there will only be these energies for any initial and final values of ℓ. The change in the frequency of the emitted spectral line is the energy change divided by h. The frequency changes are therefore $\pm eB/2m_e$ or 0.

$\vec{J} = \vec{L} + \vec{S}$ non-zero

Anomalous Zeeman Effect

$|\vec{\mu}\cdot\vec{B}| < |\xi(r)\,\vec{S}\cdot\vec{l}|$

As stated above, the anomalous Zeeman effect occurs when the spin of either the initial or the final states, or both, is nonzero. The calculation of the energy-level splitting is complicated a bit by the fact that the magnetic moment due to spin is 1 rather than $\frac{1}{2}$ Bohr magneton, and as a result the total magnetic moment is not parallel to the total angular momentum. Consider an atom with orbital angular momentum **L** and spin **S**. Its total angular momentum is

$$\mathbf{J} = \mathbf{L} + \mathbf{S}$$

whereas the total magnetic moment is

$$\boldsymbol{\mu} = -g_\ell \mu_B \frac{\mathbf{L}}{\hbar} - g_s \mu_B \frac{\mathbf{S}}{\hbar}$$

Since $g_\ell = 1$ and $g_s = 2$ (approximately—see Equation 7-47), we have

$$\boldsymbol{\mu} = -\frac{\mu_B}{\hbar}(\mathbf{L} + 2\mathbf{S}) \qquad \textbf{7-70}$$

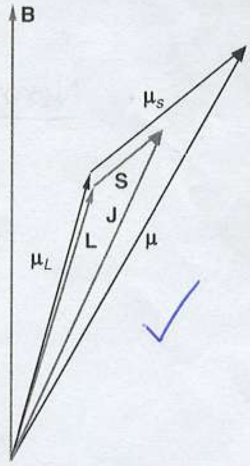

FIGURE 7-29 Vector diagram for the total magnetic moment when S is not zero. The moment is not parallel to the total angular momentum **J**, because μ_S/S is twice μ_L/L. (The directions of $\boldsymbol{\mu}_L$, $\boldsymbol{\mu}_s$, and $\boldsymbol{\mu}$ have been reversed in this drawing for greater clarity.)

Figure 7-29 shows a vector model diagram of the addition of **L** + **S** to give **J**. The magnetic moments are indicated by the darker vectors. Such a vector model can be used to calculate the splitting of the levels, but since the calculation is rather involved, we will discuss only the results.[19]

Each energy level is split into $2j + 1$ levels, corresponding to the possible values of m_j. For the usual laboratory magnetic fields, which are weak compared with the internal magnetic field associated with the spin-orbit effect, the level splitting is small compared with the fine-structure splitting. Unlike the case of the singlet levels in the normal effect, the Zeeman splitting of these levels depends on j, ℓ, and s, and in general there are more than three different transition energies due to the fact that the upper and lower states are split by different amounts. The level splitting, that is, the energy shift relative to the position of the no-field energy level, can be written

$$\Delta E = g m_j \left(\frac{e\hbar B}{2m_e}\right) = g m_j \mu_B B \qquad \textbf{7-71}$$

where g, called the Landé g factor,[20] is given by

$$g = 1 + \frac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)} \qquad \textbf{7-72}$$

Note that for $s = 0$, $j = 1$, and $g = 1$, Equation 7-71 also gives the splitting in the normal Zeeman effect, as you would expect. Figure 7-30 shows the splitting of sodium doublet levels ${}^2P_{1/2}$, ${}^2P_{3/2}$, and ${}^2S_{1/2}$. The selection rule $\Delta m_j = \pm 1$ or 0 gives four lines for the transition ${}^2P_{1/2} \to {}^2S_{1/2}$ and six lines for the transition ${}^2P_{3/2} \to {}^2S_{1/2}$, as indicated. The energies of these lines can be calculated in terms of $e\hbar B/2m_e$ from Equations 7-71 and 7-72.

If the external magnetic field is sufficiently large, the Zeeman splitting is greater than the fine-structure splitting. If B is large enough so that we can neglect the fine-structure splitting, the Zeeman splitting is given by

