Indian Institute of Technology Roorkee
MAN-001 (Mathematics-I)
Autumn Semester: 2019-20
Assignment-8: (Applications of Multiple Integrals)

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1. Find the area inside the circle $r = a\sin\theta$ and outside the cardioid $r = a(1-\cos\theta)$.

2. Find the area bounded by the circles $r = a\cos\theta$ and $r = a\sin\theta$.
3. Find the value of the triple integral $\iiint_R xydxdydz$ where R is the cylindrical solid $x^2 + y^2 \le 1$ with $0 \le z \le 1$.
4. Evaluate $\iiint_R \sqrt{x^2 + y^2}\,dxdydz$, where R is the solid bounded by the right circular cone $x^2 + y^2 = z^2$, $z > 0$ and the planes $z = 0$ and $z = 1$.

5. Using the definition of average value of a function $f(x,y,z)$ over a solid region D as $\frac{1}{vol.\ of\ D}\iiint_D f(x,y,z)dv$, find the average value of the function $f(x,y,z) = x + y + z$ over the sphere $x^2 + y^2 + z^2 = 4$.
6. Find the volume bounded above by the sphere $x^2 + y^2 + z^2 = a^2$ and below by the cone $x^2 + y^2 = z^2$.
7. Find the volume bounded by the surfaces $z = 4 - x^2 - \frac{1}{4}y^2$ and $z = 3x^2 + \frac{y^2}{4}$.
8. Find the volume of the wedge intercepted between the cylinder $x^2 + y^2 = 2ax$ and planes $z = mx,\ z = nx,\ (m > n)$.
9. Evaluate $\iiint z^2 dxdydz$ over the volume bounded by the cylinder $x^2 + y^2 = a^2$ and the paraboloid $x^2 + y^2 = z$ and the plane $z = 0$.
10. Find the centre of gravity of the area bounded by the parabola $y^2 = x$ and the line $x + y = 2$.
11. Find the mass of a plate in the form of one loop of lemniscate $r^2 = a^2\cos 2\theta$ if the density varies as the square of the distance from the pole.

12. A solid has the shape determined by the graphs of the cylinder $|x| + |y| = 1$ and the planes $z = 2$ and $z = 4$. Find its mass if the density is given by $\rho(x,y,z) = kz$, where k is a constant. Also find the moment of inertia of the solid about the z-axis.

Answers: **1.** $a^2\left(1 - \frac{\pi}{4}\right)$; **2.** $\frac{a^2}{4}\left(\frac{\pi}{2} - 1\right)$; **3.** 0; **4.** $\frac{\pi}{6}$; **5.** 0; **6.** $\frac{2\pi a^3}{3}\left(1 - \frac{1}{\sqrt{2}}\right)$;

7. $4\sqrt{2}\pi$; **8.** $(m-n)\pi a^3$; **9.** $\frac{\pi}{12}a^8$; **10.** $\left(\frac{8}{5}, -\frac{1}{2}\right)$; **11.** $\frac{ka^4\pi}{16}$; **12.** $12k,\ 4k$..