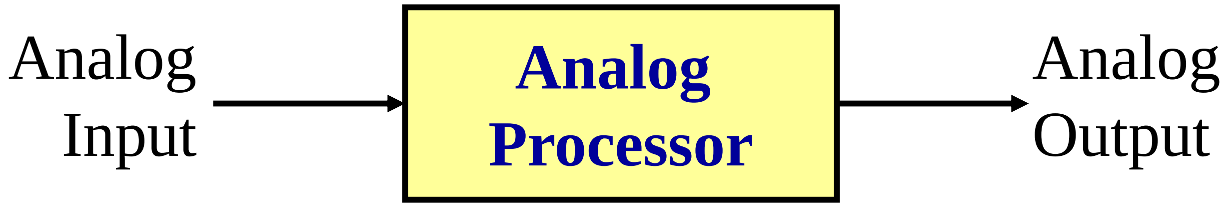
Signal processing stages

- **Analog processing**



- **Digital processing**

Why Digital?

- Does not depend on precise signal values – More tolerant to noise, independent of aging, temperature and other external parameters.
- Long lasting storage of digital data.
- Error detection and correction possible.
- Advances in VLSI technology –
  - easy fabrication and reproduction of complex and sophisticated digital circuits.
  - Small sized large memory storage media.
  - Small sized processor chips compared to large sized inductors and capacitors particularly at low frequency.

Why Digital? (contd.)

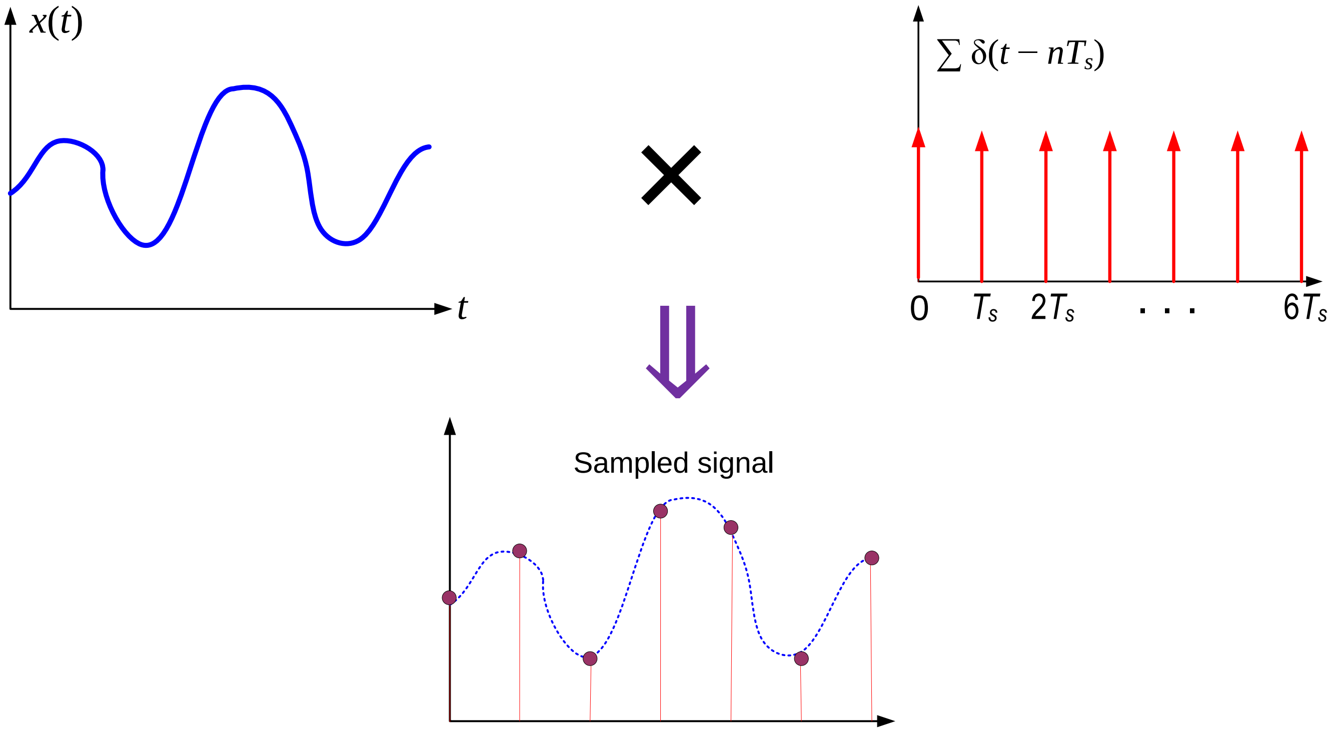
- Desirable accuracy / precision can be achieved by increasing word-length. Even floating-point value possible.
- Processor sharing by TDM.
- Easy adjustment / adaptation of processor characteristic by changing program / algorithm parameters.
- Multi-rate processing possible only in digital domain.
- Cascading of digital circuits without overloading.

