Signals & Systems (ECN-203)

Lecture 2
(Classification and representation of signals and systems)

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July 30, 2020

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Information contained in a signal

- Signals describe a wide variety of physical phenomena
  - Information in a signal is contained in a pattern of variations of some form

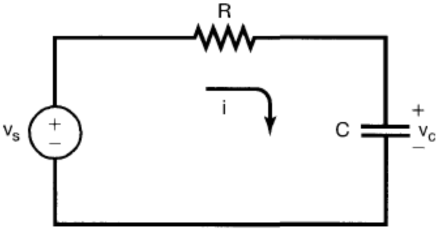

Figure 1.1 A simple *RC* circuit with source voltage v_s and capacitor voltage v_c.

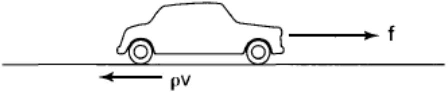

Figure 1.2 An automobile responding to an applied force f from the engine and to a retarding frictional force ρv proportional to the automobile's velocity v.

- Patterns of variation over time in the source and capacitor voltages (V_S and V_c)
- Variations over time of the applied force f and the resulting automobile velocity v

Example: Speech

- Human vocal mechanism produces speech by creating fluctuations in acoustic pressure
- Microphone sense variations in acoustic pressure and convert it into electrical signals
- Different sounds correspond to different patterns in the variations of acoustic pressure

Example: Speech

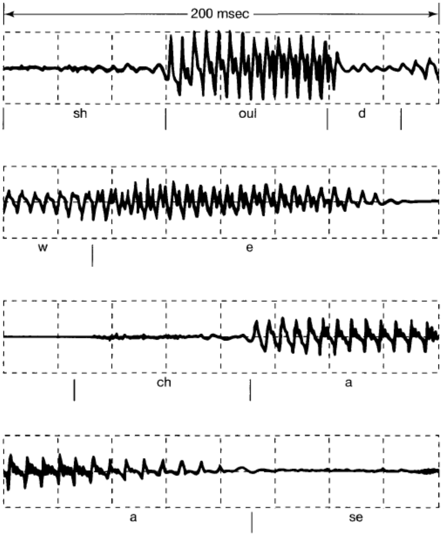

Figure 1.3 Example of a recording of speech. [Adapted from *Applications of Digital Signal Processing*, A.V. Oppenheim, ed. (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1978), p. 121.] The signal represents acoustic pressure variations as a function of time for the spoken words "should we chase." The top line of the figure corresponds to the word "should," the second line to the word "we," and the last two lines to the word "chase." (We have indicated the approximate beginnings and endings of each successive sound in each word.)

Example: Image

- Pattern of variations in brightness/color across the image

Mathematical representation: Signal as a funct

- Signals are represented mathematically as functions of one or more independent variables
  - Speech: Acoustic pressure as a function of time
  - Image: Brightness as a function of two spatial variables, x and y
- In this course, we will focus on signals having a single independent variable
  - And will generally refer to the independent variable as time
  - Although it may not in fact represent the physical quantity "time" in specific applications

Examples where the independent variable is not "time"

- Geophysics: signals representing variations of density, porosity, and electrical resistivity with change in depth
- Meteorological investigations: variations of air pressure, temperature, and wind speed with changes in altitude
- Aircraft control system: average vertical wind profile as a function of height

Two basic types of signals

- In this course, we will study two basic types of signals:
  - Continuous-time signals
  - Discrete-time signals
- Continuous-time signals: The independent variable is continuous
  - These signals are defined for a continuum of values of the independent variable
  - e.g., a speech signal as a function of time and atmospheric pressure as a function of altitude
- Discrete-time signals: Independent variable takes on only a discrete set of values
  - Defined only at discrete times
  - e.g., daily/weekly stock market index

Notations for the two types of signals

- Symbol (t): continuous-time independent variable
- Symbol [n]: discrete-time independent variable

