Chapter 3:

Mortgage Foundations
Time Value of Money

Financial Calculators

“Sign” convention (+,-). Enter outflows as negative numbers; inflows as positive numbers.

Future Value

- Compound Interest
  - Earning Interest on Interest
- Basic Components
  - PV = Initial Deposit
  - i = Interest Rate
  - n = Number of Years *(or periods)*
  - FV_n = Value at a specified future period

Future Value

General equation:

$$FV_n = PV(1+i)^n$$

Invest \$1 for 1 year @ 6%

$= 1(1+.06)^1$

= \$1.06

Future Value

- Example 3-1:
  - What is the value at the end of year 5 of $100 deposited today if the interest rate is 10% compounded annually?

$$
\begin{aligned}
FV_5 &= \$100(1.10)^5 \\
&= \$100(1.61051) \\
&= \$161.05
\end{aligned}
$$

Future Value

- Example 3-1 Using a Financial Calculator:

PV		= -100
n		= 5
i		= 10
CPT	**FV**	= $161.05

Future Value

- Monthly Compounding
  - In Example 3-1, what if interest were paid (compounded) <u>*monthly*</u> instead of annually?
    - There would be ***12*** compounding periods in each year.
    - There would be a periodic rate to match the multiple compounding periods.
    - *Most importantly, the future value would be <u>higher</u>. Additional compounding periods will affect the final result.*

Future Value

Monthly compounding

PV = -100

n = 60 *(5years x 12 periods)*

i = 10% ÷ 12 *(or .8333% per period)*

PMT = $0 *(no entry)*

CPT **FV** = $164.53

Future Value

- Notice the difference in Future Value when multiple compounding periods are used:

  $164.53 vs. $161.05

- This shows the effect of earning interest on interest. The more compounding periods there are per year, the higher the future value will be.

Compounding

Nominal versus Effective

Rate	Interval	***Effective*** **Annual Rate**
6%	Annual	6.00%
6%	Semi-annual	6.09%
6%	Quarterly	6.14%
6%	Monthly	6.17%
6%	Daily	6.18%

Note on Quoted Rates

- Bankers quote Annual Percentage Rate (APR) on their loans and credit cards. This *excludes* compounding making them appear lower than the true “effective rate”.

- The same bankers quote Annual Percentage Yield (APY)* rates on their CD’s. This *includes* the effect of compounding – thus making them higher than the “nominal rate”.

* Roughly equivalent to “effective rate” but, as a legal term, may require adjustment for fees.

