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Tutorial (8)

1) Pigeon-hole principle

1) There are 50 baskets of apples. Each basket contains no more than 24 apples. Show that there are atleast 3 baskets containing the same no. of apples.

→ According to the Pigeon-hole principle, the least no. of baskets that have the same no. of apples $= \left\lceil \frac{n}{k} \right\rceil$

$n = 50 \quad k = 24$

$\left\lceil \frac{n}{k} \right\rceil = \left\lceil \frac{50}{24} \right\rceil = 3$

2) Show that among any 4 digit nos., one can find 2 nos. so that their difference is divisible.

→ If 2 nos. when divided by 3, results in the same remainder, then their difference is divisible by 3.

The possible remainders when divided by 3 are 3.

$n = 4, k = 3$

Thus, by pigeonhole principle, the no. whose difference is divisible by 3

$= \left\lceil \frac{n}{k} \right\rceil = \left\lceil \frac{4}{3} \right\rceil = 2$

(4) Given 12 different 2-Digit nos., sh
that one can choose two of them
that their difference is a 2-Digit
with identical first and second di

→ Divisor = 11

if 2 no. when divided by 11 have t
Same remainder, than in their dif
the first and second digit will be
identical.

$n = 12$
$k = 11$

Thus, by pigeonhole principle, the no
differences having identical first
second digit $\Rightarrow \left\lceil \frac{n}{k} \right\rceil$

$\left\lceil \frac{12}{11} \right\rceil = 2$

(5) There are 13 squares of side 1 posi
inside a circle of radius 2. show
atleast (2) of the squares have a co
point.

→ Area of square $\Rightarrow 13 \times 1 = 13$ sq. unit
Area of circle $\Rightarrow \pi(r)^2 \Rightarrow \pi.(2)^4 \Rightarrow 12.$
sq.

Since, the area of square is more
area of circle.

Thus, by pigeonhole principle, the no. of square having a common point

$\Rightarrow \left\lceil \frac{n}{k} \right\rceil$

$n = 13$
$k = 12.56$

$\left\lceil \frac{13}{12.56} \right\rceil = 2$

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Tutorial (9)

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3) Recurrence Relations

1) What is the solution of the recurrence relation
$a_n = 6a_{n-1} - 9a_{n-2}$
with initial conditions $a_0 = 1$ and $a_1 = 6$?

→ $a_n = 6a_{n-1} - 9a_{n-2}$

(i) Put $a_n = r^n$

$r^n = 6r^{n-1} - 9r^{n-2}$

$r^n - 6r^{n-1} + 9r^{n-2} = 0$

(ii) Divide LHS by r^{n-2}

$r^2 - 6r + 9 = 0$ → characteristic Equation

$(r-3)^2 = 0$

$r = 3, 3$ (Roots are repeated)

The solution is of the form -

$a_n = \alpha_1 (r_1)^n + \alpha_2 n (r_2)^n$ — ①

$a_n = \alpha_1 (3)^n + \alpha_2 n (3)^n$

(iii) $a_0 = 1$

$1 = \alpha_1 . 3^0 + \alpha_2 . (0) . 3^0$

$\boxed{\alpha_1 = 1}$

$a_1 = 6$

$6 = \alpha_1 . (3)^1 + \alpha_2 (1) . 3^1$

$\boxed{\alpha_2 = 1}$

Put value of α_1 and α_2 in equation (1)

