ECN-203 - Assignment 2

Name: Shashank Aital

Enr No: 19114076

Batch: 04

Branch: CSE

Q.1. (a) $x_1(t) = 2je^{j12t}$

$\Rightarrow \omega = 12 \Rightarrow \frac{2\pi}{T_0} = 12 \Rightarrow \boxed{T_0 = \frac{\pi}{6}}$

(b) $x_2[n] = e^{-0.7jn}$

$\Rightarrow \omega = 0.7 \Rightarrow \frac{2\pi}{T} = 0.7 \Rightarrow T = \frac{2\pi}{0.7}$ (irrational)

$\Rightarrow x_2[n]$ is apriodic (T is undefined)

(c) $x_3[n] = 3e^{j\frac{3\pi}{5}(n+\frac{1}{2})}$

$\Rightarrow \omega = \frac{3\pi}{5} \Rightarrow \frac{2\pi}{T} = \frac{3\pi}{5} \Rightarrow T = \frac{10}{3}$

But T must be an integer

$\Rightarrow$ $\boxed{\text{Fundamental period, } T_0 = 10}$

(d) $x_4(t) = 5e^{j2\pi t}$

$\Rightarrow \omega_0 = 2\pi \Rightarrow \frac{2\pi}{T_0} = 2\pi \Rightarrow \boxed{T_0 = 1}$

Q-2 (a) $x(t) = 2\cos(7t+3) + 3\sin(3t+4)$

$\cos(7t+3)$: $T_{0_1} = \frac{2\pi}{\omega_{0_1}} = \frac{2\pi}{7}$

$\cos(3t+4)$: $T_{0_2} = \frac{2\pi}{\omega_{0_2}} = \frac{2\pi}{3}$

$\Rightarrow$ Period of $x(t)$ is $\mathrm{LCM}\left(\frac{2\pi}{7}, \frac{2\pi}{3}\right) = \underline{\underline{2\pi}}$,

(b) $x[n] = 1 + e^{j\frac{4\pi n}{7}} - e^{j\frac{2n}{5}}$

$\rightarrow$ Period of 1 = undefined (But it is periodic) ✓

$\rightarrow$ Period of $e^{j\frac{4\pi n}{7}}$ $\Rightarrow T_{0_2} = \frac{2\pi}{\omega_{0_2}} = \frac{2\pi}{\left(\frac{4\pi}{7}\right)} = \frac{7}{2}$

