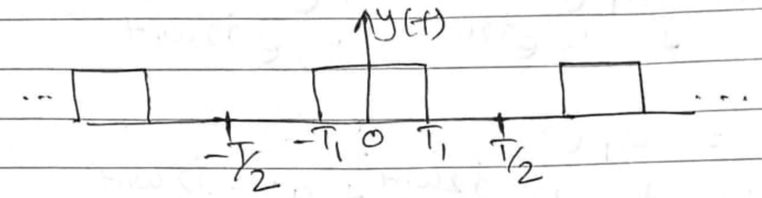
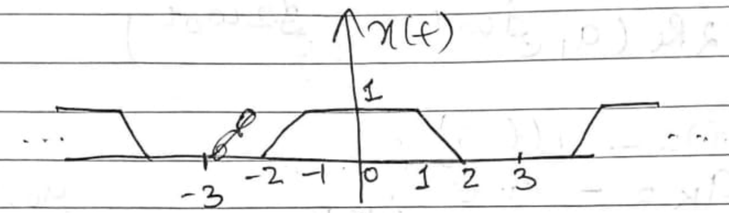
Saathi

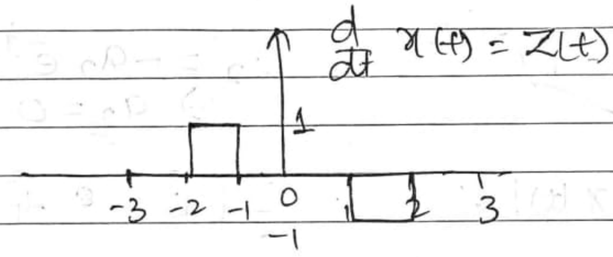
Date ___/___/_____

Q1 $y(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases} \longleftrightarrow a_k$

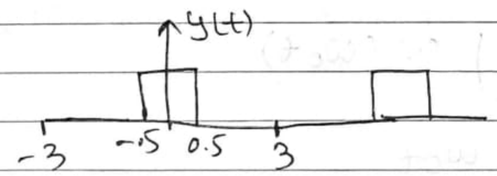




$T = 6$



$z(t) = y(t+1.5) - y(t-1.5)$ $\qquad T_1 = 0.5$



$$z(t) \leftrightarrow a_k e^{jk\omega_0 1.5} - a_k e^{-jk\omega 1.5}$$
$$= a_k (2j \sin(k\omega_0 1.5))$$
$$= 2j a_k \sin\left(\frac{3}{2} k\omega_0\right)$$

Page No. ____

Date ___/___/___

Saathi

$$x(t) = \int_{-\infty}^{t} z(t)\,dt$$

$$b_0 = \frac{1}{6}\,\frac{1}{2} \times 4 \times 2 \times 1 = \frac{2}{3}$$

$$b_k = \frac{1}{jk\omega_0} \times 2j a_k \sin\left(\frac{3}{2}k\omega_0\right)$$

$$= \frac{2}{k\omega_0} \sin\left(\frac{3}{2}k\omega_0\right) a_k.$$

Q2 $x(t) = \cos 4\pi t \qquad T = \frac{1}{2}$

$= \frac{1}{2}(e^{j4\pi t} + e^{-j4\pi t})$

$a_1 = \frac{1}{2}, \quad a_{-1} = \frac{1}{2}$

$y(t) = \sin 6\pi t \qquad T = \frac{1}{3}$

$= \frac{1}{2j}(e^{j6\pi t} - e^{-j6\pi t})$

$b_1 = \frac{1}{2j}, \quad b_{-1} = \frac{-1}{2j}$

$z(t) = x(t)\,y(t) = \cos 4\pi t \sin 6\pi t$

$= \frac{1}{2}(\sin 10\pi t + \sin 2\pi t) \Rightarrow T = 1$

$= \frac{1}{2}\left(\frac{1}{2j}(e^{j10\pi t} - e^{-j10\pi t}) + \frac{1}{2j}(e^{j2\pi t} - e^{-j2\pi t})\right)$

$= \frac{1}{4j} e^{j10\pi t} - \frac{1}{4j} e^{-j10\pi t} + \frac{1}{4j} e^{j2\pi t} - \frac{1}{4j} e^{-j2\pi t}$

$a_1 = \frac{1}{4j}, \quad a_{-1} = \frac{-1}{4j}, \quad a_5 = \frac{1}{4j}, \quad a_{-5} = \frac{1}{4j}$

Page No.

Saathi

Date ___/___/_____

Q3 $T = 6 \Rightarrow \omega_0 = \frac{2\pi}{6}$

$x(t) \rightarrow$ Real $\Rightarrow x(t) = x^*(t)$

$a_{-k} = a_k^*$

$x(t) = -x(t-3)$

$a_k = -a_k e^{-jk\omega_0 3}$

$a_k = -a_k e^{-jk\frac{2\pi}{6}\times 3} = a_k e^{-jk\pi}$

$a_k = -a_k e^{-jk\pi}$

$a_0 = -a_0 \Rightarrow a_0 = 0$

$a_1 = -a_1(-1)$ ✓ $\qquad a_{-1} = -a_{-1}$

$a_2 = -a_2 \Rightarrow a_2 = 0 \qquad a_{-2} = -a_{-2} \Rightarrow a_{-2} = 0$

$x(t) = a_1 e^{j\omega_0 t} + a_{-1} e^{-j\omega_0 t}$

$= a_1 e^{j\omega_0 t} + a_1^* e^{-j\omega_0 t}$

$= 2R_e(a_1 e^{j\omega_0 t})$

$= 2a_1 \cos\omega_0 t$

Average Power $= \frac{1}{T}\int_T |x(t)|^2 dt = \sum |a_k|^2$

$= a_1^2 + a_{-1}^2$

$= 2a_1^2 = 1$

$a_1 = \frac{1}{\sqrt{2}}$

$x(t) = 2 \cdot \sqrt{2}\cos\omega_0 t = \sqrt{2}\cos\frac{\pi}{3}t$

Page No.