Disadvantages of Digital Processing

- Extra processing stages – A/D and D/A.
- Limitation in operating frequency – higher sampling rate → lower resolution in ADC.
- More of active devices → more power consumption.
- Quantization error due to finite length words.

Digitization of Analog Signals

- **Sampling:** Signal discretized in time
  - Accomplished by multiplying the input analog signal by a train of impulses



Digitization of Analog Signals

- **Quantization:** Sample Discretized in amplitude for appropriate binary encoding.
  - Input samples: $X = \left\{ x \;\middle|\; x_{\min} \le x \le x_{\max} \right\}$
  - Quantizer output: $Q : X \to Y; \quad Y = \left\{ y_1, y_2, \ldots\ldots, y_L \right\}$
  - So, it is many-to-one mapping
- Simple example: Signal ranges from –2 V to +2 V
- Samples discretized into 4 levels: –1.5, –0.5, +0.5, +1.5
- The y's are called reconstruction level and boundary values between the intervals are called decision level. **Why?**

Digitization of Analog Signals

- **Encoding:** Assigning binary code to every quantized sample for digital transmission and/or storage.
  - We may directly convert the amplitude value to its corresponding binary number.
  - But in that case required number of bits will depend on the range of the discrete amplitude values → encoder parameter needs to be adjusted with the input signal range.
  - So, instead we code each level with corresponding level no. (or index): 0, 1, 2, …, $L - 1$. Decoder uses a LUT for the corresponding sample value.

Digitization of Analog Signals

- Accordingly, number of bits required will depend on the designed quantization levels: $R = \lceil \log_2 L \rceil$
- So, for the given example,

  $-1.5 \rightarrow 00, \quad -0.5 \rightarrow 01, \quad +0.5 \rightarrow 10, \quad +1.5 \rightarrow 11$

- Say the receiver receives a bit string 1001100000101101
- It is decoded into a set of samples which form the corresponding digital signal:

  $\{+0.5, -0.5, +0.5, -1.5, -1.5, +0.5, +1.5, -0.5\}$

- We will study later how the original analog signal (approximate) is reconstructed back from these samples.

Sampling

- Sampling is used to accomplish time-discretization of continuous-time (analog) signals.

$$x_\delta(t) = x(t) \times \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \Leftrightarrow X_\delta(f) = X(f) * \Im\left[\sum_{n=-\infty}^{+\infty} \delta(t - nT_s)\right]$$

$$X_\delta(f) = X(f) * \left\{\frac{1}{T_s}\sum_{n=-\infty}^{+\infty} \delta(f - nf_s)\right\} = \frac{1}{T_s}\sum_{n=-\infty}^{+\infty} X(f - nf_s) = X_\delta(f)$$

