ECN-203
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Assignment 2

1) a) $x_1(t) = 2je^{j(12t)}$

$= 2j(\cos(12t) + j\sin(12t))$
$= -2\sin(12t) + 2j\cos(12t)$

$12t_0 = 2\pi$

$$\boxed{t_0 = \frac{\pi}{6}}$$

b) $x_2[n] = e^{-0.7jn}$

$e^{-0.7j(n+n_0)} = e^{-0.7jn}$

$e^{0.7jn_0} = 1$

$\frac{7n_0}{10} = 2\pi m$

$n_0 = \frac{20\pi m}{7}$ } Irrational

$$\boxed{\text{Period not defined}}$$

c) $x_3[n] = 3e^{\frac{3\pi}{5}(j(n+1/2))}$

$\frac{3\pi}{5}n_0 = 2\pi m$

$n_0 = \frac{10m}{3}$

$$\boxed{n_0 = 10}$$

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(d) $x_4(t) = 5e^{j(2\pi t)}$

$T_0 = 1 \quad \left(\frac{2\pi}{2\pi}\right)$

2) a) $x(t) = \underbrace{2\cos(7t+3)}_{\frac{2\pi m}{7}} + \underbrace{3\sin(3t+4)}_{\frac{2\pi n}{3}}$

$\boxed{LCM = 2\pi}$

Periodic w/ period 2π

b) $x[n] = 1 + e^{j\left(\frac{4\pi n}{7}\right)} - \underbrace{e^{j\left(\frac{2n}{5}\right)}}$

