Chapter
9
Elasticity

Interatomic Forces

The forces between the atoms due to electrostatic interaction between the charges of the atoms are called interatomic forces. These forces are electrical in nature and these are active if the distance between the two atoms is of the order of atomic size *i.e.* 10^{-} *metre*.

(1) Every atom is electrically neutral, the number of electrons (negative charge) orbiting around the nucleus is equal to the number of protons (positive charge) in the nucleus. So if two atoms are placed at a very large distance from each other then there will be a very small (negligible) interatomic force working between them.

(2) When two atoms are brought closer to each other to a distance of the order of 10^{-} *m*, the distances between their positive nuclei and negative electron clouds get disturbed, and due to this, attractive interatomic force is produced between two atoms.

(3) This attractive force increases continuously with decrease in r and becomes maximum for one value of r called critical distance, represented by x (as shown in the figure). Beyond this the attractive force starts decreasing rapidly with further decrease in the value of r.

(4) When the distance between the two atoms becomes r_0, the interatomic force will be zero. This distance r_0 is called normal or equilibrium distance.

(r_0 = 0.74 Å for hydrogen).

(5) When the distance between the two atoms further decreased, the interatomic force becomes repulsive in nature and increases very rapidly with decrease in distance between two atoms.

(6) The potential energy U is related with the interatomic force F by the following relation.

$$F = \frac{-dU}{dr}$$

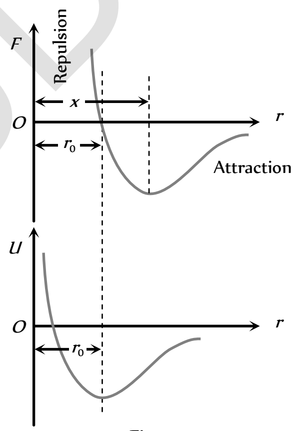


Fig. 9.1

(i) When two atoms are at very large distance, the potential energy is negative and becomes more negative as r is decreased.

(ii) When the distance between the two atoms becomes r_0, the potential energy of the system of two atoms becomes minimum (*i.e.* attains maximum negative value). As the state of minimum potential energy is the state of equilibrium, hence the two atoms at separation r_0 will be in a state of equilibrium.

($U_0 = -7.2 \times 10^{-19}$ *Joule* for hydrogen).

(iii) When the distance between the two atoms is further decreased (*i.e.* $r < r_0$) the negative value of potential energy of the system starts decreasing. It becomes zero and then attains positive value with further decrease in r (as shown in the figure).

Intermolecular Forces

The forces between the molecules due to electrostatic interaction between the charges of the molecules are called intermolecular forces. These forces are also called Vander Waal forces and are quite weak as compared to inter-atomic forces. These forces are also electrical in nature and these are active if the separation between two molecules is of the order of molecular size *i.e.* $\approx 10^{-}$ *m*.

(1) It is found that the force of attraction between molecules varies inversely as seventh power of the distance between them *i.e.*

$$F_{att} \propto \frac{1}{r^7} \quad \text{or} \quad F_{rep} = \frac{-a}{r^7}$$

The negative sign indicates that the force is attractive in nature.

(2) When the distance between molecules becomes less than r_0, the forces becomes repulsive in nature and is found to vary inversely as ninth power of the distance between them *i.e.*

$$F_{rep} \propto \frac{1}{r^9} \quad \text{or} \quad F_{rep} = \frac{b}{r^9}.$$

Therefore force between two molecules is given by

$$F = F_{att} + F_{rep} = \frac{-a}{r^7} + \frac{b}{r^9}$$

The value of constants a and b depend upon the structure and nature of molecules.

(3) Intermolecular forces between two molecules has the same general nature as shown in the figure for interatomic forces.

(4) Potential Energy : Potential energy can be approximately expressed by the formula $U = \frac{A}{r^n} - \frac{B}{r^m}$

where the term $\frac{A}{r^n}$ represents repulsive contribution and term $\frac{B}{r^m}$ represents the attractive contribution. Constants *A*, *B* and numbers *m* and *n* are different for different molecules.

For majority of solids $n = 12$ and $m = 6$.

So potential energy can be expressed as $U = \frac{A}{r^{12}} - \frac{B}{r^6}$

Comparison Between Interatomic and Intermolecular Forces

(1) **Similarities**

(i) Both the forces are electrical in origin.

(ii) Both the forces are active over short distances.

(iii) General shape of force-distance graph is similar for both the forces.

(iv) Both the forces are attractive up to certain distance between atoms/molecules and become repulsive when the distance between them become less than that value.

(2) **Dissimilarities**

(i) Interatomic force depends upon the distance between the two atoms, whereas the intermolecular force depends upon the distance between the two molecules as well as their relative orientation.

(ii) Interatomic forces are about 50 to100 times stronger than intermolecular forces.

(iii) The value of r_0 for two atoms is smaller than the corresponding value for the molecules. Therefore one molecule is not restricted to attract only one molecule, but can attract many molecule. It is not so incase of atoms, since the atoms of one molecule cannot bind the atoms of other molecules.

States of Matter

The three states of matter differ from each other due to the following two factors.

(1) The different magnitudes of the interatomic and intermolecular forces.

(2) The extent of random thermal motion of atoms and molecules of a substance (which depends upon temperature).

Comparison Chart of Solid, Liquid and Gaseous States

Property	Solid	Liquid	Gas
Shape	Definite	Not definite	Not definite
Volume	Definite	Definite	Not definite
Density	Maximum	Less than solids but more than gases.	Minimum
Compressibility	Incompressible	Less than gases but more than solids.	Compressible
Crystallinity	Crystalline	Non-crystalline	
Interatomic or intermolecular distance	Constant	Not constant	Not constant
Relation between kinetic energy *K* and potential energy (*U*)	$K < U$	$K > U$	$K >> U$
Intermolecular force	Strongest	Less than solids but more than gases.	Weakest
Freedom of motion	Molecules vibrate about their mean position but cannot move freely.	Molecules have limited free motion.	Molecules are free to move.
Effect of temperature	Matter remains in solid form below a certain temperature.	Liquids are found at temperatures more than that of solid.	These are found at temperatures greater than that of solids and liquids.

