19114079 Shreyas Dodamani

Page :
Date : / /

Assignment 1

1) a) $x_1(t) = \begin{cases} e^{-2t} & t \geq 0 \\ 0 & 0 \text{ otherwise} \end{cases}$

$$E_\infty = \int_{-\infty}^{\infty} |x_1(t)|^2 dt$$

$$= \int_{-\infty}^{0} 0\, dt + \int_{0}^{\infty} e^{-4t} dt$$

$$= \frac{-1}{4}\left(e^{-4t}\Big|_0^\infty\right)$$

$E_\infty = 1/4$

$P_\infty = 0$ b/c E_∞ is finite

b) $x_2[n] = \cos\left(\frac{\pi}{4} n\right) \quad -\infty < n < \infty \quad n \in I$

$$E_\infty = \lim_{n\to\infty} \sum_{-n}^{n} \cos^2\left(\frac{\pi n}{4}\right)$$

$$P_\infty = \frac{1}{2n+1} \lim_{n\to\infty} \sum_{-n}^{n} \cos^2\left(\frac{\pi n}{4}\right)$$

$$= \lim_{n\to\infty} \frac{1}{2n+1}\left[1+\frac{1}{2}+0+\frac{1}{2}\right]\frac{2n+1}{4}$$

$$= \lim_{n\to\infty} \frac{2}{4} = \frac{1}{2}$$

$\therefore P_\infty = 1/2$

$\Rightarrow E_\infty = \infty$

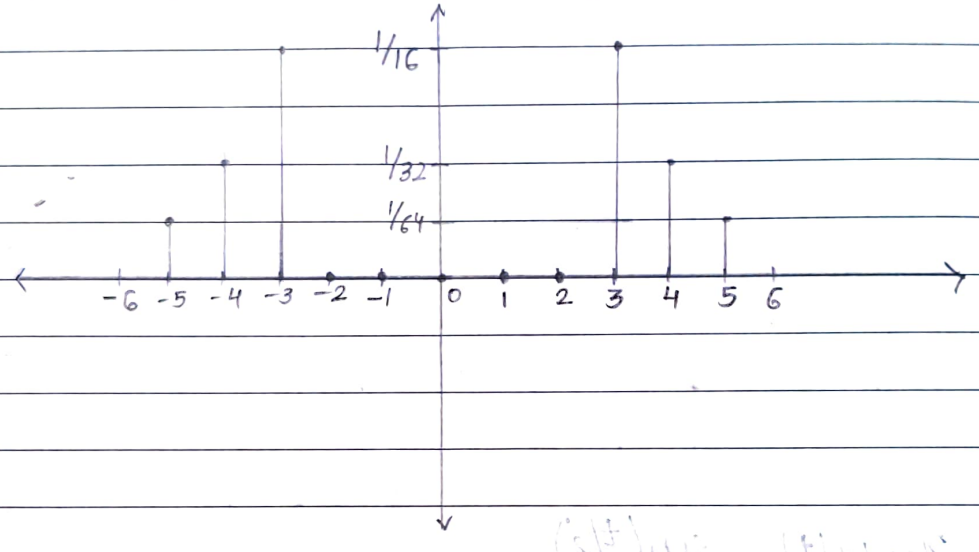
$\cos^2\left(\frac{\pi n}{4}\right)$
$n=0 \to 1$
$n=1 \to 1/2$
$n=2 \to 0$
$n=3 \to 1/2$
$n=4 \to 1$
$n=5 \to 1/2$
$n=6 \to 0$
$n=7 \to 1/2$
...

