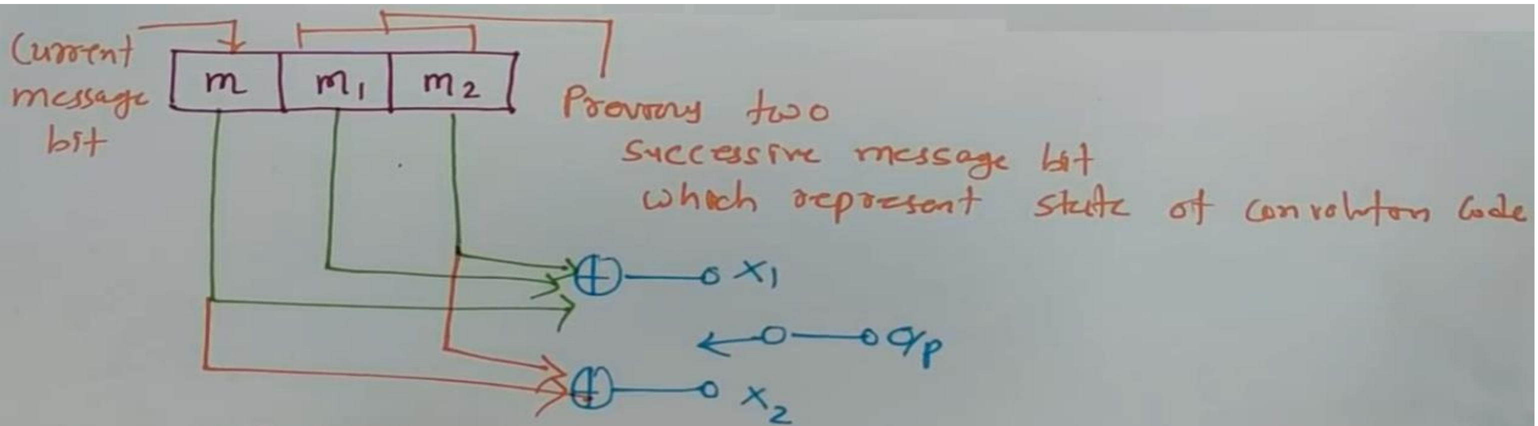
Convolutional codes basics, parameters & designing

- In Convolutional Codes, block of 'n' code digits generated by the encoder in time unit depends on not only block of 'k' message digits with in that time unit but also on the preceding (m-1) blocks of message digits.



$$x_1 = m \oplus m_1 \oplus m_2$$

$$x_2 = m \oplus m_2$$

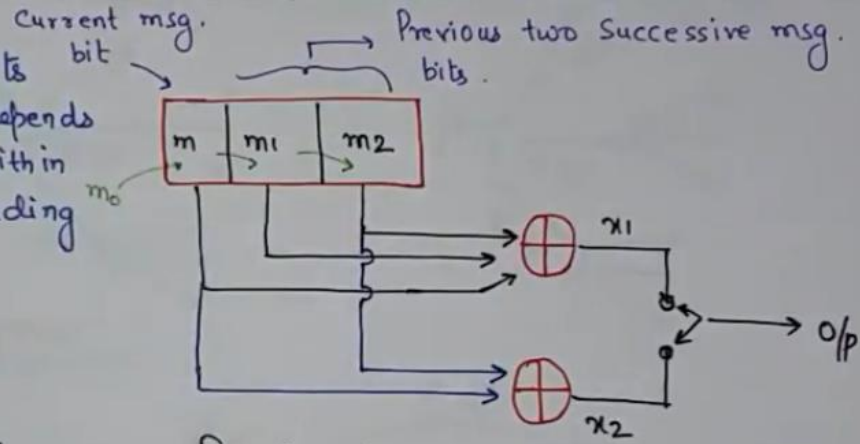
- k = no of message bits = 1
  n = no of encoded o/p bits = 2
  K = Constraint Length = 3
- Here o/p will switch in betn x_1 & x_2 so o/p will be

$$X = x_1 x_2 x_1 x_2 \ldots$$

- Code rate $\quad r = \frac{k}{n} = \frac{1}{2}$
- Constraint Length (K)
- single message bit influences encoder o/p for different successive shift.
- Code dimensions $(n, k) = (2, 1)$

CONVOLUTIONAL CODES

In Convolutional Codes, Block of 'n' Code digits generated by the encoder in time unit depends on not only block of 'K' message digits within that time unit but also on the preceding (m-1) blocks of message digits.



i/p bits are stored in fixed length Shift register.

$$x_1 = m \oplus m_1 \oplus m_2 \quad \text{—— (i)}$$

$$x_2 = m \oplus m_2 \quad \text{—— (ii)}$$

Fig. Convolution Encoder.

