ECN-203 - Quiz 1

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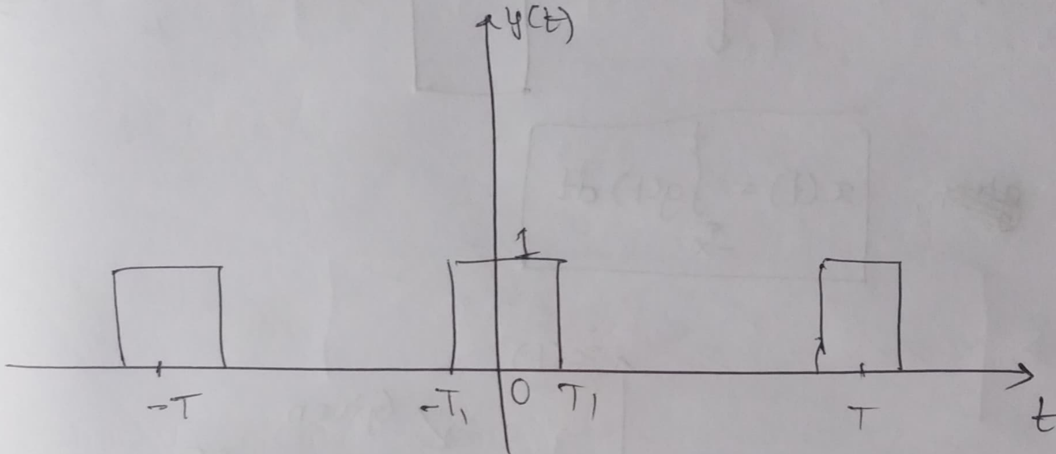
Branch: CSE

Q.1.



