

Future Distribution Grids

Finite Element Modelling for Electrical Energy Applications

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STEDIN Transformer Parameters ¹

- Power rating, $S = 400 \text{ kVA}$
- Line to Line Parameters :
 - Primary voltage, $V_{pri,line,rms} = 10750 \text{ V}$
 - Secondary voltage, $V_{sec,line,rms} = 420 \text{ V}$
 - Primary current, $I_{pri,line,rms} = 21.48 \text{ A}$
 - Secondary current, $I_{sec,line,rms} = 549.86 \text{ A}$
- Phase to Phase Parameters :
Considering a delta-wye connected transformer,
 - Primary voltage, $V_{pri,phase,rms} = 10750 \text{ V}$
 - Secondary voltage, $V_{sec,phase,rms} = 242.49 \text{ V}$
 - Primary current, $I_{pri,phase,rms} = 12.40 \text{ A}$
 - Secondary current, $I_{sec,phase,rms} = 549.86 \text{ A}$

¹M. van Dijk (2022). "A theoretical approach towards digital twins: A balance between an empirical and a fundamental model for distribution transformers". In: *Delft University of Technology*

STEDIN Transformer Parameters ²

- Phase to Phase Parameters :
 - Primary voltage, $V_{pri,phase,peak} = 15202.40 \text{ V}$
 - Secondary voltage, $V_{sec,phase,peak} = 342.93 \text{ V}$
 - Primary current, $I_{pri,phase,peak} = 17.54 \text{ A}$
 - Secondary current, $I_{sec,phase,peak} = 777.62 \text{ A}$
- Turns Ratio -
 - Primary turns, $N_{pri} = 266$
 - Secondary turns, $N_{sec} = 6$
- Resistance and inductance of windings :
 - $R_{ext,pri} = 1.8131 \text{ } \Omega$
 - $R_{ext,sec} = 1.299 \text{ m}\Omega$
 - $L_{ext,pri} = L_{ext,sec} = 1 \text{ } \mu\text{H}$
 - $CFF_p = CFF_s = 0.3$
- Depth of core (z-axis length) $l_z = 0.4 \text{ m}$

²M. van Dijk (2022). "A theoretical approach towards digital twins: A balance between an empirical and a fundamental model for distribution transformers". In: *Delft University of Technology*

Non-Linearity in the Core

Core Parameters ³

- Electrical Resistivity = $45 \times 10^{-8} \Omega\text{m}$
- Electrical Conductivity = $2.22 \times 10^6 \Omega^{-1}\text{m}^{-1}$
- Saturation Polarization = 2.03 T
- Maximum Relative Permeability = 8×10^4
- Density = $7.65 \times 10^3 \text{ kg/m}^3$
- 50 Hz Power Loss at 1.4 T = 0.6 W/kg
- 50 Hz Power Loss at 1.7 T = 1.21 W/kg

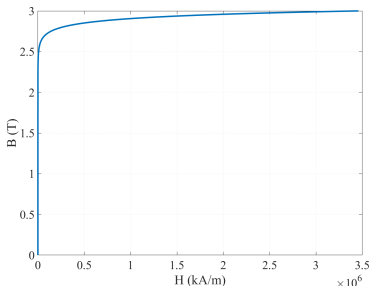
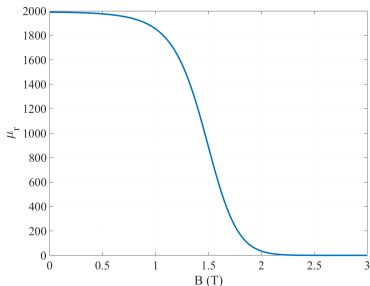
³F. Fiorillo (2004). *Characterization and measurement of magnetic materials*. Academic Press

Non-Linear Characteristics of Core

- The non-linear magnetic characteristic with hysteresis neglected can be approximated as given below ⁴.

$$\frac{1}{\mu} = k_1 e^{k_2 B^2} + k_3 \quad (1)$$

where, $k_1 = 3.8$, $k_2 = 2.17$ and $k_3 = 396.2$.