$K = 3,\ k = 1,\ n = 2.$

o/p. Switch first Samples x_1 and then x_2.

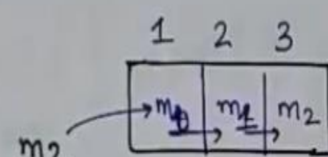
$$X = x_1 x_2 x_1 x_2 x_1 x_2 \ldots$$

CONVOLUTIONAL CODES

CODE RATE:- $r = \dfrac{K}{n}$ — K → no. of message bits; n → no. of encoded o/p bits for one message bit.

$$\boxed{r = \frac{1}{2}}$$

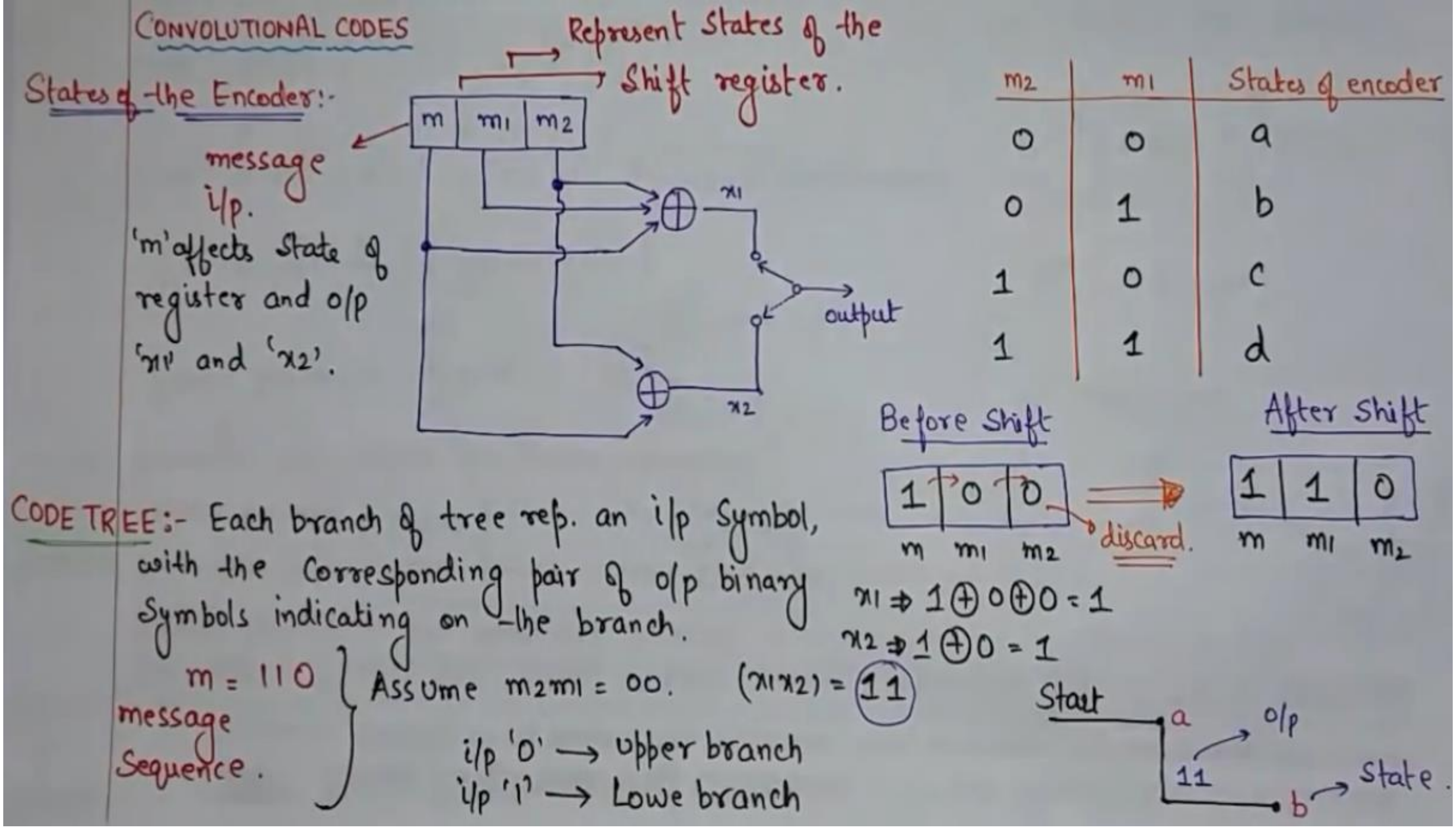
CONSTRAINT LENGTH (K):- It is defined as the number of Shifts over which a Single message bit Can influence the encoder output.

