Jitesh Jain ECN-203 Assignment-3
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Ans.1 (a) let $X(t) = a x_1(t) + b x_2(t)$

$$\Rightarrow Y(t) = t^2 X(t-1)$$
$$= t^2 [a x_1(t-1) + b x_2(t-1)]$$
$$= t^2 a x_1(t-1) + b t^2 x_2(t-1)$$
$$= a y_1(t) + b y_2(t) \quad [\text{Linear}]$$

If $y(t)$ is response to $x(t)$ then $y(t-T)$ must be response to $x(t-T)$ for time invariant systems

$$\Rightarrow y(t-T) = (t-T)^2 x(t-T)$$

let response to $x(t-T)$ be $Y(t)$

$$\therefore Y(t) = t^2 x(t-T-1)$$
$$\Rightarrow Y(t) \neq y(t-T) \longrightarrow \text{Time variant}$$

$\boxed{\text{Linear \& time variant system}}$

Ans.1 (b) $y[n] = (x[n-2])^2$

let $X[n] = a x_1[n] + b x_2[n]$

$$Y[n] = (a x_1[n-2] + b x_2[n-2])^2$$
$$= a^2 y_1[n] + b^2 y_2[n] + 2ab\, x_1[n-2] x_2[n-2]$$
$$\neq a^2 y_1[n] + b^2 y_2[n] \quad (\text{non-linear})$$

~~If $X[n-k] = x$~~

If $X[n] = x[n-k] \rightarrow$ input

$$Y[n] = (X[n-2])^2$$
$$= (x[n-k-2])^2$$
$$= y[n-k]$$

(Time invariant)

$\boxed{\text{System is non-linear \& time invariant}}$

Ans. 01(c) $y[n] = x[n+1] - x[n-1]$

Let $X[n] = ax_1[n] + bx_2[n]$

$\rightarrow$ ~~$y[n] = a[x_1[n+1] - x_1[n-1]] + b[x_2[n+1] -$~~

$\rightarrow$ $Y[n] = ax_1[n+1] + bx_2[n+1] - ax_1[n-1] - bx_2[n-1]$
$$= ay_1[n] + by_2[n]$$

(Linear)

Let $X[n] = x[n-k]$

$$Y[n] = X[n+1] - X[n-1]$$
$$= x[n-k+1] - x[n-k-1]$$
$$= y[n-k]$$

(Time Invariant)

$\boxed{\text{System is linear \& time invariant}}$

Ans 1(d) $y(t) = Ev(x(t))$

$$y(t) = \frac{x(t) + x(-t)}{2}$$

let $X(t) = a x_1(t) + b x_2(t)$

$$Y(t) = \frac{X(t) + X(-t)}{2}$$

$$= \frac{a x_1(t) + b x_2(t) + a x_1(-t) + b x_2(-t)}{2}$$

$$= a y_1(t) + b y_2(t)$$ (Linear)

let: $X(t) = x(t-k)$

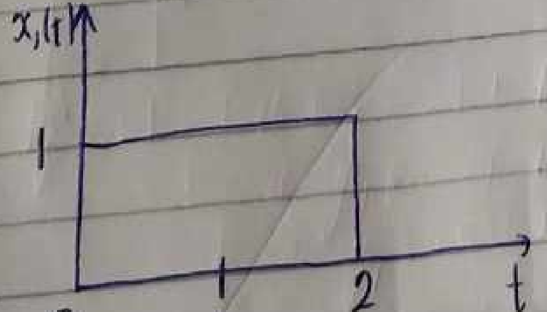
$$Y(t) = \frac{X(t) + X(-t)}{2} = \frac{x(t-k) + x(-t-k)}{2}$$

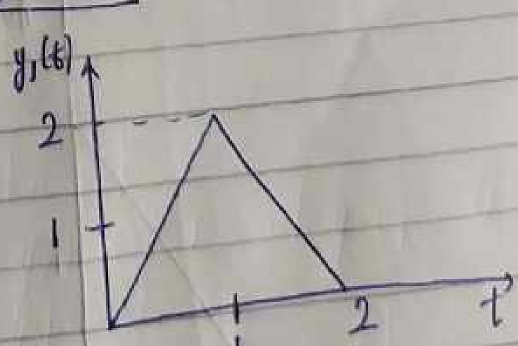
$$\neq y(t-k) \quad \left[\because y(t-k) = \frac{x(t-k) + x(-t+k)}{2}\right]$$

(Time variant)

$\boxed{\text{System is linear \& time variant}}$

Ans. 2



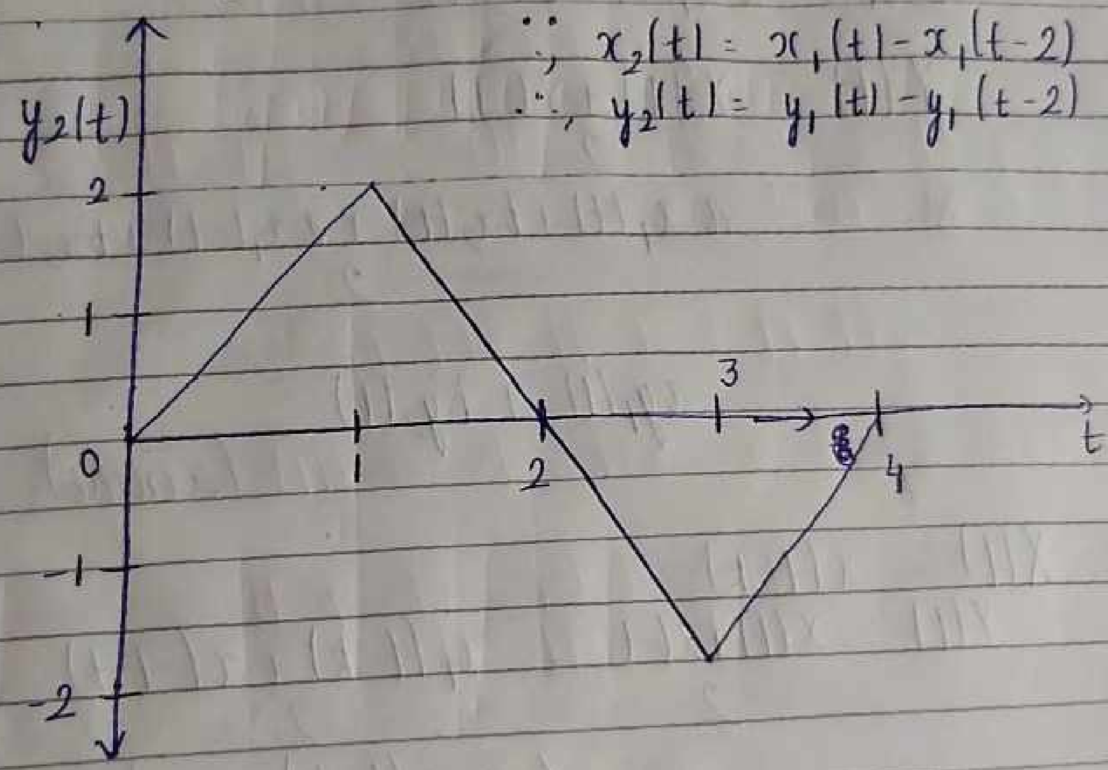


⇒ **(a)** First half is trivial. Second half is a shifted version and reversed version of the input.

