Finite Element Method : Future Distribution Grids

Contents

Chapter 1

STEDIN Transformer Parameters

This chapter contains transformer parameters as given in [1].

1.1 Power Rating

Power rating = 400 kVA

1.2 Line to Line Parameters

Primary voltage, $V_{pri,line,rms} = 10750$ V
Secondary voltage, $V_{sec,line,rms} = 420$ V
Primary current, $I_{pri,line,rms} = 21.48$ A
Secondary current, $I_{sec,line,rms} = 549.86$ A

1.3 Phase to Phase Parameters

Considering a delta-wye connected transformer,
Primary voltage, $V_{pri,phase,rms} = 10750$ V
Secondary voltage, $V_{sec,phase,rms} = 242.49$ V
Primary current, $I_{pri,phase,rms} = 12.40$ A
Secondary current, $I_{sec,phase,rms} = 549.86$ A

Corresponding peak values will be,
Primary voltage, $V_{pri,phase,peak} = 15202.40$ V
Secondary voltage, $V_{sec,phase,peak} = 342.93$ V
Primary current, $I_{pri,phase,peak} = 17.54$ A
Secondary current, $I_{sec,phase,peak} = 777.62$ A

1.4 Turns Ratio

Primary turns, $N_{pri} = 266$
Secondary turns, $N_{sec} = 6$

1.5 Core Parameters

The values are as given in Section 2.3.2 of [2] for grain-oriented silicon steel (Fe-(3 wt%)Si alloys).
Electrical Resistivity = 45×10^{-8} Ωm
Electrical Conductivity = 2.22×10^{6} $\Omega^{-1}\mathrm{m}^{-1}$
Saturation Polarization = 2.03 T
Maximum Relative Permeability = 8×10^{4}
Density = 7.65×10^{3} kg/m^3
50 Hz Power Loss at 1.4 T = 0.6 W/kg
50 Hz Power Loss at 1.7 T = 1.21 W/kg

Chapter 2

Magnetic Field Simulations

2.1 General Expression for FEM

$$\nabla \times \left[\frac{1}{\mu}\nabla \times u\right] = J_0 + J_c \tag{2.1}$$

where,
J_0 = source current density,
J_c = conduction current density.

Since, current flows along the z-axis and the geometry is in xy plane,

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla u_z\right] = J_0 + J_c \tag{2.2}$$

In frequency domain,

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla u_z\right] + j\omega\sigma u_z = J_0 \tag{2.3}$$

In time domain,

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla u_z\right] + \sigma\frac{\partial u_z}{\partial t} = J_0 \tag{2.4}$$

2.2 Without Eddy Currents

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla u_z\right] = J_0 \tag{2.5}$$

2.3 With Eddy Currents

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla u_z\right] + j\omega\sigma u_z = J_0 \tag{2.6}$$

2.4 Non-Linear Characteristics of Core

The non-linear magnetic characteristic with hysteresis neglected can be approximated as given in [3].

$$\frac{1}{\mu} = k_1 e^{k_2 B^2} + k_3 \tag{2.7}$$

where, k_1, k_2 and k_3 are constants equal to 3.8, 2.17, and 396.2, respectively.

2.5 Voltage-fed Couple Circuit Field Analysis

In frequency domain,

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla u_z\right] + j\omega\sigma u_z = J_0 = \frac{NI}{S} \tag{2.8}$$

where,
N = number of turns,
S = cross-sectional area.

Circuit equation for an R-L equivalent network,

$$V = j\omega N\psi + R_{ext}I + j\omega L_{ext}I \tag{2.9}$$
$$V = j\omega G' u + R_{ext}I + j\omega L_{ext}I \tag{2.10}$$

Combining the diffusion equation and circuit equation,

$$\begin{bmatrix} A & -f \\ j\omega G^T & R_{ext} + j\omega L_{ext} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix} = \begin{bmatrix} 0 \\ V \end{bmatrix} \tag{2.11}$$

where, V = known voltages, and

$$G = f = \frac{N}{S} \times \frac{area\ of\ element}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Note : The above formulation is only valid in case of per unit z-direction axis.