Repeats with a period of 4

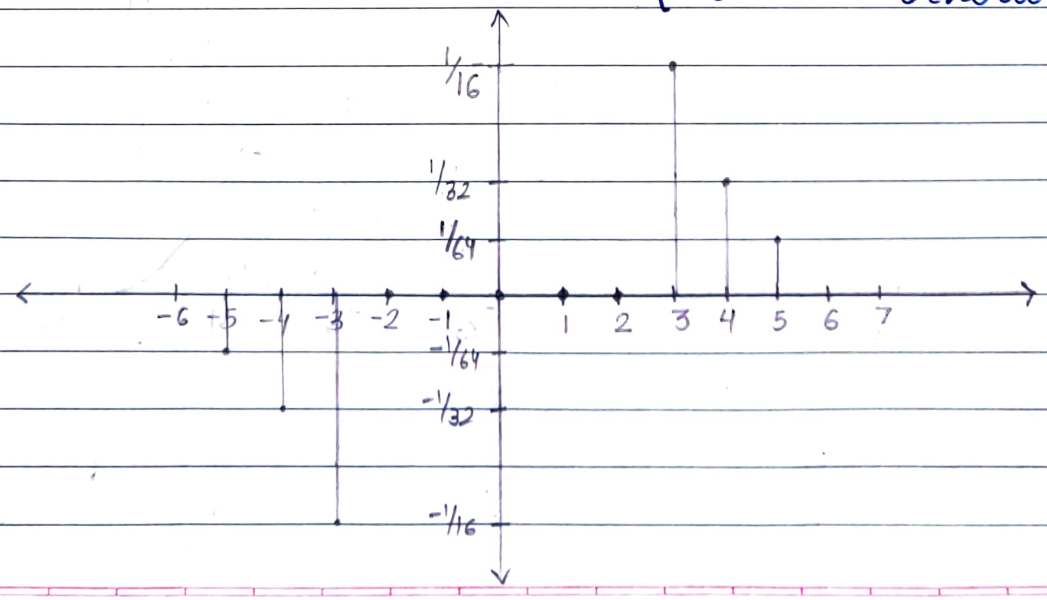
c) $x_3(n) = \begin{cases} (1/2)^n & n \geq 3 \\ 0 & \text{otherwise} \end{cases}$

$x_3[-n] = \begin{cases} (1/2)^{-n} & n \leq -3 \\ 0 & \text{otherwise} \end{cases}$

$x_{3even}[n] = \dfrac{x_3[n] + x_3[-n]}{2} = \begin{cases} (1/2)^{n+1} & n \geq 3 \\ (1/2)^{1-n} & n \leq -3 \\ 0 & \text{otherwise} \end{cases}$



$x_{3odd}[n] = \dfrac{x_3[n] - x_3[-n]}{2} = \begin{cases} (1/2)^{n+1} & n \geq 3 \\ -(1/2)^{1-n} & n \leq -3 \\ 0 & \text{otherwise} \end{cases}$



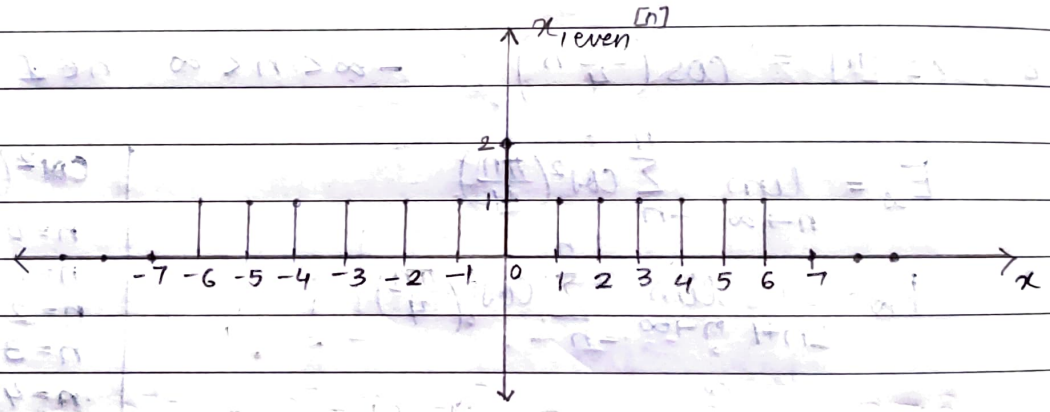
Page :
Date : / /

2) a) $x_1(n) = \begin{cases} 2 & 0 \le n \le 6 \\ 0 & \text{otherwise} \end{cases} \quad n \in I$

