Analysis of Algorithms

[5CS4-05/5IT4-05]

Unit 3: Backtracking

N Queens Problem

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Backtracking Approach

- Backtracking is an algorithm for finding all (or some) solutions to constraint satisfaction problems.

- It incrementally builds candidate solutions, and abandons a candidate ("backtracks") if it is determined that the candidate cannot possibly be completed to a valid solution.

- There are three types of problems in backtracking:
  - Decision Problem – In this, we search for a feasible solution.
  - Optimization Problem – In this, we search for the best solution.
  - Enumeration Problem – In this, we find all feasible solutions.

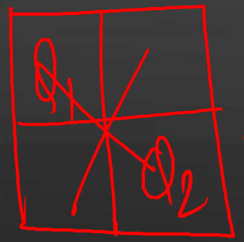
N Queens Problem

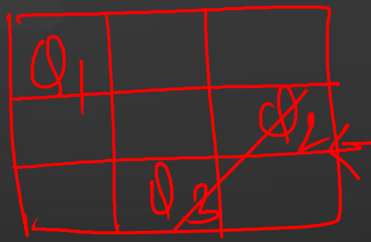
The N Queen is the problem of placing N chess queens on an N×N chessboard so that no two queens attack each other.

Thus, a solution requires that no two queens share the same row, column, or diagonal.

Solutions exist for all natural numbers n with the exception of n = 2 and n = 3.

