SVKM'S NMIMS STME, Navi-Mumbai Campus
Sem III, Ay 23-24
Discrete Mathematics Tutorial 1

Course: MBA Tech CE/ B Tech CE/ B Tech AIDS	SEM:III
Unit :1 (Set Theory, Relations and Functions)	Marks: 5 Marks
Duration: 1 hour	Week : 1

Instruction to students:

1. Solution sheet should have all the questions mentioned before the answer

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Q1.) If $A = \{x|6x^2 + x - 15 = 0\}$, $B = \{x|2x^2 - 5x - 3 = 0\}$ and $C = \{x|2x^2 - x - 3 = 0\}$ then find
1. $A \cup B \cup C$
2. $A \cap B \cap C$

Q2.) If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}, C = \{4, 5, 6, 7, 8\}$ and universal set $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then verify the following:

1. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
2. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
3. $\overline{A \cup B} = \bar{A} \cap \bar{B}$
4. $\overline{A \cap B} = \bar{A} \cup \bar{B}$
5. $A = (A \cap B) \cup (A \cap \bar{B})$
6. $B = (A \cap B) \cup (\bar{A} \cap B)$
7. $|A \cup B| = |A| + |B| - |A \cap B|$

Q3.) If A, B, C are the sets of the letters in the words "college", "marriage" and "luggage" respectively, then verify that $[A - (B \cup C)] = [(A - B) \cap (A - C)]$

Q4.) If A and B are subsets of the universal set X and $n(X) = 50, n(A) = 35, n(B) = 20, n(\bar{A} \cap \bar{B}) = 5,$ find

1. $n(A \cup B)$
2. $n(A \cap B)$
3. $n(A \cap \bar{B})$
4. $n(\bar{A} \cap B)$

Q5.) Given that $A = \{x :$ x is a even natural number less than or equal to 10$\}$ and
$B = \{x :$ x is an odd natural number less than or equal to 10$\}$
Find (i) A-B (ii) B-C (iii) is A-B=B-A ?

Q6.) In a class of 200 students who appeared certain examinations, 35 students failed in MHT-CET, 40 in AIEEE and 40 in IIT entrance, 20 failed in MHT-CET and AIEEE, 17 in AIEEE and IIT entrance,15 in MHT-CET and IIT entrance and 5 failed in all three examinations. Find how many students
i. did not fail in any examination.
ii. failed in AIEEE or IIT entrance.

Q7.) From amongst 2000 literate individuals of a town, 70% read Marathi newspapers, 50% read English newspapers and 32.5% read both Marathi and English newspapers. Find the number of individuals who read,
i. at least one of the newspapers.
ii. neither Marathi nor English newspaper.
iii. only one of the newspapers.

Q8.)Out of forty students, 14 are taking English Composition and 29 are taking Chemistry. If five students are in both classes, a) how many students are in neither class? b) How many are in either class?

Q9.) Let $S = \{1, 2, 3\}$, find power set of S?

Q10.) There is a group of 80 persons who can drive scooter or car or both. Out of these, 35 can drive scooter and 60 can drive car. Find how many can drive both scooter and car? How many can drive scooter only? How many can drive car only?

Q11.) It was found that out of 45 girls, 10 joined singing but not dancing and 24 joined singing. How many joined dancing but not singing? How many joined both?

Q12.) In a class of 40 students, 15 like to play cricket and football and 20 like to play cricket. How many like to play football only but not cricket?

Q13.) If $A = \{1, 3, 5\}$, then write all the possible subsets of A.

Q14.) Given three sets P, Q and R such that:
$P = \{x :$ x is a natural number between 10 and 16$\}$,
$Q = \{y :$ y is a even number between 8 and 20$\}$
and $R = \{7, 9, 11, 14, 18, 20\}$
(i) Find the difference of two sets P and Q
(ii) Find Q - R
(iii) Find R – P
(iv) Find Q – P

Q.15) It is known that at the university, 60 percent of the professors play tennis, 50 percent of them play bridge, 70% jog, 20% play tennis and bridge, 30% play tennis and jog and 40% play bridge and jog. If someone claimed that 20% of the professors jog and play bridge and tennis, would you believe this claim ? Why ?