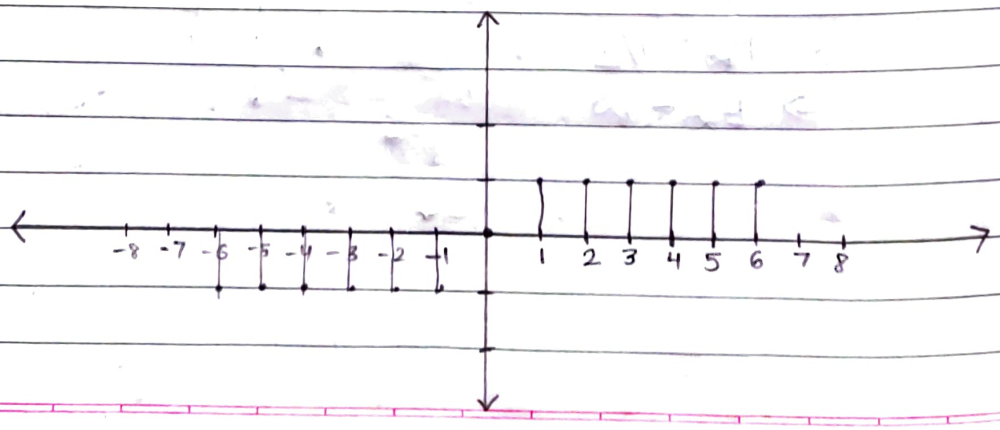
$x_1(-n) = \begin{cases} 2 & -6 \le n \le 0 \\ 0 & \text{otherwise} \end{cases}$

$x_{1even}[n] = \frac{x_1[n] + x_1[-n]}{2}$, $x_{1odd}[n] = \frac{x_1[n] - x_1[-n]}{2}$

$x_{1even}[n] = \begin{cases} 2 & n = 0 \\ 1 & n \in [-6,0) \cup (0,6] \\ 0 & \text{otherwise} \end{cases}$



$x_{1odd}[n] = \begin{cases} 1 & 0 < x \le 6 \\ -1 & -6 \le x < 0 \\ 0 & \text{otherwise} \end{cases}$



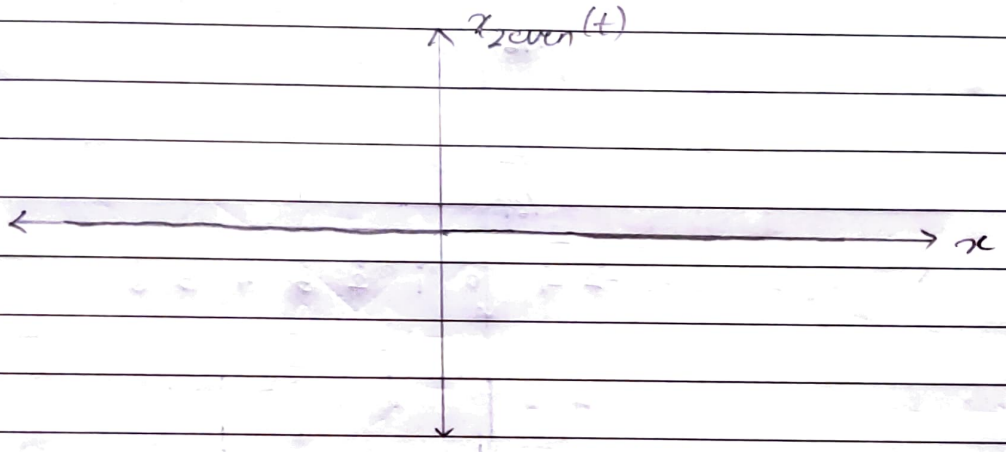
b) $x_2(t) = \sin(t/2)$, $t \in \mathbb{R}$