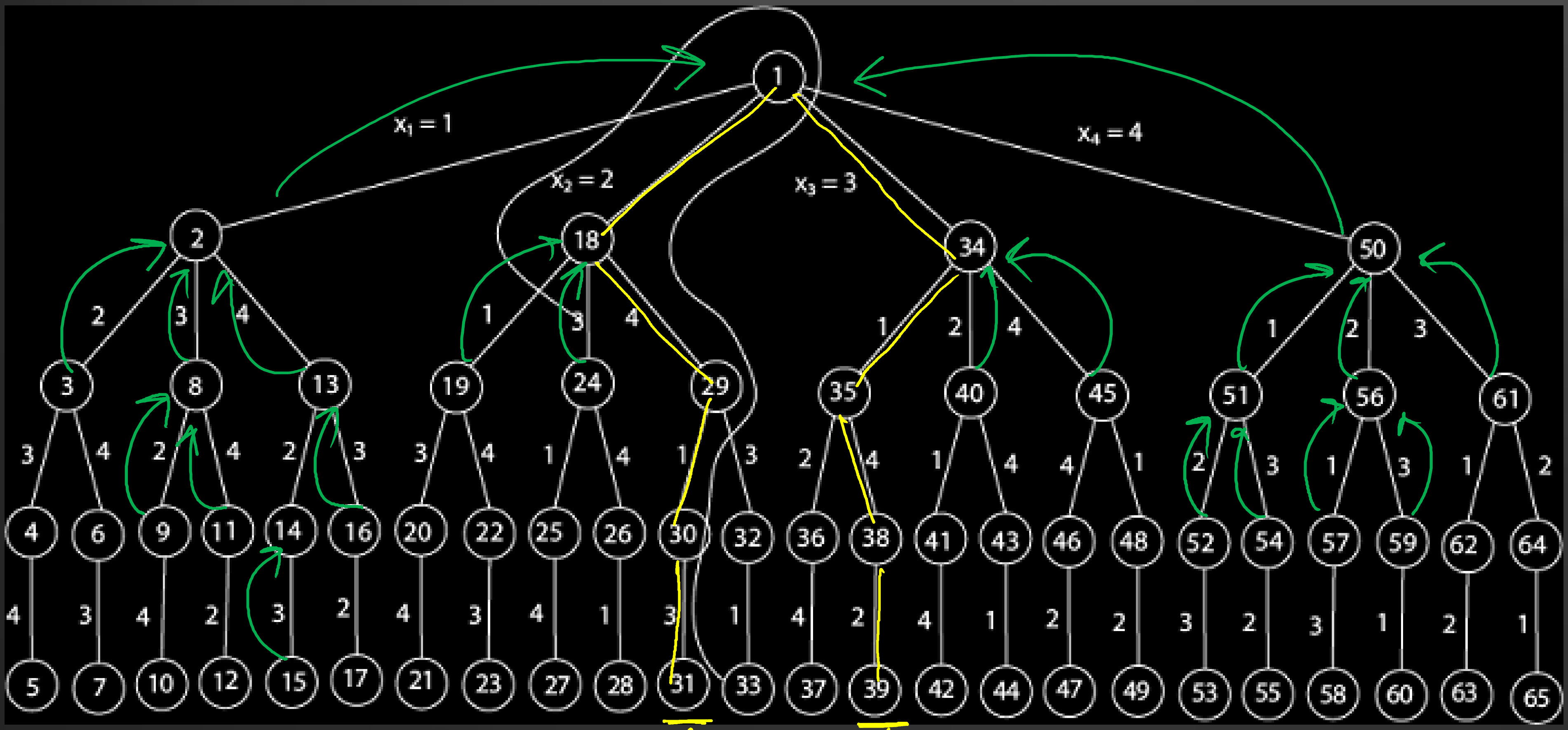
n=2 X

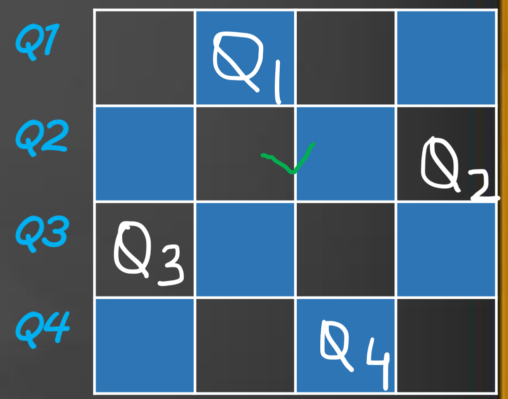


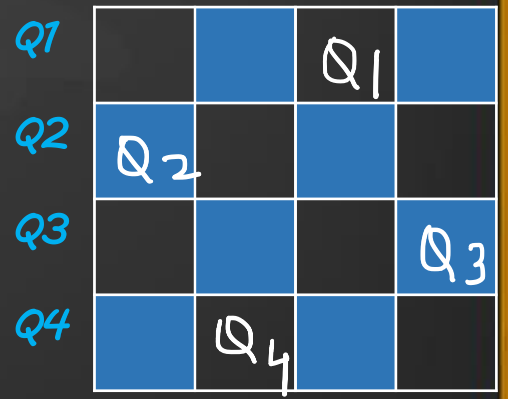


n=3 X

4 Queens Problem







Possible arrangements = $n!$ i.e. for 4 Queens $4! = \underline{24}$ arrangements

