ECN-203 Assignment 1

Ans. 1 (a)

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x_1(t)|^2 \, dt \qquad \Bigg| \qquad x_1(t) = \begin{cases} e^{-2t}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$= \lim_{T\to\infty} \left[\int_{-T}^{0} (0)\, dt + \int_{0}^{T} (e^{-2t})^2 dt \right]$$

$$= \lim_{T\to\infty} \left[\int_{0}^{T} -\frac{1}{4} (e^{-4t}) \, dt \right]$$

$$E_\infty = -\frac{1}{4} \left[\lim_{T\to\infty} \left(e^{-4T} - e^{0} \right) \right] = \frac{1}{4}$$

$$\Rightarrow \boxed{E_\infty = \frac{1}{4}}$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x_1(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left(\frac{1}{4} \right) = 0$$

$$\Rightarrow \boxed{P_\infty = 0}$$

Ans. 1 (b) $x_2[n] = \cos\left(\frac{\pi}{4} n\right), \quad -\infty < n < \infty \; ; \; n \in I$

$$\therefore E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} \left| x_2[n] \right|^2$$

$$= \lim_{N\to\infty} \sum_{n=-N}^{N} \left[\cos^2\left(\frac{\pi n}{4} \right) \right]$$

$$= \lim_{N\to\infty} \sum_{n=-N}^{N} \left[\frac{1 + \cos(\pi/2\, n)}{2} \right]$$

$$= \lim_{N\to\infty} \left[N + \sum_{n=-N}^{N} \frac{\cos(\pi/2\, n)}{2} \right] = \cancel{\lim_{N\to\infty} \left[\frac{N + \sin\left((2N+1)\frac{\pi}{4}\right) \cos}{\sin\left(\frac{\pi}{4}\right)} \right]}$$

$$\Rightarrow \lim_{N\to\infty}\left[N + \frac{1}{2}\left(\frac{\sin\left((2n+1)\frac{\pi}{4}\right)}{\sin(\pi/4)}\cos\left((2n)\frac{\pi}{4}\right)\right)\right]$$

$$\rightarrow \boxed{E_\infty \rightarrow \infty}$$

Now, $P_\infty = \lim_{N\to\infty}\frac{1}{(2N+1)}\sum_{n=-N}^{N}\left|x_2[n]\right|^2$

$$\boxed{P_\infty = \frac{1}{2}}$$

Ans. 2 (a) $x_1(n) = \begin{cases} 2, & 0 \le n \le 6 \\ 0, & \text{otherwise} \end{cases}$ | $x_1(-n) = \begin{cases} 0, & \cancel{0 \le n} \text{ othe} \\ 2, & 0 \ge n \end{cases}$

$n \in$ ...

$$= \left\{ 1, \quad 0 \le n \le 6 \right\}$$

$$= \begin{cases} 0, & n \\ -1, & -6 \le n \le -1 \\ 0, & n \le -7 \end{cases}$$

Ans.2 (a) $x_1(n) = \begin{cases} 2, & 0 \le n \le 6 \\ 0, & \text{otherwise} \end{cases}$ | $x_1(-n) = \begin{cases} 0, & \text{otherwise} \\ 2, & -6 \le n \le 0 \end{cases}$

$$* \; OD\{x_1(n)\} = \begin{cases} 0, & n \ge 7 \\ 1, & 1 \le n \le 6 \\ 0, & n = 0 \\ -1, & -6 \le n \le -1 \\ 0, & n \le -7 \end{cases}$$

* $ED\{x_1(n)\} = \begin{cases} 0, & n \geq 7 \\ 1, & 1 \leq n \leq 6 \\ 2, & n = 0 \\ 1, & -6 \leq n \leq -1 \\ 0, & n \leq -7 \end{cases}$