$a_0 = \frac{2T_1}{T}$ $\qquad a_k = \frac{\sin(k\omega_0 T_1)}{k\pi}$

For required signal, $T=6$

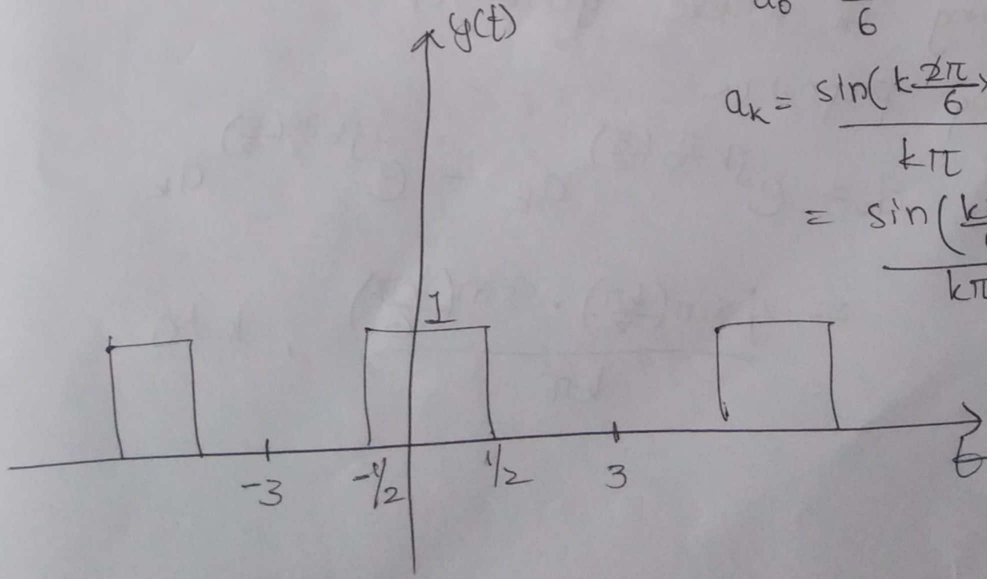
Consider $T_1 = 1/2$

$\Rightarrow y(t)$ becomes,

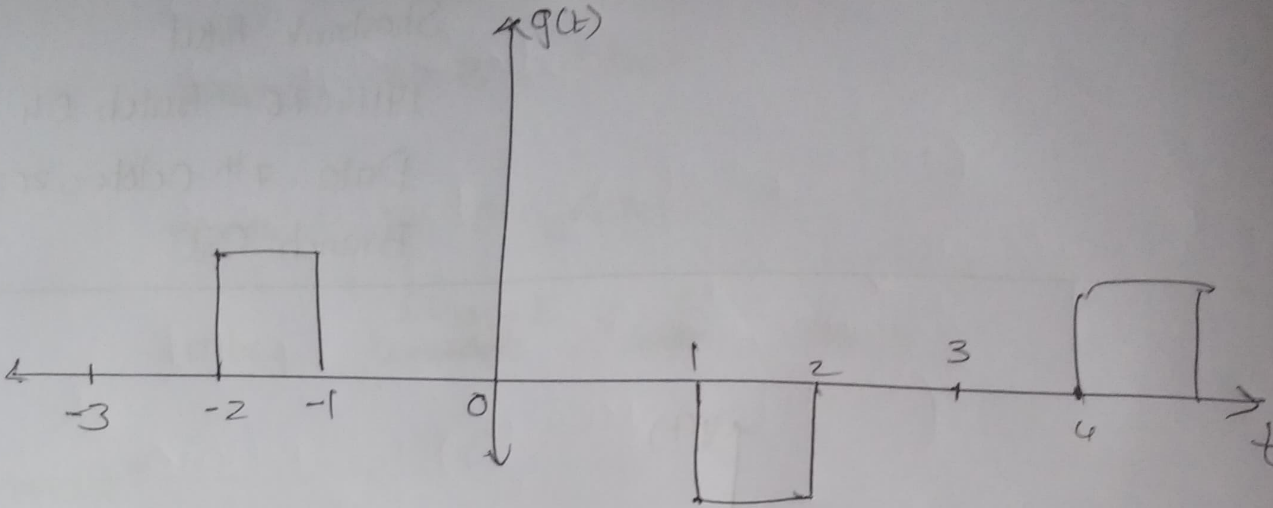
$a_0 = \frac{1}{6}$

$a_k = \frac{\sin\left(k \cdot \frac{2\pi}{6} \times \frac{1}{2}\right)}{k\pi}$

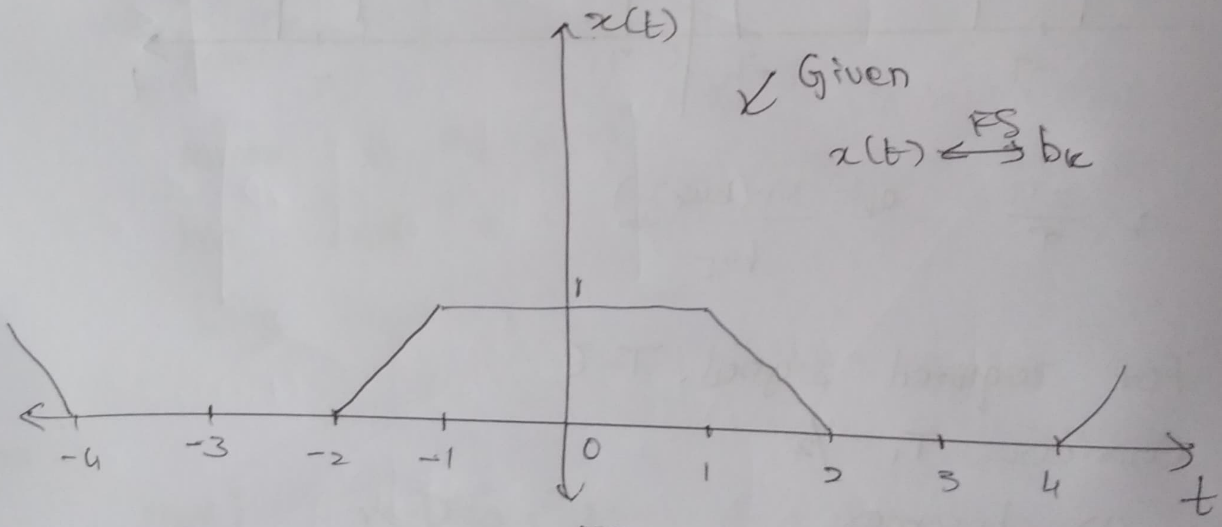
$= \frac{\sin\left(\frac{k\pi}{6}\right)}{k\pi}$