$x_2(-t) = \sin(-t/2) = -\sin(t/2)$

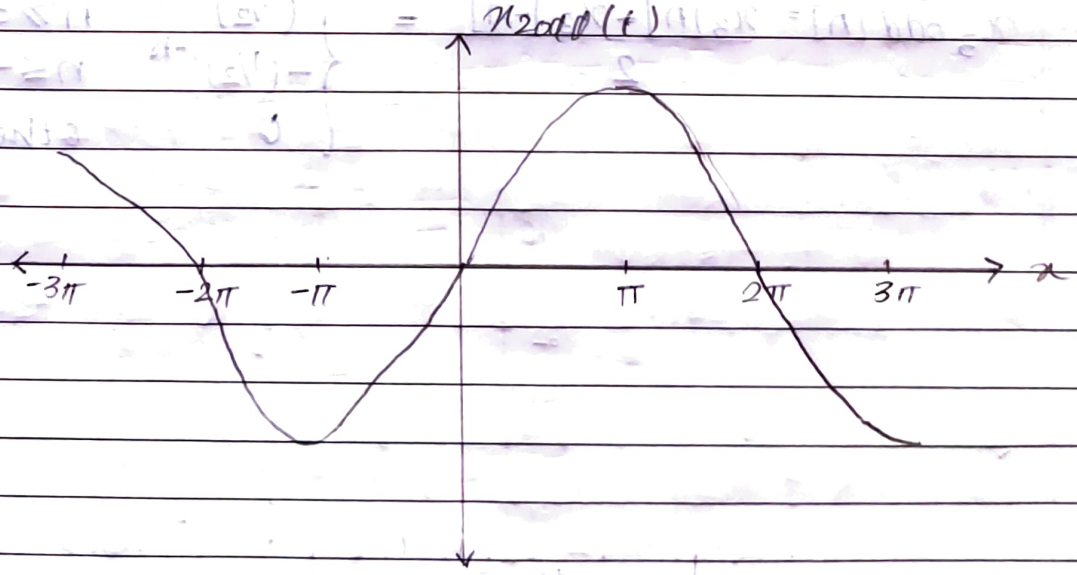
$\Rightarrow x_{2\,even}(t) = \frac{x_2(t) + x_2(-t)}{2} = 0$