$a_n = 1 . 3^n + 1 . n . 3^n$

$\boxed{a_n = 3^n + 3^n n}$

(2) Find the solution of recurrence relat

$a_n = 5a_{n-1} - 6a_{n-2} + 7^n$

→ $a_n = 5a_{n-1} - 6a_{n-2} + 7^n$

$a_n - 5a_{n-1} + 6a_{n-2} = 7^n$

$a_n = a_n^{(h)} + a_n^{(p)}$

(i) put $a_n = r^n$

$r^n - 5r^{n-1} + 6r^{n-2} = 0$

Divide the term by r^{n-2}

characteristic eqn →

$r^2 - 5r + 6 = 0$

$r = 2, 3$

The solution is of the form

$a_n^{(h)} = \alpha_1 2^n + \alpha_2 3^n$ —①

(ii) $F(n) = 7^n$

Solution is of the form $A \cdot 7^n$

$a_n - 5a_{n-1} + 6a_{n-2} = 7^n$

$A \cdot 7^n - 5A7^{n-1} + 6A7^{n-2} = 7^2$

Divide by 7^{n-2}

$49A - 35A + 6A = 49$

$20A = 49$

$\boxed{A = \frac{49}{20}}$

$a_n^{(p)} = \left(\frac{49}{20}\right) 7^n$

Soln. $\boxed{a_n = \alpha_1 \cdot 2^n + \alpha_2 \cdot 3^n + \left(\frac{49}{20}\right) \cdot 7^n}$

Solve the recurrence relation
$d_n = 4(d_{n-1} - d_{n-2})$
Subject to the initial conditions $d_0 = 1 = d_1$.

→ $d_n = 4(d_{n-1} - d_{n-2})$
$d_n - 4d_{n-1} + 4d_{n-2} = 0$
(i) Put $d_n = r^n$
$r^n - 4r^{n-1} + 4r^{n-2} = 0$
Divide LHS by r^{n-2},
characteristic eqn →
$r^2 - 4r + 4 = 0$
$r = 2, 2$ (Roots are repeated)
The solution is of the form -

$$d_n = \alpha_1 (2)^n + \alpha_2 . n . (2)^n \quad \text{—} ①$$

(ii) $d_0 = 1$
$1 = \alpha_1 (2)^0 + \alpha_2 (2)^0 . (0)$
$\boxed{\alpha_1 = 1}$

$d_1 = 1$
$1 = \alpha_1 (2)^1 + \alpha_2 . 1 . (2)^1$
$2 + \alpha_2 . (2) = 1$
$\boxed{\alpha_2 = \frac{-1}{2}}$

Solution →
$$\boxed{d_n = 1 . 2^n + \left(\frac{-1}{2}\right) . n . 2^n}$$

(4) Determine whether the sequence is solution of recurrence relation $a_n =$ for $n = 2, 3, 4, \ldots$ where $a_n = 3n$ for every non-negative integer (n). Answer the same question for $a_n = 5$.

→ $a_1 = 3(1) = 3$

$a_2 = 3(2) = 6$

$a_3 = 3(3) = 9$

$a_3 = 2a_2 - a_1$
$= 2(6) - 3 = 9$

$a_1 = 5$

$a_2 = 5$

$a_3 = 5$

$a_3 = 2a_2 - a_1$
$= 2 \times 5 - 5 = 10 - 5$
$= 5$

Hence, yes, the sequence a_n is the solution of recurrence relation for $a_n = 3n$ and $a_n = 5$ for every non negative integer (n).

(5) Find the solution of $a_{n+2} + 2a_{n+1} - 3$ that satisfies $a_0 = 1$, $a_1 = 2$.

→ $a_{n+2} + 2a_{n+1} - 3a_n = 0$

(i) Put $a_n = r^n$

$r^{n+2} + 2r^{n+1} - 3r^n = 0$

Divide LHS by r^n,

characteristics eq^n -

$$r^2 + 2r - 3 = 0$$
$$r = 1, -3$$

the solution is of the form -

$$a_n = \alpha_1 (1)^n + \alpha_2 (-3)^n \quad \text{--- (1)}$$

(ii) $a_0 = 1$

$$\alpha_1 (1)^0 + \alpha_2 (-3)^0 = 1$$
$$\alpha_1 + \alpha_2 = 1 \quad \text{--- (2)}$$

$a_1 = 2$

$$\alpha_1 (1)^1 + \alpha_2 (-3)^1 = 2$$
$$\alpha_1 - 3\alpha_2 = 2 \quad \text{--- (3)}$$

put eqn (2) in (3)

$\alpha_1 = 5/4$ $\qquad$ $\alpha_2 = \frac{-1}{4}$

Solution $\Longrightarrow$

$$\boxed{a_n = \left(\frac{5}{4}\right) \cdot (1)^n + \left(\frac{-1}{4}\right) \cdot (-3)^n \quad \text{or} \quad a_n = \frac{5}{4} \cdot (1)^n - \left(\frac{1}{4}\right) \cdot (-3)^n}$$

3) Find the solution of Fibonacci relation $a_n = a_{n-1} + a_{n-2}$ with the initial conditions $a_0 = 0$, $a_1 = 1$.

