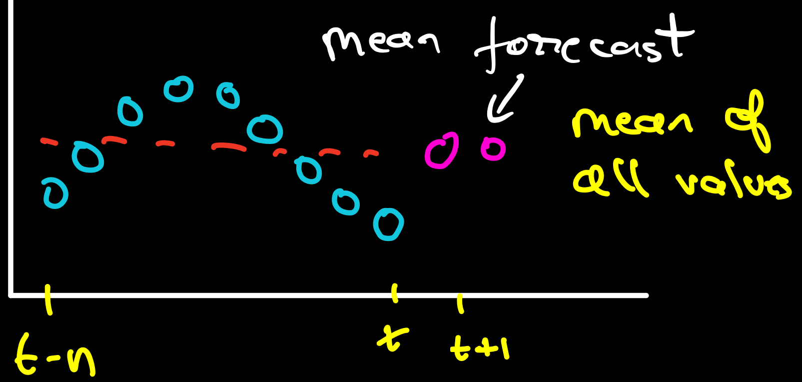
Time Series Forecasting - 2

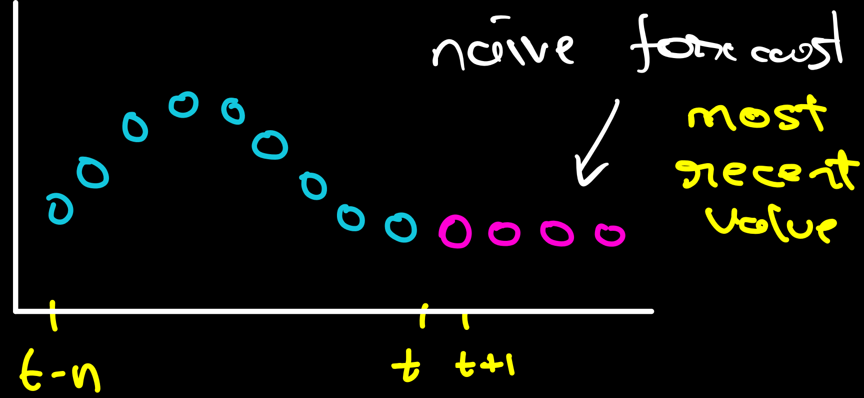
- → Decomposition from scratch
- → Simple Methods for forecasting
- → MA for forecasting
- → Smoothing Methods

Simple Forecasts

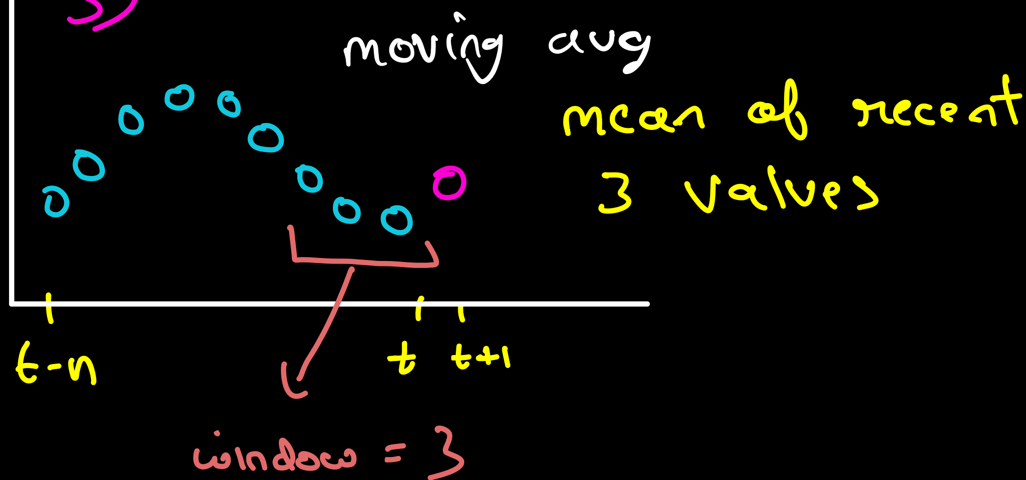
1)

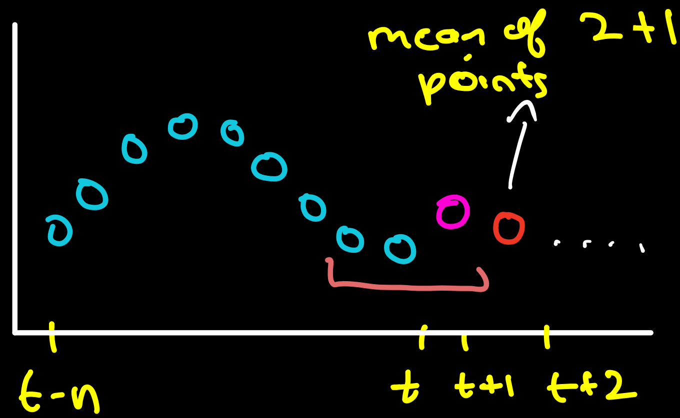


2)

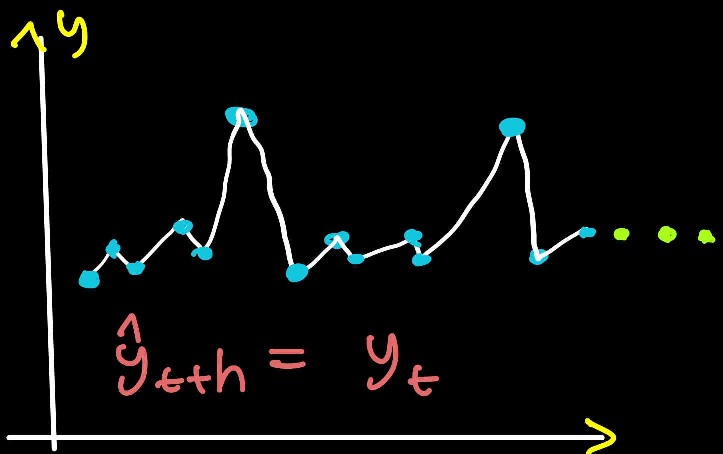


3)