<u>Ans. 2 (b)</u> $x_2(t) = \sin\left(\frac{t}{2}\right), \; -\infty < t < \infty, \; t \in R$

$$x_2(-t) = -\sin\left(\frac{t}{2}\right)$$

$$OD\{x_2(t)\} = \sin\left(\frac{t}{2}\right)$$

$$ED\{x_2(t)\} = 0$$

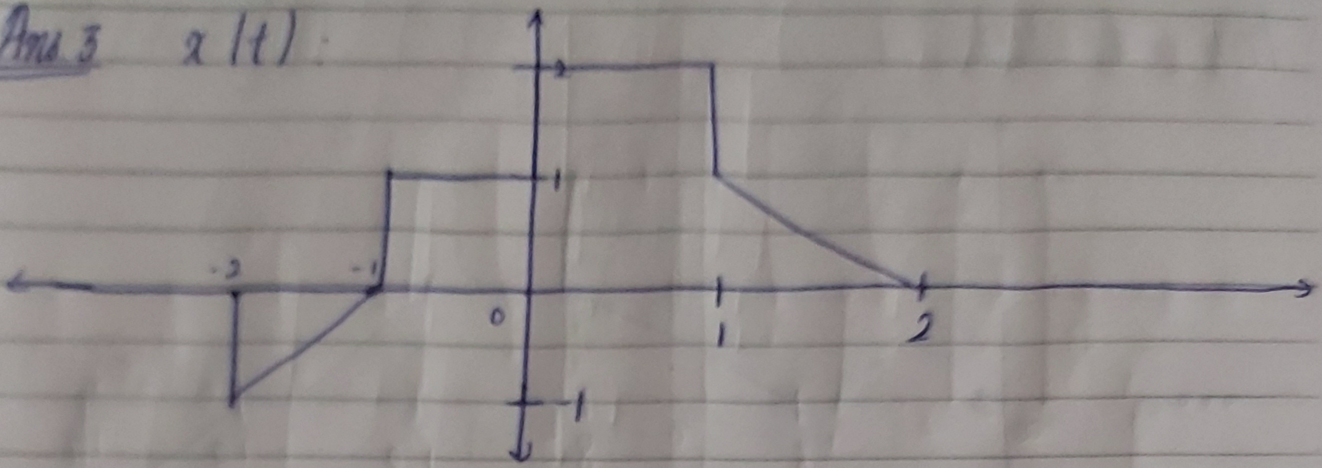
<u>Ans. 2 (c)</u> $x_3(n) = \begin{cases} (1/2)^n, & n \geq 3 \\ 0, & \text{otherwise} \end{cases}, \; n \in I$

$$x_3(-n) = \begin{cases} (1/2)^{-n}, & -n \geq 3 \\ 0, & \text{otherwise} \end{cases}$$

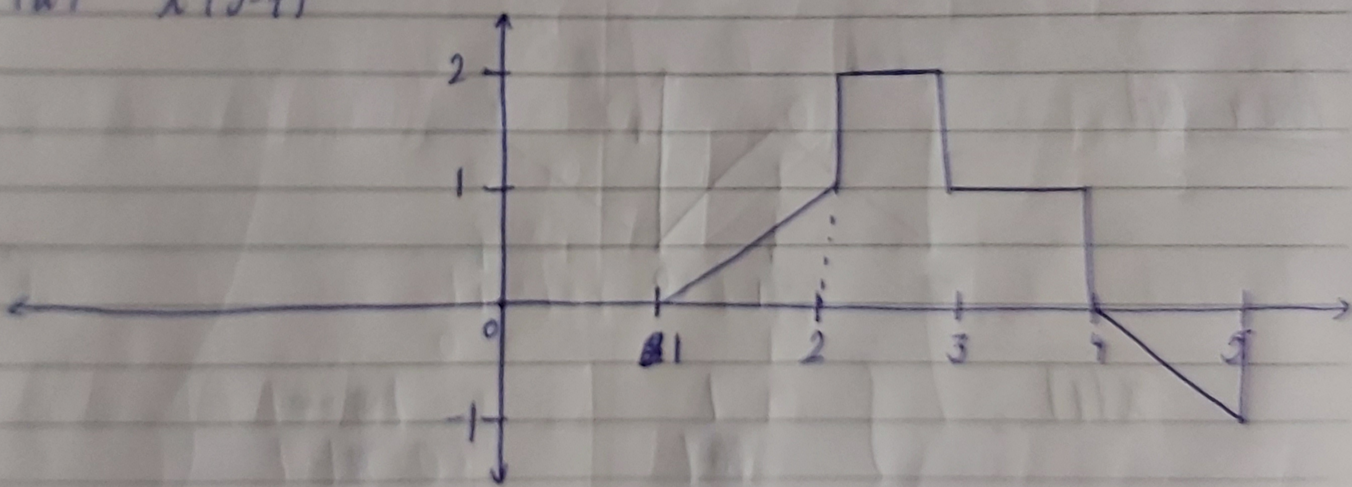
$$OD\{x_3(n)\} = \begin{cases} (1/2)^{n+1}, & n \geq 3 \\ 0, & -2 \leq n \leq 2 \\ -(1/2)^{-n+1}, & n \leq -3 \end{cases}$$