Note : ❑ The fourth state of matter in which the medium is in the form of positive and negative ions, is known as plasma. Plasma occurs in the atmosphere of stars (including the sun) and in discharge tubes.

Types of Solids

A solid is that state of matter in which its constituent atoms or molecules are held strongly at the position of minimum potential energy and it has a definite shape and volume. The solids can be classified into two categories, crystalline and glassy or amorphous solids.

Comparison chart of Crystalline and Amorphous Solids

Crystalline solids	Amorphous or glassy solids
The constituent atoms, ions or molecules are arranged in a regular repeated three dimensional pattern, within the	The constituent atoms, ions or molecules are not arranged in a regular repeated three dimensional pattern, within the

solid.	solid.
Definite external geometric shape.	No regularity in external shape.
All the bonds in ions, or atoms or molecules are equally strong.	All the bonds are not equally strong.
They are anisotropic.	They are isotropic.
They have sharp melting point.	They don't have sharp melting point.
They have a long-range order of atoms or ions or molecules in them.	They don't have a long-range order.
They are considered true and stable solids.	They are not regarded as true and stable solids.

Elastic Property of Matter

(1) **Elasticity :** The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming force is called elasticity.

(2) **Plasticity :** The property of matter by virtue of which it does not regain its original shape and size after the removal of deforming force is called plasticity.

(3) **Perfectly elastic body :** If on the removal of deforming forces the body regain its original configuration completely it is said to be perfectly elastic.

A quartz fibre and phosphor bronze (an alloy of copper containing 4% to 10% tin, 0.05% to 1% phosphorus) is the nearest approach to the perfectly elastic body.

(4) **Perfectly plastic body :** If the body does not have any tendency to recover its original configuration, on the removal of deforming force, it is said to be perfectly plastic.

Paraffin wax, wet clay are the nearest approach to the perfectly plastic body.

Practically there is no material which is either perfectly elastic or perfectly plastic and the behaviour of actual bodies lies between the two extremes.

(5) **Reason of elasticity :** In a solids, atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to neighbouring molecules. These forces are known as intermolecular forces.

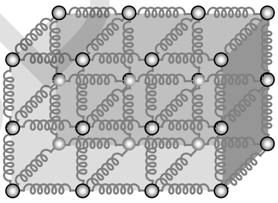


Fig. 9.2

For simplicity, the two molecules in their equilibrium positions (at inter-molecular distance $r = r$) are shown by connecting them with a spring.

In fact, the spring connecting the two molecules represents the inter-molecular force between them. On applying the deforming forces, the molecules either come closer or go far apart from each other and restoring forces are developed. When the deforming force is removed, these restoring forces bring the molecules of the solid to their respective equilibrium position ($r = r$) and hence the body regains its original form.

(6) **Elastic limit :** Elastic bodies show their property of elasticity upto a certain value of deforming force. If we go on increasing the deforming force then a stage is reached when on removing the force, the body will not return to its original state. The maximum deforming force upto which a body retains its property of elasticity is called elastic limit of the material of body.

Elastic limit is the property of a body whereas elasticity is the property of material of the body.

(7) **Elastic fatigue :** The temporary loss of elastic properties because of the action of repeated alternating deforming force is called elastic fatigue.

Due to elastic fatigue :

(i) Bridges are declared unsafe after a long time of their use.

(ii) Spring balances show wrong readings after they have been used for a long time.

(iii) We are able to break the wire by repeated bending.

(8) **Elastic after effect :** The time delay in which the substance regains its original condition after the removal of deforming force is called elastic after effect. It is the time for which restoring forces are present after the removal of the deforming force, it is negligible for perfectly elastic substance, like quartz, phosphor bronze and large for glass fibre.

Stress

When a force is applied on a body, there will be relative displacement of the particles and due to property of elasticity, an internal restoring force is developed which tends to restore the body to its original state.

The internal restoring force acting per unit area of cross section of the deformed body is called stress.

At equilibrium, restoring force is equal in magnitude to external force, stress can therefore also be defined as external force per unit area on a body that tends to cause it to deform.

If external force F is applied on the area A of a body then,

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

Unit : N/m^2 (S.I.) , $dyne/cm^2$ (C.G.S.)

Dimension : $[ML^{-1}T^{-2}]$

Stress developed in a body depends upon how the external forces are applied over it.

On this basis there are two types of stresses : Normal and Shear or tangential stress

(1) **Normal stress :** Here the force is applied normal to the surface.

It is again of two types : Longitudinal and Bulk or volume stress

(i) *Longitudinal stress*

(a) It occurs only in solids and comes in to picture when one of the three dimensions *viz.* length, breadth, height is much greater than other two.

(b) Deforming force is applied parallel to the length and causes increase in length.

(c) Area taken for calculation of stress is the area of cross section.

(d) Longitudinal stress produced due to increase in length of a body under a deforming force is called tensile stress.

(e) Longitudinal stress produced due to decrease in length of a body under a deforming force is called compressive stress.

(ii) *Bulk or Volume stress*

(a) It occurs in solids, liquids or gases.

(b) In case of fluids only bulk stress can be found.

(c) It produces change in volume and density, shape remaining same.

(d) Deforming force is applied normal to surface at all points.

(e) Area for calculation of stress is the complete surface area perpendicular to the applied forces.

(f) It is equal to change in pressure because change in pressure is responsible for change in volume.

(2) **Shear or tangential stress :** It comes into picture when successive layers of solid move on each other *i.e.* when there is a relative displacement between various layers of solid.

(i) Here deforming force is applied tangential to one of the faces.

(ii) Area for calculation is the area of the face on which force is applied.

(iii) It produces change in shape, volume remaining the same.

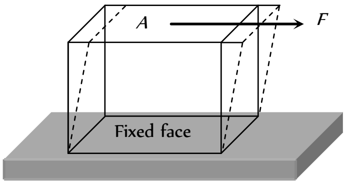


Fig. 9.3

Difference between Pressure and Stress	
Pressure	**Stress**
Pressure is always normal to the area.	Stress can be normal or tangential.
Always compressive in nature.	May be compressive or tensile in nature.

