Assignment 1

Q.1 (a) $x_1(t) = \begin{cases} e^{-2t}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \left[\int_{-T}^{0} |0|^2 dt + \int_{0}^{T} |e^{-2t}|^2 dt \right]$$

$$= \lim_{T\to\infty} \int_{0}^{T} e^{-4t} dt$$

$$= \lim_{T\to\infty} \left(\frac{e^{-4t}}{-4} \right)_0^T$$

$$= \lim_{T\to\infty} \left(\frac{1}{4} - \frac{e^{-4T}}{4} \right)$$

$$= \underline{\underline{\frac{1}{4}}}$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} e^{-4t} dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left(\frac{1}{4} \right)$$

$$= \underline{\underline{0}}.$$

(b) $x_2[n] = \cos\left(\frac{\pi}{4} n\right)$, $-\infty < n < \infty$, $n \in \mathbb{I}$

$$E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} |x_2[n]|^2$$

$$= \lim_{N\to\infty} \sum_{n=-N}^{N} \left|\cos\left(\frac{\pi}{4} n\right)\right|^2$$

$$= \lim_{N\to\infty} \sum_{n=-N}^{N} \frac{1 + \cos\frac{\pi n}{2}}{2}$$

$\cos\frac{\pi n}{2}$ is periodic with fundamental period = 4

$$\Rightarrow \lim_{N\to\infty} \sum_{n=-N}^{N} \cos\left(\frac{\pi n}{2}\right) = 0$$

$$\Rightarrow E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} \frac{1}{2}$$

$$= \lim_{N\to\infty} \frac{2N+1}{2} = \underline{\underline{\infty}}.$$

$$P_\infty = \lim_{N\to\infty} \left[\left(\sum_{n=-N}^{N} |x_2[n]|^2 \right) \times \frac{1}{2N+1} \right]$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \cos^2\left(\frac{n\pi}{4}\right)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1 + \cos\left(\frac{n\pi}{2}\right)}{2}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left(\frac{1}{2} \times (2N+1) \right)$$

$$= \underline{\underline{\frac{1}{2}}}$$

Q.2. (a) $x_1[n] = \begin{cases} 2, & 0 \le n \le 6 \\ 0, & \text{otherwise} \end{cases}, \quad n \in \mathbb{I}$

$$x_1[-n] = \begin{cases} 2, & 0 \le -n \le 6 \\ 0, & \text{otherwise} \end{cases} \Rightarrow x_1[-n] = \begin{cases} 2, & -6 \le n \le 0 \\ 0, & \text{otherwise} \end{cases}$$

<u>Decomposition</u>: Even: $\dfrac{x_1[n] + x_1[-n]}{2} \Rightarrow e_1[n] = \begin{cases} 1, & -6 \le n < 0 \\ 2, & \text{if } n = 0 \\ 1, & 0 < n \le 6 \\ 0, & \text{otherwise} \end{cases}, \; n \in \mathbb{I}$

$$\cancel{e_1[n] = x_1[n] = \begin{cases} 2, & 0 \le n \le 6 \\ 0, & \text{otherwise} \end{cases}}$$

$\cancel{\text{Odd: } o_1[n] = \dfrac{x_1[n] - x_1[-n]}{2} = 0}$ $\quad o_1[n] = \begin{cases} -1, & -6 \le n < 0 \\ 1, & 0 < n \le 6 \\ 0, & \text{otherwise} \end{cases}$

(b) $x_2(t) = \sin(t/2), \quad t \in (-\infty, \infty), \; t \in \mathbb{R}$

$$x_2(-t) = -\sin(t/2) = -x_2(t)$$

<u>Decomposition</u>: Even: $\dfrac{x_1(t) + x_2(t)}{2}$

$$\Rightarrow \boxed{e_2(t) = 0}$$

<u>Odd</u>: $\dfrac{x_1(t) - x_2(t)}{2}$

$$\Rightarrow \boxed{o_2(t) = \sin(t/2) = x_2(t)}$$

(c) $x_3[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n \ge 3 \\ 0, & \text{otherwise} \end{cases}, \quad n \in \mathbb{I}$

$$x_3[-n] = \begin{cases} \left(\frac{1}{2}\right)^{-n}, & -n \ge 3 \\ 0, & \text{otherwise} \end{cases}, \quad n \in \mathbb{I}$$

$$= \begin{cases} \left(\frac{1}{2}\right)^{-n}, & n \le -3 \\ 0, & \text{otherwise} \end{cases}, \quad n \in \mathbb{I}$$

<u>Decomposition</u>:

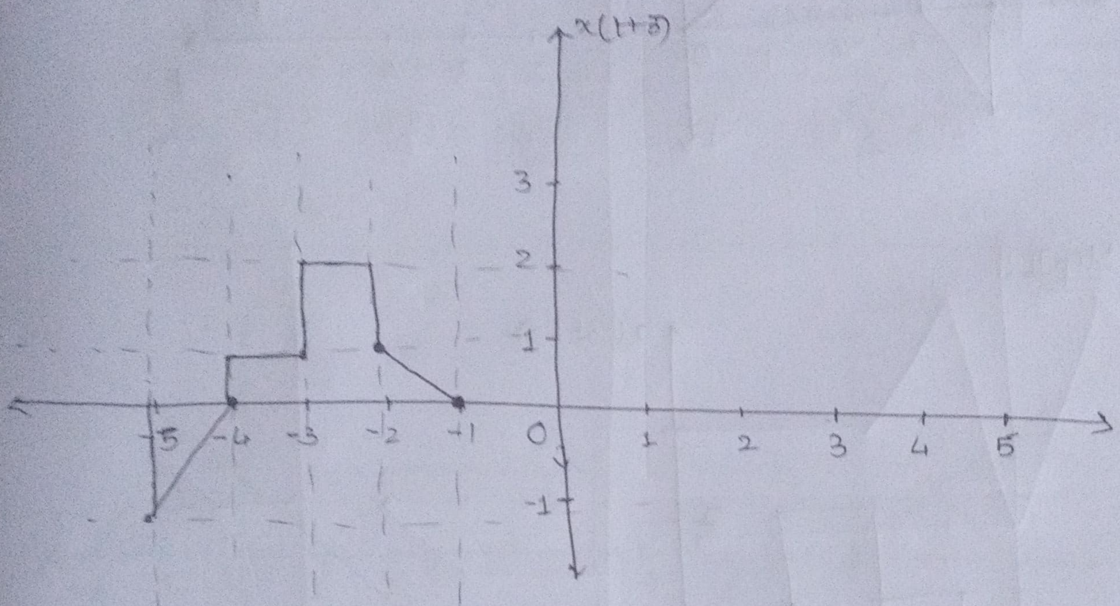
<u>Even</u>: $e_3[n] = \dfrac{x_3[n] + x_3[-n]}{2}$

$$= \begin{cases} \left(\frac{1}{2}\right)^{n+1}, & n \ge 3 \\ 0, & -3 < n < 3 \\ \left(\frac{1}{2}\right)^{-n+1}, & n \le -3 \end{cases}, \quad n \in \mathbb{I}$$

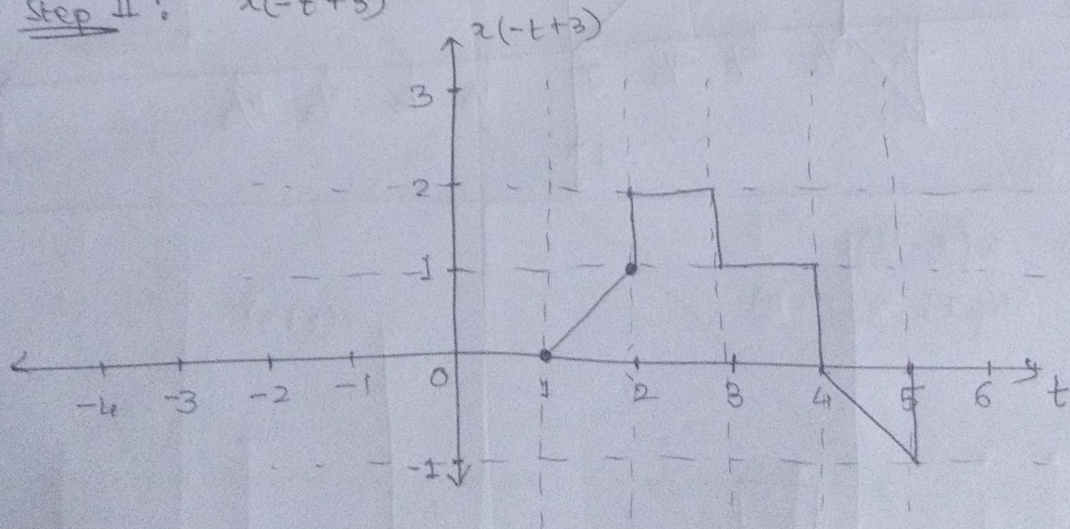
Odd: $O_3[n] = \dfrac{x_3[n] - x_3[-n]}{2}$

$$= \begin{cases} \left(\frac{1}{2}\right)^{n+1} & , n \geq 3 \\ 0 & , -3 < n < 3 \quad , n \in \mathbb{Z} \\ -\left(\frac{1}{2}\right)^{-n+1} & , n \leq -3 \end{cases}$$