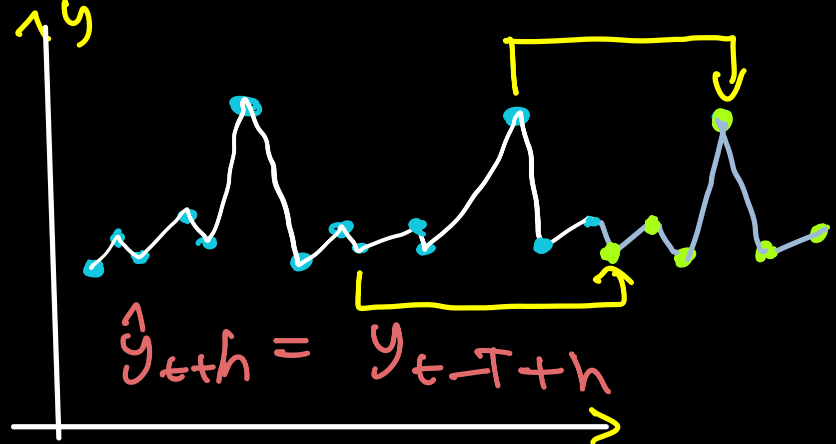


4) Seasonal Naive



$$\hat{y}_{t+h} = y_t$$

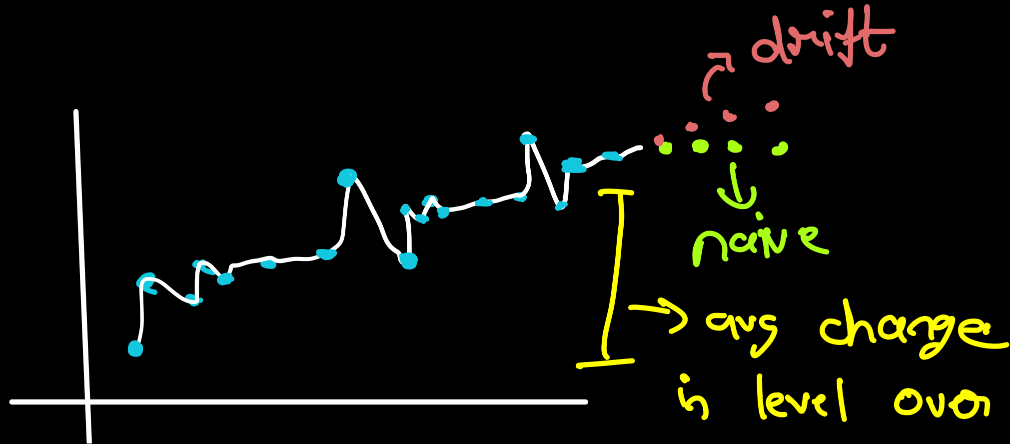
Naive Forecast



$$\hat{y}_{t+h} = y_{t-T+h}$$

Seasonal Naive

5) Drift



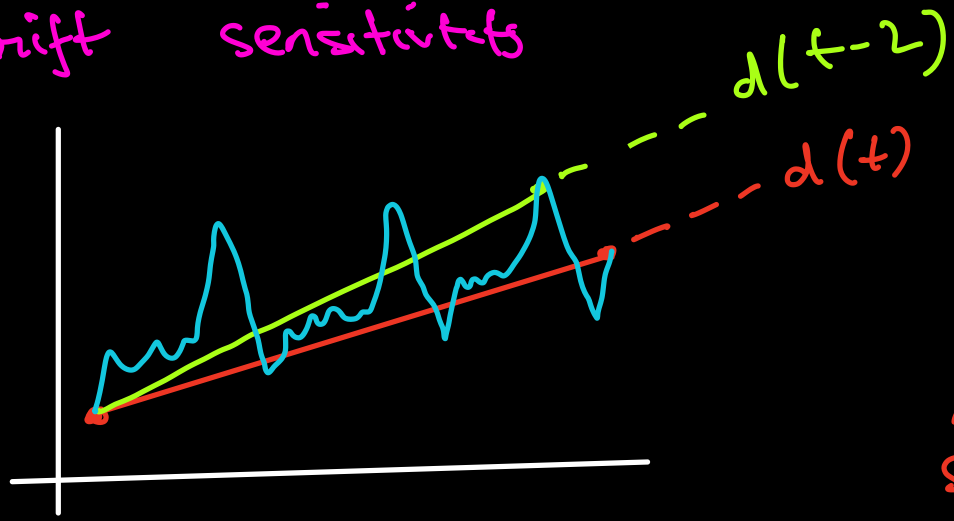
$$\hat{y}_{t+h} = y_t + h\,(\text{slope})$$

\# of in future

current level

$$\text{slope} = \frac{y_t - y_0}{t}$$

drift sensitivity



Depending on what the last point is, the drift (slope) may change significantly

Not suitable for seasonal TS.

Simple exponential smoothing (SES)

h = horizon = # steps in future

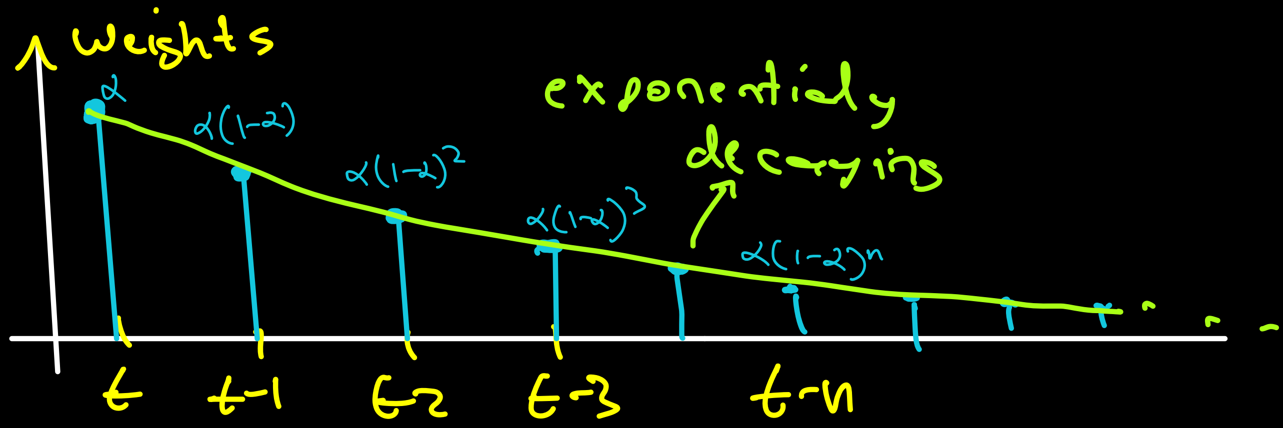
assume h=1 for easy understanding

$$\hat{y}_{t+h} = \alpha y_t + (1-\alpha)\hat{y}_t$$

$$\hat{y}_{t+h} = \alpha y_t + (1-\alpha)\left[\alpha y_{t-1} + (1-\alpha)\hat{y}_{t-1}\right]$$

$$= \underbrace{\alpha}_{\text{weigh}} \; \underbrace{y_t}_{\text{last value}} + \underbrace{(1-\alpha).\alpha.}_{\text{less weigh}} \; \underbrace{y_{t-1}}_{\text{old value}} + \underbrace{(1-\alpha)^2}_{\text{lesser weight..}} \; \underbrace{\left[\alpha y_{t-1} \cdots\right]}_{\text{older values}}$$

