Analysis of Algorithms

[5CS4-05/5IT4-05]

Unit 2: Dynamic Programming

0 / 1 Knapsack Problem

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Knapsack Problem

- Knapsack Problem: Given weights and values of n items, we need to put these items in a knapsack of capacity W to get the maximum total value in the knapsack.

- In Fractional Knapsack problem, we can break items and store in fraction to fill knapsack ← we solved using greedy approach.

- In 0-1 Knapsack problem, we are not allowed to break items. We either take the whole item or don't take it.

← we will solve using dynamic programming approach.

0/1 Knapsack Problem

0 / 1
↓
fraction values

0 → not added
1 → completely added.

In 0/1 Knapsack Problem:

- Items are indivisible.
- Items in fraction can't be added
- Items are added either completely or not at all.
- Dynamic programming approach is used to solve it.

↳ other approaches can also solve it

0/1 Knapsack Algorithm to find Max Profit

```
0_1_Knapsack (n, W)
  for w = 0 to W
      V [0,w] ← 0

  for i=0 to n
      V [i, 0] ← 0

  for w = 0 to W
      if (w_i ≤ w & v_i + V [i-1, w - w_i ] > V [i -1,w ])
          V [i, w] ← v_i + V [i - 1, w - w_i ]
      else
          V [i, w] ← V [i - 1, w]
```

0/1 Knapsack Algorithm to find items added in knapsack

```
Find_items_in_knapsack ()
  i=n, k=W
  if V [i, k] != V [i-1, k]
        mark ith item as in the knapsack
        i=i-1, k=k-wi
  else
        mark ith item as not in knapsack
        i=i-1
```

Using Dynamic Programing approach find the maximum profit and added objects for given 0/1 Knapsack Problem:

Knapsack capacity,	W	10
Weight of Objects,	w_i	5, 3, 3, 4
Profit of Objects,	v_i	90, 30, 50, 70

	w_i	v_i
O_1	5	90
O_2	3	30
O_3	3	50
O_4	4	70

W

i	0	1	2	3	4	5	6	7	8	9	10
0											
1											
2											
3											
4											

Using Dynamic Programing approach find the maximum profit and added objects for given 0/1 Knapsack Problem:

Knapsack capacity,	W	10
Weight of Objects,	w_i	5, 3, 3, 4
Profit of Objects,	v_i	90, 30, 50, 70

Step 1 :-

for $i = 0$ or $w = 0$

fill $v[i, w] = 0$

	w_i	v_i
O_1	5	90
O_2	3	30
O_3	3	50
O_4	4	70

W

i	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0										
2	0										
3	0										
4	0										

Using Dynamic Programing approach find the maximum profit and added objects for given 0/1 Knapsack Problem:

Knapsack capacity,	W	10
Weight of Objects,	w_i	5, 3, 3, 4
Profit of Objects,	v_i	90, 30, 50, 70

Here $w_i \le W \rightarrow$ True so profit added

	w_i	v_i
O_1	5	90
O_2	3	30
O_3	3	50
O_4	4	70

i \ W	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	90	90	90	90	90	90
2	0										
3	0										
4	0										

Using Dynamic Programing approach find the maximum profit and added objects for given 0/1 Knapsack Problem:

Knapsack capacity,	W	10
Weight of Objects,	w_i	5, 3, 3, 4
Profit of Objects,	v_i	90, 30, 50, 70

both object added

	w_i	v_i
O_1	5	90
O_2	3	30
O_3	3	50
O_4	4	70

W

i	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	90	90	90	90	90	90
2	0	0	0	30	30	90	90	90	120	120	120
3	0										
4	0										

Using Dynamic Programing approach find the maximum profit and added objects for given 0/1 Knapsack Problem:

Knapsack capacity,	W	10
Weight of Objects,	w_i	5, 3, 3, 4
Profit of Objects,	v_i	90, 30, 50, 70

	w_i	v_i
O_1	5	90
O_2	3	30
O_3	3	50
O_4	4	70

W

i	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	90	90	90	90	90	90
2	0	0	0	30	30	90	90	90	120	120	120
3	0	0	0	50	50	90	90	90	140	140	140
4	0										

Using Dynamic Programing approach find the maximum profit and added objects for given 0/1 Knapsack Problem:

Knapsack capacity,	W	10
Weight of Objects,	w_i	5, 3, 3, 4
Profit of Objects,	v_i	90, 30, 50, 70

	w_i	v_i
O_1	5	90
O_2	3	30
O_3	3	50
O_4	4	70

W

i	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	90	90	90	90	90	90
2	0	0	0	30	30	90	90	90	120	120	120
3	0	0	0	50	50	90	90	90	140	140	140
4	0	0	0	50	70	90	90	120	140	160	160

Maximum Profit

Using Dynamic Programing approach find the maximum profit and added objects for given 0/1 Knapsack Problem:

Knapsack capacity,	W	10
Weight of Objects,	w_i	5, 3, 3, 4
Profit of Objects,	v_i	90, 30, 50, 70

O_1 and O_4 added

90 + 70 = 160

Max. Profit we got from

	w_i	v_i
O_1	5	90
O_2	3	30
O_3	3	50
O_4	4	70

W

i	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	90	90	90	90	90	90
2	0	0	0	30	30	90	90	90	120	120	120
3	0	0	0	50	50	90	90	90	140	140	140
4	0	0	0	50	70	90	90	120	140	160	160

Object O_1 and O_4 is in knapsack   Max. Profit=160

Queries ?