Finite Element Method : Future Distribution Grids

# Contents

# Chapter 1

# STEDIN Transformer Parameters

This chapter contains transformer parameters as given in [1].

## 1.1 Power Rating

**Power rating** = 400 kVA

## 1.2 Line to Line Parameters

**Primary voltage,** $V_{pri,line,rms} = 10750$ V
**Secondary voltage,** $V_{sec,line,rms} = 420$ V
**Primary current,** $I_{pri,line,rms} = 21.48$ A
**Secondary current,** $I_{sec,line,rms} = 549.86$ A

## 1.3 Phase to Phase Parameters

Considering a delta-wye connected transformer,
**Primary voltage,** $V_{pri,phase,rms} = 10750$ V
**Secondary voltage,** $V_{sec,phase,rms} = 242.49$ V
**Primary current,** $I_{pri,phase,rms} = 12.40$ A
**Secondary current,** $I_{sec,phase,rms} = 549.86$ A

Corresponding peak values will be,
**Primary voltage,** $V_{pri,phase,peak} = 15202.40$ V
**Secondary voltage,** $V_{sec,phase,peak} = 342.93$ V
**Primary current,** $I_{pri,phase,peak} = 17.54$ A
**Secondary current,** $I_{sec,phase,peak} = 777.62$ A

## 1.4 Turns Ratio

**Primary turns,** $N_{pri} = 266$
**Secondary turns,** $N_{sec} = 6$

## 1.5 Core Parameters

The values are as given in Section 2.3.2 of [2] for grain-oriented silicon steel (Fe-(3 wt%)Si alloys).
**Electrical Resistivity** = $45 \times 10^{-8}$ Ωm
**Electrical Conductivity** = $2.22 \times 10^{6}$ $\Omega^{-1}\mathrm{m}^{-1}$
**Saturation Polarization** = 2.03 T
**Maximum Relative Permeability** = $8 \times 10^{4}$
**Density** = $7.65 \times 10^{3}$ kg/m$^3$
**50 Hz Power Loss at 1.4 T** = 0.6 W/kg
**50 Hz Power Loss at 1.7 T** = 1.21 W/kg

# Chapter 2

# Magnetic Field Simulations

## 2.1 General Expression for FEM

$$\nabla \times \left[\frac{1}{\mu}\nabla \times u\right] = J_0 + J_c \tag{2.1}$$

where,
$J_0$ = source current density,
$J_c$ = conduction current density.

Since, current flows along the z-axis and the geometry is in xy plane,

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla u_z\right] = J_0 + J_c \tag{2.2}$$

In frequency domain,

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla u_z\right] + j\omega\sigma u_z = J_0 \tag{2.3}$$

In time domain,

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla u_z\right] + \sigma\frac{\partial u_z}{\partial t} = J_0 \tag{2.4}$$

## 2.2 Without Eddy Currents

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla u_z\right] = J_0 \tag{2.5}$$

## 2.3 With Eddy Currents

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla u_z\right] + j\omega\sigma u_z = J_0 \tag{2.6}$$

## 2.4 Non-Linear Characteristics of Core

The non-linear magnetic characteristic with hysteresis neglected can be approximated as given in [3].

$$\frac{1}{\mu} = k_1 e^{k_2 B^2} + k_3 \tag{2.7}$$

where, $k_1$, $k_2$ and $k_3$ are constants equal to 3.8, 2.17, and 396.2, respectively.

## 2.5 Voltage-fed Couple Circuit Field Analysis

In frequency domain,

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla u_z\right] + j\omega\sigma u_z = J_0 = \frac{NI}{S} \tag{2.8}$$

where,
$N$ = number of turns,
$S$ = cross-sectional area.

