Chapter 5
Friction

Introduction

If we slide or try to slide a body over a surface, the motion is resisted by a bonding between the body and the surface. This resistance is represented by a single force and is called friction force.

The force of friction is parallel to the surface and opposite to the direction of intended motion.

Types of Friction

(1) **Static friction :** The opposing force that comes into play when one body tends to move over the surface of another, but the actual motion has yet not started is called static friction.

(i) If applied force is P and the body remains at rest then static friction F = P.

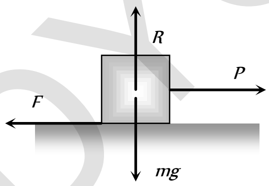


Fig. 5.1

(ii) If a body is at rest and no pulling force is acting on it, force of friction on it is zero.

(iii) Static friction is a self-adjusting force because it changes itself in accordance with the applied force and is always equal to net external force.

(2) **Limiting friction :** If the applied force is increased, the force of static friction also increases. If the applied force exceeds a certain (maximum) value, the body starts moving. This maximum value of static friction upto which body does not move is called limiting friction.

(i) The magnitude of limiting friction between any two bodies in contact is directly proportional to the normal reaction between them.

$F_l \propto R$ or $F_l = \mu_s R$

(ii) Direction of the force of limiting friction is always opposite to the direction in which one body is at the verge of moving over the other

(iii) Coefficient of static friction : (a) μ_s is called coefficient of static friction and is defined as the ratio of force of limiting friction and normal reaction $\mu_s = \dfrac{F}{R}$

(b) Dimension : $[M^0L^0T^0]$

(c) Unit : It has no unit.

(d) Value of μ depends on material and nature of surfaces in contact that means whether dry or wet ; rough or smooth polished or non-polished.

(e) Value of μ does not depend upon apparent area of contact.

(3) **Kinetic or dynamic friction :** If the applied force is increased further and sets the body in motion, the friction opposing the motion is called kinetic friction.

(i) Kinetic friction depends upon the normal reaction.

$F_k \propto R$ or $F_k = \mu_k R$ where μ_k is called the coefficient of kinetic friction

(ii) Value of μ_k depends upon the nature of surface in contact.

(iii) Kinetic friction is always lesser than limiting friction $F_k < F_l$

$\therefore \ \mu_k < \mu_s$

i.e. coefficient of kinetic friction is always less than coefficient of static friction. Thus we require more force to start a motion than to maintain it against friction. This is because once the motion starts actually ; inertia of rest has been overcome. Also when motion has actually started, irregularities of one surface have little time to get locked again into the irregularities of the other surface.

(iv) Kinetic friction does not depend upon the velocity of the body.

(v) Types of kinetic friction

(a) **Sliding friction** : The opposing force that comes into play when one body is actually sliding over the surface of the other body is called sliding friction. *e.g.* A flat block is moving over a horizontal table.

(b) **Rolling friction** : When objects such as a wheel (disc or ring), sphere or a cylinder rolls over a surface, the force of friction that comes into play is called rolling friction.

❑ Rolling friction is directly proportional to the normal reaction (*R*) and inversely proportional to the radius (*r*) of the rolling cylinder or wheel.

$$F_{rolling} = \mu_r \frac{R}{r}$$

μ_r is called coefficient of rolling friction. It would have the dimensions of length and would be measured in *metre*.

❑ Rolling friction is often quite small as compared to the sliding friction. That is why heavy loads are transported by placing them on carts with wheels.

❑ In rolling the surfaces at contact do not rub each other.

❑ The velocity of point of contact with respect to the surface remains zero all the times although the centre of the wheel moves forward.

Graph Between Applied Force and Force of Friction

(1) Part *OA* of the curve represents static friction (F_s). Its value increases linearly with the applied force

(2) At point *A* the static friction is maximum. This represent limiting friction (F_l).

(3) Beyond *A*, the force of friction is seen to decrease slightly. The portion *BC* of the curve represents the kinetic friction (F_k).

(4) As the portion *BC* of the curve is parallel to *x*-axis therefore kinetic friction does not change with the applied force, it remains constant, whatever be the applied force.

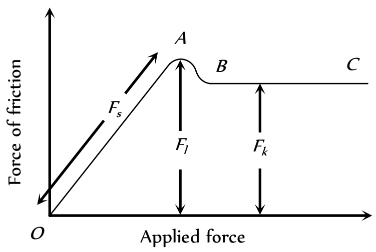


Fig. 5.2

Friction is a Cause of Motion

It is a general misconception that friction always opposes the motion. No doubt friction opposes the motion of a moving body but in many cases it is also the cause of motion. For example :

(1) While moving, a person or vehicle pushes the ground backwards (action) and the rough surface of ground reacts and exerts a forward force due to friction which causes the motion. If there had been no friction there will be slipping and no motion.