It will slowly "forget" older values



Simple way to remember:

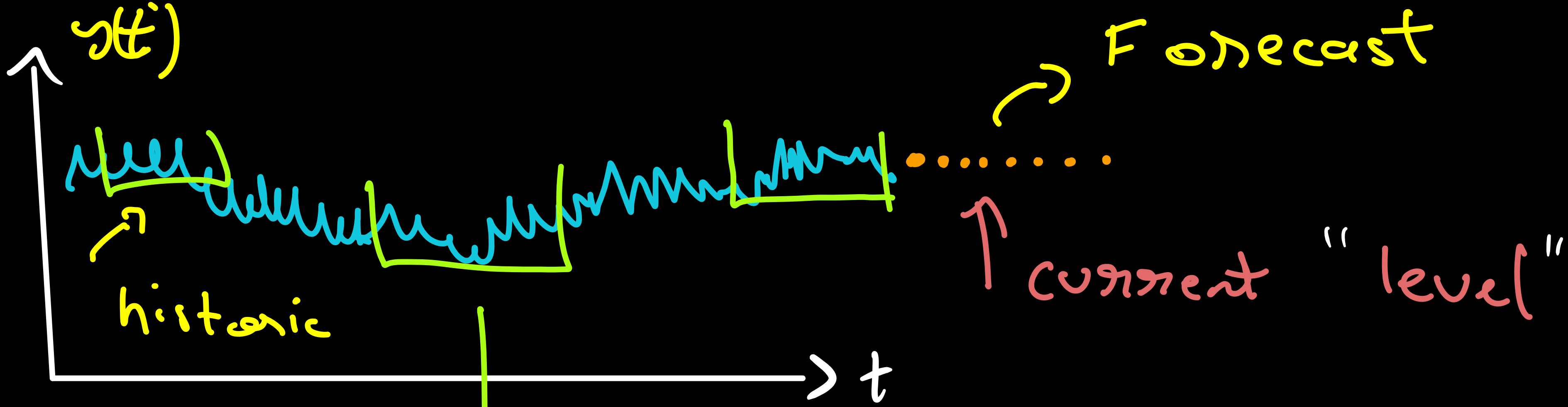
$$\hat{y}_{t+1} = \alpha y_t + \alpha(1-\alpha) y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \dots + \underbrace{\alpha(1-\alpha)^t y_0}_{\text{coeff is too small} \sim \underline{\underline{0}}}$$

if $\alpha = 0.8$ ↓

$$\hat{y}_{t+1} = 0.8 y_t + 0.16 y_{t-1} + 0.032 y_{t-2} + 0.0064 y_{t-3} \sim 0$$

↗ weighted average!

Note : All future values have the same forecast



level of TS is different at diff t

SES is good for predicting level of TS

$\alpha \to$ low $\to$ global mean

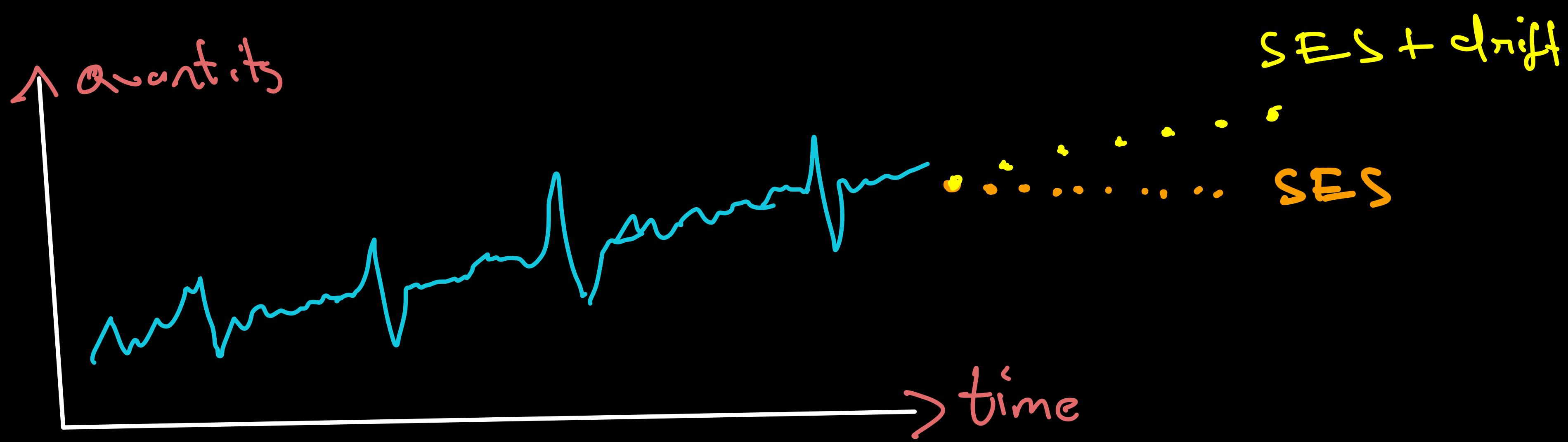
$\alpha \to$ high $\to$ naive

Double exponential smoothing DES

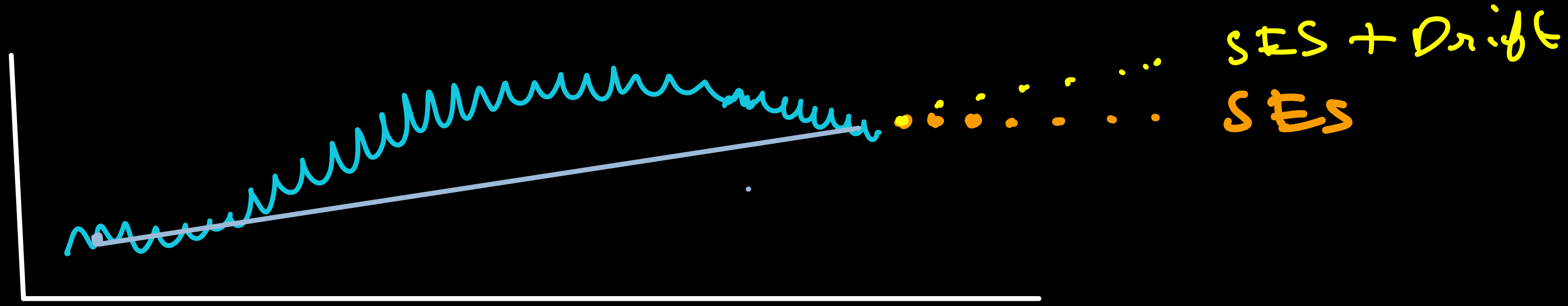
↳ The above method only captures level.
we need trend.