Circuit equation for an R-L equivalent network,

$$V = j\omega N\psi + R_{ext}I + j\omega L_{ext}I \tag{2.9}$$

$$V = j\omega G' u + R_{ext}I + j\omega L_{ext}I \tag{2.10}$$

Combining the diffusion equation and circuit equation,

$$\begin{bmatrix} A & -f \\ j\omega G^T & R_{ext} + j\omega L_{ext} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix} = \begin{bmatrix} 0 \\ V \end{bmatrix} \tag{2.11}$$

where, $V$ = known voltages, and

$$G = f = \frac{N}{S} \times \frac{area\ of\ element}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

**Note :** The above formulation is only valid in case of per unit z-direction axis.

## 2.6 Transient Analysis

In time-domain,

$$-\nabla \cdot \left[\frac{1}{\mu}\nabla u_z\right] + \sigma\frac{\partial u_z}{\partial t} = J_0 \tag{2.12}$$

Writing in matrix form,

$$Au + B\frac{\partial u}{\partial t} = f \tag{2.13}$$

Using Backward Euler method for time-stepping,

$$Au^{t+\Delta t} + B\frac{u^{t+\Delta t} - u^t}{\Delta t} = f^{t+\Delta t} \tag{2.14}$$

On simplifying, we get,

$$[B + \Delta t A]u^{t+\Delta t} = Bu^t + \Delta t f^t \tag{2.15}$$

## 2.7 Transient + Voltage-fed Couple Circuit Field Analysis

### 2.7.1 Neglecting eddy current term

General FEM equation,

$$Au = f \tag{2.16}$$

Circuit equation,

$$V = G' \frac{\partial u}{\partial t} + R_{ext} I + L_{ext} \frac{\partial I}{\partial t} \tag{2.17}$$

Combining and writing in time-stepping format,

$$\begin{bmatrix} A & -f \\ \frac{G^T}{\Delta t} & R_{ext} + \frac{L_{ext}}{\Delta t} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t+\Delta t} = \begin{bmatrix} 0 & 0 \\ \frac{G^T}{\Delta t} & \frac{L_{ext}}{\Delta t} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t} + \begin{bmatrix} 0 \\ V^{t+\Delta t} \end{bmatrix} \tag{2.18}$$

Simplifying,

$$\begin{bmatrix} \Delta t A & -\Delta t f \\ G^T & \Delta t R_{ext} + L_{ext} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t+\Delta t} = \begin{bmatrix} 0 & 0 \\ G^T & L_{ext} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t} + \begin{bmatrix} 0 \\ \Delta t V^{t+\Delta t} \end{bmatrix} \tag{2.19}$$

### 2.7.2 Considering eddy current term

General FEM equation,

$$Au + B \frac{\partial u}{\partial t} = f \tag{2.20}$$

Circuit equation,

$$V = G' \frac{\partial u}{\partial t} + R_{ext} I + L_{ext} \frac{\partial I}{\partial t} \tag{2.21}$$

Combining and writing in time-stepping format,

$$\begin{bmatrix} \frac{B}{\Delta t} + A & -f \\ \frac{G^T}{\Delta t} & R_{ext} + \frac{L_{ext}}{\Delta t} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t+\Delta t} = \begin{bmatrix} \frac{B}{\Delta t} & 0 \\ \frac{G^T}{\Delta t} & \frac{L_{ext}}{\Delta t} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t} + \begin{bmatrix} 0 \\ V^{t+\Delta t} \end{bmatrix} \tag{2.22}$$

Simplifying,

$$\begin{bmatrix} B + \Delta t A & -\Delta t f \\ G^T & \Delta t R_{ext} + L_{ext} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t+\Delta t} = \begin{bmatrix} B & 0 \\ G^T & L_{ext} \end{bmatrix} \begin{bmatrix} u \\ I \end{bmatrix}^{t} + \begin{bmatrix} 0 \\ \Delta t V^{t+\Delta t} \end{bmatrix} \tag{2.23}$$

# Chapter 3

# FEM Results

**Mesh nodes and numbers**

1. 4276 - center of the central limb
2. 4793 - center of the left limb
3. 4019 - center of the right limb
4. 4080 - top right of the core
5. 4372 - top center of the core
6. 4651 - top right horizontal area of the core