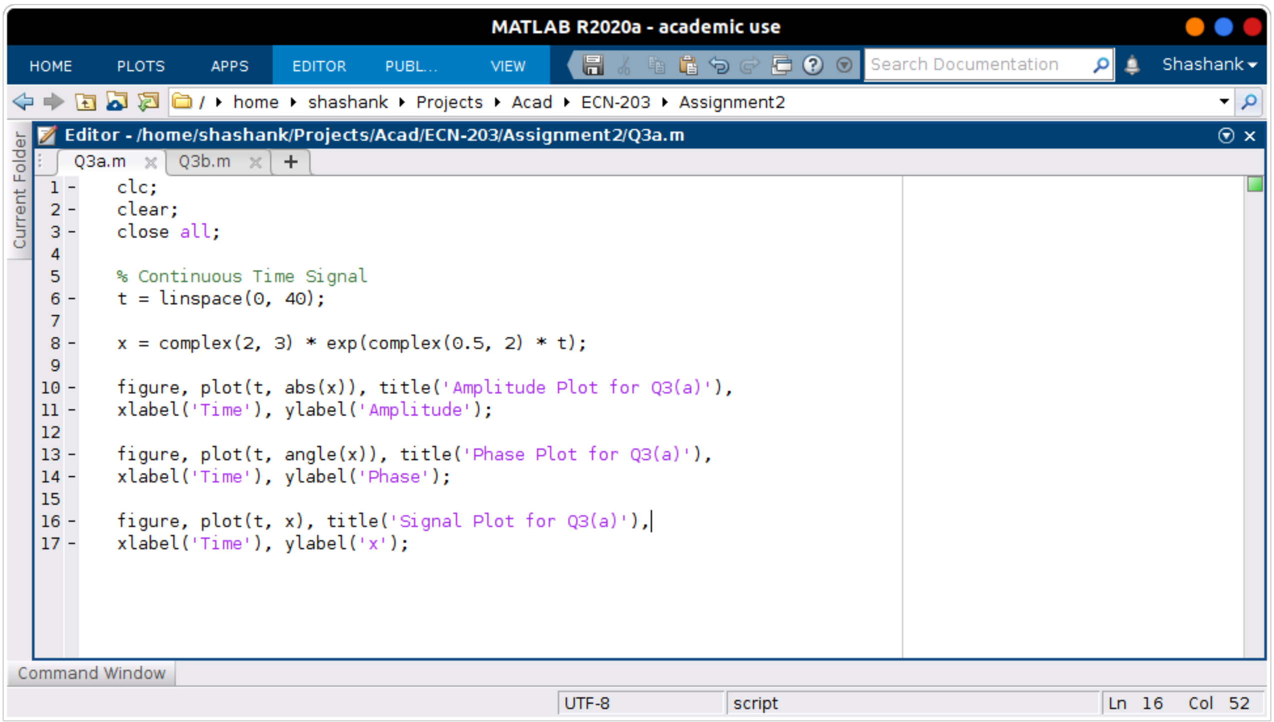
$\Rightarrow \boxed{T_{0_2} = 7}$ ✓

$\rightarrow$ Period of $e^{j\frac{2n}{5}}$ $\Rightarrow T_3 = \frac{2\pi}{(\omega_{0_3})} = \frac{2\pi}{\left(\frac{2}{5}\right)} = 5\pi$ (irrational)

$\Rightarrow e^{j\frac{2\pi}{5}}$ is aperiodic ✗

$\boxed{\text{So, } x[n] \text{ is aperiodic}}$ (Fundamental period is undefined).