$$ED\{x_3(n)\} = \begin{cases} (1/2)^{n+1}, & n \geq 3 \\ 0, & -2 \leq n \leq 2 \\ (1/2)^{-n+1}, & n \leq -3 \end{cases}$$

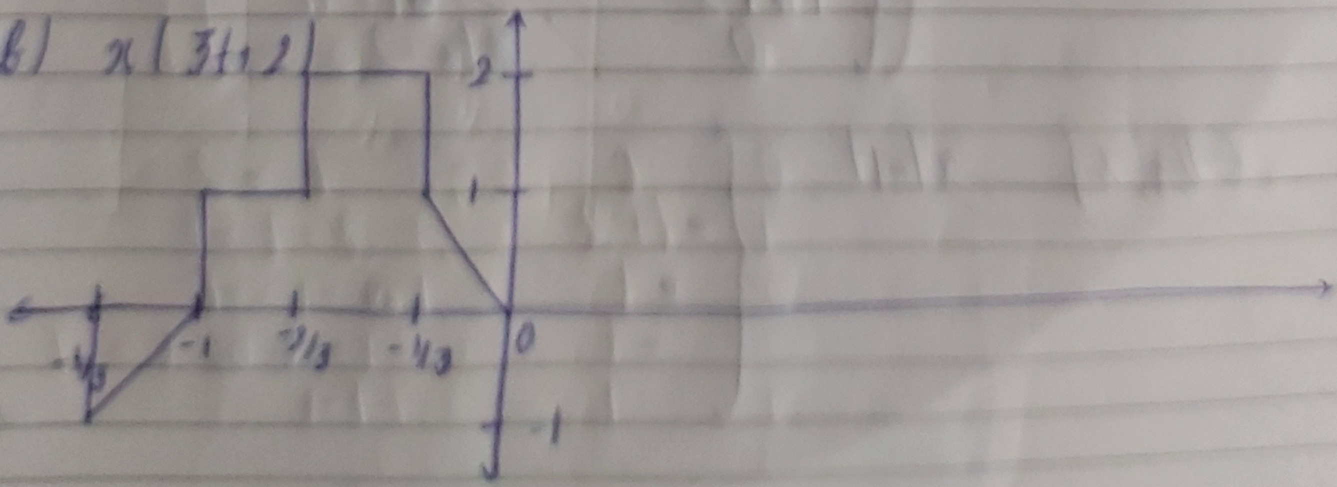
Ans 3 $x(t)$:



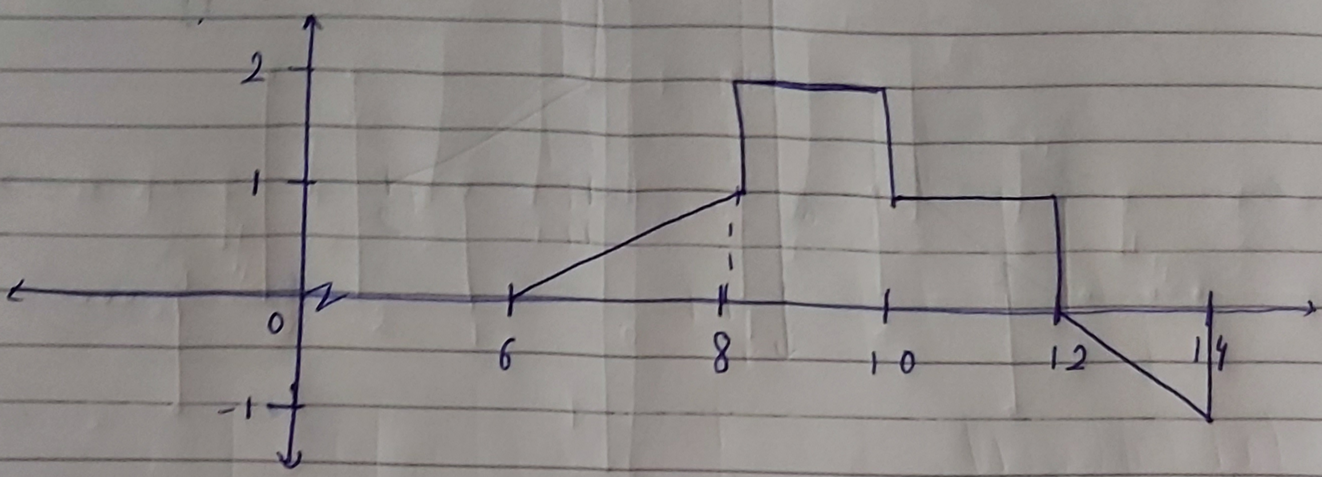
(a) $x(3-t)$



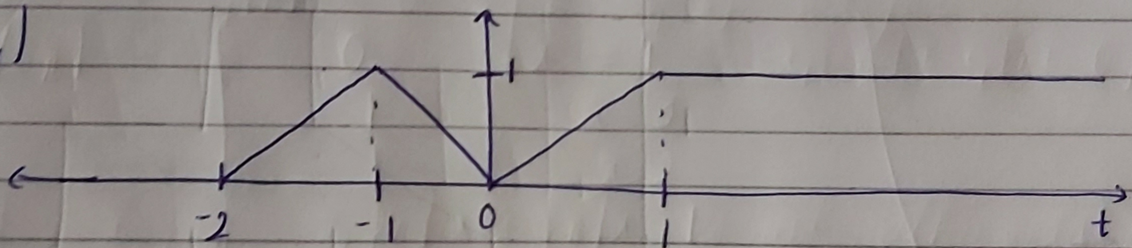
(b) $x(3t+2)$



(c) $x(5-t/2)$



Ans. 4 (a)



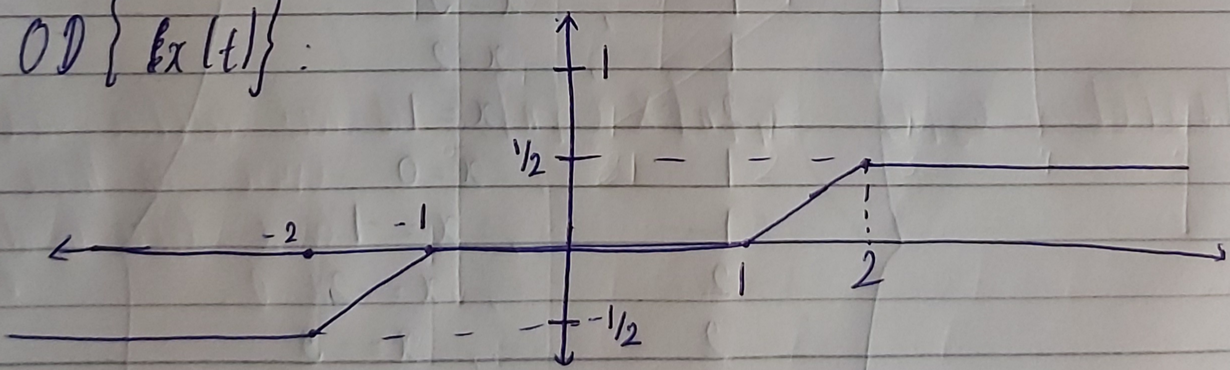
$$x(t) = \begin{cases} 1 & , \quad t \geq 1 \\ t & , \quad 0 \leq t \leq 1 \\ -t & , \quad -1 \leq t \leq 0 \\ t+2 & , \quad -2 \leq t \leq -1 \\ 0 & , \quad t \leq -2 \end{cases}$$