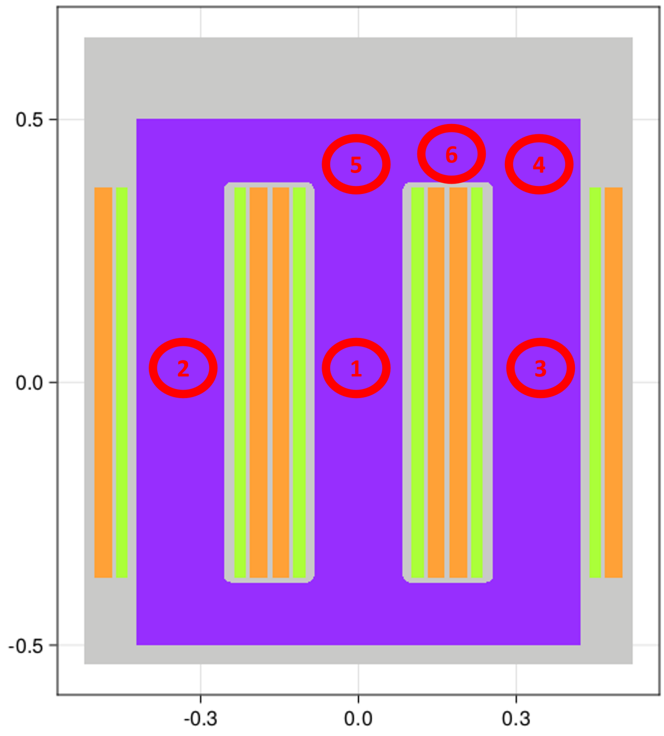


Figure 3.1: Node locations

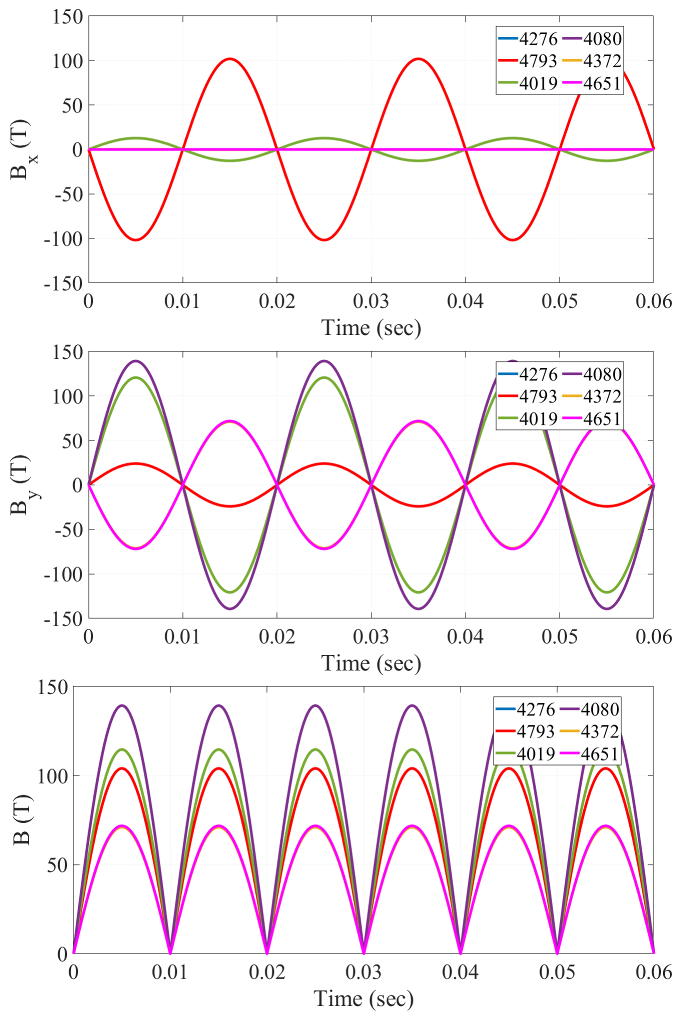


Figure 3.2: Three phase source with harmonics : B curves

## 3.1 Transient Analysis

This section deals with results using current density without harmonics as $J_0 \sin(\omega t)$.

Harmonics are given as $J_0 \sin(\omega t) + (J_0/3) J_0 \sin(3\omega t)$.