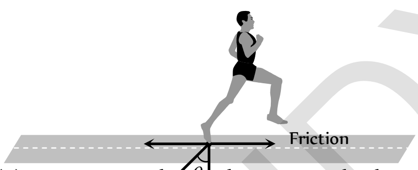


Fig. 5.3

(2) During cycling, the rear wheel moves by the force communicated to it by pedalling while front wheel moves by itself. So, when pedalling a bicycle, the force exerted by rear wheel on ground makes force of friction act on it in the forward direction (like walking). Front wheel moving by itself experience force of friction in backward direction (like rolling of a ball). [However, if pedalling is stopped both wheels move by themselves and so experience force of friction in backward direction].

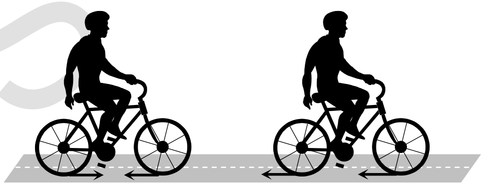


Fig. 5.4

(3) If a body is placed in a vehicle which is accelerating, the force of friction is the cause of motion of the body along with the vehicle (*i.e.*, the body will remain at rest in the accelerating vehicle until $ma < \mu_s mg$). If there had been no friction between body and vehicle, the body will not move along with the vehicle.

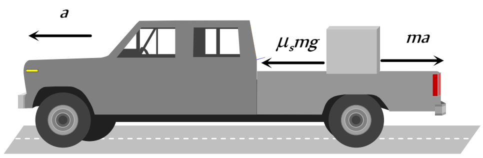


Fig. 5.5

From these examples it is clear that without friction motion cannot be started, stopped or transferred from one body to the other.

Advantages and Disadvantages of Friction

(1) **Advantages of friction**

(i) Walking is possible due to friction.

(ii) Two body sticks together due to friction.

Fig. 5.6

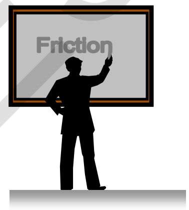


Fig. 5.7

(iii) Brake works on the basis of friction.

(iv) Writing is not possible without friction.

(v) The transfer of motion from one part of a machine to other part through belts is possible by friction.

(2) **Disadvantages of friction**

(i) Friction always opposes the relative motion between any two bodies in contact. Therefore extra energy has to be spent in over coming friction. This reduces the efficiency of machine.

(ii) Friction causes wear and tear of the parts of machinery in contact. Thus their lifetime reduces.

(iii) Frictional force result in the production of heat, which causes damage to the machinery.

Methods of Changing Friction

We can reduce friction

(1) By polishing.

(2) By lubrication.

(3) By proper selection of material.

(4) By streamlining the shape of the body.

(5) By using ball bearing.

Also we can increase friction by throwing some sand on slippery ground. In the manufacturing of tyres, synthetic rubber is preferred because its coefficient of friction with the road is larger.

Angle of Friction

Angle of friction may be defined as the angle which the resultant of limiting friction and normal reaction makes with the normal reaction.

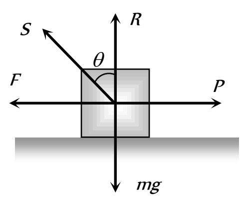


Fig. 5.8

By definition angle θ is called the angle of friction

$$\tan\theta = \frac{F_l}{R}$$

$\therefore$ $\tan\theta = \mu$ [As we know $\frac{F_l}{R} = \mu_s$]

or $\theta = \tan^{-1}(\mu_L)$

Hence coefficient of static friction is equal to tangent of the angle of friction.

Resultant Force Exerted by Surface on Block

In the above figure resultant force $S = \sqrt{F^2 + R^2}$

$$S = \sqrt{(\mu mg)^2 + (mg)^2}$$

$$S = mg\sqrt{\mu^2 + 1}$$

when there is no friction $(\mu = 0)$ S will be minimum

i.e. $S = mg$

Hence the range of S can be given by,

$$mg \le S \le mg\sqrt{\mu^2 + 1}$$

Angle of Repose

Angle of repose is defined as the angle of the inclined plane with horizontal such that a body placed on it is just begins to slide.

By definition, α is called the angle of repose.

In limiting condition $F = mg\sin\alpha$ and $R = mg\cos\alpha$

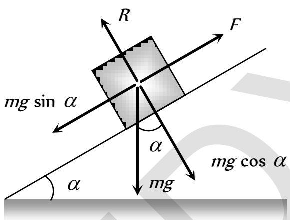


Fig. 5.9

So $\frac{F}{R} = \tan\alpha$

$\therefore$ $\frac{F}{R} = \mu_s = \tan\theta = \tan\alpha$ [As we know $\frac{F}{R} = \mu_s = \tan\theta$]

Thus the coefficient of limiting friction is equal to the tangent of angle of repose.

As well as $\alpha = \theta$ *i.e.* angle of repose = angle of friction.