2.6 Transient Analysis

In time-domain,

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla u_z\right] + \sigma\frac{\partial u_z}{\partial t} = J_0 \tag{2.12}$$

Writing in matrix form,

$$Au + B\frac{\partial u}{\partial t} = f \tag{2.13}$$

Using Backward Euler method for time-stepping,

$$Au^{t+\Delta t} + B\frac{u^{t+\Delta t} - u^t}{\Delta t} = f^{t+\Delta t} \tag{2.14}$$

On simplifying, we get,

$$[B + \Delta t A]u^{t+\Delta t} = Bu^t + \Delta t f^t \tag{2.15}$$

2.7 Transient + Voltage-fed Couple Circuit Field Analysis

2.7.1 Neglecting eddy current term

General FEM equation,

$$Au = f \tag{2.16}$$

Circuit equation,

$$V = G' \frac{\partial u}{\partial t} + R_{ext} I + L_{ext} \frac{\partial I}{\partial t} \tag{2.17}$$

Combining and writing in time-stepping format,

$$\begin{bmatrix} A & -f \\ \frac{G^T}{\Delta t} & R_{ext} + \frac{L_{ext}}{\Delta t} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t+\Delta t} = \begin{bmatrix} 0 & 0 \\ \frac{G^T}{\Delta t} & \frac{L_{ext}}{\Delta t} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t} + \begin{bmatrix} 0 \\ V^{t+\Delta t} \end{bmatrix} \tag{2.18}$$

Simplifying,

$$\begin{bmatrix} \Delta t A & -\Delta t f \\ G^T & \Delta t R_{ext} + L_{ext} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t+\Delta t} = \begin{bmatrix} 0 & 0 \\ G^T & L_{ext} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t} + \begin{bmatrix} 0 \\ \Delta t V^{t+\Delta t} \end{bmatrix} \tag{2.19}$$

2.7.2 Considering eddy current term

General FEM equation,

$$Au + B\frac{\partial u}{\partial t} = f \tag{2.20}$$

Circuit equation,

$$V = G' \frac{\partial u}{\partial t} + R_{ext} I + L_{ext} \frac{\partial I}{\partial t} \tag{2.21}$$

Combining and writing in time-stepping format,

$$\begin{bmatrix} \frac{B}{\Delta t} + A & -f \\ \frac{G^T}{\Delta t} & R_{ext} + \frac{L_{ext}}{\Delta t} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t+\Delta t} = \begin{bmatrix} \frac{B}{\Delta t} & 0 \\ \frac{G^T}{\Delta t} & \frac{L_{ext}}{\Delta t} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t} + \begin{bmatrix} 0 \\ V^{t+\Delta t} \end{bmatrix} \tag{2.22}$$

Simplifying,

$$\begin{bmatrix} B + \Delta t A & -\Delta t f \\ G^T & \Delta t R_{ext} + L_{ext} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t+\Delta t} = \begin{bmatrix} B & 0 \\ G^T & L_{ext} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t} + \begin{bmatrix} 0 \\ \Delta t V^{t+\Delta t} \end{bmatrix} \tag{2.23}$$

Chapter 3

FEM Results

Mesh elements and numbers

1. 7051 - center of the central limb
2. 6223 - center of the left limb
3. 6270 - center of the right limb
4. 8970 - top right of the core
5. 6136 - top center of the core
6. 10045 - top right horizontal area of the core