SVKM'S NMIMS STME, Navi-Mumbai Campus
Sem III, Ay 23-24
Discrete Mathematics Tutorial 2

Course: MBA Tech CE/ B Tech CE/ B Tech AIDS	SEM:III
Unit :1 (Set Theory, Relations and Functions)	Marks: 5 Marks
Duration: 1 hour	Week : 2

Instruction to students:

1. Solution sheet should have all the questions mentioned before the answer

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Q1.) Let R and S be the relations from $A = \{1, 2, 3\}$ to $B = \{a, b\}$ defined by $R = \{(1, a), (3, a), (2, b), (3, b)\}$ and $S = \{(1, b), (2, b)\}$. Find $R \cup S, R \cap S, R^C, S^{-1}$. Draw digraphs of R and S.

Q2.) Let R be a relation on the set of $+$ve integers N, defined by, $(x, y) \in R$ if and only if $x^2 + 2y = 100$. Write R as a set of ordered pairs. Find the domain and range of relation R. Find R^{-1} and describe R^{-1} as an equation.

Q3.) Let $A = \{1, 2, 3, 4, 6\}$ and let R be a relation on A defined by $(x, y) \in R$ if and only if "x divides y" written as $x|y$.
(i) Write R as a set of ordered pairs.
(ii) Draw its directed graph.
(iii) Find the inverse relation of R.

Q4.)Find domain, range, matrix and the digraph of the following relations.
a) $A = \{1, 2, 3, 4, 5\} = B$; aRb if and only if $a \leq b$.
b) $A = \{1, 3, 5, 7, 9\}, B = \{2, 4, 6, 8\}$; aRb if and only if $a < b$.
c) $A = \{1, 2, 3, 4, 8\} = B$; aRb if and only if $a + b \leq 9$.

Q5.) Let $A = B$ and R be a relation on A defined by, aRb if and only if $a^2 + b^2 = 25$. Find Dom(R) and Ran(R)

Q6.)Find the relation R defined on A and its digraph if

$$i) A = \{1, 2, 3, 4\} \text{ and } M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$ii) A = \{a, b, c, d, e\} \text{ and } M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Q7.) If $A = \{1, 2, 3, 4, 5\}$ and $R = \{(1, 2), (2, 2), (2, 3), (3, 4), (4, 4), (5, 1), (5, 4)\}$ be a relation on A. Draw the digraph and write its matrix. Find the in-degrees and out-degrees of each vertex.

Q8.) Find domain, range, matrix and the digraph of the following relations.
a) $A = \{a, b, c, d\}, B = \{1, 2, 3\}, R = \{(a, 1), (a, 2), (b, 1), (c, 2), (d, 1)\}$
b) $A = \{1, 2, 3, 4\}, B = \{1, 4, 6, 8, 9\}$; aRb if and only if $b = a^2$
c) $A = \{1, 2, 3, 4, 8\} = B$; aRb if and only if $a = b$
d) $A = \{1, 2, 3, 4, 8\}, B = \{1, 4, 6, 9\}$; aRb if and only if a divides b
e) $A = \{1, 2, 3, 4, 6\} = B$; aRb if and only if a is multiple of b

SVKM'S NMIMS STME, Navi-Mumbai Campus
Sem III, Ay 23-24
Discrete Mathematics Tutorial 3

Course: MBA Tech CE/ B Tech CE/ B Tech AIDS	SEM:III
Unit :1 (Set Theory, Relations and Functions)	Marks: 10 Marks
Duration: 1 hour	Week : 3

Instruction to students:

1. Solution sheet should have all the questions mentioned before the answer

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1. Let $A = \{1, 2, 3, 4\}$. Determine whether the relation is reflexive, symmetric, asymmetric, antisymmetric, transitive. Justify your answers.

   a)$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$

   b) $R = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$

   c) $R = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

   d) $R = \phi$

   e) $R = A \times A$

2. Determine whether the following relations R on the set A are reflexive, symmetric and transitive.

   (i) $A = \{a, b, c, d\}, R = \{(a, a), (b, a), (b, b), (c, c), (d, d), (d, c)\}$

   (ii) $A = \{1, 2, 3, 4, 5\}, R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1), (2, 3), (3, 3), (4, 4), (3, 2), (5, 5)\}$

   (iii)$A = \{1, 2, 3, 4\}, R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (3, 3), (1, 3), (4, 1), (4, 4)\}$

3. Give one example of a relation on the set $A = \{a, b, c, d\}$ with justification that is

   a) reflexive and symmetric, but not transitive.

   b) reflexive and transitive, but not symmetric.

   c) transitive, reflexive and symmetric,

   d) asymmetric and transitive.