Calculation of Required Force in Different Situation

If W = weight of the body, θ = angle of friction, $\mu = \tan\theta =$ coefficient of friction

Then we can calculate required force for different situation in the following manner :

(1) **Minimum pulling force P at an angle α from the horizontal**

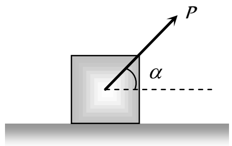


Fig. 5.10

By resolving P in horizontal and vertical direction (as shown in figure)

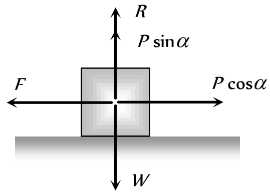


Fig. 5.11

For the condition of equilibrium

$F = P\cos\alpha$ and $R = W - P\sin\alpha$

By substituting these value in $F = \mu R$

$P\cos\alpha = \mu(W - P\sin\alpha)$

$\Rightarrow$ $P\cos\alpha = \frac{\sin\theta}{\cos\theta}(W - P\sin\alpha)$ [As $\mu = \tan\theta$]

$\Rightarrow$ $P = \frac{W\sin\theta}{\cos(\alpha - \theta)}$

(2) **Minimum pushing force P at an angle α from the horizontal**

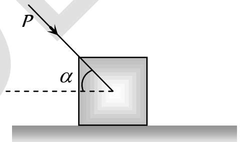


Fig. 5.12

By Resolving P in horizontal and vertical direction (as shown in the figure)

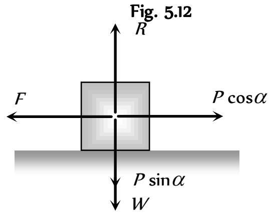


Fig. 5.13

For the condition of equilibrium

$F = P\cos\alpha$ and $R = W + P\sin\alpha$

By substituting these value in $F = \mu R$

$\Rightarrow$ $P\cos\alpha = \mu(W + P\sin\alpha)$

$\Rightarrow$ $P\cos\alpha = \frac{\sin\theta}{\cos\theta}(W + P\sin\alpha)$ [As $\mu = \tan\theta$]

$\Rightarrow$ $P = \frac{W\sin\theta}{\cos(\alpha + \theta)}$

(3) **Minimum pulling force P to move the body up on an inclined plane**

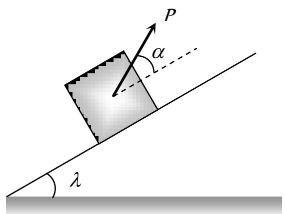


Fig. 5.14

By Resolving P in the direction of the plane and perpendicular to the plane (as shown in the figure)

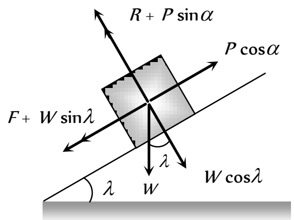


Fig. 5.15

For the condition of equilibrium

$$R + P\sin\alpha = W\cos\lambda$$

$\therefore \quad R = W\cos\lambda - P\sin\alpha$ and $F + W\sin\lambda = P\cos\alpha$

$\therefore \quad F = P\cos\alpha - W\sin\lambda$

By substituting these values in $F = \mu R$ and solving we get

$$P = \frac{W\sin(\theta+\lambda)}{\cos(\alpha-\theta)}$$

(4) **Minimum force to move a body in downward direction along the surface of inclined plane**

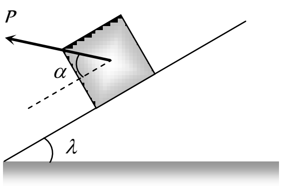


Fig. 5.16

By Resolving P in the direction of the plane and perpendicular to the plane (as shown in the figure)

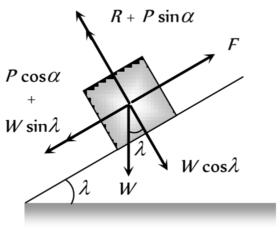


Fig. 5.17

For the condition of equilibrium

$$R + P\sin\alpha = W\cos\lambda$$

$\therefore \quad R = W\cos\lambda - P\sin\alpha$ and $F = P\cos\alpha + W\sin\lambda$

By substituting these values in $F = \mu R$ and solving we get

$$P = \frac{W\sin(\theta-\lambda)}{\cos(\alpha-\theta)}$$

(5) **Minimum force to avoid sliding of a body down on an inclined plane**

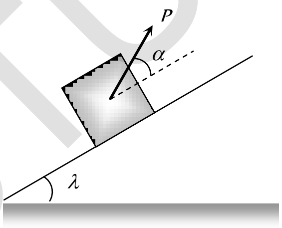


Fig. 5.18

By Resolving P in the direction of the plane and perpendicular to the plane (as shown in the figure)

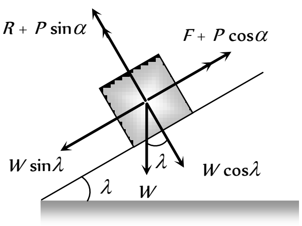


Fig. 5.19

For the condition of equilibrium

$$R + P\sin\alpha = W\cos\lambda$$

$\therefore \quad R = W\cos\lambda - P\sin\alpha$ and $P\cos\alpha + F = W\sin\lambda$

$\therefore \quad F = W\sin\lambda - P\cos\alpha$

By substituting these values in $F = \mu R$ and solving we get

$$P = W\left[\frac{\sin(\lambda-\theta)}{\cos(\theta+\alpha)}\right]$$

(6) **Minimum force for motion along horizontal surface and its direction**

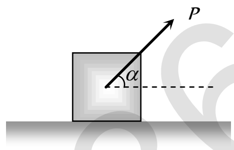


Fig. 5.20

Let the force P be applied at an angle α with the horizontal.

