Electromagnetic Induction

magnetic flux - The no. of magnetic field line crossing a surface normally.
↳ denoted by ϕ_B or ϕ

$$\boxed{\phi = \vec{B}\cdot\vec{A} = BA\cos\theta}$$

- when $\vec{B}$ is perpendicular to the surface i.e., $\theta = 0°$, $\phi = NBA$ (max.)
- when $\vec{B}$ is parallel to the surface i.e, $\theta = 90°$, $\phi = 0$ (minimum value)
- In case of a coil, having N turns, $\phi = NBA\cos\theta$.

↳ scalar quantity
↳ dimensional formula → $[ML^2T^{-2}A^{-1}]$
↳ SI unit - weber (wb)
↳ CGS unit - maxwell.

$1\ wb = 10^8$ maxwell.

Electromagnetic Induction - It is the phenomenon of generating emf (electromotive force) by changing the no. of magnetic field lines (i.e., magnetic flux). The emf generated → induced emf
↳ If the circuit is closed the current which flows in it due to induced emf → induced current.

Faraday's laws of Electromagnetic induction-

- First law - amount of magnetic flux changes → emf is induc emf persists as long as the change in magnetic flux continues.

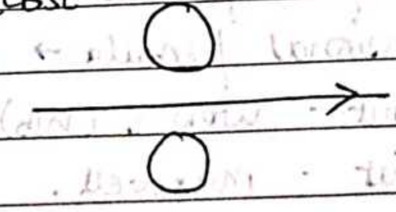
- Second law - The magnitude of induced emf is equal to the rate of change of magnetic flux.

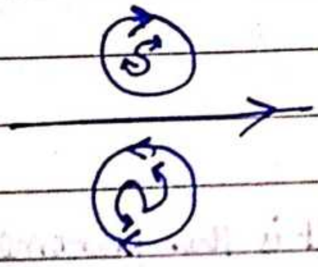
$$|\varepsilon| = \frac{d\phi}{dt}, \quad \varepsilon = -\frac{d\phi}{dt}$$

Negative sign indicates that the direction of induced emf is such that it opposes the change in magnetic flux

Lenz's law : According to this law, the polarity of induced emf in a circuit is such that it ~~opposes~~ opposes the change in magnetic flux responsible for its production.
↳ This law is in accordance with the principle of conservation of energy.

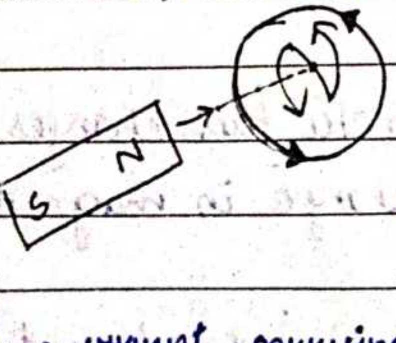
Q. Predict the direction of induced currents in metal rings 1 and 2 lying in the same plane where current I in the wire is increasing steadily. [CBSE 2012]





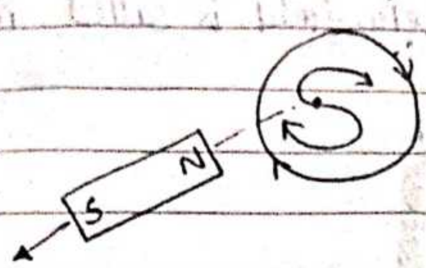
Application of Lenz's law :-

(1) When north pole of the magnet is moved towards the coil, the current induced in it will be in anticlockwise direction i.e., in North.