⁴ J Brauer (1975). "Simple equations for the magnetization and reluctivity curves of steel". In: *IEEE Transactions on Magnetics* 11.1, pp. 81–81

Finite Element Method : Frequency Domain

General Expression for FEM

- We know that

$$\nabla \times \left[\frac{1}{\mu} \nabla \times u \right] = J_0 + J_c \quad (2)$$

where,

J_0 = source current density,

J_c = conduction current density.

- Since, current flows along the z-axis and the geometry is in xy plane,

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] = J_0 + J_c \quad (3)$$

- In frequency domain,

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] + j\omega\sigma u_z = J_0 \quad (4)$$

- In time domain,

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] + \sigma \frac{\partial u_z}{\partial t} = J_0 \quad (5)$$

General Expression for FEM

- Without Eddy Currents :

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] = J_0 \quad (6)$$

- Writing in matrix form,

$$Au = f \quad (7)$$

- With Eddy Currents :

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] + j\omega\sigma u_z = J_0 \quad (8)$$

- Writing in matrix form,

$$(A + j\omega C)u = f \quad (9)$$

Parameters used in FEM Simulations

- Primary current,

$$I_{pri,a} = 0$$

$$I_{pri,b} = 0$$

$$I_{pri,c} = 0$$

- Secondary current,

$$I_{sec,a} = I_{sec,phase,peak} e^{-j\omega \frac{2\pi}{3}}$$

$$I_{sec,b} = I_{sec,phase,peak}$$

$$I_{sec,c} = I_{sec,phase,peak} e^{j\omega \frac{2\pi}{3}}$$

- Initial relative permeability $\mu_r = 2500$

Without Eddy Currents

- Equation :

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] = J_0 \quad (10)$$

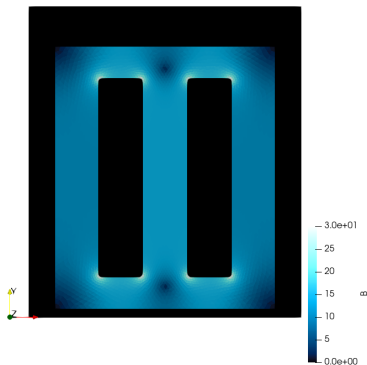


Figure: Three-Phase Transformer

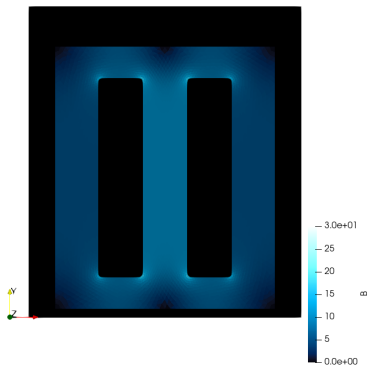


Figure: Single-Phase Transformer

With Eddy Currents

- Equation :

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] + j\omega\sigma u_z = J_0 \quad (11)$$

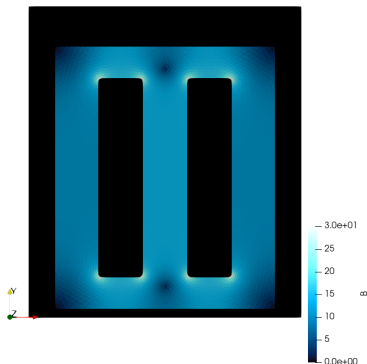


Figure: Without eddy current ($\sigma_{core} = 0$)

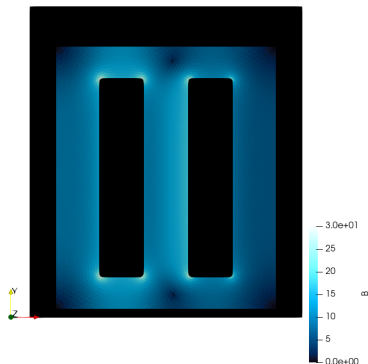


Figure: With eddy current ($\sigma_{core} = 100$)

Voltage-fed Couple Circuit Field Analysis

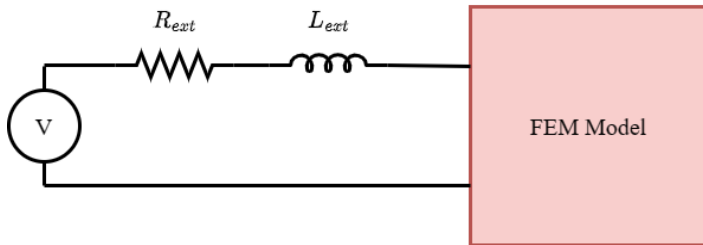


Figure: Voltage-fed source circuit representation of a single coil

where,

R_{ext} = coil resistance and any other external resistance in series,

L_{ext} = coil inductance and any other external inductance in series.