$\rightarrow$

$$a_n - a_{n-1} - a_{n-2} = 0$$

(i) put $a_n = r^n$

$$r^n - r^{n-1} - r^{n-2} = 0$$

Divide term by r^{n-2},

$r^2 - r - 1 = 0 \rightarrow$ characteristic

$r = \frac{1 \pm \sqrt{5}}{2}$

The solution is of the form -

$$a_n = \alpha_1 \left(\frac{1+\sqrt{5}}{2}\right)^n + \alpha_2 \left(\frac{1-\sqrt{5}}{2}\right)^n$$

(ii) $a_0 = 0$

$$\alpha_1 \left(\frac{1+\sqrt{5}}{2}\right)^0 + \alpha_2 \left(\frac{1-\sqrt{5}}{2}\right)^0 = 0$$

$$\alpha_1 + \alpha_2 = 0 \quad - ② \longrightarrow \alpha_1 = -\alpha_2$$

$a_1 = 1$

$$\alpha_1 \left(\frac{1+\sqrt{5}}{2}\right)^1 + \alpha_2 \left(\frac{1-\sqrt{5}}{2}\right)^1 = 1$$

$$1.6180\,\alpha_1 + (-0.6180)\,\alpha_2 = 1 \quad - ③$$

put eqn ② in ③

$$-1.6180\,\alpha_2 - 0.6180\,\alpha_2 = 1$$

$$\alpha_1 = 0.447 \approx 0.45 \quad \alpha_2 = -0.447 \approx -0.45$$

Solution $\Rightarrow$

$$\boxed{\begin{gathered} a_n = 0.45(1.6180)^n + (-0.45)(0.6180)^n \\ \text{or} \\ a_n = 0.45\left(\frac{1+\sqrt{5}}{2}\right)^n + (-0.45)\left(\frac{1-\sqrt{5}}{2}\right)^n \end{gathered}}$$

7) Given that $a_0 = 0, a_1 = 1, a_2 = 4$ and $a_3 = 12$ satisfy the recurrence relation

$$a_r + C_1 a_{r-1} + C_2 a_{r-2} = 0$$

Determine a_r.

→ $a_r + C_1 a_{r-1} + C_2 a_{r-2} = 0$

Put $r = 3$

$a_3 + C_1 a_2 + C_2 a_1 = 0$

$12 + 4C_1 + C_2 = 0$ — ①

put $r = 2$

$a_2 + C_1 a_1 + a_0 = 0$

$4 + C_1 + 0 = 0$

$\boxed{C_1 = -4}$

Put value of C_1 in eqⁿ ①

$\boxed{C_2 = 4}$

$a_r = -C_1 a_{r-1} - C_2 a_{r-2}$

$= -(-4) a_{r-1} - (4) a_{r-2}$

$$\boxed{\begin{aligned} a_r &= 4(a_{r-1} - a_{r-2}) \\ &\text{or} \\ &= 4a_{r-1} - 4a_{r-2} \end{aligned}}$$

8) Find the solution of recurrence relation

$$a_n = 5a_{n-1} - 6a_{n-2} + 7n$$

→ $a_n = a_n^{(h)} + a_n^{(p)}$

$a_n - 5a_{n-1} + 6a_{n-2} = 7n$

(i) put $a_n = r^n \longrightarrow r^n - 5r^{n-1} + 6r^{n-2} = 0$

Divide by (r^{n-2}),

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$r^2 - 5r + 6 = 0$ — characteristic

$r = 2, 3$

The solution is of the form -

$$a_n^{(h)} = \alpha_1 (2)^n + \alpha_2 (3)^n$$

(ii) $F(n)$ is of the form $7n$.

