Future Distribution Grids

Finite Element Modelling for Electrical Energy Applications

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STEDIN Transformer Parameters [1]

- Power rating, $S = 400$ kVA
- Line to Line Parameters :
  - Primary voltage, $V_{pri,line,rms} = 10750$ V
  - Secondary voltage, $V_{sec,line,rms} = 420$ V
  - Primary current, $I_{pri,line,rms} = 21.48$ A
  - Secondary current, $I_{sec,line,rms} = 549.86$ A
- Phase to Phase Parameters :
  Considering a delta-wye connected transformer,
  - Primary voltage, $V_{pri,phase,rms} = 10750$ V
  - Secondary voltage, $V_{sec,phase,rms} = 242.49$ V
  - Primary current, $I_{pri,phase,rms} = 12.40$ A
  - Secondary current, $I_{sec,phase,rms} = 549.86$ A

[1] M. van Dijk (2022). "A theoretical approach towards digital twins: A balance between an empirical and a fundamental model for distribution transformers". In: *Delft University of Technology*

STEDIN Transformer Parameters [2]

- Phase to Phase Parameters :
  - Primary voltage, $V_{pri,phase,peak} = 15202.40$ V
  - Secondary voltage, $V_{sec,phase,peak} = 342.93$ V
  - Primary current, $I_{pri,phase,peak} = 17.54$ A
  - Secondary current, $I_{sec,phase,peak} = 777.62$ A
- Turns Ratio -
  - Primary turns, $N_{pri} = 266$
  - Secondary turns, $N_{sec} = 6$
- Resistance and inductance of windings :
  - $R_{ext,pri}$ = 1.8131 Ω
  - $R_{ext,sec}$ = 1.299 mΩ
  - $L_{ext,pri}$ = $L_{ext,sec}$ = 1 μH
  - CFF_p = CFF_s = 0.3
- Depth of core (z-axis length) $l_z = 0.4$ m

[2] M. van Dijk (2022). "A theoretical approach towards digital twins: A balance between an empirical and a fundamental model for distribution transformers". In: *Delft University of Technology*

Non-Linearity in the Core

Core Parameters [3]

- Electrical Resistivity $= 45 \times 10^{-8}\ \Omega\text{m}$
- Electrical Conductivity $= 2.22 \times 10^{6}\ \Omega^{-1}\text{m}^{-1}$
- Saturation Polarization $= 2.03$ T
- Maximum Relative Permeability $= 8 \times 10^{4}$
- Density $= 7.65 \times 10^{3}\ \text{kg/m}^{3}$
- 50 Hz Power Loss at 1.4 T $= 0.6$ W/kg
- 50 Hz Power Loss at 1.7 T $= 1.21$ W/kg

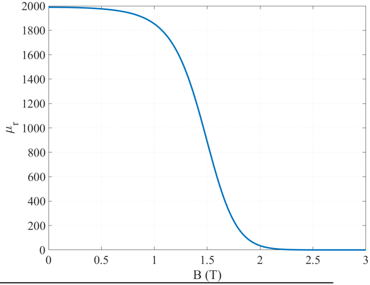
[3] F. Fiorillo (2004). *Characterization and measurement of magnetic materials*. Academic Press

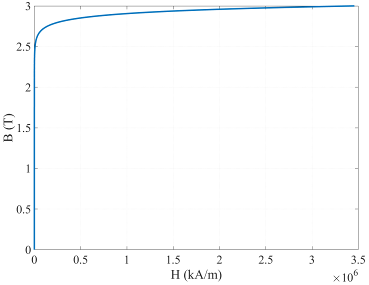
Non-Linear Characteristics of Core

- The non-linear magnetic characteristic with hysteresis neglected can be approximated as given below [4].

$$\frac{1}{\mu} = k_1 e^{k_2 B^2} + k_3 \tag{1}$$

where, $k_1 = 3.8$, $k_2 = 2.17$ and $k_3 = 396.2$.





