20.8.23. RPET – UNIT 1 : INTRO TO RANDOM PROCESS.

1. You have \$1000 to put in an account with intrest rate R compounded anually ie. if x_n is the value of the account at year n, then $x_n = 1000(1+R)^n$ for $n = 0, 1, 2 \ldots$
Then the value of R is a RV when you put the money in the bank but it does not change after a while. Then in a particular assume that R follows a uniform distribution where limits are $R \sim U(0.04, 0.05)$. Find :

a. all possible sample functions for the RP $\{x_n = 0, 1, 2 \ldots\}$
b. the expected value of your account at year 3. (ie. $E[x_3]$)

2. Let $\{x(t), t \in [0, \infty)\}$ be defined as $x(t) = A + Bt$, $\forall t \in [0, \infty)$ where A and B are independent normal RV $\sim N(1, 1)$. Find :

a. all possible sample function for this RP.
b. define the RV, $Y = x(1)$ find the PDF of Y.
c. assume $Z = x(2)$; Find $E[YZ]$

3. $\{x(t), t \in (0, 1)\}$ be defined as $x(t) = T + (1 - t)$, where T is uniform ~~random~~ RV in interval $(0, 1)$. Find :

a. CDF of $x(t)$
b. $E[x(t)]$ and $C_{xx}(t_1, t_2)$

4. Consider a RP $x(t)$ where $x(t) = A\cos(\omega t + \theta)$ where A and θ are independent and uniform RV from $(-k, k)$ and $(-\pi, \pi)$
Find the mean, autocorrelation and autocovariance.

5. $x(t) = B\cos(50t + \phi)$, where B and ϕ are independent RV, B is a RV with $E = 0$ and $var = 1$, and ϕ is $U \sim [-\pi, \pi]$.
Find the mean, auto-correlation of the RP.

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6. Suppose $x(t)$ is a RP with mean as 3 and autocorrelation $R(t_1, t_2) = 9 + 4e^{-0.2|t_1 - t_2|}$. Determine the mean, variance and covariance of RV $Z = x(5)$ and $W = x(3)$.

SSR:

1. The mean and variance of the first order stationary process and constant.

2. If $\{x(t) = A\cos\lambda t + B\sin\lambda t;\ t > 0\}$ is a RP, where A and B are independent RV, each of which assumes the value -2 and 1 with probabilities $\frac{1}{3}$ and $\frac{2}{3}$ resp. ST:

$\{x(t)\}$ is not strict sense stationary

3. Consider a RP $\{x(t) = \cos(t + \phi)\}$ where ϕ is a RV with the density func. $f(\phi) = \frac{1}{\pi}$, $\frac{-\pi}{2} < \phi < \frac{\pi}{2}$. Check whether or not process is stationary.

4. Examine the poisson process $P[x(t)] = k = \frac{e^{-\lambda t} (\lambda t)^k}{k!}$, where $k = 0, 1, 2 \ldots$ is stationary. For poisson process, mean of $x(t)$ $x(t) = \lambda t$, which is dependent of t, hence not constant, therefore, $x(t)$ is not stationary.

5. For the sine wave process $x(t) = Y\cos\omega_0 t$, $-\infty < t < \infty$. ω_0 is constant, amplitude Y is a RV with a uniform distribution in the interval of 0 to 1. Check whether process is stationary or not.

WSS:

1. consider the RP $\{x(t) : t \in T\}$ defined as $x(t) = \cos(t + A)$, A follows uniform distribution. ST, $x(t)$ is WSS.

2. Given a RV, Y with characteristic func $\phi(\omega) = E[e^{i\omega y}]$ and a RP defined by $x(t) = \cos(\lambda t + Y)$. ST, $\{x(t)\}$ is WSS, if $\phi(1) = \phi(2) = 0$.

Ergodicity:

1. A RP has sample function of the form $x(t) = A\cos(\omega t + \theta)$
   ω: constant ; A = RV that has a magnitude of 1 to −1 with equal probabilities.
   θ : RV that is uniformly distributed bet^n 0 to 2π. assume that the RV, A and θ are independent.

a. is $x(t)$ a WSS.

b. is $x(t)$ a mean ergodic process.