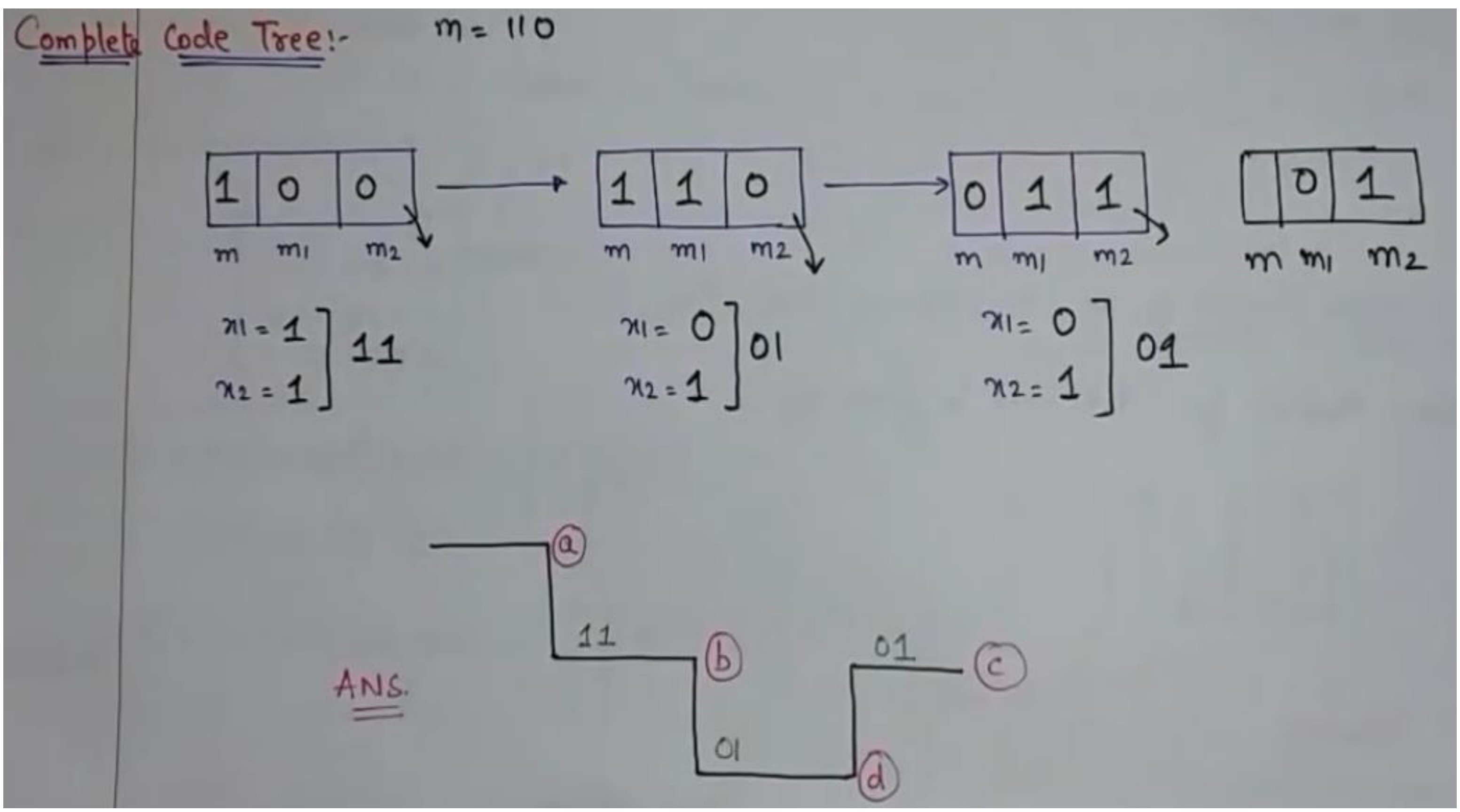


$K = 3.$ {Single message bit influences encoder o/p for 3 Successive Shifts}.

