Chapter

21

Magnetic Effect of Current

Oersted found that a magnetic field is established around a current carrying conductor.

Magnetic field exists as long as there is current in the wire.

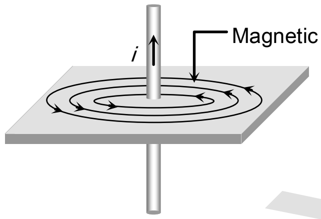


Fig. 21.1

Biot-Savart's Law

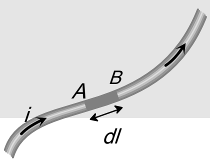
Biot-Savart's law is used to determine the magnetic field at any point due to a current carrying conductor.

This law is although for infinitesimally small conductor yet it can be used for long conductors. In order to understand the Biot-Savart's law, we need to understand the term current-element.

Current element

It is the product of current and length of infinitesimal segment of current carrying wire.

The current element is taken as a vector quantity. Its direction is same as the direction of current.



Current element AB = $i\vec{dl}$

According to Biot-Savart Law, magnetic field at point 'P due to the current element $i\vec{dl}$ is given by the expression, $d\vec{B} = k\dfrac{i\,dl\sin\theta}{r^2}\hat{n}$ also $\vec{B} = \int d\vec{B} = \dfrac{\mu_0 i}{4\pi}.\int \dfrac{dl\sin\theta}{r^2}\hat{n}$

In C.G.S. k = 1 and in S.I. : $k = \dfrac{\mu_0}{4\pi}$

where μ_0 = Absolute permeability of air or vacuum $= 4\pi \times 10^{-7}\ \dfrac{Wb}{Amp - metre}$. It's other units are

$\dfrac{Henry}{metre}$ or $\dfrac{N}{Amp^2}$ or $\dfrac{Tesla - metre}{Ampere}$

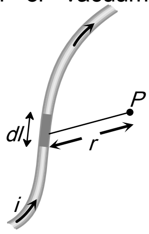


Fig. 21.2

Vectorially, $d\vec{B} = \dfrac{\mu_0}{4\pi}\cdot\dfrac{i(\vec{dl}\times\hat{r})}{r^2} = \dfrac{\mu_0}{4\pi}\cdot\dfrac{i(\vec{dl}\times\vec{r})}{r^3}$

Direction of Magnetic Field

The direction of magnetic field is determined with the help of the following simple laws :

(1) **Maxwell's cork screw rule :** According to this rule, if we imagine a right handed

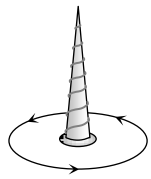


Fig. 21.3

screw placed along the current carrying linear conductor, be rotated such that the screw moves in the direction of flow of current, then the direction of rotation of the thumb gives the direction of magnetic lines of force.

(2) **Right hand thumb rule :** According to this rule if a straight current carrying conductor is held in the right hand such that the thumb of the hand represents the direction of current flow, then the direction of folding fingers will represent the direction of magnetic lines of force.

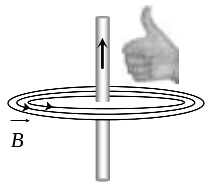


Fig. 21.4

(3) **Right hand thumb rule of circular currents :** According to this rule if the direction of current in circular conducting coil is in the direction of folding fingers of right hand, then the direction of magnetic field will be in the direction of stretched thumb.

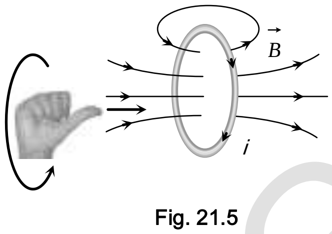


Fig. 21.5

(4) **Right hand palm rule**

If we stretch our right hand such that fingers point towards the point. At which magnetic field is required while thumb is in the direction of current then normal to the palm will show the direction of magnetic field.

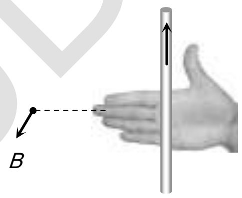


Fig. 21.6

Meaning of Cross ⊗ and dot ⊙

If magnetic field is directed perpendicular and into the plane of the paper it is represented by ⊗ (cross) while if magnetic field is directed perpendicular and out of the plane of the paper it is represented by ⊙ (dot)

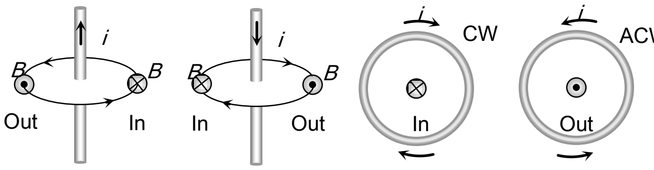


Fig. 21.7

In : Magnetic field is away from the observer or perpendicular inwards.

Out : Magnetic field is towards the observer or perpendicular outwards.

Ampere's Law

Amperes law gives another method to calculate the magnetic field due to a given current distribution.

Line integral of the magnetic field $\vec{B}$ around any closed curve is equal to μ_0 times the net current *i* threading through the area enclosed by the curve *i.e.*

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \sum i = \mu_0 (i_1 + i_3 - i_2)$$

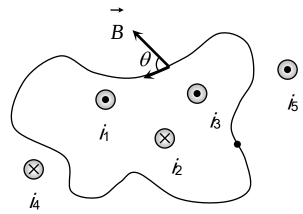


Fig. 21.8

Also using $\vec{B} = \mu_0 \vec{H}$ (where $\vec{H}$ = magnetising field)

$$\oint \mu_0 \vec{H}.\vec{dl} = \mu_0 \Sigma i \Rightarrow \oint \vec{H}.\vec{dl} = \sum i$$

Total current crossing the above area is $(i_1 + i_3 - i_2)$. Any current outside the area is not included in net current. (Outward ⊙ $\rightarrow$ +*ve*, Inward ⊗ $\rightarrow$ – *ve*)

Table 21.1 : Biot-Savart's law v/s Ampere's law

Biot-Savart's law	Ampere's law
this law is valid for all current distributions	This law is valid for symmetrical current distributions
This law is the differential form of $\vec{B}$ or $\vec{H}$	Basically this law is the integral from of $\vec{B}$ or $\vec{H}$
This law is based only on the principle of magnetism	This law is based on the principle of electromagnetism.

Magnetic Field Due to Circular Current

If a coil of radius *r*, carrying current *i* then magnetic field on it's axis at a distance *x* from its centre given by (Application of B

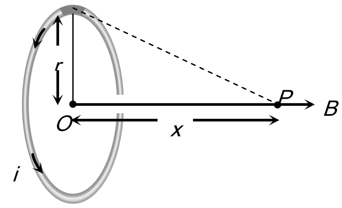


Fig. 21.9

(1) $B_{axis} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi N i r^2}{(x^2 + r^2)^{3/2}}$; where *N* = number of turns in coil.

