#### **INDIAN INSTITUTE OF TECHNOLOGY ROORKEE**



#### **CSN-101** (Introduction to Computer Science and Engineering)

#### **Lecture 15: Number Systems**

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Piazza Class Room: <a href="https://piazza.com/iitr.ac.in/fall2019/csn101">https://piazza.com/iitr.ac.in/fall2019/csn101</a>

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[Enrollment Key: csn101@2019]

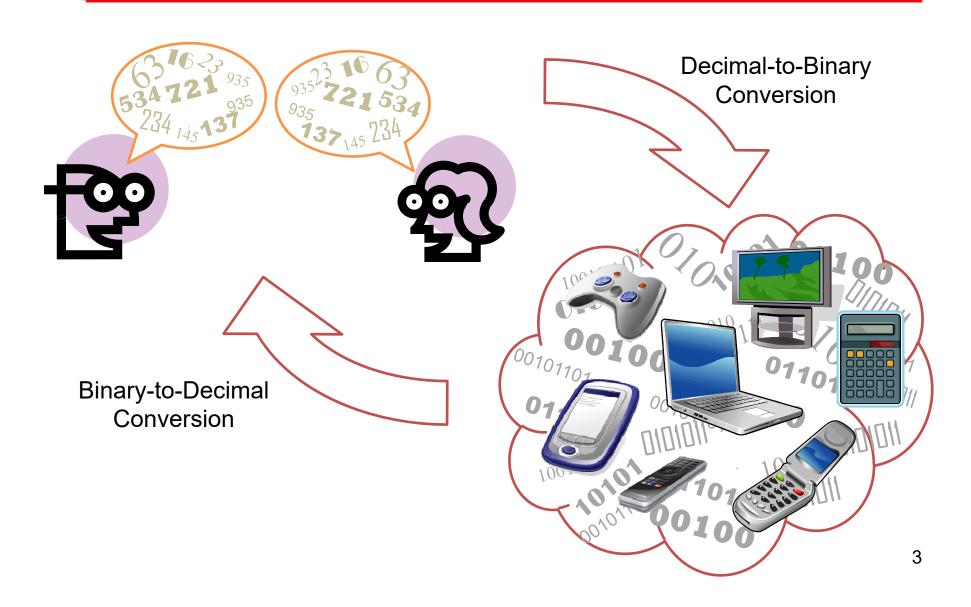


# Plan for Lecture Classes in CSN-101 (Autumn, 2019-2020)



Week	Lecture 1 (Monday 4-5 PM)	Lecture 2 (Friday 5-6 PM)	
1	Evolution of Computer Hardware and Moore's Law, Software and Hardware in a Computer	Computer Structure and Components, Operating Systems	
2	Computer Hardware: Block Diagrams, List of Components	Computer Hardware: List of Components, Working Principles in Brief, Organization of a Computer System	
3	Linux OS	Linux OS	
4	Writing Pseudo-codes for Algorithms to Solve Computational Problems	Writing Pseudo-codes for Algorithms to Solve Computational Problems	
5	Sorting Algorithms – Bubble sort, selection sort, and Search Algorithms	Sorting Algorithms – Bubble sort, selection sort, and Search Algorithms	
6	C Programming	C Programming	
7	Number Systems: Binary, Octal, Hexadecimal, Conversions among them	Number Systems: Binary, Octal, Hexadecimal, Conversions among them	
8	Number Systems: Negative number representation,	Boolean Logic: Boolean Logic Basics, De Morgan's Theorem, Logic Gates: AND, OR, NOT, NOR, NAND, XOR, XNOR: Truth-tables	
9	Computer Networking and Web Technologies: Basic concepts of networking, bandwidth, throughput	Computer Networking and Web Technologies: Basic concepts of networking, bandwidth, throughput	
10	Different layers of networking, Network components, Type of networks	Network topologies, MAC, IP Addresses, DNS, URL	
11	Different fields of CSE: Computer Architecture and Chip Design	Different fields of CSE: Data Structures, Algorithms and Programming Languages	
12	Different fields of CSE: Database management	Different fields of CSE: Operating systems and System softwares	
13	Different fields of CSE: Computer Networking, HPCs, Web technologies	Different Applications of CSE: Image Processing, CV, ML, DL	
14	Different Applications of CSE: Data mining, Computational Geometry, Cryptography, Information Security	Different Applications of CSE: Cyber-physical systems and IoTs	

# Bridging the Digital Divide



# Decimal -to- Binary Conversion

#### The Process: Successive Division

- a) Divide the *Decimal Number* by 2; the remainder is the LSB of *Binary Number*.
- b) If the quotation is zero, the conversion is complete; else repeat step (a) using the quotation as the Decimal Number. The new remainder is the next most significant bit of the *Binary Number*.

#### Example:

Convert the decimal number  $6_{10}$  into its binary equivalent.

$$2 )6$$
 r = 0 ← Least Significant Bit   
 $2 )3$  r = 1 ∴  $6_{10} = 110_2$   
 $2 )1$  r = 1 ← Most Significant Bit

#### Example:

Convert the decimal number 26<sub>10</sub> into its binary equivalent.

#### Example:

Convert the decimal number 26<sub>10</sub> into its binary equivalent.

#### Solution:

$$2 \overline{\smash{\big)}\, 26}$$
  $r = 0 \leftarrow LSB$ 

$$\frac{6}{2)13}$$
 r=1

$$\frac{3}{2 \cdot 6}$$
  $r = 0$ 

$$\frac{1}{2)3}$$
 r=1

$$2)\frac{0}{1}$$
  $r=1 \leftarrow MSB$ 

$$\therefore 26_{10} = 11010_2$$

#### Example:

Convert the decimal number 41<sub>10</sub> into its binary equivalent.

#### Example:

Convert the decimal number 41<sub>10</sub> into its binary equivalent.