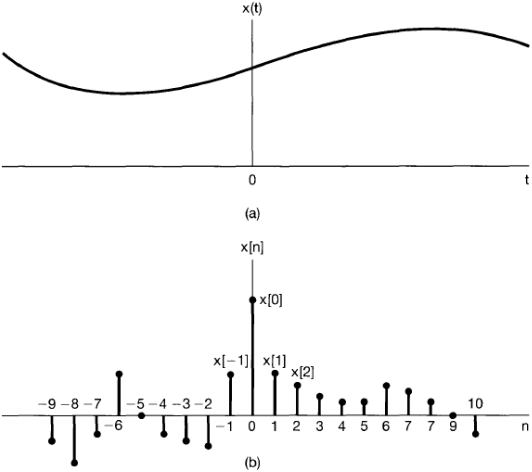

Figure 1.7 Graphical representations of (a) continuous-time and (b) discrete-time signals.

Discrete time signals

- A discrete-time signal may represent a phenomenon for which the independent variable is inherently discrete
  - e.g., demographic data
- A very important class of discrete-time signals arises from the sampling of continuous-time signals
  - $x[n]$ represents successive samples of an underlying phenomenon for which the independent variable is continuous
  - Will study this process in detail later

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Information contained in a signal

- Signals represent a broad variety of phenomena
- Usually the signals we consider are directly related to physical quantities capturing power and energy in a physical system
  - e.g., let $v(t)$ and $i(t)$ are voltage and current across a resistor with resistance R
    - Instantaneous power, $p(t) = v(t) \times i(t) = \frac{1}{R} v^2(t)$
    - Total energy dissipated in the time interval $t_1 \leq t \leq t_2$, $E = \int_{t_1}^{t_2} p(t)dt = \int_{t_1}^{t_2} \frac{1}{R} v^2(t)dt$
  - For a car moving with velocity $v(t)$
    - Frictional force, $F(t) = \rho \times v(t)$
    - Instantaneous power dissipated through friction is $p(t) = F(t) \times v(t) = \rho v^2(t)$

Energy of a signal

- We use similar terminology for power and energy for signals
- Continuous-time signal
  - Energy over the time interval $t_1 \leq t \leq t_2$ is $\int_{t_1}^{t_2} |x(t)|^2 dt$
  - The time averaged power is $\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 dt$
- Discrete-time signal
  - Energy over the time interval $n_1 \leq n \leq n_2$ is $\sum_{n=n_1}^{n_2} |x[n]|^2$
  - The time averaged power is $\frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[n]|^2$
- "Power" and "energy" defined here are independent of physical energy/power

Total energy and power of a signal

- Continuous-time signal
  - Total energy $E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$
- Discrete-time signal
  - Total energy $E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2$
- For some signals these integrals or sums might not converge
  - Infinite energy signals
- Time-averaged power over an infinite interval
  - Continuous-time signal: $P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$
  - Discrete-time signal: $P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$

Classes of signals based on energy and power

- Energy signals:
  - Signals with finite total energy, i.e., $E_\infty < \infty$
  - Such signals must have zero average power
  - e.g.,

$$x(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & Otherwise \end{cases}$$

- Power signals:
  - Signals with finite total power, i.e., $P_\infty < \infty$
  - Such signals must have infinite total energy
  - e.g.,

$$x[n] = 4$$

- Signals with infinite energy and power
  - e.g., $x(t) = t$

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Signal transformation

- A central concept in signal and system analysis is that of the transformation of a signal
- e.g., aircraft control system
  - Signals corresponding to the actions of the pilot $\rightarrow$ changes in aircraft thrust or the positions of aircraft control surfaces $\rightarrow$ changes in aircraft velocity and heading
- e.g., a sophisticated audio system
  - Input signal representing $\rightarrow$ enhance desirable characteristics
    - Remove noise
    - Balance the several components of the signal (e.g., treble and bass)
- We will focus on very limited but important signal transformations
  - Modification to the independent variable, i.e., the time axis