$\lfloor n$

$\lfloor 4 = 24$

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4 Queens Problem Solution

1st arrangement

	Q1		
			Q2
Q3			
		Q4	

2nd arrangement

		Q1	
Q2			
			Q3
	Q4		

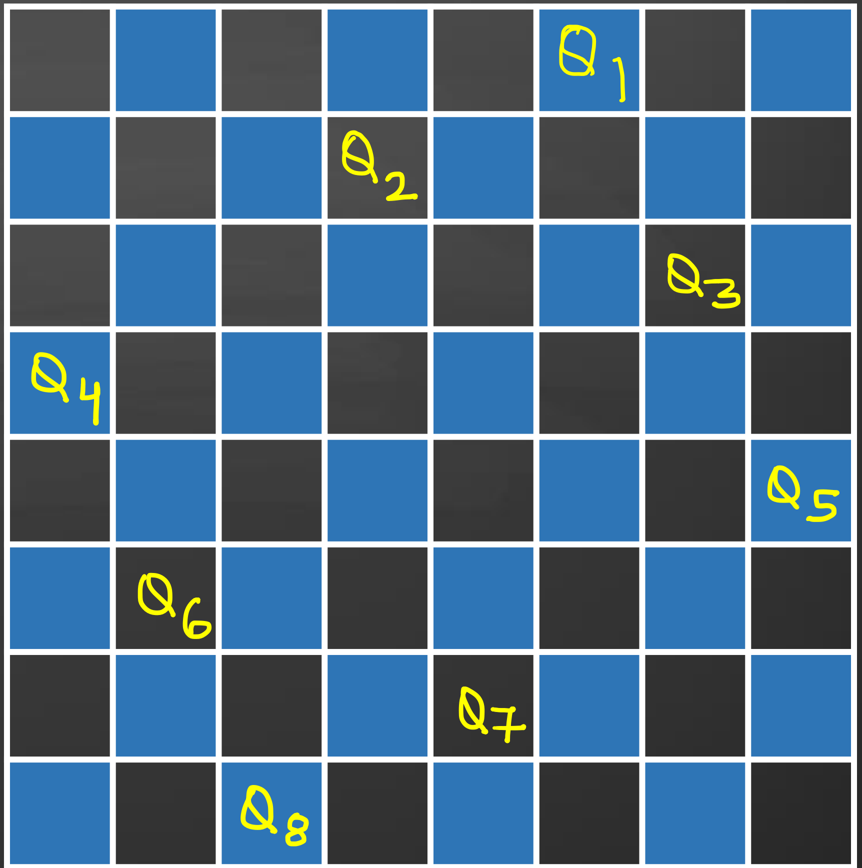
8 Queens Problem Solution

To find all the solutions to the 8- queens problem is computationally expensive, as there are 4,426,165,368 ($^{64}C_8$) possible arrangements.

By applying a rule that constrains each queen to a single row and column, its possible to reduce the arrangements to just 40,320 (that is, 8!), which are then checked for diagonal attacks. !8 = 40320

Out of these arrangements, there only 92 distinct solutions. If solutions that differ only by the symmetry operations of rotation and reflection of the board are counted as one, the puzzle has only 12 solutions.