Q.3. (a) Step I: $x(t+3)$

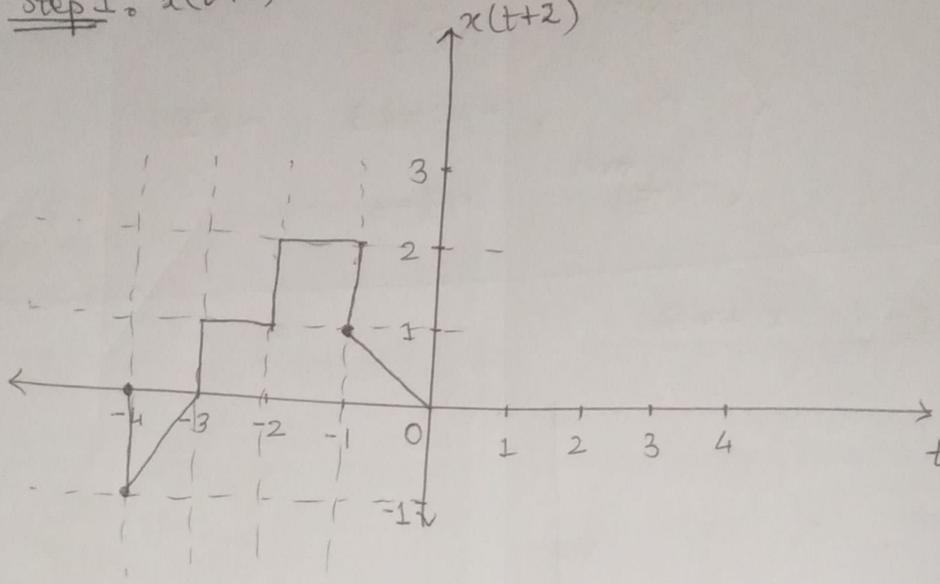


Step II: $x(-t+3)$

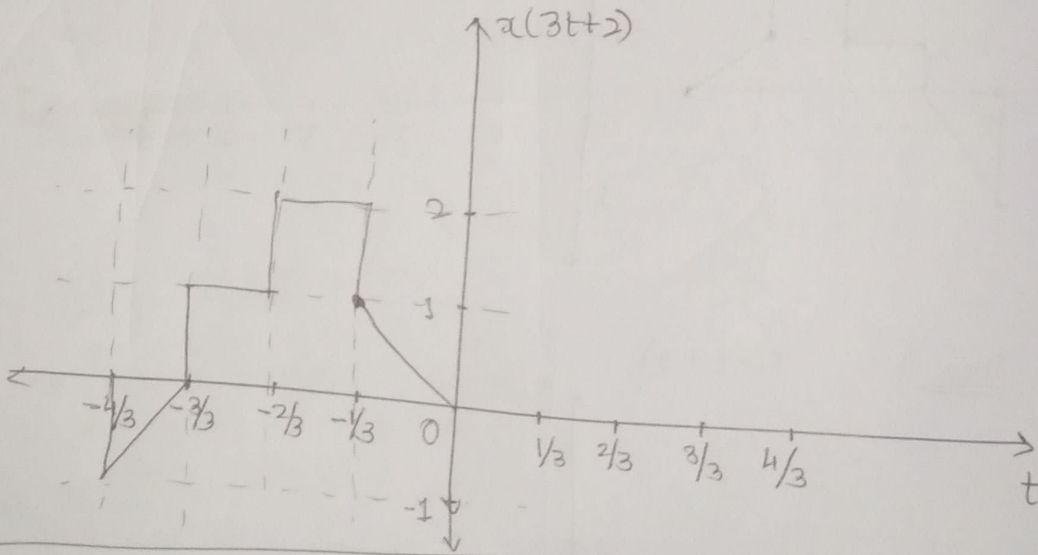


Q.3 (b) $x(3t+2)$

Step I: $x(t+2)$

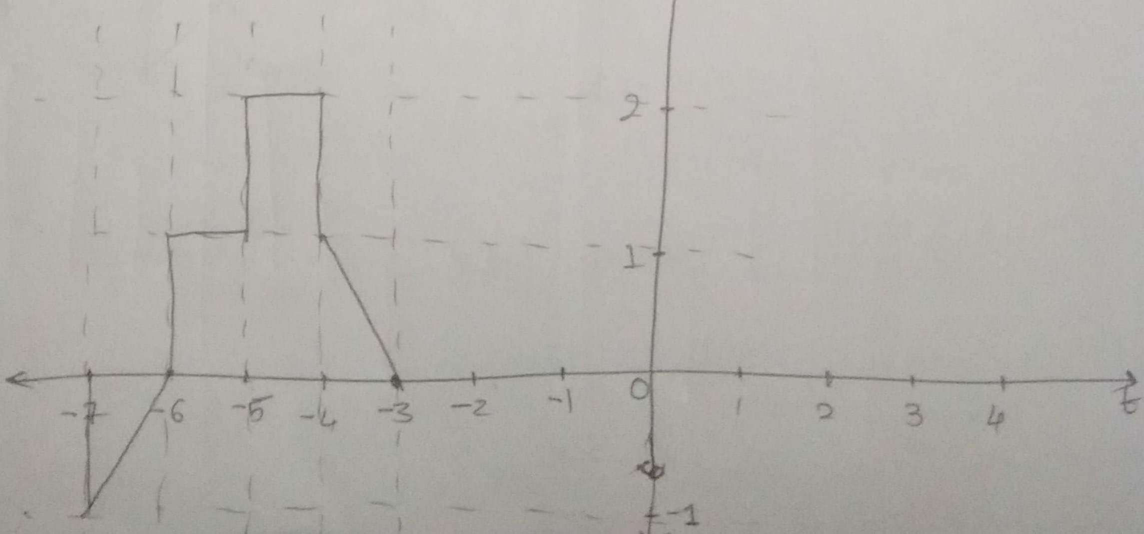


Step II:

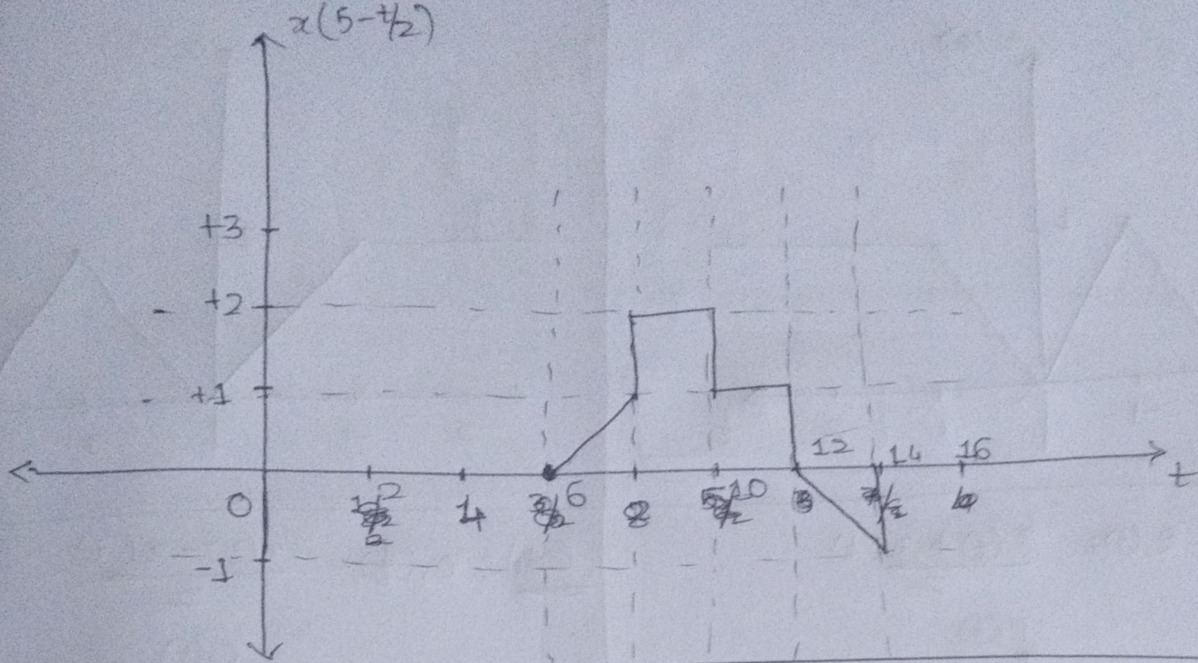


Q.3(c) $x(5-t/2)$

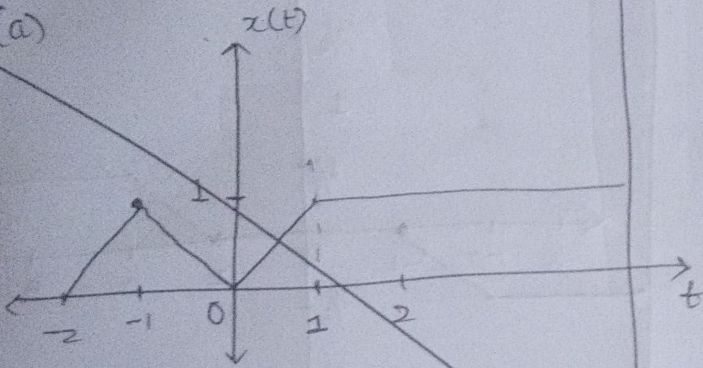
Step I: $x(t+5)$

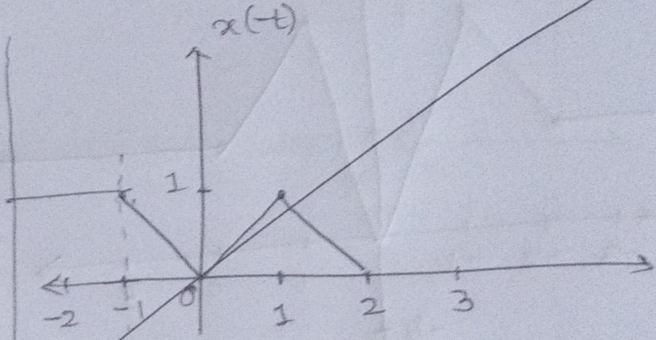


Step II: $x\left(-\frac{t}{2}+5\right)$



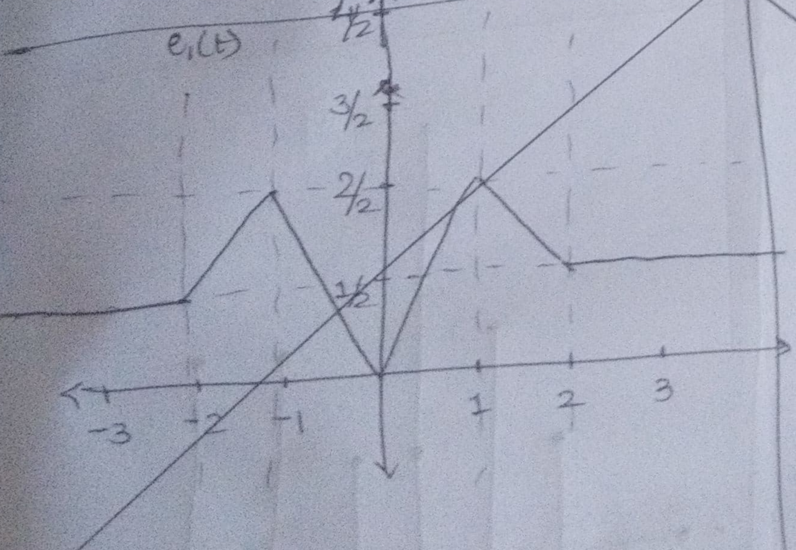
Q.4.(a)