So assuming, it is an LTI system, output after t=2 will also be shifted & reversed.

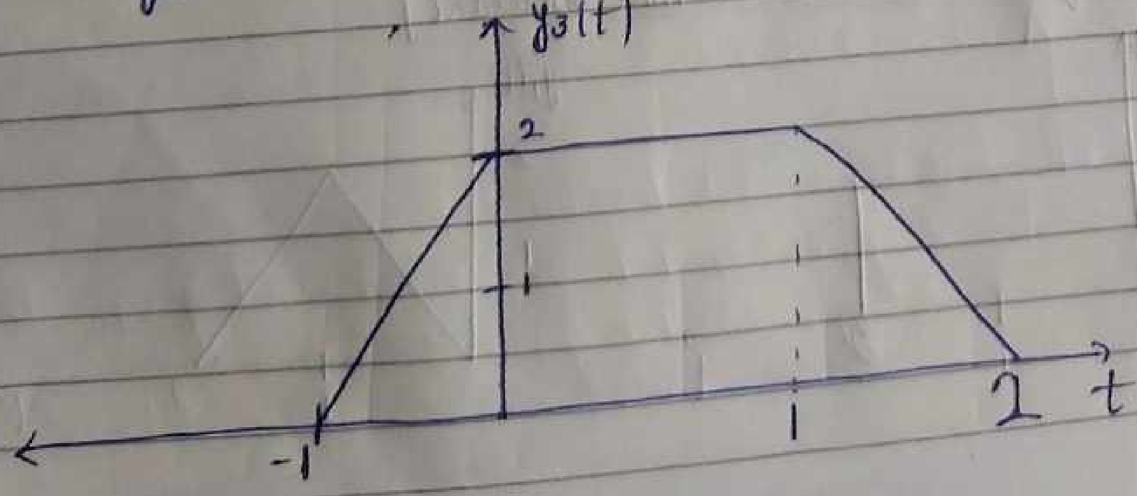
$\because$ $x_2(t) = x_1(t) - x_1(t-2)$

$\therefore$ $y_2(t) = y_1(t) - y_1(t-2)$

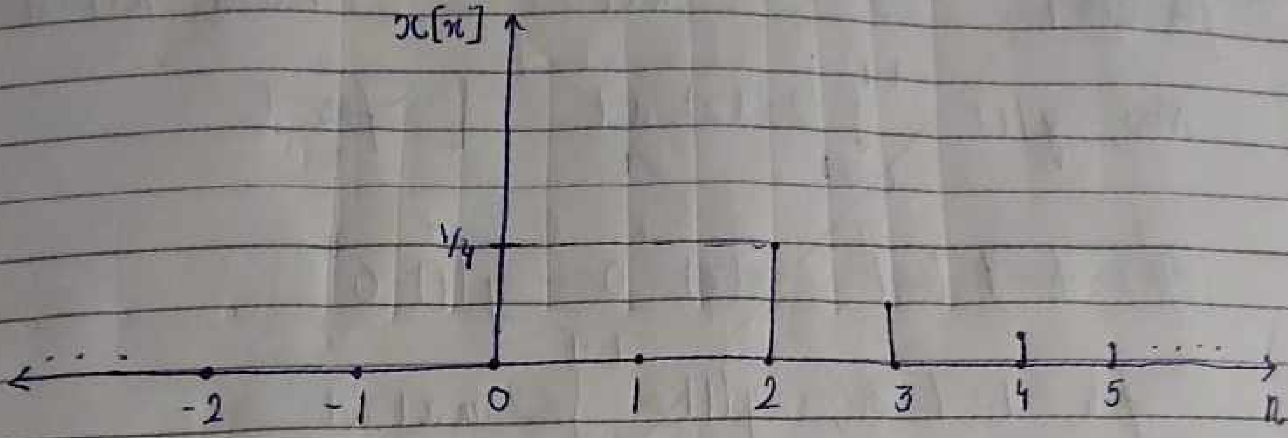


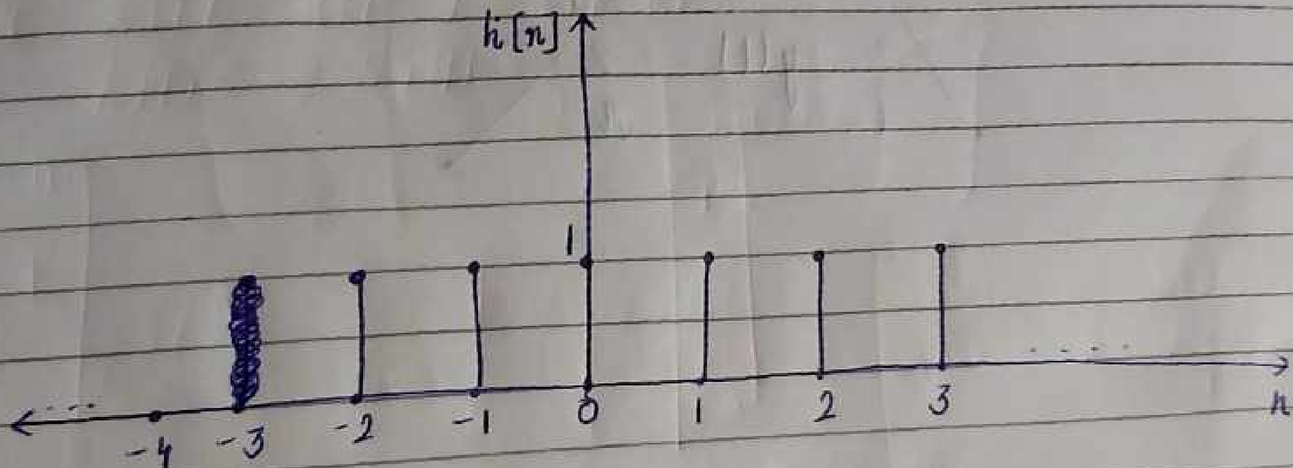