Consider $g(t) = y(t+1.5) - y(t-1.5)$



Now ~~given~~, $\boxed{x(t) = \int_{-\infty}^{t} g(t)\, dt}$



✓ Given

$x(t) \xleftrightarrow{FS} b_k$

Finding c_k where $g(t) \xleftrightarrow{FS} c_k$

Using properties,

$$c_k = e^{jk\frac{\pi}{3}\left(\frac{3}{2}\right)} a_k - e^{-jk\frac{\pi}{3}\left(\frac{3}{2}\right)} a_k$$

$$= \frac{2j \sin\left(\frac{k\pi}{2}\right) \cdot \sin\left(\frac{k\pi}{6}\right)}{k\pi}, \quad k \neq 0$$

$$\cancel{c_0 = e^{jk\frac{\pi}{3}\times\frac{3}{3}}\left(\frac{1}{6}\right) - e^{-jk\frac{\pi}{3}\times\frac{3}{3}}\left(\frac{1}{6}\right)}$$

$$c_0 = \frac{1}{T}\int_T g(t)dt = \underline{\underline{0}}.$$

$\Rightarrow$ For finding $x(t) \xleftrightarrow{FS} b_k$

$$b_0 = \frac{1}{T}\int_T y(t)dT$$

$$= \frac{1}{6}\int_{-2}^{2} y(t)dT$$

$$= \frac{1}{6}(3) = \frac{1}{2}$$

$$b_k = \frac{1}{jk\omega_0} c_k \quad , k \neq 0$$

$$= \frac{1}{\cancel{j}k\left(\frac{\pi}{3}\right)} \frac{2\cancel{j}\sin\left(\frac{k\pi}{2}\right)\sin\left(\frac{k\pi}{6}\right)}{k\pi}$$

$$= \frac{6}{k^2\pi^2}\sin\left(\frac{k\pi}{2}\right)\sin\left(\frac{k\pi}{6}\right)$$

$$\Rightarrow b_k = \begin{cases} \frac{1}{2}, & k = 0 \\ \frac{6}{k^2\pi^2}\sin\left(\frac{k\pi}{2}\right)\sin\left(\frac{k\pi}{6}\right) & \text{, otherwise} \end{cases}$$

$$\Rightarrow b_k = \begin{cases} \frac{1}{2}, & k = 0 \\ \frac{6a_k}{k\pi}\sin\left(\frac{k\pi}{2}\right) & \text{, otherwise} \end{cases}$$

Q.2, (a) $x(t) = \cos(4\pi t)$, $y(t) = \sin(6\pi t)$

Let $x(t) \xleftrightarrow{FS} a_k$

Let $y(t) \xleftrightarrow{FS} b_k$

Now, $\cos 4\pi t = \frac{1}{2}(e^{j4\pi t} + e^{-j4\pi t})$

Let us consider $T = 1$, $\omega_0 = 2\pi$ — This choice is for solving part (b) easily

$\Rightarrow \cos 4\pi t = \frac{1}{2}(e^{j2(2\pi)t} + e^{-j2(2\pi)t})$

Now $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

$\Rightarrow \cancel{a_2 = \frac{1}{2},\ a_{-2} = \frac{1}{2},\ a_k = 0, \text{ else}}$

