UNIT : 1 – LINEAR PROGRAMMING.

Formulation → Graphical

Formulation → Simplex.

conditions : simplex

1. The RHS of each constraint b_i should be non-negative.
2. each of the decision variable of a problem should be non-negative. (if it is negative, multiply with -1)

$$2x_1 + 3x_2 \leq 60$$

$$2x_1 + 3x_2 + S_1 = 60 \longrightarrow (i) ; \; S_1 : \text{slack variable}$$

$$(0 \leq S_1 \leq 60)$$

- slack variables are used to equate the LHS and RHS
- the co-eff of slack variable is zero as it is not a constraint

$$4x_1 + 3x_2 + S_2 = 96 \longrightarrow (ii) \quad (0 \leq S_2 \leq 96)$$

$$\therefore \quad x_1, x_2, S_1, S_2 \geq 0.$$

$$Z_{max} = 40x_1 + 35x_2 + 0S_1 + 0S_2.$$

Augmented form of LPP : Standardised form of LPP.

This states that : maximise : $Z_{max} = 40x_1 + 35x_2 + 0S_1 + 0S_2$

subject to : $2x_1 + 3x_2 + S_1 + 0S_2 = 60$

$4x_1 + 3x_2 + 0S_1 + S_2 = 96.$

and

$x_1, x_2, S_1, S_2 \geq 0.$

solution to simplex : to illustrate the solution of eqn (i) and (ii), let the solution found be (10, 8)

$\therefore x_1 = 10$
$x_2 = 5$

on substituting x_1, x_2 in eqn (i) and (ii) we get,
$s_1 = 25$, $s_2 = 41$.

thus the augmented form of solution is (10, 5, 25, 41)

when a LPP has 'n' variables and 'm' constraints we can have '^nC_m' basic solutions.

- in our question :
  $n = 4 \quad (x_1, x_2, s_1, s_2)$
  $m = 2 \quad (x_1, x_2)$

$\therefore$ the solutions obtained are :
$^nC_m = {}^4C_2 = 6$.

thus there are 6 basic solutions.

- In a basic solution, some variables are basic, while other are non-basic

The no. of basic variables = No. of constraints
no. of non-basic variables = No. of variables − No. of

- The non-basic variables are all set to zero and values of basic variables are obtained by solving the eqⁿ simultaneously.

- If the values of basic variables are non-negative then the solution is said to be : "BASIC FEASIBLE SOLUTION".

Steps : Simplex Method

i. obtain an initialisation
ii. test if the solution is optimal : if yes, then exit
else goto (iii)
iii. obtain an improved solution and goto step (ii)

$\Delta j = (c_j - z_j)$ [if the formula is given by $\Delta j = (z_j - c_j)$ then just follow the opp. of these rules]

- c_3 is the co-eff used to maximise or minimise the function.
- the size of the identity matrix depends on constraints in the eqn.

Q. $3x_1 + 7x_2 \geq 105 + \underline{S_3}$

↳ surplus variables

$2x_1 + 3x_2 - 15\underline{S_4} = 60$

↳ surplus variables.

moving on,

$3x_1 + 7x_2 - 15S_3 + \underline{A_1} = 105$

↳ artificial var.

$2x_1 + 3x_2 - 15S_4 + A_2 = 60$

here S_3 and S_4 are surplus variables.

we use artificial variable.

→ a simplex tableau depicts an optimal soln if all entries in Δj row are

a. zero or negative (ie. all $\Delta j \leq 0$) ⇒ LPP maximisation.

b. zero or positive (ie. all $\Delta j \geq 0$) ⇒ LPP minimisation.

⇒ Simplex Tableau : Non Optimal Solution

Basis		x_1	x_2	~~x_3~~	S_1	S_2	b_i	b_i / a_{ij}
S_1	0	2	3	~~1~~	~~0~~1	0	60	30
S_2	0	4	3	~~0~~	0	1	96	24

C_j	40	35	0	0
Soln	0	0	60	96
Δ_j	40	35	0	0

~~inc va~~

incoming var (key col.)

$$\text{New } R_2 = \frac{\text{old } R_2}{4}$$

$$\text{New } R_1 = \text{old } R_1 - 2(\text{new } R_2)$$

Q. A firm produces 3 products A, B, and C each of which passes through 3 depts: fabric^n, finishing, and packaging. Each unit of product A requires 3, 4 and 2; and a unit of product B reqs. 5, 4 and 4, while each unit of product C reqs. 2, 4 and 5 hours resp, in the 3 dept and 100 hours in the packaging dept

The unit contrib^n of product A is Rs. 5 of product B is Rs. 10 and of product C is Rs. 8

A. Let x_1 be the no. of units of product A produced,
x_2 be the no. of units of product B produced and
x_3 be the no. of units of product C produced.

objective function : $Z_{max} = 5x_1 + 10x_2 + 8x_3$ subject to,

$3x_1 + 5x_2 + 2x_3 \leq 60$ Fabric^n
$4x_1 + 4x_2 + 4x_3 \leq 72$ Finishing
$2x_1 + 4x_2 + 5x_3 \leq 100$ Packaging.

	A	B	C	
d_1	3	5	2	60 hrs
d_2	4	4	4	72 hrs
d_3	2	4	5	100 hrs.

and $x_1, x_2, x_3 \geq 0$.

s_1, s_2, s_3 are the slack variables in the 3 depts. resp.

→ Standarised OR Augmented LPP.

maximize, $Z_{max} = 5x_1 + 10x_2 + 8x_3 + 0S_1 + 0S_2 + 0S_3$

subject to,

$$3x_1 + 5x_2 + 2x_3 + 1S_1 + 0S_2 + 0S_3 = 60$$
$$4x_1 + 4x_2 + 4x_3 + 0S_1 + 1S_2 + 0S_3 = 72$$
$$2x_1 + 4x_2 + 5x_3 + 0S_1 + 0S_2 + 1S_3 = 100.$$

and

$$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0.$$

→ Simplex Table No. 1.

		5	10	8	0	0	0		
basis	c_j	x_1	x_2	x_3	S_1	S_2	S_3	b_i	θ
var	c_i								
S_1	0	3	5	2	1	0	0	60	12 ← key row.
S_2	0	4	4	4	0	1	0	72	18
S_3	0	2	4	5	0	0	1	100	25
z_j		0	0	0	0	0	0		
$\Delta_j (c_j - z_j)$		5	10	8	0	0	0		
			↑ key column						

∴ $\Delta_j \nleq 0$, sol^n is not optimal.

Simplex Table : Non Optimal Solution.

basis var	c_i	x_1	x_2	x_3	S_1	S_2	S_3	Sol. b_i	θ
x_2	10	$\frac{3}{5}$	1	$\frac{2}{5}$	$\frac{1}{5}$	0	0	12	30
S_2	0	$\frac{8}{5}$	0	$\frac{12}{5}$	$\frac{-4}{5}$	1	0	24	10
S_3	0	$\frac{-2}{5}$	0	$\frac{17}{5}$	$\frac{-4}{5}$	0	1	52	$\frac{260}{17}$
z_j		6	10	4	2	0	0		
Δ_j		-1	0	4 ⇒	-2	0	0		

new R_2 = old R_2 − 4 (new R_2)

new R_3 = old R_3 − 4 (new R_2)

Simplex Table No. 3.

		5	10	8	0	0	0		
var	c_i	x_1	x_2	x_3	S_1	S_2	S_3	sol. b_i	θ
x_2	-10	$\frac{1}{3}$	1	0	$\frac{1}{3}$	$\frac{-1}{6}$	0	8	
x_3	8	$\frac{2}{3}$	0	1	$\frac{-1}{3}$	$\frac{5}{12}$	0	10	
S_3	0	$\frac{-8}{3}$	0	0	$\frac{1}{3}$	$\frac{-17}{12}$	1	18	
z_j		$\frac{26}{3}$	10	8	$\frac{2}{3}$	$\frac{5}{3}$	0		
$\Delta_j (c_j - z_j)$		$\frac{-11}{3}$	0	0	$\frac{-2}{3}$	$\frac{-5}{3}$	0		

new R_2 = old $R_2 \times \frac{5}{12}$

new R_3 = old R_3 − (new R_2) $\times \frac{17}{5}$

new R_4 = old $R_1 - \frac{2}{5}$ (~~ot~~ new R_2)

$\because$ all $\Delta_j \leq 0$; optimality is achieved.

soln : $x_1 = 0$; $x_2 = 8$; $x_3 = 10$.