(2) At centre *x* = 0 $\Rightarrow B_{centre} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi Ni}{r} = \frac{\mu_0 Ni}{2r} = B_{\max}$

(3) The ratio of magnetic field at the centre of circular coil and on it's axis is given by $\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{r^2}\right)^{3/2}$

(4) If *x* >> *r* $\Rightarrow B_{axis} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi\, N i r^2}{x^3} = \frac{\mu_0}{4\pi} \cdot \frac{2NiA}{x^3}$

where $A = \pi r^2$ = Area of each turn of the coil.

(5) ***B-x* curve** : The variation of magnetic field due to a circular coil as the distance *x* varies as shown in the figure.

B varies non-linearly with distance *x* as shown in figure and is maximum when $x^2 = \min = 0$, *i.e.*, the point is at the centre of the coil and it is zero at $x = \pm\infty$.

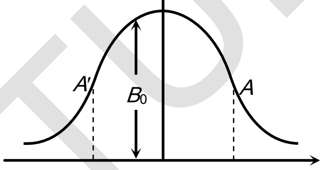
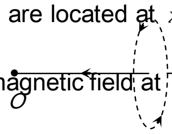


Fig. 21.10

(6) **Point of inflection** (*A* and *A*′) **:** Also known as points of curvature change or points of zero curvature.

(i) At these points *B* varies linearly with $x \Rightarrow \frac{dB}{dx} =$ constant $\Rightarrow \frac{d^2B}{dx^2} = 0$.

(ii) These are located at $x = \pm\frac{r}{2}$ from the centre of the coil and the magnetic field at $x = \frac{r}{2}$ is $B = \frac{4\mu_0 Ni}{5\sqrt{5}\, r}$



(7) **Helmholtz coils**

(i) This is the set-up of two coaxial coils of same radius such that distance between their centres is equal to their radius.

(ii) At axial mid point *O*, magnetic field is given by $B = \frac{8\mu_0 Ni}{5\sqrt{5}R} = 0.716\frac{\mu_0 Ni}{R} = 1.432\, B$, where $B = \frac{\mu_0 Ni}{2R}$

(iii) Current direction is same in both coils otherwise this arrangement is not called Helmholtz's coil arrangement.

(iv) Number of points of inflextion ⇒ Three (*A*, *A*′, *A*′′)

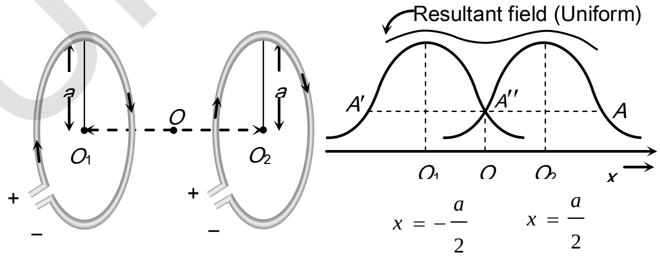
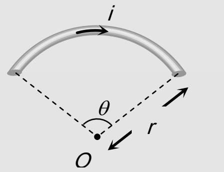
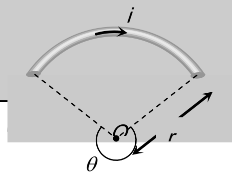
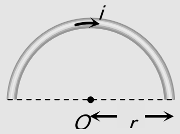
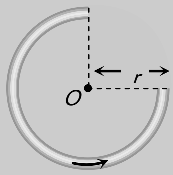
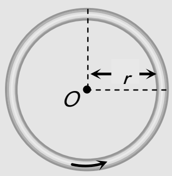
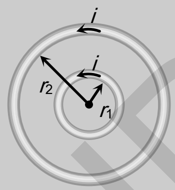
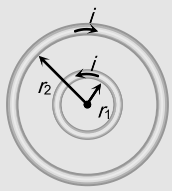
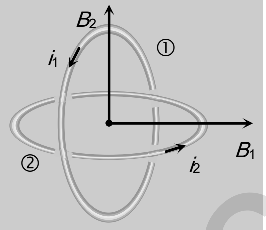
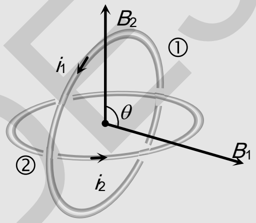
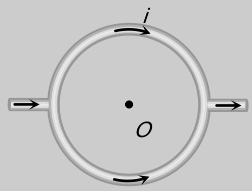
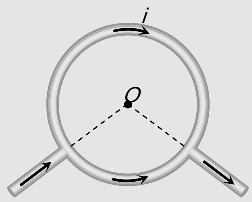


Fig. 21.11

Magnetic Field at Centre *O* in Different Conditions of Circular Current

Condition	Figure	Magnetic field
Arc subtends angle θ at the centre		$B = \frac{\mu_0}{4\pi} \cdot \frac{\theta\, i}{r}$
Arc subtends		

angle $(2\pi - \theta)$ at the centre		$B = \frac{\mu_0}{4\pi} \cdot \frac{(2\pi - \theta)i}{r}$
Semi-circular arc		$B = \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{r} = \frac{\mu_0 i}{4r}$
Three quarter semi-circular current carrying arc		$B = \frac{\mu_0}{4\pi} \cdot \frac{\left(2\pi - \frac{\pi}{2}\right)i}{r}$ $= \frac{3\mu_0 i}{8r}$
Circular current carrying arc		$B = \frac{\mu_0}{4\pi} \frac{2\pi i}{r}$ $= \frac{\mu_0 i}{2r}$
Concentric co-planer circular loops carries current in the same direction		$B_1 = \frac{\mu_0}{4\pi} 2\pi i \left(\frac{1}{r_1} + \frac{1}{r_2}\right)$
Concentric co-planer circular loops carries current in the opposite direction		$B_2 = \frac{\mu_0}{4\pi} 2\pi i \left[\frac{1}{r_1} - \frac{1}{r_2}\right]$
Concentric loops but their planes are perpendicular to each other		$B = \sqrt{B_1^2 + B_2^2}$ $= \frac{\mu_0}{2r}\sqrt{i_1^2 + i_2^2}$
Concentric loops but their planes are at an angle θ with each other		$B = \sqrt{B_1^2 + B_2^2 + 2B_1B_2\cos\theta}$
Distribution of current across the diameter		$B = 0$
Distribution of current between any two points on the circumference		$B = 0$