Strain

The ratio of change in configuration to the original configuration is called strain.

Being the ratio of two like quantities, it has no dimensions and units.

Strain are of three types :

(1) **Linear strain :** If the deforming force produces a change in length alone, the strain produced in the body is called linear strain or tensile strain.

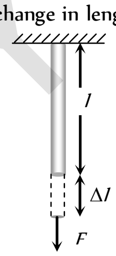


Fig. 9.4

$$\text{Linear strain} = \frac{\text{Change in length}(\Delta l)}{\text{Original length}(l)}$$

Linear strain in the direction of deforming force is called longitudinal strain and in a direction perpendicular to force is called lateral strain.

(2) **Volumetric strain :** If the deforming force produces a change in volume alone the strain produced in the body is called volumetric strain.

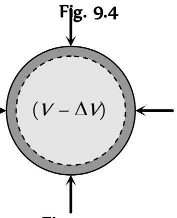


Fig. 9.5

$$\text{Volumetric strain} = \frac{\text{Change in volume}(\Delta V)}{\text{Original volume}(V)}$$

(3) **Shearing strain :** If the deforming force produces a change in the shape of the body without changing its volume, strain produced is called shearing strain.

It is defined as angle in radians through which a plane perpendicular to the fixed surface of the cubical body gets turned under the effect of tangential force.

$$\phi = \frac{x}{L}$$

Note : ❑ When a beam is bent both compression strain as well as an extension strain is produced.

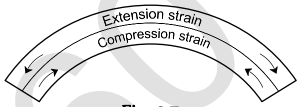


Fig. 9.7

Stress-strain Curve

If by gradually increasing the load on a vertically suspended metal wire, a graph is plotted between stress (or load) and longitudinal strain (or elongation) we get the curve as shown in figure. From this curve it is clear that :

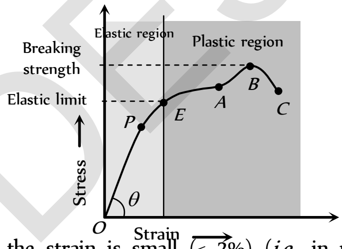


Fig. 9.8

(1) When the strain is small (< 2%) (*i.e.*, in region *OP*) stress is proportional to strain. This is the region where the so called Hooke's law is obeyed. The point *P* is called limit of proportionality and slope of line *OP* gives the Young's modulus *Y* of the material of the wire. If θ is the angle of *OP* from strain axis then $Y = \tan\theta$.

(2) If the strain is increased a little bit, *i.e.*, in the region *PE*, the stress is not proportional to strain. However, the wire still regains its original length after the removal of stretching force. This behaviour is shown up to point *E* known as elastic limit or yield-point. The region *OPE* represents the elastic behaviour of the material of wire.

(3) If the wire is stretched beyond the elastic limit *E*, *i.e.*, between *EA*, the strain increases much more rapidly and if the stretching force is removed the wire does not come back to its natural length. Some permanent increase in length takes place.

(4) If the stress is increased further, by a very small increase in it a very large increase in strain is produced (region *AB*) and after reaching point *B*, the strain increases even if the wire is unloaded and ruptures at *C*. In the region *BC* the wire literally flows. The maximum stress corresponding to *B* after which the wire begins to flow and breaks is called breaking or ultimate tensile strength. The region *EABC* represents the plastic behaviour of the material of wire.

(5) Stress-strain curve for different materials are as follows :

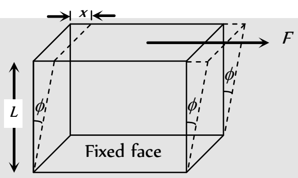
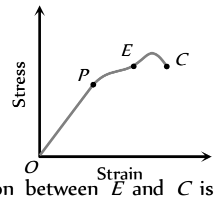
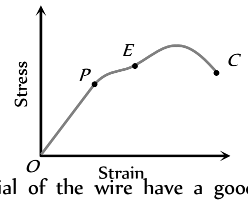
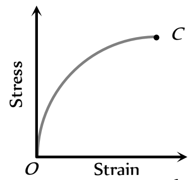


Fig. 9.6

Brittle material	Ductile material	Elastomers

 The plastic region between E and C is small for brittle material and it will break soon after the elastic limit is crossed. Example : Glass, cast iron.	 The material of the wire have a good plastic range and such materials can be easily changed into different shapes and can be drawn into thin wires Example. Mild steel	 Stress-strain curve is not a straight line within the elastic limit for elastomers and strain produced is much larger than the stress applied. Such materials have no plastic range and the breaking point lies very close to elastic limit. Example rubber

Hooke's law and Modulus of Elasticity

According to this law, within the elastic limit, stress is proportional to the strain.

i.e. stress $\propto$ strain or $\frac{\text{stress}}{\text{strain}} = \text{constant} = E$

The constant E is called modulus of elasticity.

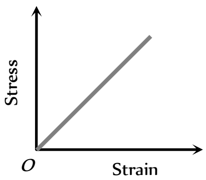


Fig. 9.9

(1) It's value depends upon the nature of material of the body and the manner in which the body is deformed.

(2) It's value depends upon the temperature of the body.

(3) It's value is independent of the dimensions (length, volume etc.) of the body.

There are three modulii of elasticity namely Young's modulus (Y), Bulk modulus (K) and modulus of rigidity (η) corresponding to three types of the strain.

Young's Modulus (Y)

It is defined as the ratio of normal stress to longitudinal strain within limit of proportionality.

$$Y = \frac{\text{Normal stress}}{\text{longitudinal strain}} = \frac{F/A}{l/L} = \frac{FL}{Al}$$

If force is applied on a wire of radius r by hanging a weight of mass M, then

$$Y = \frac{MgL}{\pi r^2 l}$$

(i) If the length of a wire is doubled,

Then longitudinal strain = $\frac{\text{change in length}(l)}{\text{initial length}(L)}$

$$= \frac{\text{final length} - \text{initial length}}{\text{Initial length}} = \frac{2L - L}{L} = 1$$

$\therefore$ Young's modulus = $\frac{\text{stress}}{\text{strain}}$ $\Rightarrow$ Y = stress

[As strain = 1]

So young's modulus is numerically equal to the stress which will double the length of a wire.

