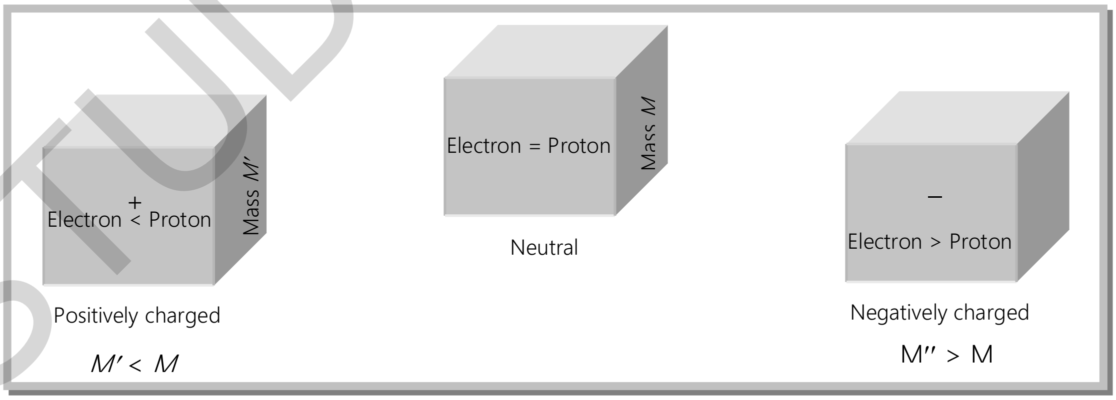
# Electrostatics

## Electric Charge

(1) **Definition :** Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects.

(2) **Origin of electric charge :** It is known that every atom is electrically neutral, containing as many electrons as the number of protons in the nucleus.

Charged particles can be created by disturbing neutrality of an atom. Loss of electrons gives positive charge (as then $n_p > n_e$) and gain of electrons gives negative charge (as then $n_e > n_p$) to a particle. When an object is negatively charged it gains electrons and therefore its mass increases negligibly. Similarly, on charging a body with positive electricity its mass decreases. Change in mass of object is equal to $n \times m_e$. Where, $n$ is the number of electrons transferred and $m_e$ is the mass of electron $= 9.1 \times 10^{-31}\, Kg$.




(3) **Type** : There exists two types of charges in nature (i) Positive charge (ii) Negative charge

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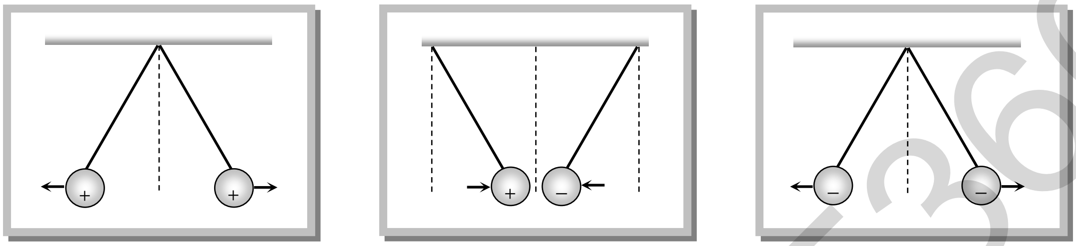
Charges with the same electrical sign repel each other, and charges with opposite electrical sign attract each other.




(4) **Unit and dimensional formula :** Rate of flow of electric charge is called electric current *i.e.,* $\boldsymbol{i} = \frac{\boldsymbol{dQ}}{\boldsymbol{dt}} \Rightarrow$ $\boldsymbol{dQ} = \boldsymbol{idt}$, hence S.I. unit of charge is – *Ampere* × *sec* = *coulomb* (*C*), smaller S.I. units are *mC*, *μC*, *nC* $(1mC = 10^{-3}C, 1\mu C = 10^{-6}C, 1nC = 10^{-9}C)$. C.G.S. unit of charge is – *Stat coulomb* or *e.s.u.* Electromagnetic unit of charge is – *ab coulomb* $1C = 3 \times 10^9$ $stat\,coulomb = \frac{1}{10} ab\,coulomb$. Dimensional formula $[Q] = [AT]$

Note : ❑ Benjamin Franklin was the first to assign positive and negative sign of charge.

❑ The existence of two type of charges was discovered by Dufog.

❑ Franklin (*i.e.*, *e.s.u.* of charge) is the smallest unit of charge while faraday is largest (1 *Faraday* = 96500 *C*).

❑ The *e.s.u.* of charge is also called stat coulomb or Franklin (*Fr*) and is related to *e.m.u.* of charge through the relation $\frac{\text{emu of charge}}{\text{esu of charge}} = 3 \times 10^{10}$

(5) **Point charge :** A finite size body may behave like a point charge if it produces an inverse square electric field. For example an isolated charged sphere behave like a point charge at very large distance as well as very small distance close to it's surface.

(6) **Properties of charge**

(i) **Charge is transferable :** If a charged body is put in contact with an uncharged body, uncharged body becomes charged due to transfer of electrons from one body to the other.

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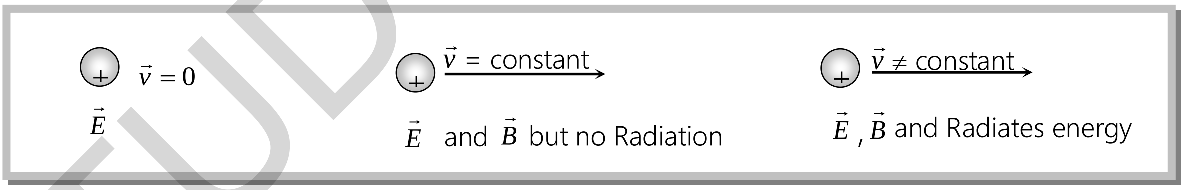
(ii) **Charge is always associated with mass**, *i.e.*, charge can not exist without mass though mass can exist without charge.

(iii) **Charge is conserved :** Charge can neither be created nor be destroyed. *e.g.* In radioactive decay the uranium nucleus (charge $= +92e$) is converted into a thorium nucleus (charge $= +90e$) and emits an $\alpha$-particle (charge $= +2e$)

${}_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_{2}He^{4}$. Thus the total charge is $+92e$ both before and after the decay.

(iv) **Invariance of charge** : The numerical value of an elementary charge is independent of velocity. It is proved by the fact that an atom is neutral. The difference in masses on an electron and a proton suggests that electrons move much faster in an atom than protons. If the charges were dependent on velocity, the neutrality of atoms would be violated.

(v) **Charge produces electric field and magnetic field** : A charged particle at rest produces only electric field in the space surrounding it. However, if the charged particle is in unaccelerated motion it produces both electric and magnetic fields. And if the motion of charged particle is accelerated it not only produces electric and magnetic fields but also radiates energy in the space surrounding the charge in the form of electromagnetic waves.




(vi) **Charge resides on the surface of conductor :** Charge resides on the outer surface of a conductor because like charges repel and try to get as far away as possible from one another and stay at the farthest distance from each other which is outer surface of the conductor. This is why a solid and hollow conducting sphere of same outer radius will hold maximum equal charge and a **soap bubble expands on charging.**

(vii) **Charge leaks from sharp points :** In case of conducting body no doubt charge resides on its outer surface, if surface is uniform the charge distributes uniformly on the surface and for irregular surface the

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distribution of charge, *i.e.*, charge density is not uniform. It is maximum where the radius of curvature is minimum and vice versa. *i.e.*, $\sigma \propto (\mathbf{1/R})$. This is why charge leaks from sharp points.

(viii) **Quantization of charge** : When a physical quantity can have only discrete values rather than any value, the quantity is said to be quantised. The smallest charge that can exist in nature is the charge of an electron. If the charge of an electron $(= 1.6 \times 10^{-19} C)$ is taken as elementary unit *i.e.* quanta of charge the charge on any body will be some integral multiple of *e i.e.*,

$\boxed{\boldsymbol{Q = \pm ne}}$ with $n = 1, 2, 3 ....$

Charge on a body can never be $\pm \frac{2}{3}e$, $\pm 17.2e$ or $\pm 10^{-5}e$ *etc.*

*Note* : ❑ Recently it has been discovered that elementary particles such as proton or neutron are composed of quarks having charge $(\pm 1/3)e$ and $(\pm 2/3)e$. However, as quarks do not exist in free state, the quanta of charge is still *e*.

❑ Quantization of charge implies that there is a maximum permissible magnitude of charge.

## Comparison of Charge and Mass

We are familiar with role of mass in gravitation, and we have just studied some features of electric charge. We can compare the two as shown below

| Charge | Mass |
|---|---|
| (1) Electric charge can be positive, negative or zero. | (1) Mass of a body is a positive quantity. |
| (2) Charge carried by a body does not depend upon velocity of the body. | (2) Mass of a body increases with its velocity as |

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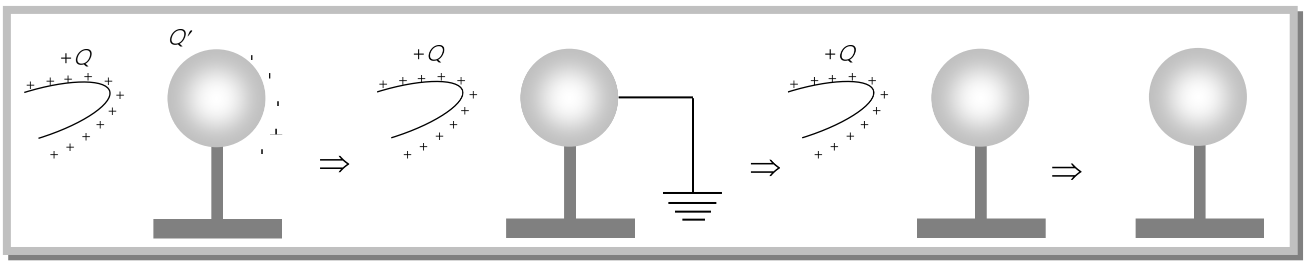
| | |
|---|---|
| | $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$ where $c$ is velocity of light in vacuum, $m$ is the mass of the body moving with velocity $v$ and $m_0$ is rest mass of the body. |
| (3) Charge is quantized. | (3) The quantization of mass is yet to be established. |
| (4) Electric charge is always conserved. | (4) Mass is not conserved as it can be changed into energy and vice-versa. |
| (5) Force between charges can be attractive or repulsive, according as charges are unlike or like charges. | (5) The gravitational force between two masses is always attractive. |

## Methods of Charging

A body can be charged by following methods :

(1) **By friction :** In friction when two bodies are rubbed together, electrons are transferred from one body to the other. As a result of this one body becomes positively charged while the other negatively charged, e.g., when a glass rod is rubbed with silk, the rod becomes positively charged while the silk negatively. However, ebonite on rubbing with wool becomes negatively charged making the wool positively charged. Clouds also become charged by friction. In charging by friction in accordance with conservation of charge, both positive and negative charges in equal amounts appear simultaneously due to transfer of electrons from one body to the other.

(2) **By electrostatic induction :** If a charged body is brought near an uncharged body, the charged body will attract opposite charge and repel similar charge present in the uncharged body. As a result of this one side of neutral body (closer to charged body) becomes oppositely charged while the other is similarly charged. This process is called electrostatic induction.



*Note* : ❑ Inducting body neither gains nor loses charge.

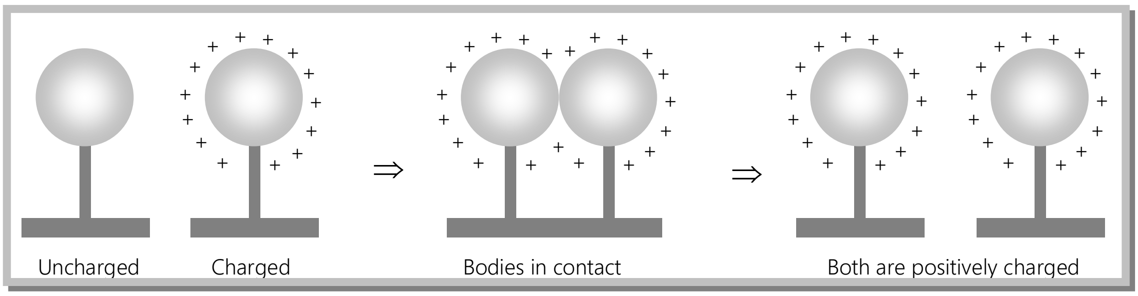
❑ Induced charge can be lesser or equal to inducing charge (but never greater) and its maximum value is given by $\boldsymbol{Q'} = -\boldsymbol{Q}\left[\mathbf{1} - \frac{\mathbf{1}}{\boldsymbol{K}}\right]$ where $Q$ is the inducing charge and $K$ is the dielectric constant of the material of the uncharged body. Dielectric constant of different media are shown below

| Medium | $K$ |
|---|---|
| Vacuum / air | 1 |
| Water | 80 |
| Mica | 6 |
| Glass | 5–10 |
| Metal | $\infty$ |

❑ Dielectric constant of an insulator can not be $\infty$

❑ For metals in electrostatics $K = \infty$ and so $Q' = -Q$; i.e. in metals induced charge is equal and opposite to inducing charge.

(3) **Charging by conduction :** Take two conductors, one charged and other uncharged. Bring the conductors in contact with each other. The charge (whether $-ve$ or $+ve$) under its own repulsion will spread over both the conductors. Thus the conductors will be charged with the same sign. This is called as charging by conduction (through contact).

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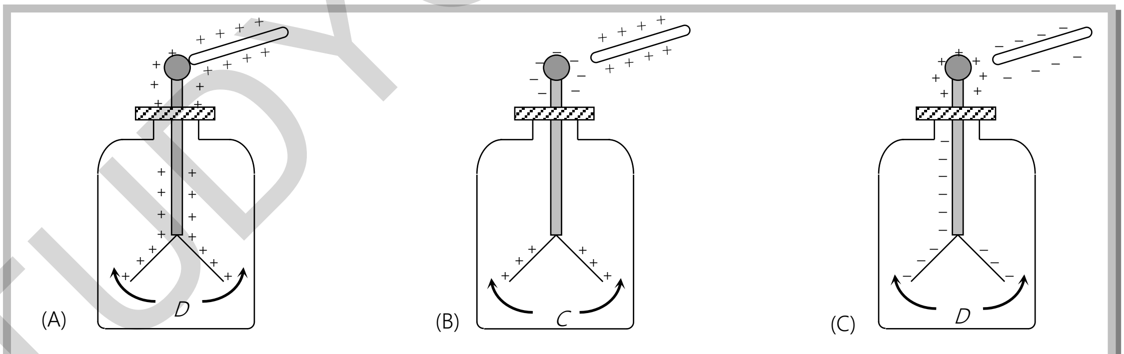
**Note** : ❑ A truck carrying explosives has a metal chain touching the ground, to conduct away the charge produced by friction.

## Electroscope

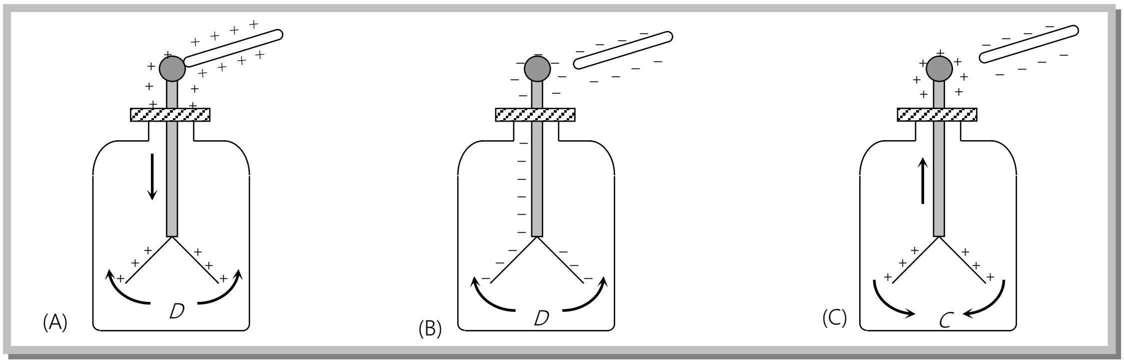
It is a simple apparatus with which the presence of electric charge on a body is detected (see figure). When metal knob is touched with a charged body, some charge is transferred to the gold leaves, which then diverges due to repulsion. The separation gives a rough idea of the amount of charge on the body. If a charged body brought near a charged electroscope the leaves will further diverge. If the charge on body is similar to that on electroscope and will usually converge if opposite. If the induction effect is strong enough leaves after converging may again diverge.

(1) **Uncharged electroscope**




Charging by conduction        Charging by conduction

(2) **Charged electroscope**

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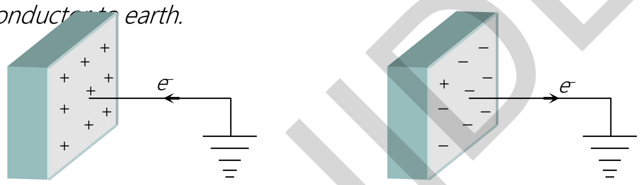
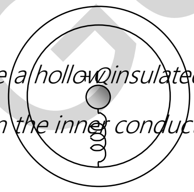
## Concepts

- After earthing a positively charged conductor electrons flow from earth to conductor and if a negatively charged conductor is earthed then electrons flows from conductor to earth.



- When a charged spherical conductor placed inside a hollow insulated conductor and connected if through a fine conducting wire the charge will be completely transferred from the inner conductor to the outer conductor.

- Lightening-rods arrestors are made up of conductors with one of their ends earthed while the other sharp, and protects a building from lightening either by neutralising or conducting the charge of the cloud to the ground.
- With rise in temperature dielectric constant of liquid decreases.
- Induction takes place only in bodies (either conducting or non-conducting) and not in particles.
- If X-rays are incident on a charged electroscope, due to ionisation of air by X-rays the electroscope will get discharged and hence its leaves will collapse. However, if the electroscope is evacuated. X-rays will cause photoelectric effect with gold and so the leaves will further diverge if it is positively charged (or uncharged) and will converge if it is negatively charged.
- If only one charge is available than by repeating the induction process, it can be used to obtain a charge many times greater than it's equilibrium. (High voltage generator)

## Examples based on properties of charge

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**Example: 1** A soap bubble is given negative charge. Its radius will [DCE 2000; RPMT 1997; CPMT 1997; MNR 1988]

(a) Increase (b) Decrease (c) Remain unchanged (d) Fluctuate

**Solution:** (a) Due to repulsive force.

**Example: 2** Which of the following charge is not possible

(a) $1.6\times10^{-18}C$ (b) $1.6\times10^{-19}C$ (c) $1.6\times10^{-20}C$ (d) None of these

**Solution:** (c) $1.6\times10^{-20}C,$ because this is $\frac{1}{10}$ of electronic charge and hence not an integral multiple.

**Example: 3** Five balls numbered 1 to 5 balls suspended using separate threads. Pair (1,2), (2,4) and (4,1) show electrostatic attraction, while pair (2,3) and (4,5) show repulsion. Therefore ball 1 must be [NCERT 1980]

(a) Positively charged (b) Negatively charged (c) Neutral (d) Made of metal

**Solution:** (c) Since 1 does not enter the list of repulsion, it is just possible that it may not be having any charge. Moreover, since ball no. 1 is being attracted by 2 and 4 both. So 2 and 4 must be similarly charged, but it is also given that 2 and 4 also attract each other. So 2 and 4 are certainly oppositely charged.

Since 1 is attracting 2, either 1 or 2 must be neutral but since 2 is already in the list of balls repelling each other, it necessarily has some charge, similarly 4 must have some charge. It means that though 1 is attracting 2 and 4 it does not have any charge.

**Example: 4** If the radius of a solid and hollow copper spheres are same which one can hold greater charge

[BHU 1999; KCET 1994; IIT-JEE 1974]

(a) Solid sphere (b) Hollow sphere

(c) Both will hold equal charge (d) None of these

**Solution:** (c) Charge resides on the surface of conductor, since both the sphere having similar surface area so they will hold equal charge.

**Example: 5** Number of electrons in one coulomb of charge will be [RPET 2001; MP PMT/PET 1998]

(a) $5.46\times10^{29}$ (b) $6.25\times10^{18}$ (c) $1.6\times10^{19}$ (d) $9\times10^{11}$

**Solution:** (b) By using $Q = ne \Rightarrow n = \frac{Q}{e} \Rightarrow n = \frac{1}{1.6\times10^{-19}} = 6.25\times10^{18}$

**Example: 6** The current produced in wire when $10^7$ electron/sec are flowing in it [CPMT 1994]

(a) $1.6\times10^{-26}$ *amp* (b) $1.6\times10^{12}$ *amp* (c) $1.6\times10^{26}$ *amp* (d) $1.6\times10^{-12}$ *amp*

**Solution:** (d) $i = \frac{Q}{t} = \frac{ne}{t} = 10^7\times1.6\times10^{-19} = 1.6\times10^{-12}\ amp$

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**Example: 7** A table-tennis ball which has been covered with a conducting paint is suspended by a silk thread so that it hangs between two metal plates. One plate is earthed. When the other plate is connected to a high voltage generator, the ball

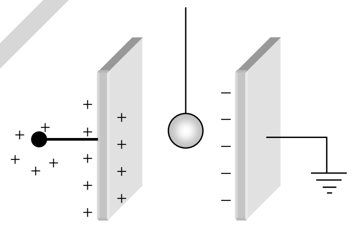
(a) Is attracted to the high voltage plate and stays there

(b) Hangs without moving

(c) Swings backward and forward hitting each plate in turn

(d) None of these

**Solution:** (c) The table tennis ball when slightly displaced say towards the positive plate gets attracted towards the positive plate due to induced negative charge on its near surface.



The ball touches the positive plate and itself gets positively charged by the process of conduction from the plate connected to high voltage generator. On getting positively charged it is repelled by the positive plate and therefore the ball touches the other plate (earthed), which has negative charge due to induction. On touching this plate, the positive charge of the ball gets neutralized and in turn the ball shares negative charge of the earthed plate and is again repelled from this plate also, and this process is repeated again and again.

Here it should be understood that since the positive plate is connected to high voltage generator, its potential and hence its charge will always remain same, as soon as this plate gives some of its charge to ball, excess charge flows from generator to the plate, and an equal negative charge is always induced on the other plate.

**Tricky example: 1**

In 1 *gm* of a solid, there are $5 \times 10^{21}$ *atoms*. If one electron is removed from everyone of 0.01% atoms of the solid, the charge gained by the solid is (given that electronic charge is $1.6 \times 10^{-19}$ *C*)

(a) + 0.08 *C* (b) + 0.8 *C* (c) – 0.08 *C* (d) – 0.8 *C*

**Solution:** (a) To calculate charge, we will apply formula $Q = ne$ for this, we must have number of electrons. Here, number of electrons $n = .01\%$ of $5 \times 10^{21}$

*i.e.* $n = \frac{5 \times 10^{21} \times .01}{100} = 5 \times 10^{21} \times 10^{-4} = 5 \times 10^{17}$

So $Q = 5 \times 10^{17} \times 1.6 \times 10^{-19} = 8 \times 10^{-2} = 0.08$ *C*

Since electrons have been removed, charge will be positive *i.e.* $Q = +\,0.08$ *C*

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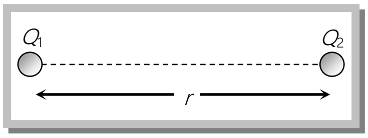
## Coulomb's Law

If two stationary and point charges $Q_1$ and $Q_2$ are kept at a distance *r*, then it is found that force of attraction




or repulsion between them is $F \propto \frac{Q_1 Q_2}{r^2}$ *i.e.*, $F = \frac{kQ_1 Q_2}{r^2}$ ; (*k* = Proportionality constant)

(1) **Dependence of $k$:** Constant *k* depends upon system of units and medium between the two charges.

(i) **Effect of units**

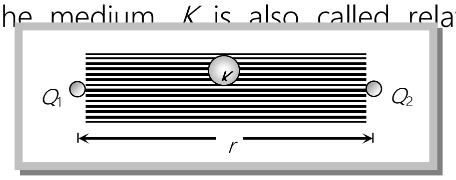
(a) In C.G.S. for air $k = 1$, $F = \frac{Q_1 Q_2}{r^2}$ *Dyne*

(b) In S.I. for air $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{N-m^2}{C^2}$, $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r^2}$ *Newton* (1 *Newton* = $10^5$ *Dyne*)

*Note* :
- ❑ $\varepsilon_0$ = Absolute permittivity of air or free space = $8.85 \times 10^{-12} \frac{C^2}{N-m^2} \left(= \frac{Farad}{m}\right)$. It's Dimension is $[ML^{-3}T^4A^2]$
- ❑ $\varepsilon_0$ Relates with absolute magnetic permeability ($\mu_0$) and velocity of light (*c*) according to the following relation $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

(ii) **Effect of medium**

(a) When a dielectric medium is completely filled in between charges rearrangement of the charges inside the dielectric medium takes place and the force between the same two charges decreases by a factor of *K* known as **dielectric constant** or specific inductive capacity **(SIC)** of the medium, *K* is also called relative permittivity $\varepsilon_r$ of the medium (relative means with respect to free space).




Hence in the presence of medium $F_m = \frac{F_{air}}{K} = \frac{1}{4\pi\varepsilon_0 K} \cdot \frac{Q_1 Q_2}{r^2}$

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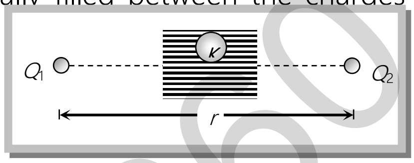
Here $\varepsilon_0 K = \varepsilon_0\ \varepsilon_r = \varepsilon$ (permittivity of medium)

(b) If a dielectric medium (dielectric constant *K*, thickness *t*) is partially filled between the charges then effective air separation between the charges becomes $(r - t + t\sqrt{K})$




Hence force $F = \dfrac{1}{4\pi\varepsilon_0} \dfrac{Q_1 Q_2}{(r - t + t\sqrt{K})^2}$

(2) **Vector form of coulomb's law :** Vector form of Coulomb's law is $\vec{F}_{12} = K.\dfrac{q_1 q_2}{r^3}\vec{r}_{12} = K.\dfrac{q_1 q_2}{r^2}\hat{r}_{12}$, where $\hat{r}_{12}$ is the unit vector from first charge to second charge along the line joining the two charges.

(3) **A comparative study of fundamental forces of nature**

| S.No. | Force | Nature and formula | Range | Relative strength |
|---|---|---|---|---|
| (i) | Force of gravitation between two masses | Attractive $F = Gm_1m_2/r^2$, obey's Newton's third law of motion, it's a conservative force | Long range (between planets and between electron and proton) | 1 |
| (ii) | Electromagnetic force (for stationary and moving charges) | Attractive as well as repulsive, obey's Newton's third law of motion, it's a conservative force | Long (upto few *kelometers*) | $10^{37}$ |
| (iii) | Nuclear force (between nucleons) | Exact expression is not known till date. However in some cases empirical formula $U_0 e^{r/r_0}$ can be utilized for nuclear potential energy $U_0$ and $r_0$ are constant. | Short (of the order of nuclear size $10^{-15}$ *m*) | $10^{39}$ (strongest) |
| (iv) | Weak force (for processes like $\beta$ decay) | Formula not known | Short (upto $10^{-15}$ *m*) | $10^{24}$ |

*Note* : ❑ Coulombs law is not valid for moving charges because moving charges produces magnetic field also.

❑ Coulombs law is valid at a distance greater than $10^{-15}\,m$.

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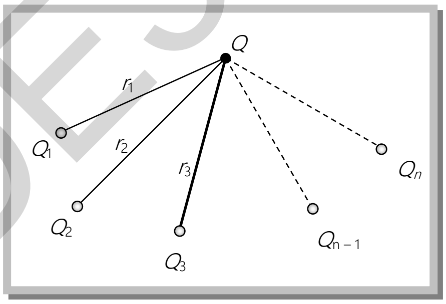
- A charge $Q_1$ exert some force on a second charge $Q_2$. If third charge $Q_3$ is brought near, the force of $Q_1$ exerted on $Q_2$ remains unchanged.
- Ratio of gravitational force and electrostatic force between (i) Two electrons is $10^{-43}/1$. (ii) Two protons is $10^{-36}/1$ (iii) One proton and one electron $10^{-39}/1$.
- Decreasing order to fundamental forces $F_{Nuclear} > F_{Electromagnetic} > F_{Weak} > F_{Gravitational}$

(4) **Principle of superposition :** According to the principle of super position, total force acting on a given charge due to number of charges is the vector sum of the individual forces acting on that charge due to all the charges.