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RPET- UNIT 2 : POWER SPECTRAL DENSITY.

1. Find the PSD of the WSS, RP whose autocorrelation func is $R(\tau) = e^{-\alpha|\tau|} \cos b\tau$

2. For a RP $x(t)$, $R(\tau) = 1 + e^{-\alpha|\tau|}$. Find PSD

3. The auto-correlation function of the random telegraph signal process is given by $R(\tau) = a^2 e^{-2Y|\tau|}$

4. Find the PSD of RP $x(t) = a\cos(bt + Y)$; Y: RV uniformly distributed over 0 to 2π, A and B are constant.

5. A RP has the autocorrelation function $R_{xx}(\tau) = \frac{4\tau^2+6}{\tau^2+1}$. Find μ_x^2 value, the mean value and the variance of the process.

6. compute the variance of RP $x(\tau)$, whose auto-correlation function is given by $R_{xx}(\tau) = 25 + \frac{4}{1+6\tau^2}$.

7. The CPS of real RP $x(t)$ and $y(t)$ is defined by,
$$S_{xy}(\omega) = \begin{cases} a + jb\omega, & |\omega| < 1 \\ 0, & \text{otherwise} \end{cases}$$
Find the cross-correlation function.

8. Find the cross-correlation function of a process whose CPSD is $S_{xy}(\omega) = \begin{cases} p + \frac{jq\omega}{B}, & |\omega| < B \\ 0, & \text{otherwise.} \end{cases}$

9. The PSD of a RP, $x(t)$ is given by, $S_{xx} = \begin{cases} \pi, & |\omega| < 1 \\ 0, & \text{otherwise} \end{cases}$. Find autocorrelation.

10. If $y(t) = x(t+a) - x(t-a)$. PT:

a. $R_{yy}(\tau) = 2R_{xx}(\tau) - R_{xx}(\tau+2a) - R_{xx}(\tau-2a)$; Hence PT:

b. $S_{yy}(\omega) = 4\sin^2 a\omega \cdot S_{xx}(\omega)$.

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Unit - 3: SPECIAL RANDOM PROCESS.

Bernoulli:

Q1. If $Y[n] = 3X[n]+1$; $X[n]$ is a bernoulli process with a success probability p. Find mean and variance of $Y[n]$.

Q2. A sequence of bernoulli trial consist of choosing 7 components at random from a batch of component a selected component is classified as either defective or non-defective. A non-defective component is considered as success and defective as failure. If a probability that a selected component is not defective is 0.8. What is the probability of 3 success.

Q3. The probability that a patient recovers from a rare blood disease is 0.3. If 15 people are known to have contracted this disease. Find the following probabilities:

a. atleast 10 survive.

b. from 3 to 8 survive.

c. exactly 6 survive.

Q4. A sequence of bernoulli trial of choosing components at random from a batch of component. A selected components ~~at~~ are classified either defective or non-defective. Here defective → failure, non-defective → success. The probability of success = 0.8. Determine the probabilities of the following event:

a. first success occurs at 5^{th} trial.

b. third success occurs on 8^{th} trial.

c. there are 2 successes by the 4^{th} trial. there ~~are~~ as 4 success by the 10^{th} trial, there are 10 success by the 18^{th} trial.

Q5. consider a sequence of independent tosses of coin with probability p of heads in any toss. Let Y_N denote no. of heads in n consecutive tosses. Evaluate the probability of the following event ; $Y_5 = 3$, $Y_8 = 5$, $Y_{14} = 9$

Hint : $\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)\, dt$.

Markov :

Q1. Let X_n be the weather conditions in Mumbai on n^{th} day of rainy season. Suppose the chance, it will rain tomorrow depends on whether it is rainy today and not on the previous days. Suppose it rains today, the probability that it will rain tomorrow is p. Therefore, the probability that it will not rain tommorrow is $1-p$. Suppose, also that if it does not rain today, the probability that it will rain tomorrow is p'. Therefore, the probability that it will not rain tomorrow is $1-p'$. Write transition probability matrix.