[4] J Brauer (1975). "Simple equations for the magnetization and reluctivity curves of steel". In: *IEEE Transactions on Magnetics* 11.1, pp. 81–81

Finite Element Method : Frequency Domain

General Expression for FEM

- We know that

$$\nabla \times \left[\frac{1}{\mu} \nabla \times u \right] = J_0 + J_c \tag{2}$$

where,
J_0 = source current density,
J_c = conduction current density.

- Since, current flows along the z-axis and the geometry is in xy plane,

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] = J_0 + J_c \tag{3}$$

- In frequency domain,

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] + j\omega\sigma u_z = J_0 \tag{4}$$

- In time domain,

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] + \sigma \frac{\partial u_z}{\partial t} = J_0 \tag{5}$$

General Expression for FEM

- Without Eddy Currents :

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] = J_0 \tag{6}$$

- Writing in matrix form,

$$Au = f \tag{7}$$

- With Eddy Currents :

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] + j\omega\sigma u_z = J_0 \tag{8}$$

- Writing in matrix form,

$$(A + j\omega C)u = f \tag{9}$$

Parameters used in FEM Simulations

- Primary current,
  $I_{pri,a} = 0$
  $I_{pri,b} = 0$
  $I_{pri,c} = 0$
- Secondary current,
  $I_{sec,a} = I_{sec,phase,peak} e^{-j\omega \frac{2\pi}{3}}$
  $I_{sec,b} = I_{sec,phase,peak}$
  $I_{sec,c} = I_{sec,phase,peak} e^{j\omega \frac{2\pi}{3}}$
- Initial relative permeability $\mu_r = 2500$

Without Eddy Currents

- Equation :

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] = J_0 \tag{10}$$

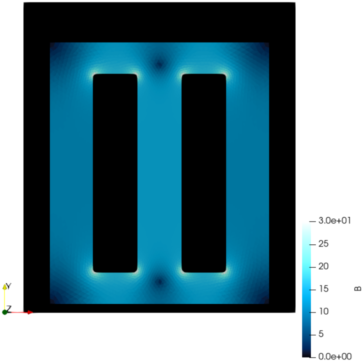


Figure: Three-Phase Transformer

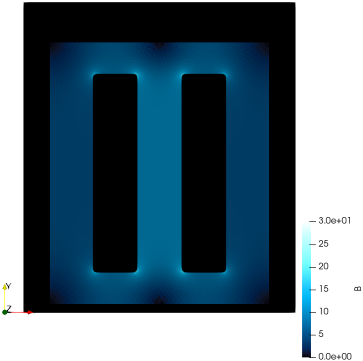


Figure: Single-Phase Transformer

With Eddy Currents

- Equation :

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] + j\omega\sigma u_z = J_0 \tag{11}$$

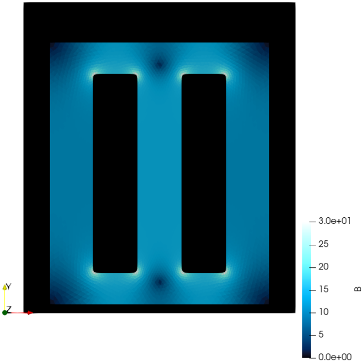


Figure: Without eddy current ($\sigma_{core} = 0$)

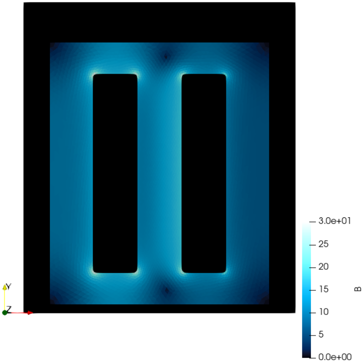


Figure: With eddy current ($\sigma_{core} = 100$)

Voltage-fed Couple Circuit Field Analysis

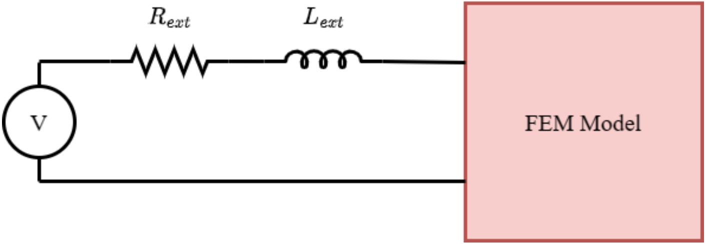


Figure: Voltage-fed source circuit representation of a single coil

where,
R_{ext} = coil resistance and any other external resistance in series,
L_{ext} = coil inductance and any other external inductance in series.