Consider number of charge $Q_1, Q_2, Q_3$ ...are applying force on a charge $Q$

Net force on $Q$ will be

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + .......... + \vec{F}_{n-1} + \vec{F}_n$$

### Concepts

☞ *Two point charges separated by a distance r in vacuum and a force F acting between them. After filling a dielectric medium having dielectric constant K completely between the charges, force between them decreases. To maintain the force as before separation between them changes to* $r\sqrt{K}$ *. This distance known as effective air separation.*

### *Examples based on Coulomb's law*

**Example: 8** Two point charges $+3\mu C$ and $+8\mu C$ repel each other with a force of 40 *N*. If a charge of $-5\mu C$ is added to each of them, then the force between them will become

(a) $-10\,N$ (b) $+10\,N$ (c) $+20\,N$ (d) $-20\,N$

**Solution:** (a) Initially $F = k \times \dfrac{3 \times 8 \times 10^{-12}}{r^2}$ and Finally $F' = -k\dfrac{2 \times 3 \times 10^{-12}}{r^2}$ so $\dfrac{F'}{F} = -\dfrac{1}{4} \Rightarrow F' = -10\,N$

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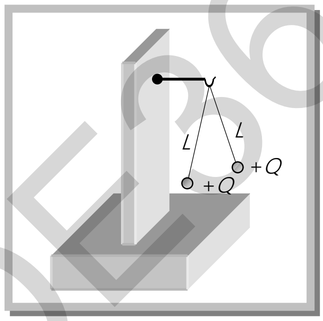
**Example: 9** Two small balls having equal positive charge $Q$ (coulomb) on each are suspended by two insulated string of equal length $L$ meter, from a hook fixed to a stand. The whole set up is taken in satellite into space where there is no gravity (state of weight less ness). Then the angle between the string and tension in the string is **[IIT-JEE 1986]**



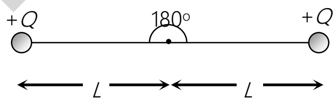

(a) $180^{o}, \frac{1}{4\pi\varepsilon_0}.\frac{Q^2}{(2L)^2}$

(b) $90^{\circ}, \frac{1}{4\pi\varepsilon_0}.\frac{Q^2}{L^2}$

(c) $180^{\circ}, \frac{1}{4\pi\varepsilon_0}.\frac{Q^2}{2L^2}$

(d) $180^{o}, \frac{1}{4\pi\varepsilon_0}.\frac{QL}{4L^2}$

**Solution:** (a) In case to weight less ness following situation arises




So angle $\theta = 180^{\circ}$ and force $F = \frac{1}{4\pi\varepsilon_0}.\frac{Q^2}{(2L)^2}$

**Example: 10** Two point charges $1\mu C$ & $5\mu C$ are separated by a certain distance. What will be ratio of forces acting on these two **[CPMT 1979]**

(a) $1:5$ (b) $5:1$ (c) $1:1$ (d) $0$

**Solution:** (c) Both the charges will experience same force so ratio is 1:1

**Example: 11** Two charges of $40\,\mu C$ and $-20\,\mu C$ are placed at a certain distance apart. They are touched and kept at the same distance. The ratio of the initial to the final force between them is **[MP PMT 2001]**

(a) $8:1$ (b) $4:1$ (c) 1 : 8 (d) 1 : 1

**Solution:** (a) Since only magnitude of charges are changes that's why $F \propto q_1 q_2 \Rightarrow \frac{F_1}{F_2} = \frac{q_1 q_2}{q'_1\, q'_2} = \frac{40 \times 20}{10 \times 10} = \frac{8}{1}$

**Example: 12** A total charge $Q$ is broken in two parts $Q_1$ and $Q_2$ and they are placed at a distance $R$ from each other. The maximum force of repulsion between them will occur, when

(a) $Q_2 = \frac{Q}{R}, Q_1 = Q - \frac{Q}{R}$ (b) $Q_2 = \frac{Q}{4}, Q_1 = Q - \frac{2Q}{3}$ (c) $Q_2 = \frac{Q}{4}, Q_1 = \frac{3Q}{4}$ (d) $Q_1 = \frac{Q}{2}, Q_2 = \frac{Q}{2}$

**Solution:** (d) Force between charges $Q_1$ and $Q_2$ $F = k\frac{Q_1 Q_2}{R^2} = k\frac{Q_1(Q - Q_1)}{R^2}$

For F to be maximum, $\frac{dF}{dQ_1} = 0$ *i.e.,* $\frac{d}{dQ_1}\left\{k\frac{\left(Q_1 Q - Q_1^2\right)}{R^2}\right\} = 0$ or $Q - 2Q_1 = 0, Q_1 = \frac{Q}{2}$

Hence $Q_1 = Q_2 = \frac{Q}{2}$

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**Example: 13** The force between two charges 0.06*m* apart is 5 *N*. If each charge is moved towards the other by 0.01*m*, then the force between them will become [SCRA 1994]

(a) 7.20 *N* (b) 11.25 *N* (c) 22.50 *N* (d) 45.00 *N*

**Solution:** (b) Initial separation between the charges = 0.06*m*

Final separation between the charges = 0.04*m*

Since $F \propto \frac{1}{r^2} \Rightarrow \frac{F_1}{F_2} = \left(\frac{r_2}{r_1}\right)^2 \Rightarrow \frac{5}{F_2} = \left(\frac{0.04}{0.06}\right)^2 = \frac{4}{9} \Rightarrow F_2 = 11.25\,N$

**Example: 14** Two charges equal in magnitude and opposite in polarity are placed at a certain distance apart and force acting between them is *F*. If 75% charge of one is transferred to another, then the force between the charges becomes

(a) $\frac{F}{16}$ (b) $\frac{9F}{16}$ (c) $F$ (d) $\frac{15}{16}F$

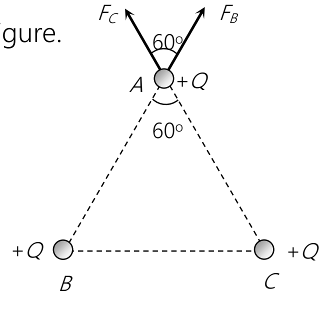
**Solution:** (a)


Initially $F = k\frac{Q^2}{r^2}$ Finally $F' = \frac{k.\left(\frac{Q}{4}\right)^2}{r^2} = \frac{F}{16}$

**Example: 15** Three equal charges each +*Q*, placed at the corners of on equilateral triangle of side a what will be the force on any charge $\left(k = \frac{1}{4\pi\varepsilon_0}\right)$ [RPET 2000]

(a) $\frac{kQ^2}{a^2}$ (b) $\frac{2kQ^2}{a^2}$ (c) $\frac{\sqrt{2}kQ^2}{a^2}$ (d) $\frac{\sqrt{3}kQ^2}{a^2}$

**Solution:** (d) Suppose net force is to be calculated on the charge which is kept at *A*. Two charges kept at *B* and *C* are applying force on that particular charge, with direction as shown in the figure.




Since $F_b = F_c = F = k\frac{Q^2}{a^2}$

So, $F_{net} = \sqrt{F_B^2 + F_C^2 + 2F_BF_C\cos 60}$

$F_{net} = \sqrt{3}F = \frac{\sqrt{3}kQ^2}{a^2}$

**Example: 16** Equal charges *Q* are placed at the four corners *A*, *B*, *C*, *D* of a square of length *a*. The magnitude of the force on the charge at *B* will be [MP PMT 1994]

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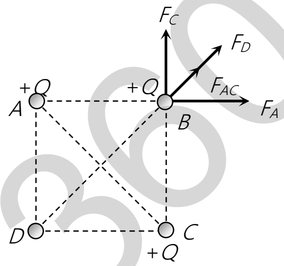
(a) $\frac{3Q^2}{4\pi\varepsilon_0 a^2}$ (b) $\frac{4Q^2}{4\pi\varepsilon_0 a^2}$ (c) $\left(\frac{1+1\sqrt{2}}{2}\right)\frac{Q^2}{4\pi\varepsilon_0 a^2}$ (d) $\left(2+\frac{1}{\sqrt{2}}\right)\frac{Q^2}{4\pi\varepsilon_0 a^2}$

**Solution:** (c) After following the guidelines mentioned above




$$F_{net} = F_{AC} + F_D = \sqrt{F_A^2 + F_C^2} + F_D$$

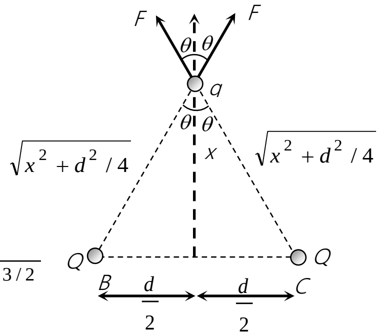
Since $F_A = F_C = \frac{kQ^2}{a^2}$ and $F_D = \frac{kQ^2}{(a\sqrt{2})^2}$

$$F_{net} = \frac{\sqrt{2}kQ^2}{a^2} + \frac{kQ^2}{2a^2} = \frac{kQ^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right) = \frac{Q^2}{4\pi\varepsilon_0 a^2}\left(\frac{1+2\sqrt{2}}{2}\right)$$

**Example: 17** Two equal charges are separated by a distance *d*. A third charge placed on a perpendicular bisector at *x* distance, will experience maximum *coulomb* force when

(a) $x = \frac{d}{\sqrt{2}}$ (b) $x = \frac{d}{2}$ (c) $x = \frac{d}{2\sqrt{2}}$ (d) $x = \frac{d}{2\sqrt{3}}$

**Solution:** (c) Suppose third charge is similar to *Q* and it is *q*




So net force on it $F_{net} = 2F\cos\theta$

Where $F = \frac{1}{4\pi\varepsilon_0}.\frac{Qq}{\left(x^2+\frac{d^2}{4}\right)}$ and $\cos\theta = \frac{x}{\sqrt{x^2+\frac{d^2}{4}}}$

$$\therefore \quad F_{net} = 2\times\frac{1}{4\pi\varepsilon_0}.\frac{Qq}{\left(x^2+\frac{d^2}{4}\right)}\times\frac{x}{\left(x^2+\frac{d^2}{4}\right)^{1/2}} = \frac{2Qqx}{4\pi\varepsilon_0\left(x^2+\frac{d^2}{4}\right)^{3/2}}$$

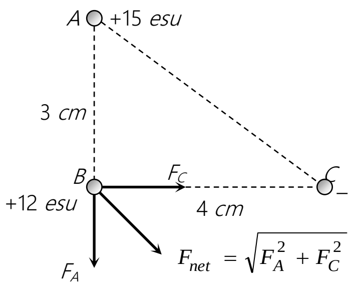
for $F_{net}$ to be maximum $\frac{dF_{net}}{dx} = 0$ i.e. $\frac{d}{dx}\left[\frac{2Qqx}{4\pi\varepsilon_0\left(x^2+\frac{d^2}{4}\right)^{3/2}}\right] = 0$

or $\left[\left(x^2+\frac{d^2}{4}\right)^{-3/2} - 3x^2\left(x^2+\frac{d^2}{4}\right)^{-5/2}\right] = 0$ i.e. $x = \pm\frac{d}{2\sqrt{2}}$

**Example: 18** *ABC* is a right angle triangle in which *AB* = 3 *cm*, *BC* = 4 cm and $\angle ABC = \frac{\pi}{2}$. The three charges +15, +12 and – 20 e.s.u. are placed respectively on *A*, *B* and *C*. The force acting on *B* is

(a) 125 *dynes* (b) 35 *dynes* (c) 25 *dynes* (d) Zero

**Solution:** (c) Net force on B $F_{net} = \sqrt{F_A^2 + F_C^2}$




$$F_A = \frac{15\times12}{(3)^2} = 20 \ \ dyne$$

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$$F_C = \frac{12 \times 20}{(4)^2} = 15 \ \ dyne$$

$$F_{net} = 25 \ \ dyne$$

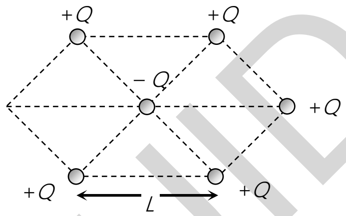
**Example: 19** Five point charges each of value $+Q$ are placed on five vertices of a regular hexagon of side $L$. What is the magnitude of the force on a point charge of value $-q$ placed at the centre of the hexagon **[IIT-JEE 1992]**

(a) $k\dfrac{Q^2}{L^2}$ (b) $k\dfrac{Q^2}{4L^2}$

(c) Zero (d) Information is insufficient

**Solution:** (a) Four charges cancels the effect of each other, so the net force on the charge placed at centre due to remaining fifth charge is

$$F = k\frac{Q^2}{L^2}$$



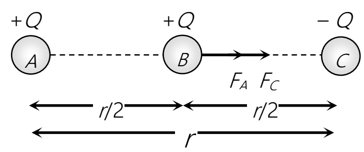

**Example: 20** Two small, identical spheres having $+Q$ and $-Q$ charge are kept at a certain distance. $F$ force acts between the two. If in the middle of two spheres, another similar sphere having $+Q$ charge is kept, then it experience a force in magnitude and direction as **[MP PET 1996]**

(a) Zero having no direction (b) $8F$ towards $+Q$ charge

(c) $8F$ towards $-Q$ charge (d) $4F$ towards $+Q$ charge

**Solution:** (c) Initially, force between $A$ and $C$ $\quad F = k\dfrac{Q^2}{r^2}$

When a similar sphere $B$ having charge $+Q$ is kept at the mid point of line joining $A$ and $C$, then Net force on $B$ is




$$F_{net} = F_A + F_C = k\frac{Q^2}{(r/2)^2} + \frac{kQ^2}{(r/2)^2} = 8\frac{kQ^2}{r^2} = 8F.$$ (Direction is shown in figure)

**Tricky example: 2**

Two equal spheres are identically charged with $q$ units of electricity separately. When they are placed at a distance $3R$ from centre-to-centre where $R$ is the radius of either sphere the force of repulsion between them is

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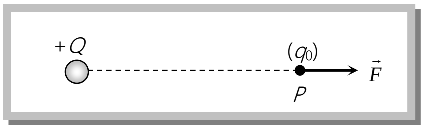
(a) $\frac{1}{4\pi\varepsilon_0}.\frac{q^2}{R^2}$ (b) $\frac{1}{4\pi\varepsilon_0}.\frac{q^2}{9R^2}$ (c) $\frac{1}{4\pi\varepsilon_0}.\frac{q^2}{4R^2}$ (d) None of these

**Solution:** (a) Generally students give the answer $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(3R)^2}$ but it is not true. Since the charges are not uniformly distributed, they cannot be treated as point charges and so we cannot apply coulombs law which is a law for point charges. The actual distribution is shown in the figure above.

## Electrical Field

A positive charge or a negative charge is said to create its field around itself. If a charge $Q_1$ exerts a force on charge $Q_2$ placed near it, it may be stated that since $Q_2$ is in the field of $Q_1$, it experiences some force, or it may also be said that since charge $Q_1$ is inside the field of $Q_2$, it experience some force. Thus space around a charge in which another charged particle experiences a force is said to have electrical field in it.

(1) **Electric field intensity ($\vec{E}$):** The electric field intensity at any point is defined as the force experienced by a unit positive charge placed at that point. $\vec{E} = \frac{\vec{F}}{q_0}$




Where $q_0 \to 0$ so that presence of this charge may not affect the source charge $Q$ and its electric field is not changed, therefore expression for electric field intensity can be better written as $\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$

(2) **Unit and Dimensional formula :** It's S.I. unit – $\frac{Newton}{coulomb} = \frac{volt}{meter} = \frac{Joule}{coulomb \times meter}$ and

C.G.S. unit – *Dyne/stat coulomb*.

Dimension : $[E] = [MLT^{-3}A^{-1}]$

(3) **Direction of electric field :** Electric field (intensity) $\vec{E}$ is a vector quantity. Electric field due to a positive charge is always away from the charge and that due to a negative charge is always towards the charge

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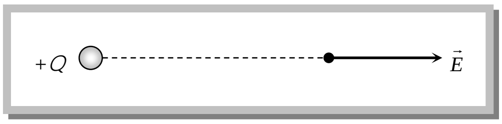
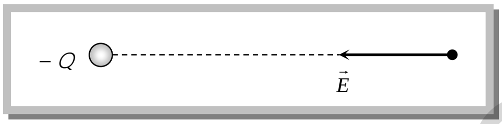
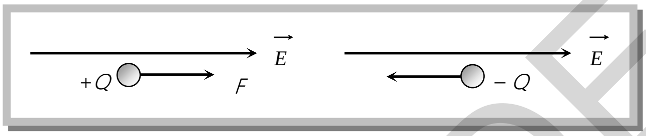


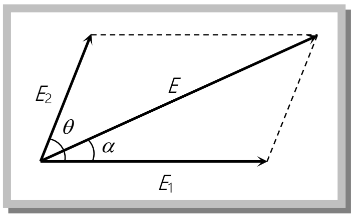




(4) **Relation between electric force and electric field :** In an electric field $\vec{E}$ a charge ($Q$) experiences a force $F = QE$. If charge is positive then force is directed in the direction of field while if charge is negative force acts on it in the opposite direction of field




(5) **Super position of electric field** (electric field at a point due to various charges) : The resultant electric field at any point is equal to the vector sum of electric fields at that point due to various charges.

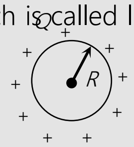
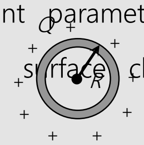
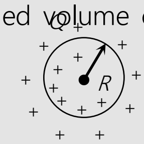
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$$




The magnitude of the resultant of two electric fields is given by

$E = \sqrt{E_1^2 + E_2^2 + 2E_1 E_2 \cos\theta}$ and the direction is given by $\tan\alpha = \dfrac{E_2 \sin\theta}{E_1 + E_2 \cos\theta}$

(6) **Electric field due to continuous distribution of charge :** A system of closely spaced electric charges forms a continuous charge distribution

| Continuous charge distribution | | |
|---|---|---|
| Linear charge distribution | Surface charge distribution | Volume charge distribution |
| In this distribution charge distributed on a line. For example : charge on a wire, charge on a ring etc. Relevant parameter is $\lambda$ which is called linear charge density *i.e.,* $\lambda = \dfrac{\text{charge}}{\text{length}}$   Circular charged | In this distribution charge distributed on the surface. For example : Charge on a conducting sphere, charge on a sheet etc. Relevant parameter is $\sigma$ which is called surface charge density *i.e.,*   Spherical shall | In this distribution charge distributed in the whole volume of the body. For example : Non conducting charged sphere. Relevant parameter is $\rho$ which is called volume charge density *i.e.,* $\rho = \dfrac{\text{charge}}{\text{volume}}$   Non conducting |

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| | | |
|---|---|---|
| $\lambda = \frac{Q}{2\pi R}$ | $\sigma = \frac{\text{charge}}{\text{area}}$<br>$\sigma = \frac{Q}{4\pi R^2}$ | $\rho = \frac{Q}{\frac{4}{3}\pi R^3}$ |

To find the field of a continuous charge distribution, we divide the charge into infinitesimal charge elements. Each infinitesimal charge element is then considered, as a point charge and electric field $\overrightarrow{dE}$ is determined due to this charge at given point. The Net field at the given point is the summation of fields of all the elements. *i.e.*, $\vec{E} = \int \overrightarrow{dE}$

## Electric Potential

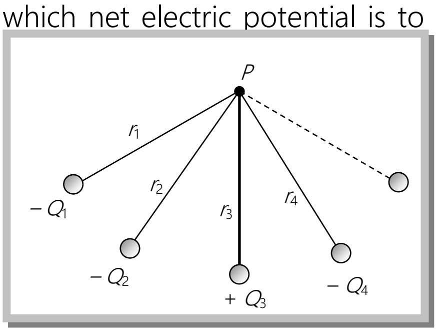
(1) **Definition :** Potential at a point in a field is defined as the amount of work done in bringing a unit positive test charge, from infinity to that point along any arbitrary path (infinity is point of zero potential). Electric potential is a scalar quantity, it is denoted by *V*; $V = \frac{W}{q_0}$

(2) **Unit and dimensional formula :** S. I. unit – $\frac{Joule}{Coulomb} = volt$ C.G.S. unit – *Stat volt* (e.s.u.); 1 *volt* $= \frac{1}{300}$ *Stat volt* Dimension – $[V] = [ML^2T^{-3}A^{-1}]$

(3) **Types of electric potential :** According to the nature of charge potential is of two types

(i) Positive potential : Due to positive charge.  (ii) Negative potential : Due to negative charge.

(4) **Potential of a system of point charges :** Consider *P* is a point at which net electric potential is to be determined due to several charges. So net potential at *P*




$$V = k\frac{Q_1}{r_1} + k\frac{Q_2}{r_2} + k\frac{Q_3}{r_3} + k\frac{(-Q_4)}{r_4} + \ldots$$

In general $V = \sum_{i=1}^{X} \frac{kQ_i}{r_i}$

*Note* : ❑ At the centre of two equal and opposite charge *V* = 0 but $E \neq 0$

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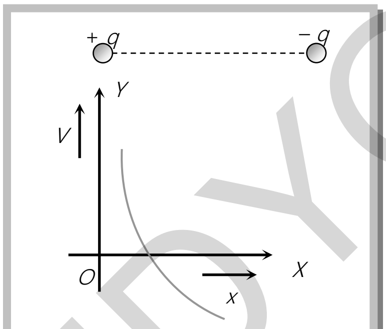
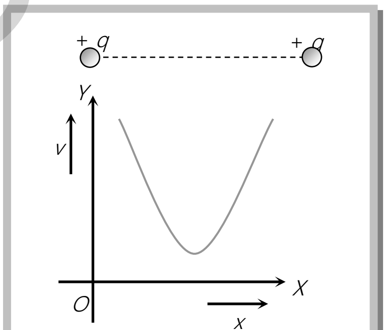
- At the centre of the line joining two equal and similar charge $V \neq 0, E = 0$

(5) **Electric potential due to a continuous charge distribution :** The potential due to a continuous charge distribution is the sum of potentials of all the infinitesimal charge elements in which the distribution may be divided *i.e.*, $\boldsymbol{V = \int dV}, \ = \int \frac{\boldsymbol{dQ}}{\boldsymbol{4\pi\varepsilon_0 r}}$

(6) **Graphical representation of potential :** When we move from a positive charge towards an equal negative charge along the line joining the two then initially potential decreases in magnitude and at centre become zero, but this potential is throughout positive because when we are nearer to positive charge, overall potential must be positive. When we move from centre towards the negative charge then though potential remain always negative but increases in magnitude fig. (A). As one move from one charge to other when both charges are like, the potential first decreases, at centre become minimum and then increases Fig. (B).



(A)



(B)

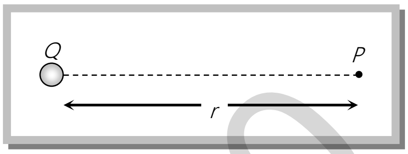
(7) **Potential difference :** In an electric field potential difference between two points *A* and *B* is defined as equal to the amount of work done (by external agent) in moving a unit positive charge from point *A* to point *B*.

*i.e.,* $V_B - V_A = \frac{W}{q_0}$ in general $W = Q.\Delta V$; $\Delta V =$ Potential difference through which charge *Q* moves.

## Electric Field and Potential Due to Various Charge Distribution

(1) **Point charge :** Electric field and potential at point *P* due to a point charge *Q* is

$$E = k\frac{Q}{r^2} \text{ or } \vec{E} = k\frac{Q}{r^2}\hat{r} \quad \left(k = \frac{1}{4\pi\varepsilon_0}\right), \quad V = k\frac{Q}{r}$$

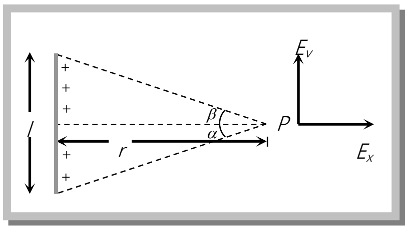



Note : ❑ Electric field intensity and electric potential due to a point charge $q$, at a distance $t_1 + t_2$ where $t_1$ is thickness of medium of dielectric constant $K_1$ and $t_2$ is thickness of medium of dielectric constant $K_2$ are :

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(t_1\sqrt{K_1} + t_2\sqrt{K_2})^2} \quad ; \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(t_1\sqrt{K_1} + t_2\sqrt{K_2})}$$

(2) **Line charge**

(i) **Straight conductor :** Electric field and potential due to a charged straight conducting wire of length $l$ and charge density $\lambda$




(a) **Electric field :** $E_x = \frac{k\lambda}{r}(\sin\alpha + \sin\beta)$ and $E_y = \frac{k\lambda}{r}(\cos\beta - \cos\alpha)$

If $\alpha = \beta$; $E_x = \frac{2k\lambda}{r}\sin\alpha$ and $E_y = 0$

If $l \to \infty$ *i.e.* $\alpha = \beta = \frac{\pi}{2}$; $E_x = \frac{2k\lambda}{r}$ and $E_y = 0$ so $E_{net} = \frac{\lambda}{2\pi\varepsilon_0 r}$

If $\alpha = 0$, $\beta = \frac{\pi}{2}$; $|E_x| = |E_y| = \frac{k\lambda}{r}$ so $E_{net} = \sqrt{E_x^2 + E_y^2} = \frac{\sqrt{2}\,k\lambda}{r}$

(b) **Potential :** $V = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left[\frac{\sqrt{r^2 + l^2} - 1}{\sqrt{r^2 + l^2} + 1}\right]$ for infinitely long conductor $V = \frac{-\lambda}{2\pi\varepsilon_0}\log_e r + c$

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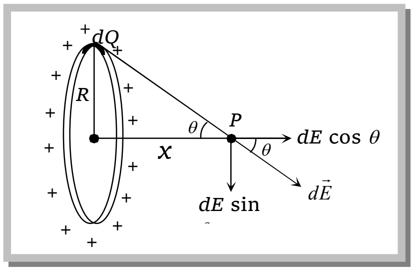
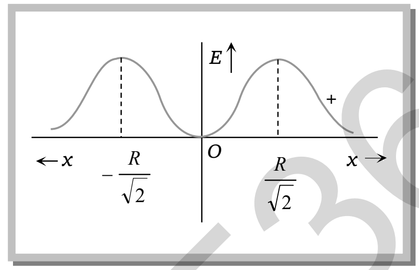
(ii) **Charged circular ring :** Suppose we have a charged circular ring of radius $R$ and charge $Q$. On it's axis electric field and potential is to be determined, at a point '$x$' away from the centre of the ring.




(a) **Electric field :** Consider an element carrying charge $dQ$. It's electric field $dE = \frac{KdQ}{(R^2+x^2)}$ directed as shown. It's component along the axis is $dE\cos\theta$ and perpendicular to the axis is $dE\sin\theta$. By symmetry $\int dE\sin\theta = 0$, hence $E = \int dE\cos\theta = \int \frac{kdQ}{(R^2+x^2)}\cdot\frac{x}{(R^2+x^2)^{1/2}}$

$$\boldsymbol{E} = \frac{\boldsymbol{kQx}}{(\boldsymbol{R}^2+\boldsymbol{x}^2)^{3/2}}$$ directed away from the centre if $Q$ is positive

(b) **Potential :** $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{\sqrt{x^2+R^2}}$

*Note* : ❑ At centre $x = 0$ so $E_{centre} = 0$ and $V_{centre} = \frac{kQ}{R}$

❑ At a point on the axis such that $x >> R$ $E = \frac{kQ}{x^2}$ and $V = \frac{kQ}{x}$

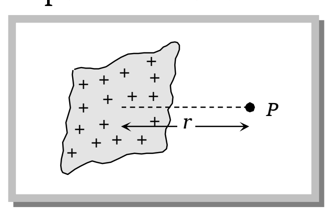
❑ At a point on the axis if $x = \pm\frac{R}{\sqrt{2}}$, $E_{\max} = \frac{Q}{6\sqrt{3}\pi\varepsilon_0 a^2}$

(3) **Surface charge :**

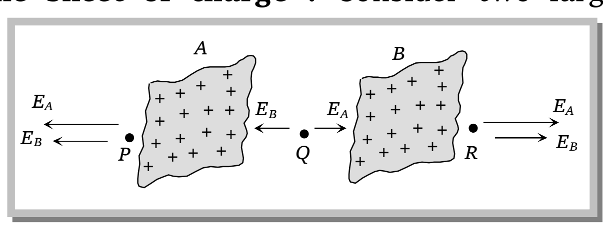
(i) **Infinite sheet of charge :** Electric field and potential at a point $P$ as shown

$$E = \frac{\sigma}{2\varepsilon_0} \quad (E \propto r^o)$$

and $$V = -\frac{\sigma r}{2\varepsilon_0} + C$$




(ii) **Electric field due to two parallel plane sheet of charge :** Consider two large, uniformly charged parallel. Plates $A$ and $B$, having surface charge densities are $\sigma_A$ and $\sigma_B$ respectively. Suppose net electric field at points $P$, $Q$ and $R$ is to be calculated.




At $P$, $E_P = (E_A + E_B) = \frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$

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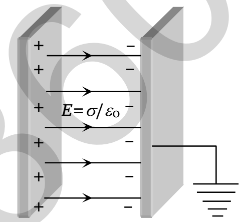
At $Q$, $E_Q = (E_A - E_B) = \frac{1}{2\varepsilon_0}(\sigma_A - \sigma_B)$; At $R$, $E_R = -(E_A + E_B) = -\frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$

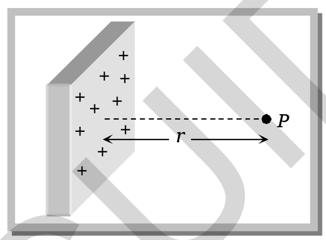
**Note:** ❑ If $\sigma_A = +\sigma$ and $\sigma_B = -\sigma$ then $E_p = 0, E_Q = \frac{\sigma}{\varepsilon_0}, E_R = 0$. Thus in case of two infinite plane sheets of charges having equal and opposite surface charge densities, the field is non-zero only in the space between the two sheets and is independent of the distance between them *i.e.,* field is uniform in this region. It should be noted that this result will hold good for finite plane sheet also, if they are held at a distance much smaller then the dimensions of sheets *i.e.,* parallel plate capacitor.