(ii) Increment in the length of wire $l = \frac{FL}{\pi r^2 Y}$ $\left[\text{As } Y = \frac{FL}{Al}\right]$

So if same stretching force is applied to different wires of same material, $l \propto \frac{L}{r^2}$ [As F and Y are constant]

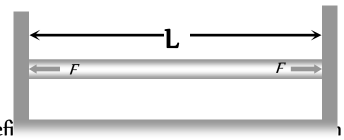
i.e., greater the ratio $\frac{L}{r^2}$, greater will be the elongation in the wire.

(iii) Elongation in a wire by its own weight : The weight of the wire Mg act at the centre of gravity of the wire so that length of wire which is stretched will be L/2.

$\therefore$ Elongation $l = \frac{FL}{AY} = \frac{Mg(L/2)}{AY} = \frac{MgL}{2AY} = \frac{L^2 dg}{2Y}$

[As mass (M) = volume (AL) $\times$ density (d)]

(iv) Thermal stress : If a rod is fixed between two rigid supports, due to change in temperature its length will change and so it will exert a normal stress (compressive if temperature increases and tensile if temperature decreases) on the supports. This stress is called thermal stress.



As by defi ... $\alpha = \frac{l}{L\Delta\theta}$

Fig. 9.10

$\Rightarrow$ thermal strain $\frac{l}{L} = \alpha\Delta\theta$

So thermal stress = Yα $\Delta\theta$ [As Y = stress/strain]

And tensile or compressive force produced in the body = YAα $\Delta\theta$

Note : In case of volume expansion Thermal stress $= K\gamma\Delta\theta$

Where K = Bulk modulus, γ = coefficient of cubical expansion

(v) **Force between the two rods** : Two rods of different metals, having the same area of cross section A, are placed end to end between two massive walls as shown in figure. The first rod has a length L_1, coefficient of linear expansion α_1 and young's modulus Y_1. The corresponding quantities for second rod are L_2, α_2 and Y_2. If the temperature of both the rods is now raised by T degrees.

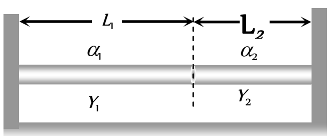


Fig. 9.11

Increase in length of the composite rod (due to heating) will be equal to

$$l_1 + l_2 = [L_1\alpha_1 + L_2\alpha_2]T \qquad [\text{As } l = L\,\alpha\,\Delta\theta]$$

and due to compressive force F from the walls due to elasticity, decrease in length of the composite rod will be equal to

$$\left[\frac{L_1}{Y_1} + \frac{L_2}{Y_2}\right]\frac{F}{A} \qquad \left[\text{As} l = \frac{FL}{AY}\right]$$

as the length of the composite rod remains unchanged the increase in length due to heating must be equal to decrease in length due to compression *i.e.* $\frac{F}{A}\left[\frac{L_1}{Y_1} + \frac{L_2}{Y_2}\right] = [L_1\alpha_1 + L_2\alpha_2]T$

or $F = \dfrac{A[L_1\alpha_1 + L_2\alpha_2]T}{\left[\dfrac{L_1}{Y_1} + \dfrac{L_2}{Y_2}\right]}$

(vi) **Force constant of wire** : Force required to produce unit elongation in a wire is called force constant of material of wire. It is denoted by k.

$$\therefore\ k = \frac{F}{l} \qquad \text{...(i)}$$

but from the definition of young's modulus

$$\frac{F}{l} = \frac{YA}{L} \qquad \text{...(ii)}$$

from (i) and (ii) $k = \dfrac{YA}{L}$

It is clear that the value of force constant depends upon the dimension (length and area of cross section) and material of a substance.

(vii) Actual length of the wire : If the actual length of the wire is L, then under the tension T_1 its length becomes L_1 and under the tension T_2 its length becomes L_2.

$$L_1 = L + l_1 \Rightarrow L_1 = L + \frac{T_1}{k} \qquad \text{...(i)}$$

and $L_2 = L + l_2 \Rightarrow L_2 = L + \dfrac{T_2}{k}$ ...(ii)

From (i) and (ii) we get $L = \dfrac{L_1T_2 - L_2T_1}{T_2 - T_1}$

Work Done in Stretching a Wire

In stretching a wire work is done against internal restoring forces. This work is stored in the wire as elastic potential energy or strain energy.

If a force F acts along the length L of the wire of cross-section A and stretches it by x then

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{x/L} = \frac{FL}{Ax} \Rightarrow F = \frac{YA}{L}x$$

So the work done for an additional small increase dx in length, $dW = Fdx = \dfrac{YA}{L}x\,.dx$

Hence the total work done in increasing the length by l, $W = \int_0^l dW = \int_0^l F dx = \int_0^l \dfrac{YA}{L}.x\,dx = \dfrac{1}{2}\dfrac{YA}{L}l^2$

This work done is stored in the wire.

$\therefore$ Energy stored in wire $U = \dfrac{1}{2}\dfrac{YAl^2}{L} = \dfrac{1}{2}Fl \quad \left[\text{As} F = \dfrac{YAl}{L}\right]$

Dividing both sides by volume of the wire we get energy stored in unit volume of wire.

$$U_V = \frac{1}{2}\times\frac{F}{A}\times\frac{l}{L} = \frac{1}{2}\times\text{stress}\times\text{strain} = \frac{1}{2}\times Y\times(\text{strain})^2$$

$$= \frac{1}{2Y}(\text{stress})^2 \qquad [\text{As AL = volume of wire}]$$

Total energy stored in wire (U)	Energy stored in per unit volume of wire (U)
$\frac{1}{2}Fl$	$\frac{1}{2}\frac{Fl}{\text{volume}}$
$\frac{1}{2}\times\text{stress}\times\text{strain}\times\text{volume}$	$\frac{1}{2}\times\text{stress}\times\text{strain}$
$\frac{1}{2}\times Y\times(\text{strain})^2\times\text{volume}$	$\frac{1}{2}\times Y\times(\text{strain})^2$
$\frac{1}{2Y}\times(\text{stress})^2\times\text{volume}$	$\frac{1}{2Y}\times(\text{stress})^2$