Q.3. (a)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Q3a.m Q3b.m
clc;
clear;
close all;
% Continuous Time Signal
t = linspace(0, 40);
x = complex(2, 3) * exp(complex(0.5, 2) * t);
figure, plot(t, abs(x)), title('Amplitude Plot for Q3(a)'),
xlabel('Time'), ylabel('Amplitude');
figure, plot(t, angle(x)), title('Phase Plot for Q3(a)'),
xlabel('Time'), ylabel('Phase');
figure, plot(t, x), title('Signal Plot for Q3(a)'),
xlabel('Time'), ylabel('x');
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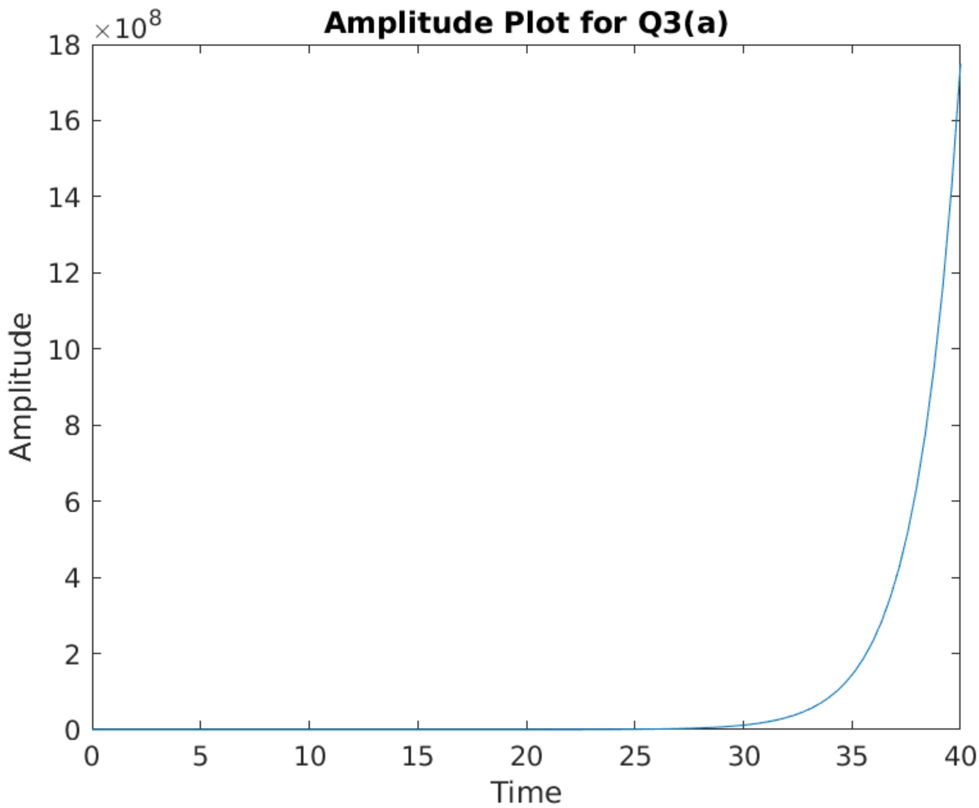
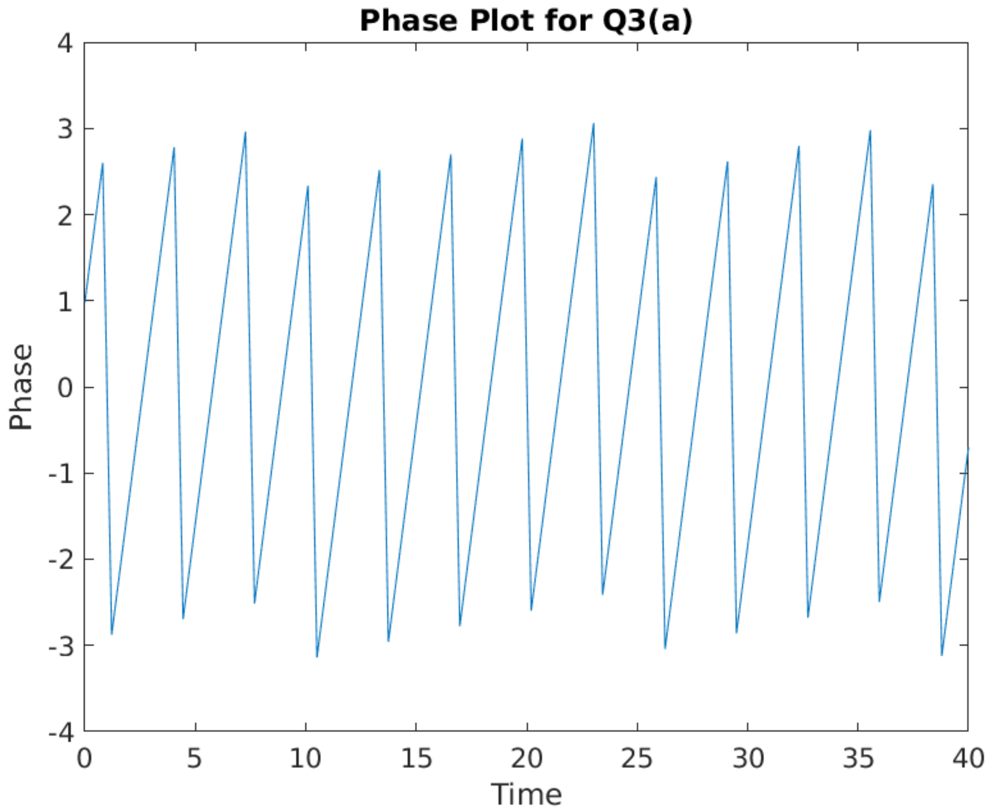
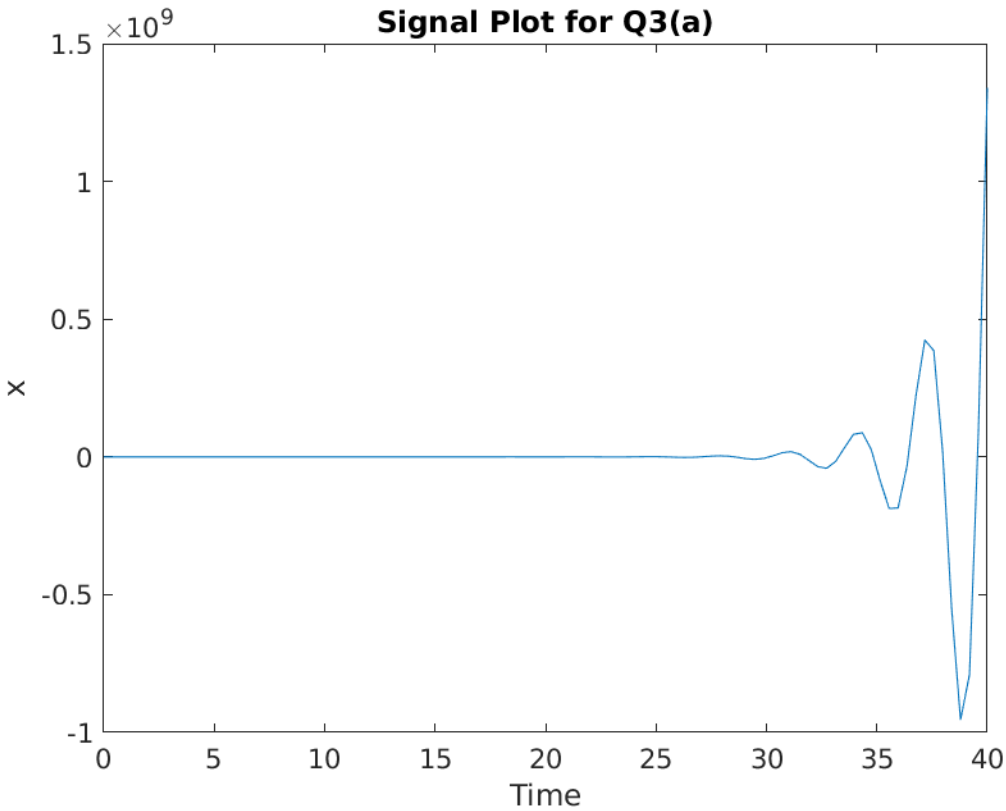
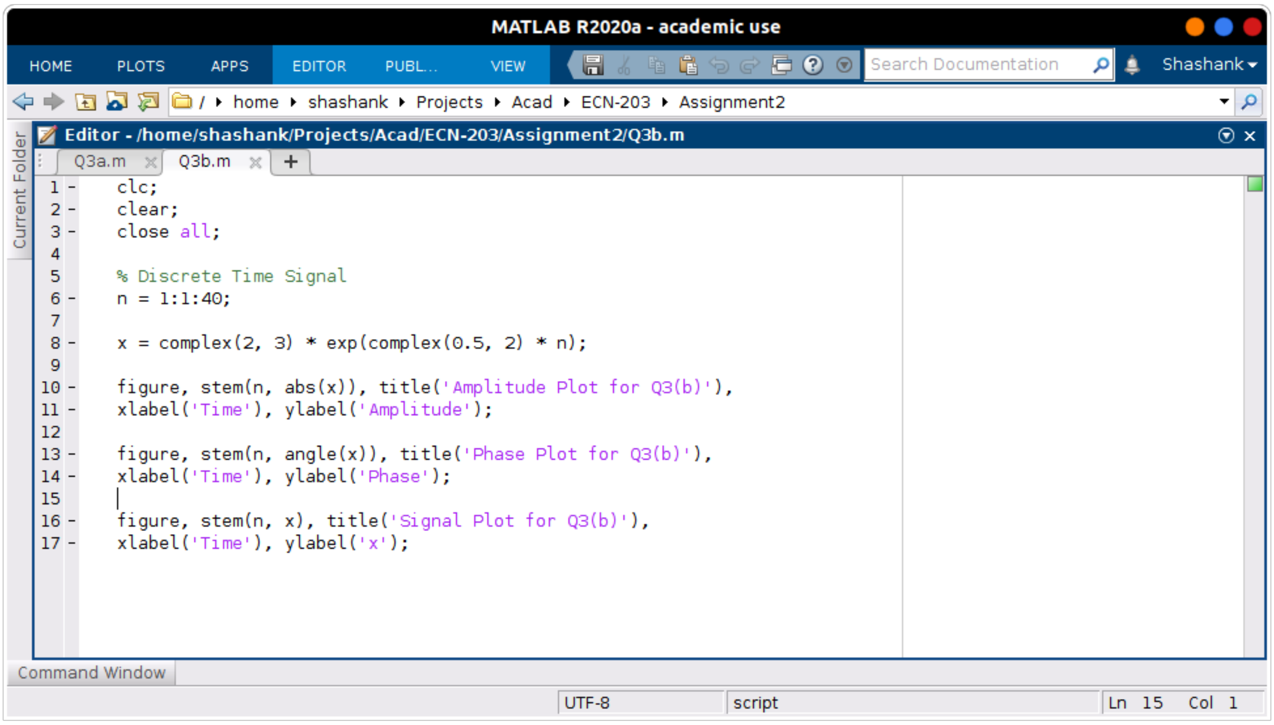