$x_{2\,odd}(t) = \frac{x_2(t) - x_2(-t)}{2} = \sin(t/2)$

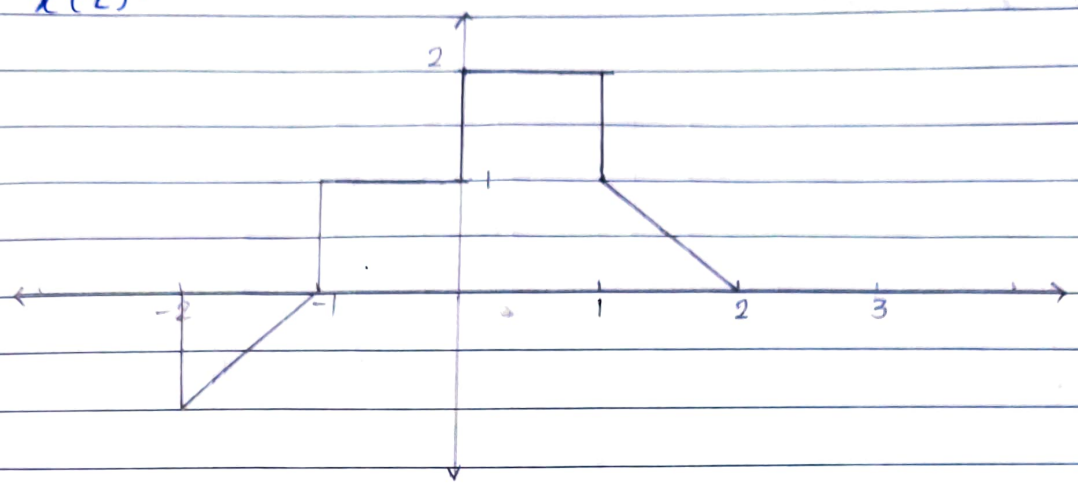
$x_{2\,even}(t) = 0$



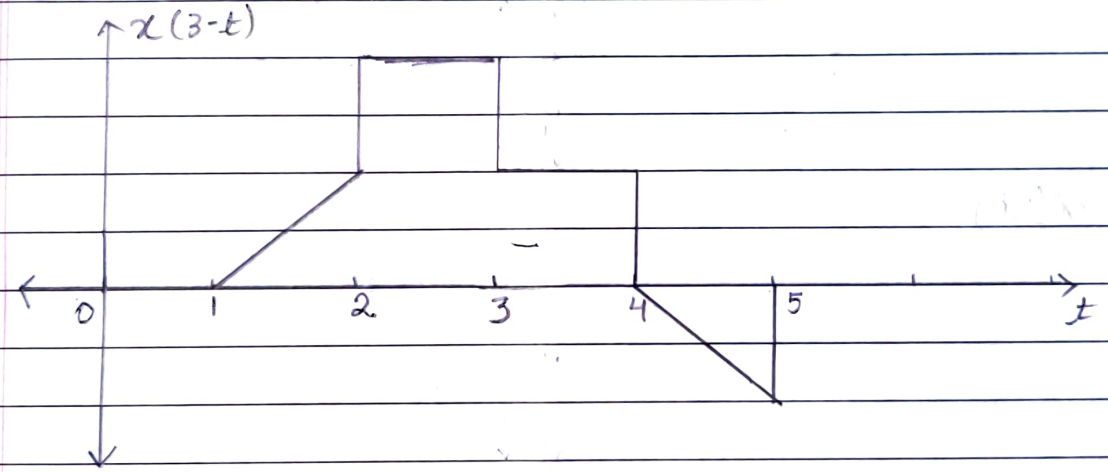
$x_{2\,odd}(t) = \sin(t/2)$



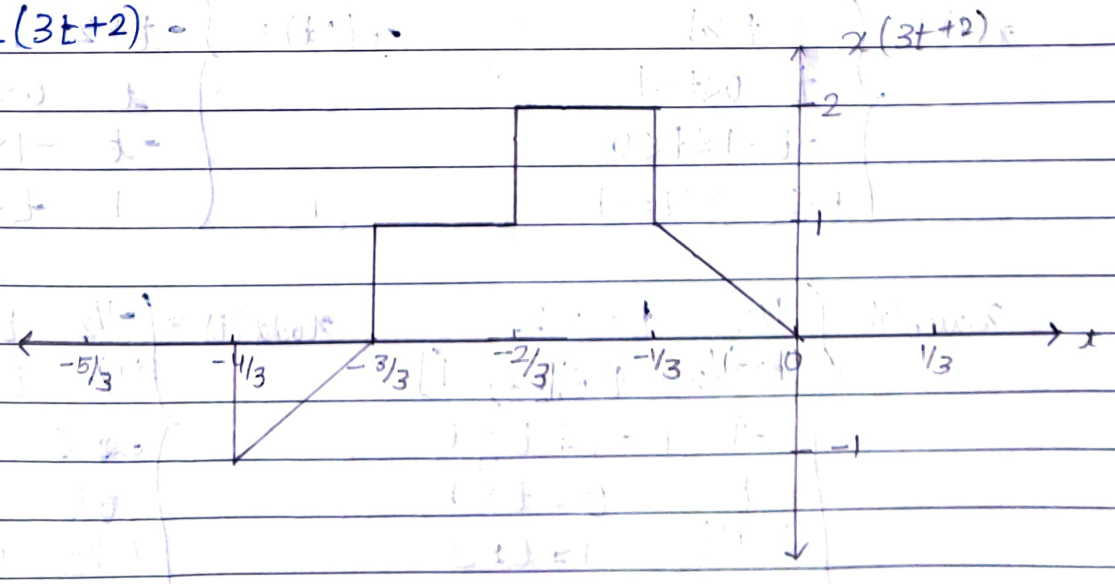
3) (a) $x(t)$:



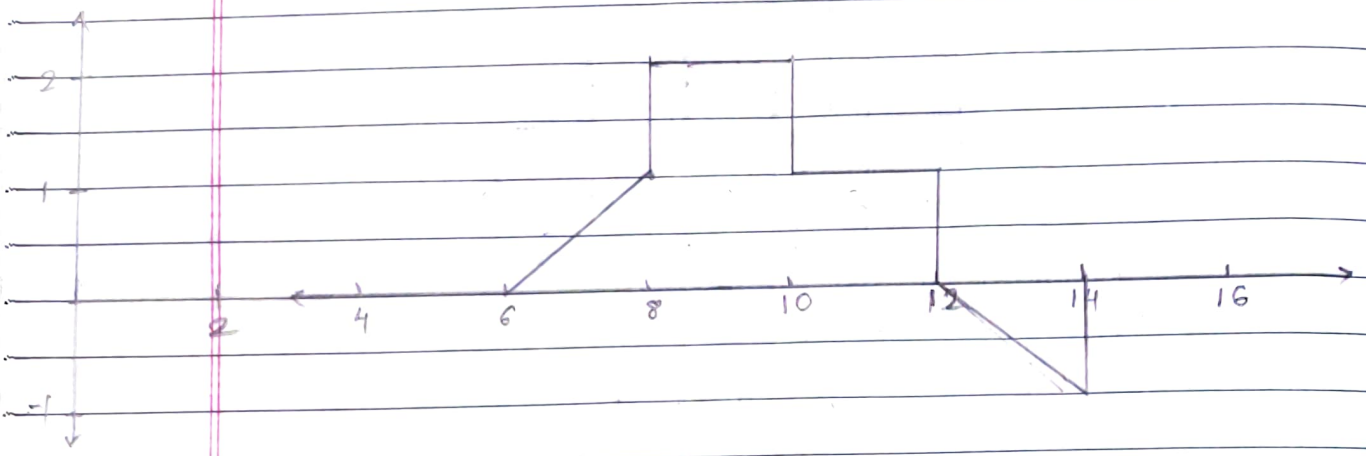
(b) $x(3-t)$:



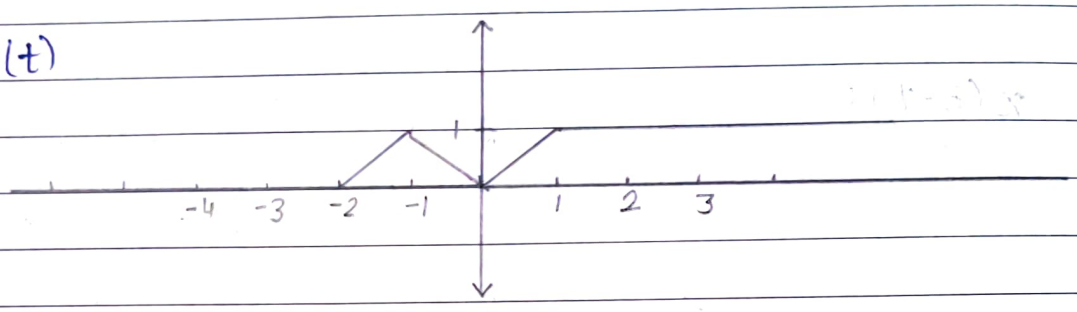
(c) $x(3t+2)$:



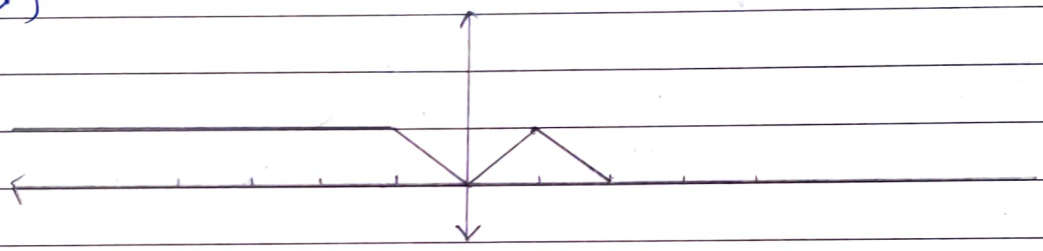
c) $x(5 - t/2)$



4) a) $x(t)$



$x(-t)$



$$x(t) = \begin{cases} 1 & t \geq 1 \\ t & 0 \leq t < 1 \\ -t & -1 \leq t \leq 0 \\ t+2 & -2 \leq t \leq -1 \end{cases}$$

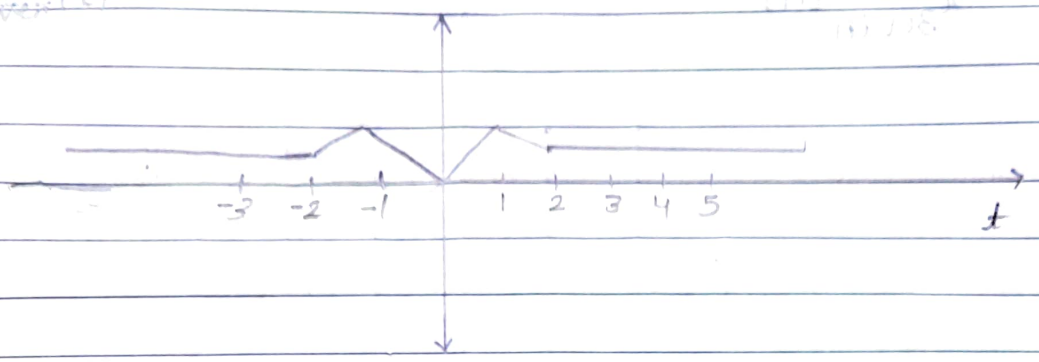
$$x(-t) = \begin{cases} -t+2 & 1 \leq t \leq 2 \\ t & 0 \leq t \leq 1 \\ -t & -1 \leq t \leq 0 \\ 1 & t \leq -1 \end{cases}$$