By resolving P in horizontal and vertical direction (as shown in figure)

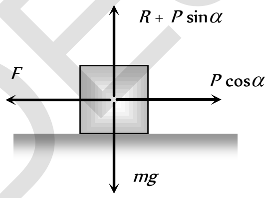


Fig. 5.21

For vertical equilibrium

$$R + P\sin\alpha = mg$$

$\therefore \quad R = mg - P\sin\alpha$ ...(i)

and for horizontal motion

$$P\cos\alpha \geq F$$

i.e. $P\cos\alpha \geq \mu R$ ...(ii)

Substituting value of R from (i) in (ii)

$$P\cos\alpha \geq \mu(mg - P\sin\alpha)$$

$$P \geq \frac{\mu mg}{\cos\alpha + \mu\sin\alpha} \quad \text{...(iii)}$$

For the force P to be minimum $(\cos\alpha + \mu\sin\alpha)$ must be maximum *i.e.*

$$\frac{d}{d\alpha}[\cos\alpha + \mu\sin\alpha] = 0$$

$\Rightarrow \quad -\sin\alpha + \mu\cos\alpha = 0$

$\therefore \quad \tan\alpha = \mu$

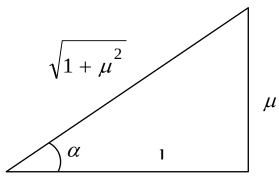


Fig. 5.22

or $\quad \alpha = \tan^{-1}(\mu) = \text{angle of friction}$

i.e. For minimum value of P its angle from the horizontal should be equal to angle of friction

As $\tan\alpha = \mu$ so from the figure, $\sin\alpha = \dfrac{\mu}{\sqrt{1+\mu^2}}$

and $\cos\alpha = \dfrac{1}{\sqrt{1+\mu^2}}$

By substituting these value in equation (iii)

$$P \geq \frac{\mu mg}{\frac{1}{\sqrt{1+\mu^2}} + \frac{\mu^2}{\sqrt{1+\mu^2}}} \geq \frac{\mu mg}{\sqrt{1+\mu^2}}$$

$$\therefore \quad P_{\min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

Acceleration of a Block Against Friction

(1) **Acceleration of a block on horizontal surface**

When body is moving under application of force *P*, then kinetic friction opposes its motion.

Let a is the net acceleration of the body

From the figure

$ma = P - F_k$

$\therefore \ a = \dfrac{P - F_k}{m}$

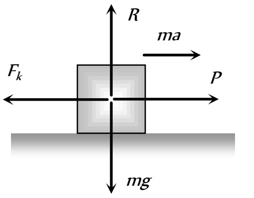


Fig. 5.23

(2) **Acceleration of a block sliding down over a rough inclined plane**

When angle of inclined plane is more than angle of repose, the body placed on the inclined plane slides down with an acceleration *a*.

From the figure $ma = mg\sin\theta - F$

$\Rightarrow \ ma = mg\sin\theta - \mu R$

$\Rightarrow \ ma = mg\sin\theta - \mu mg\cos\theta$

$\therefore$ Acceleration $a = g[\sin\theta - \mu\cos\theta]$

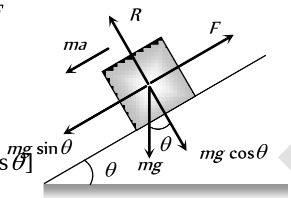


Fig. 5.24

Note : ❑ For frictionless inclined plane $\mu = 0 \ \therefore a = g\sin\theta$.

(3) **Retardation of a block sliding up over a rough inclined plane**

When angle of inclined plane is less than angle of repose, then for the upward motion

$ma = mg\sin\theta + F$

$ma = mg\sin\theta + \mu mg\cos\theta$

Retardation $a = g[\sin\theta + \mu\cos\theta]$

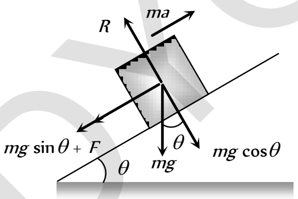


Fig. 5.25

Note : ❑ For frictionless inclined plane $\mu = 0 \ \therefore a = g\sin\theta$

Work done against friction

(1) **Work done over a rough inclined surface**

If a body of mass *m* is moved up slowly on a rough inclined plane through distance *s*, then