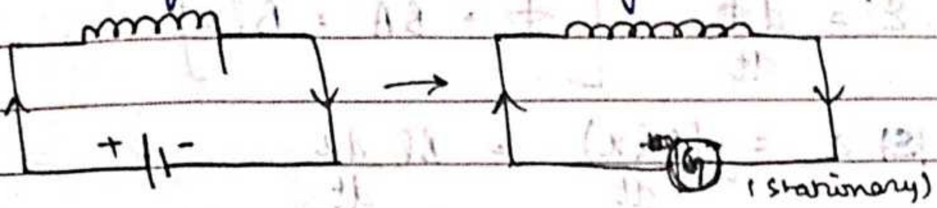


(2) ~~When a current carrying coil is moved towards a stationary coil, the direction of current induced~~

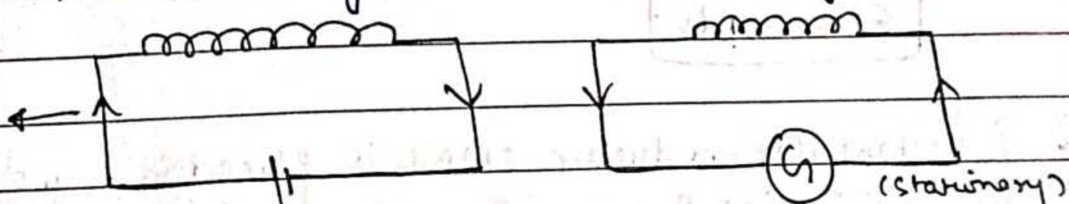
(2) When a north pole of bar magnet is moved away from the coil, the current induced in the coil will be in clockwise direction, i.e, south direction.



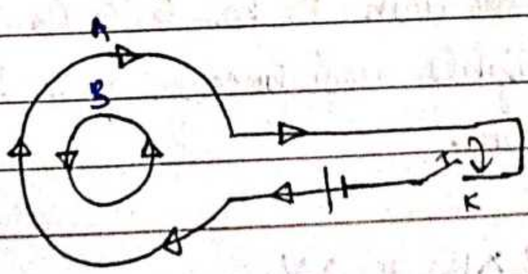
(3) When the current carrying coil is moved towards the stationary coil the direction of the current induced is



(4) When the current carrying coil is moved away from the stationary coil the direction of the current induced is



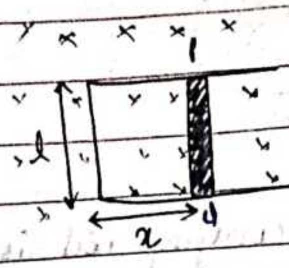
(5) When two coils A and B are arranged, then on pressing K current A increases in clockwise → induced current in B increases anticlockwise.



However when key K is released current in A decreases in clockwise direction → induced current in B will be clockwise direction.

Motional EMF:

The EMF induced across the ends of the conductor due to its motion in a magnetic field is called motional emf.



$$\varepsilon = \frac{d\phi}{dt} \quad \left[\phi = BA = Blx\right]$$

$$\varepsilon = \frac{d(Blx)}{dt} = Bl\frac{dx}{dt}$$

$$\cancel{\varepsilon = \frac{Blx}{dt}} \quad \boxed{|\varepsilon| = BlV} \quad \left[\frac{dx}{dt} = v\right]$$

$$\boxed{\varepsilon = -BlV}$$

Q. A rectangular conductor LMNO is placed in a uniform magnetic field of 0.5 T. The field is directed perpendicular to the plane of the conductor. When the arm MN of length of 20 cm is moved towards left with velocity of 10m/s. Calculate the emf induced in the arm. Given the resistance of the arm to be 5 Ω (assuming that other arms are of negligible resistance). Find the value of the current in the arm.

$\varepsilon = 0.5 \times 0.2 \times 10 = 1V$

$R = 5\,\Omega$

$V = IR, \quad \varepsilon = IR$

$$I = \frac{\varepsilon}{R} = \frac{1}{5} = 0.2\,A$$

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current Induced in the loop:

$$I = \frac{\varepsilon}{R} = \frac{Blv}{R}.$$

Force on the movable arm:

$$F = IlB \sin 90^\circ = \left(\frac{Blv}{R}\right)lB = \frac{B^2l^2v}{R}.$$

Power delivered by the external force:

$$P = Fv = \frac{B^2l^2v^2}{R}.$$

Method of generating Induced EMF.