Time shift

- Time shift transformation: The output signal is identical in shape, but a shifted version (along time axis) of the input signal
- Notation: $x(t - t_0)$, or $x[n - n_0]$, $-\infty < t_0, n_0 < \infty$
- Applications: Radar, sonar, and seismic signal processing
  - Several receivers at different locations observe a signal being transmitted through a medium (water, rock, air, etc.)
  - Difference in propagation time due to different paths/medium
    - Results in a time shift between the signals at the two receivers

Time shift

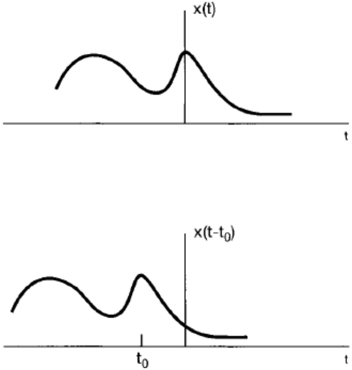


Figure 1.9 Continuous-time signals related by a time shift. In this figure $t_0 < 0$, so that $x(t - t_0)$ is an advanced version of $x(t)$ (i.e., each point in $x(t)$ occurs at an earlier time in $x(t - t_0)$).

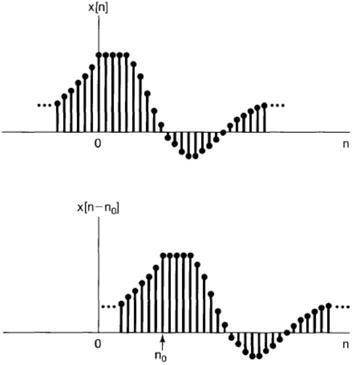
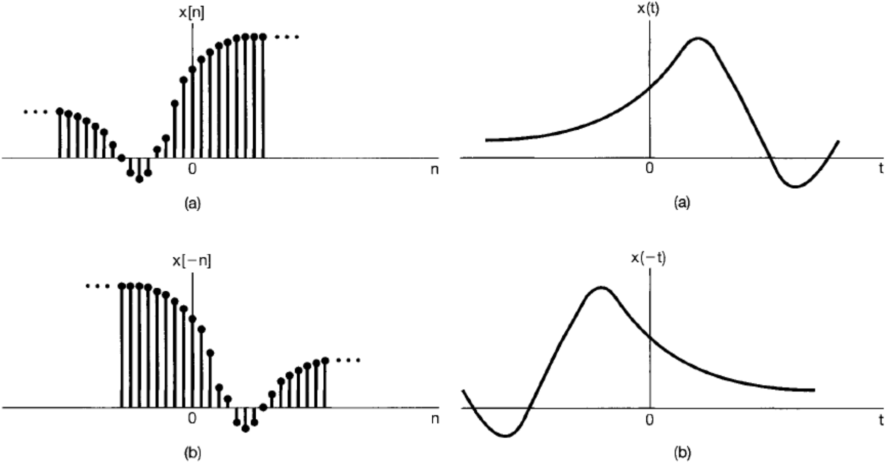


Figure 1.8 Discrete-time signals related by a time shift. In this figure $n_0 > 0$, so that $x[n - n_0]$ is a delayed verson of $x[n]$ (i.e., each point in $x[n]$ occurs later in $x[n - n_0]$).

Time reversal

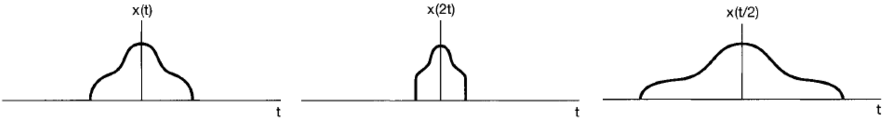
- Time reversal transformation: The output signal is obtained by reflecting the input signal about $t = 0$ (the y axis)
- Notation: $x(-t)$, or $x[-n]$
- If $x(t)$ represents an audio tape recording, then $x(-t)$ is the same tape recording played backward

