Finite Element Method : Future Distribution Grids

Contents

Chapter 1

STEDIN Transformer Parameters

This chapter contains transformer parameters as given in [1].

1.1 Power Rating

Power rating = 400 kVA

1.2 Line to Line Parameters

Primary voltage, $V_{pri,line,rms} = 10750$ V
Secondary voltage, $V_{sec,line,rms} = 420$ V
Primary current, $I_{pri,line,rms} = 21.48$ A
Secondary current, $I_{sec,line,rms} = 549.86$ A

1.3 Phase to Phase Parameters

Considering a delta-wye connected transformer,
Primary voltage, $V_{pri,phase,rms} = 10750$ V
Secondary voltage, $V_{sec,phase,rms} = 242.49$ V
Primary current, $I_{pri,phase,rms} = 12.40$ A
Secondary current, $I_{sec,phase,rms} = 549.86$ A

Corresponding peak values will be,
Primary voltage, $V_{pri,phase,peak} = 15202.40$ V
Secondary voltage, $V_{sec,phase,peak} = 342.93$ V
Primary current, $I_{pri,phase,peak} = 17.54$ A
Secondary current, $I_{sec,phase,peak} = 777.62$ A

1.4 Turns Ratio

Primary turns, $N_{pri} = 266$
Secondary turns, $N_{sec} = 6$

1.5 Core Parameters

The values are as given in Section 2.3.2 of [2] for grain-oriented silicon steel (Fe-(3 wt%)Si alloys).
Electrical Resistivity = 45×10^{-8} Ωm
Electrical Conductivity = 2.22×10^{6} $\Omega^{-1}\text{m}^{-1}$
Saturation Polarization = 2.03 T
Maximum Relative Permeability = 8×10^{4}
Density = 7.65×10^{3} kg/m^3
50 Hz Power Loss at 1.4 T = 0.6 W/kg
50 Hz Power Loss at 1.7 T = 1.21 W/kg

Chapter 2

Magnetic Field Simulations

2.1 General Expression for FEM

$$\nabla \times \left[\frac{1}{\mu}\nabla \times A\right] = J_0 + J_c \tag{2.1}$$

where,
J_0 = source current density,
J_c = conduction current density.

Since, current flows along the z-axis and the geometry is in xy plane,

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla A_z\right] = J_0 + J_c \tag{2.2}$$

In frequency domain,

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla A_z\right] + j\omega\sigma A_z = J_0 \tag{2.3}$$

In time domain,

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla A_z\right] + \sigma\frac{\partial A_z}{\partial t} = J_0 \tag{2.4}$$

2.2 Without Eddy Currents

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla A_z\right] = J_0 \tag{2.5}$$

2.3 With Eddy Currents

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla A_z\right] + j\omega\sigma A_z = J_0 \tag{2.6}$$

2.4 Non-Linear Characteristics of Core

The non-linear magnetic characteristic with hysteresis neglected can be approximated as given in [3].

$$\frac{1}{\mu} = k_1 e^{k_2 B^2} + k_3 \tag{2.7}$$

where, k_1, k_2 and k_3 are constants equal to 3.8, 2.17, and 396.2, respectively.

2.5 Voltage-fed Couple Circuit Field Analysis

In frequency domain,

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla A_z\right] + j\omega\sigma A_z = J_0 = \frac{NI}{S} \tag{2.8}$$

where,
N = number of turns,
S = cross-sectional area.

Circuit equation for an R-L equivalent network,

$$V = j\omega N\psi + R_{ext}I + j\omega L_{ext}I \tag{2.9}$$

$$V = j\omega G'u + R_{ext}I + j\omega L_{ext}I \tag{2.10}$$

Combining the diffusion equation and circuit equation,

$$\begin{bmatrix} A & -f \\ j\omega G^T & R_{ext} + j\omega L_{ext} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix} = \begin{bmatrix} 0 \\ V \end{bmatrix} \tag{2.11}$$

where, V = known voltages, and

$$G = f = \frac{N}{S} \times \frac{area\ of\ element}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Chapter 3

COMSOL Simulation

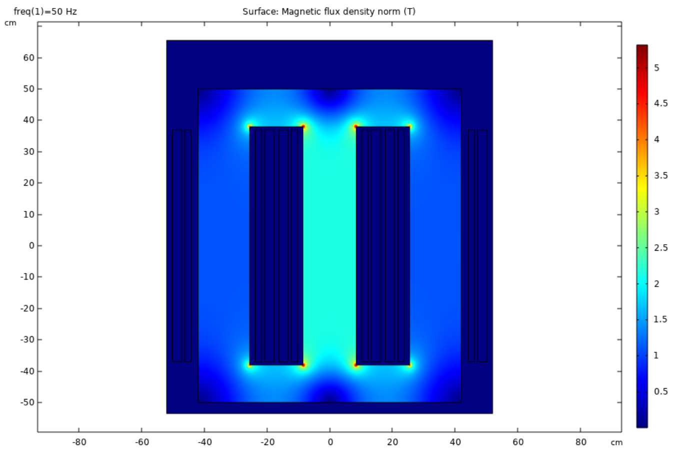

Figure 3.1: B plot for single phase system (COMSOL)

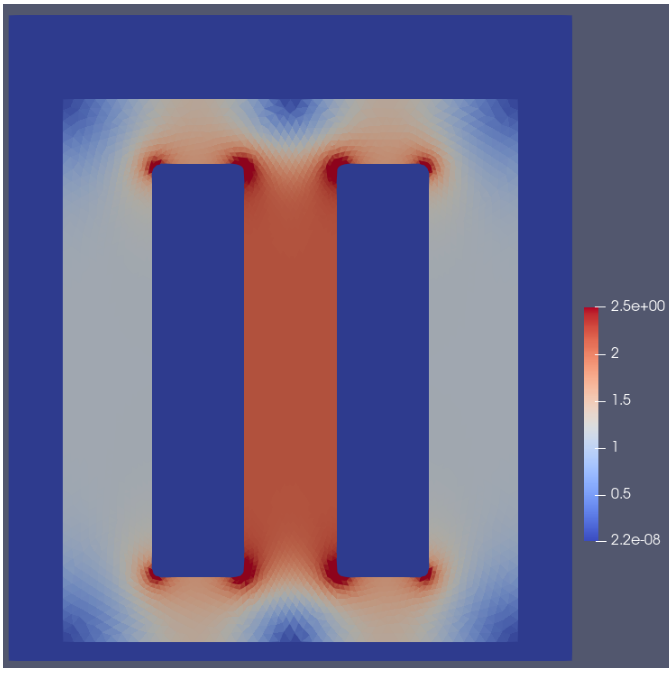

Figure 3.2: B plot for single phase system (Julia)

References

[1] M. van Dijk, "A theoretical approach towards digital twins: A balance between an empirical and a fundamental model for distribution transformers," *Delft University of Technology*, 2022.

[2] F. Fiorillo, *Characterization and measurement of magnetic materials*. Academic Press, 2004.

[3] J. Brauer, "Simple equations for the magnetization and reluctivity curves of steel," *IEEE Transactions on Magnetics*, vol. 11, no. 1, pp. 81–81, 1975.