Solution is of the form $A_0 + A_1 n$

$$a_n - 5a_{n-1} + 6a_{n-2} = 7n$$

$$A_0 + A_1 n - 5(A_0 + A_1(n-1)) + 6(A_0 + A_1(n-2))$$

$$A_0 + A_1 n - 5A_0 + 5A_1 - 5A_1 n + 6A_0 + 6A_1 n = 7n$$

$$2A_0 + 2A_1 n - 7A_1 = 7n$$

$$2A_0 = 0$$

$$\boxed{A_0 = 0}$$

$$2A_1 n = 7n$$

$$\boxed{A_1 = \frac{7}{2}}$$

$$a_n^{(p)} = A_0 + A_1 n$$
$$= 0 + \frac{7}{2} n$$

The solution is $\rightarrow$

$$\boxed{a_n = \alpha_1 (2)^n + \alpha_2 (3)^n + \frac{7}{2} n}$$

(a) Determine the sequence whose recurrence relation is $a_n = 4a_{n-1} + 5a_{n-2}$ with $a_1 = 2$ and $a_2 = 6$.

→ $a_n - 4a_{n-1} - 5a_{n-2} = 0$

(i) put $a_n = r^n$

$r^n - 4r^{n-1} - 5r^{n-2} = 0$

Divide by r^{n-2} on LHS,

$r^2 - 4r - 5 = 0$ → characteristic eqⁿ

$r = 5, -1$

The solution is of the form -

$a_n = \alpha_1 (5)^n + \alpha_2 (-1)^n$

(ii) $a_1 = 2$

$\alpha_1 (5)^1 + \alpha_2 (-1)^1 = 2$

$5\alpha_1 - \alpha_2 = 2$ — ①

$a_2 = 6$

$\alpha_1 (5)^2 + \alpha_2 (-1)^2 = 6$

$25\alpha_1 + \alpha_2 = 6$ — ②

By eqⁿ ① and ② —

we get,

$\alpha_1 = \frac{4}{15}$ $\alpha_2 = \frac{-2}{3}$

Solution ⇒

$$\boxed{a_n = \frac{4}{15}(5)^n - \frac{2}{3}(-1)^n}$$

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(10) Solve the following recurrence rela

(i) $a_r - 7a_{r-1} + 10a_{r-2} = 0$
given that $a_0 = 0$; $a_1 = 3$

→ $a_r - 7a_{r-1} + 10a_{r-2} = 0$

(i) Put $a_r = n^r$
$n^r - 7n^{r-1} + 10n^{r-2} = 0$
Divide by n^{r-2},

$n^2 - 7n + 10 = 0$ → characteristic

$\boxed{n = 5, 2}$

The solution is of the form -

$a_r = \alpha_1 (5)^r + \alpha_2 (2)^r$

(ii) $a_0 = 0$
$\alpha_1 (5)^0 + \alpha_2 (2)^0 = 0$
$\alpha_1 + \alpha_2 = 0$ — (1)

$a_1 = 3$
$\alpha_1 (5)^1 + \alpha_2 (2)^1 = 3$
$5\alpha_1 + 2\alpha_2 = 3$ — (2)

from eqn (1) and (2), we get -
$\alpha_1 = 1$ $\alpha_2 = -1$

Solution ⇒
$\boxed{a_r = 1.(5)^r - 1.(2)^r}$ or

$\boxed{a_r = 5^r - 2^r}$

(ii) $a_r - 4a_{r-1} + 4a_{r-2} = 0$

given that $a_0 = 1$, $a_1 = 6$

→ $a_r - 4a_{r-1} + 4a_{r-2} = 0$

(i) Put $a_r = n^r$

$n^r - 4n^{r-1} + 4n^{r-2} = 0$

Divide by n^{r-2},

$n^2 - 4n - 4 = 0$ ← characteristic eq^n

$\boxed{n = 2, 2}$ (repeated roots)