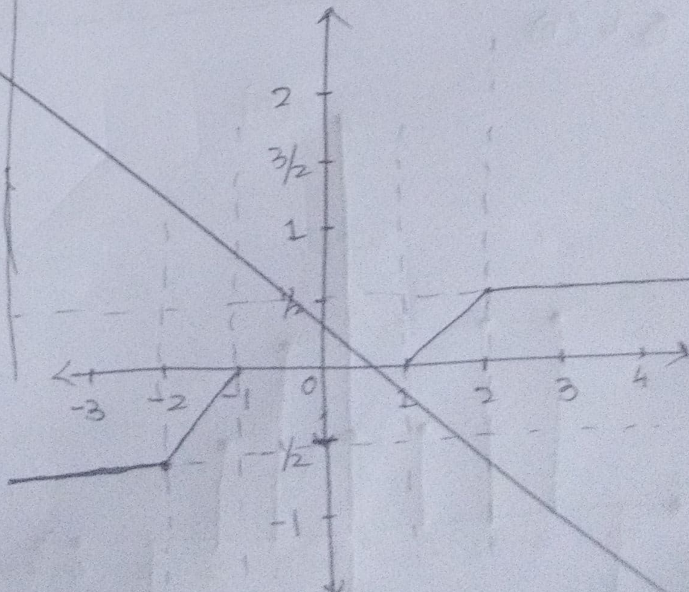




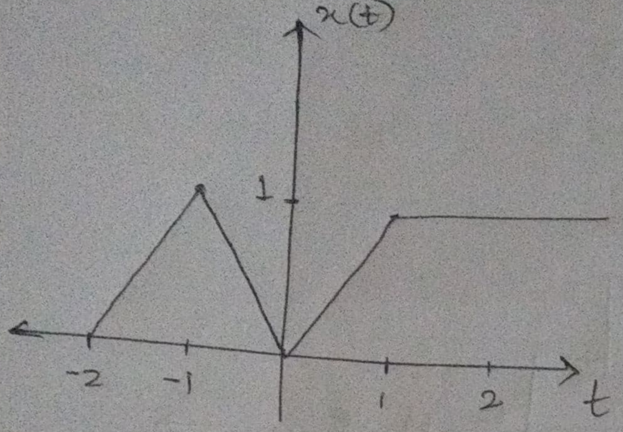
Even: $\frac{x(t)+x(-t)}{2} = e_1(t)$

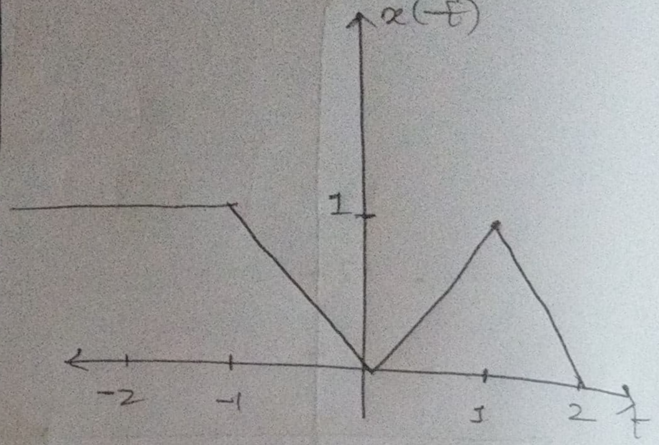
Odd: $\frac{x(t)-x(-t)}{2} = o_1(t)$