4. Determine whether the relation R on the set A is an equivalence relation.

   a. $A = \{a, b, c, d\}, R = \{(a, a,), (b, a), (b, b), (c, c), (d, d), (d, c)\}$

   b. $A = \{1, 2, 3, 4, 5\}, R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1), (2, 3), (3, 3), (4, 4), (3, 2), (5, 5)\}$

   c.$A = \{1, 2, 3, 4\}, R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (3, 3), (1, 3), (4, 1), (4, 4)\}$

   d. Given a set $S = \{1, 2, 3, 4, 5\}$, find the equivalence relation on S which generated by the partition $\{\{1, 2\}, \{3\}, \{4, 5\}\}$. Draw the graph of the relation.

5. Define equivalence relation on a set. Let R be a relation on the set of integers defined by aRb iff $a - b$ is multiple of 5. Prove that R is equivalence relation.

6. Let R be the following equivalence relation on the set $A = \{1, 2, 3, 4, 5, 6\}, R = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (1, 2), (2, 1), (2, 3), (3, 2), (1, 3), (3, 1), (5, 6), (6, 5)\}$. Find the partitions of A induced by R i.e. A/R.

7.

   a) Let $A = \{1, 2, 3, 4, 5, 6\}$ and let R be the equivalence on A defined by $R = \{(1, 1), (1, 5), (2, 2), (2, 3), (2, 6), (3, 2), (3, 3), (3, 6), (4, 4), (5, 1), (5, 5), (6, 2), (6, 3), (6, 6)\}$

      (i) Find the equivalence classes of elements of A.

(ii) Find the partition of A induced by R.

b) If $\{\{a, c, e\}, \{b, d, f\}\}$ is a partition of the set $A = \{a, b, c, d, e, f\}$, determine the corresponding equivalence relation R.

8. If $A = \{1, 2, 3\}$. How many relations are possible on ‘A’ and how many of them are equivalence.

9. If $A = \{1, 2, 3, 4\}$. How many relations exists on A and how many of them are equivalence relation.

10. Determine whether the relation R on the set A is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive.

    (a) $A = \mathbb{Z}$; aRb if and only if $a \leq b + 1$

    (b) $A = \mathbb{Z}^+$; aRb if and only if $|a - b| \leq 2$

    (c) $A = \mathbb{Z}^+$; aRb if and only if $a = b^k$ for some $k \in \mathbb{Z}^+$.

    (d) $A = \mathbb{Z}^+$; aRb if and only if $a + b$ is even.

    (e) $A = \mathbb{Z}; aRb$ if and only if $|a - b| = 2$

    (f) A = the set of real numbers; aRb if and only if $a^2 + b^2 = 4$.

11. Let R be a relation on a set of positive integers Z defined by xRy iff $3x + 4y$ is divisible by 7. Prove that R is an equivalence relation.

12. Define ’congruence modulo 5’ relation on $\mathbb{Z}$. Show that it is equivalence relation.

13. Find partition of set $\mathbb{Z}$ under equivalence relation ’congruence modulo 5’.

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Sem III, Ay 23-24
Discrete Mathematics Tutorial 4

Course: MBA Tech CE/ B Tech CE/ B Tech AIDS	SEM:III
Unit :1 (Set Theory, Relations and Functions)	Marks: 10 Marks
Duration: 1 hour	Week : 4

Instruction to students:

1. Solution sheet should have all the questions mentioned before the answer

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1. If $f(x) = 6x^3 + 4x - 5$, find $f(1), f(-2)$
2. For the function $f(x) = 2x + 1$, find the range if domain $= \{-3, -2, -1, 0, 1, 2, 3\}$
3. If $f(x) = x^2, -3 \leq x \leq 3$, find the range
4. Check if the following functions are odd or even
    1. $x^2 + 1$
    2. $x \cdot sin(x)$
    3. $x^2 \cdot cos(x)$
    4. x^3
5. if $f(x) = 3x^2 - 5x + 7$. Find $f(x - 1)$.
6. If $f(x) = f(2x + 1)$ such that $f(x) = x^2 - 3x + 4$, find (x).
7. If $f(x) = f(3x - 1)$ such that $f(x) = x^2 - 4x + 11$, find (x).
8. If $f(x) = 3x + a$ such that $f(1) = 7$ then find a & $f(4)$.
9. If $f(x) = \sqrt{x + 1}$ and $g(x) = x^2 + 2$, find $f \circ g$ and $g \circ f$
10. Evaluate $f(g(3))$ given that $f(x) = |x - 6| + x^2 - 1$ and $g(x) = 2x$
11. Find the composite function $(f \circ g)(x)$ given that $f = \{(3, 6), (5, 7), (9, 0)\}$ and $g = \{(2, 3), (4, 5), (6, 7)\}$
12. Find the composite function $(f \circ g)(x)$ given that $f = \{(1, 6), (4, 7), (5, 0)\}$ and $g = \{(6, 1), (7, 4), (0, 5)\}$
13. If $f = \frac{2x+3}{3x-2}$ such that $x \neq \frac{2}{3}$ then show that $(f \circ f)(x) = x$.
14. If $f = \frac{3x+2}{4x-1}$ such that $x \neq \frac{1}{4}$ then show that $(f \circ f)(x) = x$.
15. If $f(x) = 2x + 4$ and $g(x) = 4x^2 - 2x$, what is the value of:
    1. $(f + g)(2)$
    2. $(f + g)(-1)$
    3. $(f + g)(0)$
    4. $(f + g)(x)$
    5. $(f + g)(2x)$
16. If $f(x) = 3x - 2$ and $g(x) = \frac{1}{x}$, what is the value of:

1. $2f(x) + 4g(x)$?
2. $3f(x) - g(x)$?
3. $f \circ g(x)$?

17. Evaluate $f(3)$ given that $f(x) = |x - 6| + x^2 - 1$.

18. Find $f(x + h) - f(x)$ given that $f(x) = ax + b$

19. Find $(f \circ g)(x)$ given that $f(x) = \sqrt{x}$ and $g(x) = x^2 - 2x + 1$

20. Is the following function even, odd, or neither?

$$f(x) = 12x^{11} - 6x^7 - 5x^3$$

21. If $f(x) = x^2, 0 \leq x \leq 3$, find its range. Determine whether it is even, odd or neither.

22. If $f : \mathbb{R} - \left\{\frac{7}{3}\right\} \to \mathbb{R} - \left\{\frac{4}{3}\right\}$ defined as $f(x) = \frac{4x-5}{3x-7}$. Prove that f is bijective, find rule for f^{-1}.

23. If $f : \mathbb{R} - \left\{\frac{7}{5}\right\} \to \mathbb{R} - \left\{\frac{2}{5}\right\}$ defined as $f(x) = \frac{2x-3}{5x-7}$. Prove that f is bijective, find rule for f^{-1}.

24. If f and g be the functions from set of integers to set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. Find $(f \circ g)$ and $(g \circ f)$.

25. Let the functions f and g be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. Find the formula defining the composition function $(g \circ f)$.

26. Let $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = 2x^3 - 7$,
    $g : \mathbb{R} \to \mathbb{R}$ defined as $g(x) = 3x^2$,
    $h : \mathbb{R} \to \mathbb{R}$ defined as $h(x) = 5x + 4$.
    Find the rule of defining (i) $(g \circ f) \circ h$ (ii) $f \circ (h \circ g)$.

27. Let $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^3$,
    $g : \mathbb{R} \to \mathbb{R}$ defined as $g(x) = 4x^2 + 1$,
    $h : \mathbb{R} \to \mathbb{R}$ defined as $h(x) = 7x - 1$.
    Find the rule of defining (i) $g \circ (h \circ f)$ (ii) $(h \circ g) \circ f$.

Tutorial 5 : Logic

1) Which of these sentences are propositions? Mention the truth values of those that are propositions?
   i) 2+3=5
   ii) Miami is the capital of Florida
   iii) Answer this question
2) What is the negation of each of these propositions?
   i) Today is Thursday
   ii) There is no pollution in New Jersey
   iii) 2+1=3
3) Let p and q be the propositions "Swimming at New Jersey shore is allowed" and "Sharks have been spotted near the shore" respectively. Express each of these compound propositions as an English sentence.
   a. $\sim p$
   b. $p \vee q$
   c. $\sim p \to \sim q$
   d. $\sim q \vee (\sim p \wedge q)$
4) Let p, q and r be the propositions

p : Grizzly bears have been seen in the area

q : Hiking is safe on the trail

r : Berries are ripe along the trail

Write these propositions using p, q, and r and logical connectives (including negations).
   a. Berries are ripe along the trail but grizzly bears have not been seen in the area
   b. It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.
   c. For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to be seen in the area
5) Construct a truth table for $(p \to q) \vee (\sim p \to r)$
6) State the converse, inverse and contrapositive of the following :
   (i) If it is cold, then he wears a hat.
   (ii) If an integer is a multiple of 2, then it is even.
7) Translate the following statement into symbolic form :