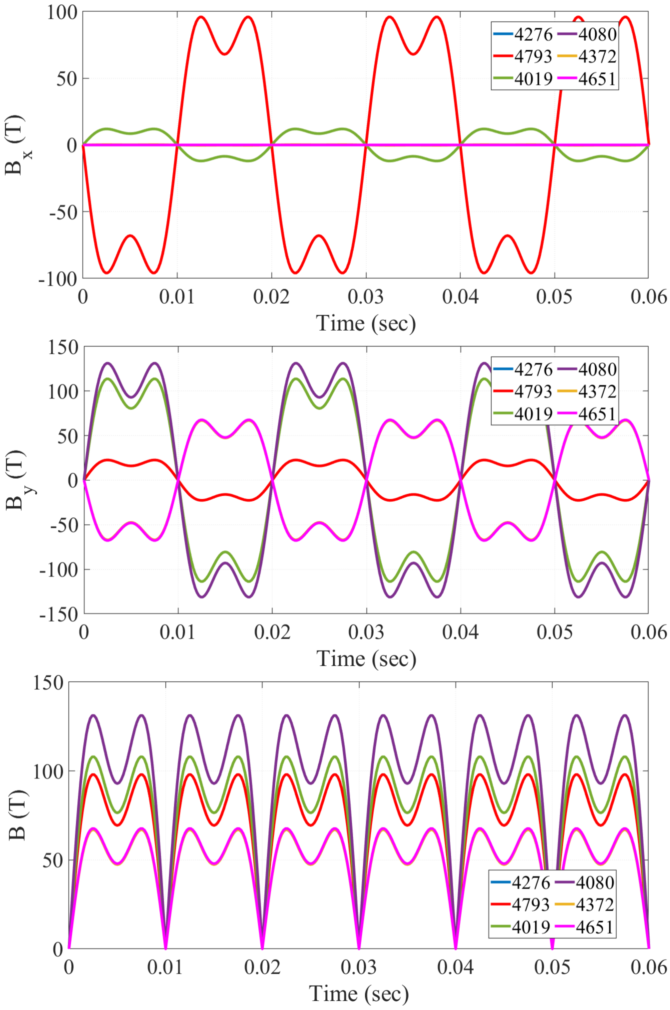


Figure 3.3: Three phase source with harmonics : B curves

# Chapter 4

# Things to do

## 4.1 Report by Auke and Philip

- check value of the amplitude of the applied current density
- check values of (in-plane) magnetic permeability (linear and non-linear) and (out-of-plane) electrical conductivity. Take laminanations into account, values for steel-ony are likely not representative;
- FFT of time traces of the magnetic flux - do higher order harmonics appear in the non-linear case? What happens in the non-linear in case that the applied current has a frequency of 150 Hz. Is a 50 Hz component in the potential and fields generated?
- other items?

## 4.2 Skin depth consideration

Extend computation of skin-depth delta with value of Bmax in the thin layer. Model using dsolve? Show that higher frequency cause material to go into saturation.

## 4.3 Extend to voltage driven winding by adding 2*3 = 6 electrical circuit equations

- What element to add in the circuit equations for the primary and secondary winding to simulate relevant scenarios (no load and short circuit)
- How to perform time-integration in the linear case? How to extend to the non-linear case

## 4.4 Extend to second order elements to reach same spatial accuracy using less degree of freedom?

## 4.5 Optimize Julia code further

- employ structarrays.jl to arrive at type-stable and thus more performant implementation. See e.g. Table 4 in https://www.sciencedirect.com/science/article/pii/S0965
- employ staticarrays.jl to represent local identity matrix, local vector and local matrix

## 4.6 Extend to harmonic balance method

Verify computations in time-domain by frequency domain approach. In linear case at single frequency. In non-linear case by taking multiple frequencies and their couplings into account.

## 4.7 Extend post-processing with copper and iron losses.

## 4.8 Work done thus far using Comsol Multiphyisics

- check what the effective BH curve is;

# References


[1] M. van Dijk, “A theoretical approach towards digital twins: A balance between an empirical and a fundamental model for distribution transformers,” *Delft University of Technology*, 2022.

[2] F. Fiorillo, *Characterization and measurement of magnetic materials*. Academic Press, 2004.

[3] J. Brauer, “Simple equations for the magnetization and reluctivity curves of steel,” *IEEE Transactions on Magnetics*, vol. 11, no. 1, pp. 81–81, 1975.