(b) $x_3(t) = x_1(t) + x_1(t+1)$

$y_3(t) = y_1(t) + y_1(t+1)$



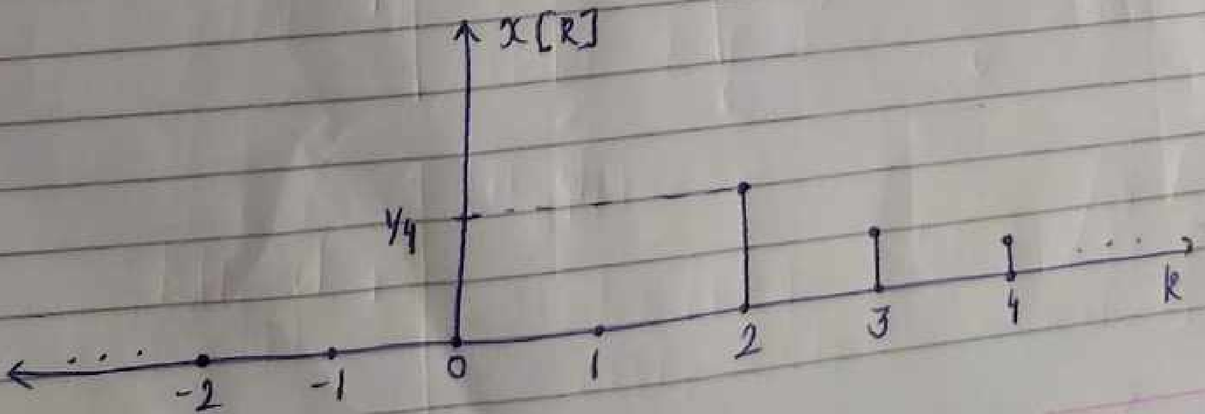
Ans 3 $x[n] = \left(\frac{1}{2}\right)^n u[n-2]$

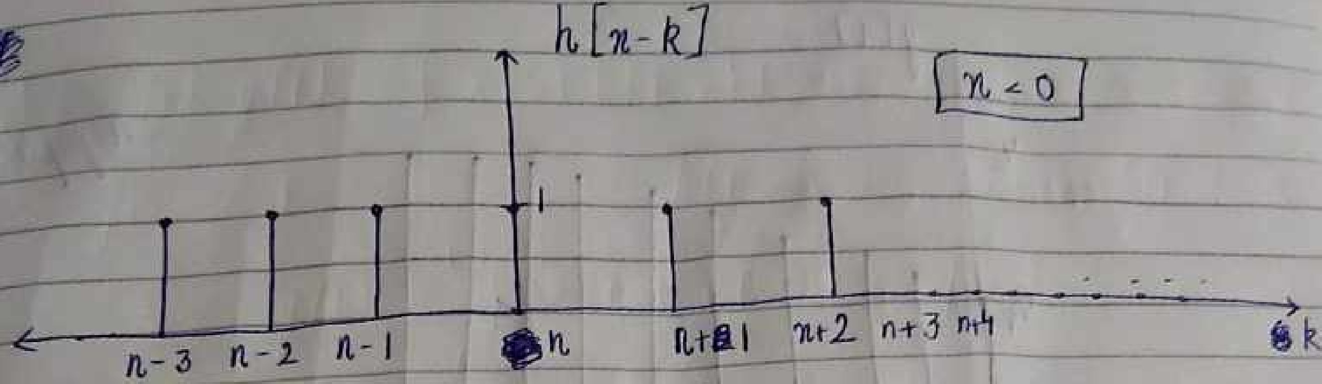




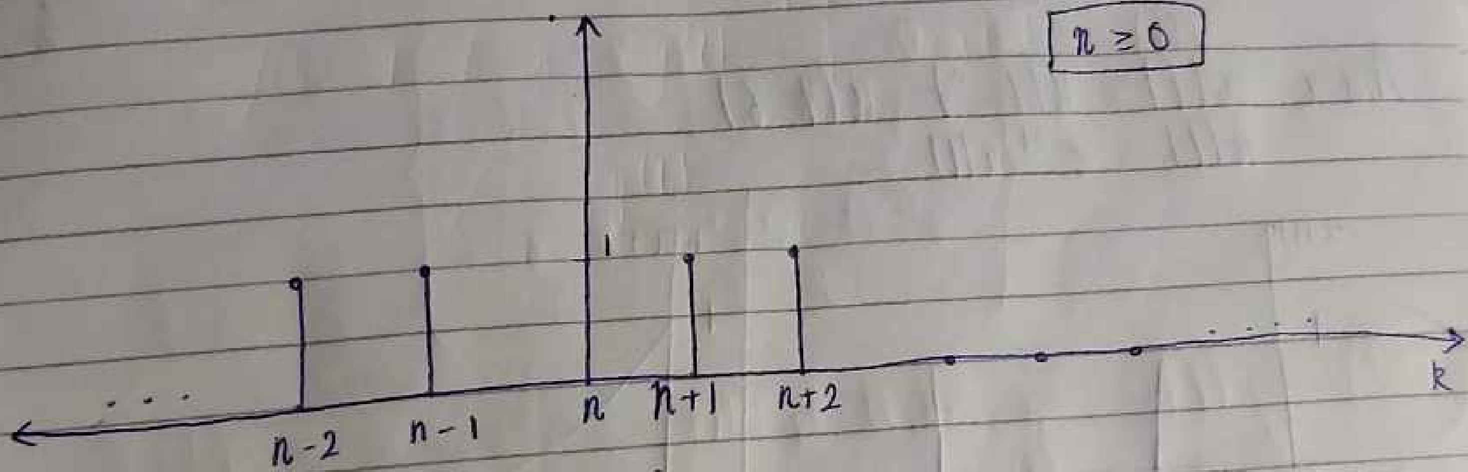
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$= x[n] * h[n]$$