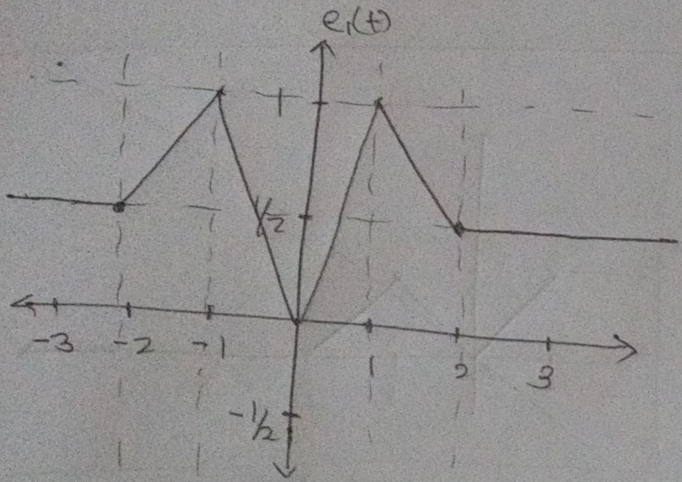


Q 4. (a)

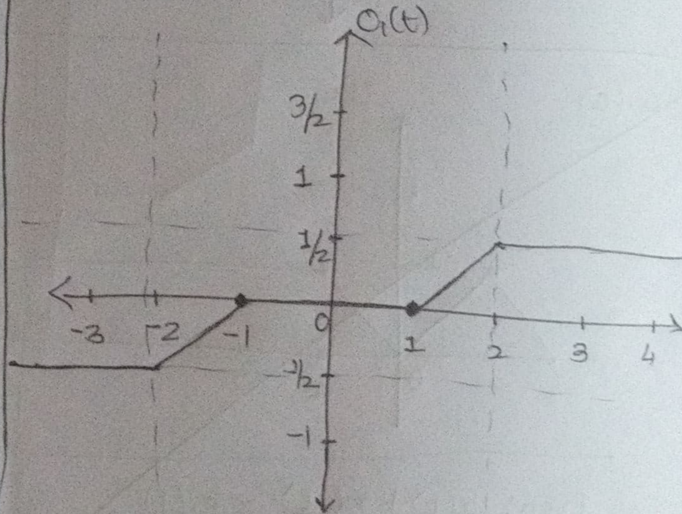




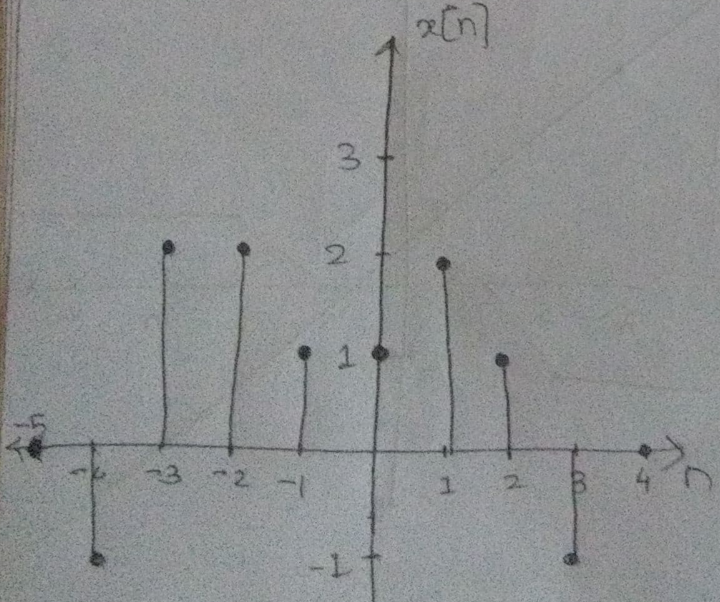
Even : $e_1(t) = \dfrac{x(t) + x(-t)}{2}$