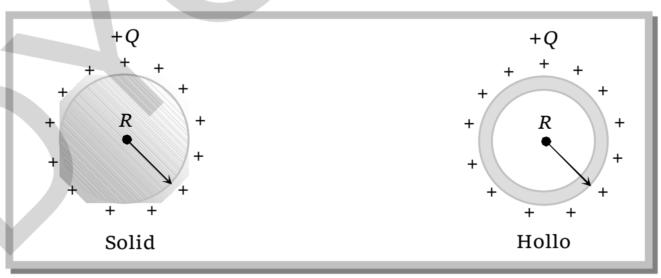

(iii) **Conducting sheet of charge :**

$$E = \frac{\sigma}{\varepsilon_0}$$

$$V = -\frac{\sigma r}{\varepsilon_0} + C$$




(iv) **Charged conducting sphere :** If charge on a conducting sphere of radius $R$ is $Q$ as shown in figure then electric field and potential in different situation are –




(a) **Out side the sphere :** $P$ is a point outside the sphere at a distance $r$ from the centre at which electric field and potential is to be determined.

Electric field at $P$

$$E_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2} \text{ and } V_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r} = \frac{\sigma R^2}{\varepsilon_0 r} \quad \begin{cases} Q = \sigma \times A \\ \;\;= \sigma \times 4\pi R^2 \end{cases}$$

(b) **At the surface of sphere :** At surface $r = R$

So, $$E_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^2} = \frac{\sigma}{\varepsilon_0} \text{ and } V_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{\sigma R}{\varepsilon_0}$$

(c) **Inside the sphere :** Inside the conducting charge sphere electric field is zero and potential remains constant every where and equals to the potential at the surface.

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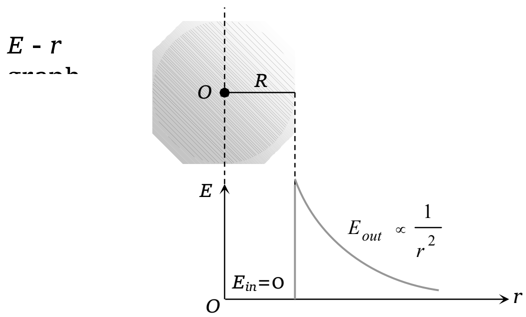
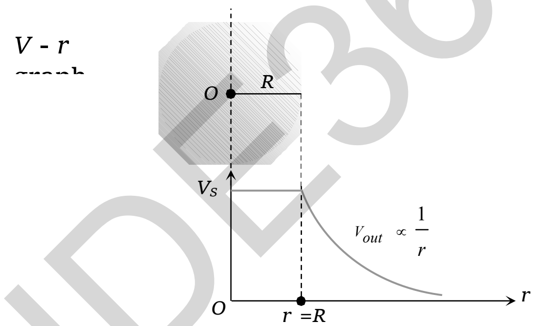
$E_{in} = 0$ and $V_{in}$ = constant $= V_s$

*Note* : ❑ Graphical variation of electric field and potential of a charged spherical conductor with distance

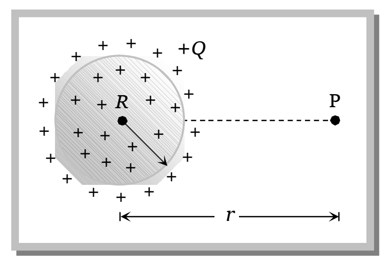



(4) **Volume charge (charged non-conducting sphere) :**

Charge given to a non conducting spheres spreads uniformly throughout it's volume.

(i) **Outside the sphere at *P***




$$E_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2} \text{ and } V_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r} \text{ by using } \rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$E_{out} = \frac{\rho R^3}{3\varepsilon_0 r^2} \text{ and } V_{out} = \frac{\rho R^3}{3\varepsilon_0 r}$$

(ii) **At the surface of sphere :** At surface $r = R$

$$E_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^2} = \frac{\rho R}{3\varepsilon_0} \qquad \text{and} \qquad V_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{\rho R^2}{3\varepsilon_0}$$

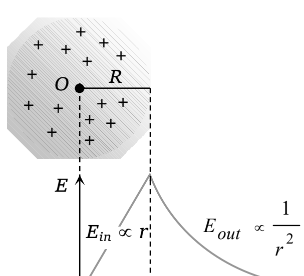
(iii) **Inside the sphere :** At a distance *r* from the centre

$$E_{in} = \frac{1}{4\pi\varepsilon_0}.\frac{Qr}{R^3} = \frac{\rho r}{3\varepsilon_0} \ \{E_{in} \propto r\} \qquad \text{and} \qquad V_{in} = \frac{1}{4\pi\varepsilon_0}\frac{Q[3R^2 - r^2]}{2R^3} = \frac{\rho(3R^2 - r^2)}{6\varepsilon_0}$$

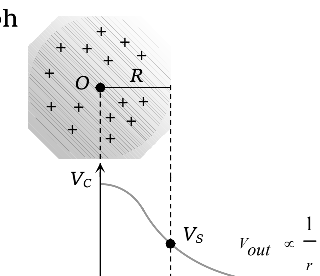
*Note* : ❑ At centre $r = 0$ So, $V_{\text{centre}} = \frac{3}{2} \times \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{3}{2} V_s$ *i.e.*, $\boldsymbol{V_{centre} > V_{surface} > V_{out}}$

❑ Graphical variation of electric field and potential with distance

*E*-*r* graph




*V*-*r* graph

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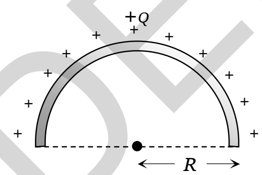
(5) **Electric field and potential in some other cases**

(i) **Uniformly charged semicircular ring :** $\lambda = \frac{\text{charge}}{\text{length}}$

At centre :

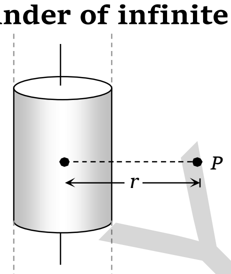



$$E = \frac{2K\lambda}{R} = \frac{Q}{2\pi^2 \varepsilon_0 R^2}$$

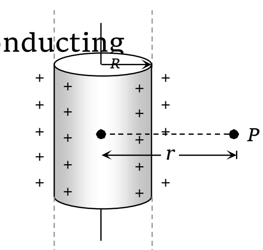
$$V = \frac{KQ}{R} = \frac{Q}{4\pi\varepsilon_0 R}$$

(iii) **Charged cylinder of infinite length**

(a) Conducting




(b) Non-conducting

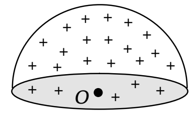



For both type of cylindrical charge distribution $E_{out} = \frac{\lambda}{2\pi\varepsilon_0 r}$, and $E_{suface} = \frac{\lambda}{2\pi\varepsilon_0 R}$ but for conducting $E_{in} = 0$ and for non-conducting $E_{\text{in}} = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$. (we can also write formulae in form of $\rho$ *i.e.,* $E_{\text{out}} = \frac{\rho R^2}{2\varepsilon_0 r}$ *etc.*)

(ii) **Hemispherical charged body :**

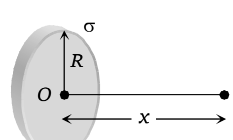
At centre $O$, $E = \frac{\sigma}{4\varepsilon_0}$

$$V = \frac{\sigma R}{2\varepsilon_0}$$




(iv) **Uniformly charged disc**

At a distance $x$ from centre $O$ on it's axis

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$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

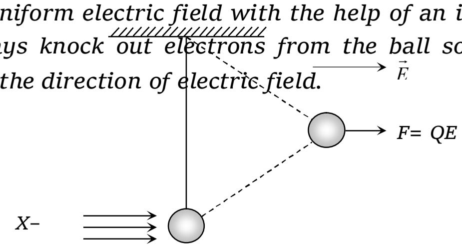
$$V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2 + R^2} - x\right]$$

*Note* :
- ❑ Total charge on disc $Q = \sigma\pi R^2$
- ❑ If $x \to 0$, $E \simeq \dfrac{\sigma}{2\varepsilon_0}$ *i.e.* for points situated near the disc, it behaves as an infinite sheet of charge.

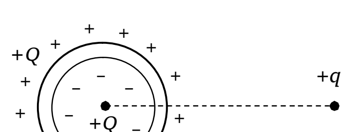
## *Concepts*

- ☞ *No point charge produces electric field at it's own position.*
- ☞ *Since charge given to a conductor resides on it's surface hence electric field inside it is zero.*


- ☞ *The electric field on the surface of a conductor is directly proportional to the surface charge density at that point i.e,* $E \propto \sigma$
- ☞ *Two charged spheres having radii* $r_1$ *and* $r_2$ *charge densities* $\sigma_1$ *and* $\sigma_2$ *respectively, then the ratio of electric field on their surfaces will be* $\dfrac{E_1}{E_2} = \dfrac{\sigma_1}{\sigma_2} = \dfrac{r_2^2}{r_1^2}$ $\left\{\sigma = \dfrac{Q}{4\pi r^2}\right.$
- ☞ *In air if intensity of electric field exceeds the value* $3 \times 10^6\ N/C$ *air ionizes.*
- ☞ *A small ball is suspended in a uniform electric field with the help of an insulated thread. If a high energy x–ray bean falls on the ball, x-rays knock out electrons from the ball so the ball is positively charged and therefore the ball is deflected in the direction of electric field.*




- ☞ *Electric field is always directed from higher potential to lower potential.*
- ☞ *A positive charge if left free in electric field always moves from higher potential to lower potential while a negative charge moves from lower potential to higher potential.*

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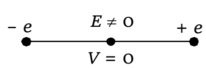
- *The practical zero of electric potential is taken as the potential of earth and theoretical zero is taken at infinity.*
- *An electric potential exists at a point in a region where the electric field is zero and it's vice versa.*
- *A point charge +Q lying inside a closed conducting shell does not exert force another point charge q placed outside the shell as shown in figure*

*Actually the point charge +Q is unable to exert force on the charge +q because it can not produce electric field at the position of +q. All the field lines emerging from the point charge +Q terminate inside as these lines cannot penetrate the conducting medium (properties of lines of force).*

*The charge q however experiences a force not because of charge +Q but due to charge induced on the outer surface of the shell.*



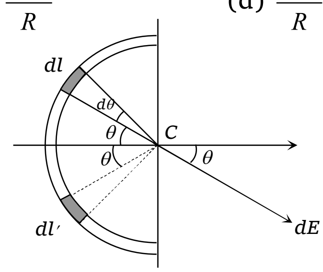

### *Examples based on electric field and electric*

**Example: 21** A half ring of radius *R* has a charge of $\lambda$ per unit length. The electric field at the centre is $\left(k = \frac{1}{4\pi\varepsilon_0}\right)$

**[CPMT 2000; CBSE PMT 2000; REE 1999]**

(a) Zero (b) $\frac{k\lambda}{R}$ (c) $\frac{2k\lambda}{R}$ (d) $\frac{k\pi\lambda}{R}$

**Solution:** (c) $dl = Rd\theta$




Charge on $dl = \lambda Rd\theta$.

Field at *C* due to $dl = k\frac{\lambda Rd\theta.}{R^2} = dE$

We need to consider only the component $dE\cos\theta$, as the component $dE\sin\theta$ will cancel out because of the field at *C* due to the symmetrical element *dl′*,

The total field at *C* is $= 2\int_0^{\pi/2} dE\cos\theta = 2\frac{k\lambda}{R}\int_0^{\pi/2}\cos\theta d\theta = 2k\frac{\lambda}{R}$ $\left\{= \frac{Q}{2\pi\varepsilon_0 R^2}\right\}$

**Example: 22** What is the magnitude of a point charge due to which the electric field 30 cm away has the magnitude 2 *newton/coulomb* $[1/4\pi\varepsilon_0 = 9\times10^9\ Nm^2]$

(a) $2\times10^{-11}$ *coulomb* (b) $3\times10^{-11}$ *coulomb* (c) $5\times10^{-11}$ *coulomb* (d) $9\times10^{-11}$ *coulomb*

**Solution:** (a) By using $E = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2}$; $2 = 9\times10^9\times\frac{Q}{(30\times10^{-2})^2} \Rightarrow Q = 2\times10^{-11}C$

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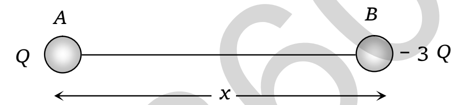
**Example: 23** Two point charges $Q$ and $-3Q$ are placed at some distance apart. If the electric field at the location of $Q$ is $E$, then at the locality of $-3Q$, it is

(a) $-E$ (b) $E/3$ (c) $-3E$ (d) $-E/3$

**Solution:** (b) Let the charge $Q$ and $-3Q$ be placed respectively at $A$ and $B$ at a distance $x$

Now we will determine the magnitude and direction to the field produced by charge $-3Q$ at A, this is $E$ as mentioned in the Example.




$\therefore E = \frac{3Q}{x^2}$ (along AB directed towards negative charge)

Now field at location of $-3Q$ i.e. field at $B$ due to charge $Q$ will be $E' = \frac{Q}{x^2} = \frac{E}{3}$ (along $AB$ directed away from positive charge)

**Example: 24** Two charged spheres of radius $R_1$ and $R_2$ respectively are charged and joined by a wire. The ratio of electric field of the spheres is

(a) $\frac{R_1}{R_2}$ (b) $\frac{R_2}{R_1}$ (c) $\frac{R_1^2}{R_2^2}$ (d) $\frac{R_2^2}{R_1^2}$

**Solution:** (b) After connection their potential becomes equal *i.e.*, $k.\frac{Q_1}{R_1} = \frac{k.Q_2}{R_2}$; $\Rightarrow$ $\frac{Q_1}{Q_2} = \frac{R_1}{R_2}$

Ratio of electric field $\frac{E_1}{E_2} = \frac{Q_1}{Q_2} \times \left(\frac{R_2}{R_1}\right)^2 = \frac{R_2}{R_1}$.

**Example: 25** The number of electrons to be put on a spherical conductor of radius 0.1$m$ to produce an electric field of 0.036 $N/C$ just above its surface is

(a) $2.7 \times 10^5$ (b) $2.6 \times 10^5$ (c) $2.5 \times 10^5$ (d) $2.4 \times 10^5$

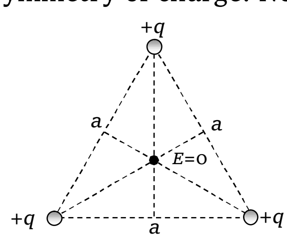
**Solution:** (c) By using $E = k\frac{Q}{R^2}$, where $R$ = radius of sphere so 0.036 $= 9 \times 10^9 \times \frac{ne}{(0.1)^2} \Rightarrow n = 2.5 \times 10^5$

**Example: 26** Eight equal charges each $+Q$ are kept at the corners of a cube. Net electric field at the centre will be $\left(k = \frac{1}{4\pi\varepsilon_0}\right)$

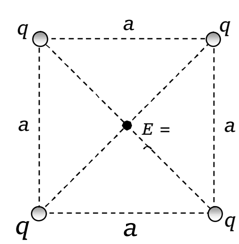
(a) $\frac{kQ}{r^2}$ (b) $\frac{8kQ}{r^2}$ (c) $\frac{2kQ}{r^2}$ (d) Zero

**Solution:** (d) Due to the symmetry of charge. Net Electric field at centre is zero.

Note : ❑

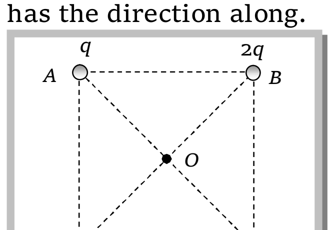



Equilateral




Square

**Example: 27** $q$, $2q$, $3q$ and $4q$ charges are placed at the four corners $A$, $B$, $C$ and $D$ of a square. The field at the centre $O$ of the square has the direction along.

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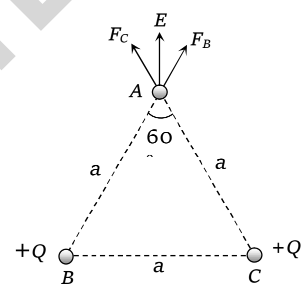
(a) $AB$ (b) $CB$ (c) $AC$ (d) $BD$

**Solution:** (b) By making the direction of electric field due to all charges at centre. Net electric field has the direction along $CB$

**Example: 28** Equal charges $Q$ are placed at the vertices $A$ and $B$ of an equilateral triangle $ABC$ of side $a$. The magnitude of electric field at the point $A$ is

(a) $\frac{Q}{4\pi\varepsilon_0 a^2}$ (b) $\frac{\sqrt{2}Q}{4\pi\varepsilon_0 a^2}$ (c) $\frac{\sqrt{3}Q}{4\pi\varepsilon_0 a^2}$ (d) $\frac{Q}{2\pi\varepsilon_0 a^2}$

**Solution:** (c) As shown in figure Net electric field at $A$

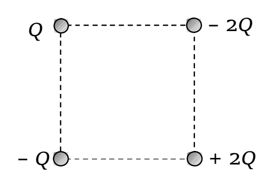



$$E = \sqrt{E_B^2 + E_C^2 + 2E_B E_C \cos 60}$$

$$E_B = E_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{a^2}$$

So, $E = \frac{\sqrt{3}Q}{4\pi\varepsilon_0 a^2}$

**Example: 29** Four charges are placed on corners of a square as shown in figure having side of 5 cm. If $Q$ is one micro coulomb, then electric field intensity at centre will be



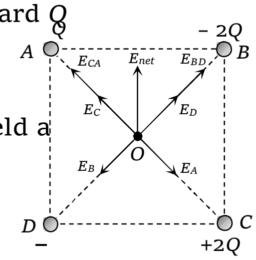

(a) $1.02 \times 10^7\ N/C$ upwards

(b) $2.04 \times 10^7\ N/C$ downwards

(c) $2.04 \times 10^7\ N/C$ upwards

(d) $1.02 \times 10^7\ N/C$ downwards

**Solution:** (a) $|E_C| > |E_A|$ so resultant of $E_C$ & $E_A$ is $E_{CA} = E_C - E_A$ directed toward $Q$




Also $|E_B| > |E_D|$ so resultant of $E_B$ and $E_D$ i.e.

$E_{BD} = E_B - E_D$ directed toward – 2$Q$ charge hence Net electric field a

$$E = \sqrt{(E_{CA})^2 + (E_{BD})^2} \quad \text{.... (i)}$$

By proper calculations $|E_A| = 9 \times 10^9 \times \frac{10^{-6}}{\left(\frac{5}{\sqrt{2}} \times 10^{-2}\right)^2} = 0.72 \times 10^7\ N/C$

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$$|E_B| = 9\times10^9 \times \frac{2\times10^{-6}}{\left(\frac{5}{\sqrt{2}}\times10^{-2}\right)^2} = 1.44\times10^7\,N/C\,; \quad |E_C| = 9\times10^9\times\frac{2\times10^{-6}}{\left(\frac{5}{\sqrt{2}}\times10^{-2}\right)^2} = 1.44\times10^7\,N/C$$

$$|E_D| = 9\times10^9\times\frac{10^{-6}}{\left(\frac{5}{\sqrt{2}}\times10^{-2}\right)^2} = 0.72\times10^7\,N/C\,; \quad \text{So,}\ |E_{CA}| = |E_C| - |E_A| = 0.72\times10^7\,N/C$$

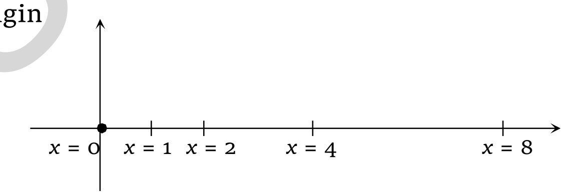
and $|E_{BD}| = |E_B| - |E_D| = 0.72\times10^7\,N/C.$ Hence from equation – (i) $E = 1.02\times10^7\,N/C$ upwards

**Example: 30** Infinite charges are lying at $x$ = 1, 2, 4, 8...*meter* on $X$-axis and the value of each charge is $Q$. The value of intensity of electric field and potential at point $x$ = 0 due to these charges will be respectively

(a) $12\times10^9 Q$ $N/C$, 1.8 × 10⁴ $V$ (b) Zero, 1.2 × 10⁴$V$

(c) $6\times10^9 Q$ $N/C$, 9 × 10³ $V$ (d) $4\times10^9 Q$ $N/C$, 6 × 10³ $V$

**Solution:** (a) By the superposition, Net electric field at origin




$$E = kQ\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{8^2}+...\infty\right]$$

$$E = kQ\left[1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+...\infty\right]$$

$1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+...\infty$ is an infinite geometrical progression it's sum can be obtained by using the formula $S_\infty = \frac{a}{1-r}$ ; Where $a$ = First term, $r$ = Common ratio.

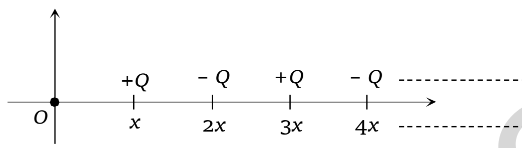
Here $a = 1$ and $r = \frac{1}{4}$ so, $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+.....\infty = \frac{1}{1-1/4} = \frac{4}{3}$.

Hence $E = 9\times10^9\times Q\times\frac{4}{3} = 12\times10^9\,Q\,N/C$

Electric potential at origin $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{1\times10^{-6}}{1}+\frac{1\times10^{-6}}{2}+\frac{1\times10^{-6}}{4}+\frac{1\times10^{-6}}{8}+.......\infty\right]$

$$= 9\times10^9\times10^{-6}\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+..........\ ..\infty\right] = 9\times10^3\left[\frac{1}{1-\frac{1}{2}}\right] = 1.8\times10^4\ volt$$

*Note* : ❑ In the arrangement shown in figure $+Q$ and $-Q$ are alternatively and equally spaced from each other, the net potential at the origin $O$ is $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \log_e 2}{x}$ **[IIT 1998]**




**Example: 31** Potential at a point $x$-distance from the centre inside the conducting sphere of radius $R$ and charged with charge $Q$ is **[MP PMT 2001]**

(a) $\frac{Q}{R}$ (b) $\frac{Q}{x}$ (c) $\frac{Q}{x^2}$ (d) $xQ$

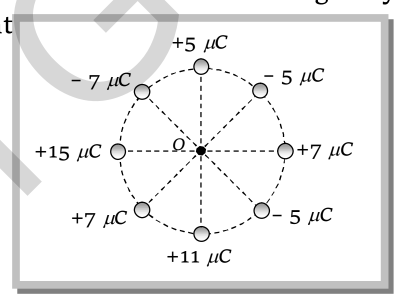
**Solution:** (a) Potential inside the conductor is constant.

**Example: 32** The electric potential at the surface of an atomic nucleus ($Z$ = 50) of radius $9 \times 10^5\ V$ is

(a) 80 $V$ (b) $8 \times 10^6\ V$ (c) 9 $V$ (d) $9 \times 10^5\ V$

**Solution:** (b) $V = 9 \times 10^9 \times \frac{ne}{r} = 9 \times 10^9 \times \frac{50 \times 1.6 \times 10^{-19}}{9 \times 10^{-15}} = 8 \times 10^6\ V$

**Example: 33** Eight charges having the valves as shown are arranged symmetrically on a circle of radius 0.4$m$ in air. Potential at cent

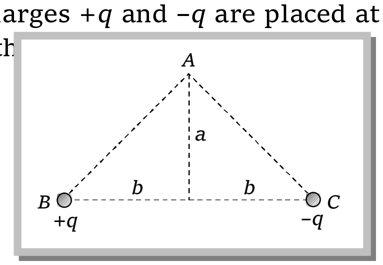



(a) $63 \times 10^4$ *volt* (b) $63 \times 10^{10}$ *volt* (c) $63 \times 10^6$ *volt* (d) Zero

**Solution:** (a) Due to the principle of superposition potential at $O$

$$V = \frac{1}{4\pi\varepsilon_0} \times \frac{28 \times 10^{-6}}{0.4} = 9 \times 10^9 \times \frac{28 \times 10^{-6}}{0.4} = 63 \times 10^4\ volt$$

**Example: 34** As shown in the figure, charges $+q$ and $-q$ are placed at the vertices $B$ and $C$ of an isosceles triangle. The potential at th

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(a) $\frac{1}{4\pi\varepsilon_0}.\frac{2q}{\sqrt{a^2+b^2}}$ (b) $\frac{1}{4\pi\varepsilon_0}.\frac{q}{\sqrt{a^2+b^2}}$ (c) $\frac{1}{4\pi\varepsilon_0}.\frac{(-q)}{\sqrt{a^2+b^2}}$ (d) Zero

**Solution:** (d) Potential at $A$ = Potential due to $(+q)$ charge + Potential due to $(-q)$ charge

$$=\frac{1}{4\pi\varepsilon_0}.\frac{q}{\sqrt{a^2+b^2}}+\frac{1}{4\pi\varepsilon_0}\frac{(-q)}{\sqrt{a^2+b^2}}=0$$

**Example: 35** A conducting sphere of radius $R$ is given a charge $Q$. consider three points $B$ at the surface, $A$ at centre and $C$ at a distance $R/2$ from the centre. The electric potential at these points are such that **[DCE 1994]**

(a) $V_A = V_B = V_C$ (b) $V_A = V_B \neq V_C$ (c) $V_A \neq V_B \neq V_C$ (d) $V_A \neq V_B = V_C$

**Solution:** (a) Potential inside a conductor is always constant and equal to the potential at the surface.

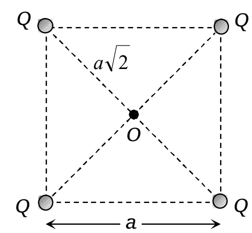
**Example: 36** Equal charges of $\frac{10}{3}\times10^{-9}$ coulomb are lying on the corners of a square of side 8 $cm$. The electric potential at the point of intersection of the diagonals will be

(a) 900 $V$ (b) $900\sqrt{2}\ V$ (c) $150\sqrt{2}\ V$ (d) $1500\sqrt{2}\ V$

**Solution:** (d) Potential at the centre $O$

$$V=4\times\frac{1}{4\pi\varepsilon_0}.\frac{Q}{a/\sqrt{2}} \text{ given } Q=\frac{10}{3}\times10^{-9}\,C \Rightarrow a=8\,cm=8\times10^{-2}\,m$$

$$V=5\times9\times10^9\times\frac{\frac{10}{3}\times10^{-9}}{\frac{8\times10^{-2}}{\sqrt{2}}}=1500\sqrt{2}\ volt$$



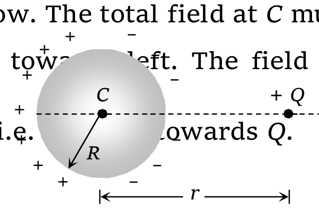

**Tricky example: 3**

A point charge $Q$ is placed outside a hollow spherical conductor of radius $R$, at a distance $(r > R)$ from its centre $C$. The field at $C$ due to the induced charges on the conductor is $\left(K=\frac{1}{4\pi\varepsilon_0}\right)$

(a) Zero (b) $K\frac{Q}{(r-R)^2}$

(c) $K\frac{Q}{r^2}$ directed towards $Q$ (d) $K\frac{Q}{r^2}$ directed away from $Q$

**Solution:** (c) A according to the figure shown below. The total field at $C$ must be zero. The field at $C$ due to the point charge is $E=K\frac{Q}{r^2}$ towards left. The field at $C$ due to the induced charges must be $\frac{KQ}{r^2}$ towards right i.e. towards $Q$.




**Tricky example: 4**

A point charge $q$ is placed at a distance of $r$ from the centre of an uncharged conducting sphere of radius $R$ ($< r$). The potential at any point on the sphere is

(a) Zero (b) $\frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$ (c) $\frac{1}{4\pi\varepsilon_0}.\frac{qR}{r^2}$ (d) $\frac{1}{4\pi\varepsilon_0}.\frac{qr^2}{R}$

**Solution:** (c) Since, potential $V$ is same for all points of the sphere. Therefore, we can calculate its value at the centre of the sphere.

$\therefore \; V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r} + V'$; where $V'$ = potential at centre due to induced charge = 0 (because net induced charge will be zero) $\therefore \; V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$.

## Potential Due to Concentric Spheres

To find potential at a point due to concentric sphere following guideline are to be considered

**Guideline 1:** Identity the point ($P$) at which potential is to be determined.

**Guideline 2:** Start from inner most sphere, you should know where point ($P$) lies *w.r.t.* concerning sphere/shell (*i.e.* outside, at surface or inside)

**Guideline 3:** Then find the potential at the point ($P$) due to inner most sphere and then due to next and so on.

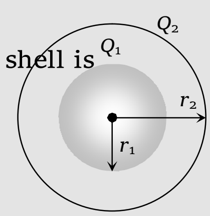
**Guideline 4:** Using the principle of superposition find net potential at required shell/sphere.