The solution is of the form –

$a_r = \alpha_1 (2)^r + \alpha_2 \cdot r \cdot (2)^r$

(ii) $a_0 = 1$

$\alpha_1 (2)^0 + \alpha_2 (0)(2)^0 = 1$

$\boxed{\alpha_1 = 1}$

$a_1 = 6$

$\alpha_1 (2)^1 + \alpha_2 (1) \cdot (2)^1 = 6$

$2\alpha_1 + 2\alpha_2 = 6$ — ①

Put α_1 value in eq^n ② –

$\boxed{\alpha_2 = 2}$

Solution ⇒

$$\boxed{\begin{array}{c} a_r = (1)(2)^r + 2 \cdot r \cdot (2)^r \\ \text{or} \\ a_r = 2^r (1 + 2r) \end{array}}$$

(iii) $a_r + 6a_{r-1} + 9a_{r-2} = 3$

given that $a_0 = 0$, $a_1 = 1$

$a_r = a_r^{(h)} + a_r^{(p)}$

$a_r + 6a_{r-1} + 9a_{r-2} = 0$

→ (i) put $a_r = n^r$

$n^r + 6n^{r-1} + 9n^{r-2} = 0$

Divide by n^{r-2},

$n^2 + 6n + 9 = 0$ → characteristic eqn

$\boxed{n = -3, -3}$

the solution is of the form

$a_r^{(h)} = \alpha_1 (-3)^r + \alpha_2 (-3)^r \cdot r$

(ii) $F(r) = 3$

Solution is of $A \cdot 3$

$a_r + 6a_{r-1} + 9a_{r-2} = 3$

$A \cdot 3 + 6 \cdot A \cdot 3 + 9 \cdot A \cdot 3 = 3$

Divide by 3 both sides,

$A + 6A + 9A = 1$

$\boxed{A = \frac{1}{16}}$

$a_r^{(p)} = A \cdot 3 = \frac{3}{16}$

$a_r = \alpha_1 (-3)^r + \alpha_2 (-3)^r \cdot r + \frac{3}{16}$

(iii) $a_0 = 0$

$\alpha_1 (-3)^0 + \alpha_2 (-3)^0 \cdot (0) + \frac{3}{16} = 0$

$\alpha_1 + \frac{3}{16} = 0$

$\boxed{\alpha_1 = -3/16}$ (1)

$a_1 = 1$

$\alpha_1 (-3)^1 + \alpha_2 (-3)^1 \cdot 1 + \frac{3}{16} = 1$

$$\boxed{-3\alpha_1 - 3\alpha_2 = \frac{13}{16}} \quad \text{--- (2)}$$

By solving eqn (1) and (2), we get-

$\alpha_1 = \frac{-3}{16}$ $\qquad$ $\alpha_2 = \frac{1}{4}$

solution $\Rightarrow$

$$\boxed{a_r = \left(\frac{-3}{16}\right)(-3)^r + \left(\frac{1}{4}\right) \cdot r \cdot (-3)^r}$$

v) $a_r + a_{r-1} + a_{r-2} = 0$

given that $a_0 = 0$; $a_1 = 2$.

$\rightarrow$ $a_r + a_{r-1} + a_{r-2} = 0$

(i) Put $a_r = n^r$

$n^r + n^{r-1} + n^{r-2} = 0$

Divide term by n^{r-2};

$n^2 - n + 1 = 0 \rightarrow$ characteristic eqn

$n = \frac{-1}{2} \pm \frac{\sqrt{3}}{2} i$

The solution is of the term -

$a_r = \alpha_1 \left(\frac{-1}{2} + \frac{\sqrt{3}}{2} i\right)^r + \alpha_2 \left(\frac{-1}{2} - \frac{\sqrt{3}}{2} i\right)^r$

(ii) $a_0 = 0$

$\alpha_1 + \alpha_2 = 0$

$\alpha_1 = -\alpha_2$

$a_1 = 2$

$\alpha_1 \left(\frac{-1}{2} + \frac{\sqrt{3}}{2} i\right)^1 + \alpha_2 \left(\frac{-1}{2} - \frac{\sqrt{3}}{2} i\right)^1 =$