Work done = force × distance

= *ma* × *s* = *mg* $[\sin\theta + \mu\cos\theta]s = mg\,s[\sin\theta + \mu\cos\theta]$

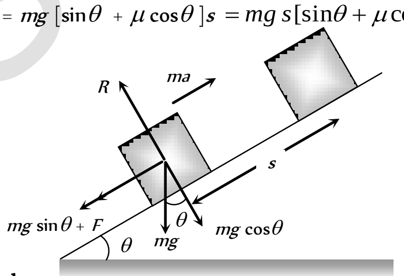


Fig. 5.26

(2) **Work done over a horizontal surface**

In the above expression if we put $\theta = 0$ then

Work done = force × distance = $F \times s = \mu\, mg\, s$

It is clear that work done depends upon

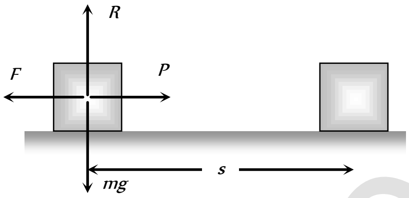


Fig. 5.27

(i) Weight of the body.

(ii) Material and nature of surface in contact.

(iii) Distance moved.

Motion of Two Bodies one Resting on the Other

When a body *A* of mass *m* is resting on a body *B* of mass *M* then two conditions are possible

(1) A force *F* is applied to the upper body, (2) A force *F* is applied to the lower body

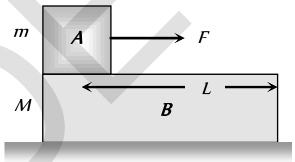


Fig. 5.28

We will discuss above two cases one by one in the following manner :

(1) **A force *F* is applied to the upper body, then following four situations are possible**

(i) **When there is no friction**

(a) The body *A* will move on body *B* with acceleration (*F*/*m*).

$$a_A = F/m$$

(b) The body *B* will remain at rest

$$a_B = 0$$

(c) If *L* is the length of *B* as shown in figure, *A* will fall from *B* after time *t*

$$t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mL}{F}} \qquad \left[\text{As } s = \frac{1}{2}at^2 \text{ and } a = F/m\right]$$

(ii) **If friction is present between *A* and *B* only and applied force is less than limiting friction ($F < F_l$)**

(F = Applied force on the upper body, F_l = limiting friction between *A* and *B*, F_k = Kinetic friction between *A* and *B*)

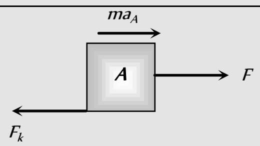
(a) The body *A* will not slide on body *B* till $F < F_l$ *i.e.* $F < \mu_s mg$

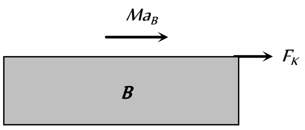
(b) Combined system (*m* + *M*) will move together with common acceleration $a_A = a_B = \dfrac{F}{M+m}$

(iii) **If friction is present between *A* and *B* only and applied force is greater than limiting friction ($F > F_l$)**

In this condition the two bodies will move in the same direction (*i.e.* of applied force) but with different acceleration. Here force of kinetic friction $\mu_k mg$ will oppose the motion of *A* while cause the motion of *B*.

$F - F_k = m a_A$	Free body diagram of *A*



i.e. $a_A = \dfrac{F - F_k}{m}$	
$a_A = \dfrac{(F - \mu_k mg)}{m}$	
$F_k = M a_B$	Free body diagram of *B*
i.e. $a_B = \dfrac{F_k}{M}$	
$\therefore$ $a_B = \dfrac{\mu_k mg}{M}$	

Note : ❑ As both the bodies are moving in the same direction.

Acceleration of body *A* relative to *B* will be

$$a = a_A - a_B = \frac{MF - \mu_k mg\,(m+M)}{mM}$$

So, A will fall from *B* after time

$$t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2\,m\,ML}{MF - \mu_k mg\,(m+M)}}$$

(iv) **If there is friction between *B* and floor**

(where $F_l' = \mu'(M+m)g$ = limiting friction between *B* and floor, F_k = kinetic friction between *A* and *B*)

B will move only if $F_k > F_l'$ and then $F_k - F_l' = M\,a_B$

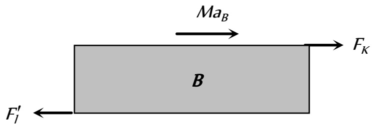


Fig. 5.29

However if *B* does not move then static friction will work (not limiting friction) between body *B* and the floor *i.e.* friction force = applied force (= *F*) not F_l'.

(2) **A force *F* is applied to the lower body, then following four situations are possible**

(i) **When there is no friction**

(a) *B* will move with acceleration (*F*/*M*) while *A* will remain at rest (relative to ground) as there is no pulling force on *A*.

$$a_B = \left(\frac{F}{M}\right) \text{ and } a_A = 0$$

(b) As relative to *B*, *A* will move backwards with acceleration (*F*/*M*) and so will fall from it in time *t*.