Q2. Let X_n be the weather in Mumbai on n^{th} day. Let there be 3 states, 1 : cold, 2 : warm, 3 : hot. Lets assume that if it is cold today, probability that it will be cold, warm or hot tommorrow are 0.4, 0.4 and 0.2 respectively. The probability that it will be cold, warm or hot tomorrow are 0.1, 0.4 and 0.5 given today is warm. If it is hot today, the probability it will be cold, warm or hot are 0., 0.6, 0.4 respectively

Q3. consider a computer system in which digits 0, 1 are transmitted through several stages. At each stage the prob that the entering digit is unchanged is p. Let x_n denote the digit entering with n^{th} step then $\{x_n, n = 0, 1, 2 \ldots\}$ is a Markov chain with 2 states. Write transition probability matrix.

Q4. A machine in a factory has 2 states, 0, 1. 0 indicates it is working. 1 indicates not working. The probability that the machine will not change it's ~~cha~~ state of working or not working tommorrow on any day is p. Write the transition probability matrix and explain it fully.

Q5. Suppose there are 2 urns labelled 0, 1. Urn 0 contains 3 balls of which 2 are marked with no. 1 and third is marked with no. 0. The urn 1 contains 4 balls of which 3 are marked with 1 and 4^{th} is marked with 0. From an urn, a ball is drawn the no. on the ball is noted and returned to the urn. The urn from which the first ball is drawn is selected by tossing a fair coin. If the outcome is H, then urn 0 is selected. If the outcome is T, then urn 1 is selected. Therefter, the urn shown by the no. of ball on the previous draw is choosen for the next draw, ie. if the ball shows no. 0, then the next ball is drawn from urn 0, for the next ball draw. Show the sequence of events by a diagram and write a transition probability matrix. Find the probability of. 1, 1, 0, 1.

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Random Walk:

Q1. A certain student wanted to travel during a break to his parents. The bus fare $20. But the student had only $10. He figured there was a bar nearby where people played card games for money. The student signed up for a game where he would bet $1 per game. If he won, he would gain $1 and if lost, lose $1. If the probability of win is 0.6 independent of other game. What is the probability that he was not able to make the trip.

Q2. A bag contains 3 red balls, 6 green balls and 2 blue balls. Jack plays a game in which he bets $1 to draw a ball from the bag. If he draws a green ball, he wins $1, else losses $1. Assume that the balls are drawn with replacement and that jack starts the game with $50, with the hope of reaching $100, before going bankrupt. What is the probability that he will succeed.

Q3. Ben and Jerry play a series of game of checkers, during each game, player bets Rs. 1 and the winner gets Rs. 2. Ben is a better player than Jerry and has a probability of 0.6 of winning each game. Initially Ben had Rs. 9, while Jerry has Rs. 6. The game is over when either player is wiped out.

a. What is the probability that Ben is ruined, ie. Jerry wipes him out?

b. What is the probability that Jerry is ruined?

Poisson Distribution:

Q1. Suppose certain bank knows from it's past ~~costomers~~ experience that between 11am to 12 noon on each day, the mean arrival rate of customers at withdrawl window is 60. Find the probability that:

a. exactly 2 ; customers will arrive in a given 1 min. interval

b. atmost 2 ; customers will arrive in a given 1 min. interval between 11am to 12noon.

Q2. If a bank recieves on an avg. 6 bad cheques per day. What is the probability that it will recieve 4 bad cheques on any given day.

b. 10 bad cheques on any 2 consecutive days.

Q3. The no. of telephone calls arriving at a switch board within a time interval of length measured in minutes is a PP $X(t)$ with parameter $\lambda = 2$. Find the probability that no telephone call arrives in 5 mins. time.

Q4. A radioactive substance emits particle at a rate of 6 particles per min. as per a PP. The probability that an emitted particle is ~~recorded~~ recorded is 0.5. Find the probability of 10 particles will be recorded in 4 mins.