#### Solution:

$$2 \overline{\smash{\big)}\, 41}$$
 r=1  $\leftarrow$  LSB

$$\frac{10}{2 \cdot 20}$$
  $r = 0$ 

$$\frac{5}{2)10}$$
  $r=0$ 

$$\frac{2}{2)5}$$
 r=1

$$2)\frac{1}{2}$$
  $r=0$ 

$$2)\frac{0}{1}$$
  $r=1 \leftarrow MSB$ 

$$\therefore$$
 41<sub>10</sub> = 101001<sub>2</sub>

# Dec → Binary : More Examples

a) 
$$13_{10} = ?$$

b) 
$$22_{10} = ?$$

c) 
$$43_{10} = ?$$

d) 
$$158_{10} = ?$$

## Dec → Binary : More Examples

a) 
$$13_{10} = ?$$
 1 1 0 1 2

b) 
$$22_{10} = ?$$
 1 0 1 1 0  $_2$ 

c) 
$$43_{10} = ?$$
 101011<sub>2</sub>

d) 
$$158_{10} = ?$$
  $10011110_2$ 

# Binary -to- Decimal Process

#### The Process: Weighted Multiplication

- a) Multiply each bit of the *Binary Number* by it corresponding bitweighting factor (i.e. Bit-0 $\rightarrow$ 2<sup>0</sup>=1; Bit-1 $\rightarrow$ 2<sup>1</sup>=2; Bit-2 $\rightarrow$ 2<sup>2</sup>=4; etc).
- b) Sum up all the products in step (a) to get the *Decimal Number*.

#### Example:

Convert the decimal number 0110<sub>2</sub> into its decimal equivalent.

$$\therefore$$
 0110<sub>2</sub> = 6<sub>10</sub>

#### Example:

Convert the binary number 10010<sub>2</sub> into its decimal equivalent.

#### Example:

Convert the binary number 10010<sub>2</sub> into its decimal equivalent.

#### Solution:

$$\therefore 10010_2 = 18_{10}$$

#### Example:

Convert the binary number 0110101<sub>2</sub> into its decimal equivalent.

#### Example:

Convert the binary number 0110101<sub>2</sub> into its decimal equivalent.

#### Solution:

$$\therefore 0110101_2 = 53_{10}$$

# Binary → Dec : More Examples

a) 
$$0110_2 = ?$$

c) 
$$0110101_2 = ?$$

# Binary → Dec : More Examples

a) 
$$0110_2 = ?$$
 6 <sub>10</sub>

c) 
$$0110101_2 = ?$$
 53 <sub>10</sub>

## Summary & Review

# Base<sub>10</sub>

Successive Division



- a) Divide the *Decimal Number* by 2; the remainder is the LSB of *Binary Number*.
- b) If the Quotient Zero, the conversion is complete; else repeat step (a) using the Quotient as the Decimal Number. The new remainder is the next most significant bit of the *Binary Number*.

# Base<sub>2</sub>

Weighted Multiplication

Base<sub>10</sub>

- a) Multiply each bit of the *Binary Number* by it corresponding bit-weighting factor (i.e. Bit-0 $\rightarrow$ 2<sup>0</sup>=1; Bit-1 $\rightarrow$ 2<sup>1</sup>=2; Bit-2 $\rightarrow$ 2<sup>2</sup>=4; etc).
- b) Sum up all the products in step (a) to get the Decimal Number.

## **Common Number Systems:**



System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, 7	No	No
Hexa- decimal	16	0, 1, 9, A, B, F	No	No

## Quantities/Counting (1 of 3):



Decimal	Binary	Octal	Hexa- decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	-5	5
6	110	6	6
7	111	7	7

## Quantities/Counting (2 of 3):



Decimal	Binary	Octal	Hexa- decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	В
12	1100	14	С
13	1101	15	D
14	1110	16	Е
15	1111	17	F

## Quantities/Counting (2 of 3):



Decimal	Binary	Octal	Hexa- decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	В
12	1100	14	С
13	1101	15	D
14	1110	16	Е
15	1111	17	F

## Quantities/Counting (3 of 3):

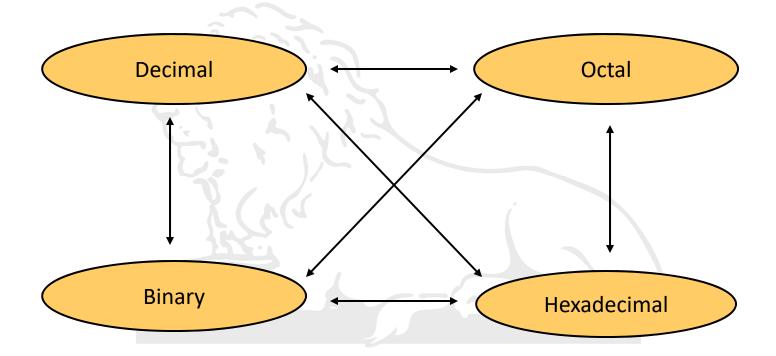


Decimal	Binary	Octal	Hexa- decimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17

## **Conversion Among Bases:**

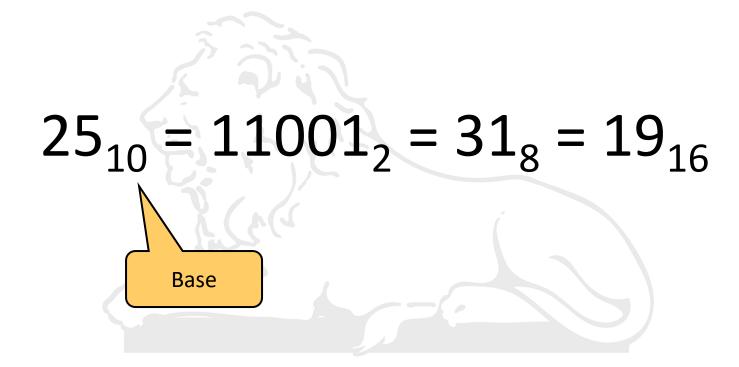


• The possibilities:



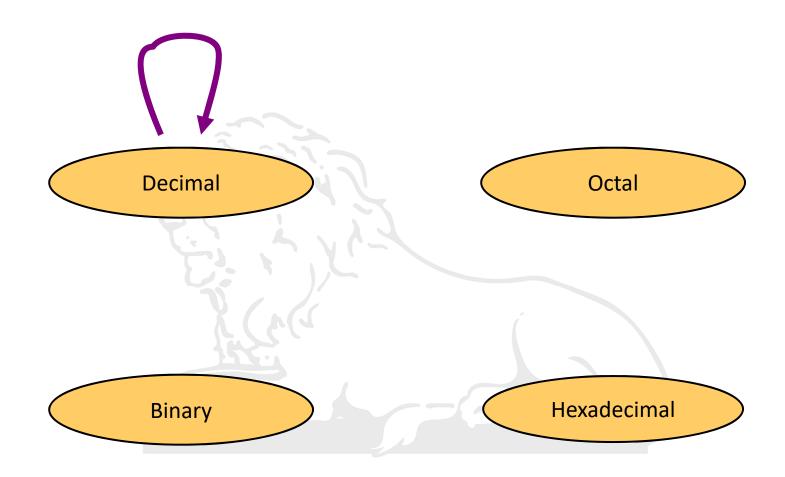
## **Quick Example:**





## **Decimal to Decimal (just for fun):**





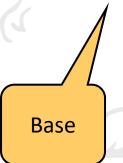




$$5 \times 10^{0} = 5$$
 $2 \times 10^{1} = 20$ 
 $1 \times 10^{2} = 100$ 

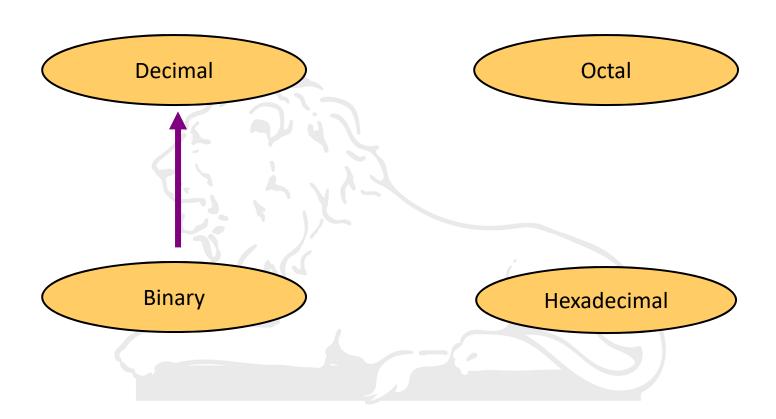
$$2 \times 10^1 = 20$$

$$1 \times 10^2 = 100$$



## **Binary to Decimal:**



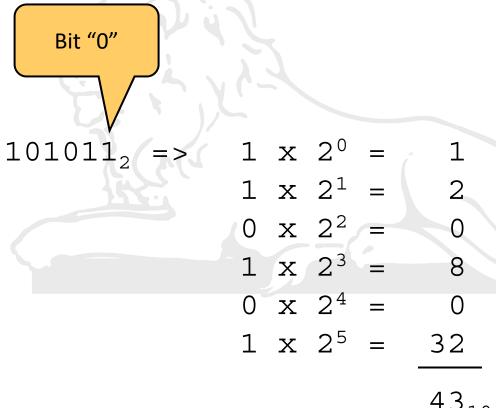


## **Binary to Decimal:**



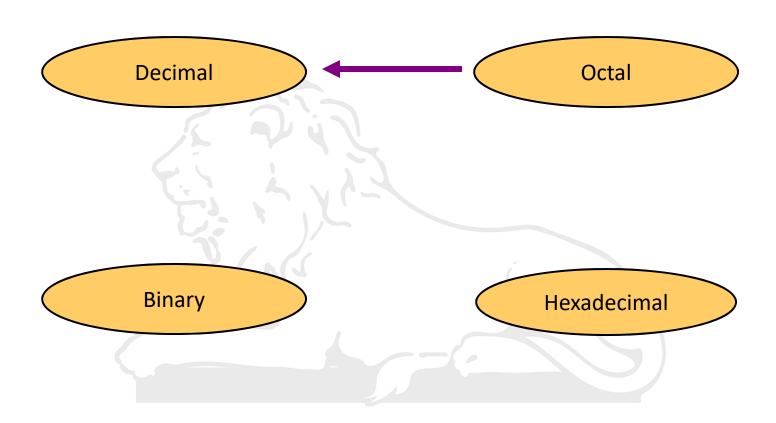
#### Technique

- Multiply each bit by  $2^n$ , where n is the "weight" of the bit
- The weight is the position of the bit, starting from 0 on the right
- Add the results



## **Octal to Decimal:**





### **Octal to Decimal:**



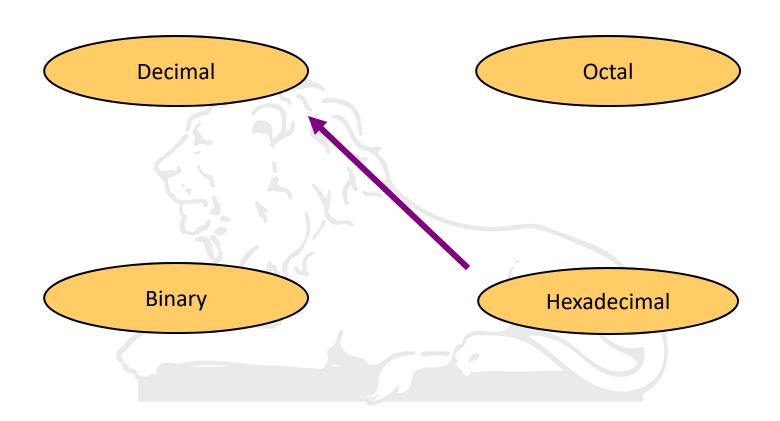
#### Technique

- Multiply each bit by  $8^n$ , where n is the "weight" of the bit
- The weight is the position of the bit, starting from 0 on the right
- Add the results

$$724_8 \Rightarrow 4 \times 8^0 = 4$$
 $2 \times 8^1 = 16$ 
 $7 \times 8^2 = 448$ 
 $468_{10}$ 