Time reversal



Time scaling

- Time scaling transformation: The output signal is obtained by linear scale changes in the independent variable
- Notation: $x(2t)$, or $x[\frac{n}{2}]$
- If $x(t)$ represents an audio tape recording, then $x(2t)$ is that recording played at twice the speed, and $x(\frac{t}{2})$ is the recording played at half-speed



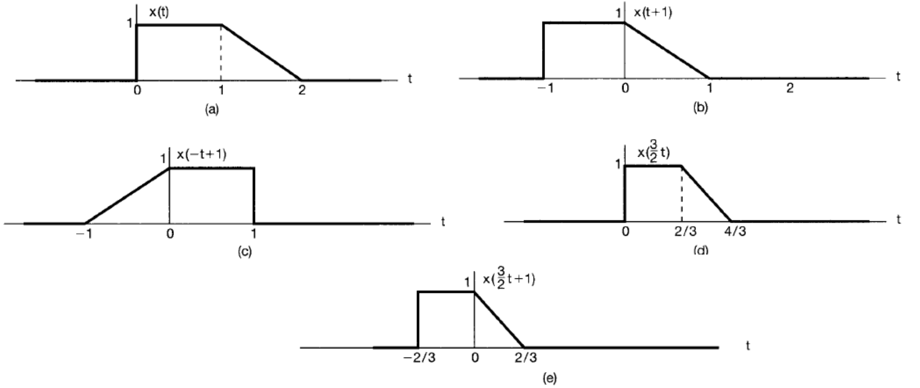
A combination of all three signal transformation operations

- $y(t) = x(\alpha t + \beta),\ \alpha, \beta \in \mathbb{R}$
- Such a transformation of the independent variable preserves the general shape of $x(t)$
  - $y(t)$ would be linearly stretched if $|\alpha| < 1$
  - $y(t)$ would be linearly compressed if $|\alpha| > 1$
  - $y(t)$ would be reversed in time if $\alpha < 0$
  - $y(t)$ would be shifted in time if $\beta \neq 0$

A systematic approach

- How to obtain $x(\alpha t + \beta)$ from $x(t)$?
- First delay or advance $x(t)$ in accordance with the value of β
- Then perform time scaling and/or time reversal on the resulting signal
  - As per the value of α
  - Linearly stretched if $|\alpha| < 1$
  - Linearly compressed if $|\alpha| > 1$
  - Reversed in time if $\alpha < 0$

Example


x(t)
1
0
1
2
t
(a)
x(t+1)
1
−1
0
1
2
t
(b)
x(−t+1)
1
−1
0
1
t
(c)
x(3/2 t)
1
0
2/3
4/3
t
(d)
x(3/2 t+1)
1
−2/3
0
2/3
t
(e)

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Periodic continuous-time signal

- There exists a positive value of T for which $x(t) = x(t+T)$ for all values of t
- A periodic signal is unchanged by a time shift of T
  - T is called as the time period of $x(t)$

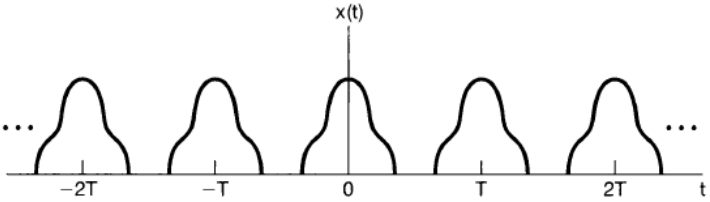

Figure 1.14 A continuous-time periodic signal.