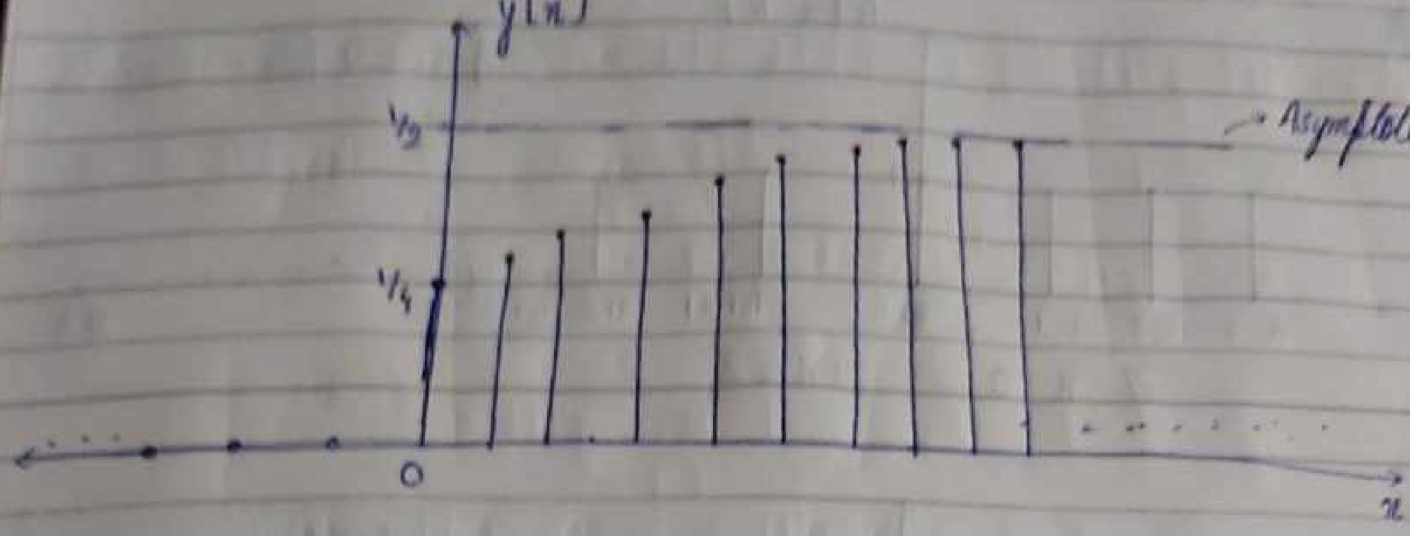
If $n < 0 \rightarrow 2+n \leq 2$



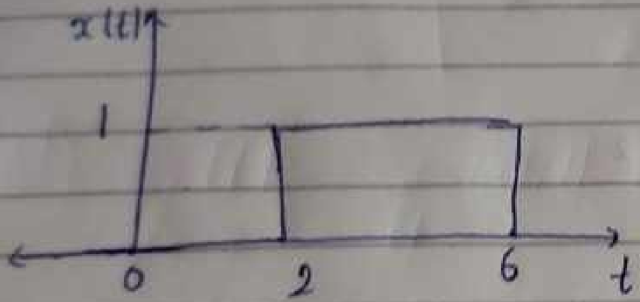
If $n \geq 0 \Rightarrow n+2 \geq 2$

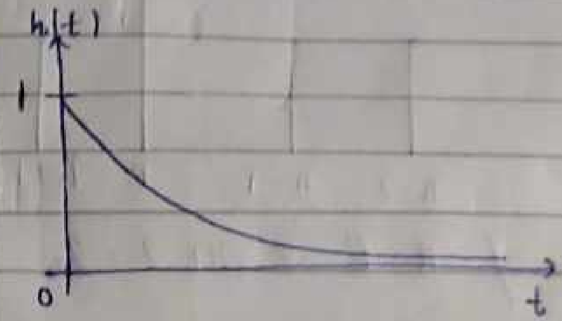
$$\therefore \; y[n] = \begin{cases} 0 , & n < 0 \\ \sum_{k=2}^{n+2} \left(\frac{1}{2}\right)^k , & n \geq 0 \end{cases}$$

$$\Rightarrow y[n] = \begin{cases} 0 , & n < 0 \\ \frac{1}{2}\left[1 - \left(\frac{1}{2}\right)^{n+1}\right] , & n \geq 0 \end{cases}$$



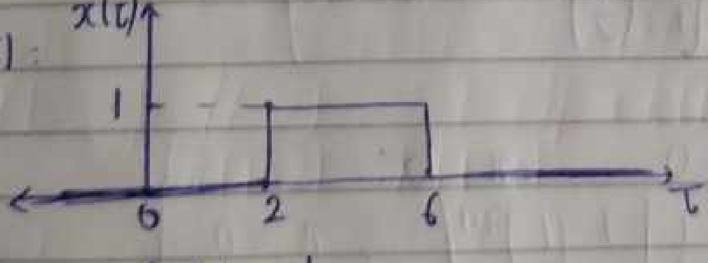
Ans 4 $x(t) = u(t-2) - u(t-6)$
$h(t) = e^{-2t} u(t)$