$$x_{even}(t) = \begin{cases} 1/2 & t \leq -2 \\ (t+3)/2 & t \in [-2, -1] \\ -t & -1 \leq t \leq 0 \\ t & 0 \leq t \leq 1 \\ (3-t)/2 & 1 \leq t \leq 2 \\ 1/2 & t \geq 2 \end{cases}$$

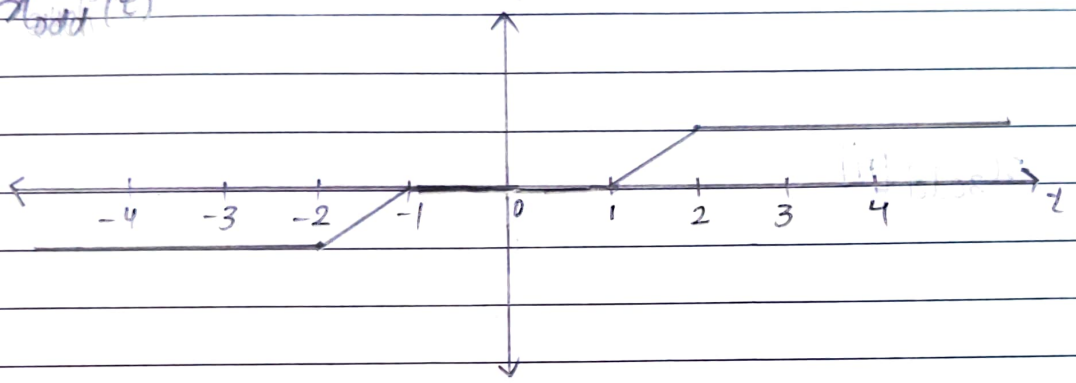
$$x_{odd}(t) = \begin{cases} -1/2 & t \leq -2 \\ (t+1)/2 & t \in [-2, -1] \\ 0 & -1 \leq t \leq 0 \\ 0 & 0 \leq t \leq 1 \\ (t-1)/2 & 1 \leq t \leq 2 \\ 1/2 & t \geq 2 \end{cases}$$

Page :
Date : / /

$x_{even}(t)$



$x_{odd}(t)$



b) $x[n] = \begin{cases} 1 & x=0 \\ 2 & x=1 \\ 1 & x=2 \\ -1 & x=3 \end{cases}$, $\begin{matrix} 1 & x=-1 \\ 2 & x=-2 \\ 2 & x=-3 \\ -1 & x=-4 \end{matrix}$

$x[-n] = \begin{cases} 1 & x=0 \\ 1 & x=1 \\ 2 & x=2 \\ 2 & x=3 \end{cases}$, $\begin{matrix} -1 & x=4 \\ 2 & x=-1 \\ 1 & x=-2 \\ -1 & x=-3 \end{matrix}$

