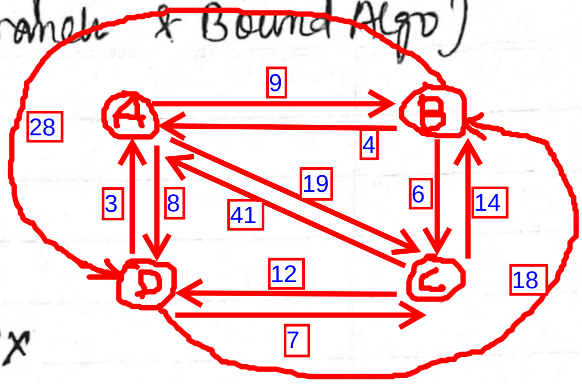
$$\begin{array}{c|c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} \infty & 9 & 19 & 8 \\ 4 & \infty & 6 & 28 \\ 41 & 14 & \infty & 12 \\ 3 & 18 & 7 & \infty \end{bmatrix} \end{array}$$

Given Cost Matrix

* Traveling Salesman Problem (Branch & Bound Algo)



Step1 :- Get the reduced Cost Matrix

1. Reduce Row - from each row reduce its minimum value

2. Reduce Column - from each column reduce respective minimum value

(1) Reducing Rows

$$\begin{bmatrix} \infty & 9 & 19 & 8 \\ 4 & \infty & 6 & 28 \\ 41 & 14 & \infty & 12 \\ 3 & 18 & 7 & \infty \end{bmatrix} \begin{array}{c} -8 \\ -4 \\ -12 \\ -3 \end{array} \quad \text{we get} \quad \begin{bmatrix} \infty & 1 & 11 & 0 \\ 0 & \infty & 2 & 24 \\ 29 & 2 & \infty & 0 \\ 0 & 15 & 4 & \infty \end{bmatrix}$$

(2) then, we reduce Columns

$$\begin{array}{c} \begin{bmatrix} \infty & 1 & 11 & 0 \\ 0 & \infty & 2 & 24 \\ 29 & 2 & \infty & 0 \\ 0 & 15 & 4 & \infty \end{bmatrix} \\ \begin{array}{cccc} & -1 & -2 & \end{array} \end{array} \quad \text{, we get} \quad \begin{bmatrix} \infty & 0 & 9 & 0 \\ 0 & \infty & 0 & 24 \\ 29 & 1 & \infty & 0 \\ 0 & 14 & 2 & \infty \end{bmatrix}$$

This is called

$$\text{Reduced Cost Matrix}, R = \begin{array}{c|c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} \infty & 0 & 9 & 0 \\ 0 & \infty & 0 & 24 \\ 29 & 1 & \infty & 0 \\ 0 & 14 & 2 & \infty \end{bmatrix} \end{array}$$

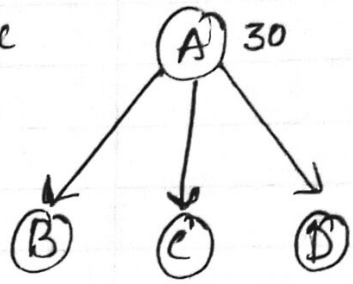
Total reduced values concludes the lower bound for this problem

$$LB = \underbrace{8 + 4 + 12 + 3}_{\text{Values reduced from rows}} + \underbrace{1 + 2}_{\text{values reduced from columns}}$$

$$LB = 30$$

This becomes initial cost to begin tour from city (A)

This will be represented using state space tree



Next we will compute cost (A, B), (A, C) and (A, D)

for city X to Y, generate a matrix using Reduced cost Matrix R with 3 changes

1. Row (X) $\leftarrow \infty$
2. Column (Y) $\leftarrow \infty$
3. Element (Y, X) $\leftarrow \infty$

~~so for (A, B)~~ and

$$\boxed{\text{Cost}(X, Y) = \text{Cost}(X) + R(X, Y) + R_{XY}}$$

$R(X, Y)$: Cost of X to Y in reduced Cost Matrix

R_{XY}: Reduction done in Matrix S_{XY}

for (A, B), we generate a matrix S_{AB}, such that

Row (A) $\leftarrow \infty$

Column (B) $\leftarrow \infty$

Element (B, A) $\leftarrow \infty$

$$S_{AB} = \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 24 \\ 29 & \infty & 1 & 0 \\ 0 & \infty & 2 & \infty \end{bmatrix}$$