$\cancel{2\pi \times \left(\frac{5}{2\pi}\right)} \neq$

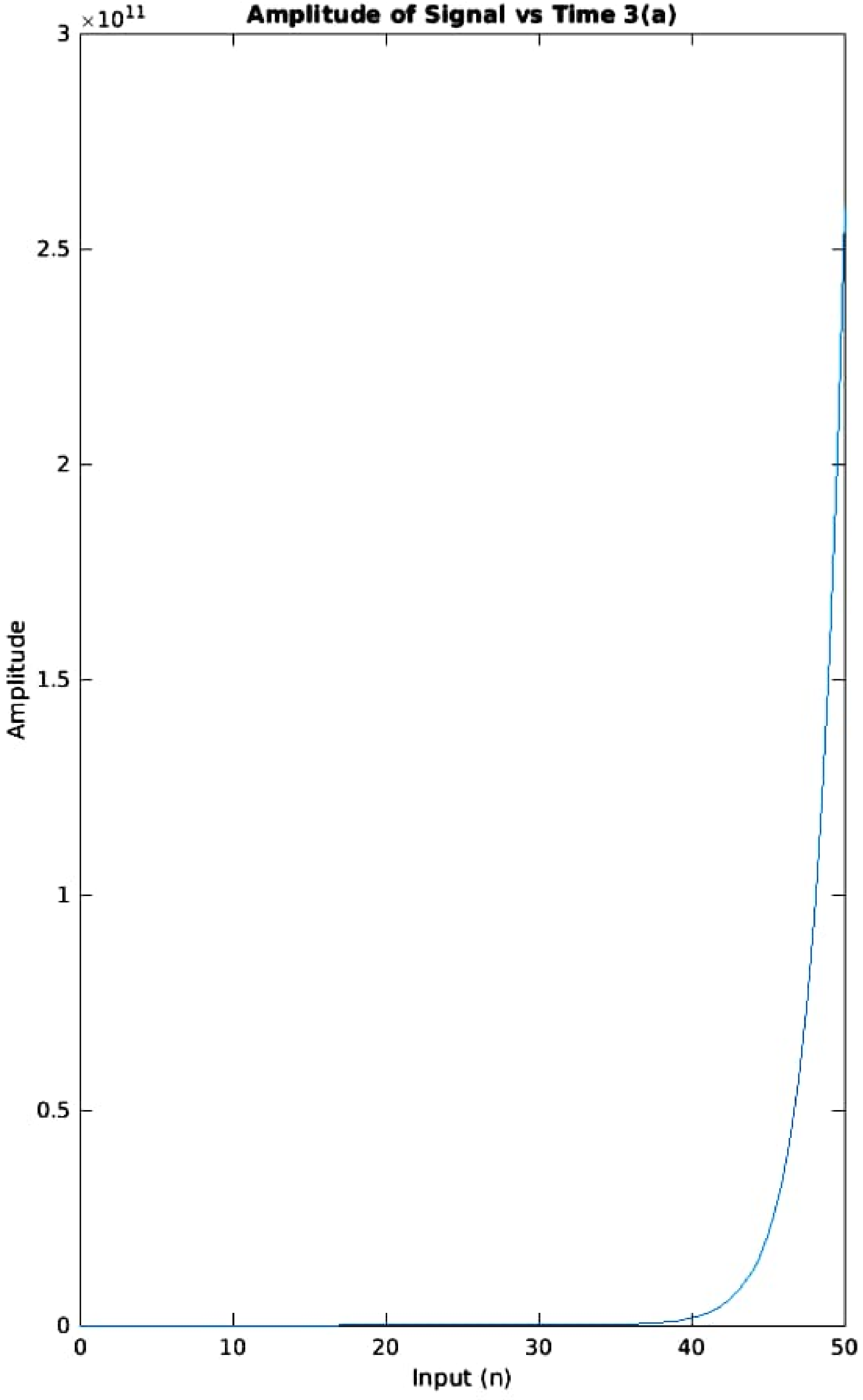
$2\pi m = \frac{2n_0}{5}$

$\frac{n_0}{m} = 5\pi \Rightarrow$ irrational

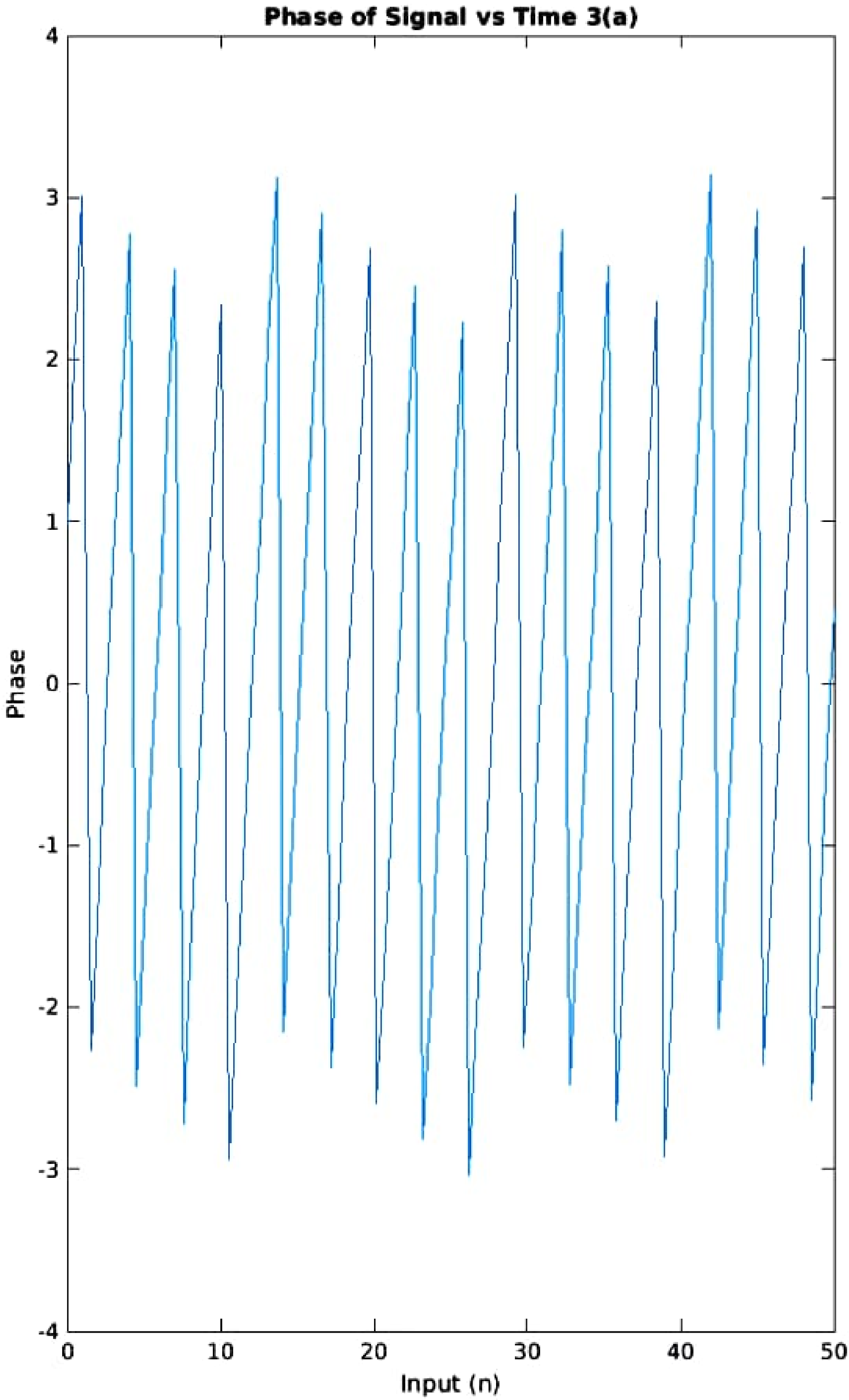
$\boxed{\Rightarrow \text{Aperiodic Fn}}$

