19.08.23 RPET: UNIT 1 - INTRODUCTION TO RANDOM PROCESS.

1. Random process : $\{ X(t), t \in T \}$

CDF : $F_X(x_1, t_1) = (X(t_1) \leq x_1)$

2. $\sim U(a, b)$: uniform distribution

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

$$E(x) = \frac{a+b}{2}$$

$$\text{var}(x) = \frac{(b-a)^2}{12}$$

3. $\sim N(\mu, \sigma^2)$: normal distribution

μ : mean

σ^2 : variance.

4. Auto-correlation : $R_{XX}(t_1, t_2) = E[X(t_1) \cdot X(t_2)]$

Auto-covariance : $C_{XX}(t_1, t_2) = R_{XX}(t_1, t_2) - \mu_X(t_1) \cdot \mu_X(t_2)$

5. Strong Stationary Random Process

$$f_X(t) = f_X(t+\Delta) \;\forall t, t+\Delta$$

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for a function to ~~SSR~~, be SSR,

Weak Sense Stationary : conditions :

i. $E[x(t)] = \text{constant}$

ii. $R_{xx}(t_1, t_2) = R_{xx}(t_1 - t_2)$

ie. should be equal to time difference.

6. ~~Prot~~ Properties : Autocorrelation : WSS

i. $|R_{xx}(\tau)| \leq R_{xx}(0)$

ii. $R_{xx}(\tau) = R_{xx}(-\tau)$: symmetric

iii. $R_{xx}(0) = E[x^2(t)]$

iv. $R_{xx}(\tau)$: no arbitary shape.

v. if $E[x(t)] \neq 0$ and ~~e~~ ergodic

$$\lim_{|\tau| \to \infty} R_{xx}(\tau) = \mu_x^2.$$

7. Ergodicity : conditions : $E[x(t)] = \frac{1}{2\pi}\int_{-T}^{T} x(t)\,d$

---x---

- $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

- $E[x(t)] \neq 0$; and,
  $E[x(t)] = \frac{1}{2\pi}\int_{-T}^{T} x(t)\,\cancel{dt} \Rightarrow$ ergodic.
  then, $\lim_{|\tau| \to \infty} R_{xx}(\tau) = \mu_x^2$.

- ND : $\frac{1}{\sigma(\sqrt{2\pi})} e^{\frac{-(y-\mu)^2}{2\sigma^2}}$

- $\sin(2\pi + \theta) = \sin\theta$
  $\sin(2\pi - \theta) = -\sin\theta$

- $\sin\left(\frac{\pi}{2} + \theta\right) = +\cos\theta$.

- $\sin\left(\frac{\pi}{2} \mp \theta\right) = \cos\theta$

- $\cos\theta$: even ; $\sin\theta$: odd.
  $\int \text{odd} = 0$.

- $a + ib = \cos\theta + i\sin\theta$

19.08.23. RPET : UNIT-2 : POWER SPECTRAL DENSITY.

1. FT of $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{is\omega x} dx$

$FT[\{f(x)\}] = F(S)$

2. $S_{xx}(\omega)$ or $S_{xx}(2\pi f)$: PSD of RP with $R_{xx}(\tau)$

★ $= \int_{-\infty}^{\infty} R(\tau) \cdot e^{-i\omega\tau} d\tau$

$= \int_{-\infty}^{\infty} R(\tau) e^{-2\pi f i \tau} d\tau$

3. Given : $S(\omega)$ or $S(f)$

$R(T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) \cdot e^{i\omega T} d\omega$

$R(T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(f) \cdot e^{2\pi f i T} df$

~~Properties:~~

i.

4. $\int_{-\infty}^{\infty} e^{-i\omega\tau} d\tau = 2\pi \delta(\omega)$

5. $e^{i\theta} - e^{-i\theta} = \cancel{\cos\theta +} 2j\sin\theta$

$e^{i\theta} + e^{-i\theta} = 2\cos\theta$

Properties: PSD.

- If the PSD at 0 freq. (ie. $\omega=0$) is the total area under the graph of the $R_{xx}(\tau)$ function.
- $R(0) = E[x^2(t)]$
- $S(-\omega) = S(\omega)$: even func / symmetric.
- PSD of RP (real / complex) is real func. of ω & +ve.
- ~~PSD and $R_{xx}(\tau)$ of real~~
  if $\int_{-\infty}^{\infty} |R(\tau)| \, d\tau < \infty$, then $S(\omega)$ is a constant func. of 'ω'.

Properties: CPSD.

- $S_{xy}(\omega) = S_{yx}(-\omega)$
- $S_{xy}(-\omega) = S^*_{xy}(\omega)$
- if $x(t)$ and $y(t)$ are orthogonal, then $S_{xy}(\omega) = 0$.

RPET : U4 – ESTIMATION THEORY.

→ **Minimum Mean Square Estimation :**

- Y estimated by X func : $g(x)$.

$MSE = E[Y - g(x)]^2$

- Y estimated by constant c.

$MSE = E[Y - c]^2$

min. $MSE = var(Y)$

- Y estimated by linear func : $aX + b$.

$MSE = E[Y - (aX + b)]^2$

min. $MSE = \hat{Y} = \rho_{XY} \frac{\sigma_Y}{\sigma_X} X + m_Y - \rho_{XY} \frac{\sigma_Y}{\sigma_X} m_X$

$$a^* = \frac{cov(x, y)}{\sigma_x^2} = \frac{E[XY] - E[X]E[Y]}{\sigma_x^2}.$$

$b^* = m_y - a^* m_x$

$Y = a^* + b^* x.$

→ **Maximum Likelihood Estimation :**

Step 1 : $L(\theta) = f(x_1, \theta) \cdot f(x_2, \theta) \cdots f(x_N, \theta)$

Step 2 : $\log(L) = \log[f(x_1, \theta)] + \log[f(x_2, \theta)] + \cdots$

Step 3 : differentiate wrt parameter partially

$$\frac{1}{L} \frac{\partial L}{\partial \theta}$$

poisson distribution:

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0, 1, 2 \cdots\cdots$$

$$MLE = \lambda = \bar{x}.$$

binomial distribution:

$$P(X=x) = {}^{n}C_{x}\, p^{x} (1-p)^{n-x}, \quad x = 0, 1, 2 \ldots n$$

$$MLE = \frac{\bar{x}}{n}.$$

normal distribution:

$$f_X(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{\frac{(x-u)^2}{2\sigma}}$$

$$MLE = \mu = \bar{x}.$$

$$= \sigma^2 = \frac{(x_1-\mu)^2 + (x_2-\mu)^2 + \cdots + (x_n-\mu)^2}{n}.$$

exponential distribution:

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0.$$

$$MLE = \lambda = \frac{1}{\bar{x}}.$$