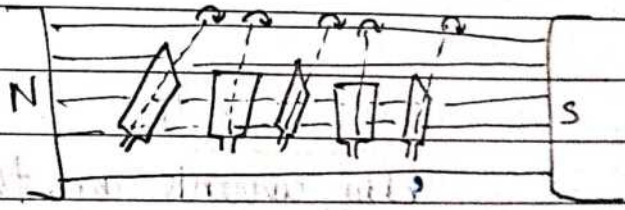
1. changing magnetic field B.
2. Changing the area of the coil
3. changing relative orientation of the coil and the magnetic field.

$\varepsilon = \varepsilon_0 \sin \omega t$

$\varepsilon_0 = NBA\omega$

$\omega = 2\pi f$

↳ angular frequency



$$\boxed{\begin{aligned}\varepsilon &= \varepsilon_0 \sin 2\pi f t \text{ ; or} \\ \varepsilon &= NBA\omega \sin \omega t\end{aligned}}$$

Q. A small flat search coil area $5\ cm^2$ w/ 140 closely wound turns is placed betn the poles of the powerful magnet producing magnetic field 0.09 T and then quickly removed out of the field region. Calculate:

(a) change of magnetic flux through the coil, and

(b) emf induced in the coil. [CBSE 2019]

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Sol^n: $B = 0.09\,T$
$A = 5\,cm^2$
$N = 140$

(a) $\phi_1 = NBA = 140 \times 0.09 \times 5 \times 10^{-4}$

$\phi_1 = 63 \times 10^{-4}\,Wb.$

$\phi_2 = 0$

Change in magnetic flux $= \phi_2 - \phi_1$
$= 63 \times 10^{-2}\,Wb.$

(b) Emf induced $= \dfrac{-d\phi}{dt} = \dfrac{-63 \times 10^{-4}}{\Delta t}$

[Here time is not given. Question is incomplete]

Eddy Currents

Eddy currents are the currents induced in solid metall masses when the magnetic flux through them changes.

↳ direction of eddy currents → Lenz's law, or Fleming's right hand rule.

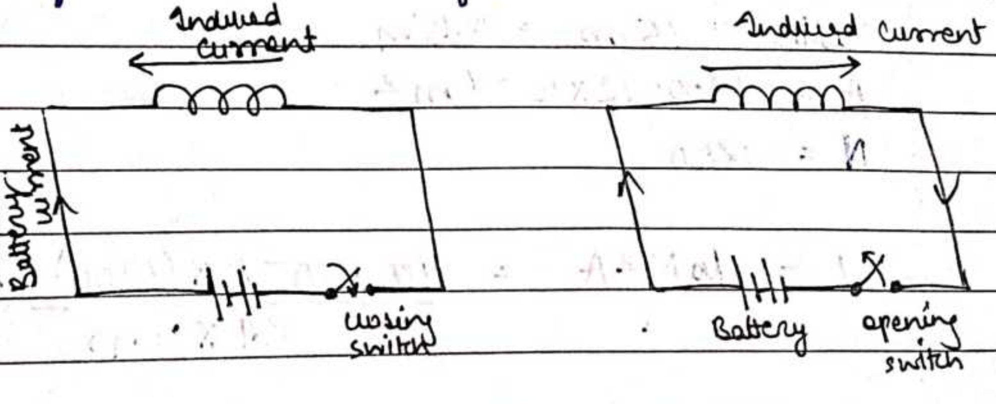
↳ Acc. to Lenz law, eddy currents set up in a metallic conductor flow in such a direction so as to oppose the change in magnetic flux linked with it.

→ Eddy currents can't be eliminated but can be minimised by-

- laminating the wire
- by taking the metallic wire in the form of thin laminated sheets attached together.