CODE DIMENSION:- Dimension of the Code is (n, K) — K → no. of message bits, n → no. of encoded o/p bits } at a time.

(2,1) dimension.





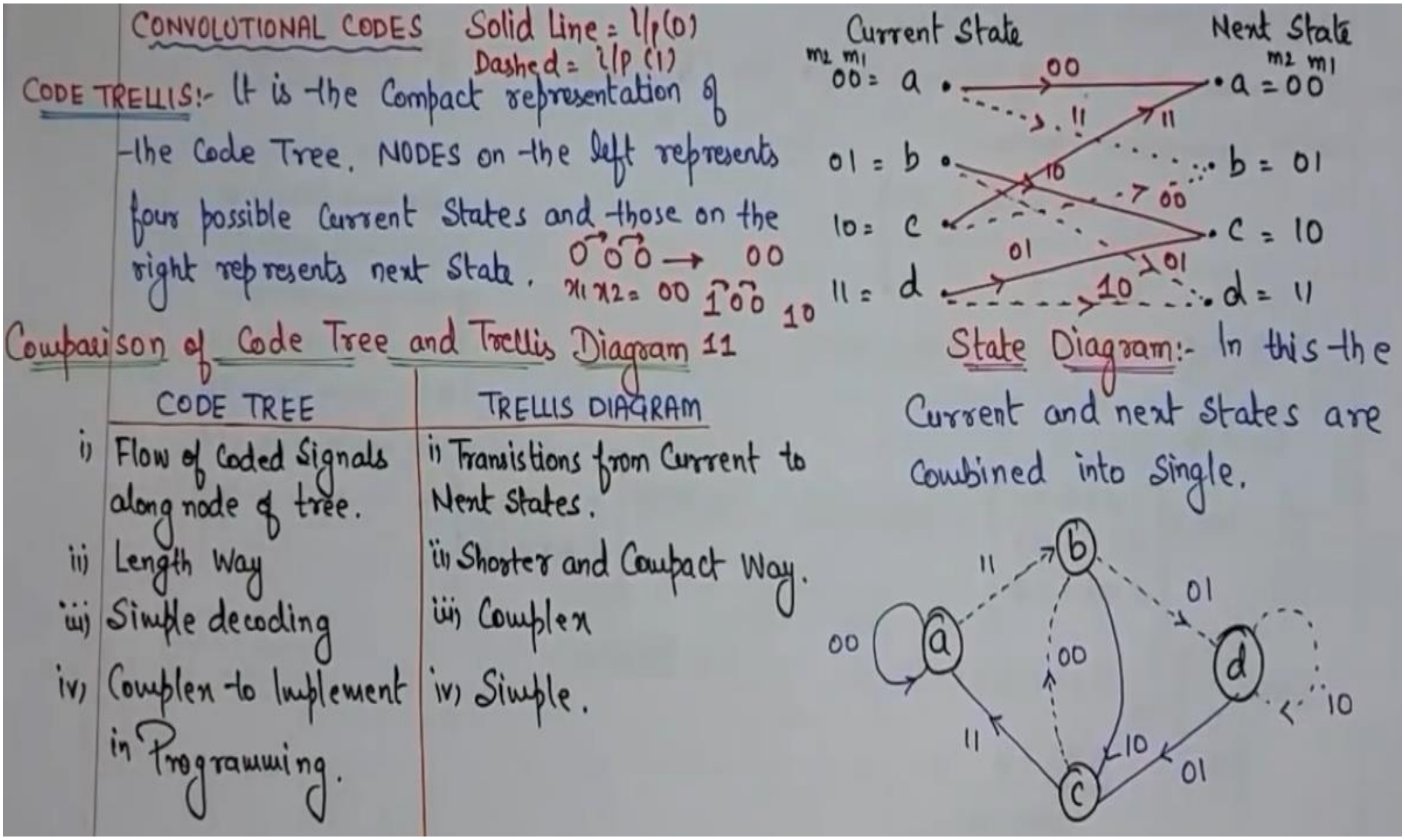
Trellis Diagram:

Another useful way of representing the working of convolution encoder is the trellis diagram. At the starting point all the stages of the register are clear i.e., they are in '0' state.