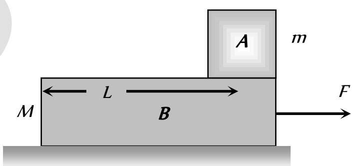


Fig. 5.30

$$\therefore \quad t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2ML}{F}}$$

(ii) **If friction is present between *A* and *B* only and $F' < F_l$**

(where F' = Pseudo force on body *A* and F_l = limiting friction between body *A* and *B*)

(a) Both the body will move together with common acceleration $a = \dfrac{F}{M+m}$

(b) Pseudo force on the body *A*,

$$F' = ma = \frac{mF}{m+M} \text{ and } F_l = \mu_s mg$$

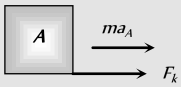
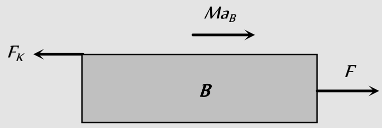
(c) $F' < F_l \Rightarrow \dfrac{mF}{m+M} < \mu_s mg \Rightarrow F < \mu_s (m+M)g$

So both bodies will move together with acceleration $a_A = a_B = \dfrac{F}{m+M}$ if $F < \mu_s[m+M]g$

(iii) **If friction is present between *A* and *B* only and $F > F_l'$**

(where $F_l' = \mu_s mg$ = limiting friction between body *A* and *B*)

Both the body will move with different acceleration. Here force of kinetic friction $\mu_k mg$ will oppose the motion of *B* while will cause the motion of *A*.

$ma_A = \mu_k mg$	Free body diagram of *A*
i.e. $a_A = \mu_k g$	
$F - F_k = Ma_B$	Free body diagram of *B*
i.e. $a_B = \dfrac{[F - \mu_k mg]}{M}$	

Note : ❑ As both the bodies are moving in the same direction

Acceleration of body *A* relative to *B* will be

$$a = a_A - a_B = -\left[\frac{F - \mu_k g(m+M)}{M}\right]$$

Negative sign implies that relative to *B*, A will move backwards and will fall it after time

$$t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2ML}{F - \mu_k g(m+M)}}$$

(iv) **If there is friction between *B* and floor and $F > F_l''$:**

(where F_l'' = $\mu(m+M)g$ = limiting friction between body *B* and surface)

The system will move only if $F > F_l''$ then replacing *F* by $F - F_l''$. The entire case (iii) will be valid.

However if $F < F_1''$ the system will not move and friction between *B* and floor will be *F* while between *A* and *B* is zero.

Motion of an Insect in the Rough Bowl

The insect crawl up the bowl, up to a certain height h only till the component of its weight along the bowl is balanced by limiting frictional force.

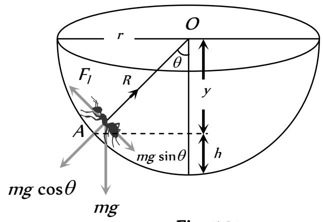


Fig. 5.31

Let m = mass of the insect, r = radius of the bowl, μ = coefficient of friction

for limiting condition at point A

$R = mg\cos\theta$ ......(i) and $F_l = mg\sin\theta$ ......(ii)

Dividing (ii) by (i)

$$\tan\theta = \frac{F_l}{R} = \mu \qquad [\text{As } F_l = \mu R]$$

$$\therefore \frac{\sqrt{r^2 - y^2}}{y} = \mu \quad \text{or} \quad y = \frac{r}{\sqrt{1+\mu^2}}$$

So $h = r - y = r\left[1 - \frac{1}{\sqrt{1+\mu^2}}\right]$, $\therefore\ h = r\left[1 - \frac{1}{\sqrt{1+\mu^2}}\right]$

Minimum Mass Hung from the String to Just Start the Motion

(1) **When a mass m_1 placed on a rough horizontal plane** Another mass m_2 hung from the string connected by frictionless pulley, the tension (T) produced in string will try to start the motion of mass m_1.

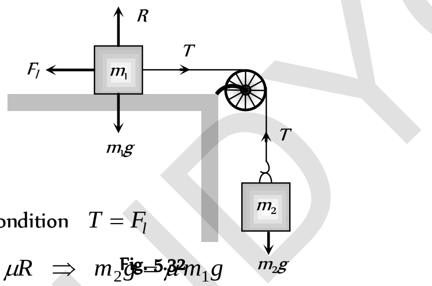


Fig. 5.32

At limiting condition $T = F_l$

$\Rightarrow\ m_2 g = \mu R \ \Rightarrow\ m_2 g = \mu m_1 g$

$\therefore\ m_2 = \mu m_1$ this is the minimum value of m_2 to start the motion.

Note : ❑ In the above condition Coefficient of friction $\mu = \dfrac{m_2}{m_1}$

(2) **When a mass m_1 placed on a rough inclined plane** Another mass m_2 hung from the string connected by frictionless pulley, the tension (T) produced in string will try to start the motion of mass m_1.