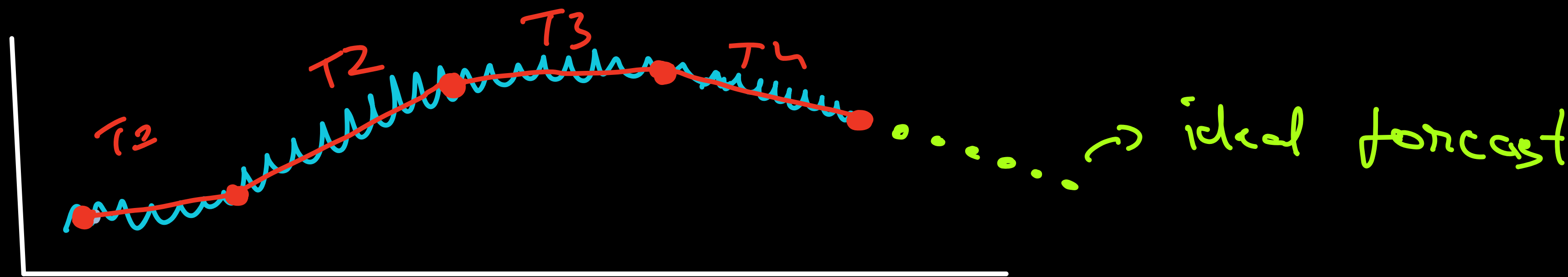
Assume: SES + drift method → DES

naive + mean



But, is the trend always constant?

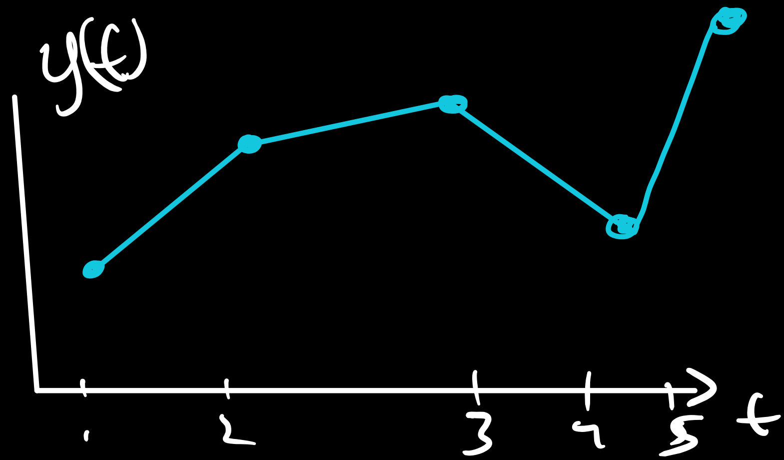




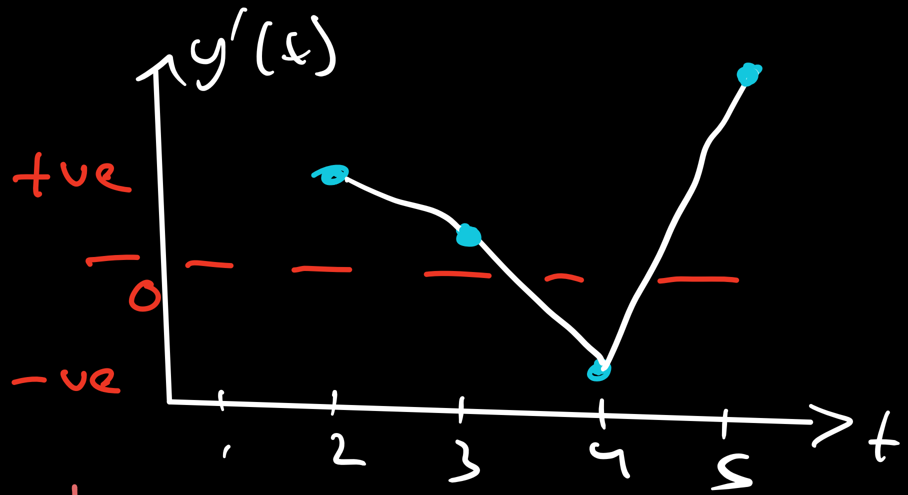
Key idea: Just as we too decaying weighted avg for "level", we should take exponential smoothing on trend too

Trend Revisit → move away from long term idea of trend.

Can I say, trend (instantaneous) = slope = rate of change?



→ Zoom in, slope is diff for every point.



$$y'(t) = \text{slope}(t)$$

$$= \frac{\Delta y}{\Delta t} = \frac{y_t - y_{t-1}}{1}$$

$$= \text{y.diff}()$$

Trend Signal!

Exponentially smooth this signal!

$$\hat{b}_{t+1} = \beta b_t + (1-\beta) \cdot \hat{b}_t$$

predicted rate of change

new hyper param

current rate of change

previous predicted

But we have to do simultaneous recursive calculation for level and trend!