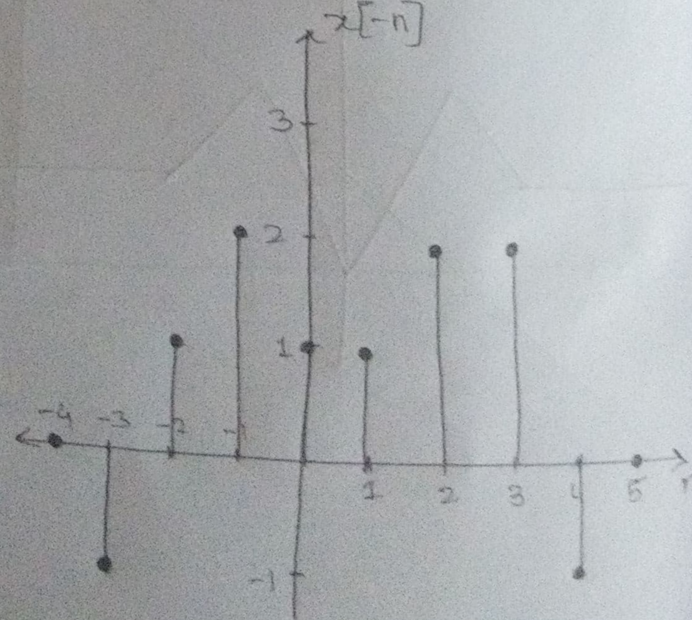


Odd : $o_1(t) = \dfrac{x(t) - x(-t)}{2}$

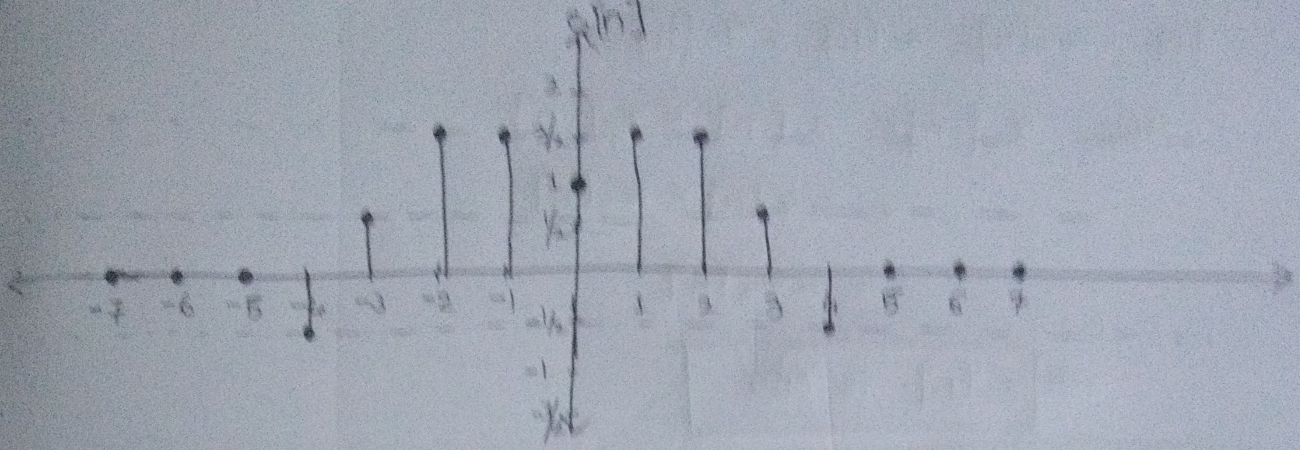


Q.4 (b)