$$\begin{aligned} \text{Cost}(A, B) &= \text{Cost}(A) + R(A, B) + R_{AB} \\ &= 30 + 0 + 0 \\ &= 30 \end{aligned}$$

similarly

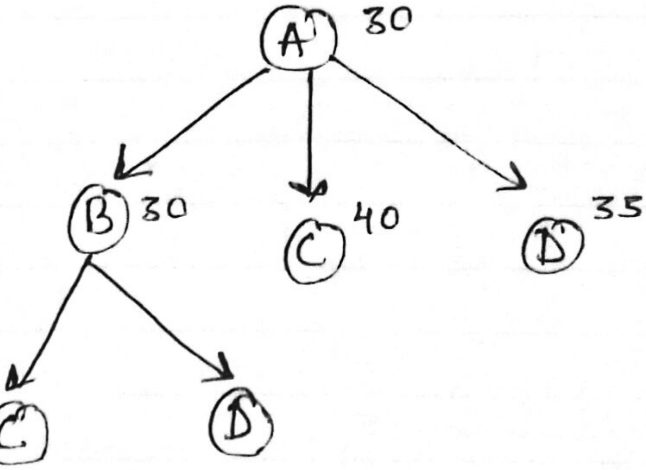
$$S_{A,C} = \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 24 \\ \infty & 1 & \infty & 0 \\ 0 & 14 & \infty & \infty \end{bmatrix} \\ \begin{array}{cccc} & -1 & & \end{array} \end{array} \rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 24 \\ \infty & 0 & \infty & 0 \\ 0 & 13 & \infty & \infty \end{bmatrix}$$

$$\text{cost}(A,C) = \text{cost}(A) + R(A,C) + R_{AC}$$
$$= 30 + 9 + 1$$
$$= 40$$

$$S_{A,D} = \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty \\ 0 & \infty & 0 & \infty \\ 29 & 1 & \infty & \infty \\ \infty & 14 & 2 & \infty \end{bmatrix} \end{array} \begin{array}{c} \\ \\ -1 \\ -2 \end{array} \rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty \\ 0 & \infty & 0 & \infty \\ 28 & 0 & \infty & \infty \\ \infty & 12 & 0 & \infty \end{bmatrix}$$

$$\text{cost}(A,D) = \text{cost}(A) + R(A,D) + R_{AD}$$
$$= 30 + 0 + 3$$
$$= 33$$

This gets us to



we see the leaf nodes and expand the one with the least value.
Here B will be expanded.

Now, we will compute cost for (A) → (B) → (C), cost (A, B, C) and (A) → (B) → (D), cost (A, B, D)

for cost (A, B, C), use S_{AB} and make changes

Row (B) ← ∞
Column (C) ← ∞
Element (C, A) ← ∞

$$S_{AB} = \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 24 \\ 29 & \infty & 1 & 0 \\ 0 & \infty & 2 & \infty \end{bmatrix}, \quad \text{So } S_{ABC} = \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 0 \\ 0 & \infty & \infty & \infty \end{bmatrix}$$

$$\begin{aligned} \text{Cost}(A,B,C) &= \text{Cost}(B) + R(B,C) + R_{ABC} \\ &= 30 + 0 + 0 \\ &= 30 \end{aligned}$$