$Z_{max} = 160$.

Q. Solve using simplex method :

$Z_{min} = 40x_1 + 24x_2$

subject to : $20x_1 + 50x_2 \geq 4800$

$80x_1 + 50x_2 \geq 7200$

and $x_1, x_2 \geq 0$.

standardised / augmented LPP format :

$Z_{min} = 40x_1 + 24x_2 + 0S_1 + 0S_2 + MA_1 + MA_2$

$20x_1 + 50x_2 - 1S_1 + 0S_2 + 1A_1 + 0_2 = 4800$.

$80x_1 + 50x_2 + 0S_1 - 1S_2 + 0A_1 + 1A_2 = 7200$.

and $x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$.

Simplex table no. 1.

basis	c_i	40	24	0	0	+M	+M		
var	c_i	x_1	x_2	S_1	S_2	A_1	A_2	sol. b_i	RR: θ
A_1	+M	20	50*	-1	0	1	0	4800	96
A_2	+M	80	50	0	-1	0	1	7200	144
	z_j	100M	100M	-M	-M	0	0	x_2 entry A_1 leave	
$\Delta j = c_j - z_j$		40-100M	24-100M	M	M	0	0		

key column.

$\because$ all $\Delta j \ngeq 0$, optimality is not reached

(*) : key element (50)

new R_1 = $\dfrac{\text{old } R_1}{50}$ new R_2 ⇒ old R_2 − 50 (new R_1)

simplex table no. 2.

basis	c_i	40	24	0	0	+M	-M		
var	c_j	x_1	x_2	S_1	S_2	A_1	A_2	sol. b_i	RR. θ
x_2	24	2/5	1	-1/50	0	1/50	0	96	
A_2	+M	60	0	1	-1	-1	1	2400	
	z_j	60M + 48/5	24	M - 24/50	-M	-M - 24/50	M	x_1 enter	
$(c_j - z_j)$		40 - 60M - 48/5	0	$\frac{24}{50}$ - M	M	2M + $\frac{24}{50}$	0	A_2 leave	

$\because$ all $\Delta_j \ngtr 0$, optimality is not reached.

key element = 60 ; new $R_2 = \frac{\text{old } R_2}{60}$

new R_1 = old $R_1 - \frac{2}{5}$ (new R_2)

basis var	c_i / c_i	40 / x_1	24 / x_2	0 / S_1	0 / S_2	+M / A_1	+M / A_2	Sol. b_i	RR
x_2	24	0	1	$-2/75$	$1/150$	$2/75$	$-1/150$	80	
x_1	40	1	0	$1/60$	$-1/60$	$-1/60$	$1/60$	40	
	z_j	40	24	$-48/75$ $2/3$	$12/75$ $-2/3$	$48/75$ $-2/3$	$-12/75$ $2/3$		
$\Delta_j = c_j - z_j$		0	0	$-2/75$	$38/75$	$M + \frac{2}{75}$	$M - \frac{38}{75}$		

key element = $\frac{1}{60}$.

new r_2 = old $r_2 \times 60$.

new r_1 = old $r_1 - \frac{-4}{150}$ new r_1.

basis var	c_i / c_j	x_1	x_2	S_1	S_2	A_1	A_2	Sol: b_j	RR
x_2	24	$\frac{8}{5}$	1	0	$\frac{-1}{50}$	0	$\frac{-1}{30}$	144	
x_1	0	60	0	1	-1	-1	1	2400	
	z_j	30.5	24	0	$\frac{-24}{50}$	0	$\frac{-4}{5}$		
$\Delta_j = c_j - z_j$		1.5	0	0	$\frac{24}{50}$	M	$M + \frac{4}{5}$		

$x_2 = 144$

$S_1 = 2400$

$x_1 = 0$; $S_2 = 0$; $A_1 = 0$; $A_2 = 0$.

$\therefore Z_{min} = 40x_1 + 24x_1 - S_1 - S_2 + MA_1 + MA_2 = 0$.

$= 0 + 24(144) - 2400 + 0 + 0 + 0$

$= \underline{\underline{3456}}$.

Duality in LPP :

$\underline{LPP} \xrightarrow{\text{ASSOCIATED}} \underline{LPP}$

↳ primal. ↳ dual ; by both primal and dual LPP, we get equivalent solution.

Main focus of dual is to find for each resource its best marginal value (dual price / shadow price).

This value reflects the scarcity of the resources, ie. max. add^n price to be paid to obtain are additional unit of the resources to maximize profits under a resource constraint.

- If a resource is not completely used, ie. there is a slack and its marginal profit is zero.

For any constraint,

Shadow price = change in optimal objective func^n unit change in the availability of resource.

NOTE: The shadow price for '≤' type constraint will always be greater than ~~as~~ ~~eo~~ or equal to zero because increasing RHS resource value cannot make the value of objective funcⁿ worse.

- Threshold price for '≥' type constraint will always be less than or equal to zero because inc. the RHS value of resources cannot improve the values of objective function.

How to convert Primal LPP ⟶ Dual LPP ?

1. Find cannonical form (symmetric) for
   objective ⟶ max Z   min
   $\leq$ $\geq$
   $\leq$ $\geq$

primal LPP : $\max Z_x = c_1x_1 + c_2x_2 + \cdots + c_nx_n$.

subject to : $a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$.

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$

$x_1, x_2, \ldots, x_n \geq 0$.

dual LPP : let $y_1, y_2, \ldots, y_m$ dual var.

$\min Z_y = b_1y_1 + b_2y_2 + \cdots + b_my_m$

sbject to : $a_{11}y_1 + a_{21}y_2 + \cdots + a_{m1}y_m \geq c_1$

$a_{12}y_1 + a_{22}y_2 + \cdots + a_{m2}y_m \geq c_2$.

$\vdots$

$a_{1n}y_1 + a_{2n}y_2 + \cdots + a_{mn}y_m \geq c_n$.

$y_1, y_2, \ldots, y_m \geq 0$.

primal LPP : $\max Z_x = \sum_{j=1}^{n} c_j x_j$

ST : $\sum_{j=1}^{n} a_{ij} x_j \le b_i \;; \; i = 1, 2, \dots, m.$

$j = 1, 2, \dots, n.$

$x_j \geq 0.$

dual LPP : $\min Z_y = \sum_{i=1}^{m} b_i y_i$

ST : $\sum_{i=1}^{m} a_{ji} y_i \geq c_j \;; \; j = 1, 2, \dots, n$

$i = 1, 2, \dots, m$

$y_i \geq 0.$

Example 1: Write dual form of the given LPP.

$Z_{min} = 3x_1 + x_2$

ST : $2x_1 + 3x_2 \geq 2 \qquad y_1$

$x_1 + x_2 \geq 1 \qquad y_2$

$x_1, x_2 \geq 0$

- In primal : no. of vars = 2
  no. of constraints = 2
- In dual : no. of vars = 2
  no. of constraints = 2.

let y_1, y_2 be dual variables

$$\left.\begin{aligned} Z_{y\,max} &= 2y_1 + y_2 \\ \text{ST} : \; 2y_1 + y_2 &\le 3 \\ 3y_1 + y_2 &\le 1 \\ y_1, y_2 &\geq 0 \end{aligned}\right\} \text{Dual.}$$

Example 2: Write dual of foll. LPP.

$$Z_{min} = 3x_1 - 2x_2 + 4x_3$$

$$ST: \; 3x_1 + 5x_2 + 7x_3 \geq 7$$
$$6x_1 + x_2 + 3x_2 \geq 4$$
$$7x_1 - 2x_2 - x_3 \leq 10$$
$$x_1 - 2x_2 + 5x_3 \geq 3$$
$$4x_1 + 7x_2 - 2x_3 \geq 2$$
$$x_1, x_2, x_3 \geq 0$$

In dual : $Z_{y\,max} = 7y_1 + 4y_2 - 10y_3 + 3y_4 + 2y_5$

$$ST: \; 3y_1 + 6y_2 - 7y_3 + y_4 + 4y_5 \leq 3$$
$$5y_1 + y_2 + 2y_3 - 2y_4 + 7y_5 \leq -2$$
$$7y_1 + 3y_2 + y_3 + 5y_4 - 2y_5 \leq 4$$
$$y_1, y_2, y_3, y_4, y_5 \geq 0.$$