$-\alpha_2 \left(\frac{-1}{2} + \frac{\sqrt{3}}{2} i\right)^1 + \alpha_2 \left(\frac{-1}{2} - \frac{\sqrt{3}}{2} i\right)^1 =$

$\frac{\cancel{\alpha_2} - \alpha_2\sqrt{3} i + \cancel{(-\alpha_2)} - \alpha_2 \sqrt{3} i}{2} = 2$

$\frac{-\cancel{2}\alpha_2 \sqrt{3} i}{\cancel{2}} = 2$

$$\boxed{\alpha_2 = \frac{-2}{\sqrt{3} i}}$$

$$\boxed{\alpha_1 = \frac{+2}{\sqrt{3} i}}$$

solution –

$$\boxed{a_n = \left(\frac{2}{\sqrt{3} i}\right)\left(\frac{-1 + \sqrt{3} i}{2}\right)^n - \left(\frac{2}{\sqrt{3} i}\right)\left(\frac{-1 - \sqrt{3} i}{2}\right)}$$

(v) $a_r - 2a_{r-1} + 2a_{r-2} - a_{r-3} = 0$
given that $a_0 = 2,\ a_1 = 1,\ a_2 = 1$

→ $a_r - 2a_{r-1} + 2a_{r-2} - a_{r-3} = 0$

(i) put $a_r = n^r$

$n^r - 2n^{r-1} + 2n^{r-2} - n^{r-3} = 0$

Divide by n^{r-3},

$n^3 - 2n^2 + 2n - 1 = 0 \rightarrow$ characteristic eq^n

$n = 1, \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

The solution is of the form-

$$a_r = \alpha_1 + \alpha_2\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)^r + \alpha_3\left(\frac{1}{2} - \frac{\sqrt{3}i}{2}\right)^r$$

(ii) $a_0 = 2$

$$\boxed{\alpha_1 + \alpha_2 + \alpha_3 = 2} \quad \text{— 1}$$

$a_1 = 1$

$$\boxed{\alpha_1 + \alpha_2\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)^1 + \alpha_3\left(\frac{1}{2} - \frac{\sqrt{3}i}{2}\right)^1 = 1} \quad \text{2}$$

$a_2 = 2$

$$\alpha_1 + \alpha_2\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)^2 + \alpha_3\left(\frac{1}{2} - \frac{\sqrt{3}i}{2}\right)^2 = 1$$

$$\alpha_1 + \frac{\alpha_2}{\cancel{4}_2}(\sqrt{3}i - 1)\cancel{\times 2} + \frac{\alpha_3}{\cancel{4}_2}(\cancel{-2})(\sqrt{3}i + 1) = 1$$

$$\boxed{\alpha_1 + \alpha_2\left(\frac{\sqrt{3}i - 1}{2}\right) - \alpha_3\left(\frac{\sqrt{3}i + 1}{2}\right) = 1} \quad \text{— 3}$$

by solving eq^n 1, 2, 3 –
we get,

$\alpha_1 = 2 \qquad \alpha_2 = \frac{1}{\sqrt{3}i} \qquad \alpha_3 = \frac{-1}{\sqrt{3}i}$

Solution $\rightarrow$

$$\boxed{a_r = 2 + \frac{1}{\sqrt{3}i}\left(\frac{1 + \sqrt{3}i}{2}\right)^r + \frac{1}{\sqrt{3}i}\left(\frac{1 - \sqrt{3}i}{2}\right)^r}$$

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(11) Find the particular solution of the