Magnetic Field Due to a Straight Wire

Magnetic field due to a current carrying wire at a point P which lies at a perpendicular distance r from the wire as shown is given as

$$B = \frac{\mu_0}{4\pi}.\frac{i}{r}(\sin\phi_1 + \sin\phi_2)$$

From figure $\alpha = (90^o - \phi_1)$

and $\beta = (90^o + \phi_2)$

Hence $B = \frac{\mu_o}{4\pi}.\frac{i}{r}(\cos\alpha - \cos\beta)$

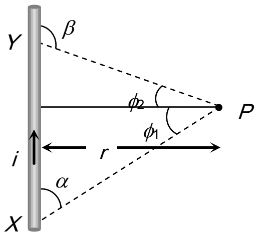


Fig. 21.12

(1) **For a wire of finite length :** Magnetic field at a point which lies on perpendicular bisector of finite length wire

$\phi_1 = \phi_2 = \phi$

So $B = \frac{\mu_0}{4\pi}.\frac{i}{r}(2\sin\phi)$

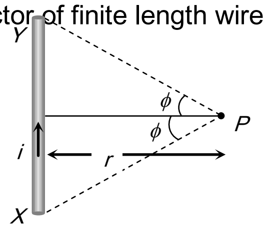


Fig. 21.13

(2) **For a wire of infinite length :** When the linear conductor XY is of infinite length and the point P lies near the centre of the conductor $\phi_1 = \phi_2 = 90^o$.

So, $B = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin 90^o + \sin 90^o]$

$= \frac{\mu_0}{4\pi}\frac{2i}{r}$

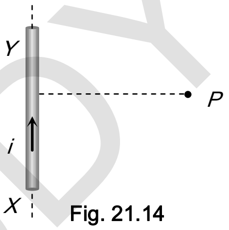


Fig. 21.14

(3) **For a wire of semi-infinite length :** When the linear conductor is of infinite length and the point P lies near the end Y or X. $\phi_1 = 90^o$ and $\phi_2 = 0^o$

So, $B = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin 90^o + \sin 0^o]$

$= \frac{\mu_0 i}{4\pi r}$

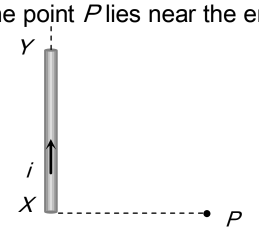


Fig. 21.15

(4) **For axial position of wire :** When point P lies on axial position of current carrying conductor then magnetic field at P

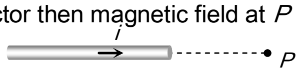


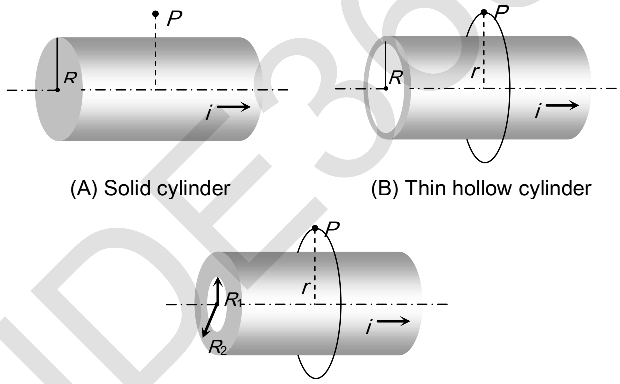
Fig. 21.16

$B = 0$

Magnetic Field Due to a Cylindrical Wire

Magnetic field due to a cylindrical wire is obtained by the application of Ampere's law

(1) **Outside the cylinder**



(A) Solid cylinder (B) Thin hollow cylinder

(C) Thick hollow cylinder

Fig. 21.17

In all above cases magnetic field outside the wire at P

$\oint \vec{B}.\vec{dl} = \mu_0 i \Rightarrow B\int dl = \mu_0 i \Rightarrow B \times 2\pi r = \mu_0 i \Rightarrow B_{out} = \frac{\mu_0 i}{2\pi r}$

In all the above cases $B_{surface} = \frac{\mu_0 i}{2\pi R}$

(2) **Inside the hollow cylinder :** Magnetic field inside the hollow cylinder is zero.

(A) Thin hollow cylinder (B) Thick hollow cylinder

Fig. 21.18

(3) **Inside the solid cylinder :** Current enclosed by loop (i') is lesser then the total current (i)

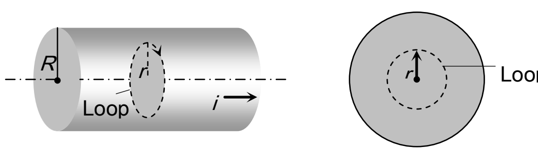


Fig. 21.19

Current density is uniform *i.e.* $J = J \Rightarrow \; i' = i \times \frac{A'}{A} = i\left(\frac{r^2}{R^2}\right)$

Hence at inside point $\oint \overrightarrow{B_{in}}.\overrightarrow{dl} = \mu_0 i' \;\Rightarrow\; B = \frac{\mu_0}{2\pi}.\frac{ir}{R^2}$

(4) **Inside the thick portion of hollow cylinder :** Current enclosed by loop is given as $i' = i \times \frac{A'}{A} = i \times \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$

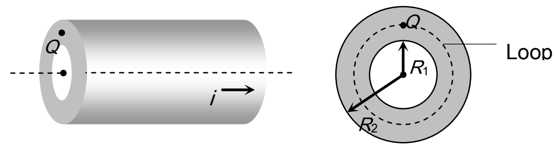


Fig. 21.20

Hence at point Q $\oint \vec{B}.\vec{dl} = \mu_0 i' \;\Rightarrow\; B = \frac{\mu_0 i}{2\pi r}.\frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$

Magnetic Field Due to an Infinite Sheet Carrying Current

The figure shows an infinite sheet of current with linear current density j (A/m). Due to symmetry the field line pattern above and below the sheet is uniform. Consider a square loop of side l as shown in the figure.