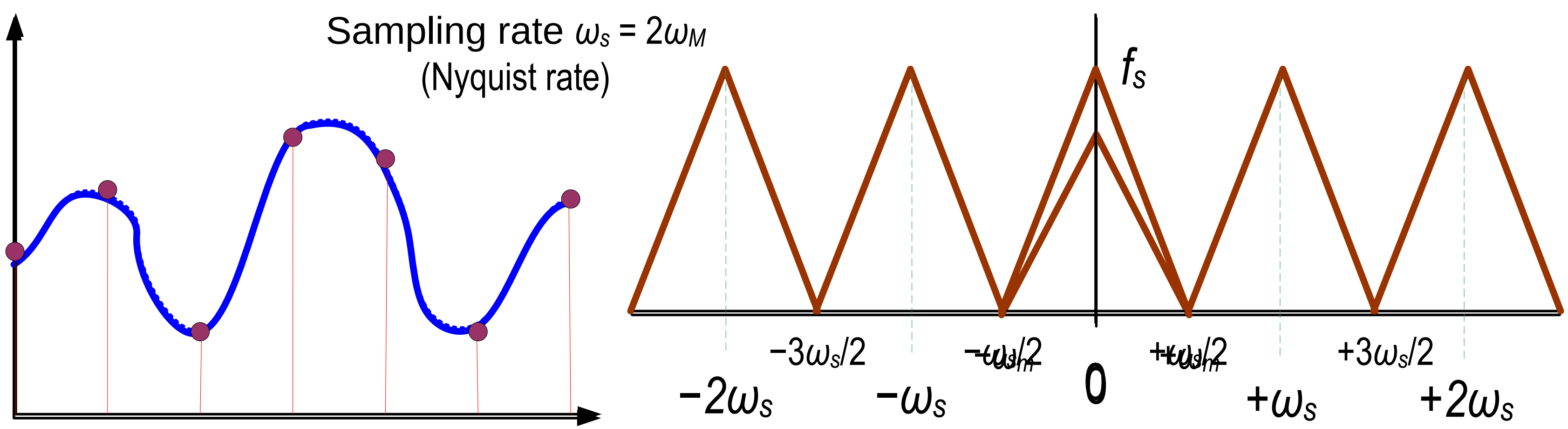
- Or

$$X_\delta(\omega) = \frac{1}{2\pi}\left[X(\omega) * \left\{\frac{2\pi}{T_s}\sum_{n=-\infty}^{+\infty} \delta(\omega - 2n\pi f_s)\right\}\right] = \frac{1}{T_s}\sum_{n=-\infty}^{+\infty} X(\omega - 2n\pi f_s) = X_\delta(\omega)$$

Sampling

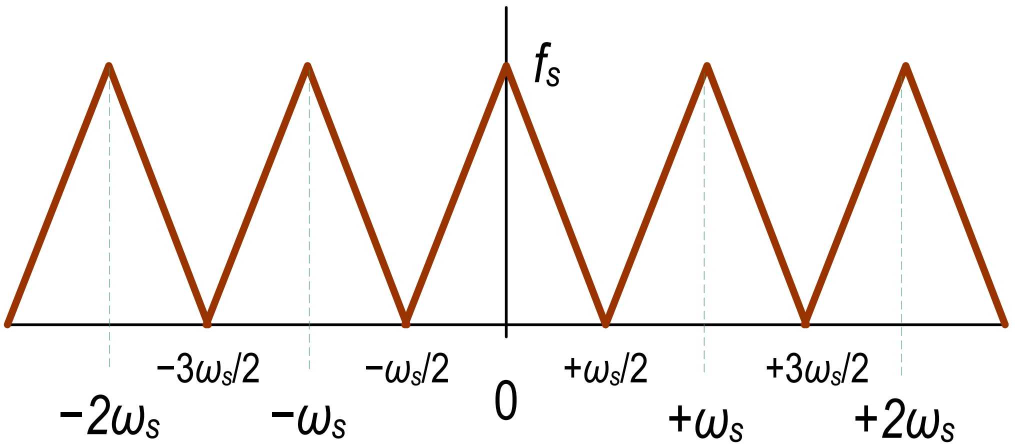
- **Spectrum of sampled signal** – Generated by periodic repetition of the input signal spectrum with period (in frequency domain) equal to the sampling rate f_s, and scaled by the reciprocal of the sampling interval, i.e.,

  $1/\ T_s$ or f_s.