Amplitude Plot for Q3(a)
×10^8
18
16
14
12
10
8
6
4
2
0
Amplitude
0 5 10 15 20 25 30 35 40
Time


Phase Plot for Q3(a)
Phase
Time


Signal Plot for Q3(a)
$\times 10^9$
x
Time

Q.3. (b)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Editor - /home/shashank/Projects/Acad/ECN-203/Assignment2/Q3b.m
Q3a.m
Q3b.m
Current Folder
clc;
clear;
close all;
% Discrete Time Signal
n = 1:1:40;
x = complex(2, 3) * exp(complex(0.5, 2) * n);
figure, stem(n, abs(x)), title('Amplitude Plot for Q3(b)'),
xlabel('Time'), ylabel('Amplitude');
figure, stem(n, angle(x)), title('Phase Plot for Q3(b)'),
xlabel('Time'), ylabel('Phase');
figure, stem(n, x), title('Signal Plot for Q3(b)'),
xlabel('Time'), ylabel('x');
Command Window
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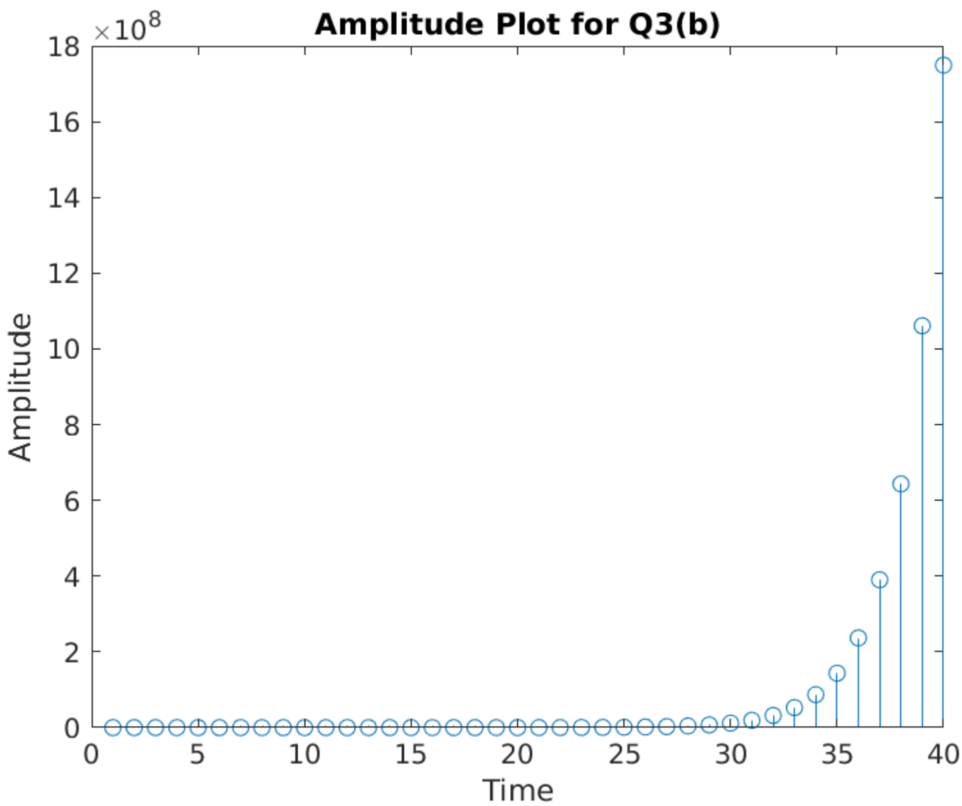
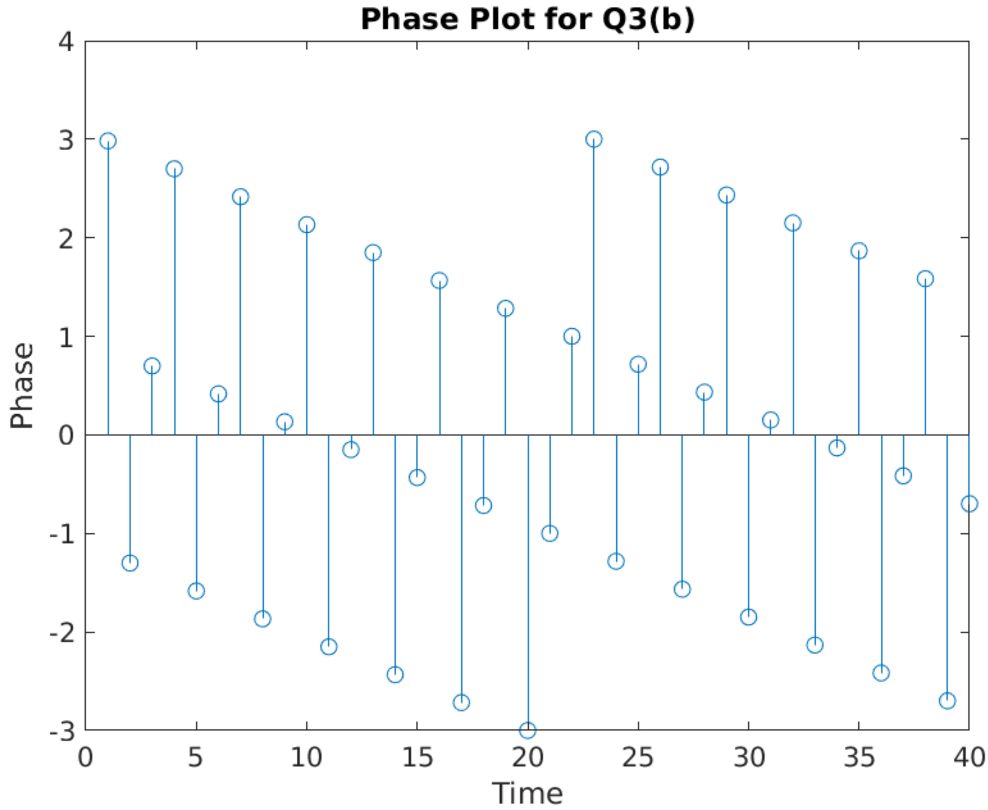
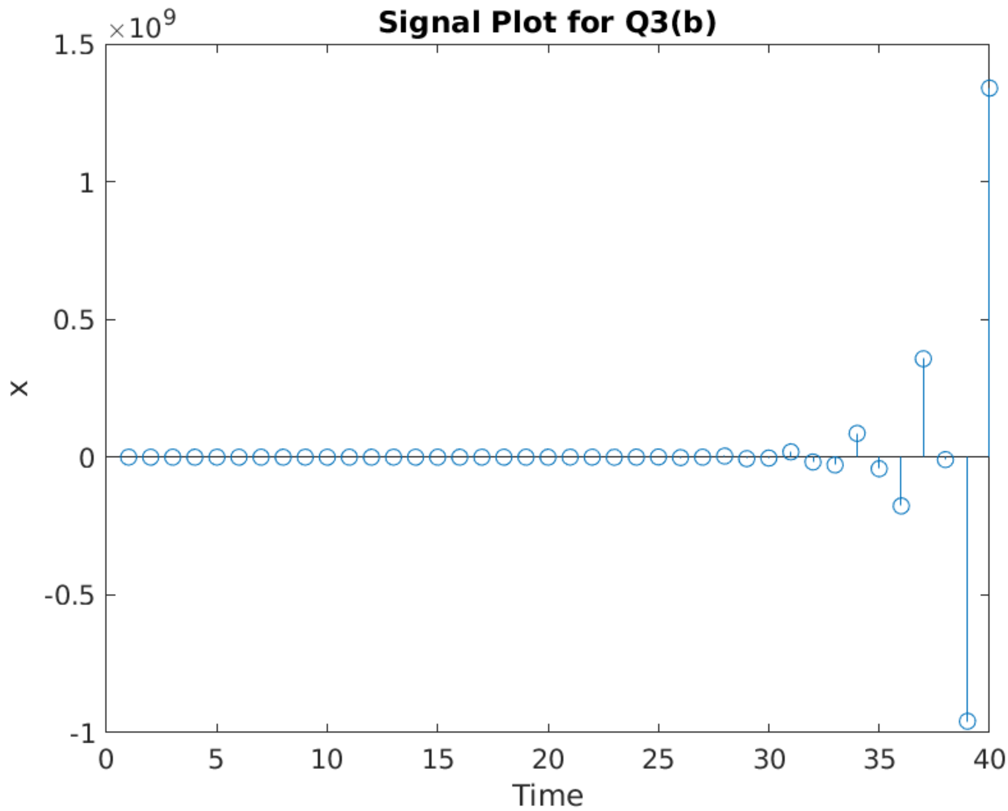