DES = Level + Trend.

current level

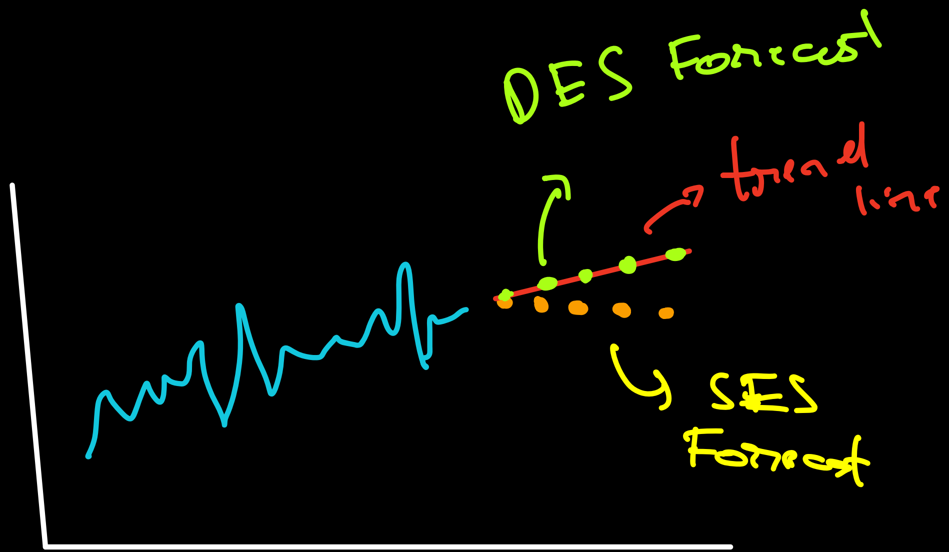
steps in future

$$\hat{y}_{t+h} = l_t + h b_t$$

current trend

$$l_t = \alpha y_t + (1-\alpha)\left[\overbrace{l_{t-1} + b_{t-1}}^{\text{Add increase due to trend}}\right]$$

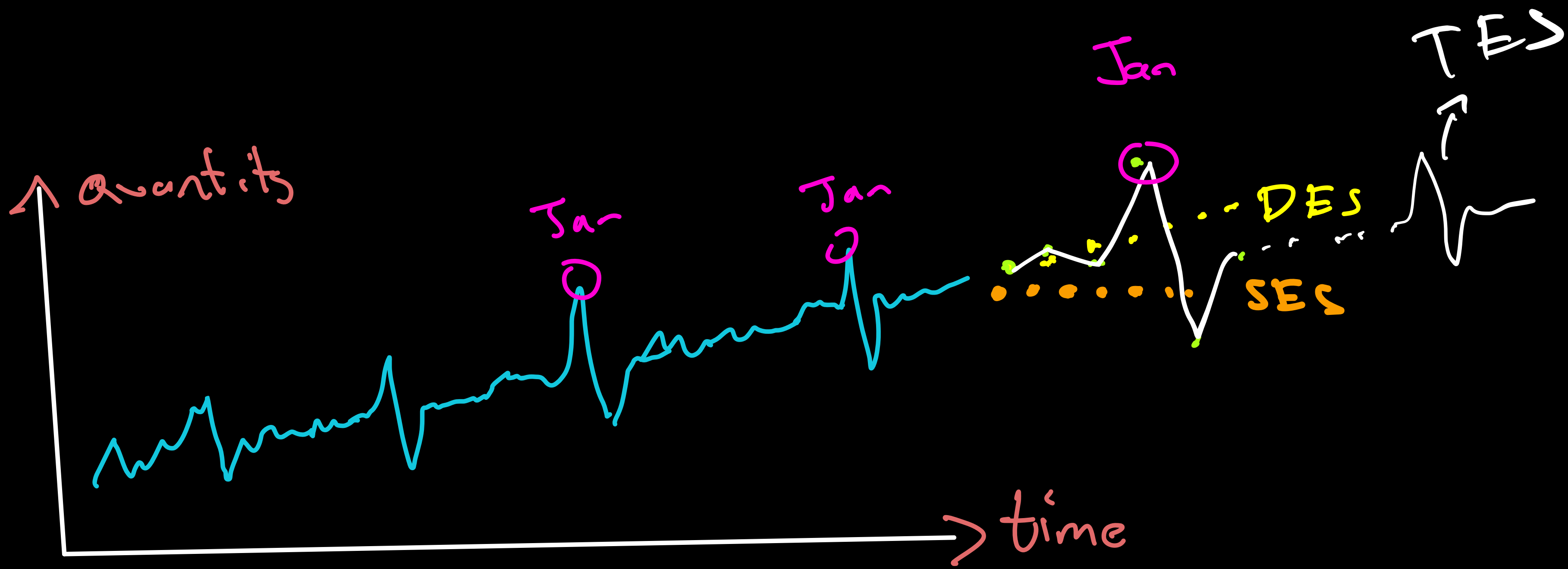
$$b_t = \beta\underbrace{\left[l_t - l_{t-1}\right]}_{\text{current actual slope}} + (1-\beta)\underbrace{b_{t-1}}_{\text{previous smoothed slope}}$$



Tripple Exponential Smoothing (TES)

Holt-Winter's method.

Q: what's left? → Seasonality



$Jan(2023) = \gamma\, Jan\,22 + \gamma(1-\gamma)\, Jan\,21 + \gamma(1-\gamma)^2\, Jan\,20.$

→ decaying weighted avg of seasonal value

Simultaneous recursive formulation:

1) $\hat{y}_{t+h} = l_t + h b_t + S_{t+h-m}$ $\longrightarrow$ m = seasonality
(assume, $h < m$) eg: monthly data, $m = \underline{12}$

S_{t+h-m} :

t = Jan 23		$t = t$
$t+h$ = Apr 23		$h = 3$
$t+h-m$ = Apr 22		$m = 12$

$\downarrow$

if you want to predict Apr 23, what is the decaying weighted avg until Apr 22 ?

$$l_t = \alpha\left[y_t - \underbrace{S_{t-m}}_{\text{Subtract effect of seasonal variation}}\right] + (1-\alpha)\left[l_{t-1} + \underbrace{b_{t-1}}_{\text{Add effect of trend}}\right]$$

$$b_t = \beta\left[l_t - l_{t-1}\right] + (1-\beta)\, b_{t-1}$$

$$S_t = \gamma\left[y_t - \underbrace{l_{t-1} - b_{t-1}}_{\text{Subtract level and trend from y; you get current season value}}\right] + (1-\gamma)\underbrace{S_{t-m}}_{\text{past season value}}$$