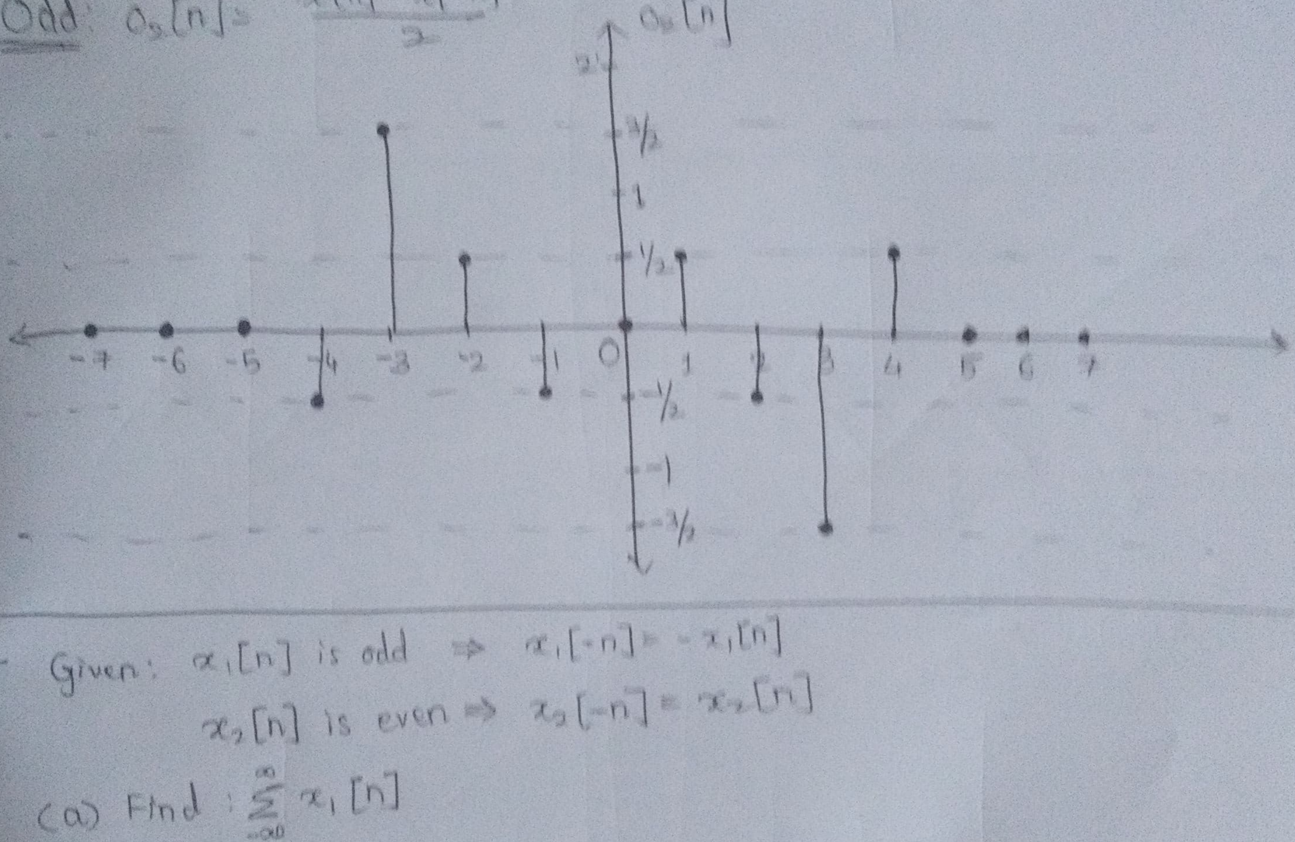




Even: $e_3[n] = \frac{x[n] + x[-n]}{2}$



Odd: $O_3[n] = \frac{x[n] - x[-n]}{2}$



Q.5 Given: $x_1[n]$ is odd $\Rightarrow x_1[-n] = -x_1[n]$

$x_2[n]$ is even $\Rightarrow x_2[-n] = x_2[n]$

(a) Find: $\sum_{-\infty}^{\infty} x_1[n]$

Now, $\sum_{-\infty}^{\infty} x_1[n] = \sum_{-\infty}^{-1} x_1[n] + x_1[0] + \sum_{1}^{\infty} x_1[n]$

$= \sum_{1}^{\infty} x_1[-n] + 0 + \sum_{1}^{\infty} x_1[n]$ ($\because x_1[0] = 0$ ... odd fu^n)

$= \sum_{1}^{\infty} \left(x_1[n] + x_1[-n] \right)$

$= \sum_{1}^{\infty} 0$

$= \underline{\underline{0}}$

(b) $x_1[n] \times x_2[n]$: Odd or even or none?

Let $x_3[n] = x_1[n] \times x_2[n]$

Now, $x_3[-n] = x_1[-n] \times x_2[-n]$

$= -x_1[n] \times x_2[n]$

$= -x_3[n]$

$\Rightarrow \boxed{x_3[n] \text{ is odd}}$