Amplitude Plot for Q3(b)
×10^8
Amplitude
Time
0
2
4
6
8
10
12
14
16
18
0
5
10
15
20
25
30
35
40


Phase Plot for Q3(b)
Phase
Time
4
3
2
1
0
-1
-2
-3
0
5
10
15
20
25
30
35
40


Signal Plot for Q3(b)
×10^9
x
Time
1.5
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30
35
40

Q4. $x[n] = 1 - \sum_{k=-2}^{\infty} \delta[n-1-k]$

Now, $u[n] = \sum_{k=0}^{\infty} \delta[n-k]$

$\Rightarrow u[n] = \sum_{k=-2}^{\infty} \delta[n-k-2]$ ... Replace k by k+2

$\Rightarrow u[n+1] = \sum_{k=-2}^{\infty} \delta[n+1-k-2]$

$\Rightarrow \boxed{u[n+1] = \sum_{k=-2}^{\infty} \delta[n-1-k]}$

$\Rightarrow x[n] = 1 - u[n+1]$

$$= \begin{cases} 0, & n+1 \geq 0 \\ 1, & n+1 < 0 \end{cases} = \begin{cases} 0, & n+1 > -1 \\ 1, & n+1 \leq -1 \end{cases}$$

$$= \begin{cases} 0, & n+2 > 0 \\ 1, & n+2 \leq 0 \end{cases}$$

~~Replace n+2 by 't'~~ $\Rightarrow$ ~~n = t~~ -2

$$\Rightarrow x[n] = \begin{cases} 0, & (-n-2) < 0 \\ 1, & -(n+2) \geq 0 \end{cases} = u[-(n+2)]$$

$\Rightarrow \boxed{x[n] = u[-n-2]}$

Q.5 We know that $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$. Here, $\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$