## **Hexadecimal to Decimal:**





#### **Hexadecimal to Decimal:**



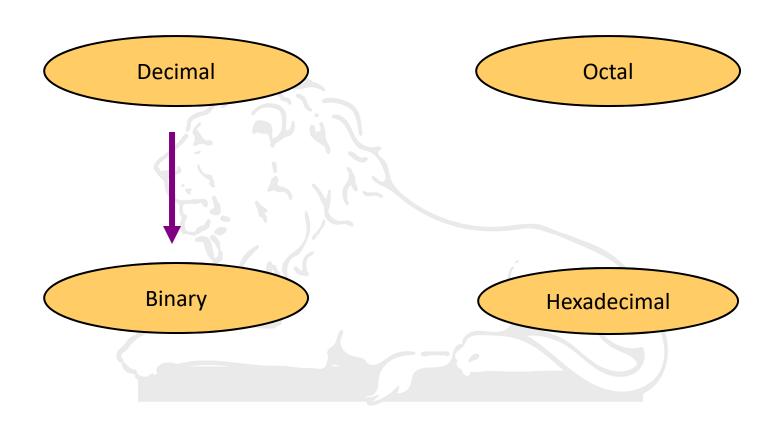
#### Technique

- Multiply each bit by  $16^n$ , where n is the "weight" of the bit
- The weight is the position of the bit, starting from 0 on the right
- Add the results

ABC<sub>16</sub> => C x 
$$16^{\circ}$$
 = 12 x 1 = 12  
B x  $16^{1}$  = 11 x  $16$  = 176  
A x  $16^{2}$  = 10 x  $256$  =  $2560$   
 $2748_{10}$ 

## **Decimal to Binary:**





## **Decimal to Binary:**

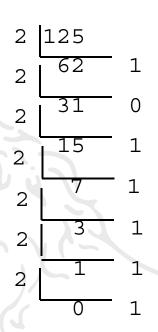


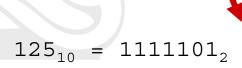
- Technique
  - Divide by two, keep track of the remainder
  - First remainder is bit 0 (LSB, least-significant bit)
  - Second remainder is bit 1
  - Etc.

## **Example:**



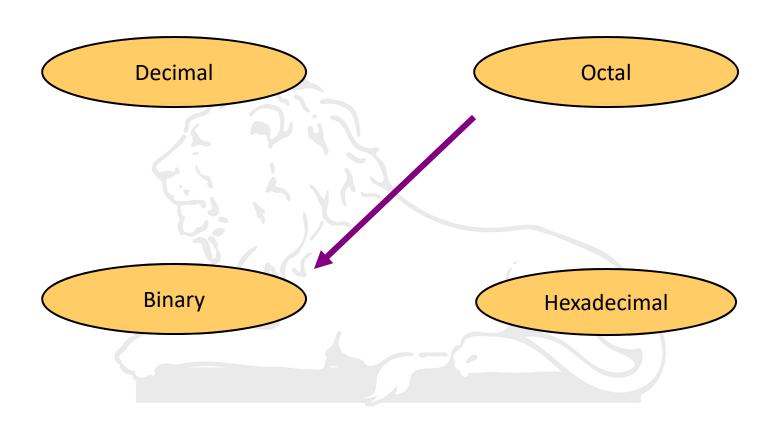
$$125_{10} = ?_2$$





# **Octal to Binary:**



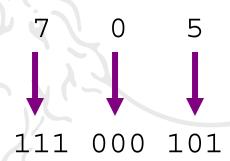


# **Octal to Binary:**



- Technique
  - Convert each octal digit to a 3-bit equivalent binary representation