Voltage-fed Couple Circuit Field Analysis [5, 6]

- In frequency domain,

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla u_z\right] + j\omega\sigma u_z = J_0 = \frac{NI}{S} \tag{12}$$

where,
N = number of turns,
S = cross-sectional area.

- Circuit equation for an R-L equivalent network,

$$V = j\omega N\psi + R_{ext}I + j\omega L_{ext}I \tag{13}$$
$$V = j\omega G' u + R_{ext}I + j\omega L_{ext}I \tag{14}$$

[5] S. V. Kulkarni and S. A. Khaparde (2017). *Transformer engineering: design, technology, and diagnostics*. CRC press

[6] Kulkarni, Shrikrishna V (2020). *Electrical Equipment and Machines: Finite Element Analysis*. https://drive.google.com/file/d/1wiyJuqohQMM8lPlCGI2hVKD1cLqO6cTP/view. [Online]

Voltage-fed Couple Circuit Field Analysis [7, 8]

- Combining the diffusion equation and circuit equation,

$$\begin{bmatrix} A + j\omega C & -f \\ j\omega G^T & R_{ext} + j\omega L_{ext} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix} = \begin{bmatrix} 0 \\ V \end{bmatrix} \tag{15}$$

$$KU = T \tag{16}$$

where, $V =$ known voltages of each coil, and

$$f = \frac{G}{l_Z} = \frac{N}{S} \times \frac{area\ of\ \ element}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

where, l_Z is the length in the z-direction.

- When $l_Z = 1$, the z-direction length is per metre.

[7] S. V. Kulkarni and S. A. Khaparde (2017). *Transformer engineering: design, technology, and diagnostics*. CRC press

[8] Kulkarni, Shrikrishna V (2020). *Electrical Equipment and Machines: Finite Element Analysis*. https://drive.google.com/file/d/1wiyJuqohQMM8lPlCGI2hVKD1cLq06cTP/view. [Online]

Parameters used in FEM Simulations

- $R_{ext,pri} = 601.8131\ \Omega$
- $R_{ext,sec} = 0.201299\ \Omega$
- $L_{ext,pri} = L_{ext,sec} = 1\ \mu\text{H}$
  Note : These values are chosen such that the time constant $\tau = \frac{L}{R}$ is minimum, so that steady state can be achieved in less number of cycles in time domain simulations.
- Open Circuit Test :
  - No primary coils present (Open circuit)
  - $V_{sec,a} = V_{sec,phase,peak}\, e^{-j\omega\frac{2\pi}{3}}$, $V_{sec,b} = V_{sec,phase,peak}$, $V_{sec,c} = V_{sec,phase,peak}\, e^{j\omega\frac{2\pi}{3}}$
- Short Circuit Test :
  - $V_{pri,a} = V_{pri,phase,peak}\, e^{-j\omega\frac{2\pi}{3}}$, $V_{pri,b} = V_{pri,phase,peak}$, $V_{pri,c} = V_{pri,phase,peak}\, e^{j\omega\frac{2\pi}{3}}$
  - $V_{sec,a} = 0$, $V_{sec,b} = 0$, $V_{sec,c} = 0$
- Initial relative permeability $\mu_r = 2500$
- Conductivity of core $\sigma_{core} = 0$

Example Representation of Matrices for Open Circuit Test

- Primary windings are open circuited and voltage is applied on secondary windings.