Consider, $I = \int_{-\infty}^{\infty} \delta(2t)\,dt$

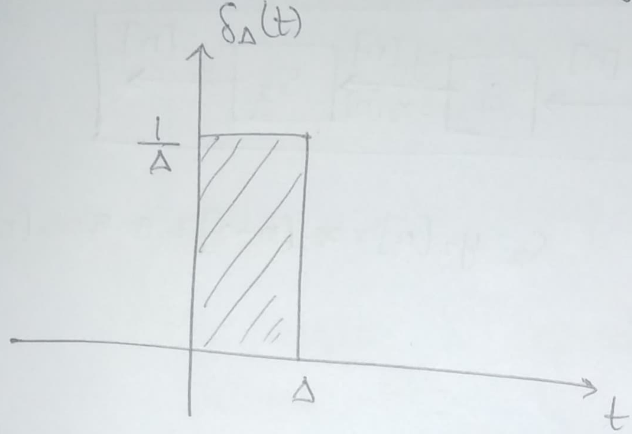
Let $2t = \tau$

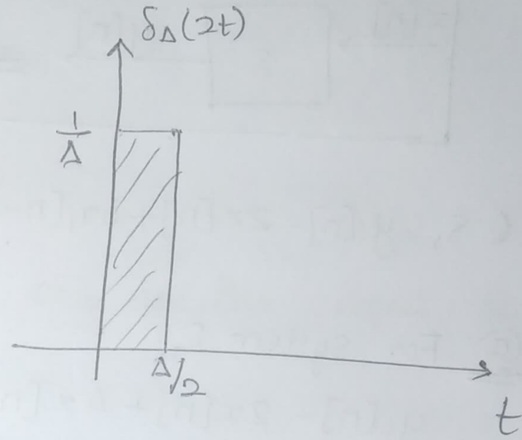
$$\Rightarrow I = \int_{-\infty}^{\infty} \delta(t) \frac{d\tau}{2} = \frac{1}{2}\int_{-\infty}^{\infty} \delta(\tau)\,d\tau = \frac{1}{2}$$

$\Rightarrow \boxed{\delta(2t) = \frac{1}{2}\delta(t)}$

In general, $\boxed{\delta(at) = \frac{1}{a}\delta(t)}$

Visualization: (Alternate proof)





As we can see from the figure too, the area covered by $\delta_\Delta(2t)$ is half of area covered by $\delta_\Delta(t)$

Proof for the above claim:

$$\cancel{\delta(2t) = \lim_{\Delta}} \qquad \delta_\Delta(t) = \begin{cases} \frac{1}{\Delta}, & 0 \le t \le \Delta \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow \delta_\Delta(2t) = \begin{cases} \frac{1}{\Delta}, & 0 \le 2t \le \Delta \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{1}{\Delta}, & 0 \le t \le \Delta/2 \\ 0, & \text{otherwise} \end{cases}$$

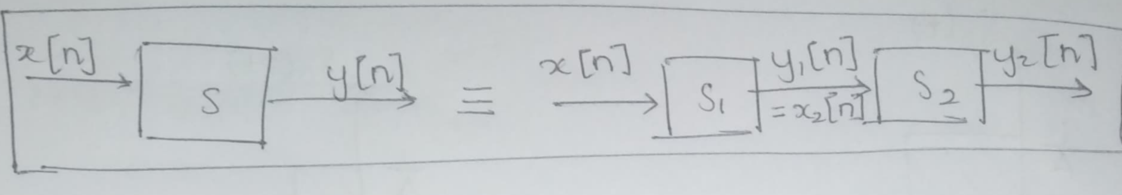
$$\Rightarrow \int_{-\infty}^{\infty} \delta_\Delta(2t)\,dt = \frac{1}{2} \int_{-\infty}^{\infty} \delta_\Delta(t)\,dt = \frac{\Delta}{2}$$

$$\Rightarrow \lim_{\Delta \to 0} \int_{-\infty}^{\infty} \delta_\Delta(2t)\,dt = \frac{1}{2} \lim_{\Delta \to 0} \int_{-\infty}^{\infty} \delta_\Delta(t)\,dt$$

$$\Rightarrow \int_{-\infty}^{\infty} \delta(2t)\,dt = \frac{1}{2} \int_{-\infty}^{\infty} \delta(t)\,dt$$

$$\Rightarrow \boxed{\delta(2t) = \frac{\delta(t)}{2}}$$

Q.6.



Given $S_1: y_1[n] = 2x_1[n] + 4x_1[n-1]$ $\qquad$ $S_2: y_2[n] = x_2[n-2] + 0.5x_2[n-3]$

(a) For system S:

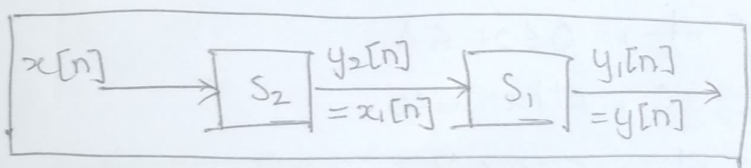
$y_1[n] = 2x[n] + 4x[n-1]$

$y[n] = y_2[n] = y_1[n-2] + 0.5y_1[n-3]$

$\Rightarrow y[n] = (2x[n-2] + 4x[n-3]) + 0.5(2x[n-3] + 4x[n-4])$

$$\boxed{\Rightarrow y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]}$$

(b)



$y_2[n] = x[n-2] + 0.5x[n-3]$

$y[n] = y_1[n] = 2y_2[n] + 4y_2[n-1]$

$\therefore y[n] = 2(x[n-2] + 0.5x[n-3]) + 4(x[n-3] + 0.5x[n-4])$

$$\boxed{\Rightarrow y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]}$$

Q. 7.

$$x(t) \rightarrow \boxed{\text{System}} \rightarrow y(t) = x(\sin(t))$$

<u>Memoryless</u> :

→ The system is <u>not memoryless</u> because ~~future~~ the output of the system at a given point of time is dependent on the past.