### Standard cases

**Case (i) : If two concentric conducting shells of radii $r_1$ and $r_2$($r_2 > r_1$) carrying uniformly distributed charges $Q_1$ and $Q_2$ respectively. What will be the potential of each shell**

To find the solution following guidelines are to be taken.

Here after following the above guideline potential at the surface of inner shell is




$$V_1 = \frac{1}{4\pi\varepsilon_0}.\frac{Q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}.\frac{Q_2}{r_2}$$

and potential at the surface of outer shell

$$V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{Q_1}{r_2} + \frac{1}{4\pi\varepsilon_0}.\frac{Q_2}{r_2}$$

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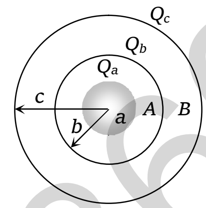
**Case (ii) : The figure shows three conducting concentric shell of radii *a, b* and c (*a* < *b* < c) having charges $Q_a$, $Q_b$ and $Q_c$ respectively what will be the potential of each shell**

After following the guidelines discussed above




Potential at *A*; $V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_a}{a} + \frac{Q_b}{b} + \frac{Q_c}{c}\right]$

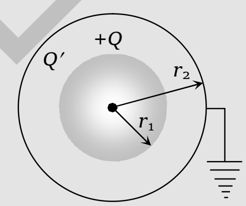
Potential at *B*; $V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_a}{b} + \frac{Q_b}{b} + \frac{Q_c}{c}\right]$

Potential at *C*; $V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_a}{c} + \frac{Q_b}{c} + \frac{Q_c}{c}\right]$

**Case (iii) : The figure shows two concentric spheres having radii $r_1$ and $r_2$ respectively ($r_2 > r_1$). If charge on inner sphere is +*Q* and outer sphere is earthed then determine.**

(a) **The charge on the outer sphere**

(b) **Potential of the inner sphere**




(*i*) Potential at the surface of outer sphere $V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r_2} + \frac{1}{4\pi\varepsilon_0}.\frac{Q'}{r_2} = 0$

$\Rightarrow Q' = -Q$

(*ii*) Potential of the inner sphere $V_1 = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{r_2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

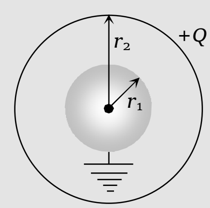
**Case (iv) : In the case III if outer sphere is given a charge +*Q* and inner sphere is earthed then**

(a) **What will be the charge on the inner sphere**

(b) **What will be the potential of the outer sphere**

(*i*) In this case potential at the surface of inner sphere is zero, so if $Q'$ is the charge induced on inner sphere

then $V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q'}{r_1} + \frac{Q}{r_2}\right] = 0$ *i.e.*, $Q' = -\frac{r_1}{r_2}Q$




(Charge on inner sphere is less than that of the outer sphere.)

(*ii*) Potential at the surface of outer sphere

$$V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{Q'}{r_2} + \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r_2}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0 r_2}\left[-Q\frac{r_1}{r_2} + Q\right] = \frac{Q}{4\pi\varepsilon_0 r_2}\left[1 - \frac{r_1}{r_2}\right]$$

***Examples based on concentric***

**Example: 37** A hollow metal sphere of radius 5 *cm* is charged such that the potential on its surface is 10 *volts*. The potential at the centre of the sphere is

(a) Zero (b) 10 *V*

(c) Same as at a point 5 *cm* away from the surface (d) Same as at a point 25 *cm* away from the surface

**Solution:** (b) Inside the conductors potential remains same and it is equal to the potential of surface, so here potential at the centre of sphere will be 10 *V*

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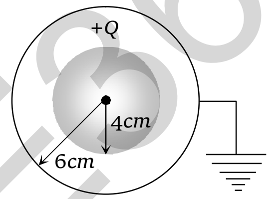
**Example: 38** A sphere of 4 *cm* radius is suspended within a hollow sphere of 6 *cm* radius. The inner sphere is charged to a potential 3 *e.s.u.* When the outer sphere is earthed. The charge on the inner sphere is **[MP PMT 1991]**

(a) 54 *e.s.u.* (b) $\frac{1}{4}$ *e.s.u.* (c) 30 *e.s.u.* (d) 36 *e.s.u.*

**Solution:** (d) Let charge on inner sphere be $+Q$. charge induced on the inner surface of outer sphere will be $-Q$.

So potential at the surface of inner sphere (in CGS)




$$3 = \frac{Q}{4} - \frac{Q}{6}$$

$$\Rightarrow \quad Q = 36 \ \ e.s.u.$$

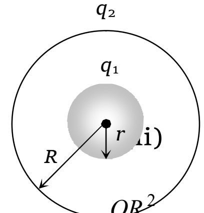
**Example: 39** A charge $Q$ is distributed over two concentric hollow spheres of radii $r$ and $(R > r)$ such that the surface densities are equal. The potential at the common centre is

(a) $\frac{Q(R^2+r^2)}{4\pi\varepsilon_0(R+r)}$ (b) $\frac{Q}{R+r}$ (c) Zero (d) $\frac{Q(R+r)}{4\pi\varepsilon_0(R^2+r^2)}$

**Solution:** (d) If $q_1$ and $q_2$ are the charges on spheres of radius $r$ and $R$ respectively, in accordance with conservation of charge

$$Q = q_1 + q_2 \quad ....\text{(i)}$$

and according to the given problem $\sigma_1 = \sigma_2$




*i.e.,* $\frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2} \Rightarrow \quad \frac{q_1}{q_2} = \frac{r^2}{R^2}$ ....(ii)

So equation (i) and (ii) gives $q_1 = \frac{Qr^2}{(R^2+r^2)}$ and $q_2 = \frac{QR^2}{(R^2+r^2)}$

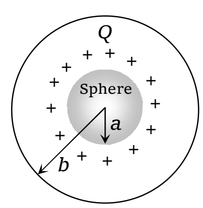
Potential at common centre $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r} + \frac{q_2}{R}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qr}{(R^2+r^2)} + \frac{QR}{(R^2+r^2)}\right] = \frac{1}{4\pi\varepsilon_0}.\frac{Q(R+r)}{(R^2+r^2)}$

**Example: 40** A solid conducting sphere having a charge $Q$ is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be $V$. If the shell is now given a charge of – $3Q$, the new potential difference between the two surfaces is

(a) $V$ (b) $2V$ (c) $4V$ (d) $-2V$

**Solution:** (a) If $a$ and $b$ are radii of spheres and spherical shell respectively, potential at their surfaces will be

$$V_{\text{sphere}} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{a} \quad \text{and} \quad V_{\text{shell}} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{b}$$




and so according to the given problem.

$$V = V_{\text{sphere}} - V_{\text{shell}} \quad = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] \quad .... \text{(i)}$$

Now when the shell is given a charge $-3Q$ the potential at its surface and also inside will change by $V_0 = \frac{1}{4\pi\varepsilon_0}\left[-\frac{3Q}{b}\right]$

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So that now $V_{\text{sphere}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{a} - \frac{3Q}{b}\right]$ and $V_{\text{shell}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{b} - \frac{3Q}{b}\right]$ hence $V_{\text{sphere}} - V_{\text{shell}} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] = V$

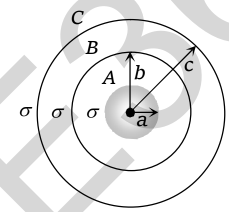
**Example: 41** Three concentric metallic spheres *A, B* and *C* have radii *a, b* and *c* $(a < b < c)$ and surface charge densities on them are $\sigma, -\sigma$ and $\sigma$ respectively. The valves of $V_A$ and $V_B$ will be

(a) $\frac{\sigma}{\varepsilon_0}(a-b-c), \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{b} - b + c\right)$

(b) $(a-b-c), \frac{a^2}{c}$

(c) $\frac{\varepsilon_0}{\sigma}(a-b-c), \frac{\varepsilon_0}{\sigma}\left(\frac{a^2}{c} - b + c\right)$

(d) $\frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$ and $\frac{\sigma}{\varepsilon_0}(a-b+c)$




**Solution:** (a) Suppose charges on *A, B* and *C* are $q_a, q_b$ and $q_c$

Respectively, so $\sigma_A = \sigma = \frac{q_a}{4\pi a^2} \Rightarrow q_a = \sigma \times 4\pi a^2$, $\sigma_B = -\sigma = \frac{q_b}{4\pi b^2} \Rightarrow q_b = -\sigma \times 4\pi b^2$

and $\sigma_C = \sigma = \frac{q_c}{4\pi c^2} \Rightarrow q_c = \sigma \times 4\pi c^2$

Potential at the surface of *A*

$$V_A = (V_A)_{\text{surface}} + (V_B)_{\text{in}} + (V_C)_{\text{in}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_a}{a} + \frac{q_b}{b} + \frac{q_c}{c}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{\sigma \times 4\pi a^2}{a} + \frac{(-\sigma) \times 4\pi b^2}{b} + \frac{\sigma \times 4\pi c^2}{c}\right]$$

$$V_A = \frac{\sigma}{\varepsilon_0}[a-b-c]]$$

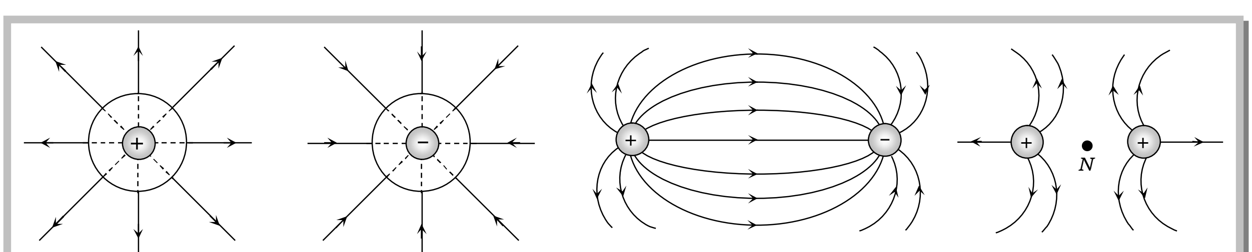
Potential at the surface of *B*

$$V_B = (V_A)_{\text{out}} + (V_B)_{\text{surface}} + (V_C)_{\text{in}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_a}{b} + \frac{q_b}{b} + \frac{q_c}{c}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{\sigma \times 4\pi a^2}{b} - \frac{\sigma \times 4\pi b^2}{b} + \frac{\sigma \times 4\pi c^2}{c}\right] = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right]$$

## Electric Lines of Force

(1) **Definition :** The electric field in a region is represented by continuous lines (also called lines of force). Field line is an imaginary line along which a positive test charge will move if left free.

Electric lines of force due to an isolated positive charge, isolated negative charge and due to a pair of charge are shown below

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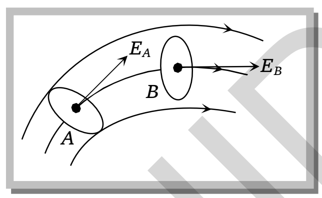
(2) **Properties of electric lines of force**

(i) Electric field lines come out of positive charge and go into the negative charge.

(ii) Tangent to the field line at any point gives the direction of the field at that point.

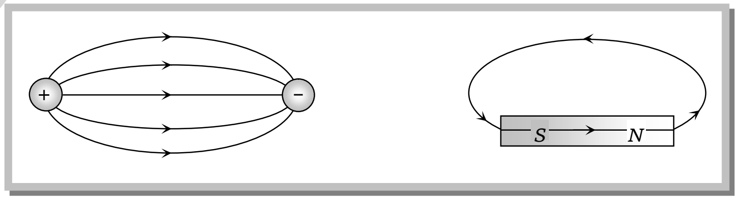



(iii) Field lines never cross each other.

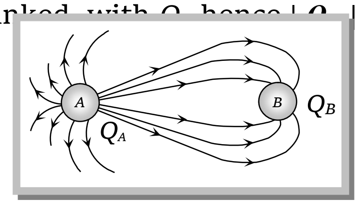
(iv) Field lines are always normal to conducting surface.


(v) Field lines do not exist inside a conductor.

(vi) The electric field lines never form closed loops. (While magnetic lines of forces form closed loop)




(vii) The number of lines originating or terminating on a charge is proportional to the magnitude of charge. In the following figure electric lines of force are originating from $A$ and terminating at $B$ hence $Q_A$ is positive while $Q_B$ is negative, also number of electric lines at force linked with $Q_A$ are more than those linked with $Q_B$ hence $|Q_A| > |Q_B|$

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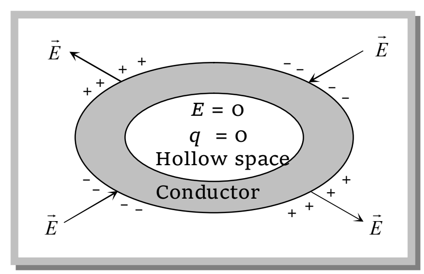
(viii) Number of lines of force per unit area normal to the area at a point represents magnitude of intensity (concept of electric flux *i.e.,* $\phi = EA$ )

(ix) If the lines of forces are equidistant and parallel straight lines the field is uniform and if either lines of force are not equidistant or straight line or both the field will be non uniform, also the density of field lines is proportional to the strength of the electric field. For example see the following figures.


(3) **Electrostatic shielding :** Electrostatic shielding/screening is the phenomenon of protecting a certain region of space from external electric field. Sensitive instruments and appliances are affected seriously with strong external electrostatic fields. Their working suffers and they may start misbehaving under the effect of unwanted fields.

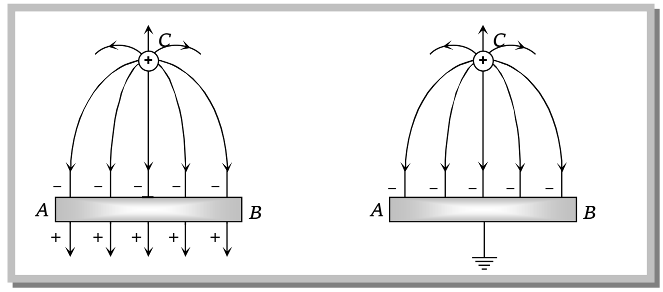



The electrostatic shielding can be achieved by protecting and enclosing the sensitive instruments inside a hollow conductor because inside hollow conductors, electric fields is zero.

(i) It is for this reason that it is safer to sit in a car or a bus during lightening rather than to stand under a tree or on the open ground.

(ii) A high voltage generator is usually enclosed in such a cage which is earthen. This would prevent the electrostatic field of the generator from spreading out of the cage.

(iii) An earthed conductor also acts as a screen against the electric field. When conductor is not earthed field of the charged body $C$ due to electrostatic induction continues beyond $AB$. If $AB$ is earthed, induced positive charge neutralizes and the field in the region beyond $AB$ disappears.

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## Equipotential Surface or Lines

If every point of a surface is at same potential, then it is said to be an equipotential surface

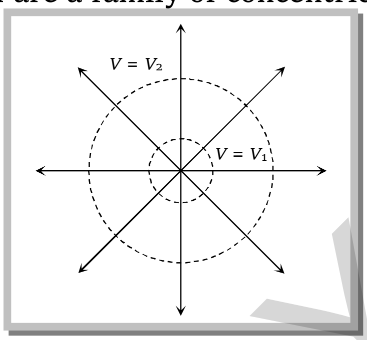
**or**

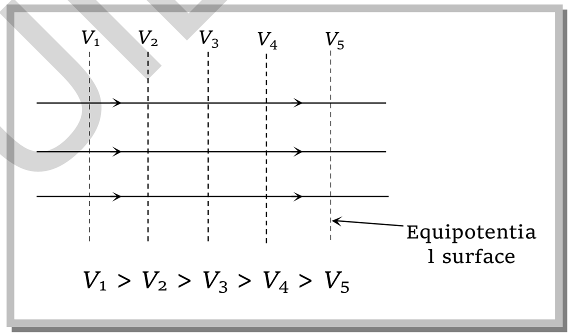
for a given charge distribution, locus of all points having same potential is called "equipotential surface" regarding equipotential surface following points should keep in mind :

(1) The density of the equipotential lines gives an idea about the magnitude of electric field. Higher the density larger the field strength.

(2) The direction of electric field is perpendicular to the equipotential surfaces or lines.

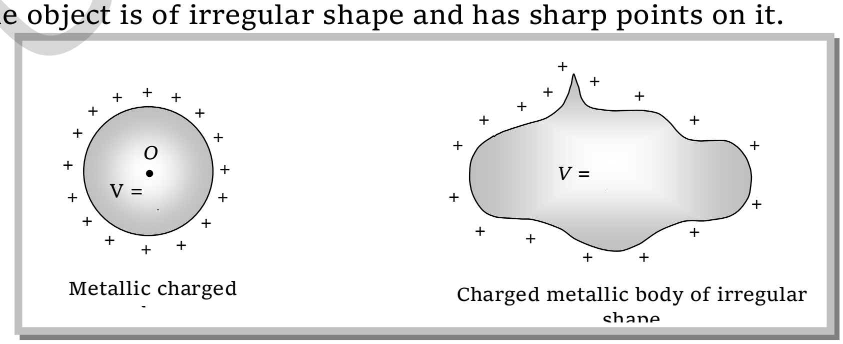
(3) The equipotential surfaces produced by a point charge or a spherically charge distribution are a family of concentric spheres.




(4) For a uniform electric field, the equipotential surfaces are a family of plane perpendicular to the field lines.

(5) A metallic surface of any shape is an equipotential surface *e.g.* When a charge is given to a metallic surface, it distributes itself in a manner such that its every point comes at same potential even if the object is of irregular shape and has sharp points on it.




If it is not so, that is say if the sharp points are at higher potential then due to potential difference between these points connected through metallic portion, charge will flow from

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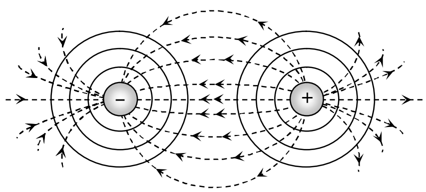
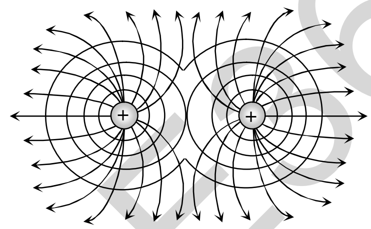
points of higher potential to points of lower potential until the potential of all points become same.

(6) Equipotential surfaces can never cross each other

(7) Equipotential surface for pair of charges



Pair of two equal and opposite



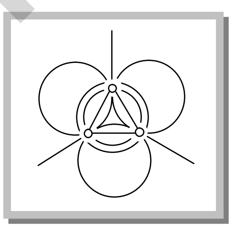
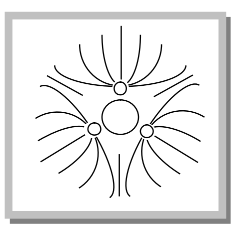
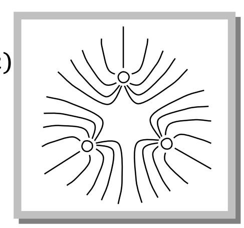
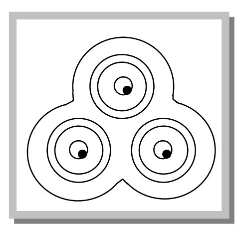
Pair of two equal and similar

### *Concepts*

- *Unit field i.e. 1N/C is defined arbitrarily as corresponding to unit density of lines of force.*
- *Number of lines originating from a unit charge is* $\frac{1}{\varepsilon_0}$
- *It is a common misconception that the path traced by a positive test charge is a field line but actually the path traced by a unit positive test charge represents a field full line only when it moves along a straight line.*
- *Both the equipotential surfaces and the lines of force can be used to depict electric field in a certain region of space. The advantage of using equipotential surfaces over the lines of force is that they give a visual picture of both the magnitude and direction of the electric field.*

## ***Examples based on electric lines of***

**Example: 42** Three positive charges of equal value $q$ are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in

(a)  (b)  (c)  (d) 

**Solution** (c) Option (a) shows lines of force starting from one positive charge and terminating at another. Option (b) has one line of force making closed loop. Option (d) shows all lines making closed loops. All these are not correct. Hence option (c) is correct

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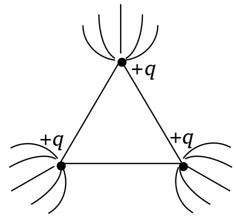
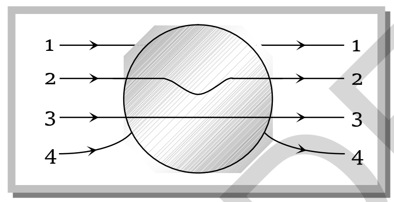



**Example: 43** A metallic sphere is placed in a uniform electric field. The lines of force follow the path (s) shown in the figure as

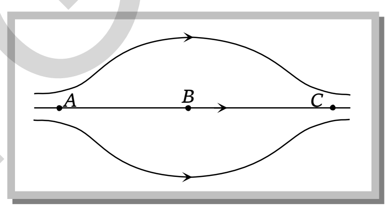



(a) 1 (b) 2 (c) 3 (d) 4

**Solution:** (d) The field is zero inside a conductor and hence lines of force cannot exist inside it. Also, due to induced charges on its surface the field is distorted close to its surface and a line of force must deviate near the surface outside the sphere.

**Example: 44** The figure shows some of the electric field lines corresponding to an electric field. The figure suggests

**[MP PMT 1999]**




(a) $E_A > E_B > E_C$ (b) $E_A = E_B = E_C$ (c) $E_A = E_C > E_B$ (d) $E_A = E_C < E_B$

**Solution:** (c)

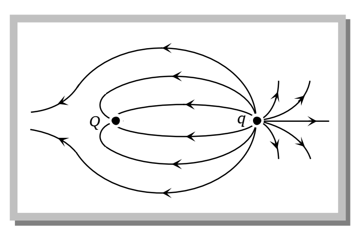
**Example: 45** The lines of force of the electric field due to two charges $q$ and $Q$ are sketched in the figure. State if

(a) $Q$ is positive and $|Q| > |q|$

(b) $Q$ is negative and $|Q| > |q|$

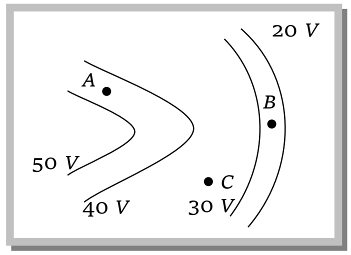
(c) $q$ is positive and $|Q| < |q|$

(d) $q$ is negative and $|Q| < |q|$




**Solution:** (c) $q$ is +*ve* because lines of force emerge from it and $|Q| < |q|$ because more lines emerge from $q$ and less lines terminate at $Q$.

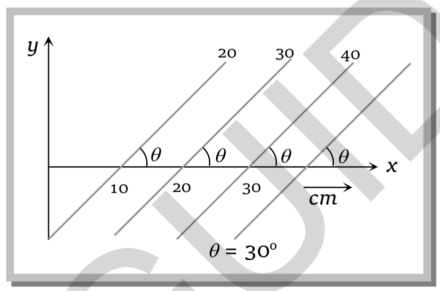
**Example: 46** The figure shows the lines of constant potential in a region in which an electric field is present. The magnitude of electric field is maximum at




(a) $A$ (b) $B$ (c) $C$ (d) Equal at $A$, $B$ and $C$

**Solution:** (b) Since lines of force are denser at $B$ hence electric field is maximum at $B$

**Example: 47** Some equipotential surface are shown in the figure. The magnitude and direction of the electric field is



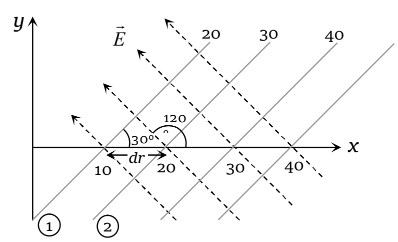

(a) 100 $V/m$ making angle 120° with the $x$-axis (b) 100 $V/m$ making angle 60° with the $x$-axis

(c) 200 $V/m$ making angle 120° with the $x$-axis (d) None of the above

**Solution:** (c) By using $dV = E\,dr\cos\theta$ suppose we consider line 1 and line 2 then

$(30 - 20) = E\cos 60^\circ\,(20 - 10) \times 10^{-2}$

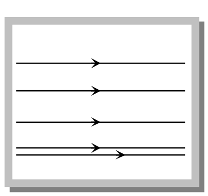
So $E = 200\ volt/m$ making in angle 120° with $x$-axis



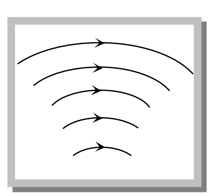

## Tricky example: 5

Which of the following maps cannot represent an electric field

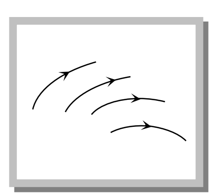
(a)



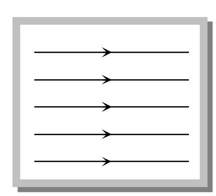
(b)

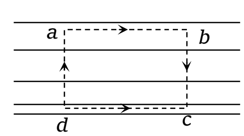


(c)



(d)



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**Solution:** (a) If we consider a rectangular closed path, two parallel sides of it considering with lines of force as shown, then we find that work done along the closed path $abcd$ is $abE_1 - cdE_2 \neq 0$. Hence the field cannot represent a conservative field. But electric field is a conservative field. Hence a field represented by these lines cannot be an electric field.

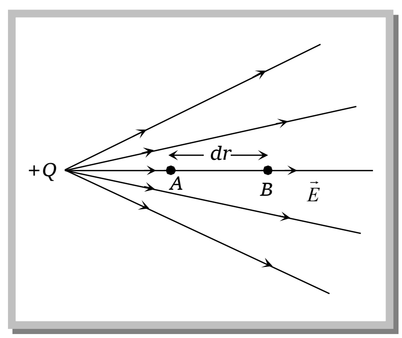
A charge $Q$ is fixed at a distance $d$ in front of an infinite metal plate. The lines of force are represented by

(a) (b) (c) (d)

**Solution:** (a) Metal plate acts as an equipotential surface, therefore the field lines should act normal to the surface of the metal plate.

## Relation Between Electric Field and Potential

In an electric field rate of change of potential with distance is known as **potential gradient.** It is a vector quantity and it's direction is opposite to that of electric field. Potential gradient relates with electric field according to the following relation $E = -\frac{dV}{dr}$; This relation gives another unit of electric field is $\frac{volt}{meter}$. In the above relation negative sign indicates that in the direction of electric field potential decreases.




In space around a charge distribution we can also write $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

where $E_x = -\frac{dV}{dx}$, $E_y = -\frac{dV}{dy}$ and $E_z = -\frac{dV}{dz}$

With the help of formula $E = -\frac{dV}{dr}$, potential difference between any two points in an electric field can be determined by knowing the boundary conditions $dV = -\int_{r_1}^{r_2} \vec{E}\,.\,\vec{dr} = -\int_{r_1}^{r_2} \vec{E}\,.\,\vec{dr}\cos\theta$.

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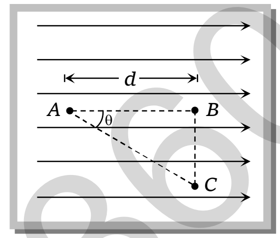
**For example:** Suppose $A$, $B$ and $C$ are three points in an uniform electric field as shown in figure.




(i) Potential difference between point $A$ and $B$ is

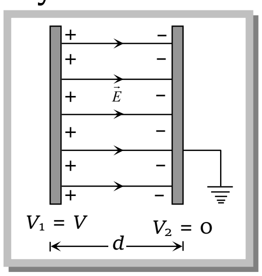
$$V_B - V_A = -\int_A^B \vec{E} \cdot \vec{dr}$$

Since displacement is in the direction of electric field, hence $\theta = 0^\circ$

So, $$V_B - V_A = -\int_A^B E \cdot dr \cos 0 = -\int_A^B E \cdot dr = -Ed$$

In general we can say that in an uniform electric field $E = -\frac{V}{d}$ or $|E| = \frac{V}{d}$

Another example




$$E = \frac{V}{d}$$

(ii) Potential difference between points $A$ and $C$ is :

$$V_C - V_A = -\int_A^C E\, dr \cos\theta = -E(AC)\cos\theta = -E(AB) = -Ed$$

Above relation proves that potential difference between $A$ and $B$ is equal to the potential difference between $A$ and $C$ *i.e.* points $B$ and $C$ are at same potential.