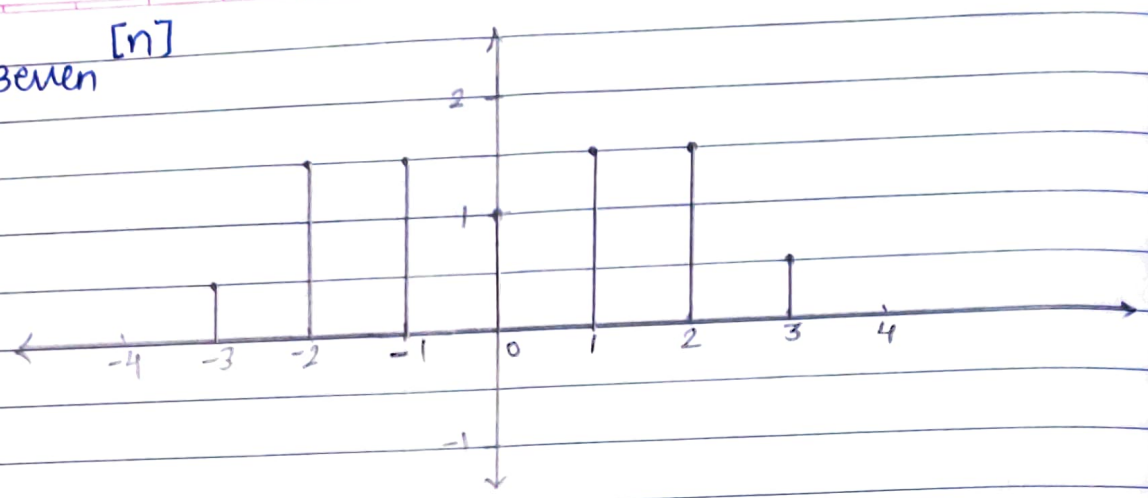
$x_{even}[n] =$

$\begin{cases} 1 & x=0 \\ 3/2 & x=1 \\ 3/2 & x=2 \\ 1/2 & x=3 \end{cases}$, $\begin{matrix} 3/2 & x=-1 \\ 3/2 & x=-2 \\ 1/2 & x=-3 \end{matrix}$

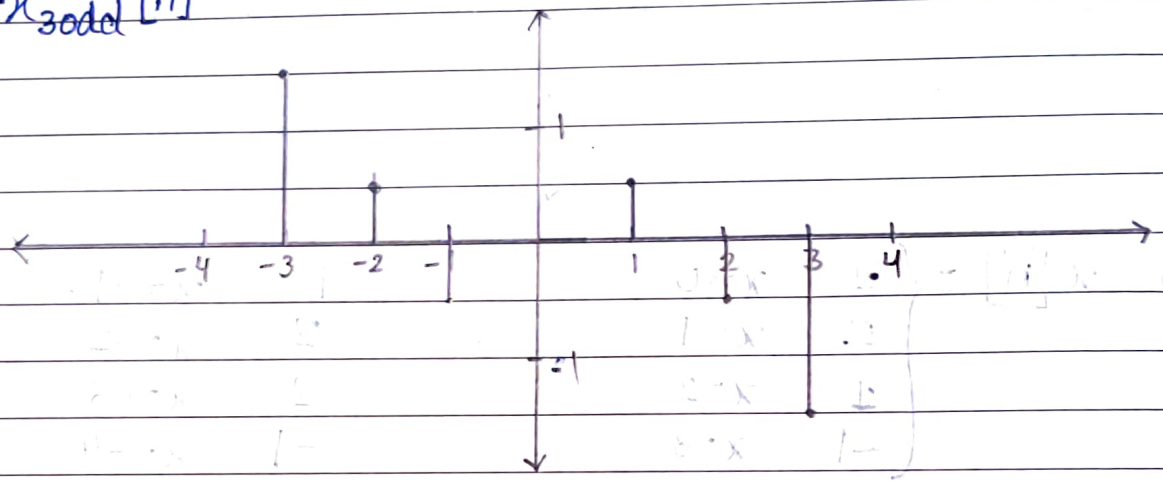
$x_{odd}[n] =$

$\begin{cases} 0 & x=0 \\ 1/2 & x=1 \\ -1/2 & x=2 \\ -3/2 & x=3 \end{cases}$, $\begin{matrix} -1/2 & x=-1 \\ 1/2 & x=-2 \\ 3/2 & x=-3 \end{matrix}$

$x_{3even}[n]$



$x_{3odd}[n]$



5] $x_1[n]$: odd $\Rightarrow$ $x_1[n] + x_1[-n] = 0$

$x_2[n]$: even $\Rightarrow$ $x_2[n] - x_2[-n] = 0$

(a) $\sum_{-\infty}^{\infty} x_1[n] = \sum_{n=0}^{\infty} (x_1[n] + x_1[-n]) = \sum_{n=0}^{\infty} 0 = 0$

(b) $g(n) = x_1[n]\, x_2[n]$

$g(n) + g(-n) = x_1[n]\, x_2[n] + x_1[-n]\, x_2[-n]$

$= x_1[n]\, x_2[n] + (-x_1[n]\, x_2[n])$

$g(n) + g(-n) = 0$

$\Rightarrow g(n) = x_1[n]\, x_2[n]$ is an odd f^{n}