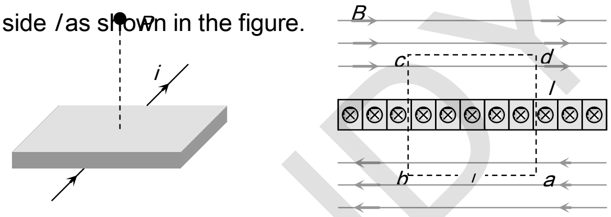


Fig. 21.21

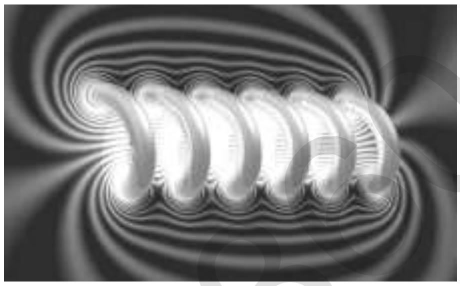
$$\int_a^b \vec{B}.\vec{dl} + \int_b^c \vec{B}.\vec{dl} + \int_c^d \vec{B}.\vec{dl} + \int_d^a \vec{B}.\vec{dl} = \mu_0 i \quad \text{(By Ampere's law)}$$

Since $B \perp dl$ along the path $b \to c$ and $d \to a$, therefore, $\int_b^c \vec{B}.\vec{dl} = 0$; $\int_d^a \vec{B}.\vec{dl} = 0$

Also, $B \parallel dl$ along the path $a \to b$ and $c \to d$, thus $\int_a^b \vec{B}.\vec{dl} + \int_d^a \vec{B}.\vec{dl} = 2Bl$

The current enclosed by the loop is $i = jl$. Therefore, according to Ampere's law $2Bl = \mu_0(jl)$ or $B = \frac{\mu_0 j}{2}$

Solenoid



A cylinderical coil of many tightly wound turns of insulated wire with generally diameter of the coil smaller than its length is called a solenoid.

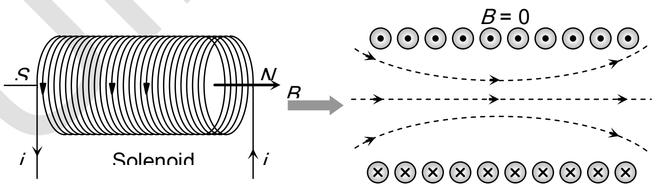


Fig. 21.22

A magnetic field is produced around and within the solenoid. The magnetic field within the solenoid is uniform and parallel to the axis of solenoid.

(1) **Finite length solenoid :** If N = total number of turns, l = length of the solenoid, n = number of turns per unit length $= \frac{N}{l}$

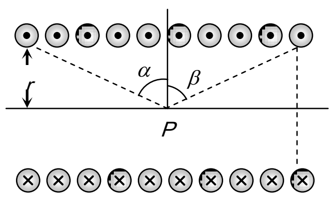


Fig. 21.23

(i) Magnetic field inside the solenoid at point P is given by $B = \frac{\mu_0}{4\pi}(2\pi ni)[\sin\alpha + \sin\beta]$

(ii) **Infinite length solenoid :** If the solenoid is of infinite length and the point is well inside the solenoid *i.e.* $\alpha = \beta = (\pi / 2)$.

So $\boldsymbol{B_{in} = \mu_0 ni}$

(iii) If the solenoid is of infinite length and the point is near one end *i.e.* $\alpha = 0$ and $\beta = (\pi/2)$ so $\boldsymbol{B_{end} = \frac{1}{2}(\mu_0 ni)}$

$(B_{end} = \frac{1}{2} B_{in})$

Toroid

A toroid can be considered as a ring shaped closed solenoid. Hence it is like an endless cylindrical solenoid.

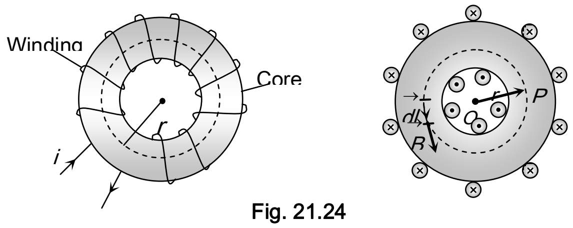


Fig. 21.24

Consider a toroid having n turns per unit length. Magnetic field at a point P in the figure is given as

$$B = \frac{\mu_0 Ni}{2\pi r} = \mu_0 ni \text{ where } n = \frac{N}{2\pi r}$$

Force On a Charged Particle in Magnetic Field

If a particle carrying a positive charge q and moving with velocity v enters a magnetic field B then it experiences a force F which is given by the expression $\vec{F} = q(\vec{v} \times \vec{B}) \Rightarrow F = qvB\sin\theta$

where $\vec{v} =$ velocity of the particle, $\vec{B} =$ magnetic field

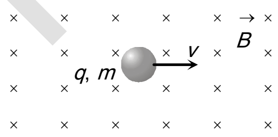


Fig. 21.25

(1) **Zero force :** Force on charged particle will be zero (*i.e.* $F = 0$) if

(i) No field *i.e.* $B = 0 \Rightarrow F = 0$

(ii) Neutral particle *i.e.* $q = 0 \Rightarrow F = 0$

(iii) Rest charge *i.e.* $v = 0 \Rightarrow F = 0$

(iv) Moving charge *i.e.* $\theta = 0^o$ or $\theta = 180^o \Rightarrow F = 0$

(2) **Direction of force :** The force $\vec{F}$ is always perpendicular to both the velocity $\vec{v}$ and the field $\vec{B}$ in accordance with Right Hand Screw Rule, though $\vec{v}$ and $\vec{B}$ themselves may or may not be p… h other.

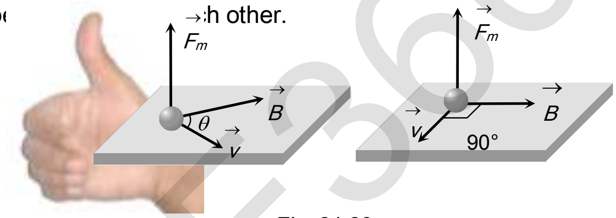


Fig. 21.26

Direction of force on charged particle in magnetic field can also be find by Fleming's Left Hand Rule (FLHR).

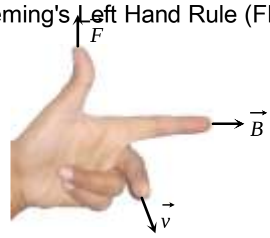


Fig. 21.27

Here, *First finger* (indicates) $\to$ Direction of magnetic field

Middle finger $\to$ Direction of motion of positive charge or direction, Opposite to the motion of negative charge.

Thumb $\to$ Direction of force

Trajectory of a Charged Particle in a Magnetic Field

(1) **Straight line :** If the direction of a $\vec{v}$ is parallel or antiparallel to $\vec{B}$, $\theta = 0$ or $\theta = 180^o$ and therefore $F = 0$. Hence the trajectory of the particle is a straight line.

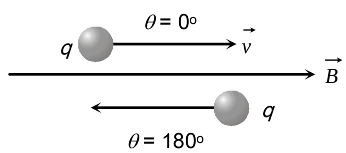


Fig. 21.28

(2) **Circular path :** If $\vec{v}$ is perpendicular to $\vec{B}$ *i.e.* θ = 90°, hence particle will experience a maximum magnetic force ***F_{max} = qvB*** which act's in a direction perpendicular to the motion of charged particle. Therefore the trajectory of the particle is a circle.

