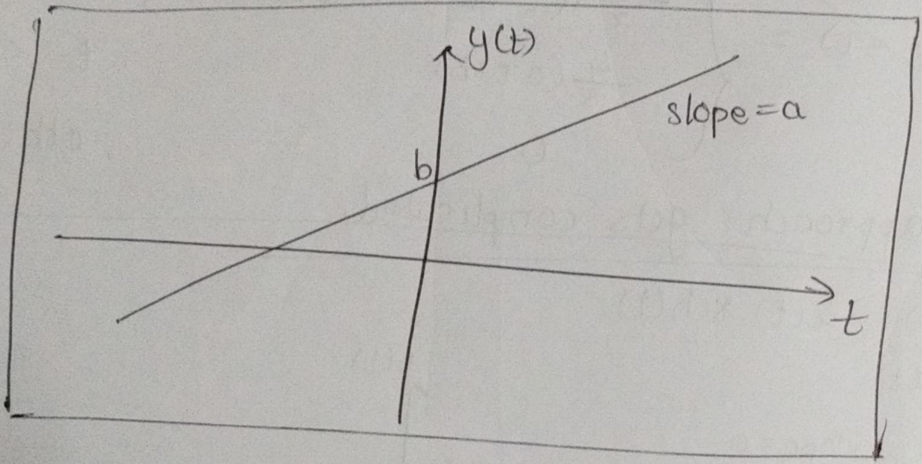
$$\Rightarrow y(t) = \frac{4}{3}\left[\frac{a}{2}\left(t^2-(t-1)^2\right)+b\right] - \frac{1}{3}\left[a(t-2)+b\right]$$

$$= \frac{4}{3}\left[\frac{a}{2}(2t-1)+b\right] - \frac{1}{3}\left[a(t-2)+b\right]$$

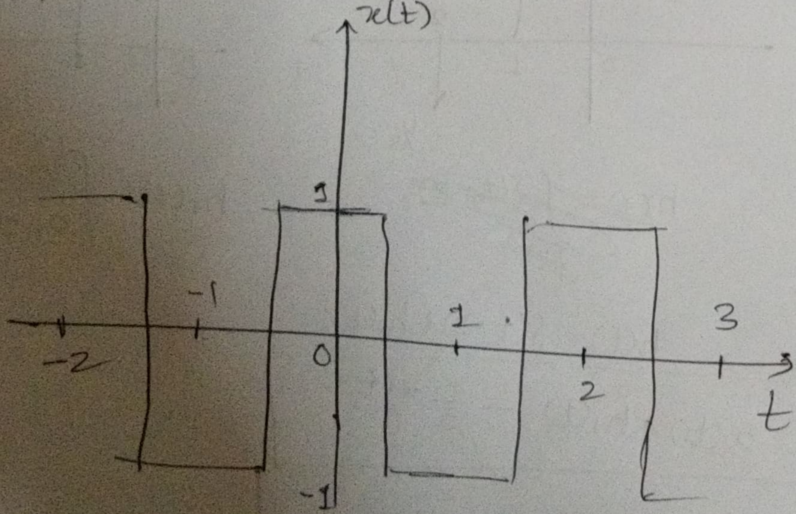
$$= \frac{4a}{3}t - \cancel{\frac{2a}{3}} + \frac{4b}{3} - \frac{at}{3} + \cancel{\frac{2a}{3}} - \frac{b}{3}$$