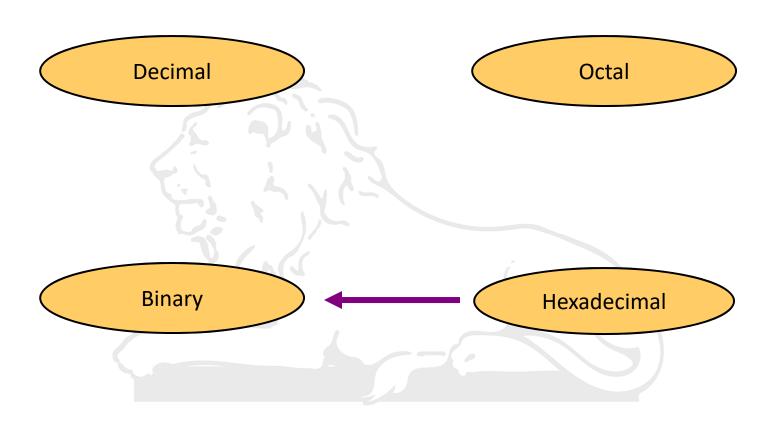
$$705_8 = ?_2$$



$$705_8 = 111000101_2$$

# **Hexadecimal to Binary:**





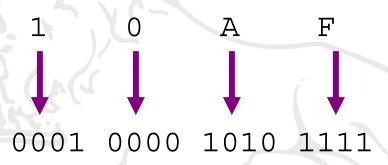
# **Hexadecimal to Binary:**



### Technique

Convert each hexadecimal digit to a 4-bit equivalent binary representation

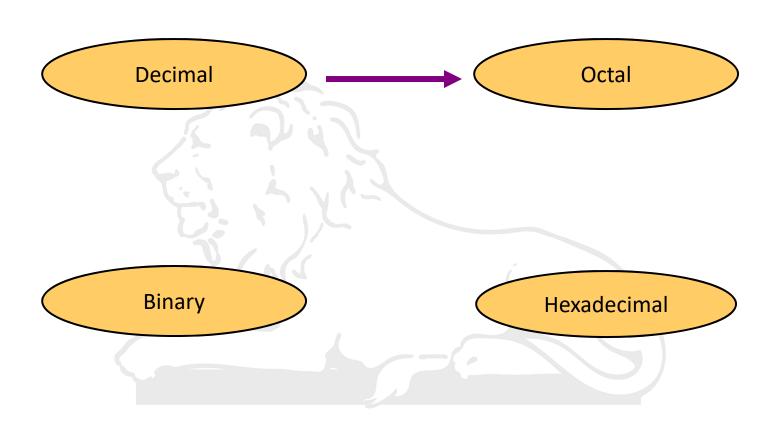
$$10AF_{16} = ?_2$$



 $10AF_{16} = 0001000010101111_2$ 

# **Decimal to Octal:**



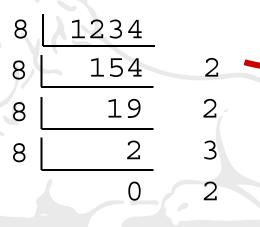


## **Decimal to Octal:**



- Technique
  - Divide by 8
  - Keep track of the remainder

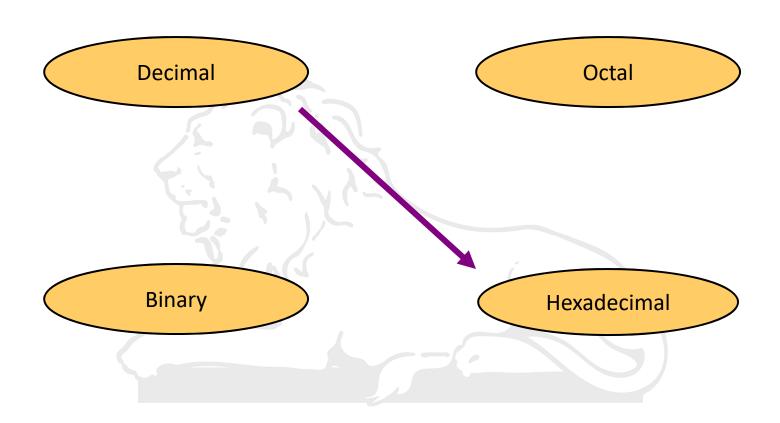
$$1234_{10} = ?_{8}$$



$$1234_{10} = 2322_{8}$$

# **Decimal to Hexadecimal:**





## **Decimal to Hexadecimal:**

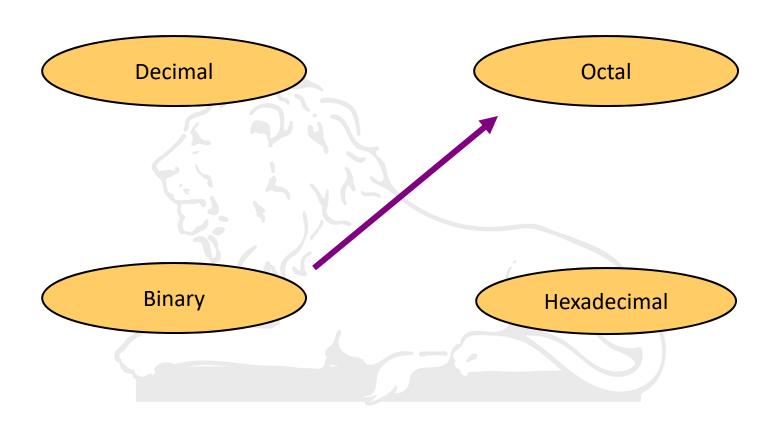


- Technique
  - Divide by 16
  - Keep track of the remainder

$$1234_{10} = 4D2_{16}$$

# **Binary to Octal:**





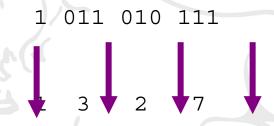
# **Binary to Octal:**



## Technique

- Group bits in threes, starting on right
- Convert to octal digits

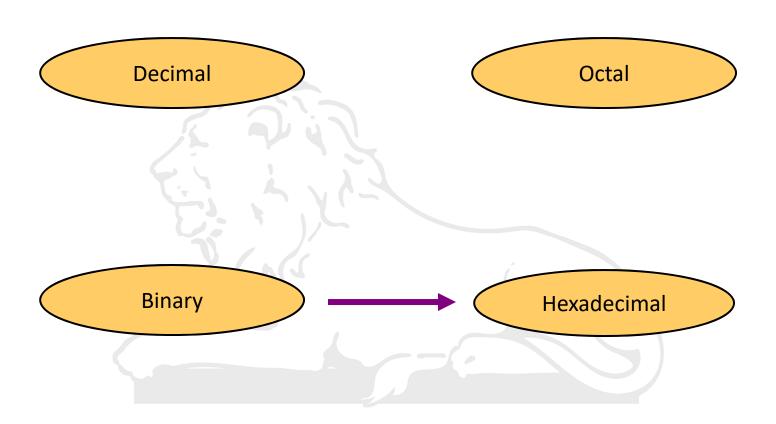
$$1011010111_2 = ?_8$$



 $1011010111_2 = 1327_8$ 

# **Binary to Hexadecimal:**





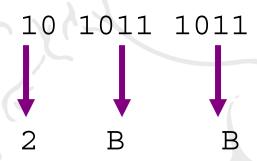
# **Binary to Hexadecimal:**



### Technique

- Group bits in fours, starting on right
- Convert to hexadecimal digits

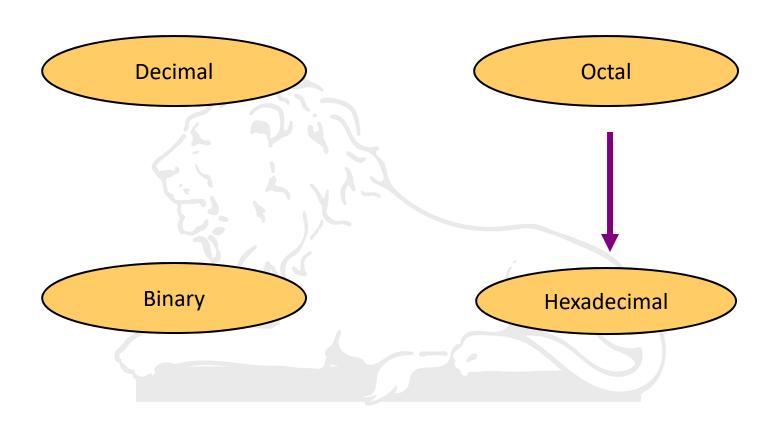
$$1010111011_2 = ?_{16}$$



$$1010111011_2 = 2BB_{16}$$

# **Octal to Hexadecimal:**





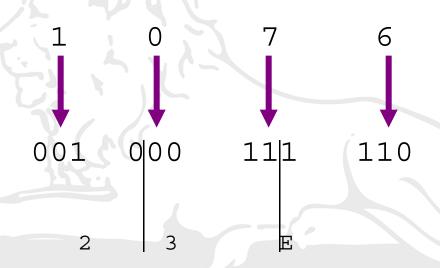