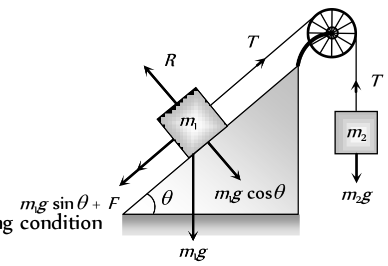


Fig. 5.33

At limiting condition

For m_2 $\quad T = m_2 g$ ...(i)

For m_1 $\quad T = m_1 g\sin\theta + F$

$\Rightarrow\ T = m_1 g\sin\theta + \mu R$

$\Rightarrow\ T = m_1 g\sin\theta + \mu m_1 g\cos\theta$ ...(ii)

From equation (i) and (ii) $\quad m_2 = m_1[\sin\theta + \mu\cos\theta]$

this is the minimum value of m_2 to start the motion

Note : ❑ In the above condition Coefficient of friction

$$\mu = \left[\frac{m_2}{m_1\cos\theta} - \tan\theta\right]$$

Maximum Length of Hung Chain

A uniform chain of length l is placed on the table in such a manner that its l' part is hanging over the edge of table without sliding. Since the chain have uniform linear density therefore the ratio of mass and ratio of length for any part of the chain will be equal.

We know $\quad \mu = \dfrac{m_2}{m_1} = \dfrac{\text{mass hanging from the table}}{\text{mass lying on the table}}$

$\therefore$ For this case we can rewrite above expression in the following manner

$$\mu = \frac{\text{length hanging from the table}}{\text{length lying on the table}}$$ [As chain have uniform linear density]

$\therefore \quad \mu = \dfrac{l'}{l - l'}$

by solving $\quad l' = \dfrac{\mu l}{(\mu + 1)}$

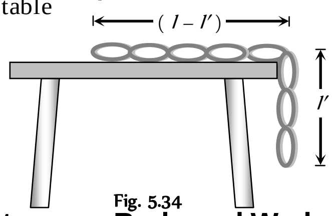


Fig. 5.34

Coefficient of Friction Between a Body and Wedge

A body slides on a smooth wedge of angle θ and its time of descent is t.

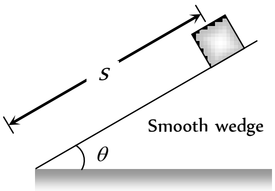


Fig. 5.35

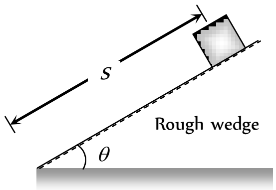


Fig. 5.36

If the same wedge made rough then time taken by it to come down becomes n times more (*i.e.* nt)

The length of path in both the cases are same.

For smooth wedge, $S = ut + \frac{1}{2}at^2$

$$S = \frac{1}{2}(g\sin\theta)t^2 \qquad \text{...(i)}$$

$$[\text{As } u = 0 \text{ and } a = g\sin\theta]$$

For rough wedge, $S = ut + \frac{1}{2}at^2$

$$S = \frac{1}{2}g(\sin\theta - \mu\cos\theta)(nt)^2 \qquad \text{...(ii)}$$

$$[\text{As } u = 0 \text{ and } a = g(\sin\theta - \mu\cos\theta)]$$

From equation (i) and (ii)

$$\frac{1}{2}(g\sin\theta)t^2 = \frac{1}{2}g(\sin\theta - \mu\cos\theta)(nt)^2$$

$$\Rightarrow \sin\theta = (\sin\theta - \mu\cos\theta)n^2$$

$$\Rightarrow \mu = \tan\theta\left[1 - \frac{1}{n^2}\right]$$

Stopping of Block Due to Friction

(1) **On horizontal road**

(i) **Distance travelled before coming to rest :** A block of mass m is moving initially with velocity u on a rough surface and due to friction, it comes to rest after covering a distance S.

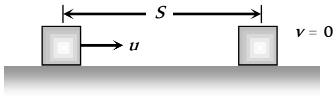


Fig. 5.37

Retarding force $F = ma = \mu R \Rightarrow ma = \mu mg$

$\therefore a = \mu g$

From $v^2 = u^2 - 2aS \Rightarrow 0 = u^2 - 2\mu g S$

[As $v = 0,\ a = \mu g$]

$$\therefore \quad S = \frac{u^2}{2\mu g} \quad \text{or} \quad S = \frac{P^2}{2\mu m^2 g}$$

[As momentum $P = mu$]

(ii) **Time taken to come to rest**

From equation $v = u - at \Rightarrow 0 = u - \mu g t$

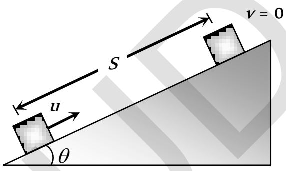
[As $v = 0, a = \mu g$]

$$\therefore \quad t = \frac{u}{\mu g}$$

(2) **On inclined road :** When block starts with velocity u its kinetic energy will be converted into potential energy and some part of it goes against friction and after travelling distance S it comes to rest *i.e.* $v = 0$.