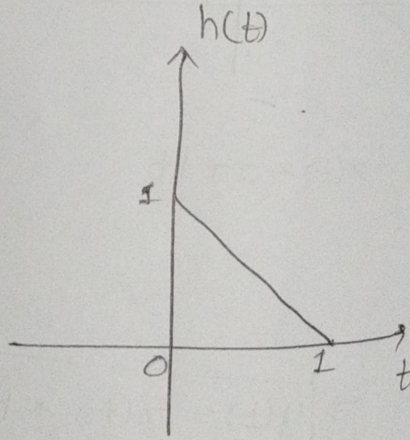
$$= at + b$$

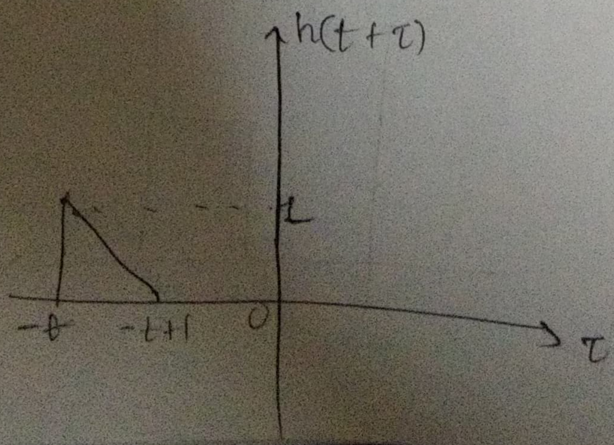
$$\Rightarrow \boxed{y(t) = x(t)}$$

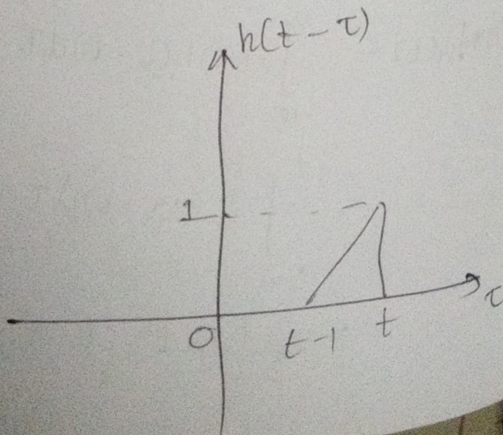


(c)





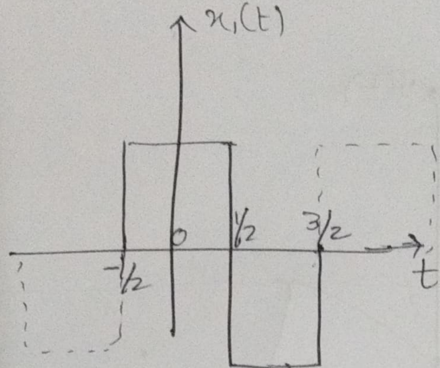




Now, $\because$ ~~x(t), fin~~

$\because$ $x(t)$ is periodic $\Rightarrow$ Finding answer for 1 period would be enough

Consider,



~~$h(t) =$~~

~~$h(t) = \tau - t + 1$~~

$h(t-\tau) = \tau - t + 1$

Now, for $-\frac{1}{2} < t < \frac{1}{2}$,

$$x(\tau)\, h(t-\tau) = \begin{cases} -(\tau - t + 1) & , \; t-1 < \tau < -1/2 \\ \tau - t + 1 & , \; -1/2 < \tau < t \end{cases}$$

For $\frac{1}{2} < t < \frac{3}{2}$,

$$x(\tau)\, h(t-\tau) = \begin{cases} \tau - t + 1 & , \; t-1 < \tau < 1/2 \\ -(\tau - t + 1) & , \; 1/2 < \tau < t \end{cases}$$

$$\Rightarrow y_1(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \begin{cases} \int_{t-1}^{-1/2} -(\tau - t + 1)\, d\tau + \int_{-1/2}^{t} (\tau - t + 1)\, d\tau & , \; -\frac{1}{2} < t < \frac{1}{2} \\ \int_{t-1}^{1/2} (\tau - t + 1)\, d\tau + \int_{1/2}^{t} -(\tau - t + 1)\, d\tau & , \; \frac{1}{2} < t < \frac{3}{2} \end{cases}$$

$$= \begin{cases} -\left(\frac{1}{2}\left(\frac{1}{4} - (t-1)^2\right) - t\left(-\frac{1}{2} - t + 1\right) + \left(-\frac{1}{2} - t + 1\right)\right) \\ \quad + \left[\frac{1}{2}\left(t^2 - \frac{1}{4}\right) - t\left(t + \frac{1}{2}\right) + t + \frac{1}{2}\right] & , \; -\frac{1}{2} < t < \frac{1}{2} \\ \left[\frac{1}{2}\left(\frac{1}{4} - (t-1)^2\right) - t\left(\frac{1}{2} - t + 1\right) + \frac{1}{2} - t + 1\right] \\ \quad - \left[\frac{1}{2}\left(t^2 - \frac{1}{4}\right) - t\left(t - \frac{1}{2}\right) + t - \frac{1}{2}\right] & , \; \frac{1}{2} < t < \frac{3}{2} \end{cases}$$

$$\therefore\ y_1(t) = \begin{cases} \frac{1}{4} + t - t^2 & , -\frac{1}{2} < t < \frac{1}{2} \\ t^2 - 3t + \frac{7}{4} & , \frac{1}{2} < t < \frac{3}{2} \end{cases}$$