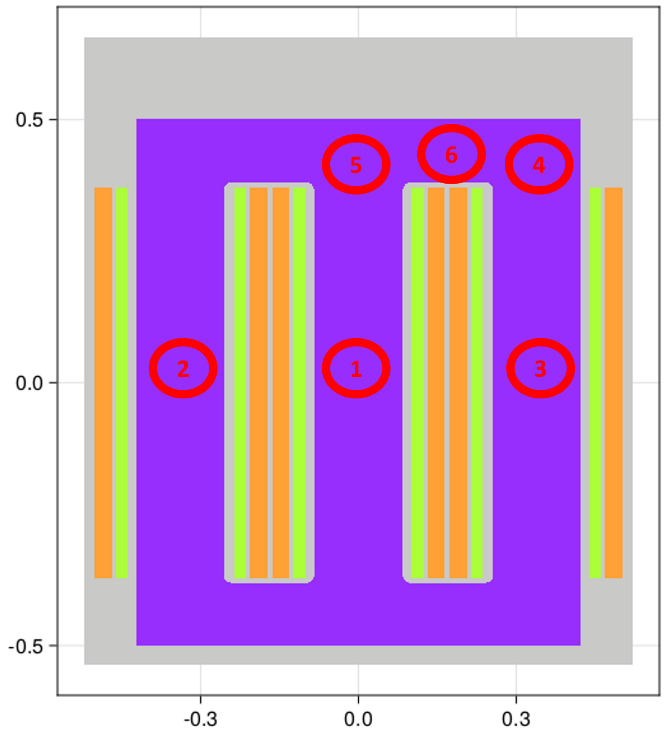

Figure 3.1: Node locations

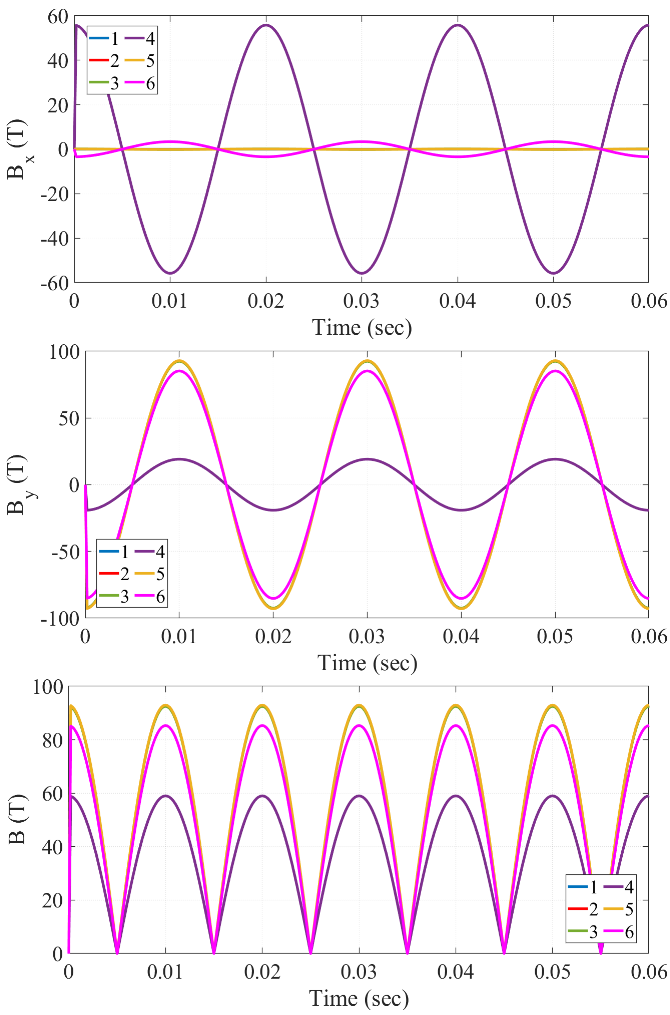

Figure 3.2: Single phase source without harmonics : B curves

3.1 Transient Analysis

This section deals with results using current density without harmonics as $J_0 \sin(\omega t)$.

Harmonics are given as $J_0 \sin(\omega t) + (J_0/3)J_0 \sin(3\omega t)$.