$$K = \begin{bmatrix} A + j\omega C & -\frac{N_{sec,a}}{S_{sec,a}} & -\frac{N_{sec,b}}{S_{sec,b}} & -\frac{N_{sec,c}}{S_{sec,c}} \\ j\omega\frac{N_{sec,a}}{S_{sec,a}} & R_{ext,sec,a} + j\omega L_{ext,sec,a} & 0 & 0 \\ j\omega\frac{N_{sec,b}}{S_{sec,b}} & 0 & R_{ext,sec,b} + j\omega L_{ext,sec,b} & 0 \\ j\omega\frac{N_{sec,c}}{S_{sec,c}} & 0 & 0 & R_{ext,sec,c} + j\omega L_{ext,sec,c} \end{bmatrix} \tag{17}$$

$$U = \begin{bmatrix} u \\ I_{sec,a} \\ I_{sec,b} \\ I_{sec,c} \end{bmatrix} \tag{18}$$

$$T = \begin{bmatrix} 0 \\ V_{sec,a} \\ V_{sec,b} \\ V_{sec,c} \end{bmatrix} \tag{19}$$

Calculation of Losses

- Core loss :
  - Steinmetz equation :

$$P_v = kf^{\alpha} B^{\beta} \qquad W/m^3 \tag{20}$$

  where, $k = 2.25$, $f = 50$ Hz, $\alpha = 1.53$, and $\beta = 1.6$.
  - Lamination factor $k_{lamination} = 0.955$
  - Total core loss :

$$P_{core} = k_{lamination} \int_V P_v \, dV \qquad W \tag{21}$$

- Windings loss :

$$P_{winding,pri} = \sum_{i=a,b,c} I^2_{pri,i} R_{ext,pri} \tag{22}$$

$$P_{winding,sec} = \sum_{i=a,b,c} I^2_{sec,i} R_{ext,sec} \tag{23}$$

B plots of Open and Short Circuit Tests (Linear)

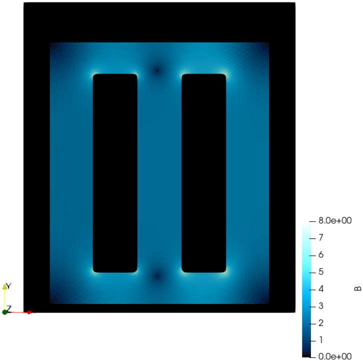


Figure: Open circuit test

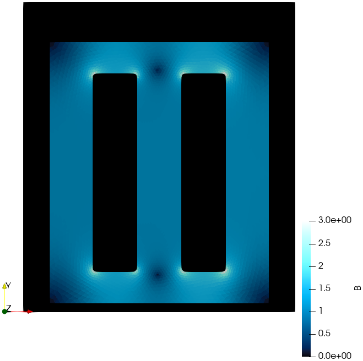


Figure: Short circuit test

- P_{core} = 939.11 W
- $P_{windings,pri}$ = 10263.99 W
- $P_{windings,sec}$ = 11784.38 W

Open Circuit Test (Non-linear)

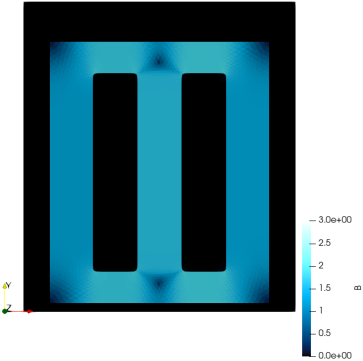


Figure: Magnetic flux density (B)

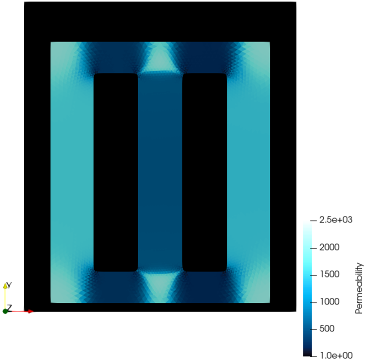


Figure: Relative permeability (μ_r)

- P_{core} = 348.27 W

Short Circuit Test (Non-linear)

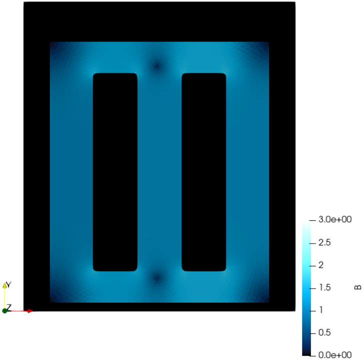


Figure: Magnetic flux density (B)

