① Given: $y(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & T_1 < |t| < \frac{T}{2} \end{cases}$ $\qquad a_0 = \frac{2T_1}{T}$

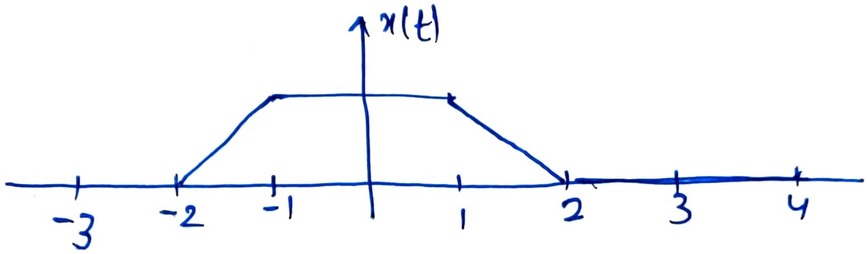
$a_k = \frac{\sin(k\omega_0 T_1)}{k\pi}$

taking $T_1 = 1/2$ and $T = 6 \Rightarrow \omega_0 = \pi/3$

$y(t) = \begin{cases} 1 & |t| < 1/2 \\ 0 & 1/2 < |t| < 3 \end{cases}$ $\qquad a_0 = \frac{1}{6}$

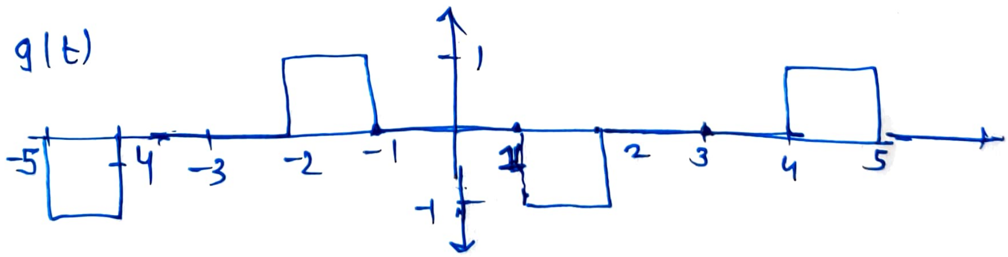
$a_k = \frac{\sin(k\omega_0/2)}{k\pi} = \frac{\sin(k\pi/6)}{k\pi}$

given: $x(t) =$

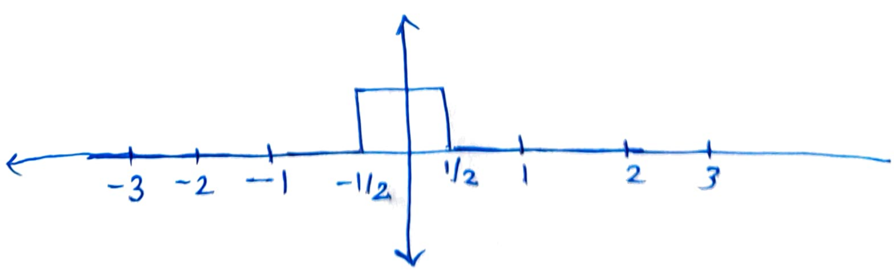


take: $g(t) = \frac{d(x(t))}{dt} = \begin{cases} 0 & 2 < |t| < 3 \\ 1 & -2 < t < -1 \\ -1 & 1 < t < 2 \\ 0 & 0 < |t| < 1 \end{cases}$

g(t)



y(t):



$\Rightarrow g(t) = y(t + 3/2) - y(t - 3/2)$

$= \frac{d(x(t))}{dt}$

$y(t) \longleftrightarrow a_k$

$y(t+3/2) \longleftrightarrow a_k e^{jk\omega_0 \frac{3}{2}}$ } Time shift property

$y(t-3/2) \longleftrightarrow a_k e^{jk\omega_0 (\frac{-3}{2})}$

$y(t+3/2) - y(t-\frac{3}{2}) \longleftrightarrow a_k (e^{jk\omega_0 \frac{3}{2}} - e^{-jk\omega_0 \frac{3}{2}})$ } linearity property.

$g(t) \longleftrightarrow a_k (e^{jk\omega_0 \frac{3}{2}} - e^{-jk\omega_0 \frac{3}{2}})$

suppose: $g(t) \longleftrightarrow C_k$

$x(t) \longleftrightarrow \frac{C_k}{jk\omega_g}$ b/c $g(t) = \frac{d(x(t))}{dt}$

$\therefore g(t) \longleftrightarrow a_k (e^{jk\omega_0 \frac{3}{2}} - e^{-jk\omega_0 \frac{3}{2}})$

$x(t) \longleftrightarrow b_k = \frac{a_k}{jk\omega_0} (2j \sin(k\omega_0 (\frac{3}{2})))$

$= \frac{2a_k \sin(k\omega_0 (3/2))}{k\omega_0}$

since $\omega_0 = \pi/3$

$$\boxed{x(t) \longleftrightarrow b_k = \frac{6a_k}{k\pi} \sin\left(\frac{k\pi}{2}\right)}$$