Example 3: Find dual,

$$Z_{min} = x_1 + x_2 + x_3$$

$$ST: \; x_1 - 3x_2 + 4x_3 = 5$$
$$x_1 - 2x_2$$
$$2x_2 - x_3 \geq 4$$
$x_1, \cancel{x_2}, x_3 \geq 0$; x_2 is unrestrict.

let $x_2 = x_2' - x_2''$ $(x_2', x_2'' \geq 0)$

$$Z_{min} = x_1 + x_2' - x_2'' + x_3$$

$$ST: \; x_1 - 3x_2' + 3x_2'' + 4x_3 = 5$$
$$x_1 - 2x_2' + 2x_2'' \leq 4$$
$$2x_2' - 2x_2'' - x_3 \geq 4$$
$$x_1, x_2', x_2'' \geq 0.$$

now we convert problem into cannonical form,

I^{st} constraint is ~~a~~ of '=' type

$$x_1 - 3x_2' + 3x_2'' + 4x_3 = 5$$
$$x_1 - 3x_2' + 3x_2'' + 4x_3 \geq 5$$
$$x_1 - 3x_2' + 3x_2'' + 4x_3 \leq 5 \quad \text{or}$$
$$-x_1 + 3x_2' - 3x_2'' - 4x_3 \geq -5$$

Find dual, let $y_1 = y_1' - y_1''$ $(y_1', y_1'' \geq 0)$

$$\max z_y = 5y_1 - 3y_2 + 4y_3$$

---x---

Assumptions of Linear Programming :

1. Linearity : the usage of resources per unit activity is independent of the activity level.
2. Certainty : assumed the data fig of cost/profit are known with certainty.
3. Additivity : total usage of a resource is the sum of resource for each of the activity.
4. Divisibility : resources may be allocated to activities in any proportion. Therefore may take any rational no.
5. Non-negativity :

General Model : ~~KKP~~. LPP :

optimize (maximum / minimum)

$$z = c_1x_1 + c_2x_2 + \cdots + c_nx_n.$$

subject to their linear constraints.

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \ (\leq, =, \geq)\ b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \ (\leq, =, \geq)\ b_2$$
$$\vdots \qquad\qquad\qquad \vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \ (\leq, =, \geq)\ b_m.$$

$$x_1, x_2, \ldots, x_n \geq 0.$$

Q. An airline agrees to charter planes for a group. The group needs atleast 160 first class seats and atleast 300 tourist class seats. The airline must use atleast two of it's model 314 planes which have 20 first class and 30 tourist class. The airline will also use some of it's model 535 planes which have 20 first class seats and 60 tourist class seats. Each flight of a model 314 plane costs the company Rs. 1 Lakh and each flight of a model 535 plane costs Rs. 1.5 lakh. How many of each type plane should be used to minimise the flight cost? Draft the LP model.

A. x_1 : flight 314.
x_2 : flight 535.

$$Z_{min} = 100000x_1 + 150000x_2$$

Subject to : $20x_1 + 20x_2 \geq 160$ [FC]
$30x_1 + 60x_2 \geq 300$ [TC]
$x_1, x_2 \geq 0$.

~~case of unrestricted variable in sign~~ $\cancel{\equiv / x_4}$.
~~let x~~

case of unrestricted variable (in sign) : $\equiv x_4$.
let $x_4 = x_5 - x_6$, x_5 & $x_6 \geq 0$.

Q. M/S Priyanshu & Co. have been asked to help a client to develop an advertising budget for the production of an ~~important~~ improved product. Advertising funds are to be spent on both tevlevision and magazine announcements. The client has specified the following requirements:

i. No more than Rs. 10000 is to be spent on television ads.

ii. Atleast Rs. 5,000 must be spent on magazine announcemen

iii. The amt. spent on magazine must not exceed Rs. 25000 less 1.5 times the amt. spent on TV. From prior marketing research studies, the agency has determined that 150 people are exposed to a msg. for each Rs. spent on TV, and 100 people are exposed to a message for each Rs. spent on magazine.

A. x_1 : amt. spent on TV
x_2 : amt. spent on magazine.

$Z_{max} = 150x_1 + 100x_2$ (no. of exposures)

Subject to: $x_1 \leq 10000$ (TV budget)
$x_2 \geq 5000$ (magazine buget)

$x_2 \ngtr 25000 - 1.5x_1$.

$1.5x_1 + x_2 \ngtr 25000$, $x_1, x_2 \geq 0$.

Q. Formulation $\begin{cases} \rightarrow \text{Graphical (2 variables)} \\ \rightarrow \text{Simplex} \end{cases}$

Q. Graphical :

Step 1: $Z_{max} = 100x_1 + 40x_2$

Step 2: subject to : $x_1 + 2x_2 \leq 60$

$2x_1 + x_2 \leq 102, \quad x_1, x_2 \geq 0$

Step 3: consider : $x_1 + 2x_2 = 60 \longrightarrow$ (i)

$2x_1 + x_2 = 102 \longrightarrow$ (ii)

Step 4: let $x_1 = 0$ in (i) $\therefore (0, 30)$

let $x_1 = 0$ in (ii) $\therefore (0, 102)$.

let $x_2 = 0$ in (i) $\therefore (60, 0)$

let $x_2 = 0$ in (ii) $\therefore (51, 0)$

Step 5: then plot a line on graph using above 4 pts.

L_1 using pts. of eqn (i).

L_2 using pts. of eqn (ii).

Step 6: • subs (0, 0) in both eqn and find common area enclosed by L_1 and L_2.

Step 7: find intersection pt. solving (i) and (ii) Simultaneously and mark the region.

step 8: list all points of the marked region and find Zmax/min for all points.

eg. $Z_{max} = 5x_1 + x_2$

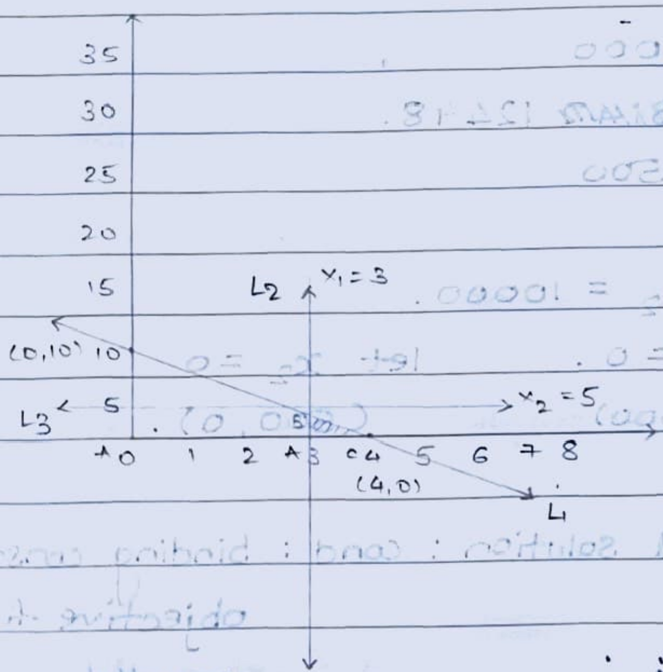
ST : $5x_1 + 2x_2 \leq 20 \longrightarrow (i)$

$x_1 \geq 3 \longrightarrow (ii)$

$(iii) \longleftarrow x_2 \leq 5$, x_1 and $x_2 \geq 0$.

let $5x_1 + 2x_2 = 20$; $x_1 = 3$, $x_2 = 5$

$(0, 10), (4, 0)$ line ∥' to y ; line ∥' to x.



$x_1 = 3$.

∴ sub x_1 in (i).

$15 + 2x_2 = 20$.

$x_2 = 2.5$.

∴ intersec^n pt : (3, 2.5)

area : ABC

∴ pts : (3, 0)

(3, 2.5)

(4, 0)

pt.	Zmax
(3, 0)	3
(3, 2.5)	17.5
(4, 0)	20

∴ $x_1 = 4$, $x_2 = 0$, Zmax = 20.

constraint $x_2 \leq 5$ is reduntant.

Q. $Z_{max} = 20x_1 + 10x_2$.

subject to : $4x_1 + 2x_2 \leq 2500$

$3x_1 + 5x_2 \leq 3000$, x_1 and $x_2 \geq 0$.

