Tutorial - 3

1 (a) $\lim_{(x,y)\to(-1,2)} \frac{xy^3}{x+y} = \frac{(-1)(2)^3}{-1+2} = -8$

(b) $\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^2}$

along $y = mx^3$

$\lim_{(x,y)\to(0,0)} \frac{x^3(mx^3)}{x^6+(mx^3)^2} = \lim_{(x,y)\to(0,0)} \frac{mx^6}{x^6+m^6x^6} = \frac{m}{1+m^6}$

So limit doesn't exist

(c) $\lim_{(x,y)\to(0,1)} \tan^{-1}\left(\frac{y}{x}\right)$

along $y = mx$

$\lim_{(x,y)\to(0,1)} \tan^{-1}\left(\frac{mx}{x}\right) = \tan^{-1}(m)$ limit doesn't exist

(d) $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = \lim_{(x,y)\to(0,0)} \frac{1}{\sqrt{\frac{1}{y^2}+\frac{1}{x^2}}} = 0$

(e) $\lim_{(x,y)\to(0,0)} \frac{\sin^2(x+y)}{|x|+|y|}$

$= \lim_{(x,y)\to(0,0)} \sin(x+y) \lim_{(x,y)\to(0,0)} \frac{\sin(x+y)}{|x|+|y|}$

Now $\left|\frac{\sin(x+y)}{|x|+|y|}\right| = \frac{|\sin(x+y)|}{|x|+|y|} \leq \frac{|x+y|}{|x|+|y|} \leq 1$

$\implies \lim_{(x,y)\to(0,0)} \frac{\sin(x+y)}{|x|+|y|}$ is finite

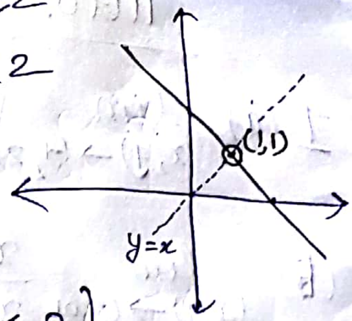
$\implies \left(\lim_{(x,y)\to(0,0)} \sin(x+y)\right) \times$ finite no $= 0$

(f). $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$ along $y = mx^2$ $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2} = \frac{2m}{1+m^2}$

So limit doesn't exist

(g) $\lim_{(x,y)\to(1,1)} f(x,y)$ where $f(x,y) = \begin{cases} 1 & \text{if } x+y \geq 2 \\ -1 & \text{if } x+y < 2 \end{cases}$

$$\lim_{(x,y)\to(1,1)} f(x,y) \qquad f(x,y) = \begin{cases} 1 & \text{if } x+y \geq 2 \\ -1 & \text{if } x+y<2 \end{cases}$$



Along the line $y=x$.

$$\lim_{(x,y)\to(1,1)} f(x,y) = \lim_{(h,k)\to(0,0)} f(1-h,1-k)$$

$$= -1 \qquad \left(\because (1-h)+(1-k) < 2\right)$$

Also

$$\lim_{(x,y)\to(1,1)} f(x,y) = \lim_{(h,k)\to(0,0)} f(1+h,1+k)$$

$$= 1 \qquad \left(\because (1+h)+(1+k) \geq 2\right)$$

So limit doesn't exist

(h) $$\lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz+zx}{\sqrt{x^2+y^2+z^2}}$$

$$= \lim_{(x,y,z)\to(0,0,0)} \left[\frac{xy}{\sqrt{x^2+y^2+z^2}} + \frac{yz}{\sqrt{x^2+y^2+z^2}} + \frac{zx}{\sqrt{x^2+y^2+z^2}}\right]$$

we have $(x-y)^2 = x^2+y^2-2xy \geq 0$

$$xy \leq \frac{1}{2}(x^2+y^2)$$

Similarly $yz \leq \frac{1}{2}(y^2+z^2)$

& $zx \leq \frac{1}{2}(z^2+x^2)$

hence $xy+yz+zx \leq (x^2+y^2+z^2)$

So

$$\left|\frac{xy+yz+zx}{\sqrt{x^2+y^2+z^2}}\right| \leq \left|\frac{x^2+y^2+z^2}{\sqrt{x^2+y^2+z^2}}\right| \leq \sqrt{x^2+y^2+z^2} < \epsilon$$