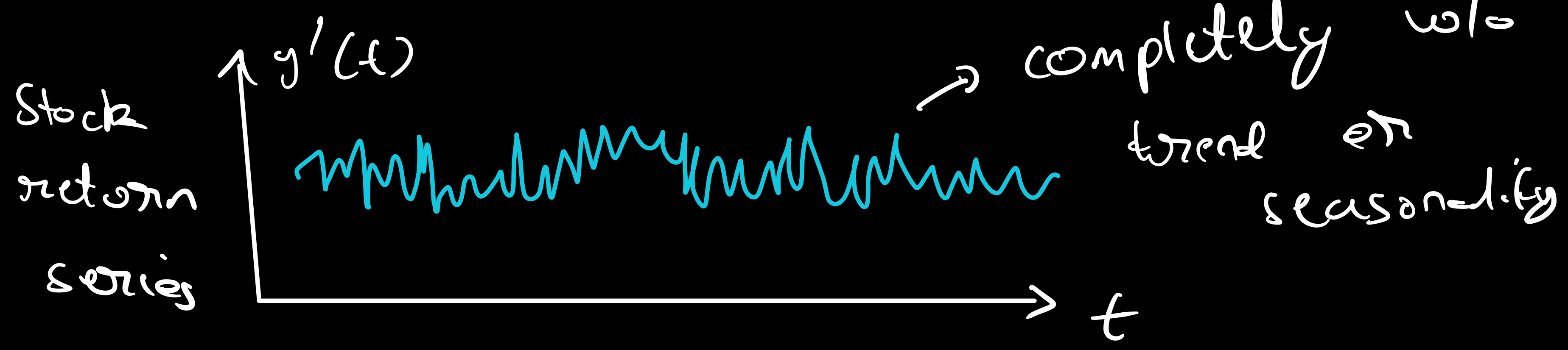
Multiplicative formulations [Not imp]

$$
\begin{aligned}
\hat{y}_{t+h|t} &= (\ell_t + hb_t)s_{t+h-m(k+1)} \\
\ell_t &= \alpha \frac{y_t}{s_{t-m}} + (1-\alpha)(\ell_{t-1} + b_{t-1}) \\
b_t &= \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1} \\
s_t &= \gamma \frac{y_t}{(\ell_{t-1} + b_{t-1})} + (1-\gamma)s_{t-m}
\end{aligned}
$$

For the next family of models we will need to study a new concept.

ARIMA models:

They are used in signals such as financial signals.



Financial analysts like to predict change in stock instead of stock.

→ Diff stocks series looks diff, but return series looks similar

Another application:

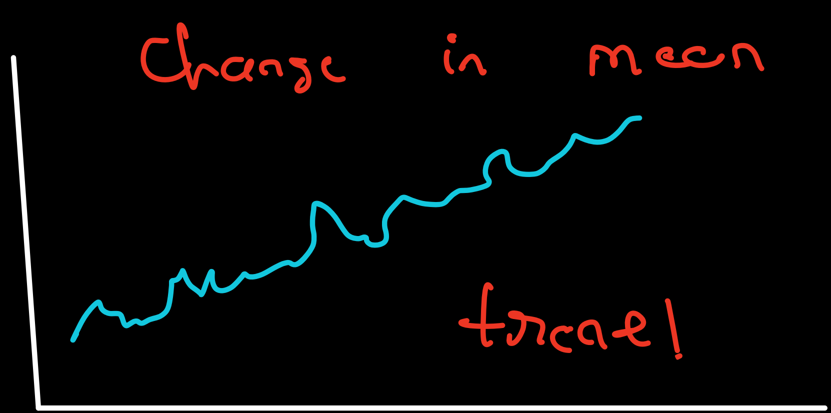
$$y(t) = b(t) + s(t) + \underbrace{e(t)}_{\substack{\text{no trend} \\ \text{no seasonality}}}$$

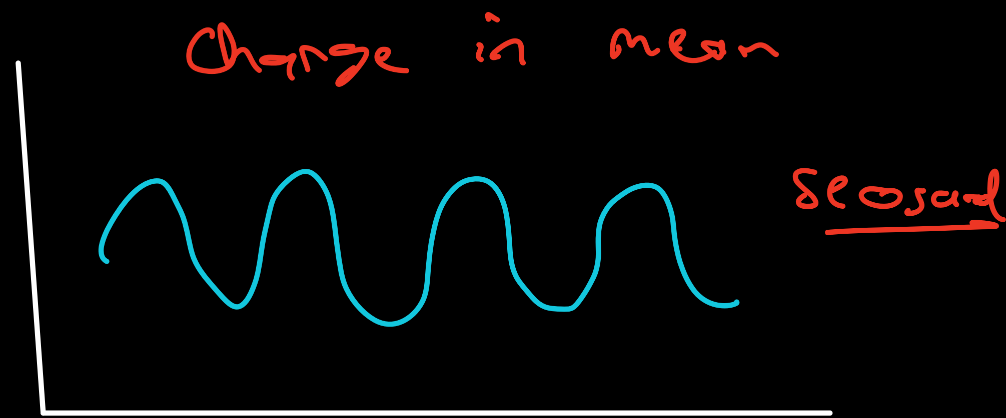
Does it have any predictable information?

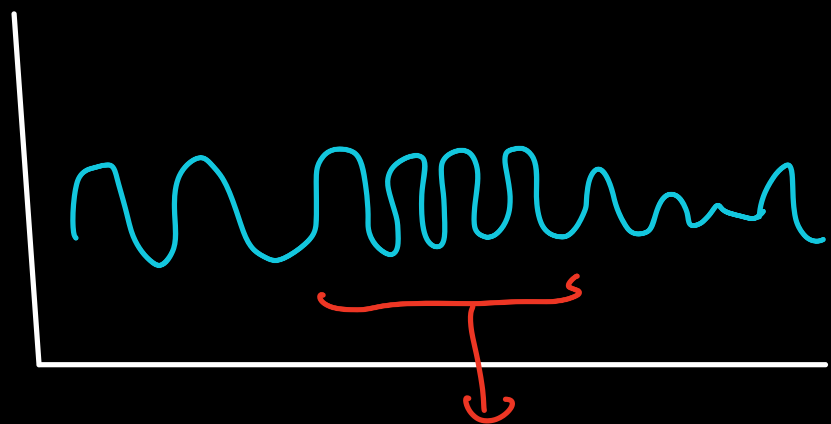
Stationarity

→ A signal is said to be stationary if its parameters such as mean, variance, amplitude, frequency do not change with time.