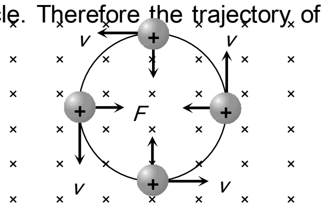


Fig. 21.29

(i) In this case path of charged particle is circular and magnetic force provides the necessary centripetal force *i.e.* $qvB = \frac{mv^2}{r}$ $\Rightarrow$ radius of path $\boldsymbol{r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}}$

where *p* = momentum of charged particle and *K* = kinetic energy of charged particle (gained by charged particle after accelerating through potential difference *V*) then $p = mv = \sqrt{2mK} = \sqrt{2mqV}$

(ii) If *T* is the time period of the particle then $T = \frac{2\pi m}{qB}$ (*i.e.*, time period (or frequency) is independent of speed of particle).

(3) **Helical path :** When the charged particle is moving at an angle to the field (other than 0°, 90°, or 180°). Particle describes a path called helix.

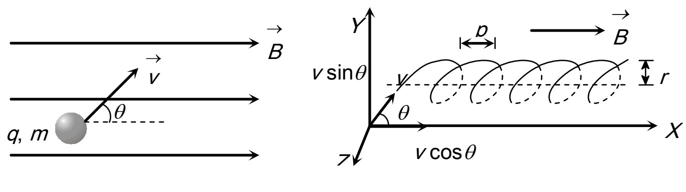


Fig. 21.30

(i) The radius of this helical path is $\boldsymbol{r = \frac{m(v\sin\theta)}{qB}}$

(ii) Time period and frequency do not depend on velocity and so they are given by $T = \frac{2\pi m}{qB}$ and $\nu = \frac{qB}{2\pi m}$

(iii) The *pitch* of the *helix*, (*i.e.*, linear distance travelled in one rotation) will be given by $p = T(v\cos\theta) = 2\pi\frac{m}{qB}(v\cos\theta)$

(iv) If pitch value is *p*, then number of pitches obtained in length *l* given as

Number of pitches $= \frac{l}{p}$ and time required $t = \frac{l}{v\cos\theta}$

Lorentz Force

When the moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$, the moving charged particle will experience electric force $\vec{F_e} = q\vec{E}$ and magnetic force $\vec{F_m} = q(\vec{v} \times \vec{B})$; so the net force on it will be $\boldsymbol{\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]}$. Which is the famous 'Lorentz-force equation'.

Depending on the directions of $\vec{v}, E$ and $\vec{B}$ following situations are possible

(i) **When $\vec{v}, \vec{E}$ and $\vec{B}$ all the three are collinear :** In this situation the magnetic force on it will be zero and only electric force will act and so $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$

(ii) The particle will pass through the field following a straight-line path (parallel field) with change in its speed. So in this situation speed, velocity, momentum and kinetic energy all will change without change in direction of motion as shown

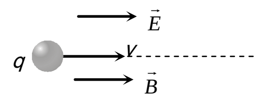


Fig. 21.31

(iii) **$\vec{v}, \vec{E}$ and $\vec{B}$ are mutually perpendicular :** In this situation if $\vec{E}$ and $\vec{B}$ are such that $\vec{F} = \vec{F_e} + \vec{F_m} = 0$ *i.e.*, $\vec{a} = (\vec{F}/m) = 0$

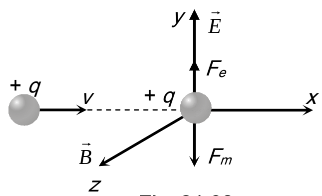


Fig. 21.32

as shown in figure, the particle will pass through the field with same velocity, without any deviation in path.

And in this situation, as $F_e = F_m$ *i.e.*, $qE = qvB$ $v = E/B$

This principle is used in 'velocity-selector' to get a charged beam having a specific velocity.

Cyclotron

Cyclotron is a device used to accelerated positively charged particles (like, α-particles, deutrons *etc.*) to acquire enough energy to carry out nuclear disintegration *etc.*

It is based on the fact that the electric field accelerates a charged particle and the magnetic field keeps it revolving in circular orbits of constant frequency.

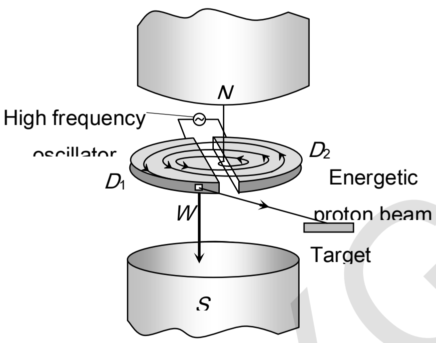


Fig. 21.33

It consists of two hollow *D*-shaped metallic chambers D_1 and D_2 called dees. The two dees are placed horizontally with a small gap separating them. The dees are connected to the source of high frequency electric field. The dees are enclosed in a metal box containing a gas at a low pressure of the order of 10^{-3} *mm* mercury. The whole apparatus is placed between the two poles of a strong electromagnet *NS* as shown in fig. The magnetic field acts perpendicular to the plane of the dees.

(1) **Cyclotron frequency :** Time taken by ion to describe a semicircular path is given by $t = \frac{\pi r}{v} = \frac{\pi m}{qB}$

If T = time period of oscillating electric field then $T = 2t = \frac{2\pi m}{qB}$ the cyclotron frequency $\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$

(2) **Maximum energy of particle :** Maximum energy gained by the charged particle $E_{\max} = \left(\frac{q^2B^2}{2m}\right)r^2$

where r_0 = maximum radius of the circular path followed by the positive ion.

Hall Effect

The Phenomenon of producing a transverse emf in a current carrying conductor on applying a magnetic field perpendicular to the direction of the current is called Hall effect.

Hall effect helps us to know the nature and number of charge carriers in a conductor.

Consider a conductor having electrons as current carriers. The electrons move with drift velocity $\vec{v}$ opposite to the direction of flow of current

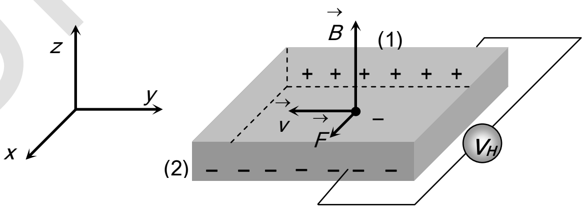


Fig. 21.34

Force acting on electron $F_m = -e(\vec{v} \times \vec{B})$. This force acts along *x*-axis and hence electrons will move towards face (2) and it becomes negatively charged.