MATLAB Drive > q3a.m

```
% Code for plotting graph for question 3(a)

clc;
clear;
close all;

t = linspace(0,50);

x = complex(2,3)*exp(complex(0.5,2)*t);

figure, plot(t, abs(x)), title('Amplitude of Signal vs Time 3(a)'),
xlabel('Input (n)'), ylabel('Amplitude');

figure, plot(t, angle(x)), title('Phase of Signal vs Time 3(a)'),
xlabel('Input (n)'), ylabel('Phase');

```



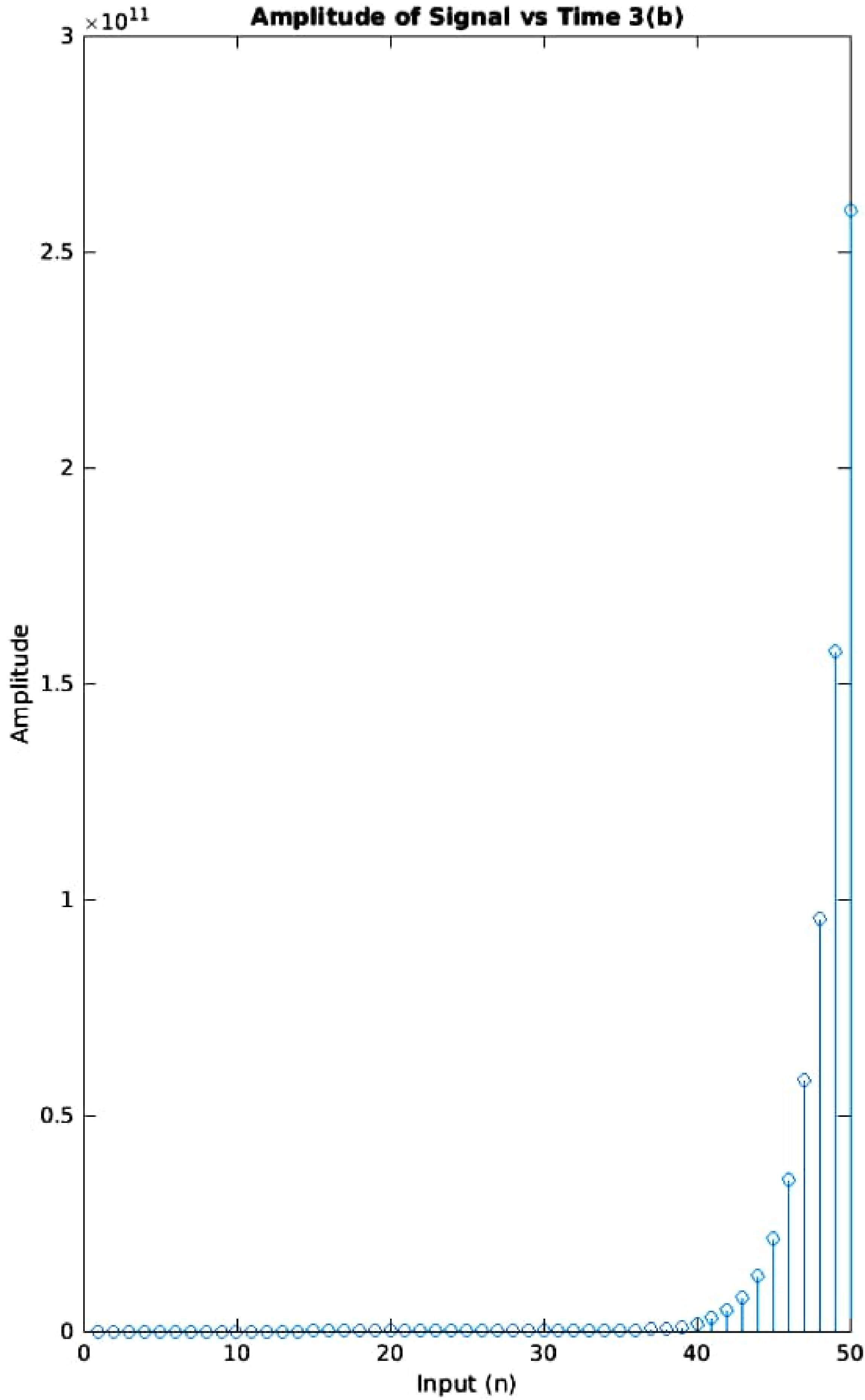
Amplitude of Signal vs Time 3(a)
×10^11
3
2.5
2
1.5
1
0.5
0
Amplitude
0
10
20
30
40
50
Input (n)