   If the utility cost goes up or the request for additional funding is desired, then a new company will be purchased if and only if we can show that the current computing facilities are indeed not adequate.
8) Show that $\sim p \to (q \to r)$ *and* $q \to (p \vee r)$ are logically equivalent
9) Show that $(p \to q) \to (r \to s)$ *and* $(p \to r) \to (q \to s)$ are not logically equivalent

10 Using laws of logic prove that following are equivalent

i) $\sim(p \to q)$ and $p \wedge \sim q$

ii) $\sim(p \vee (\sim p \wedge q))$ and $\sim p \wedge \sim q$

11) Using laws of logic prove the following

i) $\sim(p \wedge q) \to (\sim p \vee (\sim p \vee q)) \equiv \sim p \vee q$

ii) $p \to q \wedge [\sim q \wedge (T \vee \sim q)] \equiv \sim (p \vee q)$

iii) $[\sim p \wedge (\sim q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \equiv r$

iv) $((\sim p \vee q) \to (p \wedge q \wedge T)) = p$

12) Show that following statements are tautology

i) $(p \wedge q) \to (p \vee q)$

ii) $[p \wedge (p \to q)] \to q$

iii) $(p \vee q) \wedge (\sim p \vee r) \to (q \vee r)$

Tutorial 6

Q1. Express the statement "Every student in this class has studied calculus" using predicates and quantifiers.

Q2. Determine whether each of the compound propositions is satisfiable.

a. $(p \vee \sim q) \wedge (q \vee \sim r) \wedge (r \vee \sim p) \wedge (p \vee q \vee r) \wedge (\sim p \vee \sim q \vee \sim r)$
b. $(p \to q) \wedge (p \to \sim q) \wedge (\sim p \to q) \wedge (\sim p \to \sim q)$

Q3. Obtain the disjunctive normal forms of the following

a. $p \wedge (p \to q)$
b. $p \vee (\sim p \to (q \vee (q \to \sim r)))$
c. $p \to ((p \to q) \wedge \sim (\sim q \vee \sim p))$

Q4. Obtain the conjuctive normal forms of the following

d. $p \wedge (p \to q)$
e. $[q \vee (p \wedge r)] \wedge \sim [(p \vee r) \wedge q]$

Q5. Test the validity of the following arguments:

If I graduate this semester then I will pass the physics course. If I do not study physics for 10 hours a week then I will not pass physics. If I study for 10 hours a week then I cannot play volleyball. Therefore if I play volleyball then I will not graduate this semester.

Q6. Negate the following statement:

All Indians eat rice.

Q7. *Let* $n \in Z$, *if* n^2 *is odd then* n *is even.*

Q8. Prove that there is no greatest negative real number

Tutorial 7 : MATHEMATICAL INDUCTION

1) **Prove the following using Mathematical induction, for each positive integer n**
$6^{n+2} + 7^{2n+1}$ *is divisible by* 43

2) **Show that** $(1+2/3)^n \geq 1+2n/3$ *for all* $n \geq 1.$

3) **Show by mathematical induction** $n! \geq 2^{n-1}$ *for all* $n \geq 1.$

4) **Show by mathematical induction that**
$1+2+3+\ldots\ldots\ldots\ldots\ldots\ldots+n = n(\mathrm{n}+1)/2 \quad \text{for} \quad \mathrm{n} \geq 1$

5) **Show by mathematical induction** $\dfrac{1}{1.2}+\dfrac{1}{2.3}+\ldots\ldots\ldots\ldots\ldots\ldots+\dfrac{1}{n(n+1)}=\dfrac{n}{n+1}$

6) **Prove** $1.1!+2.2!+3.3!+\ldots\ldots\ldots\ldots+n.n! = (n+1)!-1,$ **for** $n \geq 1.$ **by mathematical induction.**

7) **Show that** $1+3+5+7+\ldots\ldots\ldots\ldots+(2n-1) = n^2$ **by induction**