$4x_1 + 2x_2 = 2500$ — $3x_1 + 5x_2 = 3000$

(0, 1250), (625, 0) — (0, 6000), (1000, 0).

intersec^n pt : (464.3, 321.4)

∴

pts.	Zmax
(0, 0)	0
(0, 600)	6000
(464.3, 321.4)	12498.
(625, 0)	12500

$20x_1 + 10x_2 = 10000$.

let $x_1 = 0$. — let $x_2 = 0$

(0, 1000) — (500, 0).

alternate optimal solution : cond : binding constraint and objective function are parallel.

special case :

optimal function = 10000

$20x_1 + 10x_2 = 10000 \longrightarrow (0, 1000), (500, 0)$

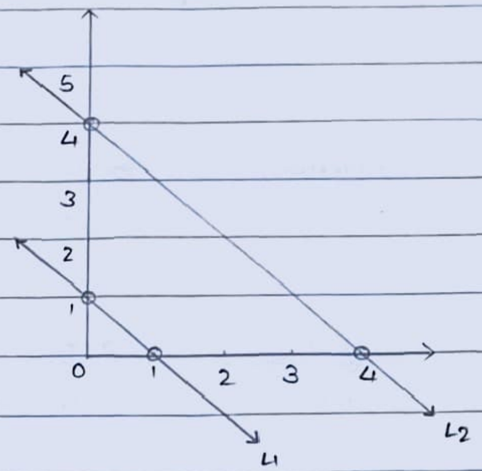
Q. $Z_{max} = 3x_1 + 3x_2$ subject to: $x_1 + x_2 \leq 1$

$x_1 + x_2 \geq 4$

$x_1, x_2 \geq 0$

A. $L_1 = x_1 + x_2 = 1$ $\quad L_2 = x_1 + x_2 = 4$

$(0, 1)\ (1, 0)$ $\quad (0, 4)\ (4, 0)$



there is no common feasible region for all constraints.
Hence it is said to be an infeasible solution.

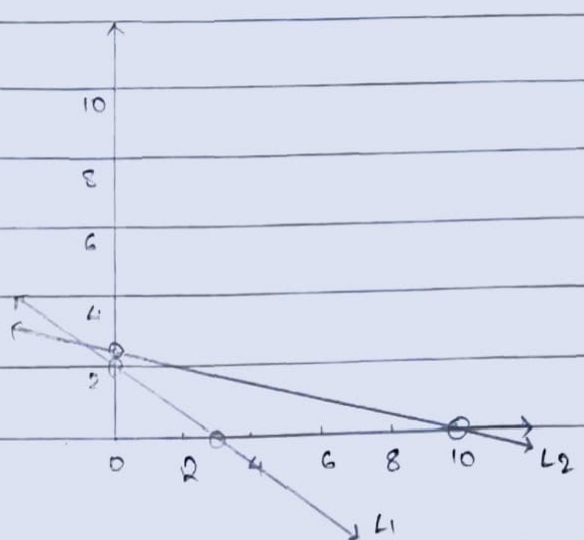
Q. $Z_{max} = 4x_1 + 3x_2$ subject to: $2x_1 + 3x_2 \leq 6$

$x_1 + 4x_2 \geq 10$

$x_1, x_2 \geq 0$.

A. $L_1 = 2x_1 + 3x_2 = 6$. $\quad L_2 = x_1 + 4x_2 \geq 10$.

$(0, 2), (3, 0)$ $\quad (0, 2.5)\ (10, 0)$.



Q. $Z_{max} = 3x_1 + 5x_2$ Subject to : $2x_1 + x_2 \geq 7$

$x_1 + x_2 \geq 6$

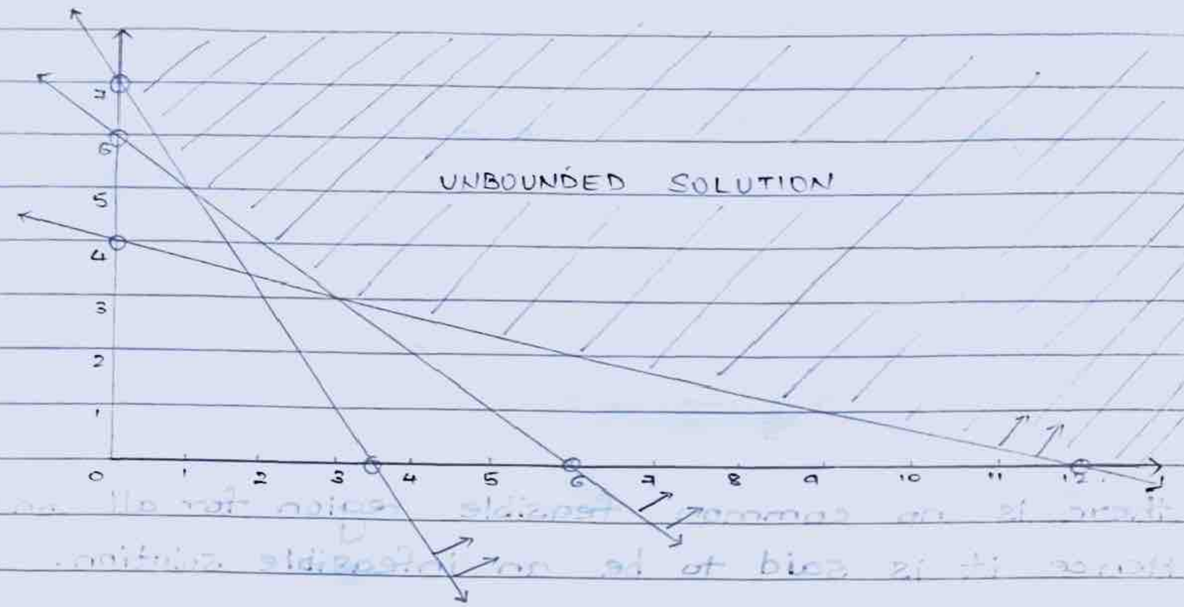
$x_1 + 3x_2 \geq 12$

$x_1, x_2 \geq 0.$

$2x_1 + x_2 = 7$ $(0, 7)\ (3.5, 0)$

$x_1 + x_2 = 6$ $(0, 6)\ (6, 0)$

$x_1 + 3x_2 = 12$ $(0, 4)\ (12, 0)$



Q. $Z_{max} = 8x_1 + 10x_2$. subject to : $6x_1 + 3x_2 \leq 24$

$4x_1 + x_2 \leq 10$

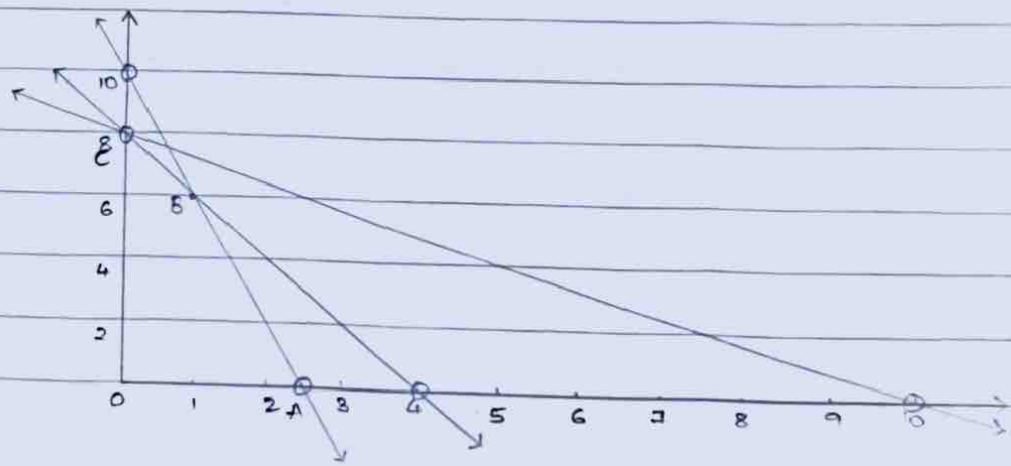
$4x_1 + 5x_2 \leq 40$

$x_1, x_2 \geq 0.$

$L_1 : 6x_1 + 3x_2 = 4$ $(0, 8)(4, 0)$

$L_2 = 4x_1 + x_2 = 10$ $(0, 10)\ (2.5, 0)$

$L_3 = 4x_1 + 5x_2 = 40$ $(0, 8)\ (10, 0)$



solving L_1 and L_2

$$6x_1 + 3x_2 = 24 \longrightarrow (i)$$
$$4x_1 + x_2 = 10 \longrightarrow (ii)$$

$$x_1 = 1, \; x_2 = 6.$$

OABC–O is the feasible region considering all constraints together.

points	Zmax
O (0, 0)	0
A (2.5, 0)	20
B (1, 6)	68
C (0, 8)	80

$\therefore x_1 = 0$

$x_2 = 8$

$Z_{max} = 80.$

The L_3 only connects to the feasible region at one pt. hence this is a case of degeneracy.