taking $\delta = \epsilon$

So $\left|\frac{xy+yz+zx}{\sqrt{x^2+y^2+z^2}}\right| < \epsilon$ whenever $\sqrt{x^2+y^2+z^2} < \delta$

$$\Rightarrow \lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz+zx}{\sqrt{x^2+y^2+z^2}} = 0$$

(i) $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^2+y^2}$ let $x = r\cos\theta$ $y = r\sin\theta$

$$\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^2+y^2} = \lim_{r\to 0} \frac{2r^3\cos^2\theta\sin\theta}{r^2} = 0$$

(2) (a) $f(x,y) = \dfrac{x+y}{x-y}$ for $(x,y) \in \mathbb{R}^2$ $\quad x-y \neq 0$

P.T. $\lim\limits_{x\to 0}\left[\lim\limits_{y\to 0} f(x,y)\right] = 1$ but $\lim\limits_{y\to 0}\left[\lim\limits_{x\to 0} f(x,y)\right] = -1$

$$\lim_{x\to 0}\left[\lim_{y\to 0}\frac{x+y}{x-y}\right] = \lim_{x\to 0}\left[\lim_{y\to 0}\frac{1+y/x}{1-y/x}\right] = 1$$

&

$$\lim_{y\to 0}\left[\lim_{x\to 0}\frac{x+y}{x-y}\right] = \lim_{y\to 0}\left[\lim_{x\to 0}\frac{x/y+1}{x/y-1}\right] = -1$$

along $y = mx$ $\quad \lim\limits_{(x,y)\to(0,0)}\dfrac{x+mx}{x-mx} = \dfrac{1+m}{1-m}$ limit doesn't exist

~~Also~~

(b). $f(x,y) = \begin{cases} x\sin\left(\frac{1}{y}\right) & ; y \neq 0 \\ 0 & ; y = 0 \end{cases}$ compute $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ and $\lim\limits_{x\to 0}\left[\lim\limits_{y\to 0} f(x,y)\right]$ $\lim\limits_{y\to 0}\left[\lim\limits_{x\to 0} f(x,y)\right]$

$$\lim_{(x,y)\to(0,0)} f(x,y) = 0 \times \text{some finite no. in } [-1,1]$$
$$= 0$$

$\lim\limits_{x\to 0}\left[\lim\limits_{y\to 0} f(x,y)\right] = \lim\limits_{x\to 0}\left[\lim\limits_{y\to 0} x\sin\left(\frac{1}{y}\right)\right] \longrightarrow$ oscillate b/w -1 and 1

So limit doesn't exist

$$\lim_{y\to 0}\left[\lim_{x\to 0} f(x,y)\right] = \lim_{y\to 0}\left[\lim_{x\to 0} x\sin\left(\frac{1}{y}\right)\right] = 0.$$

(c) $f(x,y) = \dfrac{x^2y^2}{x^2y^2+(x-y)^2}$ if $x^2y^2+(x-y)^2 \neq 0$

$$\lim_{y\to 0}\frac{x^2y^2}{x^2y^2+(x-y)^2} = \lim_{y\to 0}\frac{\cancel{0}\,y^2}{\cancel{0}\,y^2+(1-y/x)^2} = \frac{0}{0+1} = 0$$

$$\lim_{x\to 0}\left[\lim_{y\to 0} f(x,y)\right] = 0$$

Now

$$\lim_{x\to 0}\frac{x^2y^2}{x^2y^2+(x-y)^2} = \lim_{x\to 0}\frac{\cancel{y}\,x^2}{x^2+\left(\frac{x}{y}-1\right)^2} = \frac{0}{0+1} = 0$$