***Concept***

☞ *Negative of the slope of the V-r graph denotes intensity of electric field i.e.* $\tan\theta = \frac{V}{r} = -E$

### *Example based on E = – dV/dr*

**Example: 48** The electric field, at a distance of 20 *cm* from the centre of a dielectric sphere of radius 10 *cm* is 100 *V/m*. The '$E$' at 3 *cm* distance from the centre of sphere is

(a) 100 *V/m* (b) 125 *V/m* (c) 120 *V/m* (d) Zero

**Solution:** (c) For dielectric sphere *i.e.* for non-conducting sphere $E_{out} = \frac{k.q}{r^2}$ and $E_{in} = \frac{kqr}{R^3}$

$$E_{out} = 100 \frac{KQ}{(20 \times 10^{-2})^2} \Rightarrow KQ = 100 \times (0.2)^2 \text{ so } E_{in} = \frac{100 \times (0.2)^2 \times (3 \times 10^{-2})^2}{(10 \times 10^{-2})^3} = 120\ V/m$$

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**Example: 49** In $x$-$y$ co-ordinate system if potential at a point $P(x, y)$ is given by $V = axy$; where $a$ is a constant, if $r$ is the distance of point $P$ from origin then electric field at $P$ is proportional to **[RPMT 2000]**

(a) $r$ (b) $r^{-1}$ (c) $r^{-2}$ (d) $r^2$

**Solution:** (a) By using $E = -\frac{dV}{dr}$ $E_x = -\frac{dV}{dx} = -ay$, $E_y = -\frac{dV}{dy} = -ax$

Electric field at point $P$ $E = \sqrt{E_x^2 + E_y^2} = a\sqrt{x^2 + y^2} = ar$ *i.e.*, $E \propto r$

**Example: 50** The electric potential $V$ at any point $x, y, z$ (all in *metres*) in space is given by $V = 4x^2$ *volt*. The electric field at the point (1*m*, 0, 2*m*) in *volt/metre* is

(a) 8 along negative $X$-axis (b) 8 along positive $X$-axis

(c) 16 along negative $X$-axis (d) 16 along positive $Z$-axis

**Solution:** (a) By using $E = -\frac{dV}{dx} \Rightarrow E = -\frac{d}{dx}(4x^2) = -8x$. Hence at point (1*m*, 0, 2*m*). $E = -8$ *volt/m i.e.* 8 along $-ve$ $x$-axis.

**Example: 51** The electric potential $V$ is given as a function of distance $x$ (*metre*) by $V = (5x^2 + 10x - 9)$ *volt*. Value of electric field at $x = 1m$ is

(a) – 20 *V/m* (b) 6 *V/m* (c) 11 *V/m* (d) – 23 *V/m*

**Solution:** (a) By using $E = -\frac{dV}{dx}$; $E = -\frac{d}{dx}(5x^2 + 10x - 9) = (10x + 10)$,

at $x = 1m$ $E = -20\ V/m$

**Example: 52** A uniform electric field having a magnitude $E_0$ and direction along the positive $X$-axis exists. If the electric potential $V$, is zero at $X = 0$, then, its value at $X = +x$ will be

(a) $V(x) = +xE_0$ (b) $V(x) = -xE_0$ (c) $V(x) = x^2E_0$ (d) $V(x) = -x^2E_0$

**Solution:** (b) By using $E = -\frac{\Delta V}{\Delta r} = -\frac{(V_2 - V_1)}{(r_2 - r_1)}$; $E_0 = -\frac{\{V(x) - 0\}}{x - 0} \Rightarrow V(x) = -xE_0$

**Example: 53** If the potential function is given by $V = 4x + 3y$, then the magnitude of electric field intensity at the point (2, 1) will be

(a) 11 (b) 5 (c) 7 (d) 1

**Solution:** (b) By using *i.e.*, $E = \sqrt{E_x^2 + E_y^2}$; $E_x = -\frac{dV}{dx} = -\frac{d}{dx}(4x + 3y) = -4$

and $E_y = -\frac{dV}{dy} = -\frac{d}{dy}(4x + 3y) = -3$

$\therefore$ $E = \sqrt{(-4)^2 + (-3)^2} = 5\ N/C$

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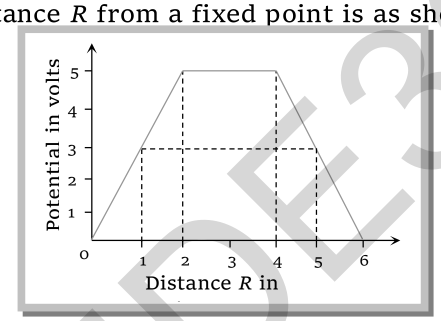
**Tricky example: 7**

The variation of potential with distance $R$ from a fixed point is as shown below. The electric field at $R = 5\,m$ is



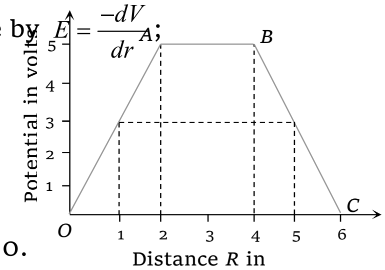

(a) 2.5 *volt/m*

(b) – 2.5 *volt/m*

(c) $\frac{2}{5}\,volt/m$

(d) $-\frac{2}{5}\,volt/m$

**Solution:** (a) Intensity at 5 *m* is same as at any point between *B* and *C* because the slope of *BC* is same throughout (i.e. electric field between *B* and *C* is uniform). Therefore electric field at *R* = 5*m* is equal to the slope of line *BC* hence by $E = \frac{-dV}{dr}$;




$$E = -\frac{(0-5)}{6-4} = 2.5\,\frac{V}{m}$$

*Note* : ❑ At $R$ = 1*m* , $E = -\frac{(5-0)}{(2-0)} = -2.5\frac{V}{m}$

and at $R = 3m$ potential is constant so $E$ = 0.

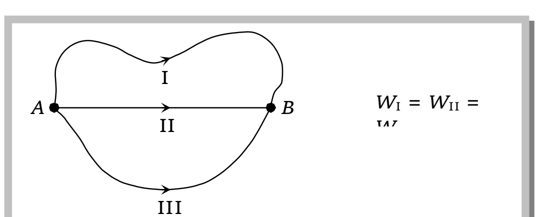
## Work Done in Displacing a Charge

(1) **Definition :** If a charge $Q$ displaced from one point to another point in electric field then work done in this process is $W = Q \times \Delta V$ where $\Delta V$ = Potential difference between the two position of charge $Q$. ($\Delta V = \vec{E}.\Delta\vec{r} = E\Delta r\cos\theta$ where $\theta$ is the angle between direction of electric field and direction of motion of charge).

(2) **Work done in terms of rectangular component of $\vec{E}$ and $\vec{r}$ :** If charge $Q$ is given a displacement $\vec{r} = (r_1\hat{i} + r_2\hat{j} + r_3\hat{k})$ in an electric field $\vec{E} = (E_1\hat{i} + E_2\hat{j} + E_3\hat{k})$. The work done is $W = Q(\vec{E}.\vec{r}) = Q(E_1r_1 + E_2r_2 + E_3r_3)$.

## Conservation of Electric Field

As electric field is conservation, work done and hence potential difference between two point is path independent and depends only on the position of points between. Which the charge is moved.

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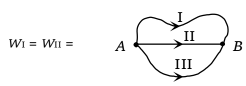
### *Concept*

☞ No work is done in moving a charge on an equipotential surface.



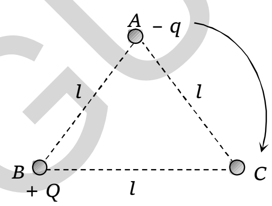

### *Examples based on work*

**Example: 54** A charge (– $q$) and another charge (+$Q$) are kept at two points $A$ and $B$ respectively. Keeping the charge (+$Q$) fixed at $B$, the charge (– $q$) at $A$ is moved to another point $C$ such that $ABC$ forms an equilateral triangle of side $l$. The network done in moving the charge (– $q$) is **[MP PET 2001]**

(a) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{l}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{l^2}$ (c) $\frac{1}{4\pi\varepsilon_0}Qql$ (d) Zero

**Solution:** (d) Since $V_A = V_C = \frac{kQ}{l}$

so $W = q(V_C - V_A) = 0$




**Example: 55** The work done in bringing a 20 *coulomb* charge from point $A$ to point $B$ for distance 0.2 $m$ is 2 *Joule*. The potential difference between the two points will be (in *volt*)

(a) 0.2 (b) 8 (c) 0.1 (d) 0.4

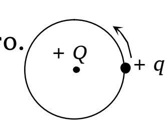
**Solution:** (c) $W = Q.\Delta V \Rightarrow 2 = 20 \times \Delta V \Rightarrow \Delta V = 0.1$ *volt*

**Example: 56** A charge $+q$ is revolving around a stationary $+Q$ in a circle of radius $r$. If the force between charges is $F$ then the work done of this motion will be

**[CPMT 1975, 90, 91, 97; NCERT 1980, 83; EAMCET 1994; MP PET 1993, 95; MNR 1998; AIIMS 1997; DCE 1995; RPET 1998]**

(a) $F \times r$ (b) $F \times 2\pi r$ (c) $\frac{F}{2\pi r}$ (d) 0

**Solution:** (d) Since $+q$ charge is moving on an equipotential surface so work done is zero.




**Example: 57** Four equal charge $Q$ are placed at the four corners of a body of side ‘a’ each. Work done in removing a charge – $Q$ from its centre to infinity is

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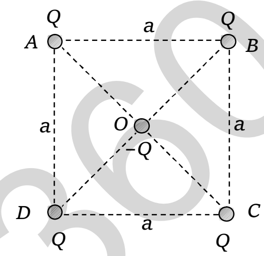
(a) 0 (b) $\frac{\sqrt{2}\,Q^2}{4\pi\varepsilon_0 a}$ (c) $\frac{\sqrt{2}\,Q^2}{\pi\varepsilon_0 a}$ (d) $\frac{Q^2}{2\pi\varepsilon_0 a}$

**Solution:** (c) We know that work done in moving a charge is $W = Q\Delta V$




Here $W = Q(V_0 - V_\infty)$ $\because$ $V_\infty = 0$ $\therefore$ $W = Q \times V_0$

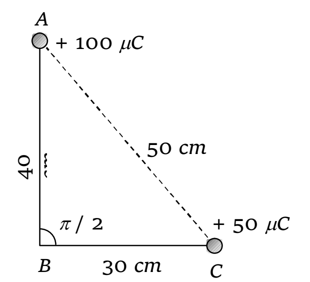
Also $V_0 = 4 \times \frac{1}{4\pi\varepsilon_0} . \frac{Q}{a/\sqrt{2}} = \frac{4\sqrt{2}\,Q}{4\pi\varepsilon_0 a} = \frac{\sqrt{2}\,Q}{\pi\varepsilon_0 a}$

So, $W = \frac{\sqrt{2}Q^2}{\pi\varepsilon_0 a}$

**Example: 58** Two point charge 100 $\mu C$ and 5 $\mu C$ are placed at point $A$ and $B$ respectively with $AB$ = 40 *cm*. The work done by external force in displacing the charge 5 $\mu C$ from $B$ to $C$, where $BC$ = 30 *cm*, angle $ABC = \frac{\pi}{2}$ and $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2/C^2$ **[MP PMT 1997]**

(a) 9 $J$ (b) $\frac{81}{20}J$ (c) $\frac{9}{25}J$ (d) $-\frac{9}{4}J$

**Solution:** (d) Potential at $B$ due to +100 $\mu C$ charge is




$V_B = 9 \times 10^9 \times \frac{100 \times 10^{-6}}{40 \times 10^{-2}} = \frac{9}{4} \times 10^6\ volt$

Potential at $C$ due to +100 $\mu C$ charge is

$V_C = 9 \times 10^9 \times \frac{100 \times 10^{-6}}{50 \times 10^{-2}} = \frac{9}{5} \times 10^6\ volt$

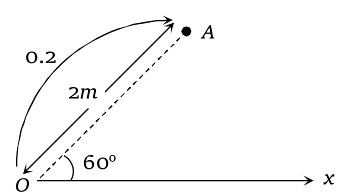
Hence work done in moving charge +5$\mu C$ from $B$ to $C$ $W = 5 \times 10^{-6}(V_C - V_B)$

$W = 5 \times 10^{-6}\left(\frac{9}{5} \times 10^6 - \frac{9}{4} \times 10^{+6}\right) = -\frac{9}{4}J$

**Example: 59** There is an electric field $E$ in $x$-direction. If the work done in moving a charge 0.2 $C$ through a distance of 2 *metres* along a line making an angle 60° with the $x$-axis is 4$J$, what is the value of $E$ **[CBSE 1995]**

(a) 4 $N/C$ (b) 8 $N/C$ (c) $\sqrt{3}\ N/C$ (d) 20 $N/C$

**Solution:** (d) By using $W = q \times \Delta V$ and $\Delta V = E\Delta r\cos\theta$




So, $W = qE\,\Delta r\cos\theta$

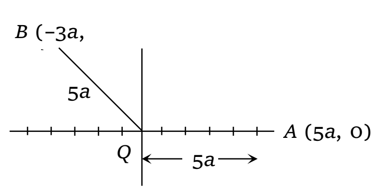
$W = 4j = 0.2 \times E \times 2 \times \cos 60$

$\Rightarrow$ $E$ = 20 $N/C$

**Example: 60** An electric charge of 20 $\mu C$ is situated at the origin of $X$-$Y$ co-ordinate system. The potential difference between the points. (5$a$, 0) and (– 3$a$, 4$a$) will be

(a) a (b) 2$a$ (c) Zero (d) $\frac{a}{\sqrt{2}}$

**Solution:** (c) $V_A = \frac{kQ}{5a}$ and $V_B = \frac{kQ}{5a}$




$\therefore$ $V_A - V_B = 0$

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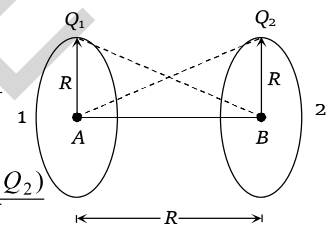
**Example: 61** Two identical thin rings each of radius $R$, are coaxially placed a distance $R$ apart. If $Q_1$ and $Q_2$ are respectively the charges uniformly spread on the two rings, the work done in moving a charge $q$ from the centre of one ring to that of the other is

(a) Zero (b) $\frac{q(Q_1 - Q_2)(\sqrt{2}-1)}{4\pi\varepsilon_0 R\sqrt{2}}$ (c) $\frac{q(Q_1 + Q_2)\sqrt{2}}{4\pi\varepsilon_0 R}$ (d) $\frac{q\left(\frac{Q_1}{Q_2}\right)(\sqrt{2}-1)}{4\pi\varepsilon_0 R\sqrt{2}}$

**Solution:** (b) Potential at the centre of first ring $V_A = \frac{Q_1}{4\pi\varepsilon_0 R} + \frac{Q_2}{4\pi\varepsilon_0\sqrt{R^2+R^2}}$



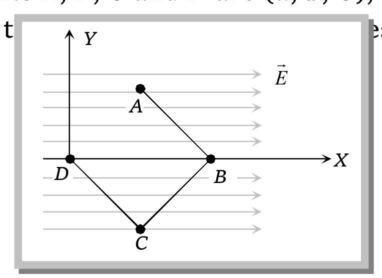

Potential at the centre of second ring $V_B = \frac{Q_2}{4\pi\varepsilon_0 R} + \frac{Q_1}{4\pi\varepsilon_0\sqrt{R^2+R^2}}$

Potential difference between the two centres $V_A - V_B = \frac{(\sqrt{2}-1)(Q_1 - Q_2)}{4\pi\varepsilon_0 R\sqrt{2}}$

$\therefore$ Work done $W = \frac{q(\sqrt{2}-1)(Q_1 - Q_2)}{4\pi\varepsilon_0 R\sqrt{2}}$

**Tricky example: 8**

A point charge $q$ moves from point $A$ to point $D$ along the path $ABCD$ in a uniform electric field. If the co-ordinates of the points $A$, $B$, $C$ and $D$ are $(a, b, 0)$, $(2a, 0, 0)$, $(a, -b, 0)$ and $(0, 0, 0)$ then the work done by t ess will be




(a) $-qEa$

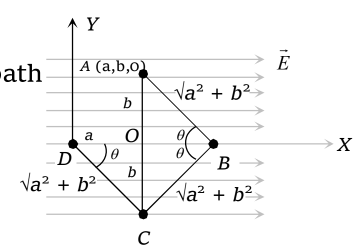
(b) Zero

(c) $2E(a+b)q$

(d) $\frac{qEa}{2b}$

**Solution:** (a) As electric field is a conservative field

Hence the work done does not depend on path




$\therefore$ $W_{ABCD} = W_{AOD} = W_{AO} + W_{OD}$

$= Fb\cos 90^\circ + Fa\cos 180^\circ = 0 + qEa(-1) = -qEa$

## Equilibrium of Charge

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(1) **Definition** : A charge is said to be in equilibrium, if net force acting on it is zero. A system of charges is said to be in equilibrium if each charge is separately in equilibrium.

(2) **Type of equilibrium :** Equilibrium can be divided in following type:

(i) **Stable equilibrium :** After displacing a charged particle from it's equilibrium position, if it returns back then it is said to be in stable equilibrium. If $U$ is the potential energy then in case of stable equilibrium $\frac{d^2U}{dx^2}$ is positive *i.e., U* is minimum.

(ii) **Unstable equilibrium :** After displacing a charged particle from it's equilibrium position, if it never returns back then it is said to be in unstable equilibrium and in unstable equilibrium $\frac{d^2U}{dx^2}$ is negative *i.e., U* is maximum.

(iii) **Neutral equilibrium :** After displacing a charged particle from it's equilibrium position if it neither comes back, nor moves away but remains in the position in which it was kept it is said to be in neutral equilibrium and in neutral equilibrium $\frac{d^2U}{dx^2}$ is zero *i.e., U* is constant

(3) **Guidelines to check the equilibrium**

(i) Identify the charge for which equilibrium is to be analysed.

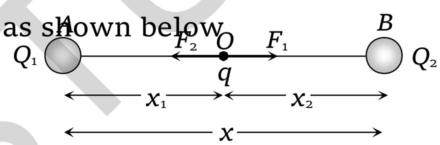
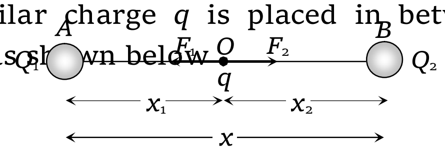
(ii) Check, how many forces acting on that particular charge.

(iii) There should be atleast two forces acts oppositely on that charge.

(iv) If magnitude of these forces are equal then charge is said to be in equilibrium then identify the nature of equilibrium.

(v) If all the charges of system are in equilibrium then system is said to be in equilibrium

(4) **Different cases of equilibrium of charge**

| | |
|---|---|
| **Case – 1 :** Suppose three similar charge $Q_1, q$ and $Q_2$ are placed along a straight line as shown below<br><br><br>Charge $q$ will be in equilibrium if $\lvert F_1 \rvert = \lvert F_2 \rvert$<br>*i.e.,* $\frac{Q_1}{Q_2} = \left(\frac{x_1}{x_2}\right)^2$ ; This is the condition of | **Case – 2 :** Two similar charge $Q_1$ and $Q_2$ are placed along a straight line at a distance $x$ from each other and a third dissimilar charge $q$ is placed in between them as shown below<br><br><br>Charge $q$ will be in equilibrium if $\lvert F_1 \rvert = \lvert F_2 \rvert$<br>*i.e.,* $\frac{Q_1}{Q_2} = \left(\frac{x_1}{x_2}\right)^2$ . |

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equilibrium of charge $q$. After following the guidelines we can say that charge $q$ is in stable equilibrium and this system is not in equilibrium

*Note* : ❑ $x_1 = \dfrac{x}{1+\sqrt{Q_2/Q_1}}$

and $x_2 = \dfrac{x}{1+\sqrt{Q_1/Q_2}}$

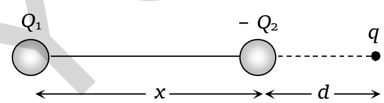
*e.g.* if two charges $+4\mu C$ and $+16\ \mu C$ are separated by a distance of 30 *cm* from each other then for equilibrium a third charge should be placed between them at a distance $x_1 = \dfrac{30}{1+\sqrt{16/4}} = 10\ cm$ or $x_2 = 20\ cm$

*Note* : ❑ Same short trick can be used here to find the position of charge $q$ as we discussed in Case–1 *i.e.,*

$$x_1 = \frac{x}{1+\sqrt{Q_2/Q_1}} \text{ and } x_2 = \frac{x}{1+\sqrt{Q_1/Q_2}}$$

❑ It is very important to know that magnitude of charge $q$ can be determined if one of the extreme charge (either $Q_1$ or $Q_2$) is in equilibrium *i.e.* if $Q_2$ is in equilibrium then $|q| = Q_1\left(\dfrac{x_2}{x}\right)^2$ and if $Q_1$ is in equilibrium then $|q| = Q_2\left(\dfrac{x_1}{x}\right)^2$ (It should be remember that sign of $q$ is opposite to that of $Q_1$ (or $Q_2$))

**Case – 3 :** Two dissimilar charge $Q_1$ and $Q_2$ are placed along a straight line at a distance $x$ from each other, a third charge $q$ should be placed out side the line joining $Q_1$ and $Q_2$ for it to experience zero net force.



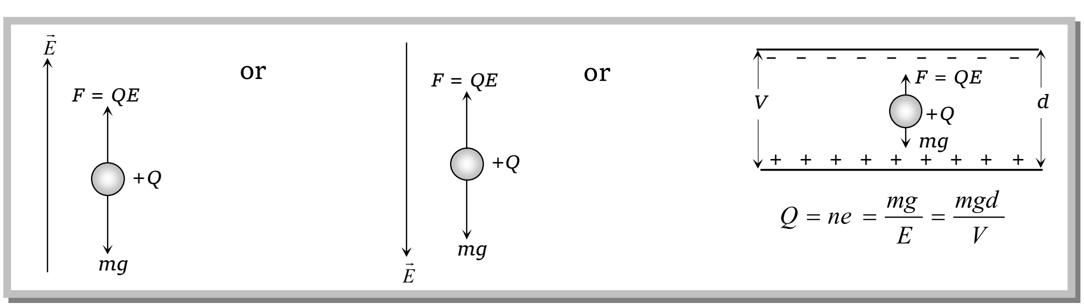

(Let $|Q_2| < |Q_1|$)

**Short Trick :**

For it's equilibrium. Charge $q$ lies on the side of chare which is smallest in magnitude and $d = \dfrac{x}{\sqrt{Q_1/Q_2} - 1}$

(5) **Equilibrium of suspended charge in an electric field**

(i) **Freely suspended charged particle :** To suspend a charged a particle freely in air under the influence of electric field it's downward weight should be balanced by upward electric force for example if a positive charge is suspended freely in an electric field as shown then

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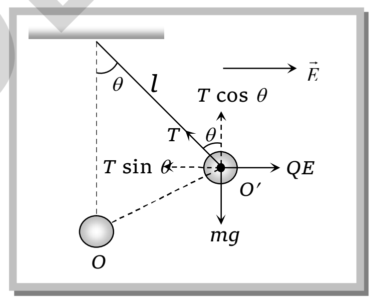
In equilibrium $QE = mg \Rightarrow \boldsymbol{E} = \frac{\boldsymbol{mg}}{\boldsymbol{Q}}$

**Note** : ❑ In the above case if direction of electric field is suddenly reversed in any figure then acceleration of charge particle at that instant will be $a = 2g$.

(ii) **Charged particle suspended by a massless insulated string** (like simple pendulum) : Consider a charged particle (like Bob) of mass $m$, having charge $Q$ is suspended in an electric field as shown under the influence of electric field. It turned through an angle (say $\theta$) and comes in equilibrium.




So, in the position of equilibrium ($O'$ position)

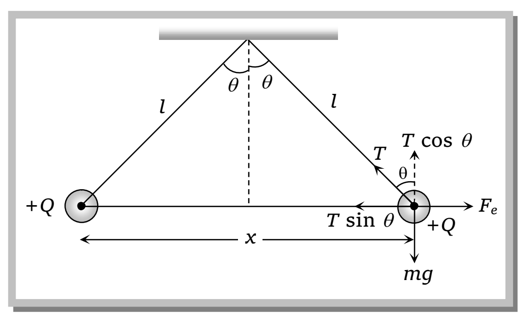
$T\sin\theta = QE$ ....(i)

$T\cos\theta = mg$ ....(ii)

By squaring and adding equation (i) and (ii) $T = \sqrt{(QE)^2 + (mg)^2}$

Dividing equation (i) by (ii) $\tan\theta = \frac{QE}{mg} \Rightarrow \theta = \tan^{-1}\frac{QE}{mg}$

(iii) **Equilibrium of suspended point charge system :** Suppose two small balls having charge $+Q$ on each are suspended by two strings of equal length $l$. Then for equilibrium position as shown in figure.



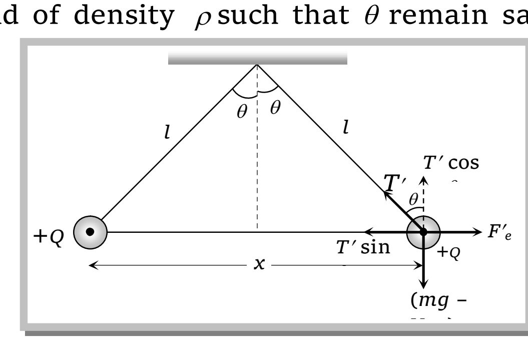

$T\sin\theta = F_e$ ....(i)

$T\cos\theta = mg$ ....(ii)

$T^2 = (F_e)^2 + (mg)^2$

and $\tan\theta = \frac{F_e}{mg}$ ; here $F_e = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{x^2}$ and $\frac{x}{2} = l\sin\theta$

(iv) **Equilibrium of suspended point charge system in a liquid :** In the previous discussion if point charge system is taken into a liquid of density $\rho$ such that $\theta$ remain same then




In equilibrium $Fe' = T'\sin\theta$ and $(mg - V\rho g) = T'\cos\theta$

$\therefore \tan\theta = \frac{Fe'}{(mg - V\rho g)} = \frac{Q^2}{4\pi\varepsilon_0 K(mg - V\rho g)x^2}$

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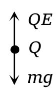
When this system was in air $\tan\theta = \dfrac{Fe}{mg} = \dfrac{Q^2}{4\pi\varepsilon_0\, mgx^2}$

$\therefore$ So equating these two gives us $\dfrac{1}{m} = \dfrac{1}{k(m - V\rho)} \Rightarrow K = \dfrac{m}{m - V\rho} = \dfrac{1}{\left(1 - \dfrac{V}{m}\rho\right)}$

If $\sigma$ is the density of material of ball then $K = \dfrac{1}{\left(1 - \dfrac{\rho}{\sigma}\right)}$




### *Examples based on equilibrium of*

**Example: 62** A charge q is placed at the centre of the line joining two equal charges $Q$. The system of the three charges will be in equilibrium. If q is equal to

**[CPMT 1999; MP PET 1999, MP PMT 1999; CBSE 1995; Bihar MEE 1995; IIT 1987]**

(a) $-\dfrac{Q}{2}$  (b) $-\dfrac{Q}{4}$  (c) $+\dfrac{Q}{4}$  (d) $+\dfrac{Q}{2}$

**Solution:** (b) By using Tricky formula $q = Q\left(\dfrac{x/2}{x}\right)^2$

$\Rightarrow$ $q = \dfrac{Q}{4}$ since $q$ should be negative so $q = -\dfrac{Q}{4}$.

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**Example: 63** Two point charges $+4q$ and $+q$ are placed at a distance $L$ apart. A third charge $Q$ is so placed that all the three charges are in equilibrium. Then location and magnitude of third charge will be **[IIT-JEE 1975]**

(a) At a distance $\frac{L}{3}$ from $+4q$ charge, $\frac{4q}{9}$

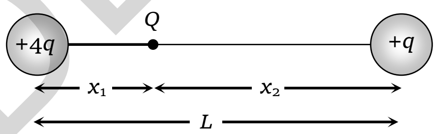
(b) At a distance $\frac{L}{3}$ from $+4q$ charge, $-\frac{4q}{9}$

(c) At a distance $\frac{2L}{3}$ from $+4q$ charge, $-\frac{4q}{9}$

(d) At a distance $\frac{2L}{3}$ from $+q$ charge, $+\frac{4q}{9}$

**Solution:** (c) Let third charge be placed at a distance $x_1$ from $+4q$ charge as shown

Now $x_1 = \dfrac{L}{1+\sqrt{\dfrac{q}{4q}}} = \dfrac{2L}{3} \Rightarrow x_2 = \dfrac{L}{3}$




For equilibrium of $q$, $Q = +4q\left(\dfrac{L/3}{L}\right)^2 = \dfrac{4q}{9} \Rightarrow Q = -\dfrac{4q}{9}$.

**Example: 64** A drop of $10^{-6}\,kg$ water carries $10^{-6}\,C$ charge. What electric field should be applied to balance it's weight (assume $g = 10\ m/sec^2$)

(a) $10\ V/m$, Upward (b) $10\ V/m$, Downward (c) $0.1\ V/m$ Downward (d) $0.1\ V/m$, Upward

**Solution:** (a) In equilibrium $QE = mg$

$E = \dfrac{mg}{Q} = \dfrac{10^{-6}\times 10}{10^{-6}} = 10\ V/m$; Since charge is positive so electric field will be upward.