8) **Show that** $1^3+2^3+3^3+4^3+\ldots\ldots\ldots\ldots+n^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

9) **Prove** $1.2+2.3+3.4+\ldots\ldots\ldots\ldots+n(n+1) = \dfrac{n(n+1)(n+2)}{3}$

10) **Show by mathematical induction**
$$\frac{1}{1.3}+\frac{1}{3.5}+.\frac{1}{5.7}\ldots\ldots\ldots\ldots\ldots\ldots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$$

11) **Prove** $1.+2.+2^2+2^3+\ldots\ldots\ldots\ldots+2^n = 2^{n+1}-1,$ *for* $n \geq 0.$

12) **Prove the following using Mathematical induction, for any integer n**
$11^{n+2} + 12^{2n+1}$ *is divisible by* 133

13) **Prove** $\dfrac{1}{n+1}+\dfrac{1}{n+2}+\ldots\ldots\ldots+\dfrac{1}{2n} > \dfrac{13}{24}$ *for* $n \geq 2.$

14) **Show that** $2^n \times 2^n - 1$ **is divisible by 3** *for all* $n \geq 1.$

15) Show that $1^2 + 3^2 + 5^2 + 7^2 + \dots\dots\dots\dots + (2n-1)^2 = \dfrac{n(2n-1)(2n+1)}{3}$ **by induction**

16) Show that $n^3 + 2n$ **is divisible by 3** $\textit{for all}\ n \geq 1.$

17) Show by mathematical induction

$$\frac{1^2}{1.3} + \frac{2^2}{3.5} + .\frac{3^2}{5.7} \dots\dots\dots\dots\dots\dots + \frac{n^2}{(2n-1)(2n+1)} = \frac{n(n+1)}{2(2n+1)}$$

Tutorial No. : _____ Topic : ____

Name : ____ Roll No. :___________

Program/Stream : _________________________________

Date of Performance : ______________________

Date of Submission : _______________________

Tutorial No. 08: Pigeonhole Principle and Recurrence Relations

Pigeon-hole principle:

1) There are 50 baskets of apples. Each basket contains no more than 24 apples. Show that there are at least 3 baskets containing the same number of apples.
2) Show that for any natural number n there is a number composed of digits 5 and 0 only and divisible by n.
3) Show that among any 4 numbers one can find 2 numbers so that their difference is divisible by 3.
4) Given 12 different 2-digit numbers, show that one can choose two of them so that their difference is a two-digit number with identical first and second digit.
5) There are 13 squares of side 1 positioned inside a circle of radius 2. Show that at least 2 of the squares have a common point.

Recurrence Relations :

1. What is the solution of the recurrence relation.
$a_n = 6a_{n-1} - 9a_{n-2}$
with initial conditions $a_0 = 1$ and $a_1 = 6$?

2. Find the solution of recurrence relation
$a_n = 5a_{n-1} - 6a_{n-2} + 7^n$

3. Solve the recurrence relation
$d_n = 4\left(d_{n-1} - d_{n-2}\right)$
Subject to the initial conditions $d_0 = 1 = d_1$.

4. Determine whether the sequence $\{a_n\}$ is solution of recurrence relation $a_n = 2a_{n-1} - a_{n-2}$ for n = 2, 3, 4, ….. where $a_n = 3n$ for every nonnegative integer n. Answer the same question for $a_n = 5$.

5. Find the solution of $a_{n+2} + 2a_{n+1} - 3a_n = 0$ that satisfies $a_0 = 1$, $a_1 = 2$.

6. Find the solution of Fibonacci relation $a_n = a_{n-1} + a_{n-2}$ with the initial conditions $a_0 = 0$, $a_1 = 1$.

7. Given that $a_0 = 0, a_1 = 1, a_2 = 4$ and $a_3 = 12$ satisfy the recurrence relation $a_r + C_1 a_{r-1} + C_2 a_{r-2} = 0$ Determine a_r.