$$\lim_{y\to 0}\left[\lim_{x\to 0} f(x,y)\right] = 0$$

However

$$\lim_{(x,y)\to(0,0)}\frac{x^2y^2}{x^2y^2+(x-y)^2} = \lim_{(x,y)\to(0,0)}\frac{x^2m^2x^2}{x^2m^2x^2+(x-mx)^2} = \lim_{(x,y)\to 0}\frac{m^2x^2}{m^2x^2+(1-m)^2}$$

along $y = mx$ $\quad$ So limit doesn't exist. $\quad = 0$ if $m \neq 1$; $= 1$ if $m = 1$

(3) $f(x,y) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$

at any point (a,b) $\lim_{(x,y)\to(a,b)} f(x,y)$ doesn't exist.

~~For any point (a,b)~~
~~If a is any real no. then $\exists$ rational seq. $\{x_n\}$~~
~~s.t. $x_n \to a$~~
~~also $f(x_n, y) = 0 \implies f(a,y) = 0$~~
~~and $\exists$ irrational seq. $\{y_n\}$ s.t.~~
~~$y_n \to b$~~
~~also $f(x, y_n) = 1 \implies f(x,b) = 0$~~

For any point $(a,b) \in \mathbb{R}^2$

$\exists$ a seq. of rational no. $\{x_n\}$ s.t. $x_n \to a$

and a seq. of irrational no. $\{y_n\}$ s.t. $y_n \to a$

Now $f(x_n, y) = 0$

$f(y_n, y) = 1$

and

$$\left.\begin{aligned} \lim_{n\to\infty} f(x_n, b) &= 0 \\ \lim_{n\to\infty} f(y_n, b) &= 1 \end{aligned}\right\} \implies \lim_{(x,y)\to(a,b)} f(x,y) \text{ doesn't exist}$$

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(4) Examine the Continuity at $(0,0)$

(a) $f(x,y) = \begin{cases} \dfrac{\sin^{-1}(x+2y)}{\tan^{-1}(2x+4y)} & (x,y) \neq 0 \\ 1/2 & (x,y) = (0,0) \end{cases}$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{\sin^{-1}(x+2y)}{\tan^{-1}(2x+4y)} \qquad \text{let } x+2y = t$$

$$= \lim_{t\to 0} \frac{\sin^{-1}(t)}{\tan^{-1}(2t)} = \lim_{t\to 0} \frac{\frac{1}{\sqrt{1-t^2}}}{\frac{2}{1+t^2}} = \frac{1}{2} = f(0,0)$$

So $f(x,y)$ is cont. at $(0,0)$.

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(b). $f(x,y) = \begin{cases} x\sin\frac{1}{x} + y\sin\frac{1}{y} & xy \neq 0 \\ 0 & xy = 0 \end{cases}$

$$|f(x,y) - f(0,0)| = \left|x\sin\tfrac{1}{x} + y\sin\tfrac{1}{y}\right|$$
$$\leq |x|\left|\sin\tfrac{1}{x}\right| + |y|\left|\sin\tfrac{1}{y}\right|$$
$$\leq |x| + |y|$$
$$\leq 2\sqrt{x^2+y^2} < \epsilon$$

taking $\delta = \epsilon/2$

So $|f(x,y) - f(0,0)| < \epsilon$ whenever $0 < \sqrt{x^2+y^2} < \delta$

Hence $f(x,y)$ is cont. at $(0,0)$.

(c). $f(x,y) = \begin{cases} xy\log(x^2+y^2) & ; (x,y) \neq (0,0) \\ 0 & ; (x,y) = (0,0) \end{cases}$

$$|f(x,y) - f(0,0)| = |xy\log(x^2+y^2)|$$
$$\leq |xy|\,|\log(x^2+y^2)|$$
$$\leq \tfrac{1}{2}(x^2+y^2)\cdot|x^2+y^2|$$
$$\leq \frac{(x^2+y^2)^2}{2} < \epsilon$$

$\left\{ \because\ xy \leq \frac{1}{2}(x^2+y^2) \ \& \ \log x \leq x \right.$

taking $\delta = (2\epsilon)^{1/4}$

$|f(x,y) - f(0,0)| < \epsilon$ whenever $0 < \sqrt{x^2+y^2} < \delta$

hence $f(x,y)$ is cont. at $(0,0)$.