Gausian :

Q1. A WSS GP has $R_{xx}(\tau) = 6e^{-|\tau|/2}$. Determine the covariance matrix of the RV $x(t+1)$, $x(t)$, $x(t+2)$, $x(t+3)$.

Q2. GP with $R_{xx}(\tau) = 4 + e^{-|\tau|}$. Determine the covariance matrix for the RV $x(0)$, $x(1)$, ~~x(2)~~, $x(3)$, $x(6)$.

Q3. Suppose $x(t)$ is a ~~RP~~ RGP with a mean 0, $R_{xx}(\tau) = e^{-|\tau|}$. Assume that the RV A is defined as $A = \int_0^1 x(t)\,dt$. Determine the following :

a. $E[A]$

b. ~~E[~~ $\sigma^2 A$.

Q4. If $x(t)$ is a GP with mean $\mu(t) = 10$ and covar $C(t_1, t_2) = 16e^{-|t_1 - t_2|}$. Find the probability that :

a. $x(10) \leq 8$

b. $|x(10) - x(6)| \leq 4$

c. $x(10) - x(6) \leq 4$.

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Unit - 4: ESTIMATION THEORY.

Q1. Prove that sample mean $\bar{x}$ is an unbiased estimator of the population mean μ.

Q2. If x is a RV having BD with parameter n and p, then the observed proportion for success in an unbiased estimator p. Prove.

Q3. The joint PDF of x, y is :

x/y	-1	0	1	T
-1	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{3}$
0	0	$\frac{1}{3}$	0	$\frac{1}{3}$
1	$\frac{1}{6}$	0	$\frac{1}{3}$	$\frac{1}{3}$
T	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1.

Find the min. mean sq. error Y given X, ie $Y = a^* + b^* x$

where $a^* = \frac{cov(x, y)}{\sigma x^2}$; $b^* = m_Y - a^*(m_X)$

Q4. If the joint PDF of x, y is given by,

x/y	-1	0	1	T
-1	0	0	$\frac{1}{3}$	$\frac{1}{3}$
0	0	$\frac{1}{3}$	0	$\frac{1}{3}$
1	$\frac{1}{3}$	0	0	$\frac{1}{3}$
T	$1/3$	$1/3$	$1/3$	1.

Find the min. sq. error linear estimator of x.

Q5. If X is uniformly distributed in interval (-1) and $Y = X^2$. Find the best linear estimator for Y in terms of X.

Q6. The following are the misprints found on 10 pages of a book. Assuming the printing mistake is a ~~poision~~ poisson distribution with parameter λ. Find the MLE(λ).

Q7. Find the MLE(p) of BD.

Q8. Find the MLE of μ and σ^2 of normal distribution.

$$f_X(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-1/2\left(\frac{x-\cancel{\sigma}\mu}{\sigma}\right)^2}$$; here $L(\theta) = f(x_1, \mu, \sigma) \cdot f(x_2, \mu, \sigma)$.

Q9. The diameter of cylinderical ~~side~~ rods coming out of production line is a normal variate with unknown mean and std. dev. A sample of 10 rods gave the following diameter. Find the MLE of mean and σ^2.

80, 84, 82, 88, 86, 88, 85, 84, 81, 84.

Q10. If $x_1, x_2, \dots x_n$ be a random sample of size n for normal population with mean $\bar{x}$ and σ^2. Show that, the estimator $S^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})$ is not an unbiased estimator of the parameter σ^2.

Q11. S^2 becomes unbiased for σ^2.

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Q12. Find the MLE of parameter λ of exp. distribution.

Q13. For the sample $x_1, x_2, \ldots, x_n$ drawn on X, which takes the values 1 and 0 with respective probabilities p, 1-p. Show that $\frac{\Sigma x_i (\Sigma x_i - 1)}{n(n-1)}$ is an unbiased estimator of p^2.

Q14. If T is the unbiased estimator for β is a unbiased estimator for β^2.

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RPET – UNIT 5 : HYPOTHESIS TESTING.