Voltage-fed Couple Circuit Field Analysis ^{5, 6}

- In frequency domain,

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] + j\omega\sigma u_z = J_0 = \frac{NI}{S} \quad (12)$$

where,

N = number of turns,

S = cross-sectional area.

- Circuit equation for an R-L equivalent network,

$$V = j\omega N\psi + R_{\text{ext}}I + j\omega L_{\text{ext}}I \quad (13)$$

$$V = j\omega G' u + R_{\text{ext}}I + j\omega L_{\text{ext}}I \quad (14)$$

⁵S. V. Kulkarni and S. A. Khaparde (2017). *Transformer engineering: design, technology, and diagnostics*. CRC press

⁶Kulkarni, Shrikrishna V (2020). *Electrical Equipment and Machines: Finite Element Analysis*.
<https://drive.google.com/file/d/1wiYJuqohQMM81P1CGI2hVKD1cLq06cTP/view>. [Online]

Voltage-fed Couple Circuit Field Analysis ^{7, 8}

- Combining the diffusion equation and circuit equation,

$$\begin{bmatrix} A + j\omega C & -f \\ j\omega G^T & R_{\text{ext}} + j\omega L_{\text{ext}} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix} = \begin{bmatrix} 0 \\ V \end{bmatrix} \quad (15)$$

$$KU = T \quad (16)$$

where, V = known voltages of each coil, and

$$f = \frac{G}{l_z} = \frac{N}{S} \times \frac{\text{area of element}}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

where, l_z is the length in the z-direction.

- When $l_z = 1$, the z-direction length is per metre.

⁷S. V. Kulkarni and S. A. Khaparde (2017). *Transformer engineering: design, technology, and diagnostics*. CRC press

⁸Kulkarni, Shrikrishna V (2020). *Electrical Equipment and Machines: Finite Element Analysis*.
<https://drive.google.com/file/d/1wiYJuqohQMM81P1CGI2hVKD1cLq06cTP/view>. [Online]

Parameters used in FEM Simulations

- $R_{ext,pri} = 601.8131 \, \Omega$
- $R_{ext,sec} = 0.201299 \, \Omega$
- $L_{ext,pri} = L_{ext,sec} = 1 \, \mu\text{H}$

Note : These values are chosen such that the time constant $\tau = \frac{L}{R}$ is minimum, so that steady state can be achieved in less number of cycles in time domain simulations.

- Open Circuit Test :
 - No primary coils present (Open circuit)
 - $V_{sec,a} = V_{sec,phase,peak} e^{-j\omega \frac{2\pi}{3}}$, $V_{sec,b} = V_{sec,phase,peak}$,
 $V_{sec,c} = V_{sec,phase,peak} e^{j\omega \frac{2\pi}{3}}$
- Short Circuit Test :
 - $V_{pri,a} = V_{pri,phase,peak} e^{-j\omega \frac{2\pi}{3}}$, $V_{pri,b} = V_{pri,phase,peak}$,
 $V_{pri,c} = V_{pri,phase,peak} e^{j\omega \frac{2\pi}{3}}$
 - $V_{sec,a} = 0$, $V_{sec,b} = 0$, $V_{sec,c} = 0$
- Initial relative permeability $\mu_r = 2500$
- Conductivity of core $\sigma_{core} = 0$

Example Representation of Matrices for Open Circuit Test

- Primary windings are open circuited and voltage is applied on secondary windings.