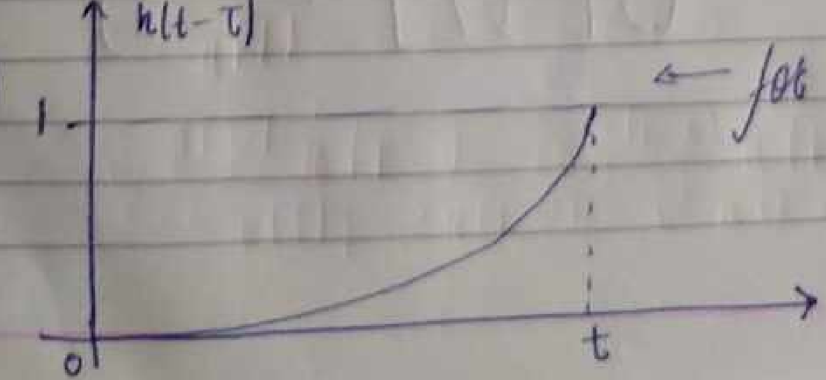


(a) $y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$

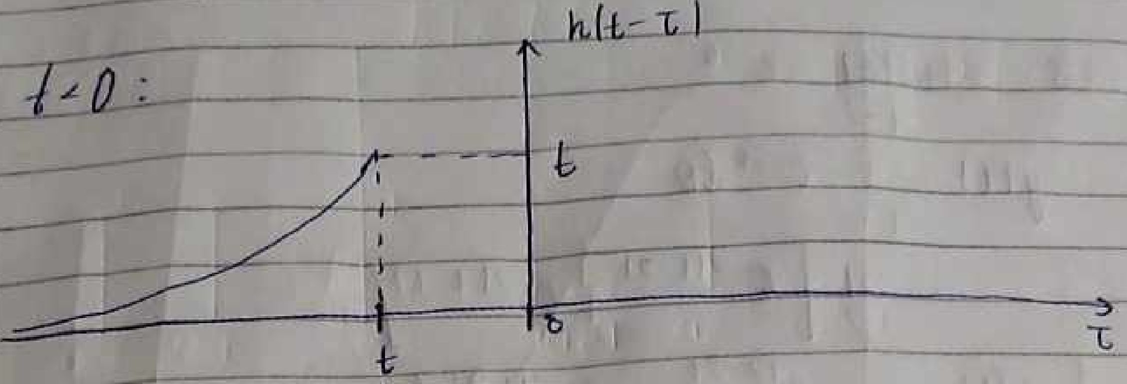
$x(\tau)$:



$h(t-\tau)$



→ for $t<0$:



Now, $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$\because$ $x(t) = 0 \quad \forall \; -\infty < t < 2 \;\&\; 6 < t < \infty$

$$y(t) = \int_{2}^{6} x(\tau)\, h(t-\tau)\, d\tau$$

$$\boxed{y(t) = \int_{t-6}^{t-2} h(\theta)\, d\theta} \qquad [\theta = t - \tau]$$

Now, $h(\theta) = \begin{cases} 0, & \theta < 0 \\ e^{-2\theta}, & \theta \geq 0 \end{cases}$

① For $t < 2 \Rightarrow h(\theta) = 0 \quad \forall \theta$

$\Rightarrow \boxed{y(t) = 0}$

② For $2 < t < 6$, $h(\theta) = \begin{cases} 0, & \theta < 0 \\ e^{-2\theta}, & \theta \geq 0 \end{cases}$

$$\Rightarrow y(t) = \int_{t-6}^{0} 0 \, d\theta + \int_{0}^{t-2} e^{-2\theta} d\theta$$

$$\Rightarrow \boxed{y(t) = \frac{1 - e^{4-2t}}{2}}$$

③ For $t > 6$, $h(\theta) = e^{-2\theta}$,

$$y(t) = \int_{t-6}^{t-2} e^{-2\theta} d\theta$$

$$= -\frac{1}{2}\left[e^{-2(t-2)} - e^{-2(t-6)}\right]$$

$$\boxed{y(t) = \frac{e^{12-2t} - e^{-2(t-2)}}{2}}$$

$$\therefore y(t) = \begin{cases} 0, & t < 2 \\ \frac{1-e^{4-2t}}{2}, & 2 \le t \le 6 \\ \frac{e^{12-2t} - e^{4-2t}}{2} & t > 6 \end{cases}$$

(b) $$g(t) = \frac{d}{dt}(x(t)) * h(t)$$

$$\frac{d(x(t))}{dt} = \frac{d}{dt}\left[u(t-2) - u(t-6)\right]$$

$$= \delta(t-2) - \delta(t-6)$$

$$g(t) = \left[\delta(t-2) - \delta(t-6)\right] * h(t)$$

$$= h(t-2) - h(t-6)$$

$$h(t) = e^{-2t} u(t)$$

$$g(t) = e^{-2(t-2)} u(t-2) - e^{-2(t-6)} u(t-6)$$

$$= e^{-2t}\left[e^{4} u(t-2) - e^{12} u(t-6)\right]$$

① $t-2<0 \Rightarrow t<2 \Rightarrow u(t-2)=0, \quad u(t-6)=0$

$g(t)=0$

② $2<t<6 \Rightarrow u(t-2)=1, \quad u(t-6)=0$

$\Rightarrow g(t) = e^{-2t}(e^{4})$

$\Rightarrow \boxed{g(t) = e^{4-2t}}$

③ $t>6 \Rightarrow u(t-2)=u(t-6)=1$

$g(t) = e^{-2t}(e^{4}-e^{12})$

$\boxed{g(t) = e^{4-2t}(1-e^{8})}$

$$\therefore \boxed{g(t) = \begin{cases} 0, & t<2 \\ e^{4-2t}, & 2\le t\le 6 \\ e^{4-2t}(1-e^{8}), & t>6 \end{cases}}$$

$\therefore$ ~~$y(t)$~~

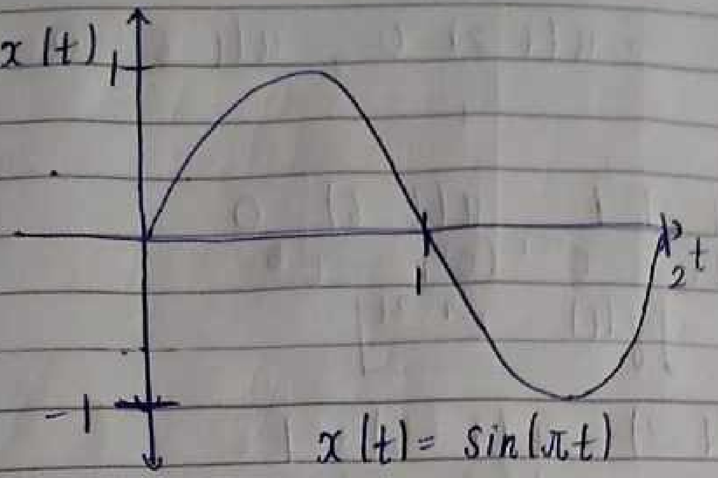
(c)

$$y(t) = \begin{cases} 0, & t<2 \\ \frac{1}{2}[1-e^{4-2t}], & 2\le t\le 6 \\ \frac{1}{2}[e^{12-2t}-e^{4-2t}], & t>6 \end{cases}$$

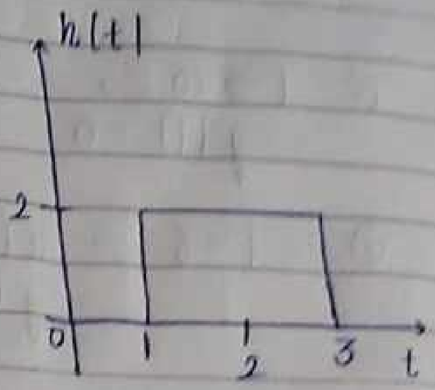
$$\boxed{\Rightarrow y(t) = \begin{cases} g(t), & t<2 \\ (\frac{1}{2})(1-g(t)), & 2\le t\le 6 \\ (-\frac{1}{2})g(t), & t>6 \end{cases}}$$

$$\Rightarrow \boxed{g(t) = \frac{d}{dt}(y(t)) = \frac{d}{dt}(x(t) * h(t))}$$

Ans. 5 (a)