similarly for cost (A, B, D), use S_{AB} and make changes

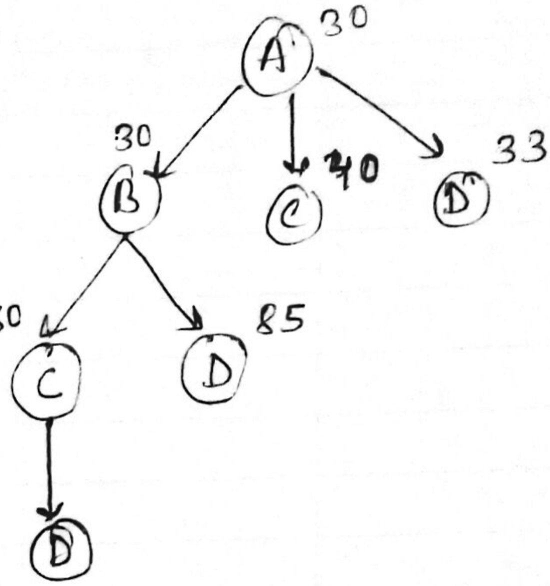
Row (B) ← ∞
Column (D) ← ∞
Element (D, A) ← ∞

$$S_{A,B,D} = \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ 29 & \infty & 1 & \infty \\ \infty & \infty & 2 & \infty \end{bmatrix} \begin{matrix} \\ \\ -1 \\ -2 \end{matrix} \\ \end{array} \rightarrow \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ 28 & \infty & 0 & \infty \\ \infty & \infty & 0 & \infty \end{bmatrix} \\ \begin{matrix} -28 & & & \end{matrix} \end{array}$$

$$= \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ 0 & \infty & 0 & \infty \\ \infty & \infty & 0 & \infty \end{bmatrix}$$

$$\text{Cost}(A,B,D) = \text{Cost}(B) + R(B,D) + R_{ABD}$$
$$= 30 + 24 + (1+2+28)$$
$$= 85$$

so, we get the state space tree as



Here we check for the min cost leaf node and expand it, Node C being the least cost node will be expanded

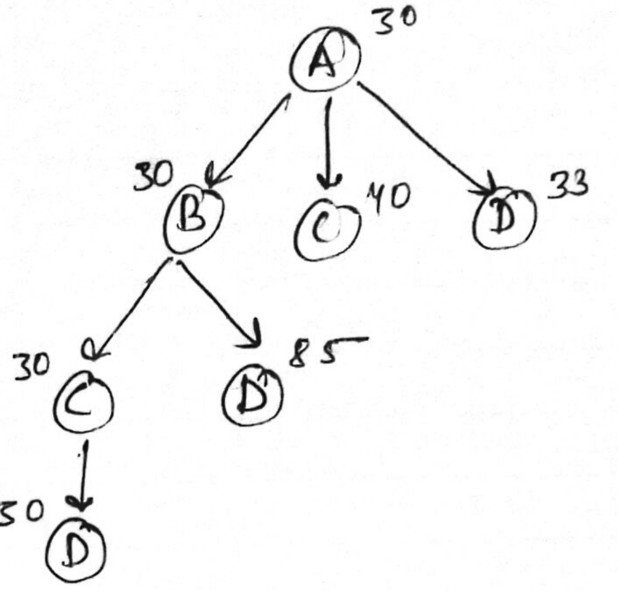
so, Next cost (A,B,C,D) will be computed, for this matrix S_{ABC} will be used with changes :-

1. Row $(C) \leftarrow \infty$
2. Column $(D) \leftarrow \infty$
3. Element $(D, A) \leftarrow \infty$

$$S_{ABCD} = \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \end{bmatrix}$$

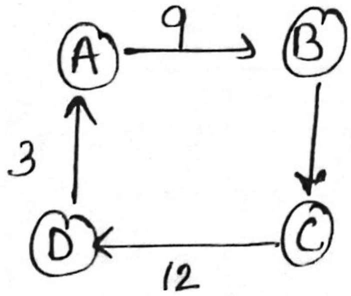
$$\text{Cost}(A,B,C,D) = \text{Cost}(C) + R(C,D) + R_{ABCD}$$
$$= 30 + 0 + 0$$
$$= 30$$

so, we get the tree as



No other node is having the minimum value and since each node (city) is travelled once, so this completes the tour for salesman.

from here we get the optimal cost tour



To get the actual cost, refer to given cost matrix and add the cost value at each edge in the generated tour.

So Min. cost = $9+6+12+3$

$\boxed{\text{Min. cost.} = 30}$