$$K = \begin{bmatrix} A + j\omega C & -\frac{N_{sec,a}}{S_{sec,a}} & -\frac{N_{sec,b}}{S_{sec,b}} & -\frac{N_{sec,c}}{S_{sec,c}} \\ j\omega \frac{N_{sec,a}}{S_{sec,a}} & R_{ext,sec,a} + j\omega L_{ext,sec,a} & 0 & 0 \\ j\omega \frac{N_{sec,b}}{S_{sec,b}} & 0 & R_{ext,sec,b} + j\omega L_{ext,sec,b} & 0 \\ j\omega \frac{N_{sec,c}}{S_{sec,c}} & 0 & 0 & R_{ext,sec,c} + j\omega L_{ext,sec,c} \end{bmatrix} \quad (17)$$

$$U = \begin{bmatrix} u \\ I_{sec,a} \\ I_{sec,b} \\ I_{sec,c} \end{bmatrix} \quad (18)$$

$$T = \begin{bmatrix} 0 \\ V_{sec,a} \\ V_{sec,b} \\ V_{sec,c} \end{bmatrix} \quad (19)$$

Calculation of Losses

- Core loss :
 - Steinmetz equation :

$$P_v = kf^\alpha B^\beta \quad W/m^3 \quad (20)$$

where, $k = 2.25$, $f = 50$ Hz, $\alpha = 1.53$, and $\beta = 1.6$.

- Lamination factor $k_{lamination} = 0.955$
- Total core loss :

$$P_{core} = k_{lamination} \int_V P_v dV \quad W \quad (21)$$

- Windings loss :

$$P_{winding,pri} = \sum_{i=a,b,c} I_{pri,i}^2 R_{ext,pri} \quad (22)$$

$$P_{winding,sec} = \sum_{i=a,b,c} I_{sec,i}^2 R_{ext,sec} \quad (23)$$

B plots of Open and Short Circuit Tests (Linear)

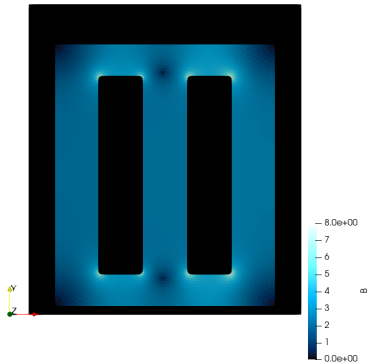


Figure: Open circuit test

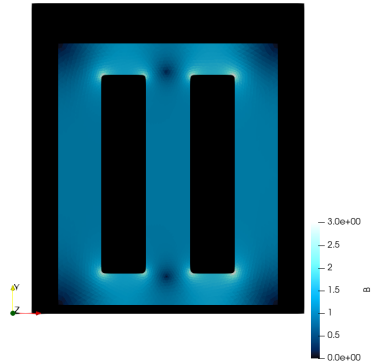


Figure: Short circuit test

- $P_{core} = 939.11 \text{ W}$

- $P_{windings,pri} = 10263.99 \text{ W}$

- $P_{windings,sec} = 11784.38 \text{ W}$

Open Circuit Test (Non-linear)

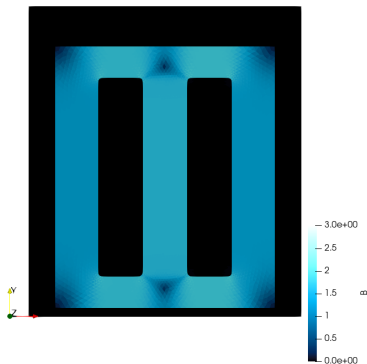


Figure: Magnetic flux density (B)

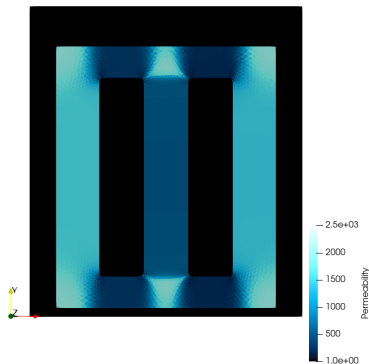


Figure: Relative permeability (μ_r)

- $P_{core} = 348.27 \text{ W}$

Short Circuit Test (Non-linear)

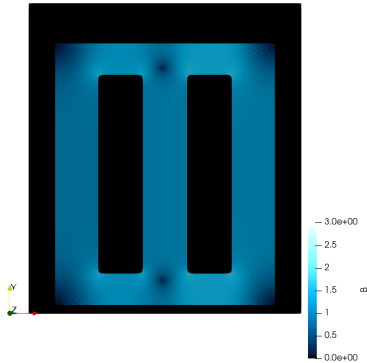


Figure: Magnetic flux density (B)