~~OD{x}~~ $x(-t) = \begin{cases} 0 & , \quad t \geq 2 \\ -t+2 & , \quad 2 \geq t \geq 1 \\ t & , \quad 1 \geq t \geq 0 \\ -t & , \quad \cancel{1 \geq t \geq 0}\ 0 \geq t \geq -1 \\ 1 & , \quad -1 \geq t \end{cases}$

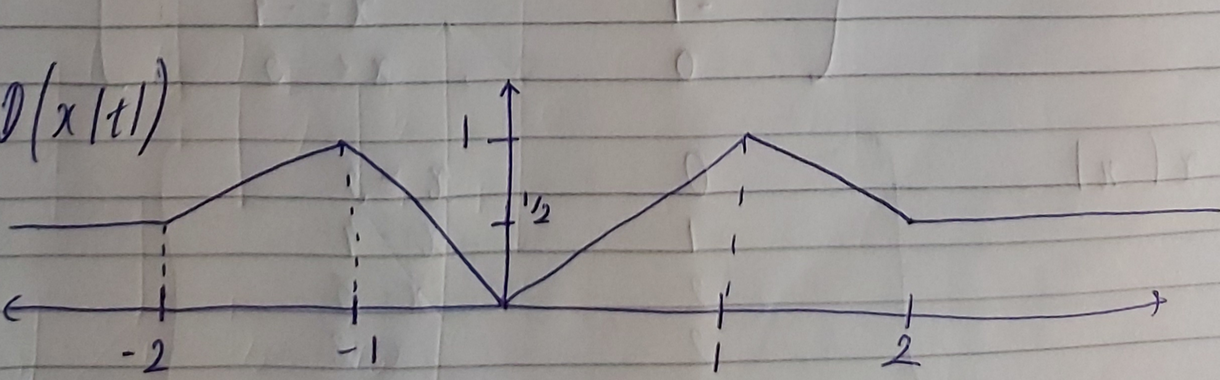
$$OD\{x(t)\} = \begin{cases} 1/2 & t \geq 2 \\ (-1+t)/2 & 2 \geq t \geq 1 \\ 0 & -1 \leq t \leq 1 \\ (t+1)/2 & , -2 \leq t \leq -1 \\ -1/2 & , t \leq -2 \end{cases}$$

$$ED\{x(t)\} = \begin{cases} 1/2 & , t \geq 2 \\ (-t+3)/2 & , 2 \geq t \geq 1 \\ t & , 1 \geq t \geq 0 \\ -t & , -1 \leq t \leq 0 \\ (t+3/2) & , -2 \leq t \leq -1 \\ 1/2 & , t \leq -2 \end{cases}$$