$x(t) = \sin(\pi t)$



Now, $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

where, $x(\tau) = \begin{cases} \sin(\pi\tau) & , 1 \le \tau \le 2 \\ 0 & , \text{otherwise} \end{cases}$

$h(t-\tau) = \begin{cases} 2 & , t-3 < \tau < t-1 \\ 0 & , \text{otherwise} \end{cases}$

$\therefore\ y(t) = \int_{0}^{2} \sin(\pi \tau)\, h(t-\tau)\, d\tau$

$\Rightarrow y(t) = \int_{t-2}^{t} \sin[\pi(t-\theta)]\, h(\theta)\, d\theta \quad [\theta = t-\tau]$

① $t<1$ and $t-2>3 \Rightarrow t>5$

$\Rightarrow h(\theta) = 0 \Rightarrow y(t) = 0$

② $1<t<3 \Rightarrow y(t) = \int_{t-2}^{1} 0\, d\tau + \int_{1}^{t} 2\sin(\pi t - \pi\theta)\, d\theta$

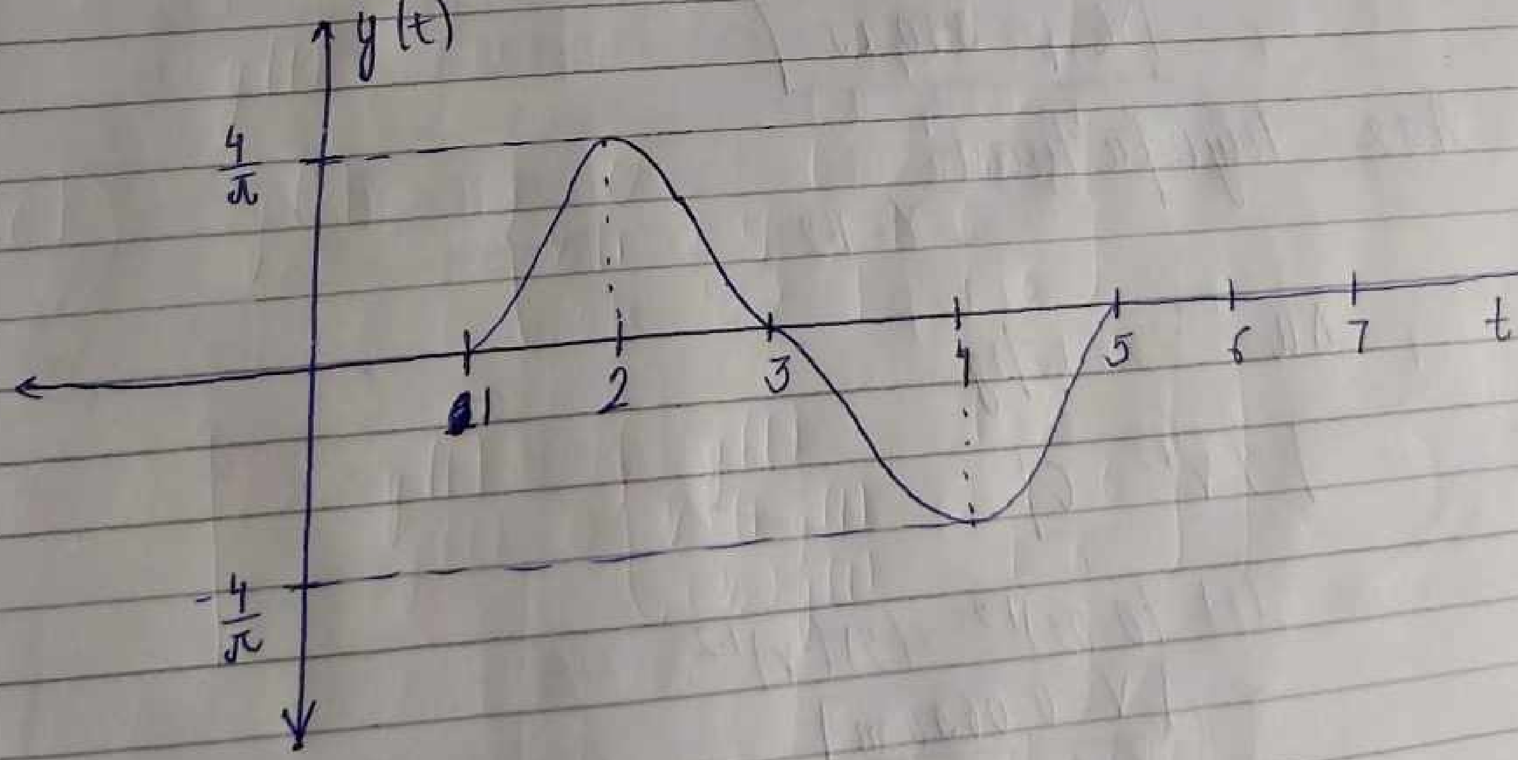
$\Rightarrow \boxed{y(t) = \frac{2}{\pi}[1+\cos(\pi t)]}$

③ $1 < t-2 < 3 \Rightarrow y(t) = \int_{t-2}^{3} [2\sin(\pi t - \pi\theta)]\,d\theta + \int_{3}^{t} 0\,d\theta$