Force On a Current Carrying Conductor In Magnetic Field

In case of current carrying conductor in a magnetic field force experienced by its small length element is $d\vec{F} = i\vec{dl} \times \vec{B}$; $i\vec{dl}$ = current element $d\vec{F} = i(\vec{dl} \times \vec{B})$

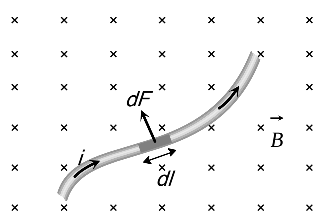


Fig. 21.35

Total magnetic force $\vec{F} = \int d\vec{F} = \int i(d\vec{l} \times \vec{B})$. If magnetic field is uniform *i.e.*, $\vec{B}$ = constant $\vec{F} = i[\int \vec{dl}] \times \vec{B} = i(\vec{L} \times \vec{B})$

$\int \vec{dl} = \vec{L'} =$ vector sum of all the length elements from initial to final point. Which is in accordance with the law of vector addition is equal to length vector $\vec{L}$ joining initial to final point.

(For a straight conductor $F = Bil\sin\theta$)

Direction of force : The direction of force is always perpendicular to the plane containing $i\vec{dl}$ and $\vec{B}$ and is same as that of cross-product of two vectors $(\vec{A} \times \vec{B})$ with $\vec{A} = i\vec{dl}$.

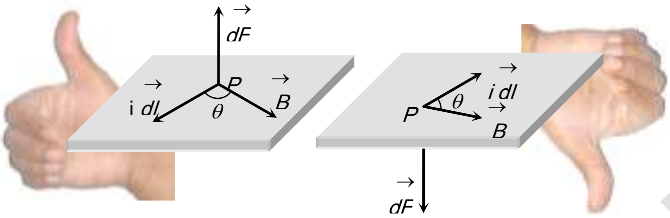


Fig. 21.36

The direction of force when current element $i\vec{dl}$ and $\vec{B}$ are perpendicular to each other can also be determined by applying either of the following rules

Fleming's left-hand rule : Stretch the fore-finger, central finger and thumb of left hand mutually perpendicular. Then if the fore-finger points in the direction of field $\vec{B}$ and the central in the direction of current *i*, the thumb will point in the direction of force.

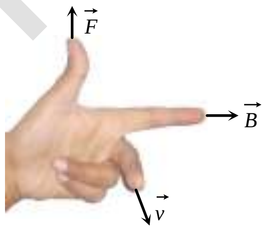


Fig. 21.37

Right-hand palm rule : Stretch the fingers and thumb of right hand at right angles to each other. Then if the fingers point in the direction of field $\vec{B}$ and thumb in the direction of current *i*, then normal to the palm will point in the direction of force

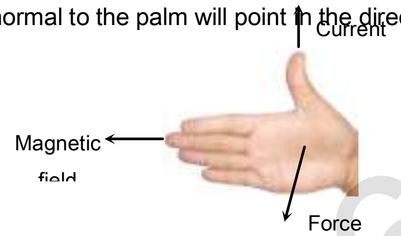


Fig. 21.38

Force Between Two Parallel Current Carrying Conductors

The force on a length *l* of each of two long, straight, parallel wires carrying currents i_1 and i_2 and separated by a distance *a* is

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2i_1i_2}{a} \times l$$

Hence force per unit length

$$\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2i_1i_2}{a}\left(\frac{N}{m}\right) \text{ or } \frac{F}{l} = \frac{2i_1i_2}{a}\left(\frac{dyne}{cm}\right)$$

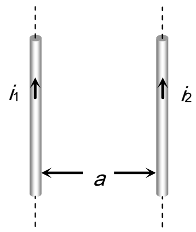


Fig. 21.39

Direction of force : If conductors carries current in same direction, then force between them will be attractive. If conductor carries current in opposite direction, then force between them will be repulsive.

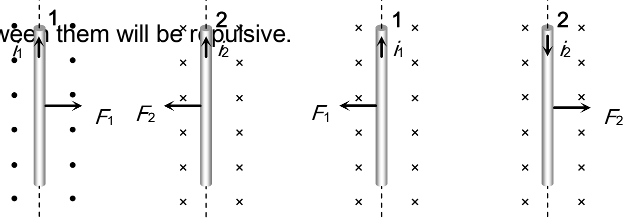


Fig. 21.40

Force Between Two Moving Charges

If two charges q_1 and q_2 are moving with velocities v_1 and v_2 respectively and at any instant the distance between them is r, then

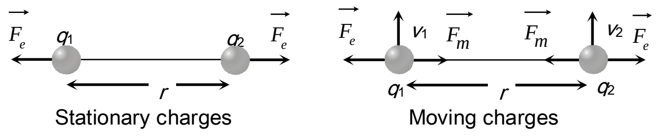


Fig. 21.41

Magnetic force between them is $F_m = \frac{\mu_0}{4\pi}.\frac{q_1 q_2 v_1 v_2}{r^2}$ .... (i)

and Electric force between them is $F_e = \frac{1}{4\pi\varepsilon_0}.\frac{q_1 q_2}{r^2}$ .... (ii)

From equation (i) and (ii) $\frac{F_m}{F_e} = \mu_0 \varepsilon_0 v^2$ but $\mu_0 \varepsilon_0 = \frac{1}{c^2}$;

where c is the velocity of light in vacuum. So $\frac{F_m}{F_e} = \left(\frac{v}{c}\right)^2$

As $v < c$ so $F_m < F_e$

Standard Cases For Force on Current Carrying Conductors

Case 1 : When an arbitrary current carrying loop placed in a magnetic field (⊥ to the plane of loop), each element of loop experiences a magnetic force due to which loop stretches and open into circular loop and tension developed in it's each part.