Applications of eddy currents -
1. Induction furnace.
2. Electromagnetic Damping
3. Electric Brakes
4. Speedometers
5. Induction motor
6. electromagnetic shielding

Self Induction - The phenomenon of production of induced emf in a coil when a changing current passes through it is called self induction.



Coefficient of self-inductance

$\phi \propto I = LI$

$$\mathcal{E} = \frac{-d\phi}{dt} = -L\frac{dI}{dt}$$

where, L is the constant for a given coil and is called coefficient of self Inductance. self Inductance also called coefficient of self Inductance.

↳ Unit → Henry(H)

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Self Inductance for long solenoid

$$L = \mu_0 n^2 l A$$

$$\left[n = \frac{N \rightarrow \text{No. of Turns.}}{l \rightarrow \text{Length}} \right.$$

$$\therefore \; L = \frac{\mu_0 N^2 A}{l}.$$

Q A toroidal solenoid with air core has an average radius of 15cm, area of cross section 12 cm^2. and has 1200 turns. Calculate the self inductance of the toroid. Assume the field to be uniform across the cross section of the toroid.

radius = 15 cm = 0.15m

$A = \cancel{1200}\ 12 \times 10^{-4} m^2$

$N = 1200$

$$L = \frac{\mu_0 N^2 A}{l} = \frac{4\pi \times 10^{-7} \times (1200)^2 \times 12 \times 10^{-4}}{2\pi \times 0.15}$$

$$= \boxed{2.3 \times 10^{-3}\ H}$$

Q. A 0.5 m long solenoid of 10 Turns/cm has area of cross section 1 cm^2. Calculate the voltage induced across its ends if the current in the solenoid is changed from 1A to 2A in 0.1s.

$l = 0.5m$ $\quad$ $n = 1000/m$

$A = 1 \times 10^{-4} m$

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change in current $dI = (2-1) = 1A$, $dt = 0.1s$
The induced voltage

$$|V| = L\frac{dI}{dt}$$

$$= \mu_0 n^2 A l \frac{dI}{dt}$$

$$= 4\pi \times 10^{-7} \times (1000)^2 \times \frac{10}{0.1}$$

$$= 4\pi \times 5 \times 10^{-5} = 20\pi \times 10^{-5}$$

$$= 6.28 \mu V$$

Factors on which self Inductance depend -

- $L \propto N^2$
- $L \propto A$
- Permeability of the core material

Mutual Inductance

- The phenomenon of production of induced emf in one coil due to a change in the current in the neighbouring coil

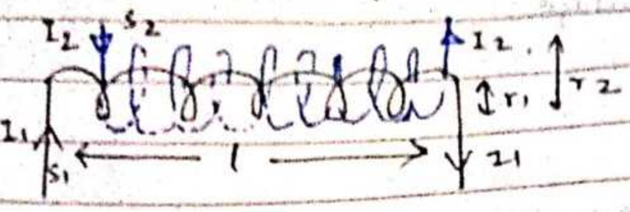
coefficient of mutual Inductance -

$$\phi \propto I, \quad \phi = MI.$$

$$\varepsilon = \frac{-d\phi}{dt} = -M \cdot \frac{dI}{dt}$$

unit of $M \rightarrow$ Henry (H)

Mutual inductance of Two solenoids



$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

Thus, the mutual inductance of two coils is the property of their combination. It does'nt matter which one of them functions as the primary or the secondry coil. This fact is known as reciprocity theorem.

Factors on which mutual inductance depend –

- $M \propto N_1 N_2$ (No. of turns)
- common area of cross section
- Relative seperation
- Relative orientation
- Permeability of the core material.

Coefficient of coupling

- It gives a measure of the manner in which two coils are coupled together. If L_1 and L_2 are the self inductance of the two coils and M is their mutual inductance, then their coefficient of coupling is

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

K lies betn 0 and 1

When there is no coupling $K=0$ and $M=0$.