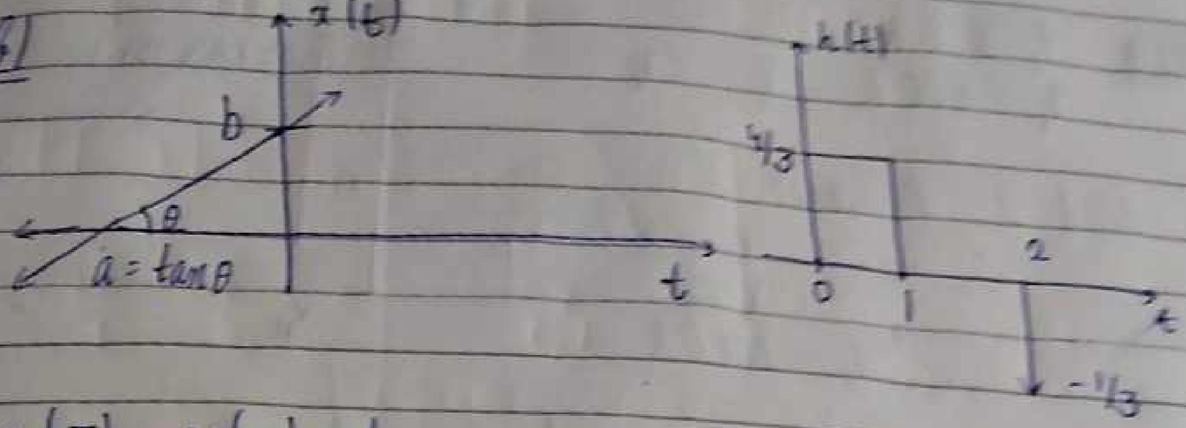
$3 < t < 5$

$$\boxed{y(t) = \frac{2}{\pi}\left[-1-\cos(\pi t)\right]}$$

$$\therefore\; y(t) = \begin{cases} \frac{2}{\pi}(1+\cos(\pi t)) & , 1<t<3 \\ \frac{-2}{\pi}(1+\cos(\pi t)) & , 3<t<5 \\ 0 & , \text{otherwise} \end{cases}$$



Ans 5 (b)



$\Rightarrow$ ~~$x(\tau) = a x(\tau) + b$~~

$\Rightarrow x(\tau) = a\tau + b$

let $h(\tau) = h_1(\tau) + \left(\frac{-1}{3}\delta(t-2)\right)$

$\therefore$ $y(t) = x(t) * h(t)$

$= x(t) * (h_1(t) - 1/3\ \delta(t-2))$

$= x(t) * h_1(t) - \frac{1}{3} x(t) * \delta(t-2)$

$= x(t) * h_1(t) - \frac{1}{3} x(t-2)$

Now, $h_1(t) = \begin{cases} 4/3 & , 0 < t \leq 1 \\ 0 & , \text{otherwise} \end{cases}$

$\therefore$ $x(t) * h_1(t) = \int_{-\infty}^{\infty} x(t-\theta)\, h(\theta)\, d\theta$ $\quad [t - \tau = \theta]$

$= \int_0^1 x(t-\theta)\left(\frac{4}{3}\right) d\theta + 0$

$= \frac{4}{3}\int_0^1 [at - a\theta + b]\, d\theta$

$\Rightarrow \boxed{x(t) * h_1(t) = \frac{4}{3}(at+b) - \frac{2a}{3}}$

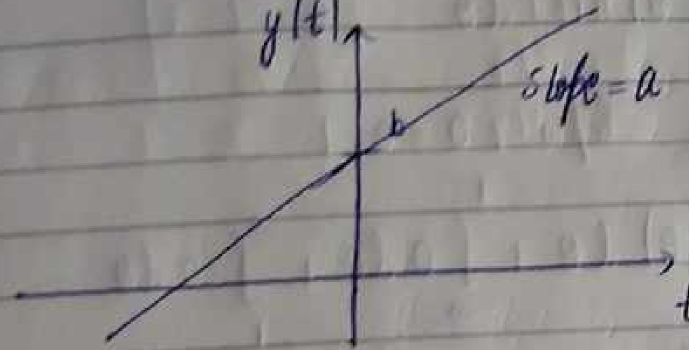
Now, $\frac{x(t-2)}{3} = \frac{1}{3}\left[a(t-2)+b\right]$

$$\therefore\ y(t) = x(t) * h_1(t) - \frac{1}{3}x(t-2)$$

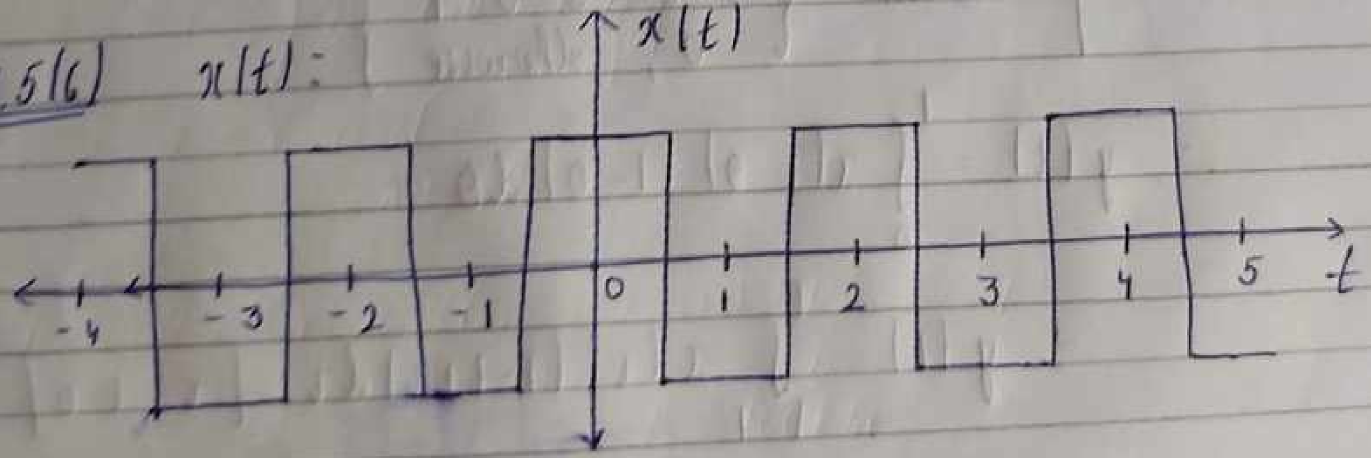
$$= \frac{4}{3}(at+b) - \frac{2a}{3} - \frac{1}{3}\left[a(t-2)+b\right]$$

$$\Rightarrow\ y(t) = at+b$$

$$\Rightarrow\ \boxed{y(t) = x(t)}$$



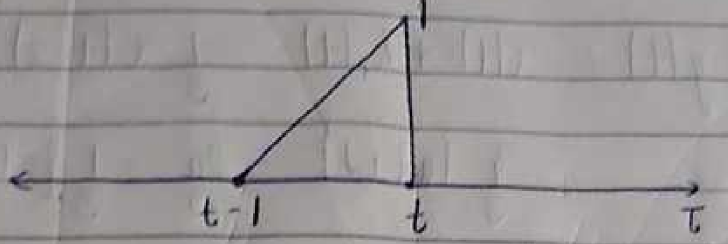
Ans. 5(c) $x(t)$:



$h(\tau)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$h(t-\tau) =$



in $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$$y(t) = \int_{-\infty}^{\infty} x(t-\theta)\, h(\theta)\, d\theta \quad [\theta = t-\tau]$$

Since, $h(\theta) \rightarrow$ non-zero only b/w $0 \le \theta \le 1$:

$$y(t) = \int_{0}^{1} x(t-\theta)\, h(\theta)\, d\theta + \int_{-\infty}^{1} 0\, d\theta + \int_{1}^{\infty} 0\, d\theta$$

$$\Rightarrow \quad h(\theta) = \begin{cases} 1-\theta & 0 \le \theta \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$$\therefore,\ y(t) = \int_{0}^{1} x(t-\theta)\,(1-\theta)\, d\theta$$

$$\Rightarrow y(t) = \int_{t-1}^{t} x(\tau)\,(1-t+\tau)\, d\tau \quad [\tau = t-\theta]$$

$$x(\tau) = \begin{cases} -1, & 2n+1/2 \le \tau \le 2n+3/2 \\ +1, & 2n-1/2 \le \tau \le 2n+1/2 \end{cases} \quad n \in I$$

① $2n+\frac{1}{2} \le t \le 2n+\frac{3}{2}$

② $2n-\frac{1}{2} \le t \le 2n+\frac{1}{2}$, $n \in I$

We can take $n=0$ as $x(t) \to$ periodic.

① $n=0 \to \frac{1}{2} \le t \le \frac{3}{2}$

$$y(t) = \int_{t-1}^{1/2} 1(1-t+\tau)\,d\tau + \int_{1/2}^{t} (-1)(1-t+\tau)\,d\tau$$

$$= (1-t)\left(\frac{1}{2}-t+1\right) - \frac{1}{8} - \frac{(t-1)^2}{2} + (-1)\left[(1-t)\left(t-\frac{1}{2}\right)+\frac{t^2}{2}-\frac{1}{8}\right]$$

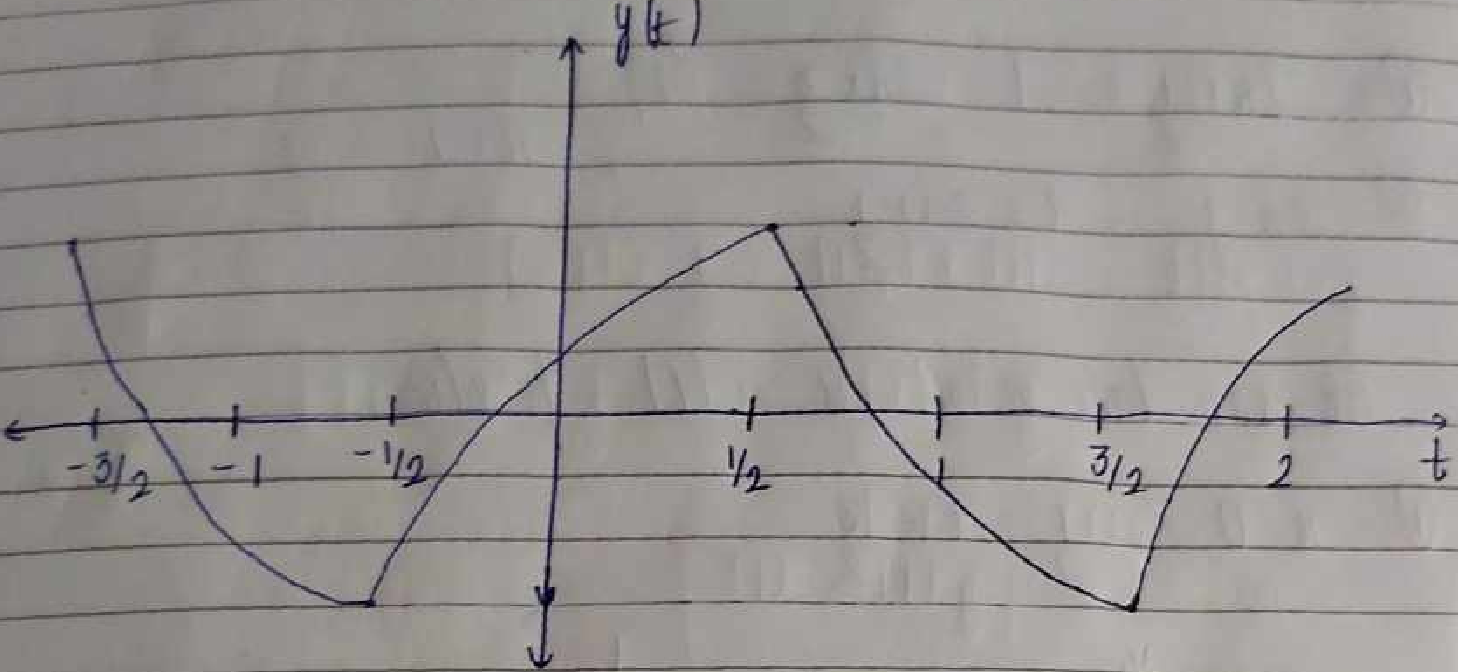
$$= t^2 - 3t + \frac{7}{4} \quad \text{for } 2n+\frac{1}{2} \le t \le 2n+\frac{3}{2},\ n \in I$$

② $n=0 \to -\frac{1}{2} \le t \le \frac{1}{2}$

$$y(t) = \int_{t-1}^{-1/2} (-1)(1-t+\tau)\,d\tau + \int_{-1/2}^{t} (1)(1-t+\tau)\,d\tau$$

$$\Rightarrow -t^2 + t + \frac{1}{4} \Rightarrow \text{for } 2n-\frac{1}{2} \le t \le 2n-\frac{3}{2},\ n \in I$$

Now, since the func^n is periodic, the graph b/w $\left[-\frac{1}{2}, \frac{3}{2}\right]$ will ~~resp~~ repeat with a period of 2.



$$\boxed{y(t) = \begin{cases} (t-2n)^2 - 3(t-2n) + 7/4 & ;\ 2n + 1/2 \le t \le 2n + 3/2 \\ -(t-2n)^2 + (t-2n) + 1/4 & ;\ 2n - 1/2 \le t \le 2n + \frac{1}{2} \end{cases}}$$