- $P_{windings,pri} = 11390.72 \text{ W}$
- $P_{windings,sec} = 10936.09 \text{ W}$

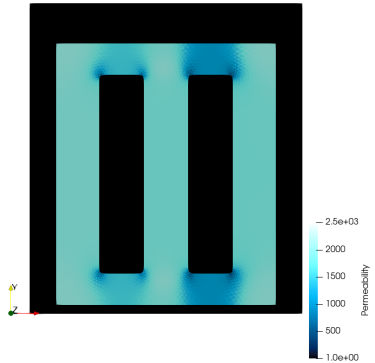


Figure: Relative permeability (μ_r)

Finite Element Method : Time Domain

Transient Analysis ^{9, 10}

- In time-domain,

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] + \sigma \frac{\partial u_z}{\partial t} = J_0 \quad (24)$$

- Writing in matrix form,

$$Au + C \frac{\partial u}{\partial t} = f \quad (25)$$

- Using Backward Euler method for time-stepping,

$$Au^{t+\Delta t} + C \frac{u^{t+\Delta t} - u^t}{\Delta t} = f^{t+\Delta t} \quad (26)$$

- On simplifying, we get,

$$[C + \Delta t A] u^{t+\Delta t} = Cu^t + \Delta t f^t \quad (27)$$

⁹S. V. Kulkarni and S. A. Khaparde (2017). *Transformer engineering: design, technology, and diagnostics*. CRC press

¹⁰Kulkarni, Shrikrishna V (2020). *Electrical Equipment and Machines: Finite Element Analysis*.
<https://drive.google.com/file/d/1wiYJuqohQMM81P1CGI2hVKD1cLq06cTP/view>. [Online]

Parameters used in FEM Simulations

- Primary current,

$$I_{pri,a} = 0$$

$$I_{pri,b} = 0$$

$$I_{pri,c} = 0$$

- Secondary current,

$$I_{sec,a} = I_{sec,phase,peak} \sin(\omega t - \frac{2\pi}{3})$$

$$I_{sec,b} = I_{sec,phase,peak} \sin(\omega t)$$

$$I_{sec,c} = I_{sec,phase,peak} \sin(\omega t + \frac{2\pi}{3})$$

- Initial relative permeability $\mu_r = 2500$
- Conductivity of core $\sigma_{core} = 0$

Transient Analysis

- Plots are shown for element present at the center of central limb.

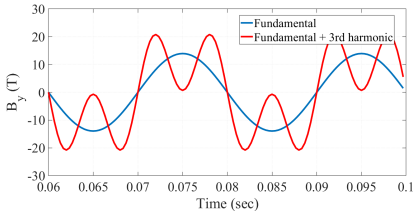


Figure: Linear analysis

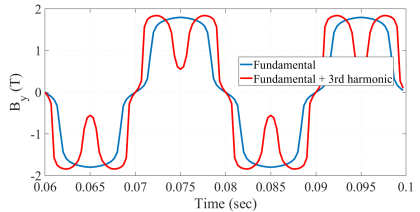


Figure: Non-linear analysis

FFT of Transient Analysis

- Plots are shown for element present at the center of central limb.
- Increase in harmonics due to non-linearity of the core.

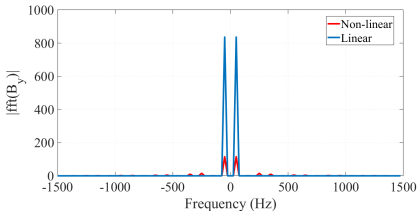


Figure: FFT of B_y with only fundamental component present

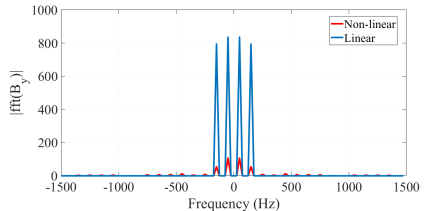


Figure: FFT of B_y in presence of third harmonic component

Specific Mesh Data Points

- Mesh node and element numbers for significant locations :

Sr. No.	Location	Node	Element
1	Center of central limb	4290	7014
2	Center of left limb	4781	6043
3	Center of right limb	4028	6000
4	Top right horizontal area	4650	4778

Transient Analysis

- Plots are shown for elements present at various locations in core.