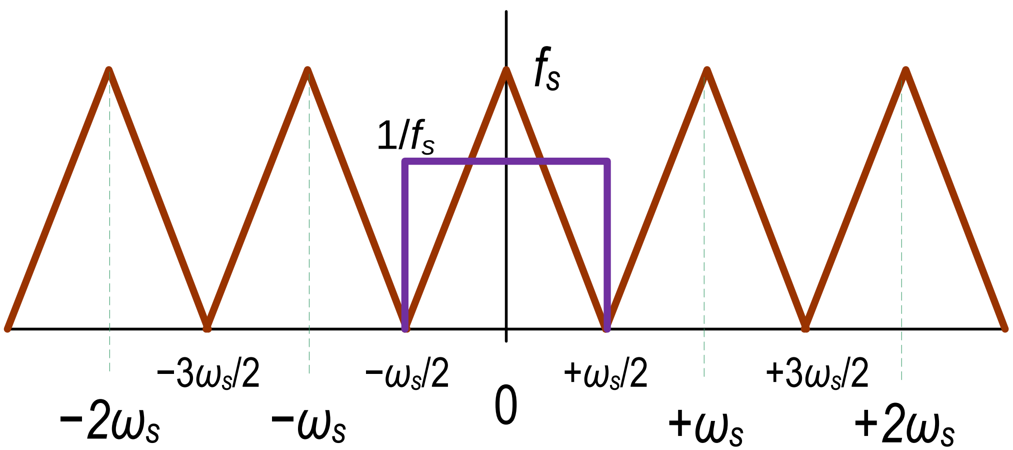
Nyquist Sampling Rate

- It can be observed that the minimum sampling rate necessary is $f_s = 2W$, where $W = f_m$ is the maximum frequency contained in the input signal.
- By this condition each spectrum component $X(f - nf_s)$ has no overlap with its adjacent spectrum components $X(f - [n-1]f_s)$ and $X(f - [n+1]f_s)$.
- This minimum sampling rate is called the Nyquist rate.



Signal Reconstruction

- Ideal reconstruction – It can also be observed that the original input analog signal may be **retrieved exactly** from the sampled signal (sampled at a rate more than or equal to Nyquist rate) when passed through an ideal low-pass filter (LPF) with gain of magnitude T_s and extending from $-f_s/2$ to $+f_s/2$ (or from $-W$ to $+W$).



Signal Reconstruction

- The original input signal may be obtained as:

$$x(t) = \int_{-\infty}^{+\infty} X(f)\exp\left[j2\pi ft\right]df \equiv \int_{-W}^{+W} X(f)\exp\left[j2\pi ft\right]df$$

$$\equiv T_s \int_{-W}^{+W} X_\delta(f)\exp\left[j2\pi ft\right]df \equiv \frac{1}{f_s}\int_{-f_s/2}^{+f_s/2} X_\delta(f)\exp\left[j2\pi ft\right]df$$

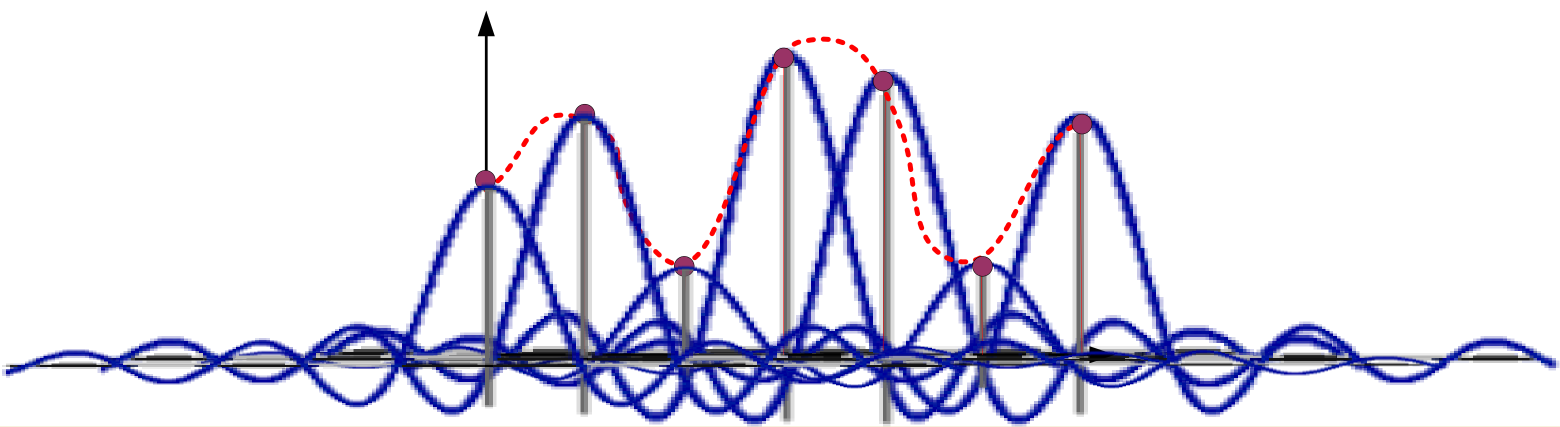
$$= \sum_{n=-\infty}^{+\infty} x(nT_s)\frac{1}{f_s}\int_{-f_s/2}^{+f_s/2} \exp\left[j2\pi f\left(t-nT_s\right)\right]df$$