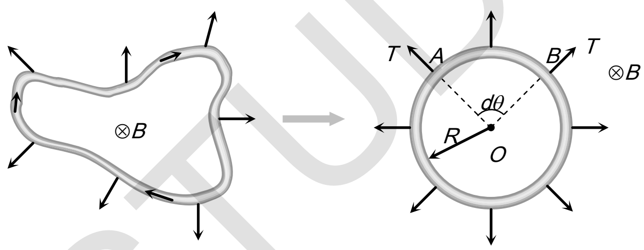


Fig. 21.42

Case 2 : **Equilibrium of a current carrying conductor :** When a finite length current carrying wire is kept parallel to another infinite length current carrying wire, it can suspend freely in air as shown below

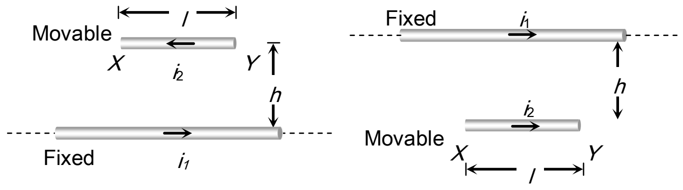
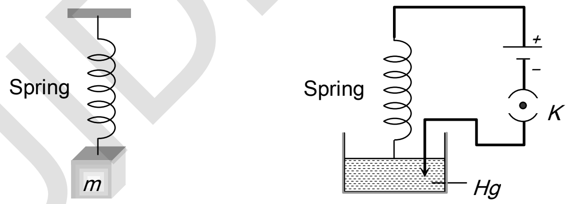


Fig. 21.43

In both the situations for equilibrium of XY it's downward weight = upward magnetic force *i.e.* $mg = \frac{\mu_0}{4\pi}.\frac{2 i_1 i_2}{h}.l$

Case 3 : **Current carrying spring :** If current is passed through a spring, then it will contract because current will flow through all the turns in the same direction.



If current makes to flow through spring, then spring will contract

If switch is closed then current start flowing, spring will execute

Fig. 21.44

Case 4 : **Tension less strings :** In the following figure the value and direction of current through the conductor XY so that strings becomes tensionless?

Strings becomes tensionless if weight of conductor XY balanced by magnetic force (F_m).

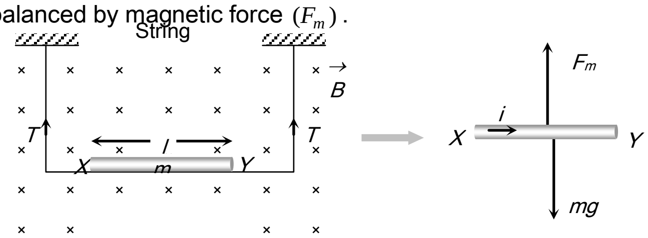


Fig. 21.45

Hence direction of current is from $X \to Y$ and in balanced condition $F_m = mg \Rightarrow Bil = mg \Rightarrow i = \frac{mg}{Bl}$

***Case* 5 : Sliding of conducting rod on inclined rails :** When a conducting rod slides on conducting rails.

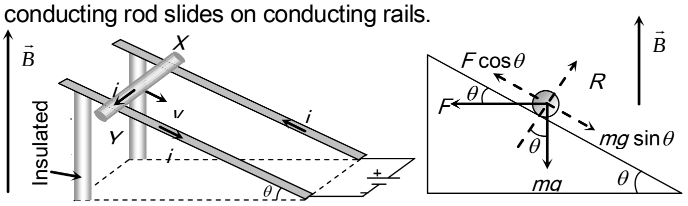


Fig. 21.46

In the following situation conducting rod (*X*, *Y*) slides at constant velocity if

$$F\cos\theta = mg\sin\theta \Rightarrow Bil\cos\theta = mg\sin\theta \Rightarrow B = \frac{mg}{il}\tan\theta$$

Current Loop as a Magnetic Dipole

A current carrying circular coil behaves as a bar magnet whose magnetic moment is *M* = *NiA*; *Where N* = Number of turns in the coil, *i* = Current through the coil and *A* = Area of the coil

Magnetic moment of a current carrying coil is a vector and it's direction is given by right hand thumb rule

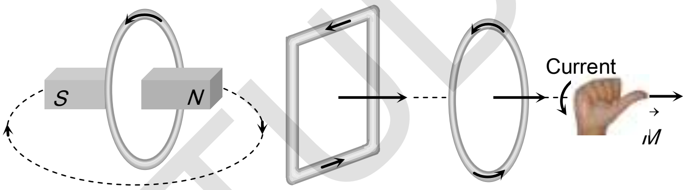


Fig. 21.47

(1) For a given perimeter circular shape have maximum area. Hence maximum magnetic moment.

(2) For a any loop or coil $\vec{B}$ at centre due to current in loop, and $\vec{M}$ are always parallel.

Fig. 21.48

Behaviour of Current Loop in a Magnetic Field

(1) **Torque :** Consider a rectangular current carrying coil *PQRS* having *N* turns and area *A*, placed in a uniform field $\vec{B}$, in such a way that the normal $(\hat{n})$ to the coil makes an angle θ with the direction of $\vec{B}$. the coil experiences a torque given by τ = ***NBiA* sin**θ. Vectorially $\vec{\tau} = \vec{M} \times \vec{B}$

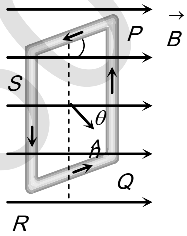


Fig. 21.49

(i) τ is zero when θ = 0, *i.e.*, when the plane of the coil is perpendicular to the field.

(ii) τ is maximum when $\theta = 90^o$, *i.e.*, the plane of the coil is parallel to the field $\tau_{\max} = NBiA$

(2) **Workdone :** If coil is rotated through an angle θ from it's equilibrium position then required work. $W = MB(1-\cos\theta)$. It is maximum when θ = 180° $\Rightarrow$ $W_{\max}$ = 2 *MB*

(3) **Potential energy :** *U* = – *MB* cosθ $\Rightarrow$ $U = -\vec{M}.\vec{B}$

Moving Coil Galvanometer

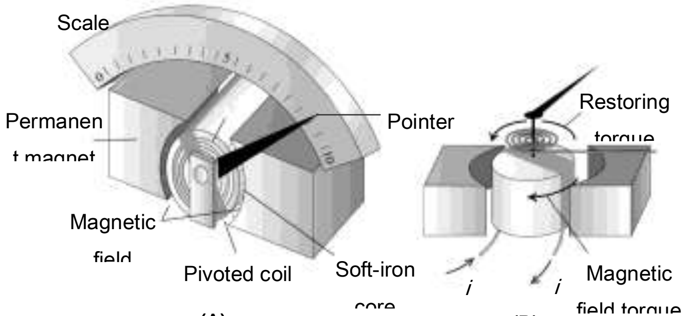


Fig. 21.50

In a moving coil galvanometer the coil is suspended between the pole pieces of a strong horse-shoe magnet. The pole pieces are made cylindrical and a soft iron cylindrical core is placed within the coil without touching it. This makes the field radial. In such a field the plane of the coil always remains parallel to the field. Therefore $\theta = 90^o$ and the deflecting torque always has the maximum value.

$$\tau_{\text{def}} = NBiA \qquad ......(i)$$

Coil deflects, a restoring torque is set up in the suspension fibre. If α is the angle of twist, the restoring torque is

$$\tau_{\text{rest}} = C\alpha \qquad .....(ii)$$

where C is the torsional constant of the fibre.