8. Find the solution of recurrence relation : $a_n = 5a_{n-1} - 6a_{n-2} + 7n$

9. Determine the sequence whose recurrence relation is $a_n = 4a_{n-1} + 5a_{n-2}$ with $a_1 = 2$ and $a_2 = 6$.

10. Solve the following recurrence relations :

(i) $a_r - 7a_{r-1} + 10a_{r-2} = 0$, given that $a_0 = 0$ & $a_1 = 3$

(ii) $a_r - 4a_{r-1} + 4a_{r-2} = 0$, given that $a_0 = 1$ & $a_1 = 6$

(iii) $a_r + 6a_{r-1} + 9a_{r-2} = 3$, given that $a_0 = 0$ & $a_1 = 1$

(iv) $a_r + a_{r-1} + a_{r-2} = 0$, given that $a_0 = 0$ & $a_1 = 2$

(v) $a_r - 2a_{r-1} + 2a_{r-2} - a_{r-3} = 0$ given that $a_0 = 2, a_1 = 1$ & $a_2 = 1$

11. Find the particular solution of the following :

(i) $a_r - 5a_{r-1} + 6a_{r-2} = 1$ (ii) $a_r + a_{r-1} = 3r\,2^r$

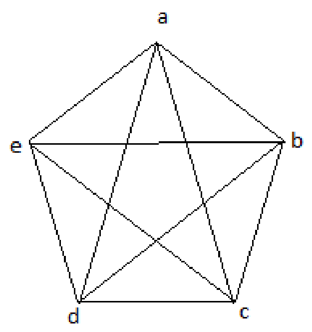
(iii) $a_r - 2a_{r-1} = 3.2^r$ (iv) $a_r - 4a_{r-1} + 4a_{r-2} = (r+1)\,2^r$

12. Find the homogeneous solution of the following :

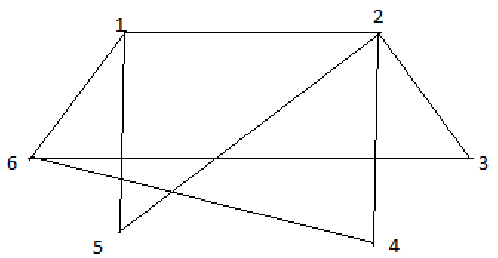
(i) $a_r + 6a_{r-1} + 12a_{r-2} + 8a_{r-3} = 0$ (ii) $4a_r - 20a_{r-1} + 17a_{r-2} - 4a_{r-3} = 0$

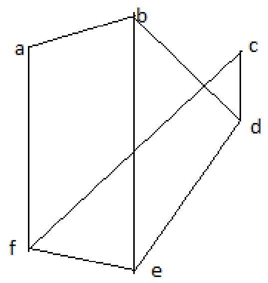
Tutorial 10 : Graphs, Paths, connectivity

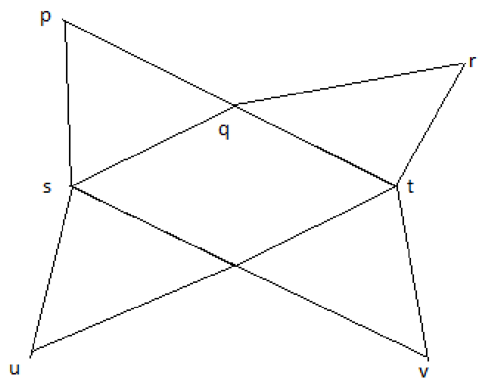
1. Can a graph exist with 15 vertices each of degree five?
2. Is it possible to draw a simple graph with 4 vertices and 7 edges? (Ans: No)
3. Give an example of a graph that is regular, but not complete, with each vertex having degree three.
4. How many vertices does a regular graph of degree four with 10 edges have?
5. Draw the subgraph induced by {a, b, c, e}



6. Determine which of the following graphs are bipartite and which are not.







7. Prove that a graph which contains a triangle can not be bipartite.

7. Write down the adjacency and incidence matrix of the graph in Fig. 6.81.

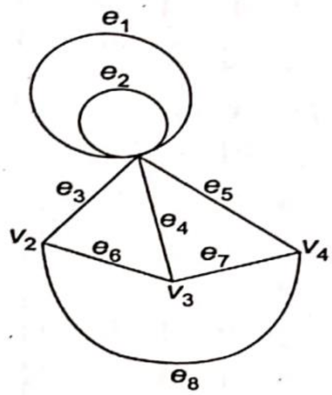


Fig. 6.81

8. Draw the graph of the given adjacency matrix.

$$\begin{bmatrix} 0 & 1 & 1 & 3 \\ 1 & 0 & 2 & 3 \\ 1 & 2 & 0 & 2 \\ 3 & 3 & 2 & 0 \end{bmatrix}$$

8.