Note : ❑ If the force on the wire is increased from F_1 to F_2 and the elongation in wire is l then energy stored in the wire $U = \dfrac{1}{2}\dfrac{(F_1+F_2)}{2}l$

❑ Thermal energy density = Thermal energy per unit volume = $\frac{1}{2}\times$ Thermal stress × strain

$$= \frac{1}{2}\frac{F}{A}\frac{l}{L} = \frac{1}{2}(Y\alpha\Delta\theta)(\alpha\Delta\theta) = \frac{1}{2}Y\alpha^2(\Delta\theta)^2$$

Breaking of Wire

When the wire is loaded beyond the elastic limit, then strain increases much more rapidly. The maximum stress corresponding to B (see stress-strain curve) after which the wire begin to flow and breaks, is called breaking stress or tensile strength and the force by application of which the wire breaks is called the breaking force.

(i) Breaking force depends upon the area of cross-section of the wire i.e., Breaking force $\propto$ A

$\therefore$ Breaking force = P × A

Here P is a constant of proportionality and known as breaking stress.

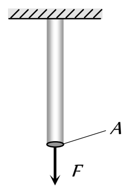


Fig. 9.12

(ii) Breaking stress is a constant for a given material and it does not depend upon the dimension (length or thickness) of wire.

(iii) If a wire of length L is cut into two or more parts, then again it's each part can hold the same weight. Since breaking force is independent of the length of wire.

(iv) If a wire can bear maximum force F, then wire of same material but double thickness can bear maximum force 4F

(v) The working stress is always kept lower than that of a breaking stress.

So that safety factor = $\dfrac{\text{breaking stress}}{\text{working stress}}$, may have large value.

(vi) Breaking of wire under its own weight.

Breaking force = Breaking stress × Area of cross section

Weight of wire = Mg = ALdg = PA [P = Breaking stress]

[As mass = volume × density = ALd]

$$\Rightarrow \; Ldg = P \;\; \therefore \; L = \frac{P}{dg}$$

This is the length of wire if it breaks by its own weight.

Bulk Modulus

When a solid or fluid (liquid or gas) is subjected to a uniform pressure all over the surface, such that the shape remains the same, then there is a change in volume.

Then the ratio of normal stress to the volumetric strain within the elastic limits is called as Bulk modulus. This is denoted by K.

$$K = \frac{\text{Normal stress}}{\text{volumetric strain}}$$

$$K = \frac{F/A}{-\Delta V/V} = \frac{-pV}{\Delta V}$$

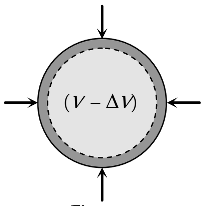


Fig. 9.13

where p = increase in pressure; V = original volume; ΔV = change in volume

The negative sign shows that with increase in pressure p, the volume decreases by ΔV i.e. if p is positive, ΔV is negative. The reciprocal of bulk modulus is called compressibility.

C = compressibility = $\dfrac{1}{K} = \dfrac{\Delta V}{pV}$

S.I. unit of compressibility is $N^{-1}m^2$ and C.G.S. unit is $dyne^{-1}cm^2$.

Gases have two bulk moduli, namely isothermal elasticity E_θ and adiabatic elasticity E_ϕ .

(1) Isothermal elasticity (E_θ) : Elasticity possess by a gas in isothermal condition is defined as isothermal elasticity.

For isothermal process, PV = constant (Boyle's law)

Differentiating both sides

PdV + VdP = 0 ⇒ PdV = – VdP

$$P = \frac{dP}{(-dV/V)} = \frac{\text{stress}}{\text{strain}} = E_\theta \quad \therefore \; E_\theta = P$$

i.e., Isothermal elasticity is equal to pressure.

(2) Adiabatic elasticity (E_ϕ) : Elasticity possess by a gas in adiabatic condition is defined as adiabatic elasticity.

For adiabatic process, PV^γ = constant (Poisson's law)

Differentiating both sides,

$$P\gamma V^{\gamma-1}dV + V^\gamma dP = 0 \;\Rightarrow\; \gamma PdV + VdP = 0$$

$$\gamma P = \frac{dP}{\left(\dfrac{-dV}{V}\right)} = \frac{\text{stress}}{\text{strain}} = E_\phi$$

$\therefore E_\phi = \gamma P$

i.e., adiabatic elasticity is equal to γ times pressure.

[Where $\gamma = \dfrac{C_p}{C_v}$]

Note : ❑ Ratio of adiabatic to isothermal elasticity

$$\frac{E_\phi}{E_\theta} = \frac{\gamma P}{P} = \gamma > 1 \quad \therefore E_\phi > E_\theta$$

i.e., adiabatic elasticity is always more than isothermal elasticity.

Density of Compressed Liquid

If a liquid of density ρ, volume V and bulk modulus K is compressed, then its density increases.

As density $\rho = \dfrac{m}{V}$ so $\dfrac{\Delta\rho}{\rho} = \dfrac{-\Delta V}{V}$ ...(i)

But by definition of bulk modulus

$K = \dfrac{-V\Delta P}{\Delta V} \Rightarrow -\dfrac{\Delta V}{V} = \dfrac{\Delta P}{K}$ ...(ii)

From (i) and (ii) $\dfrac{\Delta\rho}{\rho} = \dfrac{\rho' - \rho}{\rho} = \dfrac{\Delta P}{K}$ [As $\Delta\rho = \rho' - \rho$]

or $\rho' = \rho\left[1 + \dfrac{\Delta P}{K}\right] = \rho[1 + C\Delta P]$ $\left[\text{As} \dfrac{1}{K} = C\right]$

Fractional Change in the Radius of Sphere

A solid sphere of radius R made of a material of bulk modulus K is surrounded by a liquid in a cylindrical container.

A massless piston of area A floats on the surface of the liquid.