(d). $f(x,y) = \begin{cases} \dfrac{x^2y^2}{x^3+y^3} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

The function is not defined for $y = -x$.

It cannot be contineuous.

(e) $f(x,y) = \sqrt{|xy|}$

$|f(x,y) - f(0,0)| = \left|\sqrt{|xy|}\right| \leq \sqrt{\frac{x^2+y^2}{2}} < \epsilon$ taking $\delta = \epsilon\sqrt{2}$

$\Rightarrow f(x,y)$ is cont.

(5) $f(x,y) = \begin{cases} \dfrac{y(x^2-y^2)}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

Compute $f_x(0,y)$, $f_y(x,0)$, $f_x(0,0)$ and $f_y(0,0)$ if they exist

Sol.

$$f_x(0,y) = \lim_{\Delta x \to 0} \frac{f(\Delta x, y) - f(0,y)}{\Delta x}$$

$$= \lim_{h \to 0} \frac{\left[\frac{y(h^2-y^2)}{h^2+y^2} - \frac{y(-y^2)}{y^2}\right]}{h}$$

$$= \lim_{h \to 0} \frac{y}{h}\left[\frac{h^2-y^2}{h^2+y^2} + 1\right] = \lim_{h \to 0} \frac{y}{h}\left[\frac{2h^2}{h^2+y^2}\right]$$

$$= \lim_{h \to 0} y\left[\frac{2h}{h^2+y^2}\right] = 0$$

$$f_y(x,0) = \lim_{K \to 0} \frac{f(x,K) - f(x,0)}{K}$$

$$= \lim_{K \to 0} \frac{1}{K}\left[\frac{K(x^2-K^2)}{x^2+K^2} - 0\right] = 1$$

$$f_x(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}(0) = 0$$

$$f_y(0,0) = \lim_{K \to 0} \frac{f(0,K) - f(0,0)}{K} = \lim_{K \to 0} \frac{1}{K}\left[\frac{K(-K^2)}{K^2}\right] = -1$$

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(6) $f(x,y) = \begin{cases} -xy & |y| \geq x \\ xy & |y| < x \end{cases}$ $\quad f_{xy}(0,0)$ & $f_{yx}(0,0)$ both exist and are equal

~~$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{h \cdot 0}{h} = 0$~~

We have,

$$f_{yx}(0,0) = \lim_{K \to 0} \frac{f_x(0,K) - f_x(0,0)}{K}$$

then

$$f_x(0,y) = \lim_{h \to 0} \frac{f(h,y) - f(0,y)}{h} = \lim_{h \to 0} \frac{(-hy)}{h} = -y$$

$$f_{yx}(0,0) = \lim_{K \to 0} \frac{f_x(0,K) - f_x(0,0)}{K} = \lim_{K \to 0} \frac{(-K) - 0}{K} = -1$$

So $f_y(x,0) = \lim_{k\to 0} \frac{f(x,k) - f(x,0)}{k} = \lim_{k\to 0} \frac{xk - 0}{k} = x$

$f_{xy}(0,0) = \lim_{h\to 0} \frac{f_y(h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{h-0}{h} = 1$

$f_{xy}(0,0)$ & $f_{yx}(0,0)$ both exist but not equal.

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(7) Prove that $f(x,y) = |x| + |y|$ is cont. but not diff. at $(0,0)$.