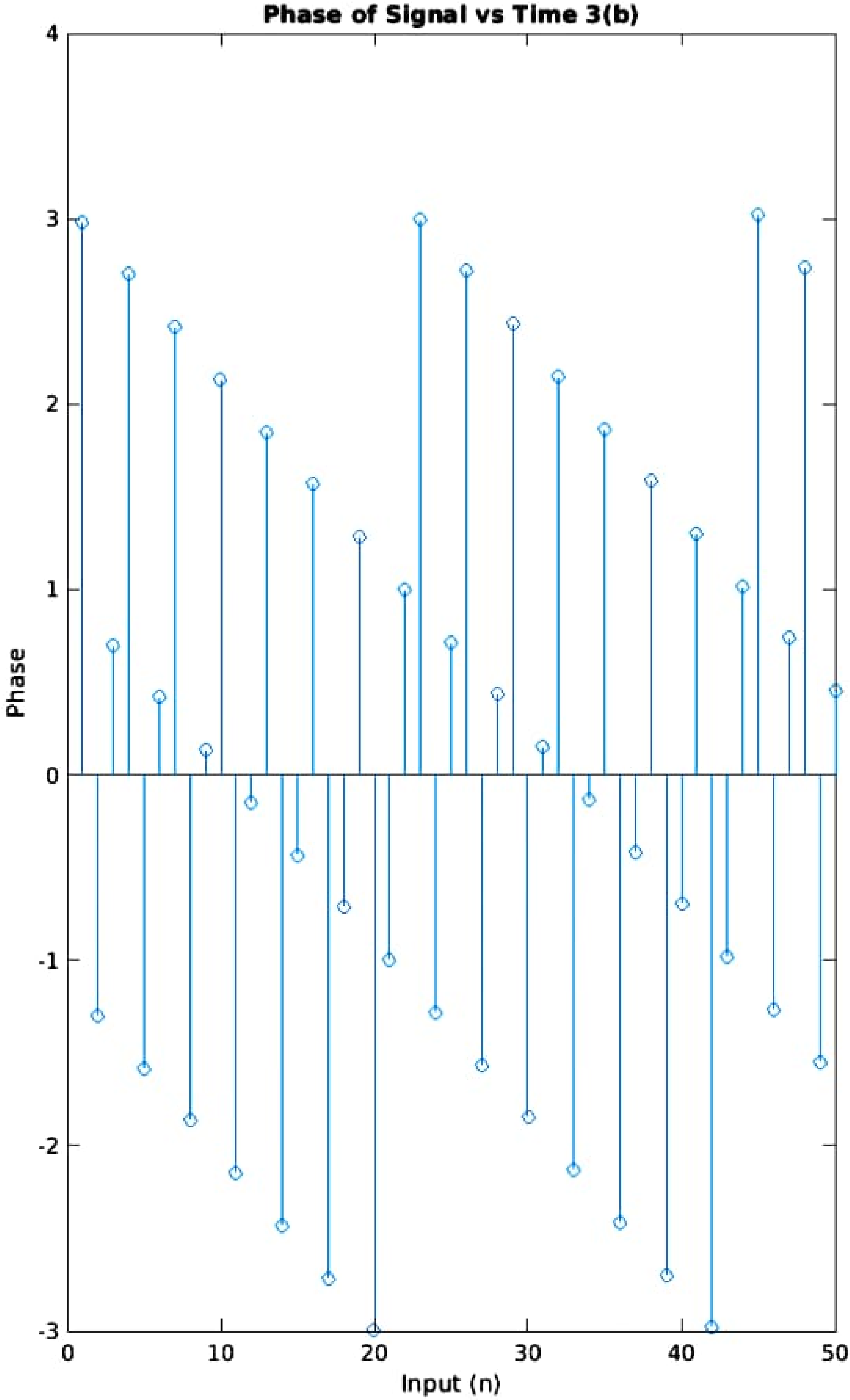
Phase of Signal vs Time 3(a)
4
3
2
1
0
-1
-2
-3
-4
Phase
0
10
20
30
40
50
Input (n)