$\Rightarrow \boxed{a_k = \begin{cases} \frac{1}{2}, & k = 2, -2 \\ 0, & \text{else} \end{cases}}$ where $\omega_0 = 2\pi$

$\cancel{\sin(6\pi t) = \frac{1}{2j}(e^{j3(2\pi)t} - e^{-j3(2\pi)t}) = \frac{-j}{2}(j e^{j3(2\pi)t} - e^{-j3(2\pi)t})}$

$\cancel{\Rightarrow b_k = \begin{cases} \frac{j}{2}, & k = 3 \\ -\frac{j}{2}, & k = -3 \\ 0, & \text{else} \end{cases}}$

$\sin(6\pi t) = \frac{j}{2}\left(e^{-j3(2\pi)t} - e^{j3(2\pi)t}\right)$

$$\Rightarrow \boxed{b_k = \begin{cases} -\frac{j}{2}, & k=3 \\ \frac{j}{2}, & k=-3 \\ 0, & \text{else} \end{cases}} , \quad \omega_0 = 2\pi$$

Q-2(b) Both $x(t)$ & $y(t)$ are periodic with period $\underline{\underline{1}}$.

$\Rightarrow z(t) = x(t)y(t)$ is periodic with period 1

$\Rightarrow z(t) = x(t)y(t) \longleftrightarrow c_k = \sum_{l} a_l b_{k-l}$

Now, $a_k = 0$ except $k = 2, -2$

& $b_k = 0$ except $k = 3, -3$

$\Rightarrow c_k = 0$ except $k = -5, -1, 1, 5$ (Rest are zero because $a_l b_{k-l}$ are zero)

$\Rightarrow c_{-5} = a_{-2} b_{-5+2} = a_{-2} b_{-3} = \frac{1}{2} \times \frac{j}{2} = \underline{\underline{\frac{j}{4}}}$

$\Rightarrow c_{-1} = a_{+2} b_{-1-2} = a_{+2} b_{-3} = \frac{1}{2} \times \frac{j}{2} = \frac{j}{4}$

$\Rightarrow c_1 = a_{-2} b_{1+2} = a_{-2} b_3 = \frac{1}{2} \times \left(-\frac{j}{2}\right) = -\frac{j}{4}$

$\Rightarrow c_5 = a_2 b_{5-2} = a_2 b_3 = \frac{1}{2} \times \left(-\frac{j}{2}\right) = -\frac{j}{4}$

$$\Rightarrow \boxed{c_k = \begin{cases} -\frac{j}{4}, & k = 1, 5 \\ \frac{j}{4}, & k = -1, -5 \\ 0, & \text{otherwise} \end{cases}}$$

Q-3. Given that $T=6$

Now $x(t)$ is real $\Rightarrow$ $a_k = a_{-k}^*$

Also, $x(t) = a_{-1} e^{-j\frac{\pi}{3}t} + a_0 + a_1 e^{j\frac{\pi}{3}t}$

Also, $1 = |a_0|^2 + |a_{-1}|^2 + |a_1|^2$

$1 = |a_0|^2 + 2|a_1|^2$

Also, $x(t) + x(t-3) = 0$

$\Rightarrow$ $x(t+3/2)$ is odd

$\Rightarrow$ $a_k e^{j\frac{3}{2}\times\omega_0}$ is odd

$\Rightarrow$ a_k is odd $\Rightarrow a_0 = 0$

$\Rightarrow$ $2|a_1|^2 = 1$

$\Rightarrow$ $a_1 = \frac{1}{\sqrt{2}} = a_{-1}$ ($\because$ a_k's are real & +ve ... given)

$\Rightarrow x(t) = \frac{1}{\sqrt{2}}\left[e^{-j\frac{\pi}{3}t} + e^{j\frac{\pi}{3}t}\right]$

$= \frac{1}{\sqrt{2}} \times 2\cos\left(\frac{\pi}{3}t\right)$

$\Rightarrow \boxed{x(t) = \sqrt{2}\cos\left(\frac{\pi}{3}t\right)}$