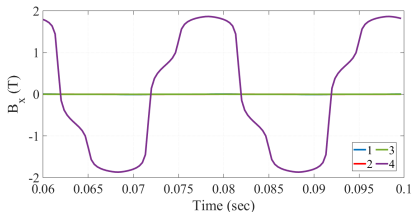


Figure: B_x

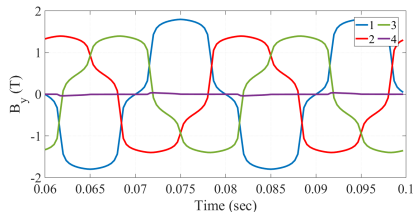


Figure: B_y

Transient Voltage-fed Couple Circuit Field Analysis ^{11, 12}

- General FEM equation,

$$Au + C \frac{\partial u}{\partial t} = f \cdot I \quad (28)$$

- Circuit equation,

$$V = G' \frac{\partial u}{\partial t} + R_{\text{ext}} I + L_{\text{ext}} \frac{\partial I}{\partial t} \quad (29)$$

- Combining and writing in time-stepping format,

$$\begin{bmatrix} \frac{C}{\Delta t} + A & -f \\ \frac{G^T}{\Delta t} & R_{\text{ext}} + \frac{L_{\text{ext}}}{\Delta t} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t+\Delta t} = \begin{bmatrix} \frac{C}{\Delta t} & 0 \\ \frac{G^T}{\Delta t} & \frac{L_{\text{ext}}}{\Delta t} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^t + \begin{bmatrix} 0 \\ V^{t+\Delta t} \end{bmatrix} \quad (30)$$

- Simplifying,

$$\begin{bmatrix} C + \Delta t A & -\Delta t f \\ G^T & \Delta t R_{\text{ext}} + L_{\text{ext}} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t+\Delta t} = \begin{bmatrix} C & 0 \\ G^T & L_{\text{ext}} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^t + \begin{bmatrix} 0 \\ \Delta t V^{t+\Delta t} \end{bmatrix} \quad (31)$$

¹¹S. V. Kulkarni and S. A. Khaparde (2017). *Transformer engineering: design, technology, and diagnostics*. CRC press

¹²Kulkarni, Shrikrishna V (2020). *Electrical Equipment and Machines: Finite Element Analysis*.

<https://drive.google.com/file/d/1wiYJuqohQMM81PlCGI2hVKD1cLq06cTP/view>. [Online]

Parameters used in FEM Simulations

- Open Circuit Test :
 - No primary coils present (Open circuit)
 - $V_{sec,a} = V_{sec,phase,peak} \sin(\omega t - \frac{2\pi}{3})$, $V_{sec,b} = V_{sec,phase,peak} \sin(\omega t)$,
 $V_{sec,c} = V_{sec,phase,peak} \sin(\omega t + \frac{2\pi}{3})$
- Short Circuit Test :
 - $V_{pri,a} = V_{pri,phase,peak} \sin(\omega t - \frac{2\pi}{3})$, $V_{pri,b} = V_{pri,phase,peak} \sin(\omega t)$,
 $V_{pri,c} = V_{pri,phase,peak} \sin(\omega t + \frac{2\pi}{3})$
 - $V_{sec,a} = 0$, $V_{sec,b} = 0$, $V_{sec,c} = 0$
- In case of presence of 3rd harmonic component, another term is added to above equations as given below,

$$V_{third_harmonic} = \frac{V_{fundamental}}{3} \sin(3\omega t + \phi_3) \quad (32)$$

where, $\phi_3 = \frac{\pi}{3}$

- Similar terms can be added for other harmonics of 5, 7, 9,

Transient Voltage-fed Couple Circuit Field Analysis

- Plots are shown for element present at the center of central limb.