MATLAB Drive > q3b.m

```matlab
% Code for plotting graph for question 3(b)

clc;
clear;
close all;

n = 1:1:50;

x = complex(2,3)*exp(complex(0.5,2)*n);

figure, stem(n, abs(x)), title('Amplitude of Signal vs Time 3(b)'),
xlabel('Input (n)'), ylabel('Amplitude');

figure, stem(n, angle(x)), title('Phase of Signal vs Time 3(b)'),
xlabel('Input (n)'), ylabel('Phase');
```



Amplitude of Signal vs Time 3(b)
×10^11
3
2.5
2
1.5
1
0.5
0
Amplitude
0
10
20
30
40
50
Input (n)



Phase of Signal vs Time 3(b)
Phase
Input (n)
4
3
2
1
0
-1
-2
-3
0
10
20
30
40
50

4) $x[n] = 1 - \sum_{k=-2}^{\infty} \delta[n-1-k]$

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

$$= \sum_{k=-2}^{\infty} \delta[n-k-2] \qquad k \to k+2$$

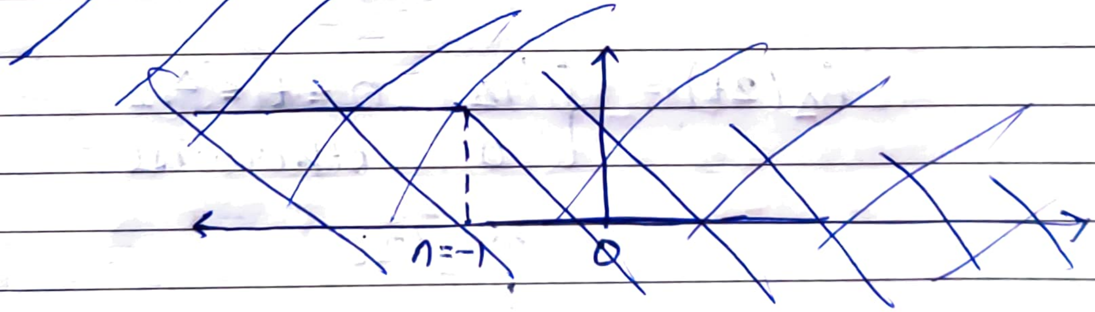
$$n \to n+1$$

$$u[n+1] = \sum_{k=-2}^{\infty} \delta[n-1-k]$$

$$x[n] = 1 - \sum_{k=-2}^{\infty} \delta[n-1-k]$$

$$= 1 - u[n+1] \qquad u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$\cancel{x[n] = \begin{cases} 0 & n+1 \geq 0 \\ 1 & n+1 < 0 \end{cases}}$$