Q. Solve by simplex method.

$Z_{max} = 30x_1 + 40x_2.$ subject to: $x_1 + 2x_2 \le 200.$

$8x_1 + 5x_2 \le 600.$

$3x_1 + 4x_2 \le 500.$

$x_1, x_2 \ge 0.$

Augmented form of LPP.

$x_1 + 2x_2 + 1S_1 + 0S_2 + 0S_3 = 200.$

$8x_1 + 5x_2 + 0S_2 + 1S_2 + 0S_3 = 600.$

$3x_1 + 4x_2 + 0S_1 + 0S_2 + 1S_3 = 500.$

$x_1, x_2, S_1, S_2, S_3 \geq 0.$

S_1, S_2, S_3 are all slack variables.

Simplex Table No. 1.

basis var	C_i c_j	x_1	x_2	S_1	S_2	S_3	Sol: b_j	θ
S_1	0	1	2	1	0	0	200	$\frac{200}{2} = 100$ ← key row
S_2	0	8	5	0	1	0	600	$\frac{600}{5} = 120$
S_3	0	3	4	0	0	1	500	$\frac{500}{4} = 125$
z_j		0	0	0	0	0		
$\Delta_j (c_j - z_j)$		30	40	0	0	0		

↑

key column.

here x_2 enters & S_1 leaves.

$r_1 = \frac{r_1}{2}$; $r_2 = r_2 - 5r_1$; $r_3 = r_3 - 4r_1$.

basis var	c_i / c_j	30 x_1	40 x_2	0 s_1	0 s_2	0 s_3	sol: b_j	$RR = \theta$
x_2	40	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	100	$\frac{100}{(1/2)} = 200$
s_2	0	$\frac{11}{2}$ *	0	$\frac{-5}{2}$	1	0	100	$\frac{100}{(\frac{11}{2})} = \frac{200}{11}$ ← KR
s_3	0	1	0	−2	0	1	100	$\frac{100}{1} = 100$
z_j		20	40	20	0	0		
$\Delta_j (c_j - z_j)$		10	0	−20	0	0		
		↑ key col.						

here s_2 leaves and x_1 enters.

new r_2 = old $r_2 \cdot \frac{2}{11}$

new r_1 = old $r_1 - \frac{1}{2}$ new r_2.

new r_3 = old r_3 − new r_2.

Simplex table no. 3.

basis var	c_j / c_i	30 x_1	40 x_2	0 S_1	0 S_2	0 S_3	Sol: b_j	RR = θ
x_2	40	0	1	$\frac{8}{11}$	$\frac{-1}{11}$	0	$\frac{1000}{11}$	
x_1	30	1	0	$\frac{-5}{11}$	$\frac{2}{11}$	0	$\frac{200}{11}$	
S_3	0	0	0	$\frac{-17}{11}$	$\frac{-2}{11}$	0	$\frac{400}{11}$	
z_j		30	40	$\frac{170}{11}$	$\frac{20}{11}$	0		
$\Delta_j (c_j - z_j)$		0	0	$\frac{-170}{11}$	$\frac{-20}{11}$	0.		

$x_1 = \frac{200}{11}$ $\quad x_2 = \frac{1000}{11}.$

$\therefore \; Z_{max} = 30 \times \frac{200}{11} + 40 \times \frac{1000}{11}.$

$= \underline{\underline{4181.8181}}.$

Q. Find the dual of,

$Z_{min} = x_1 + 2x_2$; subject to: $2x_1 + 4x_2 \leq 160$

$x_1 - x_2 = 30.$

$x_1 \geq 10$; $x_1, x_2 \geq 0.$

multiply eq^n by -1 to change inequality to '$\geq$'.

$$-2x_1 - 4x_2 \geq -160$$
$$x_1 - x_2 \geq 30$$
$$-x_1 + x_2 \geq -30$$
$$x_1 \geq 10.$$
$$x_1, x_2 \geq 0.$$

$Z_{max} = -160y_1 + 30y_2 - 30y_3 + 10y_4$; ~~subject to~~:

subject to: $-2y_1 + y_2 - y_3 + y_4 \leq 1.$
$-4y_1 - y_2 + y_3 + 0y_4 \leq 2.$
$y_1, y_2, y_3, y_4 \geq 0.$

let $y_5 = y_2 - y_3$

$Z_{max} = -160y_1 + 30y_5 + 10y_4$; subject to: $-2y_1 + y_5 + y_4 \leq 1$
$-4y_1 - y_5 \leq 2.$
$y_1, y_5, y_4 \geq 0.$

Q. Find the dual of,

$Z_{max} = 2x_1 + 5x_2 + 6x_3$; ~~subject to: $5x_1 + 6x_2 - x_3 \leq 3$~~

subject to: $5x_1 + 6x_2 - x_3 \leq 3$
$-2x_1 + x_2 + 4x_3 \leq 4$
$x_1 - 5x_2 + 3x_3 \leq 1$
$-3x_1 - 3x_2 + 7x_3 \leq 6.$
and $x_1, x_2, x_3 \geq 0.$

In dual, $Z_{min} = 3y_1 + 4y_2 + y_3 + 6y_4$
subject to: $5y_1 - 2y_2 + y_3 - 3y_4 \geq 2$
$6y_1 + y_2 - 5y_3 - 3y_4 \geq 5$
$-y_1 + 4y_2 + 3y_3 + 7y_4 \geq 6.$

Q. Find the dual of,

$$Z_{max} = 8x_1 + 10x_2 + 5x_3$$

subject to: $x_1 - 3x_3 \leq 4$

$2x_1 + 4x_2 \leq 12$

$x_1 + x_2 + x_3 \geq 2$

$3x_1 + 2x_2 - x_3 = 8.$

and $x_1, x_2, x_3 \geq 0$

A. $Z_{min} = 4y_1 + 12y_2 - 2y_3 + 8y_4 - 8y_5$

subject to: $y_1 + 2y_2 - y_3 + 3y_4 - 3y_5 \geq 8$

$0y_1 + 4y_2 - y_3 + 2y_4 - 2y_5 \geq 10$

$-3y_1 + 0y_2 - y_3 - y_4 + 3y_5 \geq 5$

and $y_1, y_2, y_3, y_4, y_5 \geq 0$

Sensitivity Analysis:

Q. $Z_{max} : 60x_1 + 50x_2.$

Subject to: $4x_1 + 10x_2 \leq 100$ hrs (assembly)

$2x_1 + x_2 \leq 22$ hrs (inspection)

$3x_1 + 3x_2 \leq 39 ft^3$ (storage).

—x—

UNIT - 2 : TRANSPORTATION AND ASSIGNMENT PROBLEMS.

TRANSPORTATION :

- essentially min. cause min. cost while max. the goods being transported while meeting all the transport needs.

- balanced TP : Σ capacity = Σ demand

- unbalanced TP : either add^n of dummy source
  ie. Σ capacity < Σ demand OR.
  Σ capacity > Σ demand

c_{ij} : cost of unit transportation from i^{th} destination.

	j_1	j_2
i_1		
i_2		

$\longrightarrow$ Σ capacity (source)

$\Sigma c = \Sigma d$

$\uparrow$

Σ demand (destination)

x_{ij} = no. of units transported from i^{th} row to the j^{th} column / destination.

objective : min. cost $z = \Sigma x_{ij} . c_{ij}$.