Column sequence



6
4
7
1
8
2
5
3

2nd arrangement → 90°
3rd arrangement → 180°
4th arrangement → 270°

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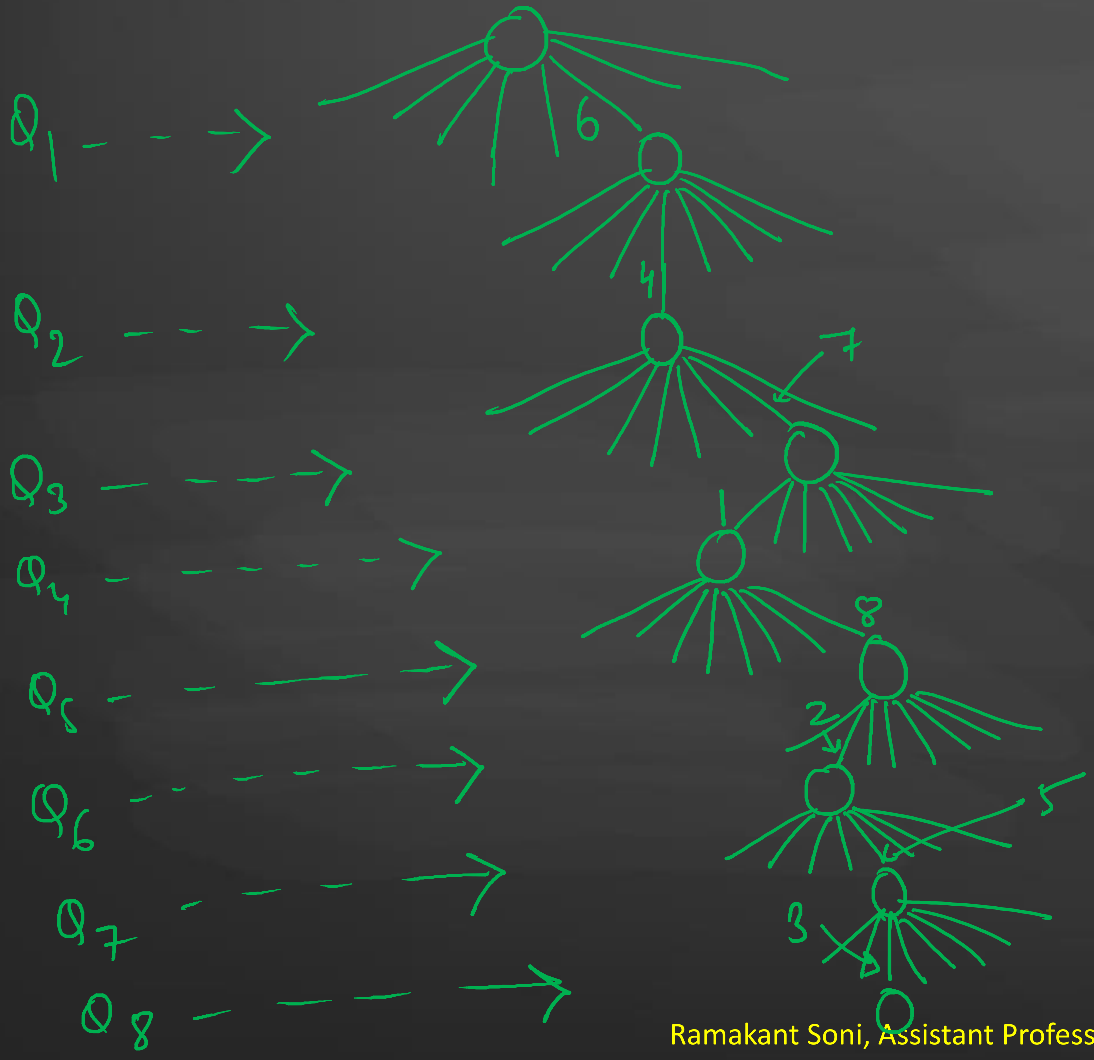
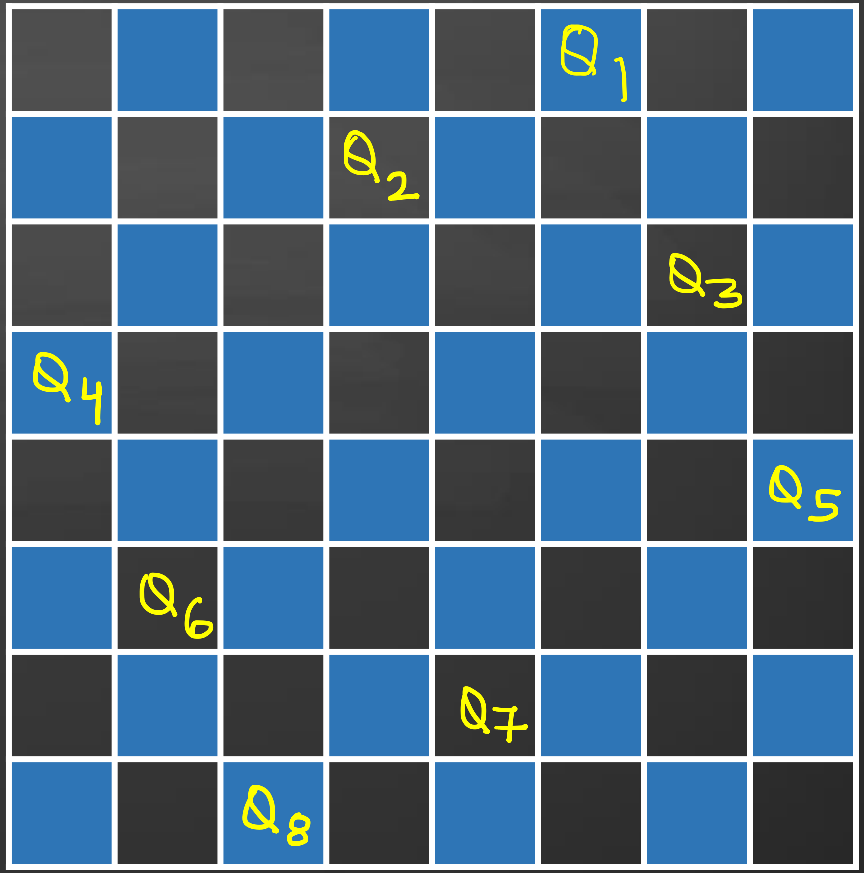
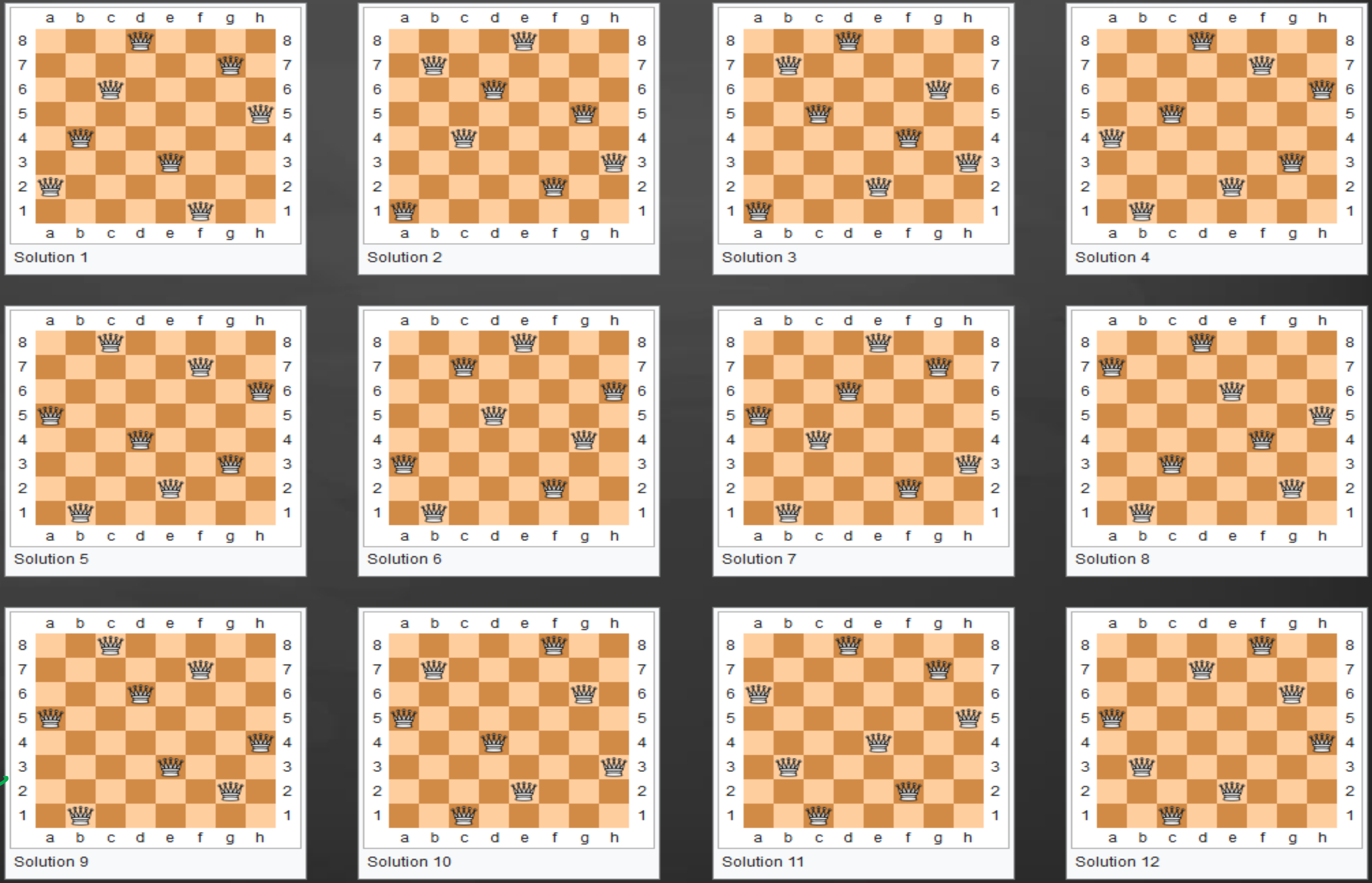
8 Queens Problem Solution





8 Queens Problem Solution

The eight queens puzzle has 92 distinct solutions. If solutions that differ only by the symmetry operations of rotation and reflection of the board are counted as one, the puzzle has 12 solutions. These are called fundamental solutions; representatives of each are shown here:



12 fundamental solutions of 8 Queen's Problem

Queries ?