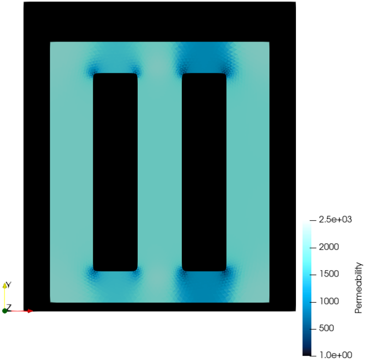


Figure: Relative permeability (μ_r)

- $P_{windings,pri}$ = 11390.72 W
- $P_{windings,sec}$ = 10936.09 W

Finite Element Method : Time Domain

Transient Analysis [9, 10]

- In time-domain,

$$-\nabla \cdot \left[\frac{1}{\mu} \nabla u_z \right] + \sigma \frac{\partial u_z}{\partial t} = J_0 \tag{24}$$

- Writing in matrix form,

$$Au + C \frac{\partial u}{\partial t} = f \tag{25}$$

- Using Backward Euler method for time-stepping,

$$Au^{t+\Delta t} + C \frac{u^{t+\Delta t} - u^t}{\Delta t} = f^{t+\Delta t} \tag{26}$$

- On simplifying, we get,

$$[C + \Delta t A] u^{t+\Delta t} = C u^t + \Delta t f^t \tag{27}$$

[9] S. V. Kulkarni and S. A. Khaparde (2017). *Transformer engineering: design, technology, and diagnostics*. CRC press

[10] Kulkarni, Shrikrishna V (2020). *Electrical Equipment and Machines: Finite Element Analysis*. https://drive.google.com/file/d/1wiyJuqohQMM8lPlCGI2hVKD1cLq06cTP/view. [Online]

Parameters used in FEM Simulations

- Primary current,
  $I_{pri,a} = 0$
  $I_{pri,b} = 0$
  $I_{pri,c} = 0$
- Secondary current,
  $I_{sec,a} = I_{sec,phase,peak} \sin(\omega t - \frac{2\pi}{3})$
  $I_{sec,b} = I_{sec,phase,peak} \sin(\omega t)$
  $I_{sec,c} = I_{sec,phase,peak} \sin(\omega t + \frac{2\pi}{3})$
- Initial relative permeability $\mu_r = 2500$
- Conductivity of core $\sigma_{core} = 0$

Transient Analysis

- Plots are shown for element present at the center of central limb.

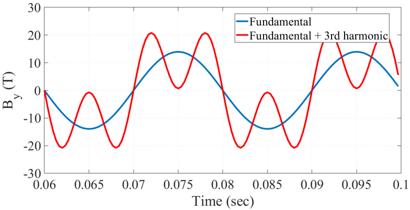


Figure: Linear analysis

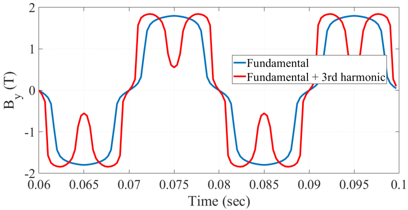


Figure: Non-linear analysis

FFT of Transient Analysis

- Plots are shown for element present at the center of central limb.
- Increase in harmonics due to non-linearity of the core.

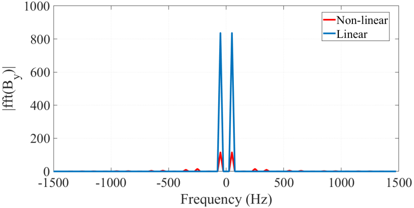


Figure: FFT of B_y with only fundamental component present

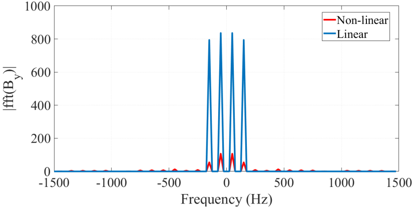


Figure: FFT of B_y in presence of third harmonic component

Specific Mesh Data Points

- Mesh node end element numbers for significant locations :

Sr. No.	Location	Node	Element
1	Center of central limb	4290	7014
2	Center of left limb	4781	6043
3	Center of right limb	4028	6000
4	Top right horizontal area	4650	4778

Transient Analysis

- Plots are shown for elements present at various locations in core.