## **Octal to Hexadecimal:**



## • Technique

Use binary as an intermediary

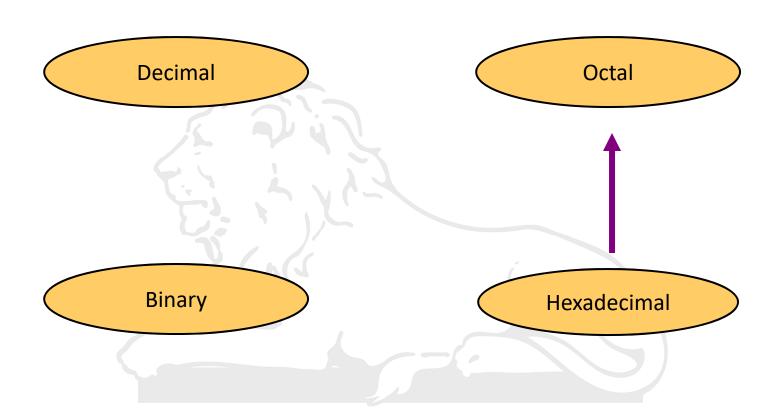
$$1076_8 = ?_{16}$$



$$1076_8 = 23E_{16}$$

# **Hexadecimal to Octal:**





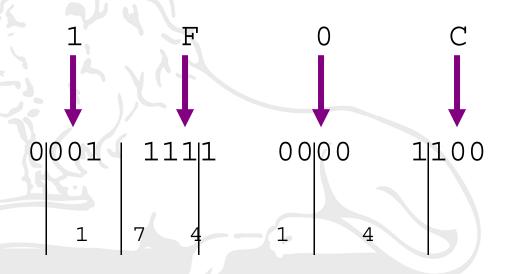
## **Hexadecimal to Octal:**



## • Technique

Use binary as an intermediary

$$1F0C_{16} = ?_{8}$$



$$1F0C_{16} = 17414_{8}$$

# **Exercise – Convert ...**



Decimal	Binary	Octal	Hexa- decimal
33	412		
	1110101		
	Ev(A	703	
		-5.	1AF

Don't use a calculator!

# **Exercise – Convert ...**



#### Answer

Decimal	Binary	Octal	Hexa- decimal
33	100001	41	21
117	1110101	165	75
451	111000011	703	1C3
431	110101111	657	1AF



# Common Powers (1 of 2):



Base 10

Power	Preface	Symbol	Value
10-12	pico	p	.000000000001
10-9	nano	n	.000000001
10-6	micro	μ	.000001
10-3	milli	m	.001
$10^3$	kilo	k	1000
$10^6$	mega	M	1000000
10 <sup>9</sup>	giga	G	1000000000
10 <sup>12</sup>	tera	Т	1000000000000

# Common Powers (2 of 2):



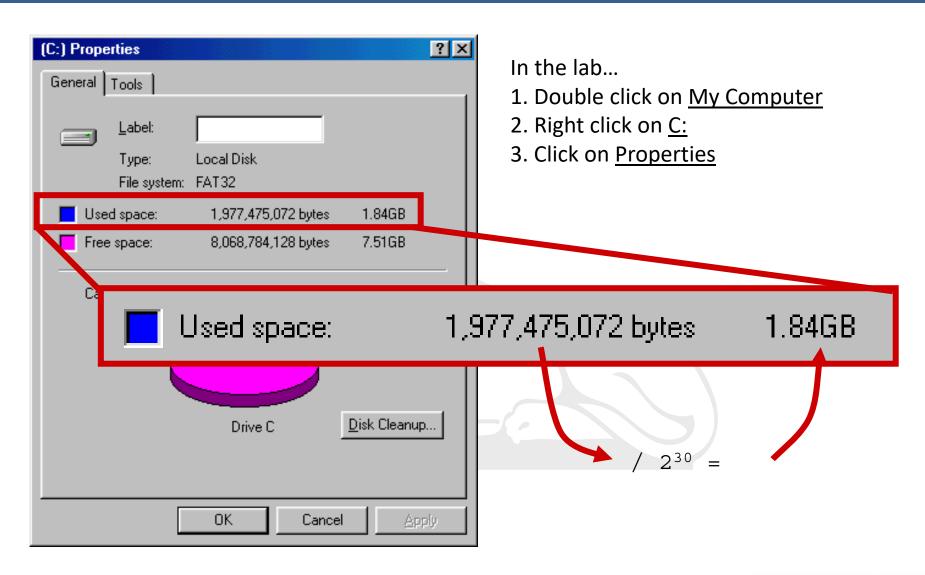
• Base 2

Power	Preface	Symbol	Value
210	kilo	k	1024
$2^{20}$	mega	M	1048576
$2^{30}$	Giga	G	1073741824

- What is the value of "k", "M", and "G"?
- In computing, particularly w.r.t. <u>memory</u>, the base-2 interpretation generally applies

# **Example:**





# Exercise – Free Space

 Determine the "free space" on all drives on a machine in the lab

	Free space		
Drive	Bytes	GB	
A:			
C:			
D:			
E:			
etc.			