$|f(x,y) - f(0,0)| = \big|\, |x| + |y| - 0 \,\big| \le 2\sqrt{x^2+y^2} < \epsilon$

taking $\delta = \epsilon/2$

hence $f(x,y)$ is Cont. at $(0,0)$

$\Delta z = f_x \Delta x + f_y \Delta y + \epsilon_1 \Delta x + \epsilon_2 \Delta y$

$\Delta z = f(0+\Delta x, 0+\Delta y) - f(0,0)$

$= |\Delta x| + |\Delta y| - 0$

$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{|h|}{h}$ doesn't exist

$f_y(0,0) = \lim_{k\to 0} \frac{f(0,k) - f(0,0)}{k} = \lim_{k\to 0} \frac{|k|}{k}$ doesn't exist

$\therefore f$ is not differentiable at $(0,0)$.

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(8) $f(x,y) = \begin{cases} \dfrac{(x+y)\{\sqrt{x^2+y^2} + xy\}}{\sqrt{x^2+y^2}}, & \text{when } x^2+y^2 \neq 0 \\ 0, & \text{when } x = y = 0 \end{cases}$

differentiable at $(0,0)$. Hence deduce $f_x(0,0) = f_y(0,0) = 1$

Sol.

$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{1}{h}\left\{\frac{(h)\sqrt{h^2}}{\sqrt{h^2}}\right\} = 1$

$f_y(0,0) = \lim_{k\to 0} \frac{f(0,k) - f(0,0)}{k} = \lim_{k\to 0} \frac{1}{k}\left\{\frac{k\sqrt{k^2}}{\sqrt{k^2}}\right\} = 1$

Now,

$\Delta z = f_x \cdot \Delta x + f_y \cdot \Delta y + \epsilon_1 \cdot \Delta x + \epsilon_2 \cdot \Delta y$

$$\Delta z = f(0+\Delta x, 0+\Delta y) - f(0,0)$$

$$= \frac{(\Delta x+\Delta y)\{\sqrt{\Delta x^2+\Delta y^2} + \Delta x . \Delta y\}}{\sqrt{\Delta x^2+\Delta y^2}}$$

$$= \Delta x + \Delta y + \frac{(\Delta x+\Delta y).\Delta x.\Delta y}{\sqrt{\Delta x^2+\Delta y^2}}$$

$$= \Delta x + \Delta y + \left(\frac{\Delta x.\Delta y}{\sqrt{\Delta x^2+\Delta y^2}}\right)\Delta x + \left(\frac{\Delta x.\Delta y}{\sqrt{\Delta x^2+\Delta y^2}}\right)\Delta y$$

Comparing with

$$\Delta z = f_x \Delta x + f_y \Delta y + \epsilon_1 \Delta x + \epsilon_2 \Delta y$$

$$\epsilon_1 = \epsilon_2 = \frac{\Delta x . \Delta y}{\sqrt{\Delta x^2+\Delta y^2}}$$

$$\lim_{(\Delta x, \Delta y)\to(0,0)} \frac{\Delta x . \Delta y}{\sqrt{\Delta x^2+\Delta y^2}} = \lim_{r\to 0} \frac{r^2 \sin\theta\cos\theta}{r} = 0$$

Both limits exist and are equal. Hence f is differentiable

(9)
$$f(x,y) = \begin{cases} x^3\sin(1/x^2) + y^3\sin(1/y^2) & \text{when } xy \neq 0 \\ x^3\sin(1/x^2) & \text{when } x\neq 0,\ y=0 \\ y^3\sin(1/y^2) & \text{when } x=0,\ y\neq 0 \\ 0 & x=0=y \end{cases}$$

is differentiable at $(0,0)$, whereas none f_x, f_y is cont. at $(0,0)$

Sol.