Feasible Solution : • all indivisual allocations are either +ve or 0
• satisfies requirement (total row sum = total col. sum)

Infeasible Solution: • Atleast 1 indivisual alloc^n is -ve.
• atleast 1 indivisual alloc^n is +ve. in a cell having high or infinite cost (μ)

- BFS if, no. of +ve alloc^n : $[(m+n)-1]$, m : no. of rows
  n : no. of columns

- ~~dexge~~ degenerate basis feasible solution : no. of alloc^n < $[(m+n)-1]$

- non-degenerate ~~basis~~ BFS : no. of +ve alloc^{n}s = $[(m+n)-1]$
  each alloc^n is in independent post^{n}s

Independent Postition of Allocation :

Alloc^n are independent only if it's impossible to form a closed path alloc^n, the following table are Independent.

	*	
*	*	*
	*	

*	*		
		*	
	*		*
		*	*

Q. A company has 3 factories that supply to 4 marketing areas. The transportation cost of shipping from each factory to different marketing areas is given below. The availability at each factory and requirement at different markets is also given.

i. NWCR
ii. LCM
iii. VAM.

Types of Transportation Problems.

- **Balanced**
  - **Min**
    - solve directly
  - **Max**
    - convert to min (regret matrix) and solve
- **Unbalanced**
  - **Min**
    - unbalanced to balanced and solve
  - **Max**
    - unbalanced to balanced, then max to min and solve.

- Optimal Solution : Transportation cost cant be reduced further
  - a. IBFS / IFS / BFS : basic feasible solution.
  - b. optimum solution.

Method to find IBFS : Vogels method, north west solution, least cost method.

Transportation Problem Types :

- unbalanced
- alternate / multiple optimal solution.
- maximisation.
- degeneracy.
- prohibited routes.

Original TP Minimisation : In maximisation problems, convert to minimisation type by suitable form transportation, then by usual TP.

i. most NW, SE check and allocate the least value and carry on.

FACTORY	MARKETING AREAS				SUPPLY
F_1	1000 / 19	~~600~~ 0 / 30	— / 50	— / 10	~~600~~ 0 / ~~1600~~
F_2	— / 70	900 / 30	~~300~~ / 40	— / 40	~~300~~ 0 / ~~1200~~
F_3	— / 40	— / 8	500 / 70	1200 / 20	~~1200~~ 0 / ~~1700~~
DEMAND	~~1000~~ 0	~~1500~~ ~~900~~ 0	~~800~~ ~~500~~	~~1200~~ 0	4500 / 4500

Total cost (NWCR) = Rs. 135000

ie. $(19 \times 1000) + (30 \times 600) + (30 \times 900) + (40 \times 30) + (70 \times 500)$
$+ (1200 \times 20)$

$[(m+n)-1] = 3+4-1$
$= 6$ occupied cells.

$\because$ no. of $alloc^n = [(m+n)-1]$
$\therefore$ non-degenerate.

ii. LCM : use the one with the highest $alloc^n$ if same cell value and the least value cell start from the least cell value and continue.

FACTORY	MARKETING AREAS				SUPPLY
	M_1	M_2	M_3	M_4	
F_1	400 19	— 30	50	1200 10	~~1600~~ ~~400~~ 0
F_2	400 70	— 30	800 40	— 40	1200 400
F_3	200 40	1500 8	70	— 20	~~1700~~ 200
DEMAND	~~1000~~ ~~600~~ ~~400~~	~~1500~~ 0	~~800~~ 0	~~1200~~ 0	4500 4500

Total cost : $(400 \times 19) + (70 \times 400) + (40 \times 200) + (10 \times 1200)$
$+ (40 \times 800) + (8 \times 150)$
$= \underline{\underline{\text{Rs. } 99600}}$

$[(m+n)-1] = 3+4-1 = 6$ occupied cells.

$\therefore$ non-degerate.

iii. VAM : penalty incurred by adopting the 2nd least costly, then select the highest penalty, then test cell cost.

FACTORY	MARKETING AREAS				SUPPLY	PENALTIES				
	M_1	M_2	M_3	M_4		I	II	III	IV	V
F_1	19 (1600)	30 (—)	50 (—)	10 (600)	~~1600~~ ~~600~~ 0	9	9	40	40	
F_2	70 (—)	30 (—)	40 (800) (5)	40 (400) (6)	~~1200~~ ~~400~~ 0	10	—	—	—	—
F_3	40 (—)	8 (1500) (1)	70 (—)	20	~~1700~~ ~~200~~ 0	12	20	50	—	—
DEMAND	~~1000~~ 0	~~1500~~ 0	~~800~~ 0	~~1200~~ ~~1000~~ 0	4500 / 4500					
I	21	22	10	10						
II	21	—	10	10						
III	—	—	10	10						
IV	—	—	10	30						
V	—	—	40	40						

- Iterate 40 is same for both the cell 5 and ultimately the answer is the same, but took for the one where higher cost can be allocated.

It operates on the basis of penalties ⟶ per penalty and least cell in the row or column is selected and then the rest are cancelled.

Total cost = 19000 + 32000 + 16000 + 6000 + 12000 + 7000
= Rs. 89000.

non-generate = [(m+n)-1 = 6].

Q. The following table gives the profit per unit of product transportation from warehouses (WH_1 to WH_3) to markets (M_1, M_2, M_3).

Profit → Matrix → Since Profit → Max, Since cost → Min

	M_1	M_2	M_3	SUPPLY
WH1	29	28	30	2000
WH2	25	27	23	2000
WH3	35	37	38	2000
DEMAND	1500	3000	1500	

∵ it is max → convert to min.

Find the optimal solution so as to get maximum profit.

(38 is subtracted)

Profits

M1	M2	M3
9	10	8
13	11	15
3	1	0

⇒ normal TP

ASSIGNMENT PROBLEMS.

	J_1	J_2	J_3	J_4
W_1	10	9	11	14
W_2	15	8	7	10
W_3	10	12	17	16
W_4	9	15	18	11

is it balanced?

ie. $m = n$?

Hungerian Method

	J_1	J_2	J_3	J_4	J_5
W_1	8	4	2	6	1
W_2	0	9	5	5	4
W_3	3	8	9	2	6
W_4	4	3	1	0	3
W_5	9	5	8	9	5

following table gives matrix, where 5 workers are working on 5 jobs.
Find their assignment such that total cost in minimum ie. minimize total cost.

A. AP is balanced $(m = n)$
row minima: find least cost el. of row and subtract every el. row from it.

	J_1	J_2	J_3	J_4	J_5
W_1	7	3	1	5	0
W_2	0	9	5	5	4
W_3	1	6	7	0	4
W_4	4	3	1	0	3
W_5	4	0	3	4	0

column minima: find least cost el. of column and subtract every el. column from it.

	J_1	J_2	J_3	J_4	J_5
W_1	7	3	$\boxed{0}$	5	$\boxed{0}$
W_2	$\boxed{0}$	9	4	5	4
W_3	1	6	6	0	4
W_4	4	3	$\boxed{0}$	$\boxed{0}$	3
W_5	4	$\boxed{0}$	2	4	$\boxed{0}$

task is to find min. no. of lines (containing 0's) with (horizontal / vertical)

- start with row having max. 0's.

∵ min. no. of lines reqd. to cover all the 0's is 5. (this equals no. of rows / columns)

∴ we can go further to obtain optimal assignment.

$W1 \rightarrow J5$		1
$W_2 \rightarrow J_1$		0
$W_3 \rightarrow J4$		2
$W_4 \rightarrow J3$		1
$W_5 \rightarrow J2$		5
total cost		9

Q2.

	A	B	C	D	E
P1	9	2	2	1	7
P2	8	6	7	1	6
P3	6	4	5	1	3
P4	3	5	8	2	4
P5	8	9	8	5	9

row minima :

	A	B	C	D	E
P1	8	1	1	0	6
P2	7	5	6	0	5
P3	5	3	4	0	2
P4	1	2	6	0	2
P5	3	4	3	0	4

column minima:

	A	B	C	D	E
P1	7	$\boxed{0}$	$\boxed{0}$	$\boxed{0}$	4
P2	6	4	5	$\boxed{0}$	3
P3	4	2	3	$\boxed{0}$	$\boxed{0}$
P4	$\boxed{0}$	1	5	$\boxed{0}$	$\boxed{0}$
P5	2	3	2	$\boxed{0}$	2

possible to cover all line zeros in 4 lines (ie. less than m/n)
hence we cannot make optimal assignment.

- find the least uncovered values. (eg. 2)
- subtract it from all uncovered el's.