$b_0 = \frac{6a_0}{\pi} \left[\frac{\pi}{2}\right] = 3a_0$

since $a_0 = \frac{2T_1}{T} = \frac{1}{6} \Rightarrow b_0 = \frac{1}{2}$

2) $x(t) = \cos(4\pi t) \longleftrightarrow a_k$

$y(t) = \sin(6\pi t) \longleftrightarrow b_k$

a) $x(t) = \cos(4\pi t) = \frac{1}{2}(e^{j4\pi t} + e^{-j4\pi t})$

$= \frac{e^{j4\pi t}}{2} + \frac{e^{-j4\pi t}}{2}$

$\Rightarrow a_4 = \frac{1}{2}, a_{-4} = \frac{1}{2}$, by taking $\omega = \pi$, $a_k = 0$ for rest $k \neq \{4, -4\}$

$y(t) = \sin(6\pi t) = \frac{1}{2j}(e^{j6\pi t} - e^{-j6\pi t})$

$= \frac{-j}{2} e^{j6\pi t} + \frac{j}{2} e^{-j6\pi t}$

$\Rightarrow a_6 = \frac{-j}{2}, a_{-6} = \frac{j}{2}, a_k = 0$ for $k \neq \{6, -6\}$

by taking $\omega = \pi$

(b) Both $x(t)$ & $y(t)$ periodic with period $\frac{2\pi}{\pi} = 2 = T$

$z(t) = x(t)\, y(t) \longleftrightarrow h_k = \sum a_l b_{k-l}$

a_l is non-zero for $l = 4, l = -4$

b_{k-l} is non-zero for $k - l = 6, -6$

① $\Rightarrow l = 4$ & $k - l = 6$

$\Rightarrow l = 4$

$k = 10$

$\Rightarrow h_k = h_{10} = a_4 b_6$

$\boxed{h_{10} = \frac{-j}{4}}$

② $l = 4$ & $k - l = -6$

$\Rightarrow l = 4$

$k = -2$

$h_{-2} = a_4 b_{-6}$

$\boxed{h_{-2} = \frac{j}{4}}$

③ $l = -4$, $k - l = 6$ $\Rightarrow l = -4$, $k = 2$

$h_6 = a_{-4} k_6$

$\boxed{h_6 = \frac{-j}{4}}$

④ $l = -4$, $k - l = -6$ $\Rightarrow l = -4$, $k = -10$

$\Rightarrow h_{-10} = a_{-4} b_{-6} = \boxed{\frac{j}{4} = h_{-10}}$

$\boxed{\text{Rest } h_k \text{ are zero because } a_l b_{k-l} = 0}$

(3) Info given:

a) $T = 6 \rightarrow \omega = \pi/3$

b) $x(t) + x(t-3) = 0$

c) a_K are real and positive

d) $a_K = 0$ for $|K| > 2$

e) $\frac{1}{T}\int_T |x(t)|^2 dt = 1$

$$x(t) = \sum_{K=-\infty}^{\infty} a_K e^{jK\omega t} = \sum_{K=-2}^{2} a_K e^{jK\frac{\pi}{3}t}$$

$a^*_K = a_{-K}$

b/c $x(t)$ is real

since $\frac{1}{T}\int_T (x(t))^2 dt = 1$

From parceval's : $1 = |a_0|^2 + |a_1|^2 + |a_2|^2 + |a_{-1}|^2 + |a_{-2}|^2$

since $a^*_K = a_{-K}$

$\Rightarrow |a_K| = |a_{-K}|$

$\Rightarrow 1 = |a_0|^2 + 2(|a_1|^2 + |a_2|^2)$

$x(t) \longleftrightarrow a_K$

$x(t-3) \longleftrightarrow a_K e^{+jK\omega_0 3}$

$a_K = -a_K e^{jK\omega_0 3}$

$\Rightarrow (a_K)(1 + e^{jK\omega_0 3}) = 0$ $\quad \because \omega_0 = \pi/3$

$(a_K)(1 + e^{jK\pi}) = 0$

$\Rightarrow e^{jK\pi} = -1$ OR $a_K = 0$

$\therefore K\pi = (2n+1)\pi$

$K = (2n+1)$ OR $a_K = 0$

$\Rightarrow |a_2| = |a_{-2}| = 0 = |a_0|$

$\Rightarrow 2|a_1|^2 = 1 \Rightarrow \boxed{|a_1| = \pm\frac{1}{\sqrt{2}}}$

$\therefore \cancel{\text{that}} \quad a_1 = \frac{1}{\sqrt{2}} = a_{-1}$

$\Rightarrow x(t) = \frac{1}{\sqrt{2}} e^{j\omega_0 t} + \frac{1}{\sqrt{2}} e^{-j\omega_0 t}$

$= \frac{1}{\sqrt{2}} 2\cos(\omega_0 t)$

$x(t) = \sqrt{2}\cos\left(\frac{\pi}{3} t\right)$