**Example: 65** A charged water drop of radii $0.1\ \mu m$ is under equilibrium in some electric field. The charge on the drop is equivalent to electronic charge. The intensity of electric field is

(a) $1.61\ N/C$ (b) $25.2\ N/C$ (c) $262\ N/C$ (d) $1610\ N/C$

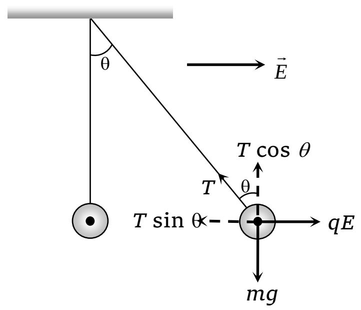
**Solution:** (c) In equilibrium $QE = mg$ ; $E = \dfrac{mg}{Q} = \dfrac{\left(\frac{4}{3}\pi r^3\rho\right).g}{Q} = \dfrac{4}{3}\dfrac{\times(3.14)(0.1\times10^{-6})^3\times10^3\times10}{1.6\times10^{-19}} = 262\ N/C$

**Example: 66** The bob of a pendulum of mass $8\ \mu g$ carries an electric charge of $39.2\times10^{-10}$ coulomb in an electric field of $20\times10^3\ volt/meter$ and it is at rest. The angle made by the pendulum with the vertical will be

(a) $27^\circ$ (b) $45^\circ$ (c) $87^\circ$ (d) $127^\circ$

**Solution:** (b) $T\sin\theta = qE$, $T\cos\theta = mg$

$\therefore \quad \tan\theta = \dfrac{qE}{mg}$

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$$\tan\theta = \frac{39.2\times10^{-10}\times20\times10^{3}}{8\times10^{-6}\times9.8} = 1$$

$\Rightarrow \quad \theta = 45^{\circ}$

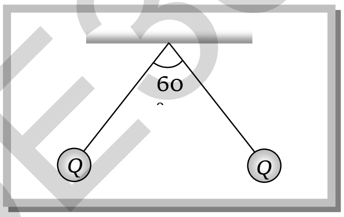
**Example: 67** Two small spherical balls each carrying a charge $Q$ = 10 $\mu C$ (10 micro-coulomb) are suspended by two insulating threads of equal lengths 1 $m$ each, from a point fixed in the ceiling. It is found that in equilibrium threads are separated by an angle 60° between them, as shown in the figure. What is the tension in the threads. (Given : $\frac{1}{(4\pi\varepsilon_0)} = 9\times10^{9}\ Nm/C^2$ )

(a) 18 $N$

(b) 1.8 $N$

(c) 0.18 $N$

(d) None of these



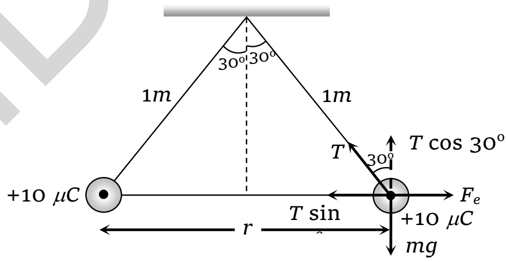

**Solution:** (b) From the geometry of figure

$r$ = 1$m$

In the condition of equilibrium $T\sin 30^o = F_e$

$$T\times\frac{1}{2} = 9\times10^{9}.\frac{(10\times10^{-6})^2}{1^2}$$

$\Rightarrow T$= 1.8 $N$



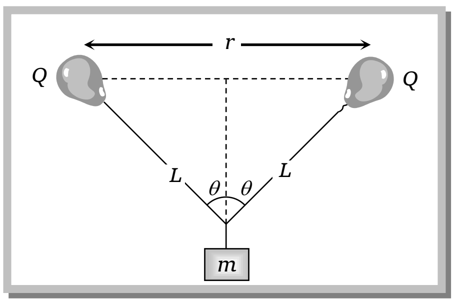

**Example: 68** Two similar balloons filled with helium gas are tied to $L$ $m$ long strings. A body of mass $m$ is tied to another ends of the strings. The balloons float on air at distance $r$. If the amount of charge on the balloons is same then the magnitude of charge on each balloon will be

(a) $\left[\frac{mgr^2}{2k}\tan\theta\right]^{1/2}$

(b) $\left[\frac{2k}{mgr^2}\tan\theta\right]^{1/2}$

(c) $\left[\frac{mgr}{2k}\cot\theta\right]^{1/2}$

(d) $\left[\frac{2k}{mgr}\tan\theta\right]^{1/2}$



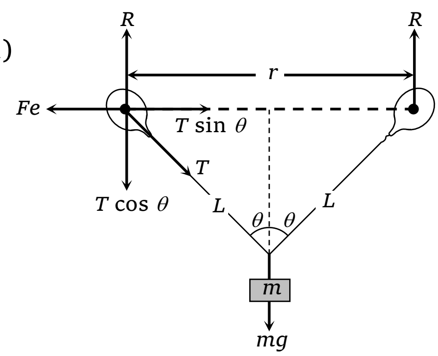

**Solution:** (a) In equilibrium

$2R = mg$ .... (i) $F_e = T\sin\theta$ .... (ii) $R = T\cos\theta$ .... (iii)

From equation (i) and (iii)

$2T\cos\theta = mg$ .... (iv)

Dividing equation (ii) by equation (iv)

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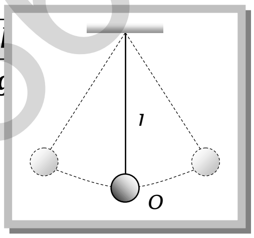
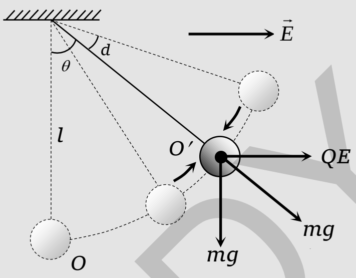
$$\frac{1}{2}\tan\theta = \frac{F_e}{mg} \Rightarrow \frac{1}{2}\tan\theta = \frac{k\frac{Q^2}{r^2}}{mg} \Rightarrow \theta = \left(\frac{mgr^2}{2k}\tan\theta\right)^{1/2}$$

## Time Period of Oscillation of a Charged Body

(1) **Simple pendulum based :** If a simple pendulum having length $l$ and mass of bob $m$ oscillates about it's mean position than it's time period of oscillation $T = 2\pi\sqrt{\frac{l}{g}}$




**Case – 1 :** If some charge say $+Q$ is given to bob and an electric field E is applied in the direction as shown in figure then equilibrium position of charged bob (point charge) changes from $O$ to $O'$.




On displacing the bob from it's equilibrium position O′. It will oscillate under the effective acceleration $g'$, where

$$mg' = \sqrt{(mg)^2 + (QE)^2}$$

$$\Rightarrow g' = \sqrt{g^2 + (QE/m)^2}$$

Hence the new time period is $T_1 = 2\pi\sqrt{\frac{l}{g'}}$

$$T_1 = 2\pi\sqrt{\frac{l}{\left(g^2 + (QE/m)^2\right)^{\frac{1}{2}}}}$$

Since $g' > g$, hence $T_1 < T$

**Case – 2 :** If electric field is applied in the downward direction then.

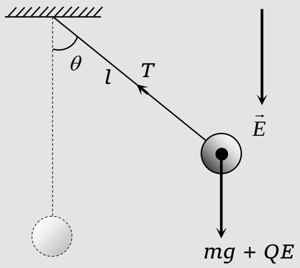
Effective acceleration

$g' = g + QE/m$

So new time period

$$T_2 = 2\pi\sqrt{\frac{l}{g + (QE/m)}}$$

$T_2 < T$



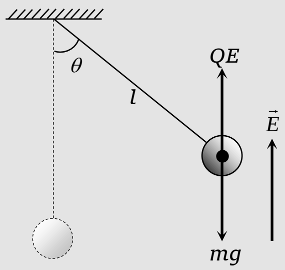

**Case – 3 :** In case 2 if electric field is applied in upward direction then, effective acceleration.

$g' = g - QE/m$

So new time period

$$T_3 = 2\pi\sqrt{\frac{l}{g - (QE/m)}}$$

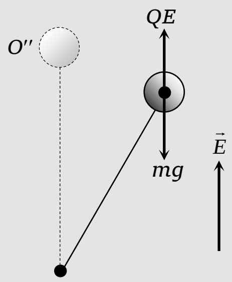
$T_3 > T$




**Case – 4 :** In the case 3,

if $T_3 = \frac{T}{2}$ *i.e.,* $2\pi\sqrt{\frac{l}{(g - QE/m)}}$

$= \frac{1}{2}2\pi\sqrt{\frac{l}{g}} \Rightarrow QE = 3$ mg




*i.e.,* effective vertical force (gravity + electric)

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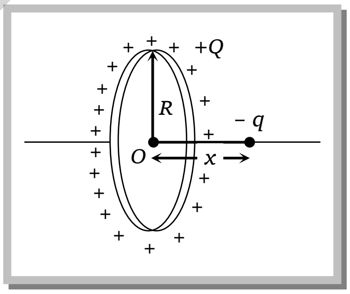
| | |
|---|---|
| i.e. time period of pendulum will decrease. | on the bob = $mg$ – 3 $mg$ = – 2 $mg$, hence the equilibrium position $O''$ of the bob will be above the point of suspension and bob will oscillate under on effective acceleration $2g$ directed upward.<br><br>Hence new time period $T_4 = 2\pi\sqrt{\frac{l}{2g}}$, $T_4 < T$ |

(2) **Charged circular ring :** A thin stationary ring of radius $R$ has a positive charge $+Q$ unit. If a negative charge – $q$ (mass $m$) is placed at a small distance x from the centre. Then motion of the particle will be simple harmonic motion.




Electric field at the location of – $q$ charge $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qx}{(x^2+R^2)^{\frac{3}{2}}}$

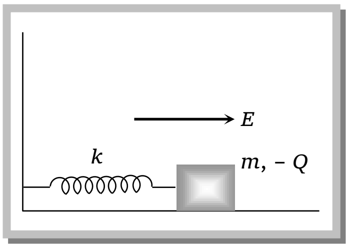
Since $x << R$, So $x^2$ neglected hence $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qx}{R^3}$

Force experienced by charge – $q$ is $F = -q\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qx}{R^3}$

$\Rightarrow F \propto -x$ hence motion is simple harmonic

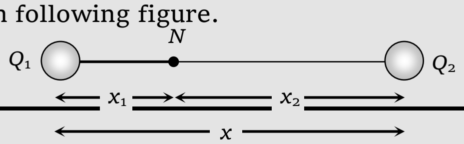
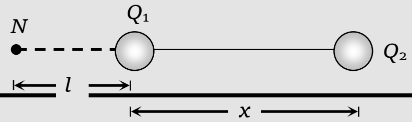
Having time period $T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mR^3}{Qq}}$

(3) **Spring mass system :** A block of mass $m$ containing a negative charge – $Q$ is placed on a frictionless horizontal table and is connected to a wall through an unstretched spring of spring constant $k$ as shown. If electric field $E$ applied as shown in figure the block experiences an electric force, hence spring compress and block comes in new position. This is called the equilibrium position of block under the influence of electric field. If block compressed further or stretched, it execute oscillation having time period $T = 2\pi\sqrt{\frac{m}{k}}$. Maximum compression in the spring due to electric field = $\frac{QE}{k}$




## Neutral Point

A neutral point is a point where resultant electrical field is zero. It is obtained where two electrical field are equal and opposite. Thus neutral points can be obtained only at those points where the resultant field is subtractive. Thus it can be obtained.

| | |
|---|---|
| (1) **At an internal point along the line joining two like charges (Due to a system of two like point charge) :** Suppose two like charges. $Q_1$ and $Q_2$ are separated by a distance x from each other along a line as shown in following figure.  | (2) **At an external point along the line joining two like charges (Due to a system of two unlike point charge) :** Suppose two unlike charge $Q_1$ and $Q_2$ separated by a distance $x$ from each other.  |

If $N$ is the neutral point at a distance $x_1$ from $Q_1$ and at a distance $x_2 (= x - x_1)$ from $Q_2$ then –

At $N$ $\quad$ **| E.F. due to $Q_1$ | = | E.F. due to $Q_2$ |**

*i.e.*, $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_1}{x_1^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_2}{x_2^2} \Rightarrow \frac{Q_1}{Q_2} = \left(\frac{x_1}{x_2}\right)^2$

**Short trick :** $x_1 = \frac{x}{1+\sqrt{Q_2/Q_1}}$ and $x_2 = \frac{x}{1+\sqrt{Q_1/Q_2}}$

*Note* : ❑ In the above formula if $Q_1 = Q_2$, neutral point lies at the centre so remember that resultant field at the midpoint of two equal and like charges is zero.

Here neutral point lies outside the line joining two unlike charges and also it lies nearer to charge which is smaller in magnitude.

If $|Q_1| < |Q_2|$ then neutral point will be obtained on the side of $Q_1$, suppose it is at a distance $l$ from $Q_1$

Hence at neutral point ; $\frac{kQ_1}{l^2} = \frac{kQ_2}{(x+l)^2} \Rightarrow \frac{Q_1}{Q_2} = \left(\frac{l}{x+l}\right)^2$

**Short trick :** $l = \frac{x}{\left(\sqrt{Q_2/Q_1} - 1\right)}$

*Note* : ❑ In the above discussion if $|Q_1| = |Q_2|$ neutral point will be at infinity.

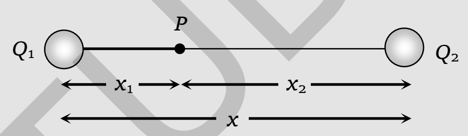
## Zero Potential Due to a System of Two Point Charge

If both charges are like then resultant potential is not zero at any finite point because potentials due to like charges will have same sign and can therefore never add up to zero. Such a point can be therefore obtained only at infinity.

If the charges are unequal and unlike then all such points where resultant potential is zero lies on a closed curve, but we are interested only in those points where potential is zero along the line joining the two charges.

Two such points exist, one lies inside and one lies outside the charges on the line joining the charges. Both the above points lie nearer the smaller charge, as potential created by the charge larger in magnitude will become equal to the potential created by smaller charge at the desired point at larger distance from it.

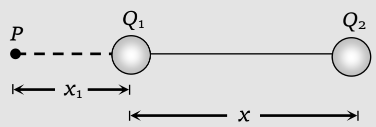
**I. For internal point :**




(It is assumed that $|Q_1| < |Q_2|$).

$\frac{Q_1}{x_1} = \frac{Q_2}{(x - x_1)} \Rightarrow x_1 = \frac{x}{(Q_2/Q_1 + 1)}$

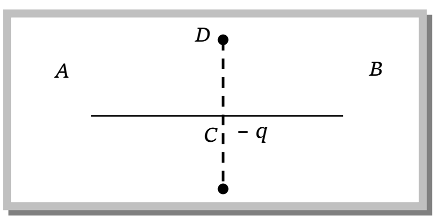
**II. For External point :**




$\frac{Q_1}{x_1} = \frac{Q_2}{(x + x_1)} \Rightarrow x_1 = \frac{x}{(Q_2/Q_1 - 1)}$

### *Examples based on oscillation of charge and neutral*

**Example: 69** Two similar charges of $+Q$ as shown in figure are placed at points $A$ and $B$. $-q$ charge is placed at point $C$ midway between $A$ and $B$. $-q$ charge will oscillate if




(a) It is moved towards $A$

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(b) It is moved towards $B$

(c) It is moved along $CD$

(d) Distance between $A$ and $B$ is reduced

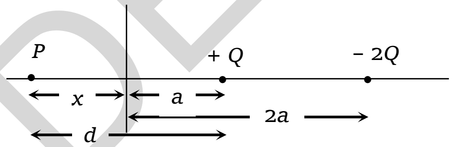
**Solution:** (c) When – $q$ charge displaced along $CD$, a restoring force act on it which causes oscillation.

**Example: 70** Two point charges $(+Q)$ and $(-2Q)$ are fixed on the $X$-axis at positions $a$ and $2a$ from origin respectively. At what position on the axis, the resultant electric field is zero

(a) Only $x=\sqrt{2}\,a$ (b) Only $x=-\sqrt{2}\,a$ (c) Both $x=\pm\sqrt{2}\,a$ (d) $x=\frac{3a}{2}$ only

**Solution:** (b) Let the electric field is zero at a point $P$ distance $d$ from the charge $+Q$ so at $P$. $\frac{k.Q}{d^2}+\frac{k(-2Q)}{(a+d)^2}=0$

$$\Rightarrow \frac{1}{d^2}=\frac{2}{(a+d)^2} \Rightarrow d=\frac{a}{(\sqrt{2}-1)}$$




Since $d > a$ *i.e.* point $P$ must lies on negative $x$-axis as shown at a distance $x$ from origin hence $x=d-a=\frac{a}{\sqrt{2}-1}-a=\sqrt{2}a$. Actually $P$ lies on negative $x$-axis so $x=-\sqrt{2}\,a$.

**Example: 71** Two charges $9e$ and $3e$ are placed at a distance $r$. The distance of the point where the electric field intensity will be zero is

(a) $\frac{r}{\left(\sqrt{3}+1\right)}$ from 9e charge

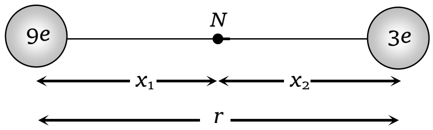
(b) $\frac{r}{1+\sqrt{1/\ 3}}$ from 9e charge

(c) $\frac{r}{\left(1-\sqrt{3}\right)}$ from 3e charge

(d) $\frac{r}{1+\sqrt{1/\ 3}}$ from 3e charge

**Solution:** (b) Suppose neutral point is obtained at a distance $x_1$ from charge 9e and $x_2$ from charge 3e

By using $x_1=\frac{x}{1+\sqrt{\frac{Q_2}{Q_1}}}=\frac{r}{1+\sqrt{\frac{3e}{9e}}}=\frac{r}{\left(1+\frac{1}{\sqrt{3}}\right)}$




**Example: 72** Two point charges – $Q$ and $2Q$ are separated by a distance $R$, neutral point will be obtained at

(a) A distance of $\frac{R}{(\sqrt{2}-1)}$ from $-Q$ charge and lies between the charges.

(b) A distance of $\frac{R}{(\sqrt{2}-1)}$ from – $Q$ charge on the left side of it

(c) A distance of $\frac{R}{(\sqrt{2}-1)}$ from $2Q$ charge on the right side of it

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(d) A point on the line which passes perpendicularly through the centre of the line joining – $Q$ and $2Q$ charge.

**Solution:** (b) As already we discussed neutral point will be obtained on the side of charge which is smaller in magnitude *i.e.* it will obtained on the left side of – $Q$ charge and at a distance.

$$l = \frac{R}{\sqrt{\frac{2Q}{Q}} - 1} \Rightarrow l = \frac{R}{(\sqrt{2} - 1)}$$

**Example: 73** A charge of + $4\mu C$ is kept at a distance of 50 $cm$ from a charge of – $6\mu C$. Find the two points where the potential is zero

(a) Internal point lies at a distance of 20 $cm$ from $4\mu C$ charge and external point lies at a distance of 100 $cm$ from $4\mu C$ charge.

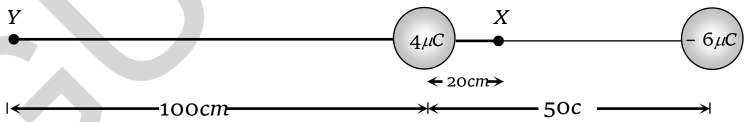
(b) Internal point lies at a distance of 30 $cm$ from $4\mu C$ charge and external point lies at a distance of 100 $cm$ from $4\mu C$ charge

(c) Potential is zero only at 20 $cm$ from $4\mu C$ charge between the two charges

(d) Potential is zero only at 20 $cm$ from – $6\mu C$ charge between the two charges

**Solution:** (a) For internal point $X$, $x_1 = \frac{x}{\left(\frac{Q_2}{Q_1} + 1\right)} = \frac{50}{\frac{6}{4} + 1} = 20\,cm$ and for external point $Y$,

$$x_1 = \frac{x}{\left(\frac{Q_2}{Q_1} - 1\right)} = \frac{50}{\frac{6}{4} - 1} = 100\ cm$$




## Tricky example: 9

Two equal negative charges – $q$ are fixed at points (0, $a$) and (0, – $a$) on the y-axis. A positive charge $Q$ is released from rest at the point ($2a$, 0) on the x-axis. The charge $Q$ will

**[IIT-JEE 1984, Bihar MEE 1995, MP PMT 1996]**

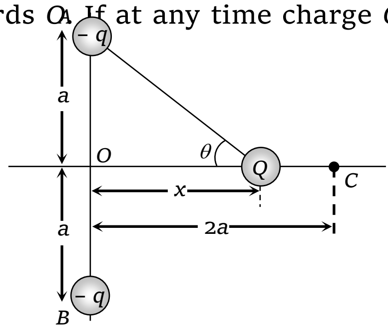
(a) Execute simple harmonic motion about the origin

(b) Move to the origin and remains at rest

(c) Move to infinity

(d) Execute oscillatory but not simple harmonic motion.

**Solution:** (d) By symmetry of problem the components of force on $Q$ due to charges at $A$ and $B$ along y-axis will cancel each other while along x-axis will add up and will be along $CO$. Under the action of this force charge $Q$ will move towards $O$. If at any time charge $Q$ is at a distance $x$ from $O$.




$$F \Rightarrow 2F\cos\theta = 2\frac{1}{4\pi\varepsilon_0}\frac{-qQ}{(a^2 + x^2)} \times \frac{x}{(a^2 + x^2)^{1/2}}$$

*i.e.*, $$F = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{2qQx}{\left(a^2 + x^2\right)^{3/2}}$$

As the restoring force F is not linear, motion will be oscillatory (with amplitude $2a$) but not simple

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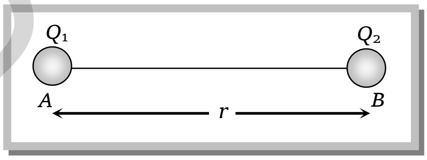
harmonic.

## Electric Potential Energy

(1) **Potential energy of a charge :** Work done in bringing the given charge from infinity to a point in the electric field is known as potential energy of the charge. Potential can also be written as potential energy per unit charge. *i.e.* $V = \frac{W}{Q} = \frac{U}{Q}$.

(2) **Potential energy of a system of two charges :** Since work done in bringing charge $Q_2$ from $\infty$ to point $B$ is $W = Q_2 V_B$, where $V_B$ is potential of point $B$ due to charge $Q_1$ i.e. $V_B = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r}$

So, $$W = U_2 = \frac{1}{4\pi\varepsilon_0}.\frac{Q_1 Q_2}{r}$$

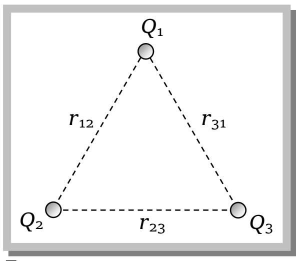



This is the potential energy of charge $Q_2$, similarly potential energy of charge $Q_1$ will be $U_1 = \frac{1}{4\pi\varepsilon_0}.\frac{Q_1 Q_2}{r}$

Hence potential energy of $Q_1$ = Potential energy of $Q_2$ = potential energy of system $U = k\frac{Q_1 Q_2}{r}$ (in C.G.S. $U = \frac{Q_1 Q_2}{r}$)

*Note :* ❑ Electric potential energy is a scalar quantity so in the above formula take sign of $Q_1$ and $Q_2$.

(3) **Potential energy of a system of *n* charges :** In a system of $n$ charges electric potential energy is calculated for each pair and then all energies so obtained are added algebraically. *i.e.* $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1 Q_2}{r_{12}} + \frac{Q_2 Q_3}{r_{23}} + .........\right]$ and in case of continuous distribution of charge. As $dU = dQ.V \Rightarrow U = \int V\,dQ$




*e.g.* Electric potential energy for a system of three charges

Potential energy $= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1 Q_2}{r_{12}} + \frac{Q_2 Q_3}{r_{23}} + \frac{Q_3 Q_1}{r_{31}}\right]$

While potential energy of any of the charge say $Q_1$ is $\frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1 Q_2}{r_{12}} + \frac{Q_3 Q_1}{r_{31}}\right]$

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*Note* : ❑ For the expression of total potential energy of a system of $n$ charges consider $\frac{n(n-1)}{2}$ number of pair of charges.

(4) **Electron volt (*eV*) :** It is the smallest practical unit of energy used in atomic and nuclear physics. As electron volt is defined as "the energy acquired by a particle having one quantum of charge 1*e* when accelerated by 1*volt*" *i.e.* $1eV = 1.6\times10^{-19}C\times\frac{1J}{C} = 1.6\times10^{-19}J$ = $1.6 \times 10^{-12}$ *erg*

Energy acquired by a charged particle in *eV* when it is accelerated by *V volt* is *E* = (charge in quanta) × (p.d. in *volt*)

**Commonly asked examples** :

| S.No. | Charge | Accelerated by p.d. | Gain in K.E. |
|---|---|---|---|
| (i) | Proton | $5 \times 10^4\ V$ | $K = e \times 5 \times 10^4\ V = 5 \times 10^4\ eV = 8 \times 10^{-15}\ J$ **[JIPMER 1999]** |
| (ii) | Electron | $100\ V$ | $K = e \times 100\ V = 100\ eV = 1.6 \times 10^{-17}\ J$ **[MP PMT 2000; AFMC 1999]** |
| (iii) | Proton | $1\ V$ | $K = e \times 1\ V = 1\ eV = 1.6 \times 10^{-19}\ J$ **[CBSE 1999]** |
| (iv) | $0.5\ C$ | $2000\ V$ | $K = 0.5 \times 2000 = 1000\ J$ **[JIPMER 2002]** |
| (v) | $\alpha$-particle | $10^6\ V$ | $K = (2e) \times 10^6\ V = 2\ MeV$ **[MP PET/PMT 1998]** |

(5) **Electric potential energy of a uniformly charged sphere :** Consider a uniformly charged sphere of radius *R* having a total charge *Q*. The electric potential energy of this sphere is equal to the work done in bringing the charges from infinity to assemble the sphere.

$$U = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

(6) **Electric potential energy of a uniformly charged thin spherical shell :**

$$U = \frac{Q^2}{8\pi\varepsilon_0 R}$$

(7) **Energy density :** The energy stored per unit volume around a point in an electric field is given by

$U_e = \frac{U}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$. If in place of vacuum some medium is present then $U_e = \frac{1}{2}\varepsilon_0\varepsilon_r E^2$

**_Concepts_**

☞ *Electric potential energy is not localised but is distributed all over the field*

☞ *If a charge moves from one position to another position in an electric field so it's potential energy change and*

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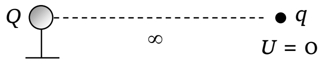
*work done in this changing is* $W = U_f - U_i$

☞ *If two similar charge comes closer potential energy of system increases while if two dissimilar charge comes closer potential energy of system decreases.*




## *Examples based on electric potential*

**Example: 74** If the distance of separation between two charges is increased, the electrical potential energy of the system

**[AMU 1998]**

(a) May increases or decrease (b) Decreases

(c) Increase (d) Remain the same

**Solution:** (a) Since we know potential energy $U = k.\frac{Q_1 Q_2}{r}$

As $r$ increases, $U$ decreases in magnitude. However depending upon the fact whether both charges are similar or disimilar, $U$ may increase or decrease.