When the coil is in equilibrium $NBiA = C\alpha \Rightarrow i = K\alpha$,

where $K = \dfrac{C}{NBA}$ is the galvanometer constant. This linear relationship between i and α makes the moving coil galvanometer useful for current measurement and detection.

Current sensitivity (S_i) : The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit current flowing through it.

$$S_i = \frac{\alpha}{i} = \frac{NBA}{C}$$

Voltage sensitivity (S_V) : Voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit voltage applied to it.

$$S_V = \frac{\alpha}{V} = \frac{\alpha}{iR} = \frac{S_i}{R} = \frac{NBA}{RC}$$

Tips & Tricks

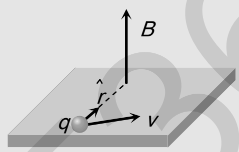
✍ The device whose working principle based on Halmholtz coils and in which uniform magnetic field is used called as "Halmholtz galvanometer".

✍ The value of magnetic field induction at a point, on the centre of separation of two linear parallel conductors carrying equal currents in the same direction is zero.

✍ If a current carrying circular loop (n = 1) is turned into a coil having n identical turns then magnetic field at the centre of the coil becomes n^2 times the previous field *i.e.* $B_{(n\ turn)} = n^2 B_{(single\ turn)}$

✍ When a current carrying coil is suspended freely in earth's magnetic field, it's plane stays in **East-West** direction.

✍ Magnetic field ($\vec{B}$) produced by a moving charge q is given by $\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\hat{r})}{r^2}$; where v = velocity of charge and $v << c$ (speed of light).



✍ If an electron is revolving in a circular path of radius r with speed v then magnetic field produced at the centre of circular path $B = \frac{\mu_0}{4\pi}.\frac{ev}{r^2} \Rightarrow r \propto \sqrt{\frac{v}{B}}$

✍ The line integral of magnetising field $(\vec{H})$ for any closed path called magnetomotive force (MMF). It's S.I. unit is amp.

✍ Ratio of dimension of e.m.f. to MMF is equal to the dimension of resistance.

✍ The positive ions are produced in the gap between the two dees by the ionisation of the gas. To produce proton, hydrogen gas is used; while for producing alpha-particles, helium gas is used.

✍ Cyclotron frequency is also known as magnetic resonance frequency.

✍ Cyclotron can not accelerate electrons because they have very small mass.

✍ The energy of a charged particle moving in a uniform magnetic field does not change because it experiences a force in a direction, perpendicular to it's direction of motion. Due to which the speed of charged particle remains unchanged and hence it's K.E. remains same.

✍ Magnetic force does no work when the charged particle is displaced while electric force does work in displacing the charged particle.

✍ Magnetic force is velocity dependent, while electric force is independent of the state of rest or motion of the charged particle.

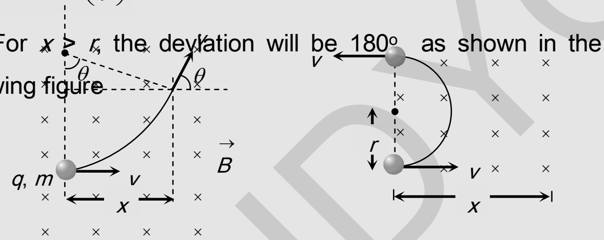
✍ If a particle enters a magnetic field normally to the magnetic field, then it starts moving in a circular orbit. The point at which it enters the magnetic field lies on the circumference. (Most of us confuse it with the centre of the orbit)

✍ Deviation of charged particle in magnetic field : If a charged particle (q, m) enters a uniform magnetic field $\vec{B}$ (extends upto a length x) at right angles with speed v as shown in figure. The speed of the particle in magnetic field does not change. But it gets deviated in the magnetic field.

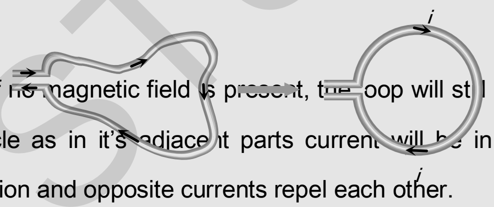
Deviation in terms of time t; $\theta = \omega t = \left(\frac{Bq}{m}\right)t$

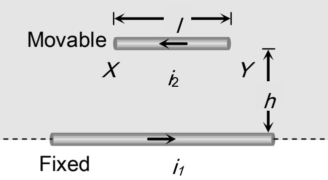
Deviation in terms of length of the magnetic field; $\theta = \sin^{-1}\left(\frac{x}{r}\right)$. This relation can be used only when $x \le r$.

For $x > r$, the deviation will be 180° as shown in the following figure



✍ If no magnetic field is present, the loop will still open into a circle as in it's adjacent parts current will be in opposite direction and opposite currents repel each other.



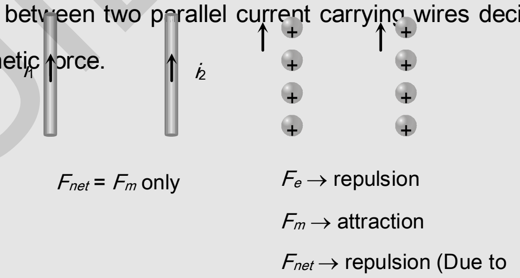


✍ In the following case if wire XY is slightly displaced from its equilibrium position, it executes SHM and it's time period is given by $T = 2\pi\sqrt{\frac{h}{g}}$.

✍ In the previous case if direction of current in movable wire is reversed then it's instantaneous acceleration produced is $2g$ ↓.

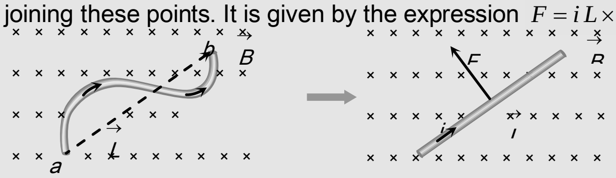
✍ Electric force is an absolute concept while magnetic force is a relative concept for an observer.

✍ The nature of force between two parallel charge beams decided by electric force, as it is dominator. The nature of force between two parallel current carrying wires decided by magnetic force.



✍ If a straight current carrying wire is placed along the axis of a current carrying coil then it will not experience magnetic force because magnetic field produced by the coil is parallel to the wire.

✍ The force acting on a curved wire joining points a and b as shown in the figure is the same as that on a straight wire joining these points. It is given by the expression $\vec{F} = i\vec{L} \times \vec{B}$



✍ If a current carrying conductor *AB* is placed transverse to a long current carrying conductor as shown then force.

Experienced by wire *AB*

$$F = \frac{\mu_0 i_1 i_2}{2\pi} \log_e \left(\frac{x+l}{x} \right)$$