	A	B	C	D	E
P1	7	0	0	2	6
P2	4	2	3	0	3
P3	2	0	1	0	0
P4	0	2	5	2	2
P5	0	1	0	0	2

- add it to all the intersec^n cell.
- retain all the covered values as they are.
- find optimal assignment.

P1 →	B	2
P2 →	D	1
P3 →	E	3
P4 →	A	3
P5 →	C	8
		17.

Maximisation Assignment.

	A	B	C	D
O1	15	9	10	6
O2	10	6	9	6
O3	25	15	15	9
O4	15	9	10	10
P	8	6	5	4

working time : 8 hr. less 30 min.

$= (8 \times 60) - 30$

$= 450$ min.

consider operator 1 and product A, it takes 15 mins of process time, to produce 1 unit of product A.

Hence in one shift operator can produce $= \frac{450}{15}$

$= 30$ products.

Profit matrix.

	A	B	C	D
P1	240	300	225	300
P2	360	450	250	300
P3	144	180	150	200
P4	240	300	225	180

Regret (cost) matrix.

	A	B	C	D
P1	210	150	225	150
P2	90	0	200	150
P3	306	270	300	250
P4	210	150	225	270

row minima :

	A	B	C	D
P1	60	0	85	0
P2	90	0	200	150
P3	106	70	100	0
P4	60	0	85	120

column matrix :

	A	B	C	D
P1	$\boxed{0}$	$\boxed{0}$	$\boxed{0}$	$\boxed{0}$
P2	30	$\boxed{0}$	115	150
P3	76	70	15	$\boxed{0}$
P4	$\boxed{0}$	$\boxed{0}$	$\boxed{0}$	120

final assignment :

P1 $\longrightarrow$ A 240

P2 $\longrightarrow$ B 450

P3 $\longrightarrow$ D 200

P4 $\longrightarrow$ C 225

1115.

UNIT-3 : Decision Making Problems.

Decision Theory :

Decision making situations : under certainty
under risk
under uncertainty

Sample : Conditional Pay-off Table.

States of Nature		STRATERGIES / ACTIONS			
Boxes	Probability	A1 2	A2 4	A3 6	A4 8
2 S_1	P_1 0.2				
4 S_2	P_2 0.2				
6 S_3	P_3 0.5				
8 S_4	P_4 0.1				

$\Sigma = 1$.

Q. A toy company is bringing out a new type of toy.

anticipated first year profit (in L Rs.)

Demand	Level of Produc^n^		
	Full	Partial	Minimal
High	180	110	60
Medium	110	90	30
Low	-70	-40	10

conditional regret matrix

PTO.

Demand	Level of Production		
	Full	Partial	Minimal.
High	0	70	120
Medium	0	20	80
Low	80	50	0
Avg.	73.3	53.3	33.3
Hurwicz ($\alpha = 0.6$)	80	50	40

Decision ~~Rules~~ Rules :

Decision under uncertainty :
1. Laplace
2. MaxMin / MinMax
3. Hurwicz
4. Savage Maximax / Minimin.

1. **Laplace Principle** : assumes equal likelihood of various states of nature.

Criterion	Decision
Laplace	full production
Maxim~~ax~~in	minimal production
Hurwicz	full production
Minimax Regret	~~full~~ production partial
Maximax	full production

Q. Given below is a pay-off table (or matrix) of a retail investor.

THEORY:

Decision under uncertainity:

1. Maximax / Optimum Criterion :
   - find the highest pay-off for each action.
   - select that action which has the maximum value of those.

2. Maximin / Pessimism Criterion :
   - find least payoff for each action.
   - select action with highest value of those.

3. Minimax / Regret Criterion :
   - consider given pay-off table or create one.
   - from each state of nature : find the highest payoff.
   - replace each value by (Highest val - that val). table hence formed is Regret table.
   - identify max. regret for each action
   - select ~~lowest~~ action with lowest value.

4. Hurwicz : Criterion :
   - compromise bet^n optimistic and pessimistic.
   - select α ; $(0 < \alpha < 1)$
   - α close to : 1 : optimistic
     0 : pessimistic.

- for each action, determine H.

$$H = [\{\alpha\} * (\text{Max. payoff})] + [\{1-\alpha\} * (\text{Max. payoff of that action})]$$

- select ~~H~~, ~~for~~ action, for max. H.

5. Laplace Criterion:
   - assign equal probability to each state of nature.
   - compute the expected value for each action, by multiplying each outcome/payoff by it's corresponding probabilities and then sum them.
   - select that action with max. value.

	A1	A2	A3	laplace prob
	bank	govt sec.	stocks	
S1 slowdown	500	100	-400	0.3333
S2 normal	500	600	650	0.3333
S3 eco boom	500	500	1000	0.3333
optimism	500	800	1000	

first : maximax A3 = 1000

pessimism 500 100 -400

second : maxmin A1 = 500

opportunity loss (or regret) table.

	A1	(A2)	A3
	bank	govt sec	stocks
S1 slowdown	0	400	900
S2 normal	150	50	0
S3 eco boom	500	200	0
column max	500	400	900

3^{rd} min. of max regret A2 = 400

laplace EMV	499.995	499.995	416.3663
hurwicz (H) α = 0.8	500	660	720

H = (α * M + L - d. min).

3. <u>Formulation</u> :

Let x_1 denote no. of units of products. A
Let x_2 denote no. of Units of products B

obj. function $Z_{cost} = 60x_1 + 80x_2$
subject to
$x_2 \geq 200$ min
$x_1 \leq 400$ machine hours
$x_1 + x_2 \leq 500$ labour hours
$x_1, x_2 \geq 0$ non negativity

x_2 m min. cost

A	B	
1 hr	(✓)	machine ($\cancel{5}$4 00 hrs)
1 hr	1 hr	labour (500 hrs)
60 RS.	80 hrs.	cost

Q. cost of producing 1 unit of product $C_1 = 50$, 25 wages, 25 material.
cost of producing 1 unit of product C_2 = 125 wages, 75 material.

SP of C_1 = 150 pu.
SP of C_2 = 350 pu.

initial bal. = 20000
machine time = 4000 hrs.
assembly time = 2800 hrs.

$C_1 = 6$
$C_2 = 4$
$C_1 = 4$
$C_2 = 6$.

C_1 profit = 100 : C_2 profit = 150.

$$\max Z = 100x_1 + 150x_2.$$

subject to : $6x_1 + 4x_2 \le 4000$

$4x_1 + 6x_2 \le 2800$

$50x_1 + 200x_2 \le 20000$

$x_1, x_2 \ge 0.$

Q.

	SP	CP	drilling	shipping	polishing
A $\longrightarrow x_1$	40	25	$\cancel{25}$ 4	$\cancel{25}$ 6	$\cancel{40}$ 3.75
B $\longrightarrow x_2$	50	30	$\cancel{40}$ 2.5	$\cancel{20}$ 7.5	$\cancel{30}$ 5
C $\longrightarrow x_3$	70	50	$\cancel{25}$ 4	$\cancel{20}$ 7.5	$\cancel{40}$ 8.75
			100	150	150

maximise : $1.25x_1 + \cancel{8}\,5x_2 + 4.75x_3$

subject to : $\dfrac{x_1}{25} + \dfrac{x_2}{40} + \dfrac{x_3}{425} \le 1$ drilling

$\dfrac{x_1}{25} + \dfrac{x_2}{20} + \dfrac{x_3}{20} \le 1.$ shipping

$\dfrac{x_1}{40} + \dfrac{x_2}{30} + \dfrac{x_3}{40} \le 1$ polishing.

$x_1, x_2, x_3 \ge 1.$

Q.

Midnight - 4am	7	x_1				
4am - 8am	20	x_1	x_2			
8am - 12noon	14		x_2	x_3		
12noon - 4pm	20			x_3	x_4	
4pm - 8pm	10				x_4	x_5
8pm - midnight	5					x_5

let x_1 be no. of person starting work at midnight

let x_2 4am (4am - 12noon)

minimize : $x_1 + x_2 + x_3 + x_4 + x_5 + x_6$.

subject to : $x_1 + x_6 \geq 7$

$x_1 + x_2 \geq 20$

$x_2 + x_3 \geq 14$

$x_3 + x_4 \geq 20$

$x_4 + x_5 \geq 10$

$x_5 + x_6 \geq 5$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$.

