

Tutorial 1 - Asymptotic Notations

Find the relationship between $f(n)$ and $g(n)$. It can be:

- $f(n) = \mathcal{O}(g(n))$
- $f(n) = \Omega(g(n))$
- $f(n) = \Theta(g(n))$

1. $f(n) = \frac{n^2-n}{2}$ and $g(n) = n$
2. $f(n) = n + 2\sqrt{n}$ and $g(n) = n^2$
3. $f(n) = n + \log n$ and $g(n) = n\sqrt{n}$
4. $f(n) = 2(\log n)^2$ and $g(n) = \log n + 1$
5. $f(n) = n^2$ and $g(n) = n^3$
6. $f(n) = 2n^2 + 1$ and $g(n) = n^2$
7. $f(n) = 3n^2 + \sqrt{n}$ and $g(n) = n + n\sqrt{n} + \sqrt{n}$
8. $f(n) = n \log n$ and $g(n) = n\sqrt{n}$
9. $f(n) = n^2 + 3n$ and $g(n) = 6n + 7$
10. $f(n) = n\sqrt{n}$ and $g(n) = n^2 - n$
11. $f(n) = n^{\log n}$ and $g(n) = (\log n)^n$
12. $f(n) = 3n \lfloor \log n \rfloor$ and $g(n) = n^2$
13. Rank the following functions by increasing order of growth (i.e., the slowest-growing first, the fastest-growing last)
 - (a) $(\log \log n)^2$
 - (b) $\log(n!)$
 - (c) \sqrt{n}
 - (d) $n^{1.1}$
 - (e) $n \log n$
 - (f) 2^n

- (g) n^2
- (h) $(\log n)^{0.3}$
- (i) $10 \log n$
- (j) $2^{\log n}$

14. Compute simple, tight upper bounds for $f(n)$, where $f(n)$ is the following:

- (a) $\log_{\ln 5}(\log^{\log 100} n)$
- (b) $5^{\log(3)}n^3 + 10^{80}n^2 + \log(3)n^{3.1} + 6006$

15. Sort the functions in increasing order of asymptotic (big-O) complexity:

- (a) $f_1(n) = n^{0.999999} \log n$
- (b) $f_2(n) = 1000000n$
- (c) $f_3(n) = 1.0000000001^n$
- (d) $f_4(n) = n^2$