Two lines emerge from the starting point-

(i) The **solid line represents** input data digit **'0'** and

(ii) **Dashed line represents** the input data digit **'1'.**



Decoding of convolutional Code:

There are several different approaches to decoding of convolutional codes. These are grouped in two basic categories:

(i). Sequential Decoding- Fano Algorithm

(ii). **Maximum Likely-hood decoding- Viterbi Decoding**

CONVOLUTIONAL CODES

VITERBI ALGORITHM :- It is used for the decoding of Convolutional Codes.

↳ Also Known as Maximum Likelihood decoding

The algorithm operates by Computing a metric or discrepancy for every possible path in the trellis. The metric for a particular path is defined as the Hamming Distance b/w the Coded Seq. represented by that path and received Sequence. Thus for each node (state) in trellis, the algorithm compares two paths entering the node. The path with lower metric is retained and other is discarded.

Algorithm:-

(i) Initialization:- Label left most State of the trellis as 0.

(ii) Computation (j+1) :- Let j = 0,1,2... and Suppose that at previous step j, we have done two things:-

- All Survivor paths are identified.
- Survivor path and its metric for each state is stored.

→ At level j+1, Compute metric for all paths, entering each state by adding metric of incoming branches.

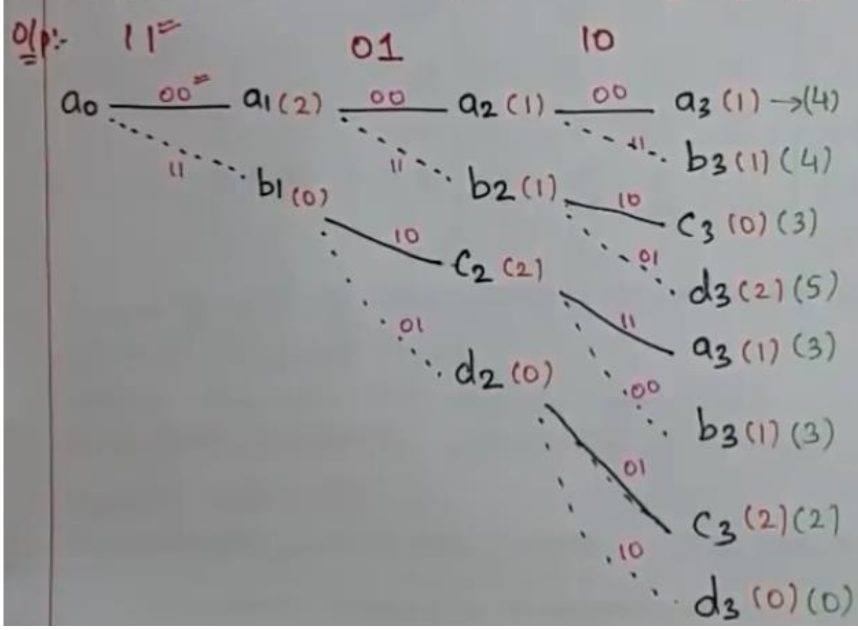
→ Identify path with lowest metric.

(iii) Final Step: Continue Computation until the algorithm Completes its forward Search through trellis.

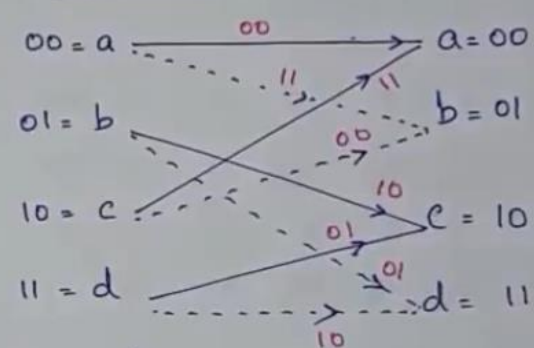
VITERBI ALGORITHM:-

Decode the Code Sequence:- Y = 11 01 10

O/P:- 11 01 10



Code trellis



a0 b1 d2 d3 (---- → 1)