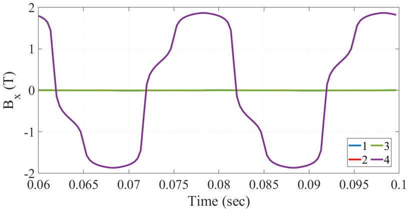


Figure: B_x

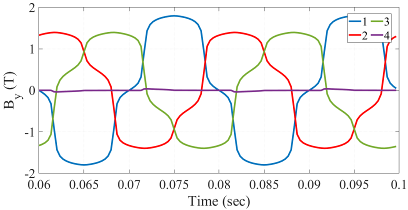


Figure: B_y

Transient Voltage-fed Couple Circuit Field Analysis [11, 12]

- General FEM equation,

$$Au + C\frac{\partial u}{\partial t} = f \cdot I \tag{28}$$

- Circuit equation,

$$V = G'\frac{\partial u}{\partial t} + R_{ext}I + L_{ext}\frac{\partial I}{\partial t} \tag{29}$$

- Combining and writing in time-stepping format,

$$\begin{bmatrix} \frac{C}{\Delta t} + A & -f \\ \frac{G^T}{\Delta t} & R_{ext} + \frac{L_{ext}}{\Delta t} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t+\Delta t} = \begin{bmatrix} \frac{C}{\Delta t} & 0 \\ \frac{G^T}{\Delta t} & \frac{L_{ext}}{\Delta t} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t} + \begin{bmatrix} 0 \\ V^{t+\Delta t} \end{bmatrix} \tag{30}$$

- Simplifying,

$$\begin{bmatrix} C + \Delta tA & -\Delta tf \\ G^T & \Delta tR_{ext} + L_{ext} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t+\Delta t} = \begin{bmatrix} C & 0 \\ G^T & L_{ext} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t} + \begin{bmatrix} 0 \\ \Delta tV^{t+\Delta t} \end{bmatrix} \tag{31}$$

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[11] S. V. Kulkarni and S. A. Khaparde (2017). *Transformer engineering: design, technology, and diagnostics*. CRC press

[12] Kulkarni, Shrikrishna V (2020). *Electrical Equipment and Machines: Finite Element Analysis*. https://drive.google.com/file/d/1wiyJuqohQMM8lPlCGI2hVKD1cLq06cTP/view. [Online]

Parameters used in FEM Simulations

- Open Circuit Test :
  - No primary coils present (Open circuit)
  - $V_{sec,a} = V_{sec,phase,peak} \sin(\omega t - \frac{2\pi}{3})$, $V_{sec,b} = V_{sec,phase,peak} \sin(\omega t)$, $V_{sec,c} = V_{sec,phase,peak} \sin(\omega t + \frac{2\pi}{3})$
- Short Circuit Test :
  - $V_{pri,a} = V_{pri,phase,peak} \sin(\omega t - \frac{2\pi}{3})$, $V_{pri,b} = V_{pri,phase,peak} \sin(\omega t)$, $V_{pri,c} = V_{pri,phase,peak} \sin(\omega t + \frac{2\pi}{3})$
  - $V_{sec,a} = 0$, $V_{sec,b} = 0$, $V_{sec,c} = 0$
- In case of presence of third harmonic component, another term given below is added to above equations,

$$V_{third\_harmonic} = \frac{V_{fundamental}}{3} \sin(3\omega t + \phi_3) \tag{32}$$

where, $\phi_3 =$ phase shift. In this study, $\phi_3 = \frac{\pi}{3}$.

- Similar terms can be added for other harmonics of 5, 7, 9, ...

Transient Voltage-fed Couple Circuit Field Analysis

- Plots are shown for element present at the center of central limb.