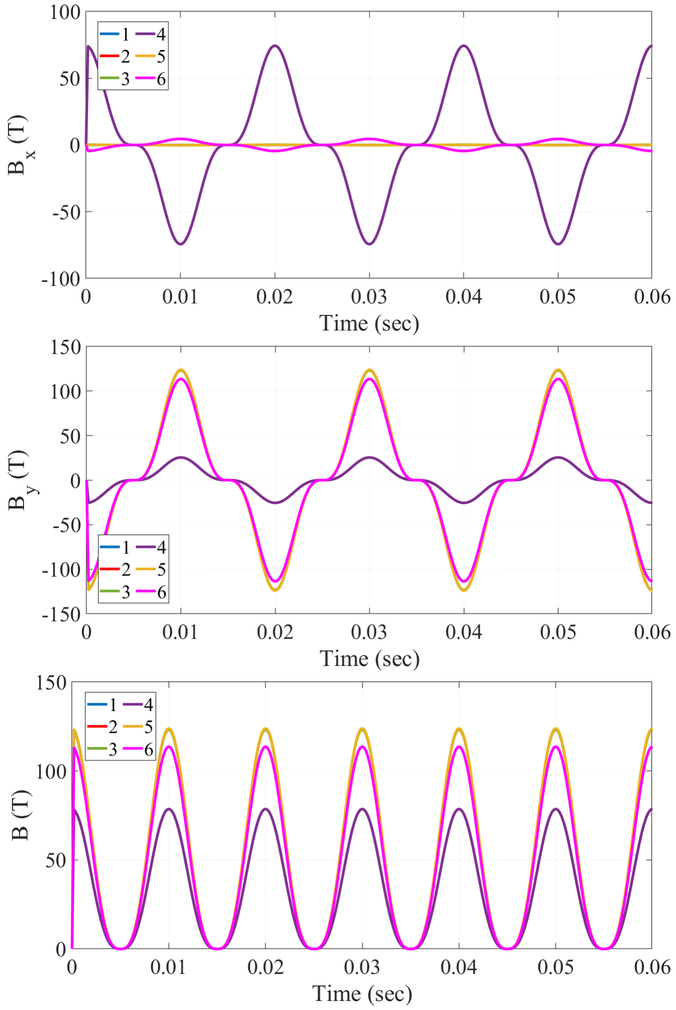

Figure 3.3: Single phase source with harmonics : B curves

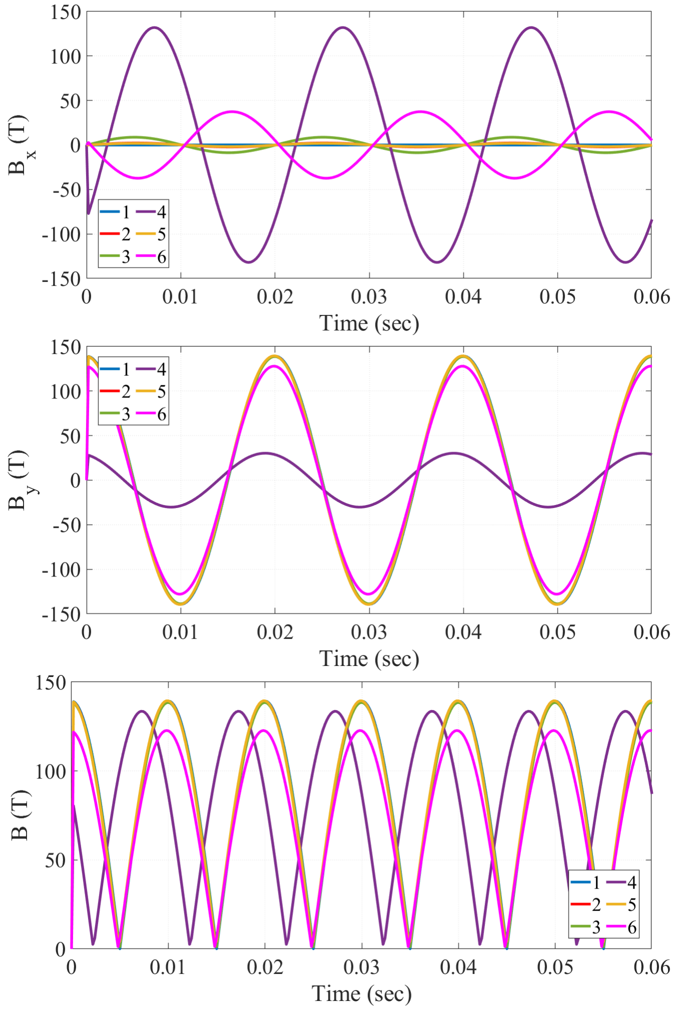

Figure 3.4: Three phase source with harmonics : B curves

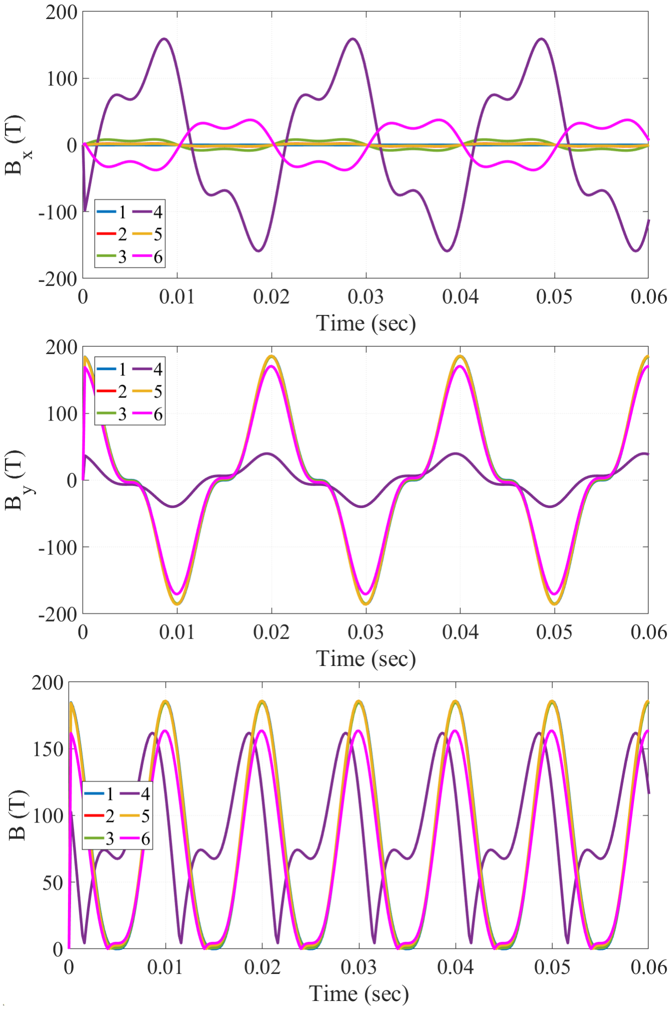

Figure 3.5: Three phase source with harmonics : B curves

Chapter 4

Things to do

4.1 Report by Auke and Philip

- check value of the amplitude of the applied current density

- check values of (in-plane) magnetic permeability (linear and non-linear) and (out-of-plane) electrical conductivity. Take laminanations into account, values for steel-ony are likely not representative;

- FFT of time traces of the magnetic flux - do higher order harmonics appear in the non-linear case? What happens in the non-linear in case that the applied current has a frequency of 150 Hz. Is a 50 Hz component in the potential and fields generated?

- other items?

4.2 Skin depth consideration

Extend computation of skin-depth delta with value of Bmax in the thin layer. Model using dsolve? Show that higher frequency cause material to go into saturation.

4.3 Extend to voltage driven winding by adding 2*3 = 6 electrical circuit equations

- What element to add in the circuit equations for the primary and secondary winding to simulate relevant scenarios (no load and short circuit)

- How to perform time-integration in the linear case? How to extend to the non-linear case

4.4 Extend to second order elements to reach same spatial accuracy using less degree of freedom?

4.5 Optimize Julia code further

- employ structarrays.jl to arrive at type-stable and thus more performant implementation. See e.g. Table 4 in https://www.sciencedirect.com/science/article/pii/S0965
- employ staticarrays.jl to represent local identity matrix, local vector and local matrix

4.6 Extend to harmonic balance method

Verify computations in time-domain by frequency domain approach. In linear case at single frequency. In non-linear case by taking multiple frequencies and their couplings into account.

4.7 Extend post-processing with copper and iron losses.

4.8 Work done thus far using Comsol Multiphyisics

- check what the effective BH curve is;

References

[1] M. van Dijk, "A theoretical approach towards digital twins: A balance between an empirical and a fundamental model for distribution transformers," *Delft University of Technology*, 2022.

[2] F. Fiorillo, *Characterization and measurement of magnetic materials*. Academic Press, 2004.

[3] J. Brauer, "Simple equations for the magnetization and reluctivity curves of steel," *IEEE Transactions on Magnetics*, vol. 11, no. 1, pp. 81–81, 1975.