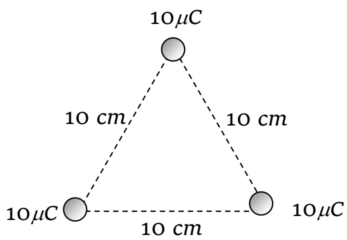
**Example: 75** Three particles, each having a charge of $10\mu C$ are placed at the corners of an equilateral triangle of side $10 cm$. The electrostatic potential energy of the system is (Given $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 N - m^2/C^2$)

**[AMU 1998]**

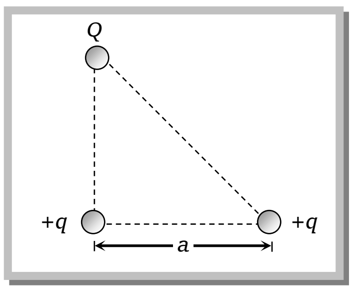
(a) Zero (b) Infinite (c) 27 $J$ (d) 100 $J$

**Solution:** (c) Potential energy of the system,

$$U = 9 \times 10^9 \left[\frac{(10 \times 10^{-6})^2}{0.1} \times 3\right] = 27\,J$$




**Example: 76** Three charges $Q$, $+q$ and $+q$ are placed at the vertices of a right-angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero if $Q$ is equal to




(a) $\frac{-q}{1+\sqrt{2}}$

(b) $\frac{-\sqrt{2}q}{1+\sqrt{2}}$

(c) – 2 $q$

(d) $+q$

**Solution:** (b) Potential energy of the configuration $U = k.\frac{Qq}{a} + \frac{k.q^2}{a} + k.\frac{Qq}{a\sqrt{2}} = 0 \Rightarrow Q = \frac{-\sqrt{2}\,q}{\sqrt{2}+1}$

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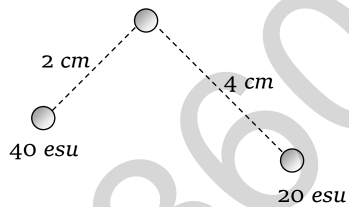
**Example: 77** A charge 10 *e.s.u.* is placed at a distance of 2 *cm* from a charge 40 *e.s.u.* and 4 *cm* from another charge of 20 *e.s.u.* The potential energy of the charge 10 *e.s.u.* is (in *ergs*)

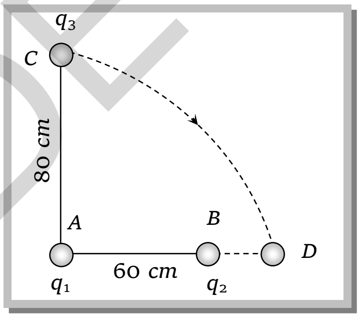
(a) 87.5 (b) 112.5 (c) 150 (d) 250

**Solution:** (d) Potential energy of 10 *e.s.u.* charge is

$$U = \frac{10 \times 40}{2} + \frac{10 \times 20}{4} = 250 \ erg.$$




**Example: 78** In figure are shown charges $q_1 = +2 \times 10^{-8}\ C$ and $q_2 = -0.4 \times 10^{-8}\ C$. A charge $q_3 = 0.2 \times 10^{-8}\ C$ in moved along the arc of a circle from $C$ to $D$. The potential energy of $q_3$



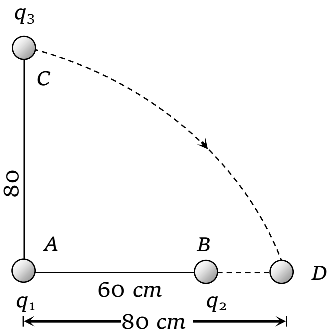

(a) Will increase approximately by 76%

(b) Will decreases approximately by 76%

(c) Will remain same

(d) Will increases approximately by 12%

**Solution:** (b) Initial potential energy of $q_3$ $U_i = \left(\frac{q_1 q_3}{0.8} + \frac{q_2 q_3}{1}\right) \times 9 \times 10^9$




Final potential energy of $q_3$ $U_f = \left(\frac{q_1 q_3}{0.8} + \frac{q_2 q_3}{0.2}\right) \times 9 \times 10^9$

Change in potential energy = $U_f - U_i$

Now percentage change in potential energy $= \frac{U_f - U_i}{u_i} \times 100$

$$= \frac{q_2 q_3 \left(\frac{1}{0.2} - 1\right) \times 100}{q_3 \left(\frac{q_1}{0.8} + \frac{q_2}{1}\right)}$$ On putting the values $\simeq -76\%$

**Tricky example: 10**

Three charged particles are initially in position 1. They are free to move and they come in position 2 after some time. Let $U_1$ and $U_2$ be the electrostatics potential energies in position 1 and 2. Then

(a) $U_1 > U_2$ (b) $U_2 > U_1$

(c) $U_1 = U_2$ (d) $U_2 \geq U_1$

**Solution:** (a) Particles move in a direction where potential energy of the system is decreased.

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## Motion of Charged Particle in an Electric Field

(1) **When charged particle initially at rest is placed in the uniform field :**

Let a charge particle of mass $m$ and charge $Q$ be initially at rest in an electric field of strength $E$

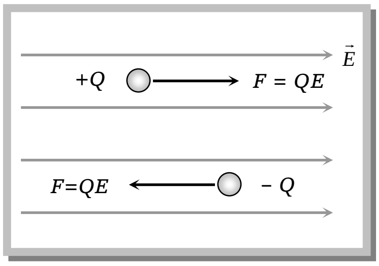


**Fig.**

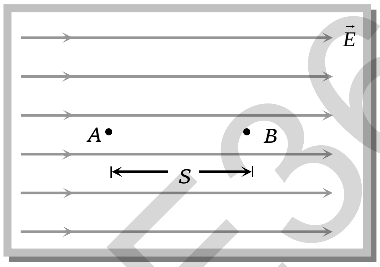


**Fig.**

(i) **Force and acceleration :** The force experienced by the charged particle is $F = QE$. Positive charge experiences force in the direction of electric field while negative charge experiences force in the direction opposite to the field. [Fig. (A)]

Acceleration produced by this force is $a = \dfrac{F}{m} = \dfrac{QE}{m}$

Since the field $E$ in constant the acceleration is constant, thus motion of the particle is uniformly accelerated.

(ii) **Velocity :** Suppose at point $A$ particle is at rest and in time $t$, it reaches the point $B$ [Fig. (B)]

$V$ = Potential difference between $A$ and $B$; $S$ = Separation between $A$ and $B$

(a) By using $v = u + at$, $\quad v = 0 + Q\dfrac{E}{m}t$, $\quad \Rightarrow \quad v = \dfrac{QEt}{m}$

(b) By using $v^2 = u^2 + 2as$, $v^2 = 0 + 2 \times \dfrac{QE}{m} \times s$ $\quad v^2 = \dfrac{2QV}{m}$ $\quad \left\{\because E = \dfrac{V}{s}\right\} \Rightarrow \quad v = \sqrt{\dfrac{2QV}{m}}$

(iii) **Momentum :** Momentum $p = mv$, $p = m \times \dfrac{QEt}{m} = QEt$ or $p = m \times \sqrt{\dfrac{2QV}{m}} = \sqrt{2mQV}$

(iv) **Kinetic energy :** Kinetic energy gained by the particle in time $t$ is $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\dfrac{(QEt)^2}{m} = \dfrac{Q^2E^2t^2}{2m}$

or $\qquad K = \dfrac{1}{2}m \times \dfrac{2QV}{m} = QV$

(2) **When a charged particle enters with an initial velocity at right angle to the uniform field :**

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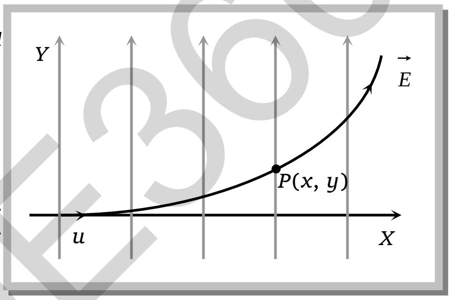
When charged particle enters perpendicularly in an electric field, it describe a parabolic path as shown

(i) Equation of trajectory : Throughout the motion particle has uniform velocity along $x$-axis and horizontal displacement ($x$) is given by the equation $x = ut$

Since the motion of the particle is accelerated along $y$–axis, we will use equation of motion for uniform acceleration to determine displacement $y$. From $S = u$



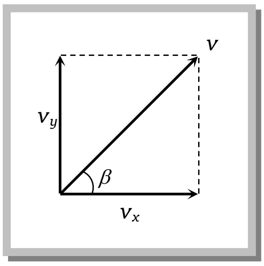

We have $u = 0$ (along $y$-axis) so $y = \frac{1}{2}at^2$

*i.e.*, displacement along $y$-axis will increase rapidly with time (

From displacement along $x$-axis $t = \frac{x}{u}$

So $y = \frac{1}{2}\left(\frac{QE}{m}\right)\left(\frac{x}{u}\right)^2$ ; this is the equation of parabola which shows $y \propto x^2$

(ii) Velocity at any instant : At any instant $t$, $v_x = u$ and $v_y = \frac{QEt}{m}$



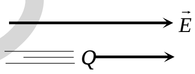

So $v = |\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + \frac{Q^2E^2t^2}{m^2}}$

If $\beta$ is the angle made by $v$ with $x$-axis than $\tan\beta = \frac{v_y}{v_x} = \frac{QEt}{mu}$.

### *Concepts*

☞ *An electric field is completely characterized by two physical quantities Potential and Intensity. Force characteristic of the field is intensity and work characteristic of the field is potential.*

☞ *If a charge particle (say positive) is left free in an electric field, it experiences a force $(F = QE)$ in the direction of electric field and moves in the direction of electric field (which is desired by electric field), so its kinetic energy increases, potential energy decreases, then work is done by the electric field and it is negative.*




### ***Examples based on motion of***

**Example: 79** An electron (mass = $9.1 \times 10^{-31}\ kg$ and charge = $1.6 \times 10^{-19}\ coul.$) is sent in an electric field of intensity $1 \times 10^6\ V/m$. How long would it take for the electron, starting from rest, to attain one–tenth the velocity of light

(a) $1.7 \times 10^{-12}\ sec$ (b) $1.7 \times 10^{-6}\ sec$ (c) $1.7 \times 10^{-8}\ sec$ (d) $1.7 \times 10^{-10}\ sec$

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**Solution:** (b) By using $v = \frac{QEt}{m} \Rightarrow \frac{1}{10} \times 3 \times 10^8 = \frac{(1.6 \times 10^{-19}) \times 10^6 \times t}{9.1 \times 10^{-31}} \Rightarrow t = 1.7 \times 10^{-10}\, sec.$

**Example: 80** Two protons are placed $10^{-10} m$ apart. If they are repelled, what will be the kinetic energy of each proton at very large distance

(a) $23 \times 10^{-19}\ J$ (b) $11.5 \times 10^{-19}\ J$ (c) $2.56 \times 10^{-19}\ J$ (d) $2.56 \times 10^{-28}\ J$

**Solution:** (d) Potential energy of the system when protons are separated by a distance of $10^{-10}$ *m* is

$$U = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{10^{-10}} = 23 \times 10^{-19}\ J$$


According to law of conservation of energy at very larger distance, this energy is equally distributed in both the protons as their kinetic energy hence *K.E.* of each proton will be $11.5 \times 10^{-19}\ J.$

**Example: 81** A particle *A* has a charge +*q* and particle *B* has charge +4*q* with each of them having the same mass *m*. When allowed to fall from rest through the same electrical potential difference, the ratio of their speeds $\frac{v_A}{v_B}$ will becomes

(a) 2 : 1 (b) 1 : 2 (c) 1 : 4 (d) 4 : 1

**Solution:** (b) We know that kinetic energy $K = \frac{1}{2} mv^2 = QV$. Since, *m* and *V* are same so, $v^2 \propto Q \Rightarrow$ $\frac{v_A}{v_B} = \sqrt{\frac{Q_A}{Q_B}} = \sqrt{\frac{q}{4q}} = \frac{1}{2}.$

**Example: 82** How much kinetic energy will be gained by an $\alpha-$particle in going from a point at 70 *V* to another point at 50 *V* **[RPET 1996]**

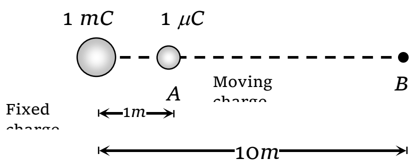
(a) 40 *eV* (b) 40 *keV* (c) 40 *MeV* (d) 0 *eV*

**Solution:** (a) Kinetic energy $K = Q\Delta V \Rightarrow K = (2e)(70 - 50)V = 40\ eV$

**Example: 83** A particle of mass 2*g* and charge $1\mu C$ is held at a distance of 1 metre from a fixed charge of $1mC$. If the particle is released it will be repelled. The speed of the particle when it is at a distance of 10 metres from the fixed charge is

(a) 100 *m/s* (b) 90 *m/s* (c) 60 *m/s* (d) 45 *m/s*

**Solution:** (b) According to conservation of energy




Energy of moving charge at $A =$ Energy of moving charge at *B*

$$9 \times 10^9 \times \frac{10^{-3} \times 10^{-6}}{1} = 9 \times 10^9 \times \frac{10^{-3} \times 10^{-6}}{10} + \frac{1}{2} \times (2 \times 10^{-3}) v^2$$

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$\Rightarrow \quad v^2 = 8100 \quad \Rightarrow \quad v = 90 \text{ m/sec}$

**Tricky example: 11**

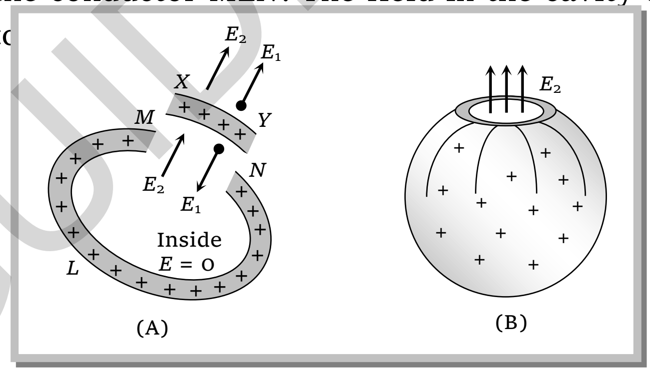
A mass of 1*g* carrying charge *q* falls through a potential difference *V*. The kinetic energy acquired by it is *E*. When a mass of 2*g* carrying the charge *q* falls through a potential difference *V*. What will be the kinetic energy acquired by it

(a) 0.25 *E* (b) 0.50 *E* (c) 0.75 *E* (d) *E*

**Solution:** (d) In electric field kinetic energy gain by the charged particle *K* = *qV*. Which depends charge and potential difference applied but not on the mass of the charged particle.

## Force on a Charged Conductor

To find force on a charged conductor (due to repulsion of like charges) imagine a small part *XY* to be cut and just separated from the rest of the conductor *MLN*. The field in the cavity due to the rest of the conductor is $E_2$, while field due to




Inside the conductor $E = E_1 - E_2 = 0$ or $E_1 = E_2$

Outside the conductor $E = E_1 + E_2 = \dfrac{\sigma}{\varepsilon_0}$

Thus $E_1 = E_2 = \dfrac{\sigma}{2\varepsilon_0}$

To find force, imagine charged part *XY* (having charge $\sigma\, dA$ placed in the cavity *MN* having field $E_2$). Thus force $dF = (\sigma\, dA)E_2$ or $dF = \dfrac{\sigma^2}{2\varepsilon_0} dA$. The force per unit area or electric pressure is $\dfrac{dF}{dA} = \dfrac{\sigma^2}{2\varepsilon_0}$

The force is always outwards as $(\pm\sigma)^2$ is positive *i.e.,* whether charged positively or negatively, this force will try to expand the charged body.

A soap bubble or rubber balloon expands on given charge to it (charge of any kind + or –).

## Equilibrium of Charged Soap Bubble

For a charged soap bubble of radius *R* and surface tension *T* and charge density $\sigma$. The pressure due to surface tension $4\dfrac{T}{R}$ and atmospheric pressure $P_{\text{out}}$ act radially inwards and the electrical pressure $(P_{el})$ acts radially outward.

The total pressure inside the soap bubble $P_{\text{in}} = P_{\text{out}} + \dfrac{4T}{R} - \dfrac{\sigma^2}{2\varepsilon_0}$

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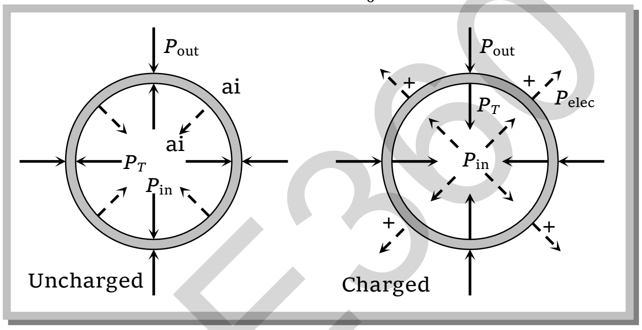
Excess pressure inside the charged soap bubble $P_{\text{in}} - P_{\text{out}} = P_{\text{excess}} = \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}$. If air pressure inside and outside are assumed equal then $P_{\text{in}} = P_{\text{out}}$ *i.e.,* $P_{\text{excess}} = 0$. So, $\frac{4T}{R} = \frac{\sigma^2}{2\varepsilon_0}$




This result give us the following formulae

(1) Radius of bubble $R = \frac{8\,\varepsilon_0\,T}{\sigma^2}$

(2) Surface tension $T = \frac{\sigma^2 R}{8\varepsilon_0}$

(3) Total charge on the bubble $Q = 8\pi R\sqrt{2\varepsilon_0 TR}$

(4) Electric field intensity at the surface of the bubble $E = \sqrt{\frac{8T}{\varepsilon_0 R}} = \sqrt{\frac{32\pi kT}{R}}$

(5) Electric potential at the surface $V = \sqrt{3\pi RTk} = \sqrt{\frac{8RT}{\varepsilon_0}}$

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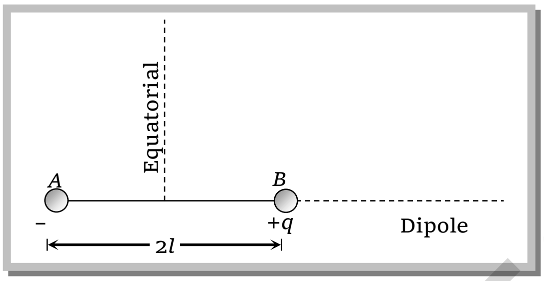
## Electric Dipole

(1) **General information :** System of two equal and opposite charges separated by a small fixed distance is called a dipole.




(i) **Dipole axis :** Line joining negative charge to positive charge of a dipole is called its axis. It may also be termed as its longitudinal axis.

(ii) **Equatorial axis :** Perpendicular bisector of the dipole is called its equatorial or transverse axis as it is perpendicular to length.

(iii) **Dipole length :** The distance between two charges is known as dipole length ($L = 2l$)

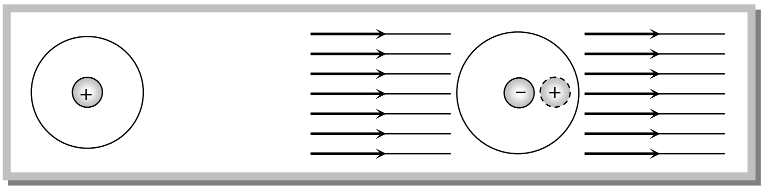
(iv) **Dipole moment :** It is a quantity which gives information about the strength of dipole. It is a vector quantity and is directed from negative charge to positive charge along the axis. It is denoted as $\vec{p}$ and is defined as the product of the magnitude of either of the charge and the dipole length.

*i.e.* $$\vec{p} = q(2\vec{l})$$

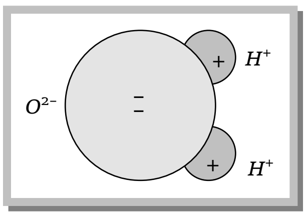
Its S.I. unit is ***coulomb-metre*** or ***Debye*** (1 *Debye* = $3.3 \times 10^{-30}$ $C \times m$) and its dimensions are $M^0L^1T^1A^1$.

*Note* : ❑ A region surrounding a stationary electric dipole has electric field only.

❑ When a dielectric is placed in an electric field, its atoms or molecules are considered as tiny dipoles.



❑ Water ($H_2O$), Chloroform ($CHCl_3$), Ammonia ($NH_3$), $HCl$, $CO$ molecules are some example of permanent electric dipole.

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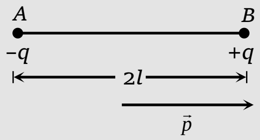
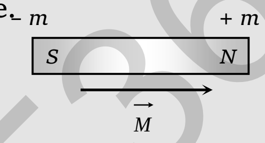
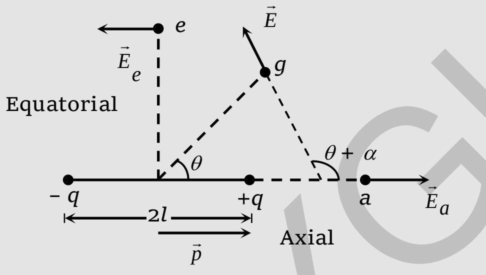
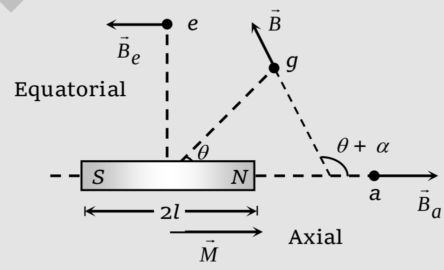
(2) **Electric field and potential due to an electric dipole :** It is better to understand electric dipole with magnetic dipole.

| S.No. | Electric dipole | Magnetic dipole |
|---|---|---|
| (i) | System of two equal and opposite charges separated by a small fixed distance.<br> | System of two equal and opposite magnetic poles (Bar magnet) separated by a small fixed distance.<br> |
| (ii) | **Electric dipole moment :** $\vec{p} = q(2\vec{l})$, directed from $-q$ to $+q$. It's S.I. unit is *coulomb* × *meter* or *Debye*. | **Magnetic dipole moment :** $\vec{M} = m(2\vec{l})$, directed from $S$ to $N$. It's S.I. unit is *ampere* × *meter*$^2$. |
| (iii) | **Intensity of electric field**<br><br>If $a$, $e$ and $g$ are three points on axial, equatorial and general position at a distance $r$ from the centre of dipole<br>on axial point $E_a = \frac{1}{4\pi\varepsilon_0}.\frac{2p}{r^3}$ (directed from $-q$ to $+q$)<br>on equatorial point $E_e = \frac{1}{4\pi\varepsilon_0}.\frac{p}{r^3}$ (directed from $+q$ to $-q$)<br>on general point $E_a = \frac{1}{4\pi\varepsilon_0}.\frac{p}{r^3}\sqrt{(3\cos^2\theta + 1)}$<br>Angle between – $\vec{E}_a$ and $\vec{p}$ is 0°, $\vec{E}_e$ and $\vec{p}$ is 180°, $\vec{E}$ and $\vec{p}$ is $(\theta + \alpha)$ (where $\tan\alpha = \frac{1}{2}\tan\theta$ ) | **Intensity of magnetic field**<br><br>If $a$, $e$ and $g$ are three points on axial, equatorial and general position at a distance $r$ from the centre of dipole<br>on axial point $B_a = \frac{\mu_0}{4\pi}.\frac{2M}{r^3}$ (directed from $S$ to $N$)<br>on equatorial point $B_e = \frac{\mu_0}{4\pi}.\frac{M}{r^3}$ (directed from $N$ to $S$)<br>on general point $B_a = \frac{\mu_0}{4\pi}.\frac{M}{r^3}\sqrt{(3\cos^2\theta + 1)}$<br>Angle between – $\vec{B}_a$ and $\vec{M}$ is 0°, $\vec{B}_e$ and $\vec{M}$ is 180°, $\vec{B}$ and $\vec{M}$ is $(\theta + \alpha)$ (where $\tan\alpha = \frac{1}{2}\tan\theta$ ) |
| | Electric Potential – At $a$ $V_a = \frac{1}{4\pi\varepsilon_0}.\frac{p}{r^2}$, At $e$ $V = 0$ | |

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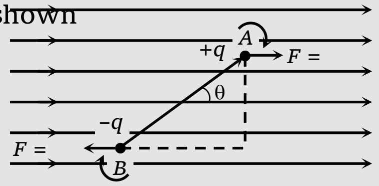
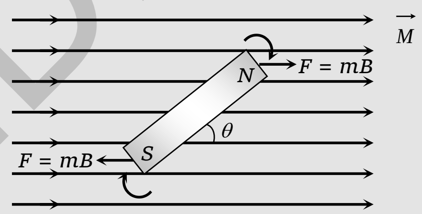
| | | |
|---|---|---|
| | At g $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p\cos\theta}{r^2}$ | |

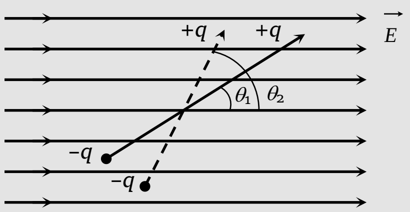
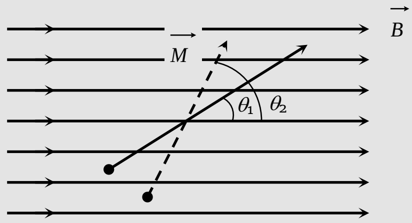
(3) **Dipole (electric/magnetic) in uniform field (electric/magnetic)**

(i) **Torque :** If a dipole is placed in an uniform field such that dipole (*i.e.* $\vec{p}$ or $\vec{M}$ ) makes an angle $\theta$ with direction of field then two equal and opposite force acting on dipole constitute a couple whose tendency is to rotate the dipole hence a torque is developed in it and dipole tries to align it self in the direction of field.

| | |
|---|---|
| Consider an electric dipole in placed in an uniform electric field such that dipole (i.e. $\vec{p}$ ) makes an angle $\theta$ with the direction of electric field as shown   | A magnetic dipole of magnetic moment $M$ is placed in uniform magnetic field $B$ by making an angle $\theta$ as shown   |
| (a) Net force on electric dipole $F_{net} = 0$ | (a) Net force on magnetic dipole $F_{net} = 0$ |
| (b) Produced torque $\tau = pE\sin\theta$ $(\vec{\tau} = \vec{P}\times\vec{E})$ | (b) torque $\tau = MB\sin\theta$ $(\vec{\tau} = \vec{M}\times\vec{B})$ |

(ii) **Work :** From the above discussion it is clear that in an uniform electric/magnetic field dipole tries to align itself in the direction of electric field (i.e. equilibrium position). To change it's angular position some work has to be done.

Suppose an electric/magnetic dipole is kept in an uniform electric/magnetic field by making an angle $\theta_1$ with the field, if it is again turn so that it makes an angle $\theta_2$ with the field, work done in this process is given by the formula

| | |
|---|---|
|   |   |
| $W = pE(\cos\theta_1 - \cos\theta_2)$ | $W = MB(\cos\theta_1 - \cos\theta_2)$ |
| If $\theta_1$ = 0° and $\theta_2$ = $\theta$ *i.e.* initially dipole is kept along the field then it turn through $\theta$ | If $\theta_1$ = 0° and $\theta_2$ = $\theta$ then $W = MB(1 - \cos\theta)$ |

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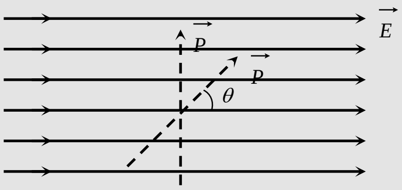
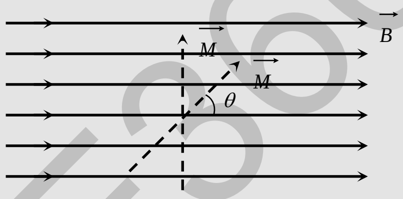
| | |
|---|---|
| so work done $W = pE(1 - \cos\theta)$ | |

(iii) **Potential energy :** In case of a dipole (in a uniform field), potential energy of dipole is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction *i.e.* if $\theta_1 = 90^o$ and $\theta_2 = \theta$ then –

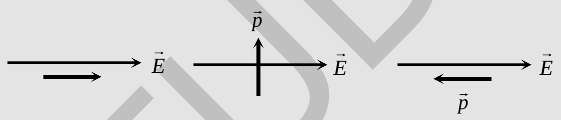
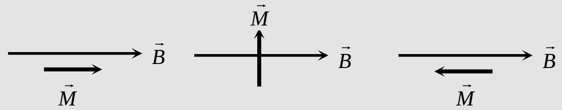
| | |
|---|---|
|  |  |
| $W = U = pE(\cos 90 - \cos\theta) \Rightarrow U = -pE\cos\theta$ | $W = U = MB(\cos 90 - \cos\theta) \Rightarrow U = -MB\cos\theta$ |

(iv) **Equilibrium of dipole :** We know that, for any equilibrium net torque and net force on a particle (or system) should be zero.

We already discussed when a dipole is placed in an uniform electric/magnetic field net force on dipole is always zero. But net torque will be zero only when $\theta = 0^o$ or $180^o$

When $\theta = 0^o$ *i.e.* dipole is placed along the electric field it is said to be in stable equilibrium, because after turning it through a small angle, dipole tries to align itself again in the direction of electric field.

When $\theta = 180^o$ *i.e.* dipole is placed opposite to electric field, it is said to be in unstable equilibrium.

| | | | | | |
|---|---|---|---|---|---|
|  | | |  | | |
| $\theta = 0^o$ | $\theta = 90^O$ | $\theta = 180^o$ | $\theta = 0^o$ | $\theta = 90^O$ | $\theta = 180^o$ |
| Stable equilibrium | | Unstable | Stable equilibrium | | |
| equilibrium | | | Unstable equilibrium | | |
| $\tau = 0$ | $\tau_{max} = pE$ | $\tau = 0$ | $\tau = 0$ | $\tau_{max} = MB$ | $\tau = 0$ |
| $W = 0$ | $W = pE$ | $W_{max} = 2pE$ | $W = 0$ | $W = MB$ | $W_{max} = 2MB$ |
| $U_{min} = -pE$ | $U = 0$ | $U_{max} = pE$ | $U_{min} = -MB$ | $U = 0$ | $U_{max} = MB$ |

(v) **Angular SHM :** In a uniform electric/magnetic field (intensity $E/B$) if a dipole (electric/magnetic) is slightly displaced from it's stable equilibrium position it executes angular SHM having period of oscillation. If $I$ = moment of inertia of dipole about the axis passing through it's centre and perpendicular to it's length.