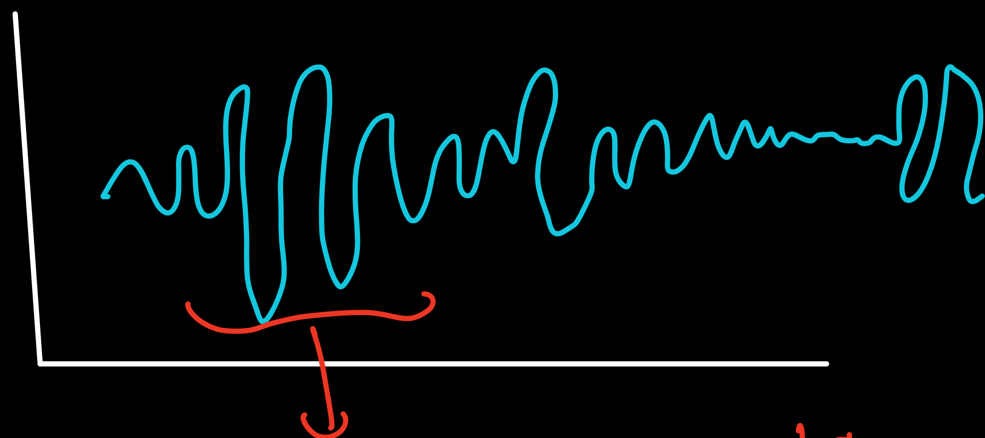
Ex: of non-stationary







change in frequency



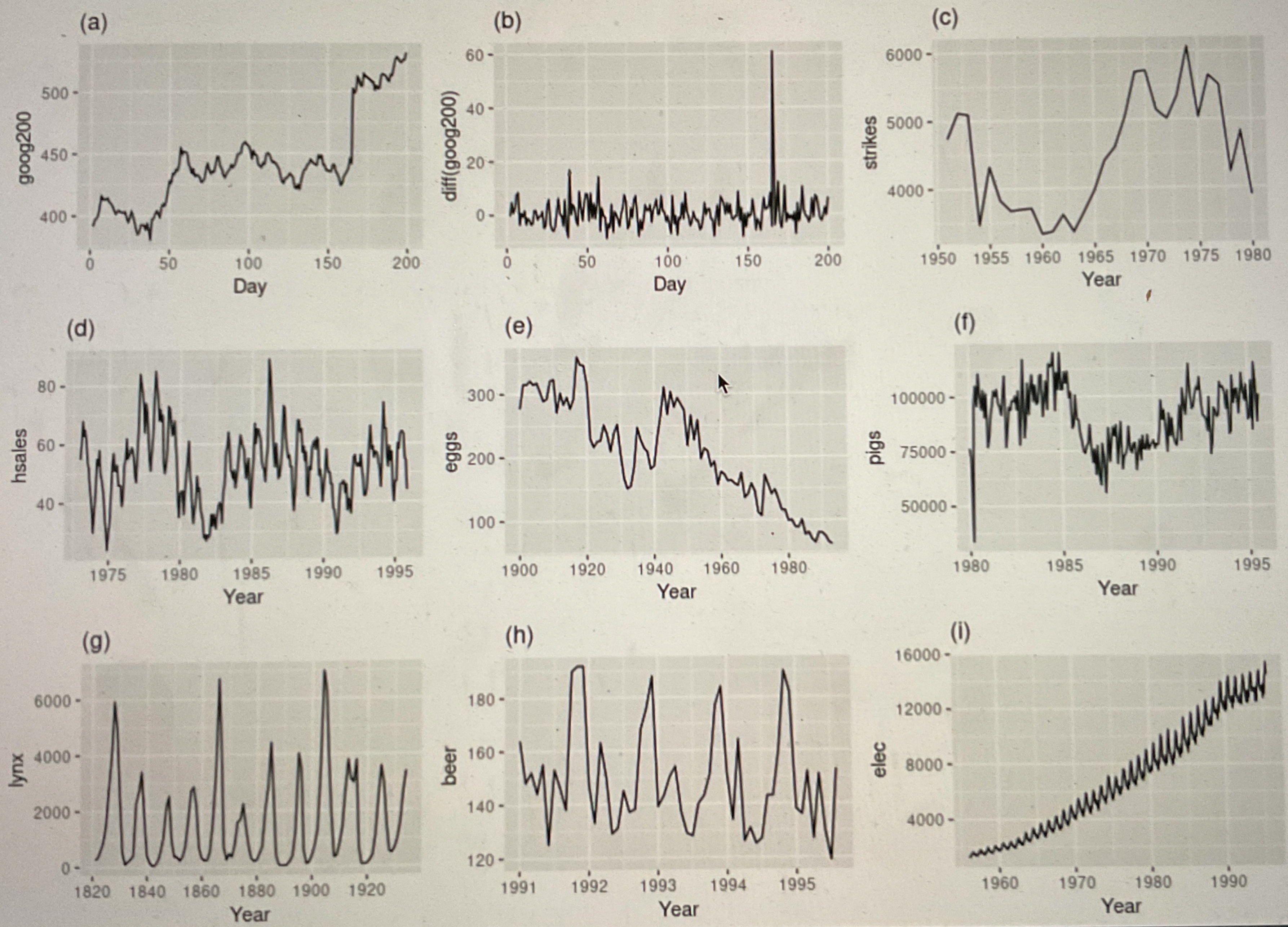
change in amplitude, variance.

How to check for stationarity using code?

→ Dickey-Fuller Test.

if p-value < significance level

→ stationary.


(a)
goog200
Day
(b)
diff(goog200)
Day
(c)
strikes
Year
(d)
hsales
Year
(e)
eggs
Year
(f)
pigs
Year
(g)
lynx
Year
(h)
beer
Year
(i)
elec
Year

Which is not stationary?

- `a, c, e, f, :` Not stationary
  - either have a trend, or
  - mean changing with time.
- `d, h:` Not stationary
  - Seasonality
- `i:` Not stationary
  - Has a trend
  - Variance is also not stable
  - Season
- `b:` Stationary
  - There is 1 outlier
  - Can't say anything about mean; seems to be just noise.
- `g:` Stationary
  - Predicting this is dicey, so we assume it to be stationary, and try building model, and seeing if it performs.
  - Looks like a cyclic time series. But these are not at regular intervals.
  - So, even though there is some seasonality, it can't be predicted.

How to make a signal stationary?