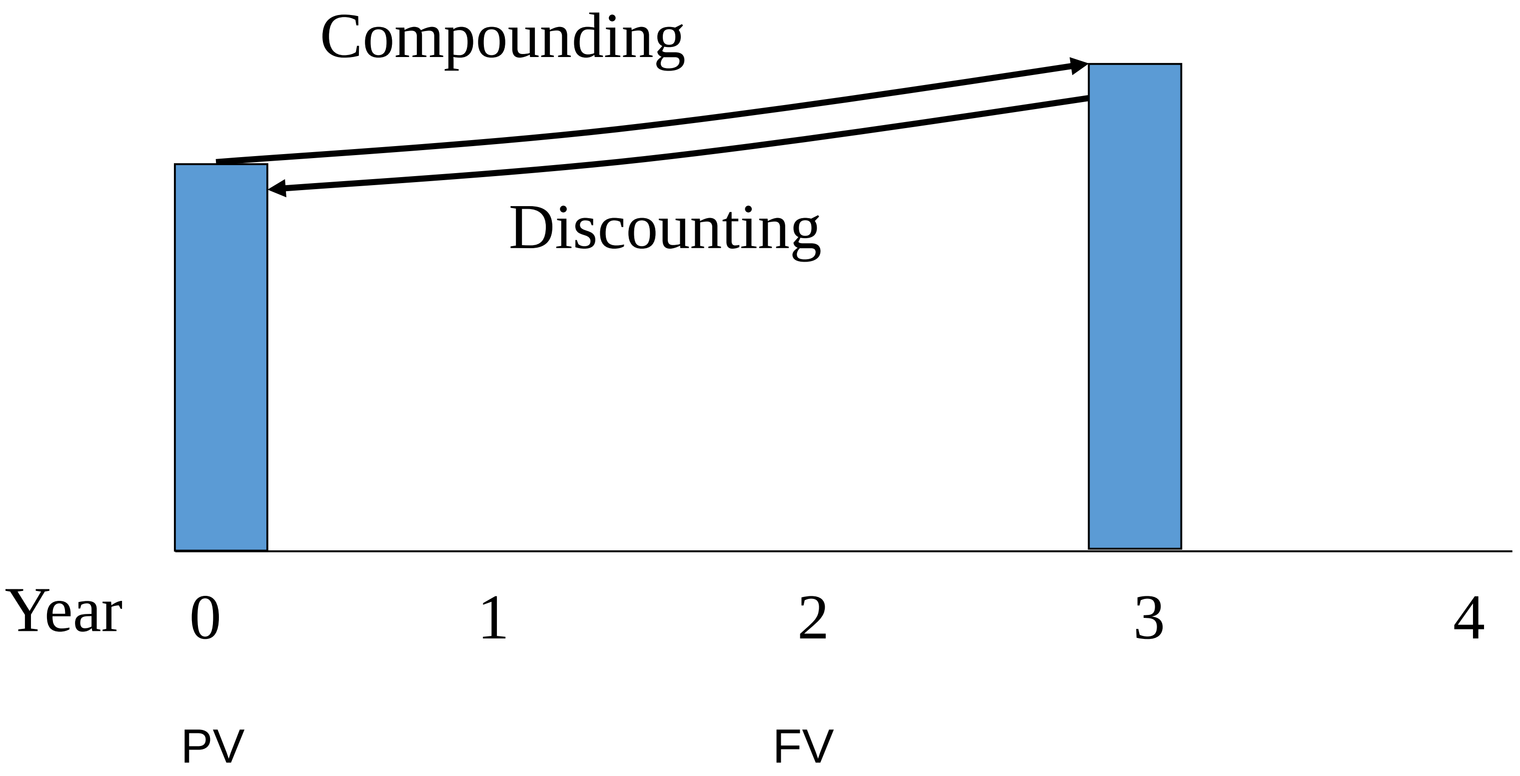
Present Value

- Discounting: Converting Future Cash Flows to the Present
- General Equation

$$PV = FV_n \frac{1}{(1+i)^n}$$

- Same equation as $FV_n = PV(1+i)^n$
- This is important

Discounting Brings Future Values Back Down the Compounding Curve



Present Value

- Example 3-2:
  - What is the value today of $2,000 you will receive at end of year 3 if the interest rate is 8% (*compounded annually)* ?

$$PV = 2000\left[\frac{1}{(1.08)^3}\right]$$

Discount factor

= 2000(.79383)

= $1587.66

Present Value

- Example 3-2 Using a Financial Calculator:

$\boxed{\textbf{FV}}$ = \$2000

$\boxed{\textbf{n}}$ = 3 periods

$\boxed{\textbf{i}}$ = 8

$\boxed{\textbf{CPT}}$ $\boxed{\textbf{PV}}$ = -1587.66

Present Value

- With monthly compounding

FV	= \$2000
n	= 36 *periods (months)*
i	= 8÷12 *(monthly rate)*
PMT	= \$0 *(no entry)*
CPT **PV**	= -1574.51

PV & Compounding

Note the effect of compounding on the PV calculation.

Annual compounding PV $1587.66

Monthly compounding PV $1574.51

Thus: DCF calculations using monthly compounding will ***discount*** results more severely than using annual numbers.

Annuity

- Level Cash Flow Stream (Periodic)
- Terminates at some point (otherwise called a perpetuity)

- Ordinary Annuity
  - Cash flows occur at the end of each period
    - Thus cash flows begin one period from today
  - Used by convention in most valuation analysis
- "Annuity Due"
  - Cash flows occur at the beginning of each period
    - Thus cash flows begin immediately

Annuity: Present Value

- Example 3-4:
  - If you had the opportunity to purchase a ten-year, $1200 annuity, what would it be worth today at an interest rate of 8%.

$$PVA = 1200 \cdot \frac{\left(1 - \frac{1}{1.08^{10}}\right)}{.08}$$

$$= 1200(6.71008)$$

$$= \$8,052.10$$

Easily done on a calculator

Annuity: Present Value

- Using the Financial Calculator:

PMT = 1200

n = 10

i = 8

FV = $0

CPT **PV** = -8052.10

Annuity: Present Value

- What if \$100 per month instead of \$1200 annually?
- (the same money but a slightly different flow)

PMT = \$100

n = 120 periods

i = 8 ÷ 12

FV = \$0

CPT **PV** = -8,242.15

Time Value of Money

- Given the basic equations that we have discussed, we can solve for any missing single variable.
- Some common applications
  - Solve for the interest rate *(or yield)*
  - Compute payments on an amortizing loan
  - Compute a future balance (loan balance for payoff)
  - Compute payments to accumulate a future sum

Time Value of Money

- Rate of Return or Discount Rate
- Example 3-5:
  - Jones Real Estate is financing a new building
  - Loan: $1,000,000
  - Term: 30 years *(fully amortizing)*
  - Payments: $7,337.65 monthly
  - What annual interest rate is the company paying?

Time Value of Money

$\boxed{\textbf{PV}}$	= $1,000,000
$\boxed{\textbf{n}}$	= 360 (30 x 12)
$\boxed{\textbf{PMT}}$	= (7337.65)
$\boxed{\textbf{FV}}$	= $0 (fully amortized)
$\boxed{\textbf{CPT}}$ $\boxed{\textbf{i}}$	= .6667/mo (8% annually)

Another Example

- Example 3-6:
    - A bank makes a $100,000 loan and will receive payments of $805 each month for 30 years as repayment. What is the rate of return to the bank for making this loan?