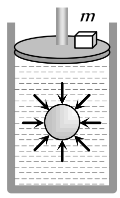


Fig. 9.14

Volume of the spherical body $V = \dfrac{4}{3}\pi R^3$

$$\frac{\Delta V}{V} = 3\frac{\Delta R}{R}$$

$\therefore \dfrac{\Delta R}{R} = \dfrac{1}{3}\dfrac{\Delta V}{V}$ ...(i)

Bulk modulus $K = -V\dfrac{\Delta P}{\Delta V}$

$\therefore \left|\dfrac{\Delta V}{V}\right| = \dfrac{\Delta P}{K} = \dfrac{mg}{AK}$ .....(ii)

$$\left[\text{As}\,\Delta P = \frac{mg}{A}\right]$$

Substituting the value of $\dfrac{\Delta V}{V}$ from equation (ii) in equation (i) we get $\dfrac{\Delta R}{R} = \dfrac{1}{3}\dfrac{mg}{AK}$

Modulus of Rigidity

Within limits of proportionality, the ratio of tangential stress to the shearing strain is called modulus of rigidity of the material of the body and is denoted by η, i.e. $\eta = \dfrac{\text{Shearing stress}}{\text{Shearing strain}}$

In this case the shape of a body changes but its volume remains unchanged.

Consider a cube of material fixed at its lower face and acted upon by a tangential force F at its upper surface having area A. The shearing stress, then, will be

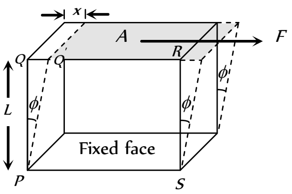


Fig. 9.15

Shearing stress $= \dfrac{F}{A}$

This shearing force causes the consecutive horizontal layers of the cube to be slightly displaced or sheared relative to one another, each line such as PQ or RS in the cube is rotated through an angle ϕ by this shear. The shearing strain is defined as the angle ϕ in radians through which a line normal to a fixed surface has turned. For small values of angle,

$$\text{Shearing strain} = \phi = \frac{QQ'}{PQ} = \frac{x}{L}$$

$$\text{So } \eta = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\phi} = \frac{F}{A\phi}$$

Only solids can exhibit a shearing as these have definite shape.

Poisson's Ratio

When a long bar is stretched by a force along its length then its length increases and the radius decreases as shown in the figure.

Lateral strain : The ratio of change in radius or diameter to the original radius or diameter is called lateral strain.

Longitudinal strain : The ratio of change in length to the original length is called longitudinal strain.

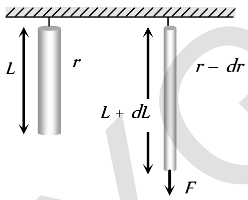


Fig. 9.16

The ratio of lateral strain to longitudinal strain is called Poisson's ratio (σ).

$$\text{i.e. } \sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\sigma = \frac{-dr/r}{dL/L}$$

Negative sign indicates that the radius of the bar decreases when it is stretched.

Poisson's ratio is a dimensionless and a unitless quantity.

Relation Between Volumetric Strain, Lateral Strain and Poisson's Ratio

If a long bar have a length L and radius r then volume $V = \pi r^2 L$

Differentiating both the sides $dV = \pi r^2 dL + \pi 2rLdr$

Dividing both the sides by volume of bar

$$\frac{dV}{V} = \frac{\pi r^2 dL}{\pi r^2 L} + \frac{\pi 2rLdr}{\pi r^2 L} = \frac{dL}{L} + 2\frac{dr}{r}$$

$\Rightarrow$ Volumetric strain = longitudinal strain + 2(lateral strain)

$$\Rightarrow \frac{dV}{V} = \frac{dL}{L} + 2\sigma\frac{dL}{L} = (1+2\sigma)\frac{dL}{L}$$

$$\left[\text{As } \sigma = \frac{dr/r}{dL/L} \Rightarrow \frac{dr}{r} = \sigma\frac{dL}{L}\right]$$

$$\text{or } \sigma = \frac{1}{2}\left[\frac{dV}{AdL} - 1\right]$$

[where A = cross-section of bar]

(i) If a material having $\sigma = -0.5$ then $\dfrac{dV}{V} = [1+2\sigma]\dfrac{dL}{L} = 0$

$\therefore$ Volume = constant or $K = \infty$ *i.e.* the material is incompressible.

(ii) If a material having $\sigma = 0$, then lateral strain is zero *i.e.* when a substance is stretched its length increases without any decrease in diameter *e.g.* cork. In this case change in volume is maximum.

(iii) Theoretical value of Poisson's ratio $-1 < \sigma < 0.5$.

(iv) Practical value of Poisson's ratio $0 < \sigma < 0.5$

Relation between Y, k, η and σ

Moduli of elasticity are three, *viz.* Y, K and η while elastic constants are four, *viz*, Y, K, η and σ. Poisson's ratio σ is not modulus of elasticity as it is the ratio of two strains and not of stress to strain. Elastic constants are found to depend on each other through the relations :

$$Y = 3K(1-2\sigma) \quad \text{...(i)}$$

$$Y = 2\eta(1+\sigma) \quad \text{...(ii)}$$

Eliminating σ or Y between these, we get

$$Y = \frac{9K\eta}{3K+\eta} \quad \text{...(iii)}$$

$$\sigma = \frac{3K-2\eta}{6K+2\eta} \quad \text{...(iv)}$$

Torsion of Cylinder

If the upper end of a cylinder is clamped and a torque is applied at the lower end the cylinder gets twisted by angle θ. Simultaneously shearing strain ϕ is produced in the cylinder.

(i) The angle of twist θ is directly proportional to the distance from the fixed end of the cylinder.

At fixed end $\theta = 0^o$ and at free end θ = maximum.

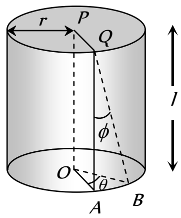


Fig. 9.17

(ii) The value of angle of shear ϕ is directly proportional to the radius of the cylindrical shell.

At the axis of cylinder ϕ = 0 and at the outermost shell ϕ = maximum.