We know that retardation $a = g[\sin\theta + \mu\cos\theta]$

By substituting the value of v and a in the following equation



$v^2 = u^2 - 2aS$

Fig. 5.38

$$\Rightarrow \quad 0 = u^2 - 2g[\sin\theta + \mu\cos\theta]S$$

$$\therefore \quad S = \frac{u^2}{2g(\sin\theta + \mu\cos\theta)}$$

Stopping of Two Blocks Due to Friction

When two masses compressed towards each other and suddenly released then energy acquired by each block will be dissipated against friction and finally block comes to rest

i.e., $F \times S = E$ [Where F = Friction, S = Distance covered by block, E = Initial kinetic energy of the block]

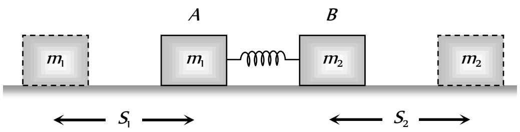


Fig. 5.39

$$\Rightarrow \quad F \times S = \frac{P^2}{2m} \qquad \text{[Where } P = \text{momentum of block]}$$

$$\Rightarrow \quad \mu mg \times S = \frac{P^2}{2m} \qquad \text{[As } F = \mu\, mg]$$

$$\Rightarrow \quad S = \frac{P^2}{2\mu m^2 g}$$

In the given condition P and μ are same for both the blocks.

So, $S \propto \dfrac{1}{m^2}$; $\therefore \dfrac{S_1}{S_2} = \left[\dfrac{m_2}{m_1}\right]^2$

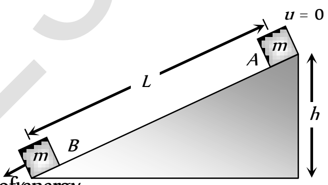
Velocity at the Bottom of Rough Wedge

A body of mass m which is placed at the top of the wedge (of height h) starts moving downward on a rough inclined plane.

Loss of energy due to friction = FL (Work against friction)

PE at point $A = mgh$

KE at point $B = \dfrac{1}{2}mu^2$



By the law of conservation of energy

Fig. 5.40

i.e. $\dfrac{1}{2}mv^2 = mgh - FL$

$$v = \sqrt{\frac{2}{m}(mgh - FL)}$$

Sticking of a Block With Accelerated Cart

When a cart moves with some acceleration toward right then a pseudo force (ma) acts on block toward left.

This force (ma) is action force by a block on cart.

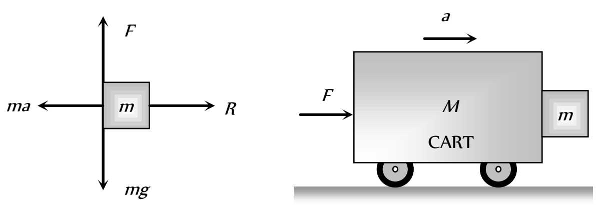


Fig. 5.41

Now block will remain static *w.r.t.* cart. If friction force $\mu R \geq mg$

$\Rightarrow \mu ma \geq mg$ [As $R = ma$]

$$\Rightarrow \quad a \geq \frac{g}{\mu}$$

$$\therefore \quad a_{\min} = \frac{g}{\mu}$$

This is the minimum acceleration of the cart so that block does not fall.

and the minimum force to hold the block together

$$F_{\min} = (M + m)a_{\min}$$

$$F_{\min} = (M + m)\frac{g}{\mu}$$

Sticking of a Person with the Wall of Rotor

A person with a mass m stands in contact against the wall of a cylindrical drum (rotor). The coefficient of friction between the wall and the clothing is μ.

If Rotor starts rotating about its axis, then person thrown away from the centre due to centrifugal force at a particular speed ω, the person stuck to the wall even the floor is removed, because friction force balances its weight in this condition.

From the figure.

Friction force (F) = weight of person (mg)

$\Rightarrow \mu R = mg \Rightarrow \mu F_c = mg$

[Here, F_c = centrifugal force]

$\Rightarrow \mu m\omega_{\min}^2 r = mg$

$\therefore \omega_{\min} = \sqrt{\dfrac{g}{\mu r}}$

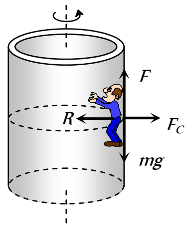


Fig. 5.42

Tips & Tricks

✍ Force of friction is non-conservative force.

✍ Force of friction always acts in a direction opposite to that of the relative motion between the surfaces.

✍ Rolling friction is much less than the sliding friction. This knowledge was used by man to invent the wheels.

✍ The friction between two surfaces increases (rather than to decrease), when the surfaces are made highly smooth.

✍ The atomic and molecular forces of attraction between the two surfaces at the point of contact give rise to friction between the surfaces.