1. 2 samples drawn from 2 different population gave the following results :

	size	mean	SD
I	400	124	14
II.	250	120	12.

Find : 95% confidence limits for the difference between the population means.

2.

	size	mean	SD
I	300	87	10
II.	250	84	8

Find: 95% confidence limits for difference between population.

3. Means of 2 samples $n_1 = 1000$ and $n_2 = 2000$ respectively are $\bar{x}_1 = 67.5$ and $\bar{x}_2 = 68$ inches. Can the sample be regarded as drawn from the sample population of SD 2.5 inches.

4. The average of marks ~~80~~ scored by 32 boys is 72 with SD 8, while that of girls (36) is 70 with SD 6. Test at 1% LOS whether boys perform better than girls.

5. 2 populations have same $(\bar{x}_1 - \bar{x}_2)$ mean but SD of one is twice that of the other $(\sigma_1 = 2\sigma_2)$. ST in samples each of size 500 $(n_1 = n_2 = 500)$ drawn under simple random conditions. The difference of the means of all probabilities will not exceed 0.3σ where σ is the smaller SD.

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6. A sample size of 10 has mean 40 and σ as 10. Construct 99% confidence interval for the population mean.

7. 7 homemakers were randomly sampled and it was determined that they walked in their housework had an avg. of 39.2 miles per week and sample SD 3.2 miles per week. Construct the 95% and 99% CI for the population.

8. A Sample of size 9 from a normal population gave $\bar{x} = 15.8$ and has $SD^2 = 10.3$. Find 99% interval of population mean.

9. A soap manufacturing company was distributing a particular brand soap through a large no. of retail. Shops. Before a heavy ad. campaign the mean ~~81~~ sales per week per soap was 140 dozen. After the campaign a sample of 20 soaps were taken and the mean sale was found to be 147 dozen. With SD of 16, can you consider the ad effective.

10. A plastic chair manufacturing company claims that the mean life of it's chair is 10 yrs with SD of 1 yr. A random Sample of 6 chair found to have mean-life with foll. values : 10, 11, 9, 12, 10, 7 yrs. Use t-test to find, if the company's claim is valid at 5% LOS.

11. A random sample of size 16 from a normal population Showed a mean of 103.75 cm and sum of sq. of dev. from the mean is 843.75 cm^2. Can we say, that the population has mean of ~~18~~ 108.75 cm.

12. 2 independent random samples of sizes 15 and 8 resp. have following means and SD's.

13. A sample of 8 students of 16 yrs each shown a mean systollic BP of 118.4mm of Hg with SD of 12.17 while a sample of 10 students of 17 yrs each shown a mean systollic BP of 121mm of Hg with SD of 12.8 during an investigation. The investigator feels that the systollic BP is related to age. Do you think that the data provide enough region to support investigator feeling at 5% LOS. [Assume the distribution of systollic BP to be normal]

14. The mean of 2 random samples of size 9 and 7 are given as 196.42 and 198.82 resp. The sum of sq. of the dev. from the means are 26.94 and 18.73. Can the sample be considered to have been drawn from the same population.

15. Samples of 2 types of electric bulbs were tested for length of life and the following data got obtained.

	Type I	Type II
No. of samples	8	7
Mean (hrs)	1134	1024
SD (hrs)	35	40

Test at 5% LOS whether the diff. in the somple mean is significant.

16. Suppose there are 2 samples of food items from first lot of 200 samples packet of food items. It is observed that the mean wt. of food items is 30gm with SD = 3.2gm. From the 2^{nd} lot of 210 samples packets of the food itema, it is observed that the mean wt. of food items is 32gms with SD = 1gm. Can we conclude that both the samples have significant diff. in mean wt. at 5% LOS.

17. In a random sample of 2000, wooden doors from India, 1900 are found to be of good quality. Do these details indicates a significant ~~a~~ difference between the quality of wooden doors from India and China? Test the significance at 1%.

18. In a recent statewide election, 55% of voters rejected lotteries. A random sample of 150 urban precints showed that 49% votes rejected lotteries. Is the diff. significant?

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