$$x[n] = 1 - u[n+1]$$

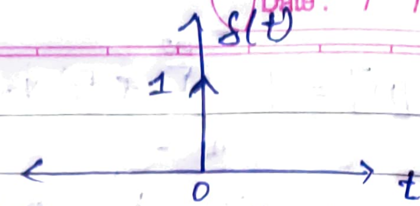
$$\begin{cases} 0 & n+1 \geq 0 \\ 1 & n+1 < 0 \end{cases} = \begin{cases} 0 & n+1 > -1 \\ 1 & n+1 \leq -1 \end{cases}$$

$$= \begin{cases} 0 & n+2 > 0 \\ 1 & n+2 \leq 0 \end{cases}$$

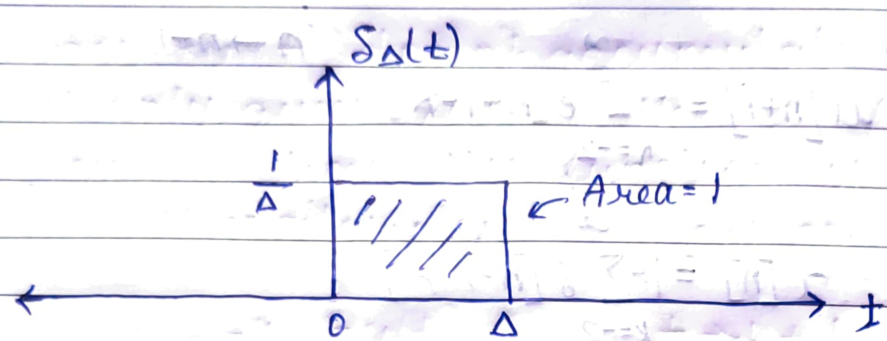
$$\Rightarrow x[n] = \begin{cases} 0 & -(n+2) \leq 0 \\ 1 & -(n+2) \geq 0 \end{cases}$$

$$\Rightarrow x[n] = u[-n-2]$$

5) $\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t) \Rightarrow$

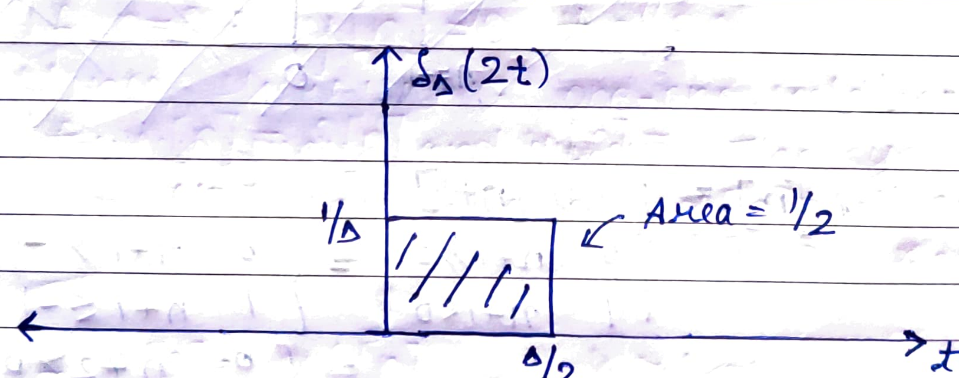


$$\delta_\Delta(t) = \begin{cases} 1/\Delta & 0 \le t \le \Delta \\ 0 & \text{otherwise} \end{cases}$$

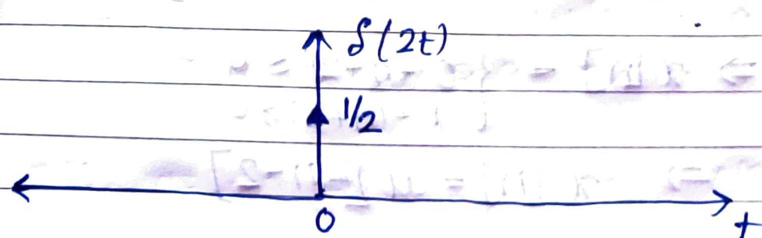


Consider: $\delta(2t) = \lim_{\Delta \to 0} \delta_\Delta(2t)$

$$\delta_\Delta(2t) = \begin{cases} 1/\Delta & 0 \le t \le \Delta/2 \\ 0 & \text{otherwise} \end{cases}$$