$$= \sum_{n=-\infty}^{+\infty} x(nT_s)\operatorname{sinc}\left(f_s t-n\right)$$

where $\operatorname{sinc}(u) = \dfrac{\sin(\pi u)}{\pi u}$

Signal Reconstruction

- Thus, the original signal can be reconstructed by adding a series of sinc functions scaled by the sample values and translated in time (with each sinc function centered at the corresponding sampling instant)
- In other words, the original signal can be retrieved from the samples only by using an interpolation function → the sinc function acts as the interpolation function.



Signal Reconstruction

- This is essentially convolution of the sampled signal with sinc function.
- So, for signal reconstruction, the sampled signal is passed through a system having impulse response equal to the sinc function.
- That is, pass the samples through an LPF extending from $-f_s / 2$ to $+f_s / 2$.
- This filter is called interpolation filter or reconstruction filter.

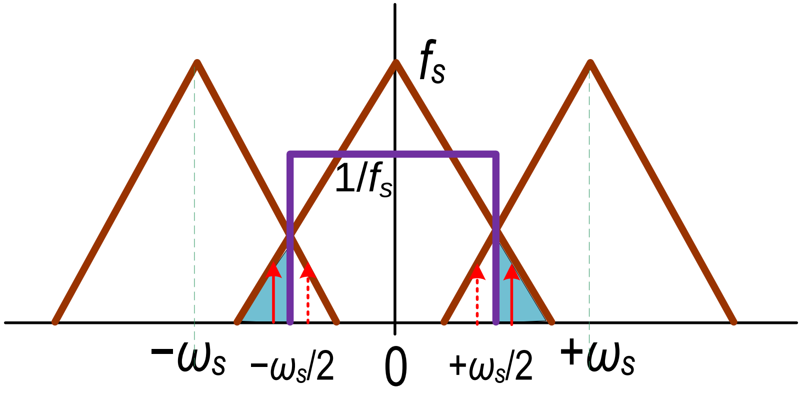
Sampling Theorem

- A band-limited signal of finite energy, which has no frequency components higher than W Hz, is completely described by specifying the values of the signal at instants of time separated by $1 / 2W$ seconds.

- A band-limited signal of finite energy, which has no frequency components higher than W Hz, may be completely recovered from a knowledge of its samples taken at the rate of $2W$ per second.