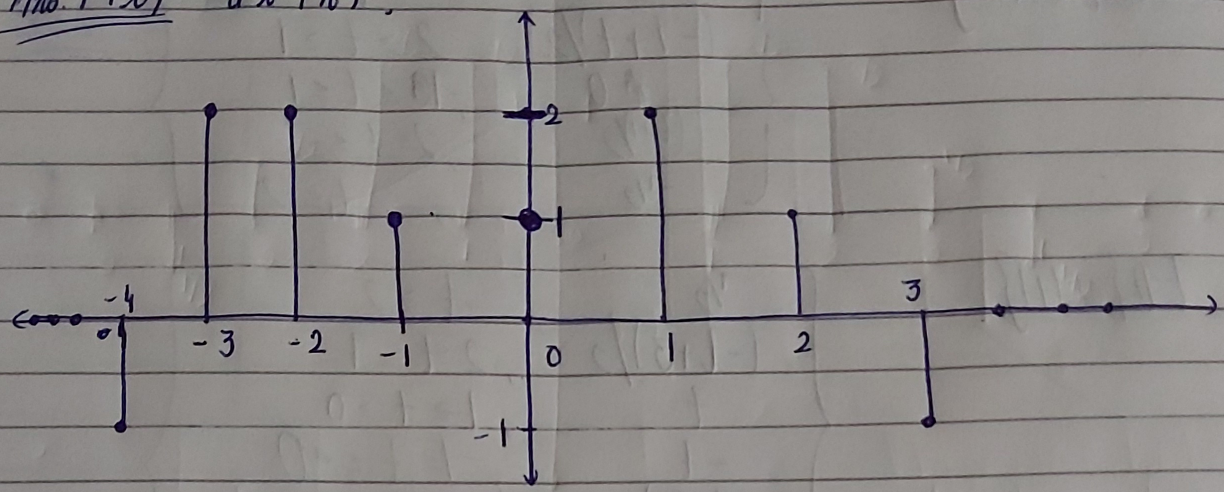
$OD\{x(t)\}$:



$ED(x(t))$



Ans. 4 (b) $x(n)$:



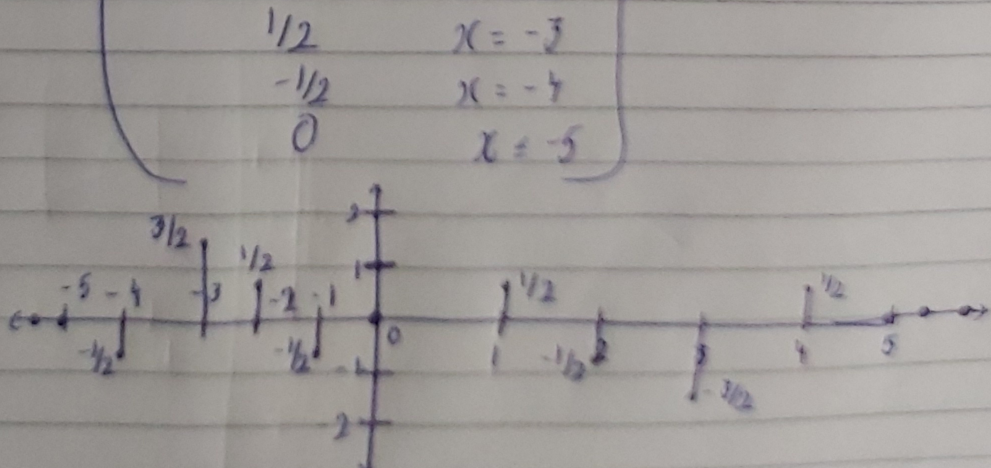
$$x(n) = \begin{cases} 0, & x \geq 4 \\ -1, & x = 3 \\ 1, & x = 2 \\ 2, & x = 1 \\ 1, & x = 0 \\ 1, & x = -1 \\ 2, & x = -2 \\ 2, & x = -3 \\ -1, & x = -4 \\ 0, & x \leq -5 \end{cases}$$

$$x(-n) = \begin{cases} 0 & x \geq 5 \\ -1 & x = 4 \\ 2 & x = 3 \\ 2 & x = 2 \\ 1 & x = 1 \\ 1 & x = 0 \\ 2 & x = -1 \\ 1 & x = -2 \\ -1 & x = -3 \\ 0 & x \leq -4 \end{cases}$$