(iii) Relation between angle of twist (θ) and angle of shear (ϕ)

$AB = r\theta = \phi l \quad \therefore \quad \phi = \dfrac{r\theta}{l}$

(iv) Twisting couple per unit twist or torsional rigidity or torque required to produce unit twist.

$$C = \frac{\pi\eta r^4}{2l} \quad \therefore \quad C \propto r^4 \propto A^2$$

(v) Work done in twisting the cylinder through an angle θ is

$$W = \frac{1}{2}C\theta^2 = \frac{\pi\eta r^4\theta^2}{4l}$$

Interatomic Force Constant

Behaviour of solids with respect to external forces is such that if their atoms are connected to springs. When an external force is applied on a solid, this distance between its atoms changes and interatomic force works to restore the original dimension.

The ratio of interatomic force to that of change in interatomic distance is defined as the interatomic force constant. $K = \dfrac{F}{\Delta r}$

It is also given by $K = Y \times r_0$ [Where Y = Young's modulus, r_0 = Normal distance between the atoms of wire]

Unit of interatomic force constant is N/m and Dimension MT^{-2}

Note : ❑ The number of atoms having interatomic distance r_0 in length l of a wire, $N = l/r_0$.

The number of atoms in area A of wire having interatomic separation r_0 is $N = A/r_0^2$.

Elastic Hysteresis

When a deforming force is applied on a body then the strain does not change simultaneously with stress rather it lags behind the stress. The lagging of strain behind the stress is defined as elastic hysteresis. This is the reason why the values of strain for same stress are different while increasing the load and while decreasing the load.

Hysteresis loop : The area of the stress-strain curve is called the hysteresis loop and it is numerically equal to the work done in loading the material and then unloading it.

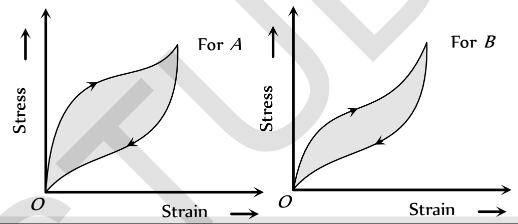


Fig. 9.18

If we have two tyres of rubber having different hysteresis loop then rubber B should be used for making the car tyres. It is because of the reason that area under the curve *i.e.* work done in case of rubber B is lesser and hence the car tyre will not get excessively heated and rubber A should be used to absorb vibration of the machinery because of the large area of the curve, a large amount of vibrational energy can be dissipated.

Factors Affecting Elasticity

(1) Hammering and rolling : Crystal grains break up into smaller units by hammering and rolling. This results in increase in the elasticity of material.

(2) Annealing **:** The metals are annealed by heating and then cooling them slowly. Annealing results in decrease in the elasticity of material.

(3) Temperature **:** Intermolecular forces decreases with rise in temperature. Hence the elasticity decreases with rise in temperature but the elasticity of invar steel (alloy) does not change with change of temperature.

(4) Impurities **:** Due to impurities in a material, elasticity can increase or decrease. The type of effect depends upon the nature of impurities present in the material.

Important Facts About Elasticity

(1) The body which requires greater deforming force to produce a certain change in dimension is more elastic.

Example : Ivory and steel balls are more elastic than rubber.

(2) When equal deforming force is applied on different bodies then the body which shows less deformation is more elastic.

Example : (i) For same load, more elongation is produced in rubber wire than in steel wire hence steel is more elastic than rubber.

(ii) Water is more elastic than air as volume change in water is less for same applied pressure.

(iii) Four identical balls of different materials are dropped from the same height then after collision, balls rises upto different heights.

The order of their height can be given by $h_{ivory} > h_{steel} > h_{rubber} > h_{clay}$ because $Y_{ivory} > Y_{steel} > Y_{rubber} > Y_{clay}$.

(3) The value of moduli of elasticity is independent of the magnitude of the stress and strain. It depends only on the nature of material of the body.

(4) For a given material there can be different moduli of elasticity depending on the type of stress applied and resulting strain.

Name of substance	Young's modulus (Y) 10 N/m	Bulk modulus (K) 10 N/m	Modulus of rigidity (η) 10 N/m
Aluminium	6.9	7.0	2.6
Brass	9.0	6.7	3.4
Copper	11.0	13.0	4.5
Iron	19.0	14.0	4.6
Steel	20.0	16.0	8.4
Tungsten	36.0	20.0	15.0
Diamond	83.0	55.0	34.0

Water	–	0.22	–
Glycerin	–	0.45	–
Air	–	1.01	–

(5) The moduli of elasticity has same dimensional formula and units as that of stress since strain is dimensionless. $\therefore$ Dimensional formula is $[ML^{-1}T^{-2}]$ while units *dyne/cm* or *Newton/m*.

(6) Greater the value of moduli of elasticity more elastic is the material. But as $Y \propto (1/l)$, $K \propto (1/\Delta V)$ and $\eta \propto (1/\phi)$ for a constant stress, so smaller change in shape or size for a given stress corresponds to greater elasticity.

(7) The moduli of elasticity Y and η exist only for solids as liquids and gases cannot be deformed along one dimension only and also cannot sustain shear strain. However K exist for all states of matter *viz.* solid, liquid or gas.

(8) Gases being most compressible are least elastic while solids are most *i.e.* the bulk modulus of gas is very low while that for liquids and solids is very high. $K_{solid} > K_{liquid} > K_{gas}$

(9) For a rigid body l, ΔV or $\phi = 0$ so Y, K or η will be ∞, *i.e.* elasticity of a rigid body is infinite.

Diamond and carborundum are nearest approach to rigid bodies.

(10) In a suspension bridge there is a stretch in the ropes by the load of the bridge. Due to which length of rope changes. Hence Young's modulus of elasticity is involved.

(11) In an automobile tyre as the air is compressed, volume of the air in tyre changes, hence the bulk modulus of elasticity is involved.

(12) In transmitting power, an automobile shaft is sheared as it rotates, so shearing strain is set up, hence modulus of rigidity is involved.

(13) The shape of rubber heels changes under stress, so modulus of rigidity is involved.

Practical Applications of Elasticity

(i) The metallic parts of machinery are never subjected to a stress beyond elastic limit, otherwise they will get permanently deformed.