Paschen-Back effect

$|\vec{\mu}\cdot\vec{B}| \sim |\xi(r)\,\vec{S}\cdot\vec{l}|$

$\vec{J} = \vec{L} + \vec{S}$ non-zero

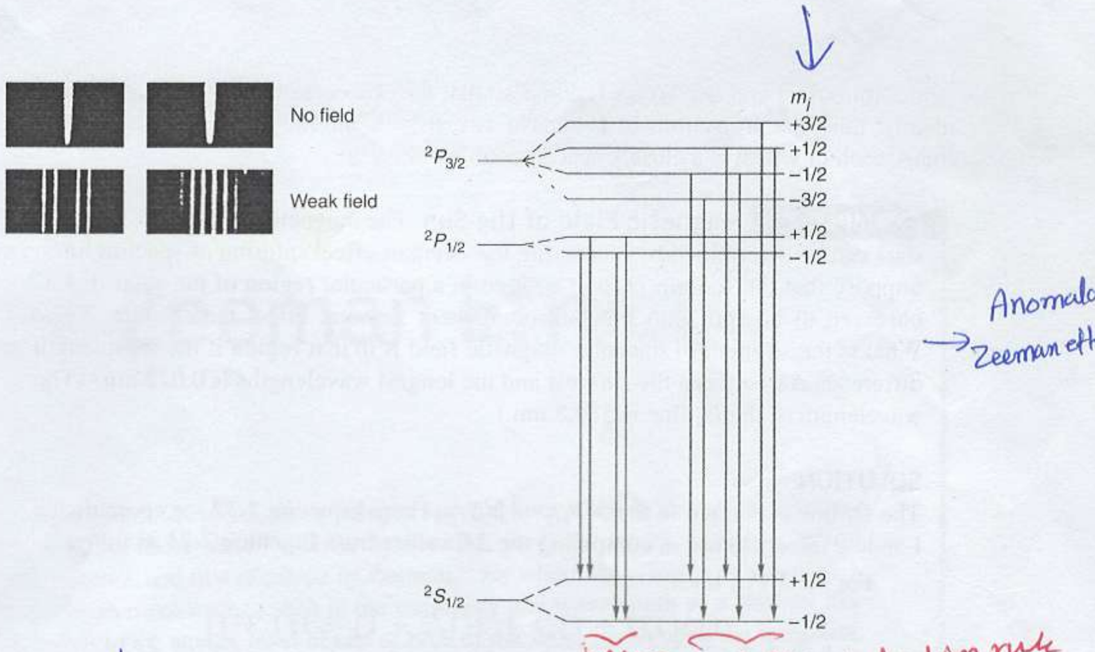

FIGURE 7-30 Energy-level splitting in a magnetic field for the $^2P_{3/2}$, $^2P_{1/2}$, and $^2S_{1/2}$ energy levels for sodium, showing the anomalous Zeeman effect. These are the D_1 and D_2 lines in Figure 7-22. The splitting of the levels depends on **L**, **S**, and **J**, leading to more than the three lines seen in the normal effect. *[Photo from H.E. White, Introduction to Atomic Spectra, New York: McGraw-Hill Book Company, 1934. Used by permission of the publisher.]*

Paschen-Bach

$$\boxed{\Delta E = (m_\ell + 2m_s)\left(\frac{e\hbar B}{2m_e}\right) = (m_\ell + 2m_s)\mu_B B}$$

The splitting is then similar to the normal Zeeman effect and only three lines are observed. This behavior in large magnetic fields is called the Paschen-Back effect after its discoverers, F. Paschen and E. Back. Figure 7-31 shows the transition of the splitting of the levels from the anomalous Zeeman effect to the Paschen-Back effect as the magnitude of **B** increases. The basic reason for the change in the appearance of the anomalous effect as **B** increases is that the external magnetic field overpowers the

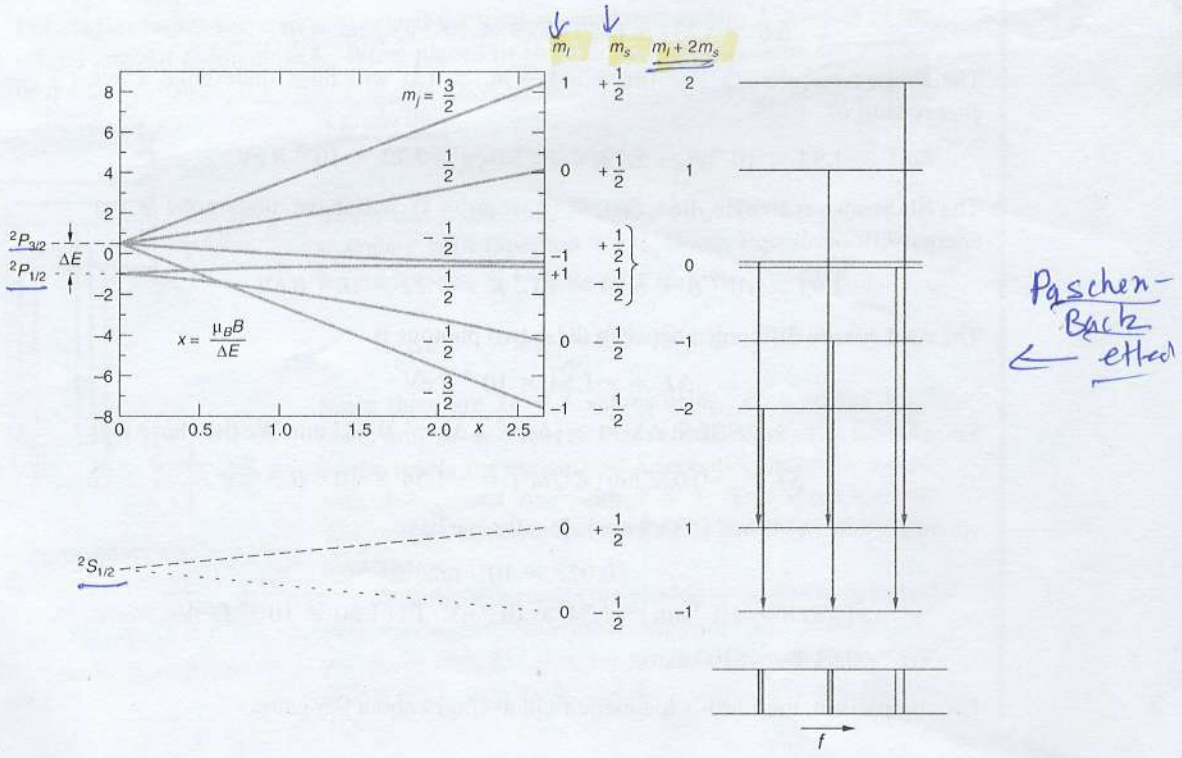

FIGURE 7-31 Paschen-Back effect. When the external magnetic field is so strong that the Zeeman splitting is greater than the spin-orbit splitting, effectively decoupling **L** and **S**, the level splitting is uniform for all atoms and only three spectral lines are seen, as in the normal Zeeman effect. Each of the three lines is actually a closely spaced doublet, as illustrated by the transitions shown at the right. These are the same transitions illustrated in Figure 7-30. Levels shown are for $x = 2.7$.