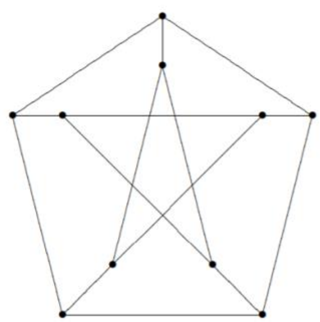
TUTORIAL 11

Answer the following questions with **justification.**

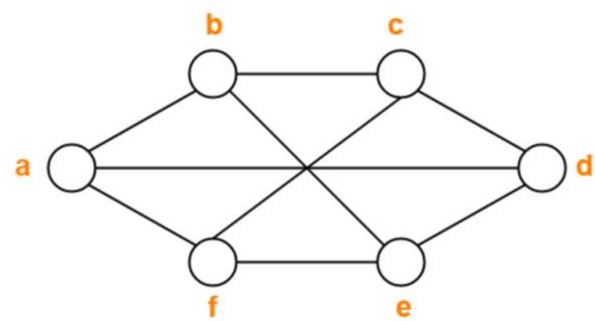
a) How many maximum edges exists in bipartite graph with 20 vertices?
b) How many maximum numbers of edges exists in simple graph with 15 vertices?
c) A connected graph has 9 vertices with degree sequence 5,4,4,3,3,3,2,2,2. How many edges are there in the graph?
d) Does there exists a graph with degree sequence 5,5,4,3,2,1? Explain.
e) In r-regular graph with n vertices, how many edges exists?

1) Using Welsh-Powel algorithm, find Chromatic number of the following graphs.

a)

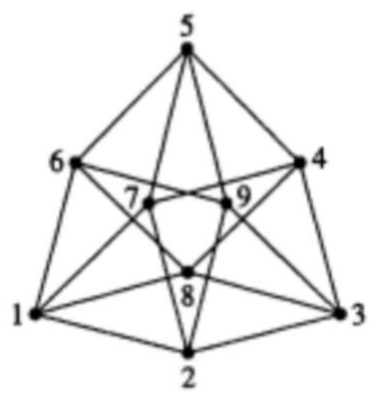


b)

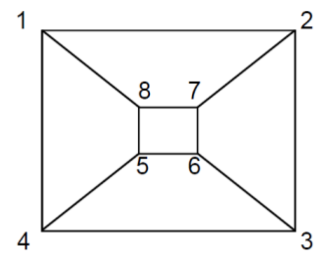


2) Determine whether the following graphs are bipartite

a)

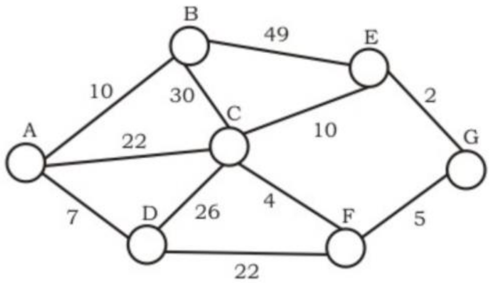


b)

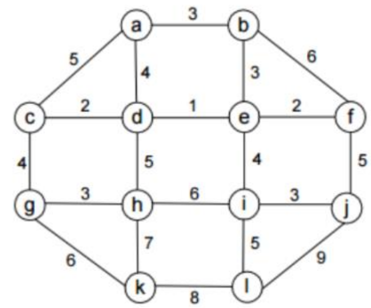


3) Find minimum spanning tree 'MST' of the following graphs using Krushkal's algorithm.

a)



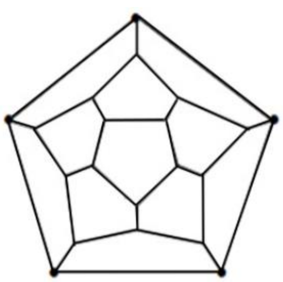
b)



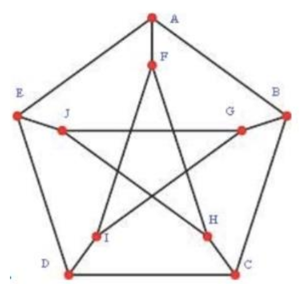
4) Find minimum spanning tree 'MST' of the graphs given in ex. 5 using Prim's algorithm.

5) Determine whether following graphs contains Eulerian path, Eulerian circuit, Hamiltonian path, Hamiltonian circuit. Also check whether they are Eulerian graph, Hamiltonian graph

a)



b)



c)