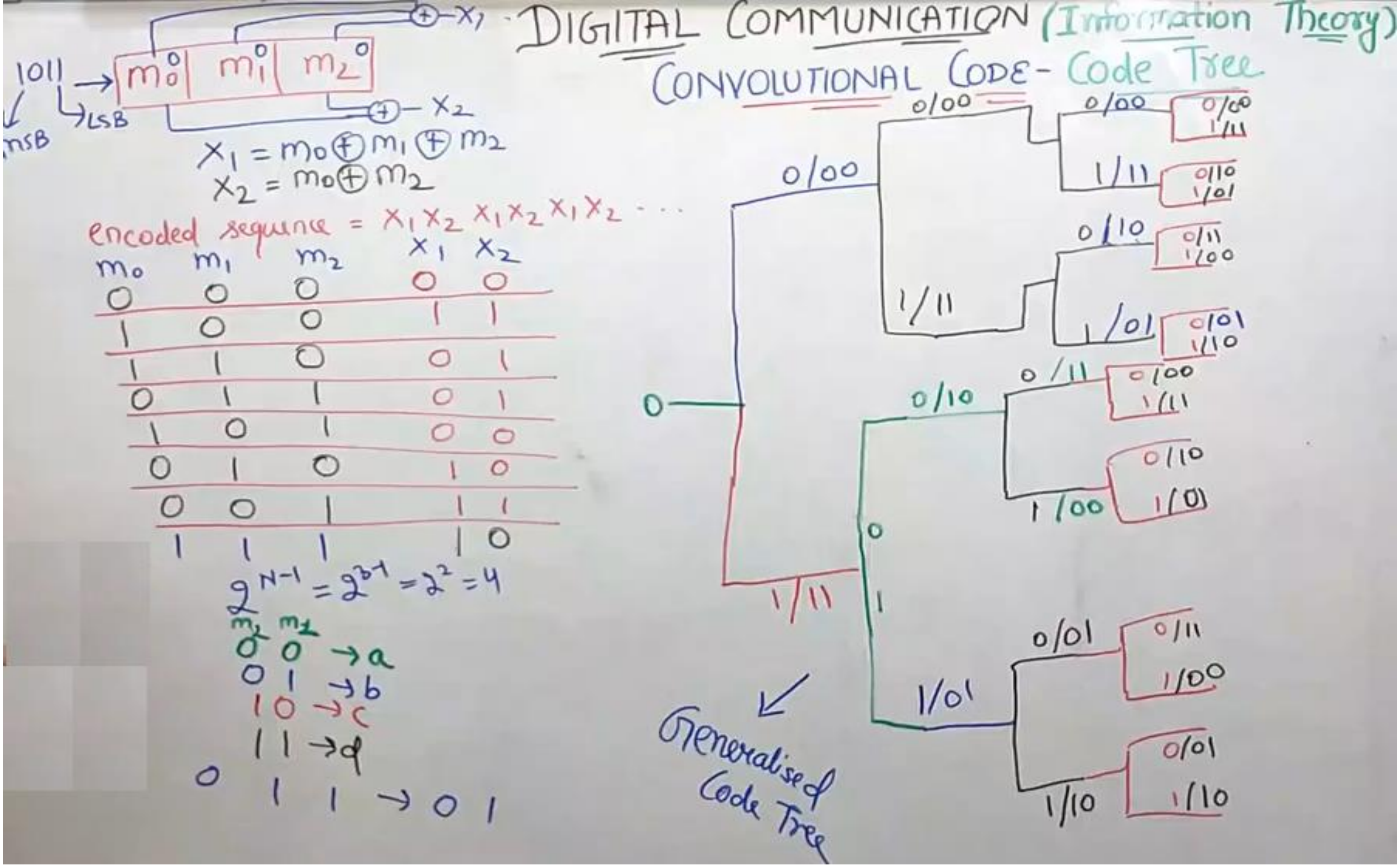
a0 ---- b1 ----- d2 ------ d3

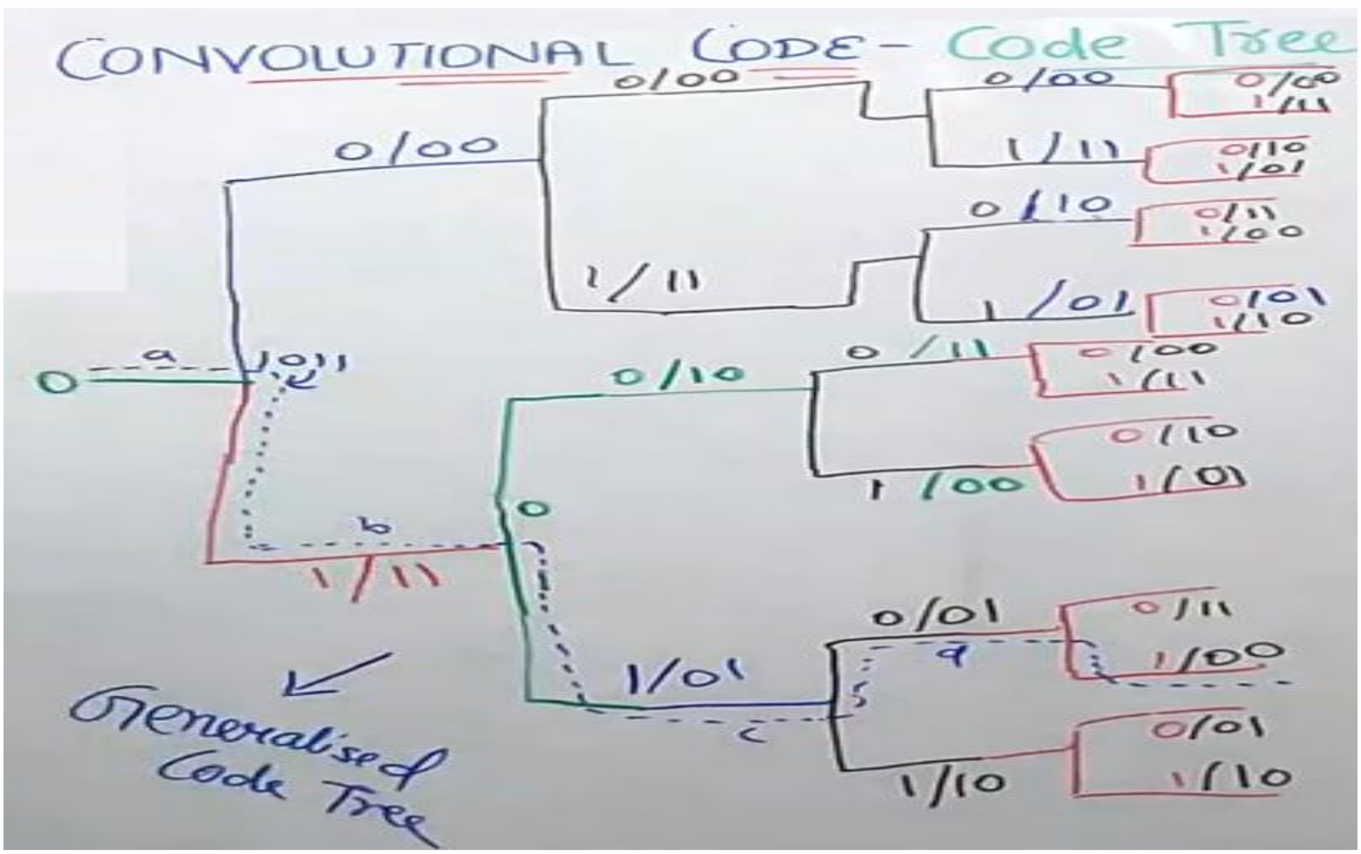
1 1 1

Message Sequence → (111) ANS.

11	01	10
11	01	10

0 → Error.


DIGITAL COMMUNICATION (Information Theory)
CONVOLUTIONAL CODE - Code Tree
1011
MSB
LSB
m0 m1 m2
X1
X2
X1 = m0 ⊕ m1 ⊕ m2
X2 = m0 ⊕ m2
encoded sequence = X1 X2 X1 X2 X1 X2 . . .
m0 m1 m2 X1 X2
0 0 0 0 0
1 0 0 1 1
1 1 0 0 1
0 1 1 0 1
1 0 1 0 0
0 1 0 1 0
0 0 1 1 1
1 1 1 1 0
2^(N-1) = 2^(3-1) = 2^2 = 4
m2 m1
0 0 → a
0 1 → b
1 0 → c
1 1 → d
0 1 1 → 0 1
Generalised Code Tree
0/00
1/11
0/10
1/01
0/11
1/00
0/01
1/10


CONVOLUTIONAL CODE - Code Tree
0/00
1/11
0/10
1/01
0/11
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a
b
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d
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Generalised Code Tree

CONVOLUTIONAL CODE - Trellis Diagram