# Review – multiplying powers

For common bases, add powers

$$a^b \times a^c = a^{b+c}$$

$$2^6 \times 2^{10} = 2^{16} = 65,536$$

$$2^6 \times 2^{10} = 64 \times 2^{10} = 64k$$

# Binary Addition (1 of 2)

• Two 1-bit values

A	В	A + B	
0	0	0	
0	1	1	
1	0	1	
1	1	10 🥆	
,			"two"

# Binary Addition (2 of 2)

- Two *n*-bit values
  - Add individual bits
  - Propagate carries
  - − E.g.,

# Multiplication (1 of 3)

Decimal (just for fun)

# Multiplication (2 of 3)

Binary, two 1-bit values

A	В	$A \times B$
0	0	0
0	1	0
1	0	0
1	1	1

# Multiplication (3 of 3)

- Binary, two *n*-bit values
  - As with decimal values

– E.g.,

# **Fractions**

Decimal to decimal (just for fun)

```
3.14 \Rightarrow 4 \times 10^{-2} = 0.04
1 \times 10^{-1} = 0.1
3 \times 10^{0} = 3
3.14
```

# Fractions

Binary to decimal

```
10.1011 => 1 x 2^{-4} = 0.0625

1 x 2^{-3} = 0.125

0 x 2^{-2} = 0.0

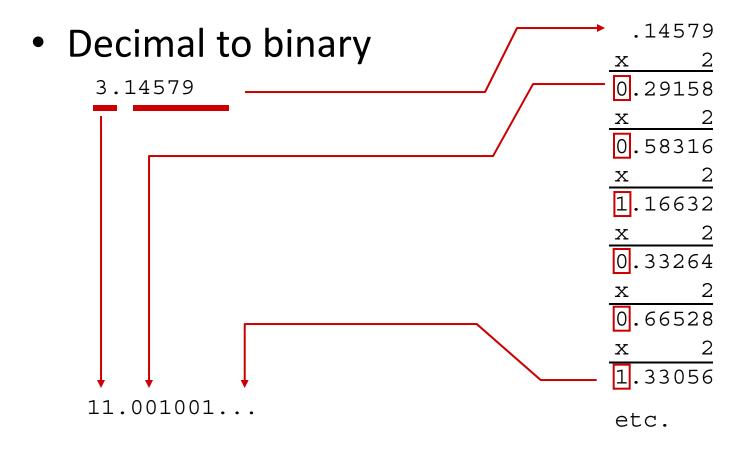
1 x 2^{-1} = 0.5

0 x 2^{0} = 0.0

1 x 2^{1} = 2.0

2.6875
```

# **Fractions**



# Exercise – Convert ...

Decimal	Binary	Octal	Hexa- decimal
29.8			
	101.1101		
		3.07	
			C.82

Don't use a calculator!

Skip answer

Answer

# Evarcica - Canvart

#### Answer

Decimal	Binary	Octal	Hexa- decimal
29.8	11101.110011	35.63	1D.CC
5.8125	101.1101	5.64	5.D
3.109375	11.000111	3.07	3.1C
12.5078125	1100.10000010	14.404	C.82



# **Test:**



• Decimal: 111

Hex: 6F

• Oct: 157

• Bin: 0110 1111

# How a computer stores information

# Binary Numbers are at the heart of how a computer stores all information

- Computers Store ALL information using Binary Numbers
- Computers use binary numbers in different ways to store different types of information.
- Common types of information that are stored by computers are :
  - Whole numbers (i.e. Integers).Examples: 8 97 -732 0 -5 etc
  - Numbers with decimal points.Examples: 3.5 -1.234 0.765 999.001 etc
  - Textual information (including letters, symbols and digits)
- Keep reading ...

### Integers

• Integers (e.g., 87)

A computer stores integer numbers (i.e. "whole" numbers) simply as the equivalent binary value for that number.

#### Numbers with Decimal Points

- Numbers with decimal points (e.g., 87.123)
  - Internally, a computer stores a number with a decimal point as two different integer numbers (each stored using binary). To get the actual value, the computer performs a mathematical calculation using the two integers to derive the number.
  - We will NOT discuss here the <u>actual</u> mathematical calculation nor how the computer breaks a number with a decimal point into two integers.

[NOTE: The two integers are NOT the whole number part and fractional part.]

## Letters and symbols

- Letters and symbols
  - To store letters and symbols, the computer assigns every character on the keyboard a numerical value.
  - Computers remember letters and other symbols by storing the binary number for the symbol.
  - For this system to work a standard numbering system needs to be defined and consistently used for all symbols that the computer needs to process.
  - See the following slide ...

### ASCII (Americal Standard Code for Information Interchange)

- ASCII (Americal Standard Code for Information Interchange) is the standard numbering given to all characters on a standard keyboard.
- "ASCII values" range in number from 1 to 128. Some "ASCII values" and their associated symbols are listed to the right.
- Note that EVERY symbol on a standard keyboard has an ASCII value. Even the digits 0,1,2,...9 have ASCII values. (see next slide)