<u>Ex</u>: $y(2\pi) = x(\sin(2\pi)) = \underline{x(0)}$.

↑ Past input

<u>Causality</u>:

→ The system is <u>not causal</u> because ~~oput~~ output at some of the times can depend upon the future.

<u>Ex</u>: $y(-2\pi) = x(\sin(-2\pi)) = x(0)$

↑ Future input

<u>Invertibility</u> :

→ The system <u>is not invertible</u> because,

If $x(t) = 0 \Rightarrow y(t) = 0 \ \forall t$

If $x(t) = u(t-k)$, for $k > 1 \Rightarrow y(t) = 0 \ \forall t$

⇒ We can't tell these 2 inputs apart ~~[illegible]~~.
if we are just given $y(t)$

Q.8. (a) $y(t) = x(t-4)$

~~$\Rightarrow x(t-4) = y(t)$~~

~~$\Rightarrow x$~~

Since distinct inputs have distinct outputs.

~~$\therefore y(t)$ is~~

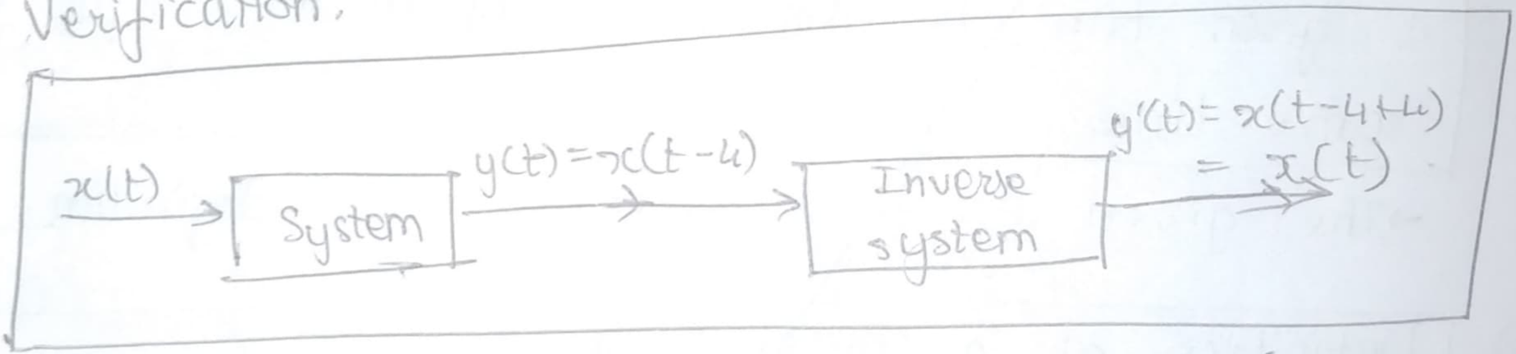
$\therefore$ The system is invertible

$\Rightarrow x(t-4) = y(t)$

$\Rightarrow \boxed{x(t) = y(t+4)}$

$\Rightarrow$ Inverse system is: $\boxed{y'(t) = x(t+4)}$

Verification:



Q.8 (b) $y[n] = n x[n]$

$\rightarrow$ Here, let's look at $n=0$

If $n=0$, irrespective of $x[n]$, $y[n]=0$

Lets take an example,

For $x[n] = \delta[n]$, $y[n]=0$

Also, for $x[n] = k\delta[n]$, $y[n]=0$, $k>1, k\in\mathbb{Z}$

$\Rightarrow$ Just by looking at $y[n]$, you can't determine if $x[n]$ was $\delta[n]$, $2\delta[n]$ or any other function.

$\therefore$ $y[n] = nx[n]$ is non-invertible $\Rightarrow$ No inverse exists.

Q.8 (c) $y[n] = \begin{cases} x[n+1], & n\geq 0 \\ x[n], & n\leq -1 \end{cases}$

If we take a closer look at $y[n]$, we come to the conclusion that $y[n]$ does not depend on $x[0]$ at any point of time.

$\Rightarrow$ Functions having same values at all points except $x[0]$ will yield the same output

Ex: Consider $x[n]=0 \ \forall n \Rightarrow y[n]=0 \ \forall n$

Also, consider $x[n] = \delta[n] \Rightarrow y[n]=0 \ \forall n$

As a matter of fact, consider $x[n] = k\delta[n]$, $k\in\mathbb{Z}$

$\Rightarrow y[n]=0 \ \forall n$

$\Rightarrow$ Multiple input functions lead to the same output $y[n]=0$

$\Rightarrow y[n] = \begin{cases} x[n+1], & n\geq 0 \\ x[n], & n\leq -1 \end{cases}$ is non-invertible and hence, no inverse exists.