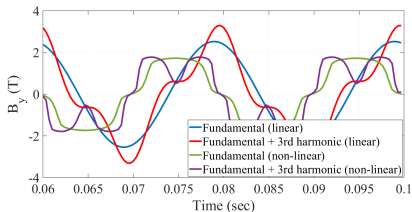


Figure: B_y (Open circuit test)

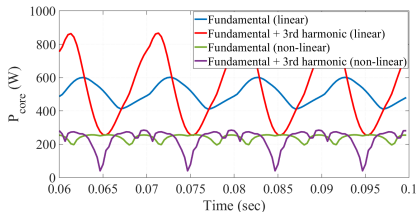


Figure: P_{core} (Open circuit test)

Transient Voltage-fed Couple Circuit Field Analysis

- Plots are shown for element present at the center of central limb.

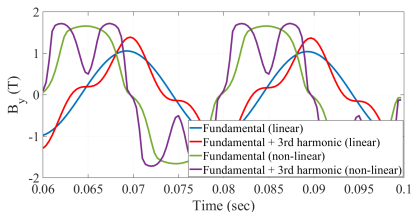


Figure: B_y (Short circuit test)

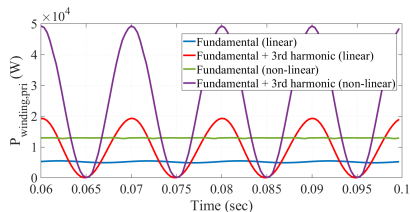


Figure: $P_{winding,pri}$ (Short circuit test)

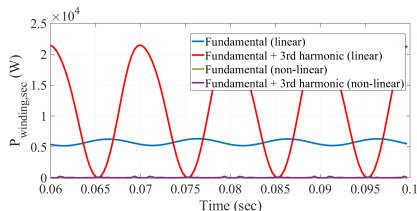


Figure: $P_{winding,sec}$ (Short circuit test)

FFT of Transient Voltage-fed Couple Circuit Field Analysis

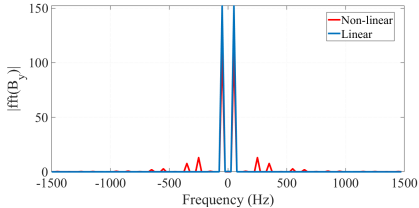


Figure: FFT of B_y with only fundamental component present (Open circuit test)

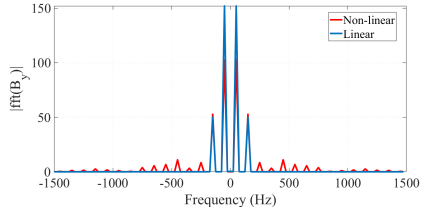


Figure: FFT of B_y in presence of third harmonic component (Open circuit test)

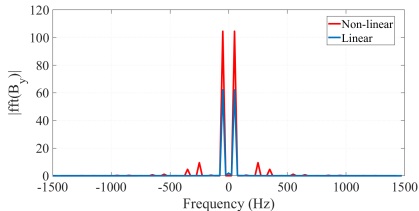


Figure: FFT of B_y with only fundamental component present (Short circuit test)

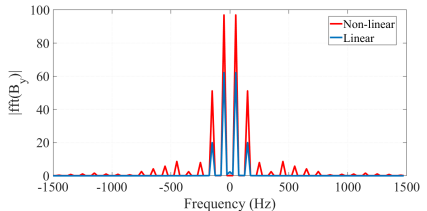


Figure: FFT of B_y in presence of third harmonic component (Short circuit test)

Julia

Upgrades in Julia

- Struct to store all nodes, elements, area values and Emat matrices
- LU decomposition for A matrix in $A \setminus f$ using *factorise()* function
- Damping factor in non-linear calculations
- FastSparseMatrix
- StaticArrays
- Makie plots
- VTK files

Appendix

Calculation using EMF Equation of Transformer

- EMF Equation of transformer :

$$V_{rms} = 4.44 B_{peak} A N f \quad (33)$$

- For primary winding,

$$V_{rms} = 10750 \text{ V}, N = 266, A = 17 \times 10^{-2} \text{ m}^2,$$

$$B_{peak} = 1.0710 \text{ T} \quad (34)$$

- For secondary winding,

$$V_{rms} = 242.49 \text{ V}, N = 6, A = 17 \times 10^{-2} \text{ m}^2,$$

$$B_{peak} = 1.0710 \text{ T} \quad (35)$$

- Calculations are done for core depth of 1m.

Thank You