#### Some ASCII values (values 1-31 and 128 are not shown)

	V CL	taob (vara
32 = Space	•	64 = @
33 = !	•	65 = A
34 = "	•	66 = B
35 = #	•	67 = C
36 = \$	•	68 = D
37 = %	•	69 = E
38 = &	•	70 = F
39 = `	•	71 = G
40 = (	•	72 = H
41 = )	•	73 = I
42 = *	•	74 = J
43 = +	•	75 = K
44 = ,	•	76 = L
45 = -	•	$77 = \mathbf{M}$
46 = .	•	78 = N
47 = /	•	79 = O
48 = 0	•	80 = P
49 = 1	•	81 = Q
50 = 2	•	82 = R
51 = 3	•	83 = S
52 = 4	•	84 = T
53 = 5	•	85 = U
54 = 6	•	86 = V
55 = 7	•	87 = W
56 = 8	•	88 = X
57 = 9	•	89 = Y
58 = :	•	90 = Z
	•	91 = [
60 = <	•	92 = \
61 = =	•	93 = ]
62 = >	•	94 = ^
	33 = ! 34 = " 35 = # 36 = \$ 37 = % 38 = & 39 = ` 40 = ( 41 = ) 42 = * 43 = + 44 = , 45 = - 46 = . 47 = / 48 = 0 49 = 1 50 = 2 51 = 3 52 = 4 53 = 5 54 = 6 55 = 7 56 = 8 57 = 9 58 = : 59 = ; 60 = < 61 = =	32 = Space 33 = ! 34 = " 35 = # 36 = \$ 37 = % 38 = & 39 = ` 40 = ( 41 = ) 42 = * 43 = + 44 = , 45 = - 46 = . 47 = / 48 = 0 49 = 1 50 = 2 51 = 3 52 = 4 53 = 5 54 = 6 55 = 7 56 = 8 57 = 9 58 = : 59 = ; 60 = < 61 = =

63 = ?

95 =

```
96 = `
97 = a
98 = b
99 = c
100 = d
101 = e
102 = f
103 = g
104 = h
105 = i
106 = i
107 = k
108 = 1
109 = m
110 = n
111 = 0
112 = p
113 = q
114 = r
115 = s
116 = t
117 = u
118 = v
119 = w
120 = x
121 = y
122 = z
123 = {
124 = |
125 = 
126 = \sim
                      76
```

#### Why do 0 through 9 have ASCII values?

- Numbers that are used in mathematical calculations
  - If a computer needs to do math with a number it will store that number using the appropriate binary representation of the number.
  - This makes it easier for the computer to perform mathematical calculations with the number.
  - Example: 5 would be stored as

#### 00000101

- Numbers that are NOT used in mathematical calculations
  - If the computer does NOT need to do math with the number (e.g. a zip code) then it will generally store the number using the ASCII values of the digits.
  - In this case using the ASCII value is more efficient (for reasons we will not explain here).
  - Example 5 would be stored using its ASCII value of 53 which is represented in binary as

00110101

# Other numbering systems (Unicode and EBCDIC)

#### ASCII

ASCII was the standard numbering system for many years and is still used widely today.

#### EBCDIC

- Is a different numbering system used by IBM Mainframe computers.
- It is very similar to ASCII but uses different numbers to represent the symbols.
- EBCDIC stands for "Extended Binary Coded Decimal Interchange Code"

#### Unicode

- ASCII and EBCDIC are limited to just the basic English letters and common symbols.
- Today computers use many different symbols including letters from languages that don't use English letters (e.g. Hebrew, Chinese, etc.) and international symbols (e.g. the English pound sign)
- Unicode defines a unique number for every symbol in all known languages (e.g. Hebrew, Chinese, etc.) and commonly used non-letter symbols (e.g. English pound sign, copyright symbol, etc).
- Modern programs are moving towards using Unicode to store letters and symbols.
- It should be noted that Unicode numbers 1-128 correspond to the EXACT SAME symbols as ASCII 1-128

# How a computer stores information (i.e., decimal vs. analog)

## How a computer stores info.

- A computer stores all information as binary numbers.
- Computer memory simply remembers ones and zeros
- Computer storage remembers ones and zeros
- Data is passed inside the computer from one portion of the computer to another (e.g. memory to CPU to graphics card, etc) by ones and zeros

# Terms (bit, byte, etc.)

- BIT
  - definition: a single Binary digIT (i.e. BIT)
- BYTE
  - definition: 8 bits
- NYBLE
  - definition: 4 bits

#### **Prefixes**

#### Prefixes

- Kilo: one thousand
- Mega: one millioin
- Giga: one billion
- Tera: one trillion
- Peta: one quadrillion
- Exa: one quintillion
- ... etc.

#### Data sizes

- Data sizes
  - Kilobyte (KB)
    - "about" one thousand bytes
    - exactly 2<sup>10</sup> or 1024 bytes
  - Megabyte (MB)
    - "about" one million bytes
    - exactly 2<sup>20</sup> or 1,048,576 bytes
  - Gigabyte (GB)
    - "about" one billion bytes
    - exactly 2<sup>30</sup> or 1,073,741,824 bytes
  - Terabyte (TB)
    - "about" one trillion bytes
    - exactly 2<sup>40</sup> or 1,099,511,627,776 bytes

# Data Sizes – bytes vs bits

- MB = one Mega Byte
- Mb = one Mega Bit

## Speeds

- MBPS = one MegaByte per second
- MbPS = one Mega Bit per second

## How data is stored using binary

- Integers are stored as a binary number
- A character is stored as the ASCII value (i.e. an integer) for that character
- A decimal number is stored using two different integer values - the mantissa and the exponent

- 1 bit
- 1 byte = 8 bits
- 1 kb =  $2^{10}$  bytes = 1024 bytes !=1000
- 1 Mb = 1 k k bytes =  $2^{10} * 2^{10}$  bytes
- 1 G b =  $2^{10} * 2^{10} * 2^{10}$  bytes
- 1 Terab =  $2^{10} * 2^{10} * 2^{10} * 2^{10}$  bytes

## Even larger capacity

- 1 petabyte =  $2^{10} * 2^{10} * 2^{10} * 2^{10} * 2^{10}$  bytes (2 to the 50th power)
- 1 exabyte= 2<sup>60</sup>
- 1 zettabyte = 2<sup>70</sup>
- 1 yottabyte =  $2^{80}$

# Some interesting facts about what these various-sized bytes can store:

- 1 bit: a binary decision
- 1 byte: a character
- 5 Megabytes: The complete works of Shakespeare
- 2 Gigabytes: 20 meters of shelved books
- 10 Terabytes: The printed collection of the US Library of Congress
- 200 Petabytes: All printed material in the whole word.
- 5 Exabytes: All words ever spoken by human beings