(ii) The thickness of the metallic rope used in the crane in order to lift a given load is decided from the knowledge of elastic limit of the material of the rope and the factor of safety.

(iii) The bridges are declared unsafe after long use because during its long use, a bridge under goes quick alternating strains continuously. It results in the loss of elastic strength.

(iv) Maximum height of a mountain on earth can be estimated from the elastic behaviour of earth.

At the base of the mountain, the pressure is given by $P = h\rho g$ and it must be less than elastic limit (K) of earth's supporting material.

$$K > P > h\rho g \quad \therefore \; h < \frac{K}{\rho g} \qquad \text{or} \qquad h_{\max} = \frac{K}{\rho g}$$

(v) In designing a beam for its use to support a load (in construction of roofs and bridges), it is advantageous to increase its depth rather than the breadth of the beam because the depression in rectangular beam.

$$\delta = \frac{Wl^3}{4Ybd^3}$$

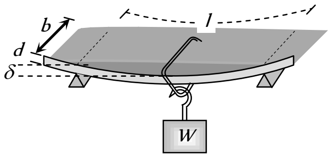


Fig. 9.19

To minimize the depression in the beam, it is designed as *I*-shaped girder.

(vi) For a beam with circular cross-section depression is given by $\delta = \frac{WL^3}{12\pi r^4 Y}$

(vii) A hollow shaft is stronger than a solid shaft made of same mass, length and material.

Torque required to produce a unit twist in a solid shaft

$$\tau_{\text{solid}} = \frac{\pi\eta r^4}{2l} \qquad \text{...(i)}$$

and torque required to produce a unit twist in a hollow shaft

$$\tau_{\text{hollow}} = \frac{\pi\eta(r_2^4 - r_1^4)}{2l} \qquad \text{...(ii)}$$

From (i) and (ii),

$$\frac{\tau_{\text{hollow}}}{\tau_{\text{solid}}} = \frac{r_2^4 - r_1^4}{r^4} = \frac{(r_2^2 + r_1^2)(r_2^2 - r_1^2)}{r^4} \qquad \text{...(iii)}$$

Since two shafts are made from equal volume $\therefore$ $\pi r^2 l = \pi(r_2^2 - r_1^2)l \;\Rightarrow\; r^2 = r_2^2 - r_1^2$

Substituting this value in equation (iii) we get, $\frac{\tau_{\text{hollow}}}{\tau_{\text{solid}}} = \frac{r_2^2 + r_1^2}{r^2} > 1 \quad \therefore \; \tau_{hollow} > \tau_{solid}$

i.e., the torque required to twist a hollow shaft is greater than the torque necessary to twist a solid shaft of the same mass, length and material through the same angle. Hence, a hollow shaft is stronger than a solid shaft.

Tips & Tricks

- Metals are polycrystalline materials.
- Metals are elastic for small strains and for large strains, metals become plastic.
- The substances having large molecular structure (formed by the union of two to several thousand simple molecules) are called polymers.
- Rubber is a polymer.
- Rubber is elastic for very large strains.
- It stretches easily at first but then becomes stiffer.
- Young's modulus is defined only for the solids.
- Bulk modulus was first defined by Maxwell.
- Bulk modulus is defined for all types of materials, solids, liquids and gases.

✍ Reciprocal of bulk modules is called compressibility.

✍ Hooke's law is obeyed only for small values of strain.

✍ Higher value of the elasticity (modulus) means greater force is required for producing a given change.

✍ The material which break as soon as the stress goes beyond the elastic limit are called brittle.

✍ The material which do not break well beyond the elastic limit are called ductile.

✍ The deformation beyond elastic limit is called plasticity.

✍ Rubber sustains elasticity even when stretched several times its length.

However it is not ductile. If breaks down as soon as the elastic limit is crossed.

✍ Within elastic limit, the force constant for a spring is given by $K = \frac{YA}{L}$

✍ Elastic after effect is a temporary absence of the elastic properties.

✍ Quartz is the best available example of perfectly elastic materials.

✍ Isothermal elasticity = pressure (P)

✍ Adiabatic elasticity = Ratio of specific heats × pressure $= \gamma P$

✍ Elasticity is meaningless for the rigid bodies. It is the property of the non rigid bodies.

✍ Diamond and carborundum are the nearest approach to the rigid body.

✍ Elastic fatigue occurs, when a metal is subjected to repeated loading and unloading.

✍ Theoretical value of Poisson's ratio lies between –1 and +1/2 but practical value lies between zero and +1/2.

✍ Negative value of poisson's ratio means that if length increases then radius decreases.

✍ Stress and pressure have the same units and dimensions, but the pressure is always normal to the surface but the stress may be parallel or perpendicular to the surface.

✍ Normal stress is also called tensile stress when the length of the body tends to increase.

✍ Normal stress is also called compressive stress when length of the body tends to decrease.

✍ Tangential stress is also called shearing stress.

✍ When the deforming force is inclined to the surface, both the tangential as well as normal stress are produced.

✍ When a body is sheared, two mutually perpendicular strains are produced. They are called longitudinal strain and compressional strain. Both are equal in magnitude.

✍ When a beam is bent, both extensional as well as compressional strain is produced.

✍ The energy stored by an elastic material is the area under the force-extension graph. The area under the stress-strain graph gives the energy stored per unit volume.

✍ Thermal stress in a rod $= Y\alpha\,\Delta\theta$. It is independent of the area of cross section or length of the wire.

✍ Breaking stress for a wire of unit cross-section is called tensile strength.

✍ Breaking stress does not depend on the length or area of cross section of the wire. However it depends on the material of the wire.

✍ Breaking force depends on the area of cross section. Breaking stress of a wire is called tensile strength.

✍ If we double the radius of rope its breaking force becomes four times. But the breaking stress remains unchanged.

✍ If a beam of rectangular cross-section is loaded its depression at the beam is inversely proportional to the cube of thickness.

✍ If a beam of circular cross-section is loaded, its depression is inversely proportional to the fourth power of radius. *i.e.* $\delta \propto \frac{1}{r^4}$