Aliasing

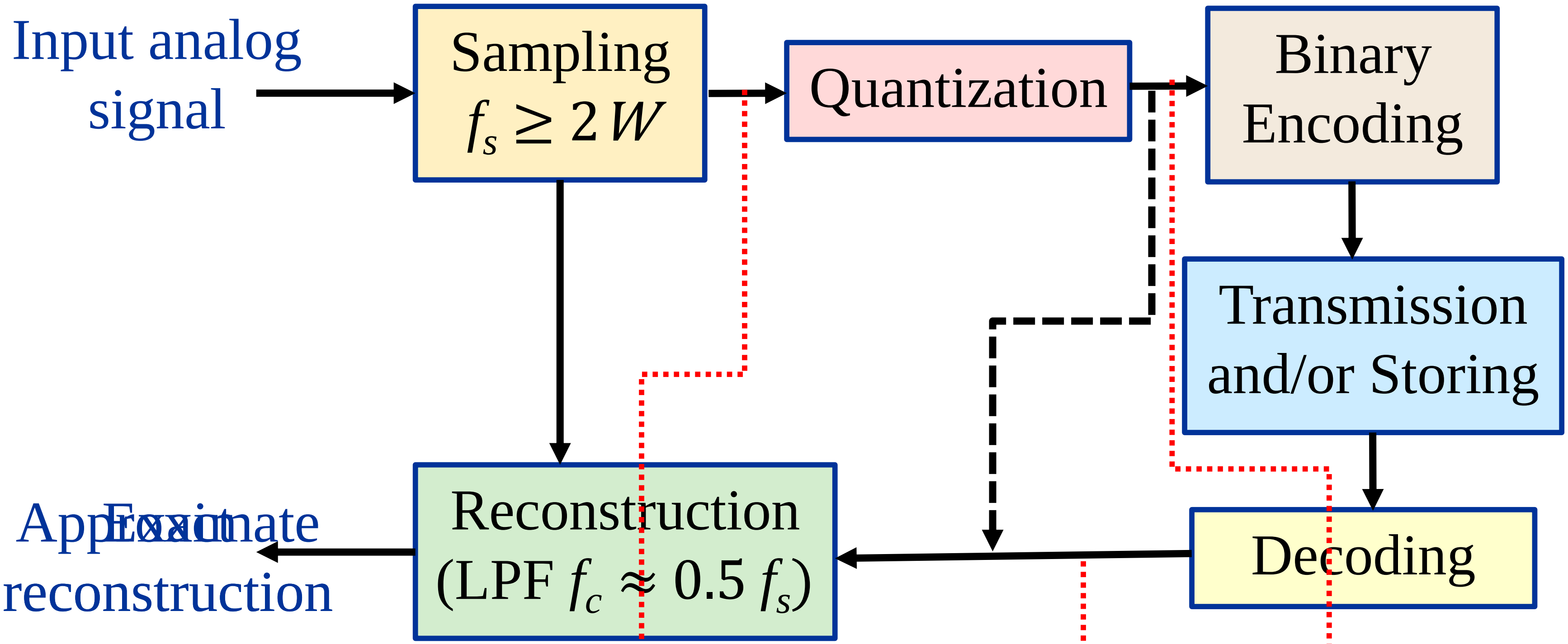
- Sampling at rate lower than the Nyquist rate causes overlapping of the spectrum components.
- When undersampled signal is passed through the reconstruction filter all high frequency components in the range $f_s/2$ to W translate to lower frequency in the reconstructed signal due to foldover of that part of the spectrum. This is aliasing – high frequency signal components appearing as lower frequency.



Aliasing

- Anti-aliasing filter – For rejecting spurious frequencies beyond the actual signal frequencies which otherwise may enter the sampler and cause distortion in the reconstructed signal due to aliasing.
- For example, speech signals band-limited to 3.4 kHz is generally sampled at 8 kHz.
- Guard band – When sampled at a rate higher than the Nyquist rate then the gap between two adjacent spectral components is the guard band.
  - Guard band provides margin for avoiding aliasing.
  - It is also necessary to avoid any distortion in reconstruction due to use of practical (non-ideal) LPF.

Summarizing …



Henceforth, when we will talk of digital signal $x[n]$, it will refer to either the set of original signal samples or the set of quantized samples

DIY Questions

- What do you mean by quantization noise? How does it affect the performance of a digital system? How can this quantization noise be reduced? At what cost?
- Why do we generally take number of quantization levels L as an integral power of 2?
- Show that the Fourier transform of a train of impulses is also a train of impulses.
- Find another signal which has the same functional form in both time and frequency domains.
- Why car wheels seem to rotate in a backward direction in movies?

DIY Questions

- When a computer screen is shown in TV, we will have clearly visible line scrolling and flicker down the screen. Why?
- A 100 kHz sine signal is sampled at the Nyquist rate of 200 kHz. The output is an all-zero sample signal from which the original signal cannot be retrieved. Where is the problem?
- It is possible to sample a band-pass signal, band-limited from 5 kHz to 7 kHz, at a rate less than 14 kHz so that the signal can be reconstructed exactly from the samples. Determine all possible sampling frequencies that are less than 14 kHz.