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[h^3 \sin\left(\frac{1}{h^2}\right)\right] = \lim_{h\to 0}\left[h^2 \sin\left(\frac{1}{h^2}\right)\right] = 0$$

$$f_y(0,0) = \lim_{K\to 0} \frac{f(0,K)-f(0,0)}{K} = \lim_{K\to 0} \frac{1}{K}\left[K^3 \sin\left(\frac{1}{K^2}\right)\right] = 0$$

$$\Delta z = f(0+\Delta x, 0+\Delta y) - f(0,0)$$

$$= \Delta x^3 \sin\left(\frac{1}{\Delta x^2}\right) + \Delta y^3 \sin\left(\frac{1}{\Delta y^2}\right) - 0$$

Comparing with $\Delta z = f_x \cdot \Delta x + f_y \cdot \Delta y + \epsilon_1 \Delta x + \epsilon_2 \Delta y$

$\epsilon_1 = \Delta x^2 \sin\left(\frac{1}{\Delta x^2}\right) \qquad \epsilon_2 = \Delta y^2 \sin\left(\frac{1}{\Delta y^2}\right)$

$$\lim_{(\Delta x, \Delta y)\to(0,0)} \epsilon_1 = \lim_{(\Delta x, \Delta y)\to(0,0)} \Delta x^2 \sin\left(\frac{1}{\Delta x^2}\right) = 0$$

Similarily $\lim\limits_{(\Delta x, \Delta y)\to(0,0)} \epsilon_2 = 0$

Hence function is differentiable at $(0,0)$.

$$f_x(x,y) = \begin{cases} 3x^2 \sin\frac{1}{x^2} + x^3\left(\frac{-2}{x^3}\right)\cos\frac{1}{x^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$= \begin{cases} 3x^2 \sin\frac{1}{x^2} - 2\cos\frac{1}{x^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$\lim_{(x,y)\to(0,0)} f_x(x,y) = \lim_{x\to 0}\left[3x^2 \sin(1/x^2) - 2\cos(1/x^2)\right]$$

$$= 0 - 2 \cdot \lim_{x\to 0} \cos\left(\frac{1}{x^2}\right)$$

Limit doesn't exist

$\Longrightarrow f_x(x,y)$ is not Cont. at $(0,0)$

Similary $f_y(x,y)$ is also not cont.

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(10) $$f(x,y) = \begin{cases} (x^2+y^2)\cos\left(\frac{1}{\sqrt{x^2+y^2}}\right), & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

is differentiable at $(0,0)$ and that f_x, f_y are not cont. at $(0,0)$

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{1}{h}\left[h^2 \cos\left(\frac{1}{h}\right)\right] = 0$$

Similarily $f_y(0,0) = 0$

$$\Delta z = (\Delta x^2 + \Delta y^2)\cos\left(\frac{1}{\sqrt{\Delta x^2 + \Delta y^2}}\right)$$

Comparing with $\Delta z = f_x \cdot \Delta x + f_y \Delta y + \epsilon_1 \Delta x + \epsilon_2 \Delta y$

$$\epsilon_1 = \Delta x \cdot \cos\left(\frac{1}{\sqrt{\Delta x^2+\Delta y^2}}\right) \;\&\; \epsilon_2 = \Delta y \cos\left(\frac{1}{\sqrt{\Delta x^2+\Delta y^2}}\right)$$

$\lim\limits_{\Delta x\to 0} \epsilon_1 \to 0$ $\qquad$ $\lim\limits_{\Delta y\to 0} \epsilon_2 \to 0$

hence $f(x,y)$ is differentiable at $(0,0)$.

Now

$$f_x(x,y) = 2x\cos\frac{1}{\sqrt{x^2+y^2}} + (x^2+y^2)\sin\frac{1}{\sqrt{x^2+y^2}}\cdot\left(\frac{2x}{2(x^2+y^2)^{3/2}}\right)$$

$$= 2x\cos\frac{1}{\sqrt{x^2+y^2}} + \frac{x}{\sqrt{x^2+y^2}}\sin\frac{1}{\sqrt{x^2+y^2}}$$

along $y=0$.

$$\lim_{x\to 0} f_x(x,y) = \lim_{x\to 0}\left(2x\cos\frac{1}{x} + \sin\frac{1}{x}\right)$$

$$= \lim_{x\to 0} 2x\cos\frac{1}{x} + \lim_{x\to 0}\sin\frac{1}{x}$$

doesn't exist

So $f_x(x,y)$ is not Cont. at $(0,0)$

Similarly for $f_y(x,y)$