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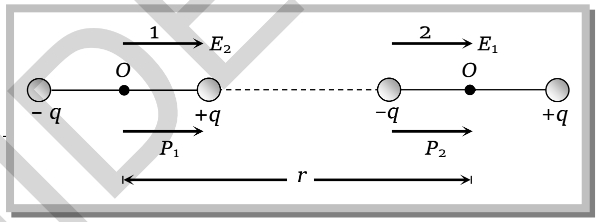
For electric dipole : $T = 2\pi\sqrt{\frac{I}{pE}}$ and For Magnetic dipole : $T = 2\pi\sqrt{\frac{I}{MB}}$

(vi) **Dipole-point charge interaction :** If a point charge/isolated magnetic pole is placed in dipole field at a distance *r* from the mid point of dipole then force experienced by point charge/pole varies according to the relation $F \propto \frac{1}{r^3}$

(vii) **Dipole-dipole interaction :** When two dipoles placed closed to each other, they experiences a force due to each other. If suppose two dipoles (1) and (2) are placed as shown in figure then

Both the dipoles are placed in the field of one another hence potential energy dipole (2) is



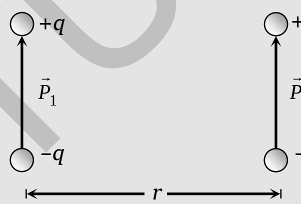
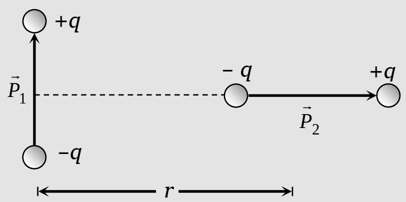

$$U_2 = -p_2 E_1 \cos 0 = -p_2 E_1 = -p_2 \times \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p_1}{r^3}$$

then by using $F = -\frac{dU}{dr}$, Force on dipole (2) is $F_2 =$

$$\Rightarrow F_2 = -\frac{d}{dr}\left\{\frac{1}{4\pi\varepsilon_0} \cdot \frac{2p_1 p_2}{r^3}\right\} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{6p_1 p_2}{r^4}$$

Similarly force experienced by dipole (1) $F_1 = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{6p_1 p_2}{r^4}$ so $F_1 = F_2 = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{6p_1 p_2}{r^4}$

Negative sign indicates that force is attractive. $|F| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{6p_1 p_2}{r^4}$ and $\boldsymbol{F \propto \frac{1}{r^4}}$

| S. No. | Relative position of dipole | Force | Potential energy |
|---|---|---|---|
| (i) |   | $\frac{1}{4\pi\varepsilon_0} \cdot \frac{6p_1 p_2}{r^4}$ (attractive) | $\frac{1}{4\pi\varepsilon_0} \cdot \frac{2p_1 p_2}{r^3}$ |
| (ii) |   | $\frac{1}{4\pi\varepsilon_0} \cdot \frac{3p_1 p_2}{r^4}$ (repulsive) | $\frac{1}{4\pi\varepsilon_0} \cdot \frac{p_1 p_2}{r^3}$ |
| (iii) |   | $\frac{1}{4\pi\varepsilon_0} \cdot \frac{3p_1 p_2}{r^4}$ (perpendicular to *r* ) | 0 |

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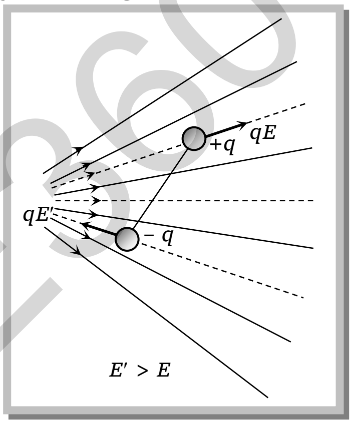
**Note :** ❑ Same result can also be obtained for magnetic dipole.

(4) **Electric dipole in non-uniform electric field :** When an electric dipole is placed in a non-uniform field, the two charges of dipole experiences unequal forces, therefore the net force on the dipole is not equal to zero. The magnitude of the force is given by the negative derivative of the potential energy w.r.t. distance along the axis of the dipole *i.e.*

$$\vec{F} = -\frac{dU}{dr} = -\vec{p}\,.\frac{d\vec{E}}{dr}.$$




Due to two unequal forces, a torque is produced which rotate the dipole so as to align it in the direction of field. When the dipole gets aligned with the field, the torque becomes zero and then the unbalanced force acts on the dipole and the dipole then moves linearly along the direction of field from weaker portion of the field to the stronger portion of the field. So in non-uniform electric field

(i) Motion of the dipole is translatory and rotatory

(ii) Torque on it may be zero.

### *Concepts*

☞ *For a short dipole, electric field intensity at a point on the axial line is double than at a point on the equatorial line on electric dipole i.e.* $E_{axial} = 2E_{equatorial}$

☞ *It is intresting to note that dipole field* $E \propto \frac{1}{r^3}$ *decreases much rapidly as compared to the field of a point charge* $\left(E \propto \frac{1}{r^2}\right)$.

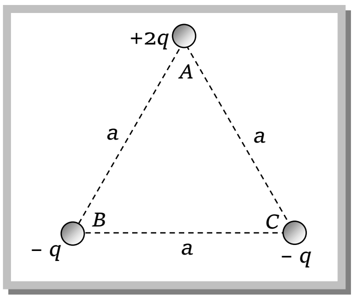

### *Examples based on electric*

**Example: 84** If the magnitude of intensity of electric field at a distance $x$ on axial line and at a distance $y$ on equatorial line on a given dipole are equal, then $x : y$ is

(a) 1 : 1 (b) $1 : \sqrt{2}$ (c) 1 : 2 (d) $\sqrt[3]{2} : 1$

**Solution:** (d) According to the question $\frac{1}{4\pi\varepsilon_0}.\frac{2p}{x^3} = \frac{1}{4\pi\varepsilon_0}.\frac{p}{y^3} \Rightarrow \frac{x}{y} = (2)^{1/3} : 1$

**Example: 85** Three charges of $(+2q)$, $(-q)$ and $(-q)$ are placed at the corners $A$, $B$ and $C$ of an equilateral triangle of side a as shown in the adjoining figure. Then the dipole moment of this combination is **[MP PMT 1994; CPMT 1994]**




(a) $qa$

(b) Zero

(c) $qa\sqrt{3}$

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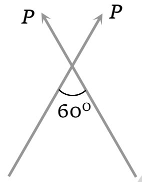
(d) $\frac{2}{\sqrt{3}}qa$

**Solution:** (c) The charge $+2q$ can be broken in $+q$, $+q$. Now as shown in figure we have two equal dipoles inclined at an angle of 60°. Therefore resultant dipole moment will be

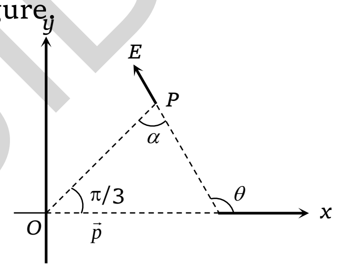



$$p_{net} = \sqrt{p^2 + p^2 + 2pp\cos 60}$$
$$= \sqrt{3}p$$
$$= \sqrt{3}\,qa$$

**Example: 86** An electric dipole is placed along the $x$-axis at the origin $O$. A point $P$ is at a distance of 20 $cm$ from this origin such that $OP$ makes an angle $\frac{\pi}{3}$ with the $x$-axis. If the electric field at $P$ makes an angle $\theta$ with $x$-axis, the value of $\theta$ would be

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{3} + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (c) $\frac{2\pi}{3}$ (d) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

**Solution:** (b) According to question we can draw following figure




As we have discussed earlier in theory $\theta = \frac{\pi}{3} + \alpha$

$$\tan\alpha = \frac{1}{2}\tan\frac{\pi}{3} \Rightarrow \alpha = \tan^{-1}\frac{\sqrt{3}}{2}$$

So, $$\theta = \frac{\pi}{3} + \tan^{-1}\frac{\sqrt{3}}{2}$$

**Example: 87** An electric dipole in a uniform electric field experiences

(a) Force and torque both (b) Force but no torque (c) Torque but no force (d) No force and no torque

**Solution:** (c) In uniform electric field $F_{net} = 0$, $\tau_{net} \neq 0$

**Example: 89** Two opposite and equal charges $4 \times 10^{-8}$ *coulomb* when placed $2 \times 10^{-2}$ *cm* away, form a dipole. If this dipole is placed in an external electric field $4 \times 10^{8}$ *newton/coulomb*, the value of maximum torque and the work done in rotating it through 180° will be

(a) $64 \times 10^{-4}$ *Nm* and $64 \times 10^{-4}$ *J* (b) $32 \times 10^{-4}$ *Nm* and $32 \times 10^{-4}$ *J*

(c) $64 \times 10^{-4}$ *Nm* and $32 \times 10^{-4}$ *J* (d) $32 \times 10^{-4}$ *Nm* and $64 \times 10^{-4}$ *J*

**Solution:** (d) $\tau_{max} = pE$ and $W_{max} = 2pE$ $\because$ $p = Q \times 2l = 4 \times 10^{-8} \times 2 \times 10^{-2} \times 10^{-2} = 8 \times 10^{-12}$ *C-m*

So, $\tau_{max} = 8 \times 10^{-12} \times 4 \times 10^{8} = 32 \times 10^{-4}$ *N-m* and $W_{max} = 2 \times 32 \times 10^{-4} = 64 \times 10^{-4}$ *J*

**Example: 90** A point charge placed at any point on the axis of an electric dipole at some large distance experiences a force $F$. The force acting on the point charge when it's distance from the dipole is doubled is

**[CPMT 1991; MNR 1986]**

(a) $F$ (b) $\frac{F}{2}$ (c) $\frac{F}{4}$ (d) $\frac{F}{8}$

**Solution:** (d) Force acting on a point charge in dipole field varies as $F \propto \frac{1}{r^3}$ where $r$ is the distance of point charge from the centre of dipole. Hence if $r$ makes double so new force $F' = \frac{F}{8}$.

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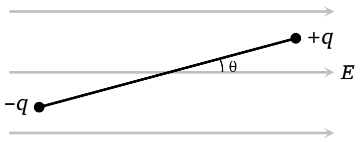
**Example: 91** A point particle of mass $M$ is attached to one end of a massless rigid non-conducting rod of length $L$. Another point particle of the same mass is attached to other end of the rod. The two particles carry charges $+q$ and $-q$ respectively. This arrangement is held in a region of a uniform electric field $E$ such that the rod makes a small angle $\theta$ (say of about 5 degrees) with the field direction (see figure). Will be minimum time, needed for the rod to become parallel to the field after it is set free **[CPMT 1995]**




(a) $t = 2\pi\sqrt{\frac{mL}{2pE}}$ (b) $t = \frac{\pi}{2}\sqrt{\frac{mL}{2qE}}$ (c) $t = \frac{3\pi}{2}\sqrt{\frac{mL}{2pE}}$ (d) $t = \pi\sqrt{\frac{2mL}{qE}}$

**Solution:** (b) In the given situation system oscillate in electric field with maximum angular displacement $\theta$.

It's time period of oscillation (similar to dipole)

$T = 2\pi\sqrt{\frac{I}{pE}}$ where $I$ = moment of inertia of the system and $p = qL$

Hence the minimum time needed for the rod becomes parallel to the field is $t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{I}{pE}}$

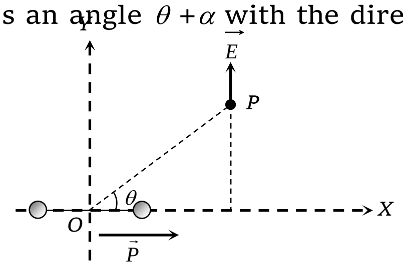
Here $I = M\left(\frac{L}{2}\right)^2 + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{2} \Rightarrow t = \frac{\pi}{2}\sqrt{\frac{ML^2}{2\times qL\times E}} = \frac{\pi}{2}\sqrt{\frac{ML}{2qE}}$

**Tricky example: 12**

An electric dipole is placed at the origin $O$ and is directed along the $x$-axis. At a point $P$, far away from the dipole, the electric field is parallel to $y$-axis. $OP$ makes an angle $\theta$ with the $x$-axis then

(a) $\tan\theta = \sqrt{3}$ (b) $\tan\theta = \sqrt{2}$ (c) $\theta = 45^\circ$ (d) $\tan\theta = \frac{1}{\sqrt{2}}$

**Solution:** (b) As we know that in this case electric field makes an angle $\theta + \alpha$ with the direction of dipole




Where $\tan\alpha = \frac{1}{2}\tan\theta$

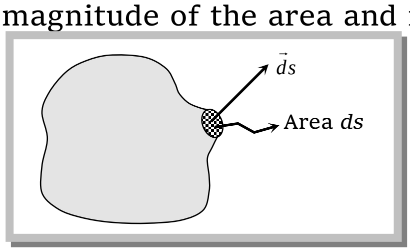
Here $\theta + \alpha = 90^\circ \Rightarrow \alpha = 90 - \theta$

Hence $\tan(90-\theta) = \frac{1}{2}\tan\theta \Rightarrow \cot\theta = \frac{1}{2}\tan\theta$

$\Rightarrow \tan^2\theta = 2 \Rightarrow \tan\theta = \sqrt{2}$

## Electric Flux

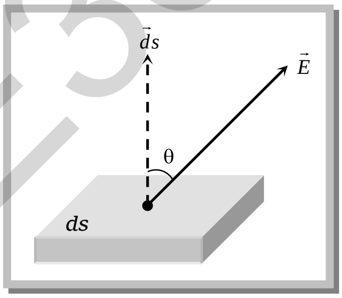
(1) **Area vector :** In many cases, it is convenient to treat area of a surface as a vector. The length of the vector represents the magnitude of the area and its direction is along the outward drawn normal to the area.



(2) **Electric flux :** The electric flux linked with any surface in an electric field is basically a measure of total number of lines of forces passing normally through the surface. **or**

Electric flux through an elementary area $\overrightarrow{ds}$ is defined as the scalar product of area of field i.e. $d\phi = \vec{E}\cdot\overrightarrow{ds} = E\,ds\,\cos\theta$

Hence flux from complete area ($S$) $\phi = \int E\,ds\cos\theta = ES\cos\theta$




If $\theta$ = 0°, *i.e.* surface area is perpendicular to the electric field, so flux linked with it will be max.

*i.e.* $\phi_{max} = E\,ds$ and if $\theta$ = 90°, $\phi_{min}$ = 0

(3) **Unit and Dimensional Formula**

S.I. unit – ($volt \times m$) or $\frac{N-C}{m^2}$

It's Dimensional formula – ($ML^3T^{-3}A^{-1}$)

(4) **Types :** For a closed body outward flux is taken to be positive, while inward flux is to be negative


Positive – flux
(A)

Negative-flux
(B)

## Gauss's Law

(1) **Definition :** According to this law, total electric flux through a closed surface enclosing a charge is $\frac{1}{\varepsilon_0}$ times the magnitude of the charge enclosed i.e. $\phi = \frac{1}{\varepsilon_0}(Q_{enc.})$

(2) **Gaussian Surface :** Gauss's law is valid for symmetrical charge distribution. Gauss's law is very helpful in calculating electric field in those cases where electric field is symmetrical around the source producing it. Electric field can be calculated very easily by the clever choice of a closed surface that encloses the source charges. Such a surface is called "Gaussian surface". This surface should pass through the point where electric field is to be calculated and must have a shape according to the symmetry of source.

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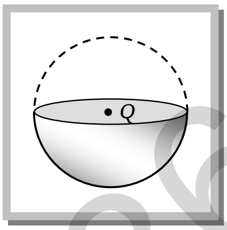
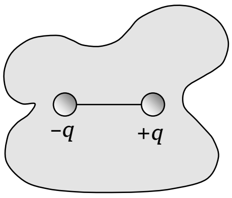
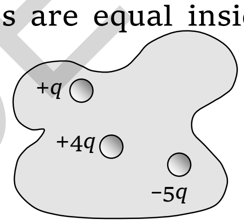
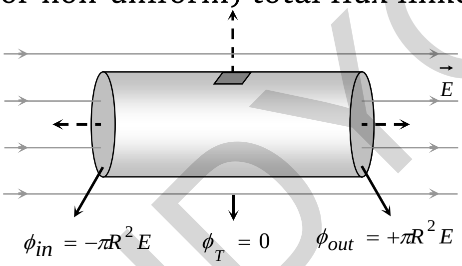
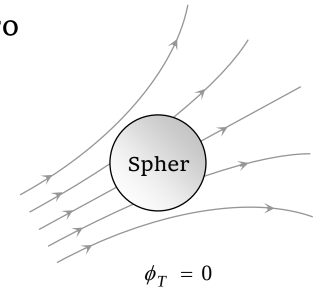
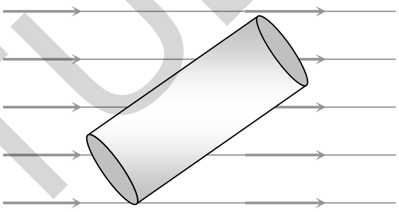
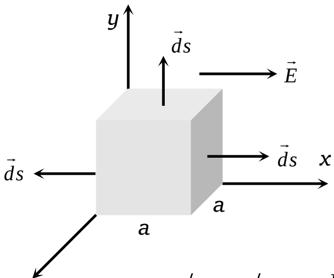
***e.g.*** If suppose a charge $Q$ is placed at the centre of a hemisphere, then to calculate the flux through this body, to encloses the first charge we will have to imagine a Gaussian surface. This imaginary Gaussian surface will be a hemisphere as shown.



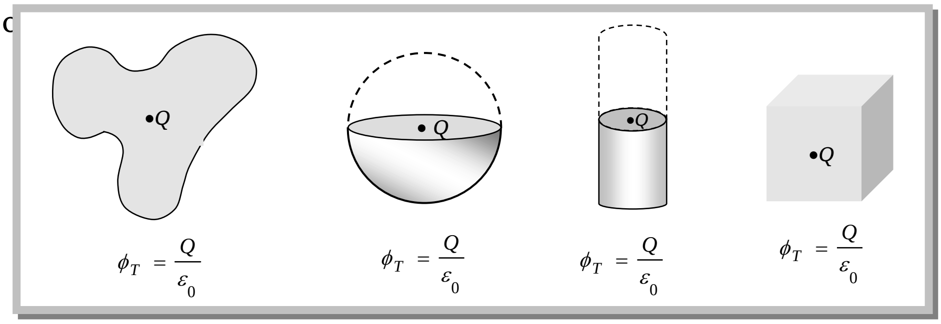

Net flux through this closed body $\phi = \frac{Q}{\varepsilon_0}$

Hence flux coming out from given hemisphere is $\phi = \frac{Q}{2\varepsilon_0}$.

(3) **Zero flux :** The value of flux is zero in the following circumstances

<table>
<tr>
<td>(i) If a dipole is enclosed by a surface<br>$\phi = 0; Q_{enc} = 0$<br>
</td>
<td>(ii) If the magnitude of positive and negative charges are equal inside a closed surface<br>
<br>$Q_{enc} = 0$,<br>so, $\phi = 0$</td>
</tr>
<tr>
<td colspan="2">(iii) If a closed body (not enclosing any charge) is placed in an electric field (either uniform or non-uniform) total flux linked with it will be zero<br>
<br>
<br><br>$\phi_T = 0$<br>
<br>$\phi_{in} = \phi_{out} = E a^2$</td>
</tr>
</table>

(4) **Flux emergence :** Flux linked with a closed body is independent of the shape and size of the body and position c

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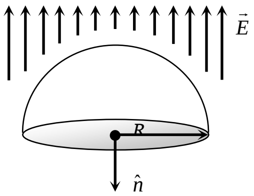
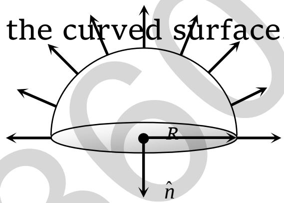
(i) If a hemispherical body is placed in uniform electric field then flux linked with the curved surface



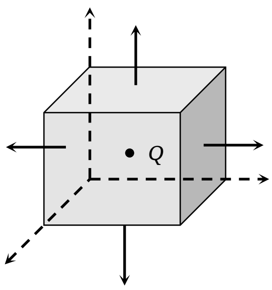

$\phi_{curved} = +\pi R^2 E$

(ii) If a hemispherical body is placed in non-uniform electric field as shown below. then flux linked with the curved surface.




$\phi_{curved} = 2\pi R^2 E$

(v) If charge is kept at the centre of cube

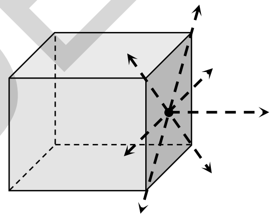



$$\phi_{total} = \frac{1}{\varepsilon_0}.(Q)$$

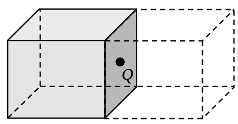
$$\phi_{face} = \frac{Q}{6\varepsilon_0}$$

$$\phi_{corner} = \frac{Q}{8\varepsilon_0} \qquad \phi_{edge} = \frac{Q}{12\varepsilon_0}$$

(iv) If charge is kept at the centre of a face



First we should enclosed the charge by assuming a Gaussian surface (an identical imaginary cube)




$$\phi_{total} = \frac{Q}{\varepsilon_0}$$

$\phi_{cube} = \dfrac{Q}{2\varepsilon_0}$ (*i.e.* from 5 face only)

$$\phi_{face} = \frac{1}{5}\left(\frac{Q}{2\varepsilon_0}\right) = \frac{Q}{10\,\varepsilon_0}.$$

***Concept***

☞ *In C.G.S.* $\varepsilon_0 = \dfrac{1}{4\pi}$. *Hence if 1C charge is enclosed by a closed surface so flux through the surface will be* $\phi = 4\pi$.

***Example based on electric flux and Gauss's***

**Example: 91** Electric charge is uniformly distributed along a long straight wire of radius 1 *mm*. The charge per *cm* length of the wire is *Q coulomb*. Another cylindrical surface of radius 50 *cm* and length 1 *m* symmetrically encloses the wire as shown in the figure. The total electric flux passing through the cylindrical surface is **[MP PET 2001]**

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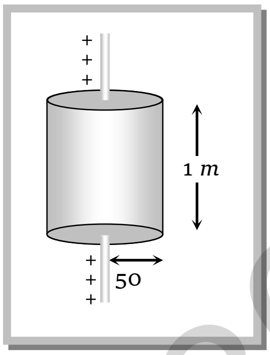
(a) $\frac{Q}{\varepsilon_0}$

(b) $\frac{100\ Q}{\varepsilon_0}$

(c) $\frac{10\ Q}{(\pi\varepsilon_0)}$

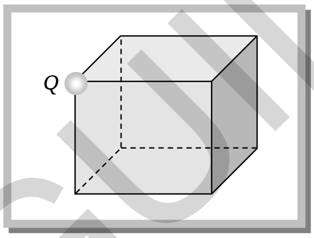
(d) $\frac{100\ Q}{(\pi\varepsilon_0)}$




**Solution:** (b) Given that charge per *cm* length of the wire is *Q*. Since 100 *cm* length of the wire is enclosed so $Q_{enc} = 100\ Q$

$\Rightarrow$ Electric flux emerging through cylindrical surface $\phi = \frac{100\ Q}{\varepsilon_0}$.

**Example: 92** A charge *Q* is situated at the corner *A* of a cube, the electric flux through the one face of the cube is **[CPMT 2000]**




(a) $\frac{Q}{6\varepsilon_0}$ (b) $\frac{Q}{8\varepsilon_0}$ (c) $\frac{Q}{24\,\varepsilon_0}$ (d) $\frac{Q}{2\varepsilon_0}$

**Solution:** (c) For the charge at the corner, we require eight cube to symmetrically enclose it in a Gaussian surface. The total flux $\phi_T = \frac{Q}{\varepsilon_0}$. Therefore the flux through one cube will be $\phi_{cube} = \frac{Q}{8\varepsilon_0}$. The cube has six faces and flux linked with three faces (through *A*) is zero, so flux linked with remaining three faces will $\frac{\phi}{8\varepsilon_0}$. Now as the remaining three are identical so flux linked with each of the three faces will be $= \frac{1}{3} \times \left[\frac{1}{8}\left(\frac{Q}{\varepsilon_0}\right)\right] = \frac{1}{24}\frac{Q}{\varepsilon_0}$.

**Example: 93** A square of side 20 *cm* is enclosed by a surface of sphere of 80 *cm* radius. Square and sphere have the same centre. Four charges $+ 2 \times 10^{-6}$ *C*, $- 5 \times 10^{-6}$ *C*, $- 3 \times 10^{-6}$ *C*, $+6 \times 10^{-6}$ *C* are located at the four corners of a square, then out going total flux from spherical surface in $N{-}m^2/C$ will be **[RPMT 1989]**

(a) Zero (b) $(16\ \pi) \times 10^{-6}$ (c) $(8\pi) \times 10^{-6}$ (d) $36\pi \times 10^{-6}$

**Solution:** (a) Since charge enclosed by Gaussian surface is

$$\phi_{enc.} = (2\times10^{-6} - 5\times10^{-6} - 3\times10^{-6} + 6\times10^{-6}) = 0 \quad \text{so} \quad \phi = 0$$

**Example: 94** In a region of space, the electric field is in the *x*-direction and proportional to *x*, *i.e.*, $\vec{E} = E_0 x\hat{i}$. Consider an imaginary cubical volume of edge *a*, with its edges parallel to the axes of coordinates. The charge inside this cube is

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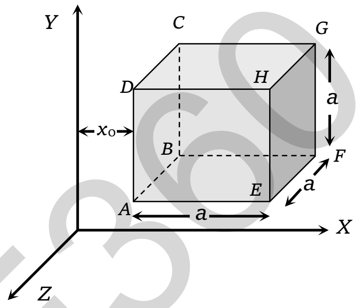
(a) Zero (b) $\varepsilon_0 E_0 a^3$ (c) $\frac{1}{\varepsilon_0} E_0 a^3$ (d) $\frac{1}{6}\varepsilon_0 E_0 a^2$

**Solution:** (b) The field at the face *ABCD* = $E_0 x_0 \hat{i}$.

$\therefore$ Flux over the face *ABCD* = – $(E_0 x_0)a^2$

The negative sign arises as the field is directed into the cube.




The field at the face EFGH = $E_0(x_0 + a)\hat{i}$.

$\therefore$ Flux over the face *EFGH* = $E_0(x_0 + a)a^2$

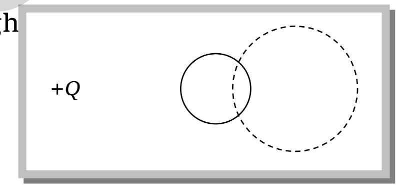
The flux over the other four faces is zero as the field is parallel to the surfaces.

$\therefore$ Total flux over the cube $= E_0 a^2 = \frac{1}{2} q$

where *q* is the total charge inside the cube. $\therefore\ q = \varepsilon_0 E_0 a^3$.

**Tricky example: 13**

In the electric field due to a point charge + *Q* a spherical closed surface is drawn as shown by the dotted circle. The electric flux through the surface drawn is zero by Gauss's law. A conducting sphere is inserted intersecting the previously drawn Gaussian surface. The electric flux through




(a) Still remains zero

(b) Non zero but positive

(c) Non-zero but negative

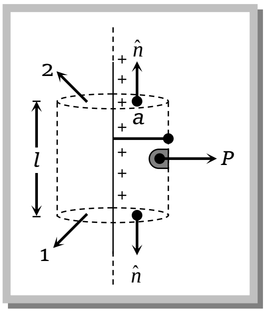
(d) Becomes infinite

**Solution:** (b) Due to induction some positive charge will lie within the Gaussian surface drawn and hence flux becomes something positive.

## Application of Gauss's Law

Gauss's law is a powerful tool for calculating electric field in case of symmetrical charge distribution by choosing a Gaussian surface in such away that $\vec{E}$ is either parallel or perpendicular to it's various faces.

***e.g.* Electric field due to infinitely long line of charge :** Let us consider a uniformly charged wire of infinite length having a constant linear charge density is $\lambda\left(\lambda = \frac{\text{charge}}{\text{length}}\right)$. Let *P* be a point distant *r* from the wire at which the electric field is to be calculated.



Draw a cylinder (Gaussian surface) of radius $r$ and length $l$ around the line charge which encloses the charge $Q$ ($Q = \lambda . l$). Cylindrical Gaussian surface has three surfaces; two circular and one curved for surfaces (1) and (2) angle between electric field and normal to the surface is 90° *i.e.*, $\theta = 90^o$.

So flux linked with these surfaces will be zero. Hence total flux will pass through curved surface and it is

$$\phi = \int E\, ds \cos\theta \qquad \text{.... (i)}$$

According to Gauss's law

$$\phi = \frac{Q}{\varepsilon_0} \qquad \text{.... (ii)}$$

Equating equation (i) and (ii) $\int E\, ds = \frac{Q}{\varepsilon_0}$

$$\Rightarrow \quad E\int ds = \frac{Q}{\varepsilon_0} \Rightarrow Ex 2\pi rl = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{Q}{2\pi\varepsilon_0 rl} = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r} \quad \left\{K = \frac{1}{4\pi\varepsilon_0}\right\}$$

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