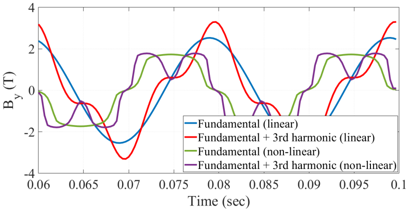

Figure: B_y (Open circuit test)

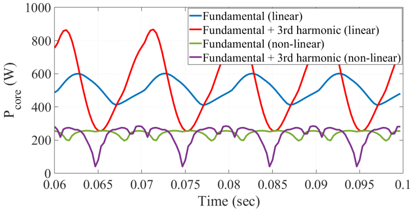

Figure: P_{core} (Open circuit test)

Transient Voltage-fed Couple Circuit Field Analysis

- Plots are shown for element present at the center of central limb.

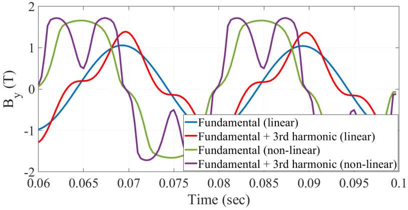

Figure: B_y (Short circuit test)

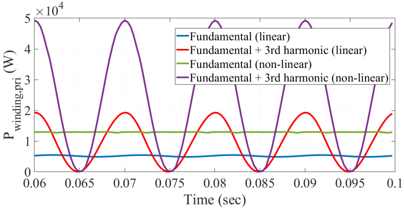

Figure: $P_{winding,pri}$ (Short circuit test)

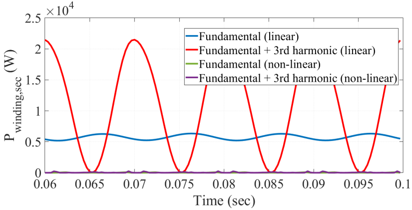

Figure: $P_{winding,sec}$ (Short circuit test)

FFT of Transient Voltage-fed Couple Circuit Field Analysis

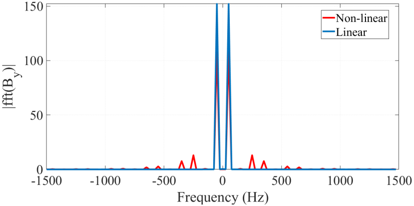

Figure: FFT of B_y with only fundamental component present (Open circuit test)

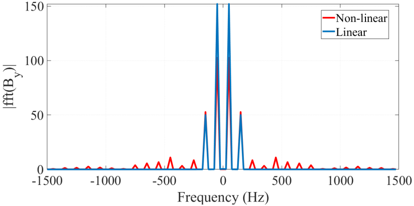

Figure: FFT of B_y in presence of third harmonic component (Open circuit test)

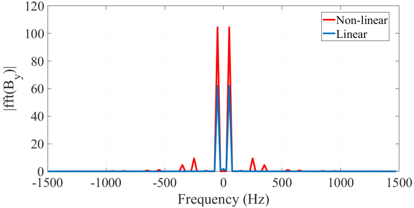

Figure: FFT of B_y with only fundamental component present (Short circuit test)

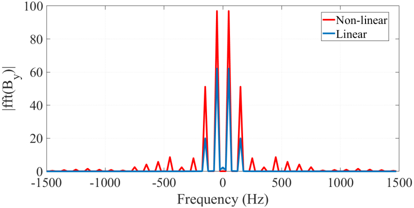

Figure: FFT of B_y in presence of third harmonic component (Short circuit test)

Julia

Upgrades in Julia

- Struct to store all nodes, elements, area values and Emat matrices
- LU decomposition for A matrix in $A\backslash f$ using $factorise()$ function
- Damping factor in non-linear calculations
- FastSparseMatrix
- StaticArrays
- Makie plots
- VTK files

Appendix

Calculation using EMF Equation of Transformer

- EMF Equation of transformer :

$$V_{rms} = 4.44 B_{peak} ANf \tag{33}$$

- For primary winding,
  $V_{rms} = 10750$ V, $N = 266$, $A = 17 \times 10^{-2}$ m^2,

$$B_{peak} = 1.0710\,T \tag{34}$$

- For secondary winding,
  $V_{rms} = 242.49$ V, $N = 6$, $A = 17 \times 10^{-2}$ m^2,

$$B_{peak} = 1.0710\,T \tag{35}$$

- Calculations are done for core depth of 1m.

Thank You