- Periodic continuous time signals arise in a variety of contexts
  - Ideal LC circuits without resistive energy dissipation
  - Ideal mass-spring system without frictional losses

The fundamental period

- If $x(t)$ is periodic with period T, then $x(t) = x(t + mT)$ for all t and for any integer m
- Thus $x(t)$ is also periodic with period $2T$, $3T$, $4T$, ...
- The fundamental period T_0 of $x(t)$ is the smallest positive value of T for which $x(t) = x(t + T)$ holds
  - Except for $x(t) = c$ (a constant signal)
  - For this case, the fundamental period is undefined

Periodic discrete-time signal

- Periodic signals are defined analogously in discrete time
- $x[n]$ is periodic with period N (N is a positive integer) if it is unchanged by a time shift of N
  - $x[n] = x[n+N]$ for all (integer) values of n
  - Then $x[n]$ is also periodic with period $2N$, $3N$, $4N$, ...
- The fundamental period N_0 is the smallest positive value of N for which $x[n] = x[n+N]$ holds

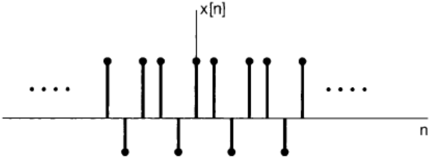

Figure 1.15 A discrete-time periodic signal with fundamental period $N_0 = 3$.

Example

- Is this signal periodic?
  - $$x(t) = \begin{cases} cos(t), & t < 0 \\ sin(t), & t \geq 0 \end{cases}$$
- Both $cos(t)$ and $sin(t)$ are periodic with time period 2π
  - Considering $t \geq 0$ and $t < 0$ separately, $x(t)$ does repeat itself over every interval of length 2π

Example

- But what about the boundary $t = 0$?

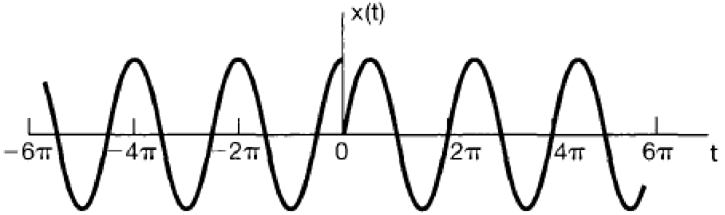

Figure 1.16 The signal $x(t)$ considered in Example 1.4.

- $x(t)$ has a discontinuity at $t = 0$ that does not recur at any other time
  - $x(t)$ is not periodic

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Symmetry under time reversal

- A useful properties of signals: Symmetry under time reversal
- Even signal: Identical to its time-reversed counterpart
  - $x(-t) = x(t)$, $x[-n] = x[n]$
- Odd signal: negative of its time-reversed counterpart
  - $x(-t) = -x(t)$, $x[-n] = -x[n]$
  - An odd signal must necessarily be 0 at $t = 0$ or $n = 0$

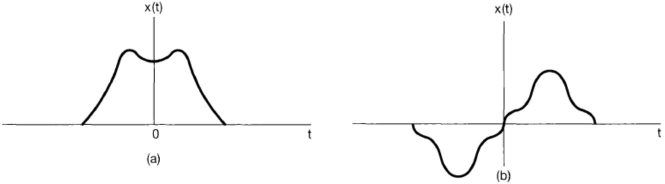
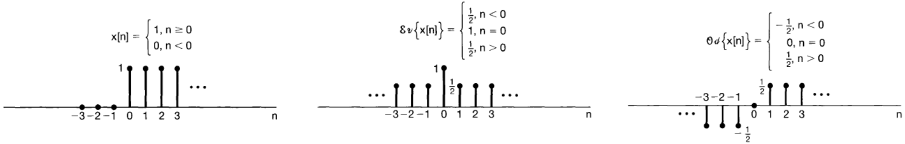


Figure 1.17 (a) An even continuous-time signal; (b) an odd continuous-time signal.

Even-odd decomposition

- Any signal $x(t)$ can be broken into a sum of two signals, one of which is even and one of which is odd
  - $\mathbb{EV}\{x(t)\} = \frac{1}{2}[x(t) + x(-t)]$ (check if this signal is even?)
  - $\mathbb{OD}\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$ (check if this signal is odd?)
- $\mathbb{EV}\{x(t)\} + \mathbb{OD}\{x(t)\} = x(t)$



Thanks.