        - This is also the cost to the borrower *(ignoring fees and closing costs)*.

Time Value of Money

PMT = $805

n = 360

PV = ($100,000)

FV = $0

CPT **i** = .7504/mo = 9% yr

Note here we have entered PMT as positive and PV as negative

Sinking (Escrow) Fund Example

- Example 3-7: Accumulating a Future Sum*
  - MegaREIT Inc has issued general obligation bonds for $10 million due in 15 years at 9%. The indenture requires semi-annual interest-only payments and requires the firm to create a secured sinking fund which will grow to an amount sufficient to pay off the principal at the end of 15 years. If the firm can earn 7% on deposits, how much do they need to pay into the Sinking Fund semi-annually.

* This example contains information beyond what is required to solve the problem as real world scenarios are not usually presented as a simple set of variables.

Sinking Fund Example

FV = \$10,000,000

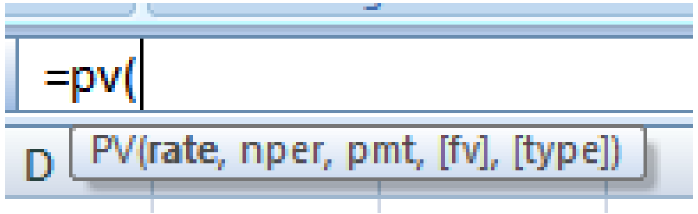
n = 30 (15 x 2)

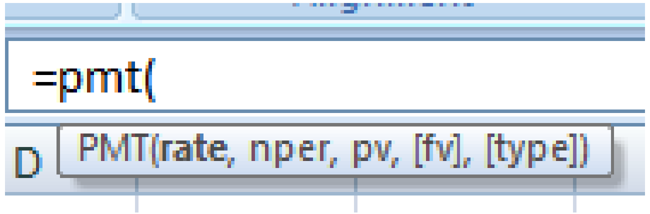
PV = \$0

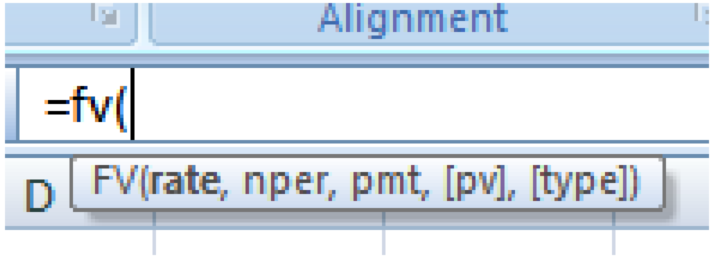
i = 7 ÷ 2

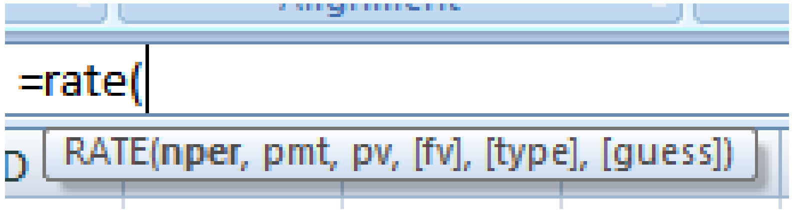
CPT **PMT** = -\$193,713.32

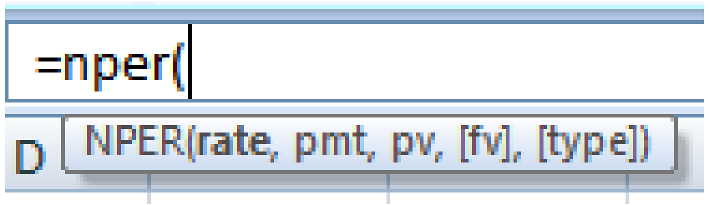
Spreadsheet Functions











For complex analysis, Excel is much better than the financial calculator. It is far more powerful.

Internal Rate of Return

Simply a way of expressing the return or yield on an investment if you are given all the cash flows.

CPT **I/Y**

For a single receipt or a series of <u>even</u> cash flows (annuity)

CPT **IRR**

For uneven cash flows, you can use cash flow registers on calculator – but better to use Excel.

Example 3-8

You represent an investment company and have been offered a right to purchase a new, performing real estate loan. For this risk, you decide you need a yield of 7.5%. How much should you "bid" for this loan?

– Loan Principal: $10,000,000

– Loan Interest rate: 7.0 % (monthly compounding)

– Terms: 30 year amortizing

– Payments: Monthly

Example 3-8

Step 1. What is the payment stream you are buying?

FV	= 0
n	= 360
PV	= 10,000,000
i	= 7 ÷ 12

Contract loan rate

CPT PMT = -$66,530.25

Example 3-8

Step 2. Calculate Maximum Price you can pay.

FV	= 0
n	= 360
PMT	= 66,530.25
i	= 7.5 ÷ 12
CPT **PV**	= -$9,514,998

"required" rate of return