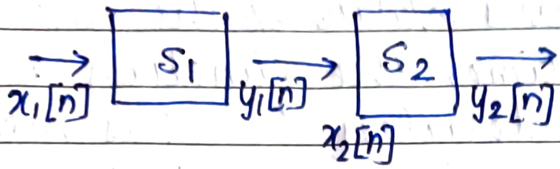


$\lim_{\Delta \to 0} \delta_\Delta(2t) = \delta(2t)$ @ $t = 0$ $= 1/2$



Hence, $\delta(2t) = \begin{cases} 1/2 & t = 0 \\ 0 & \text{otherwise} \end{cases}$

6) $S_1: \quad y_1[n] = 2x_1[n] + 4x_1[n-1]$
$S_2: \quad y_2[n] = x_2[n-2] + 0.5x_2[n-3]$

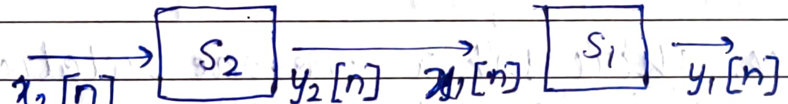


a) $y_2[n] = y_1[n-2] + 1/2\, y_1[n-3]$

$y_1[n-2] = 2x_1[n-2] + 4x_1[n-3]$
$y_1[n-3] = 2x_1[n-3] + 4x_1[n-4]$

$$\boxed{\Rightarrow y_2[n] = 2x_1[n-2] + 5x_1[n-3] + 2x_1[n-4]}$$

b)



$y_1[n] = 2y_2[n] + 4y_2[n-1]$

$y_2[n-1] = x_2[n-3] + 1/2\, x_2[n-4]$

$y_1[n] = 2x_2[n-2] + x_2[n-3] + 4x_2[n-3] + 2x_2[n-4]$

$$\boxed{y_1[n] = 2x_2[n-2] + 5x_2[n-3] + 2x_2[n-4]}$$

7a] $y(t) = x(\sin(t))$

a) If a system depends on past values, it has memory.

since $y(2\pi) = x(\sin(2\pi))$
$= x(\sin(0)) = y(0)$

$y(2\pi)$ depends on $y(0)$.
Hence, the system is NOT memoryless

b) If a system depends on future values, it is NOT causal

$y(-2\pi) = x(\sin(-2\pi))$
$= x(\sin(0)) = y(0)$

"future" input.

c) The system is not invertible

If $x(t) = 0 \Rightarrow y(t) = 0 \ \forall t$
$x(t) = u(t-K)$ for $K > 1 \Rightarrow y(t) = 0 \ \forall t$

We cannot distinguish these inputs if were just given $y(t)$

8)

a) $y(t) = x(t-4)$

$x(t-4) = y(t)$ $\quad t \Rightarrow t+4$

$x(t) = y(t+4)$

$\therefore$ Inverse system is: $\cancel{y(t) = x(t+4)}$

$x(t) = y(t+4)$

b) $y[n] = n x[n]$

If $n=0$: $y[n] = 0 \ \forall x \in \mathbb{N}$

If $x[n] = \delta[n]$, $y[n] = 0$

b/c $x[n] = 0 \ \forall n \neq 0$

$n x[n] = 0 \ \cancel{\forall}\ n = 0$

If $x[n] = K\delta[n]$ $\quad K \geq 1$

$\Rightarrow y[n] = K(n\delta[n])$, which, again is always zero as shown above.

Hence, ~~we so~~ the system is NOT invertible

8c) $y[n] = \begin{cases} x[n+1] & n \geq 0 \\ x[n] & n \leq -1 \end{cases}$

~~y[n] is defined for inputs, x[1], x[~~

for $n \geq 0$ $x[n+1]$ takes values $x[1], x[2], \ldots$
for $n < 0$ $x[n]$ takes values $x[-1], x[-2], \ldots$

$y[n]$ does not depend on the value of $x[0]$.

consider $x[n] = \delta[n]$ and $x_1[n] = k\delta[n]$, $k > 1$
Both have same value of $x[n]$ and $x_1[n]$
for $n \neq 0$. except for $n = 0$, where they differ.

→ Hence, both of them yield the same output $y[n]$, even though they are different functions.

→ Therefore, a 1-1 relationship between input and output does not exist.

Hence, the function is not invertible.