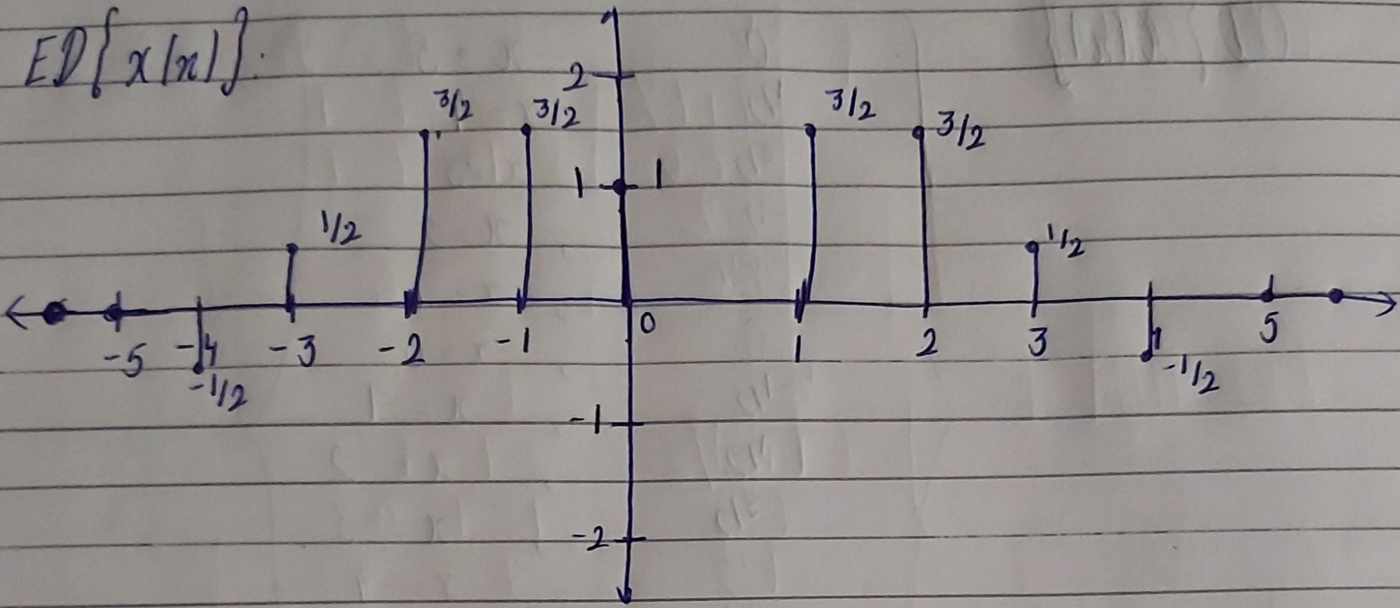
$$OD\{x(|x|)\} : \begin{cases} 0 , & x \geq 5 \\ 1/2 , & x = 4 \\ -3/2 , & x = 3 \\ -1/2 , & x = 2 \\ 1/2 , & x = 1 \\ 0 , & x = 0 \\ -1/2 , & x = -1 \\ 1/2 , & x = -2 \\ 3/2 , & x = -3 \\ -1/2 , & x = -4 \\ 0 , & x \leq -5 \end{cases}$$

$$ED\{x(|x|)\} : \begin{cases} 0 & x \geq 5 \\ -1/2 & x = 4 \\ 1/2 & x = 3 \\ 3/2 & x = 2 \\ 3/2 & x = 1 \\ 1 & x = 0 \\ 3/2 & x = -1 \\ 3/2 & x = -2 \\ 1/2 & x = -3 \\ -1/2 & x = -4 \\ 0 & x \leq -5 \end{cases}$$

$OD\{x(|x|)\}$



$ED\{x[n]\}$:



Ans 5 (a) $\sum_{-\infty}^{\infty} x_1[n] = 0$ $\left[\text{as } x_1[n] \rightarrow \text{odd func}^n \quad \therefore x_1[-n] + x_1[n] = 0\right]$

Ans 5 (b) $x_3[n] = x_1[n] \times x_2[n]$ is ~~not~~ neither odd nor even as there may be a discontinuity at $n = 0$.