1) Decomposition:

$$y(t) = b(t) + s(t) + \underbrace{e(t)}_{\text{DF test}}$$

$$e(t) = y(t) - [b(t) + s(t)]$$

2) Differencing: [De-trending]

$$y(t) = b(t) + s(t) + e(t)$$

$$y(t) = m^{\times} t + s(t) + e(t)$$

m ↑ slope (assuming linear trend)

$$y'(t) = \underbrace{m}_{\text{no trend any more.}} + s'(t) + \underbrace{e_2(t)}_{\text{diff noise = another noise}}$$

3) m-difference [De-seasonalisation]

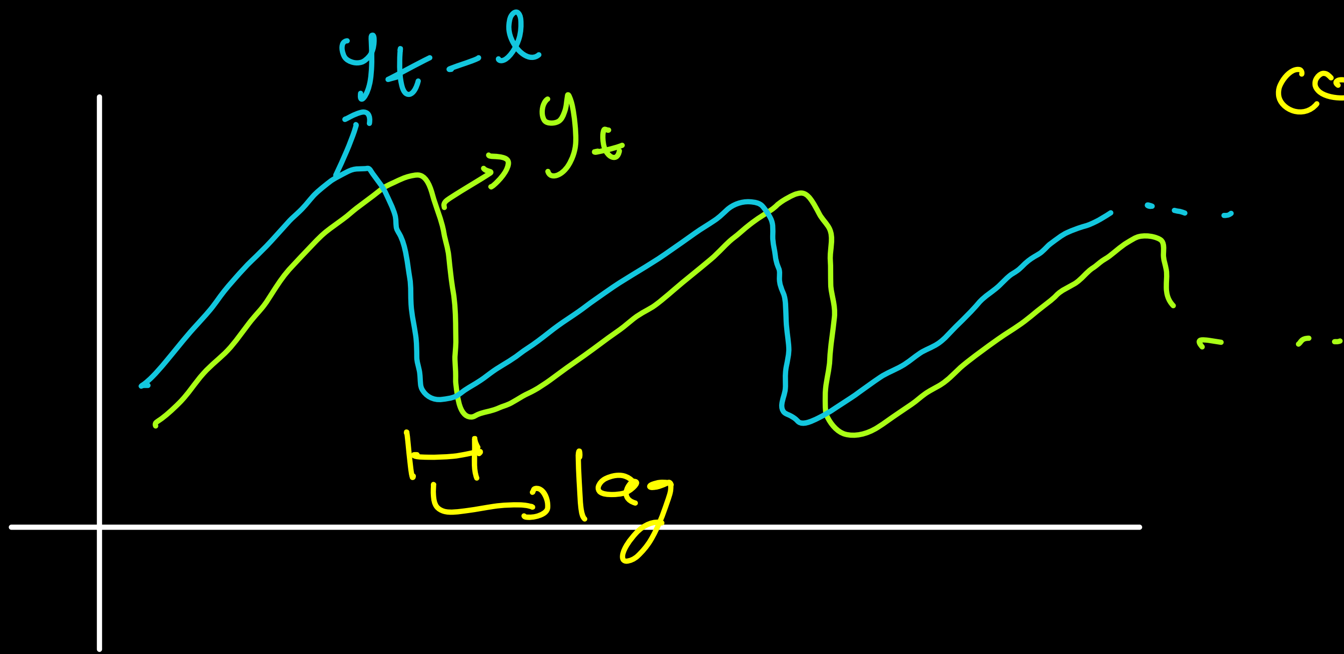
$$y''(t) = m + \underbrace{s(t) - s(t-m)}_{\text{Jan 23 - Jan 22}} + e_3(t)$$

Effect of season will be killed like this.

ACF / PACF

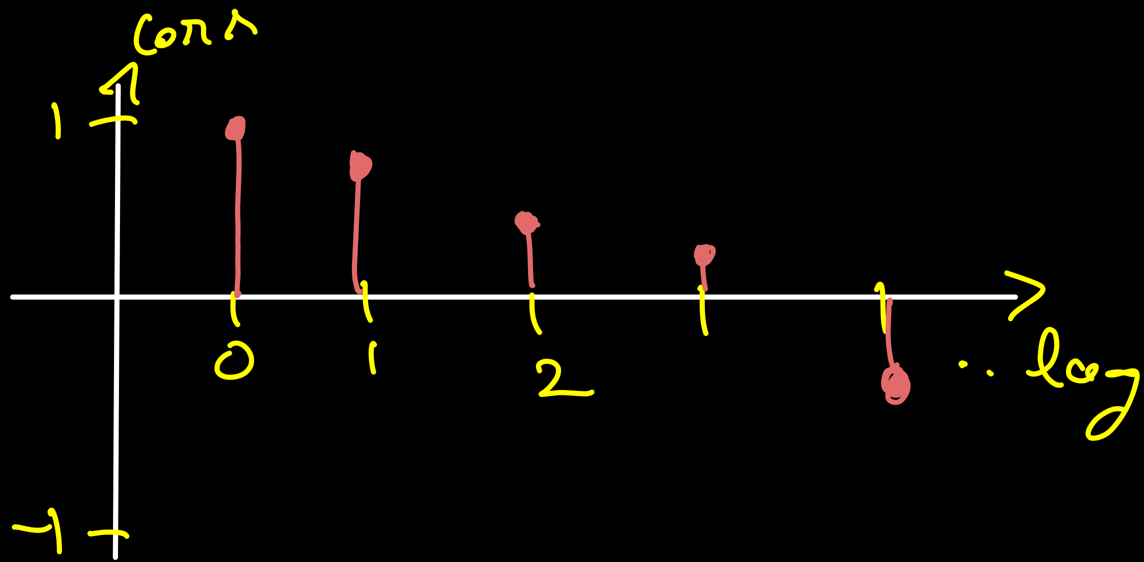
↳ Can I use previous values to forecast future values?

What is the correlation w.r.t past?



$corr(y, y_{t-1})$

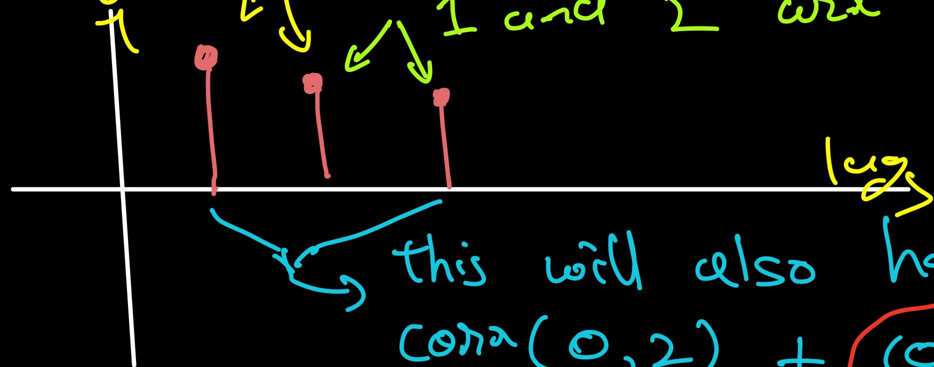
Auto-correlation Function [ACF]



PACF

0 and 1 are correlated

1 and 2 are corr



this will also have

corr(0,2) + corr(0,1) + corr(1,2)

indirect correlation

↓

removed in PACF

Post read for more info