(a) $a_r - 5a_{r-1} + 6a_{r-2} = 1$

$a_r = a_r^{(h)} + a_r^{(p)}$

(i) put $a_r = n^r$

$n^r - 5n^{r-1} + 6n^{r-2} = 1$

Divide by n^{r-2},

$n^2 - 5n + 6 = 0 \rightarrow$ characteristic e

$n = 3, 2$

The solution is of the form -

$$\boxed{a_r^{(h)} = \alpha_1 (3)^r + \alpha_2 (2)^r}$$

(ii) $F(n) = 1$

Solution is of the form $A \cdot 1$

$a_r - 5a_{r-1} + 6a_{r-2} = 1$

$A \cdot 1 - 5 \cdot A \cdot 1 + 6 \cdot A \cdot 1 = 1$

$A - 5A + 6A = 1$

$2A = 1$

$$\boxed{A = \frac{1}{2}}$$

$a_r^{(p)} = A \cdot 1 = \frac{1}{2}$

Solution $\rightarrow$

$$\boxed{a_r = \alpha_1 (3)^r + \alpha_2 (2)^r + \frac{1}{2}}$$

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b) $a_r + a_{r-1} = 3r2^r$

$a_r = a_r^{(h)} + a_r^{(p)}$

(i) Put $a_r = n^r$

$n^r + n^{r-1} = 0$

Divide by n^{r-1},

$n + 1 = 0$

$\boxed{n = -1}$

The solution is of the form -

$a_r^{(h)} = \alpha_1 (-1)^n$

(ii) $F(n) = 3r2^r$

solution is of the form $(A_1 n + A_0) 2^n$

(c) $a_r - 2a_{r-1} = 3.2^r$

→ (i) put $a_r = n^r$

$n^r - 2n^{r-1} = 0$

Divide term by n^{r-1},

$n - 2 = 0$

$\boxed{n = 2}$

The solution is of the form -

$a_n^{(h)} = \alpha_1 . (2)^n$

(ii)

$F(n) = 3.2^r$

Solution is of the form → $2^r . A$ i)

$a_r - 2a_{r-1} = 3.2^r$

$rA.2^r - 2_2^A(r-1) A \cancel{\;}(r-1) = 3.2^r$

d) $a_r - 4a_{r-1} + 4a_{r-2} = (r+1)2^r$

$a_r = a_r^{(h)} + a_r^{(P)}$

(i) Put $a_r = n^r$

$n^r - 4n^{r-1} + 4n^{r-2} = 0$

Divide term by n^{r-2},

$n^2 - 4n + 4 = 0 \rightarrow$ characteristic eq^n

$\boxed{n = 2, 2}$ (repeated roots)

The solution is of the form $\rightarrow$

$a_r^{(h)} = \alpha_1 (2)^r + \alpha_2 \cdot r \cdot (2)^r$

ii) $F(n) = (r+1)2^r$

Solution is of the form $= (A_1 n + A_0) n^2 \cdot 2^n$

$a_r - 4a_{r-1} + 4a_{r-2} = (r+1)2^r = n^2 \cdot 2^n$

$(A_1 n + A_0) n^2 2^n - 4(A_1(n-1)$

(12) Find the homogeneous solution of the following -

(a) $a_r + 6a_{r-1} + 12a_{r-2} + 8a_{r-3} = 0$

→ Put $a_r = n^r$

$n^r + 6n^{r-1} + 12n^{r-2} + 8n^{r-3} = 0$

Divide by $n^{r-3} = 0$

$n^3 + 6n^2 + 12n + 8 = 0$ → characte

The solution is of the form -

$$\boxed{\begin{gathered} a_r = \alpha_1(-2)^r + r\alpha_2(-2)^n + r^2\alpha_3 \\ \text{or} \\ a_r = (-2)^r[\alpha_1 + r\alpha_2 + r^2\alpha_3] \end{gathered}}$$

(b) $4a_r - 20a_{r-1} + 17a_{r-2} - 4a_{r-3} = 0$

→ Put $a_r = n^r$

$4n^r - 20n^{r-1} + 17n^{r-2} - 4n^{r-3} = 0$

Divide by n^{r-3},

$4n^3 - 20n^2 + 17n - 4 = 0$ → characte

The solution is of the form -

$$\boxed{a_r = (\alpha_1 + \alpha_2 r)(1/2)^n + \alpha_3 \cdot (4)^n}$$