Q. The research director of XYZ pharmaceutical laboratory has to decide about one of the 3 influenza vaccines (P1, P2, P3) which should be funded for mass production. Payoff's depend upon the type of influenza outbreak (S_1, S_2, S_3, S_4) that is most perrasive in the next year. The payoff matrix, with profits (in millions of rupees) is given below.

Prior to acquiring any additional info. about the occurence of states of nature, the director's probobility judgements are :

$P(S_1) = 0.2$ and $P(S_2) = 0.2$.

$P(S_3) = 0.5$ and $P(S_4) = 0.1$.

i. if the director could consult on authority who could tell him which state will occur, what is the expected value of this info using above pay-off matrix.

ii. verify your answer by calculating EVPI from the loss matrix.

STATES OF NATURE		COURSE OF ACTION					
	PROBABILITY	P1	P2	P3	P1	P2	P3
S1	0.2	10	8	-15	2	1.6	-3
S2	0.2	4	12	12	0.8	2.4	2.4
S3	0.5	0	-5	8	0	-2.5	4.0
S4	0.1	-2	-10	8	-0.2	-1.0	0.8
					2.6	0.5	4.2

EMV

EVPI = expected value of perfect info.

= [expected value with perfect info − EMV]

= 9.2 − 4.2

= 5.0.

∵ (10×0.2) + (12×0.2) + 8(0.5) + 8(0.1)

= expected value with perfect info

= 9.2.

STATE OF NATURE		COURSE OF ACTION						REGRET					
	PROB	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
S1	0.2	10	8	-15	2	1.6	-3	2	12	25	0	0.4	5
S2	0.2	4	12	12	0.8	2.4	2.4	0.8	0	0	1.6	0	0
S3	0.5	0	-5	8	0	-2.5	4.0	0	13	0	4	6.5	0
S4	0.1	-2	-10	8	-2	-1.0	0.8	-0.2	18	0	1	1.8	0
					2.6	0.5	4.2				6.6	8.7	5.0

EMV

EOL

EVPI

Q. A certain output is manufactured at Rs. 80 and sold at Rs. 140 per unit. The product is such that if it is produced but not sold during a day's time it becomes worthless. The daily sales records in the past are as follows.

Sales per day :	30	40	50	60	70
No. of days :	24	24	36	24	12

i. prepare a pay-off and a regret table

ii. find the expected payoffs and regret.

iii. Find the optimal act and the EVPI.

STATES OF NATURE				COURSE OF ACTION (UNITS MANUF)				
SALES (UNITS)			PROB.	$A_1(30)$	$A_2(40)$	$A_3(50)$	$A_4(60)$	$A_5(70)$
S1	30	24	0.2	1800	1000	200	-600	-1400
S2	40	24	0.2	1800	24000	1600	800	0
S3	50	36	0.3	1800	2400	3000	2200	1400
S4	60	24	0.2	1800	2400	3000	3600	2800
S5	70	12	0.1	1800	2400	3000	3600	4200

$\Sigma = 120$

$CP = 80$; $SP = 140$; $P = 60$.

Formula : $(Sales * SP) - (A_i * CP)$

if

$A_i <$ Sales : $(A_i * SP) - (A_i * CP)$

REGRET TABLE

30	40	50	60	70	30	40	50	60	70
0	800	1600	2400	3200	0	180	320	480	640
600	0	800	1600	2400	120	0	160	320	480
1200	600	0	800	1600	360	180	0	240	480
1800	1200	600	0	800	360	240	120	0	160
2400	1800	1200	600	0	240	280	120	60	0
					1080	760	720	110	1760.

regret table : highest A_i (row) − A_i.

last table : ~~Ai~~ $R_i * Prob_i$

Q. The research dept. of an F and CG company has recomended to it's marketing dept. to launch shampoos of 3 diff. type. The VP has to decide which type of shampoo to be launched under the foll. levels of scales. The table gives the payoff in lakhs. What choice will the VP marketing make, if the foll. criteria are applied.

	EGG SHAMPOO	CLINIC SHAMPOO	DELUXE SHAMPOO
HIGH LVL OF SALES	30	40	55
AVG LVL OF SALES.	10	15	20
LOW LVL OF SALES.	10	5	3

1. a. optimum : deluxe shampoo.

2. pessimistic : egg shampoo.

3. regret table : row maximum : 55, 20, 10.

∴ regret / opportunity loss :

egg	clinic	deluxe.
25	15	0
10	5	0
0	5	7
25	15	7.

∴ regret : deluxe shampoo.

4. Hurwicz max : 30 40 55

$\alpha = 0.4$ min : 10 5 3

$*H_1 = (0.4)(30) + (1-0.4)*10 = 18.$

$*H_2 = (0.4*40) + (1-0.4)*5 = 19.$

$*H_3 = (0.4*55) + (1-0.4)*3 = \boxed{23.8}$

hurwicz : deluxe shampoo.

5. Laplace :

egg	clinic	deluxe.
9.99	13.33	18.15
3.33	4.95	6.67
3.33	1.66	0.99
16.66	19.94	25.73.

∴ laplace = deluxe. shampoo.

classmate
Date
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UNIT – 4 : GAME THEORY.

Game theory deals with decision-making in situations where,

- two or more rational players.
- have a set of stratergies.
- involved in conditions of competition and conflicting intrests.
- aware of pay-offs resulting from play of various combinations of stratergies by different players.

The sol^n to a game calls for determining optimal stratergy for the players to play.

Two-person zero-sum games : there are 2 players,

- each player has a finite no. of storatergies to play.
- conditional pay-off's resulting from play of various combinations of stratergies are known.
- each pay-off is a gain for 1 player and loss for the other.
- the solution calls for determining optimal stratergies for both parties, whether pure or mixed and resulting value of game.

SADDLE POINT:

- find min. pay-off for each row.
- select largest of min. pay-off; which is the maximin stratergies of the maximising player.
- find max. pay-off in each column.
- select the smallest of these pay-off's; which is the ~~maximin~~ minimax stratergy of the minimising player.
- if maximin = minimax ; game has saddle point.

- saddle-pt ✓ : pure stratergy.
- saddle-pt ✗ : mixed stratergy.

Note: a game can have more than one saddle pt. ie. multiple optimal stratergies.

for no saddle-pt, $v = 0 \Rightarrow$ fair game.
if $v > 0$: favouring maximising player.
$v < 0$: favouring minimising player.

1. A Game : 2 Saddle Points.

		B's stratergy			Row minima
A's STRATERGY		b_1	b_2	b_3	
	a_1	12	-8	-2	-8
	a_2	6	3*	3*	3 ← maximin
	a_3	-10	-6	2	-10
col maxima		12	3	3	

→ minimax ←

2. A Game : 0 Saddle Points.

		B'S STRATERGY			ROW MINIMA
A'S STRATERGY		b_1	b_2	b_3	
	a_1	8	5	14	5
	a_2	22	-6	8	-6
	a_3	7	9	12	7 ← maximin
COL MAXIMA		22	9	14	

minimax

minimax stratergy = b_2 = 9
maximin stratergy = a_3 = 7.

Solution :

1. for 2×2 game :

		PLAYER B	
		b_1	b_2
PLAYER A	a_1	a_{11}	a_{12}
	a_2	a_{21}	a_{22}

- if A plays a_1 with probability x and a_2 with probability $1-x$; and B plays b_1 with y and b_2 with $1-y$.

$$x = \frac{a_{22} - a_{21}}{(a_{11}+a_{22}) - (a_{21}+a_{12})}$$

$$y = \frac{a_{22} - a_{12}}{(a_{11}+a_{22}) - (a_{21}+a_{12})}$$

$$v = \frac{(a_{11} \cdot a_{22}) - (a_{21} \cdot a_{12})}{(a_{11}+a_{22}) - (a_{21}+a_{2})}$$

eg. $a_{11} = 8$; $a_{12} = -7$

$a_{21} = -6$; $a_{22} = 4$

$$\therefore \; x = \frac{4+6}{(8+4)-(-6-7)} = 0.4.$$

~~y//=~~

$$y = \frac{4+7}{(8+4)-[-6-7]} = 0.44.$$

$$v = \frac{(8 \times 4) - (-6 \times -7)}{(8+4)-(-6-7)} = -0.08.$$

2. for $(m \times 2)$ or $(2 \times n)$ game :

- plot expected pay-off of each stratergy on a graph.
- locate the highest pt. in the lower envelope $(2 \times n)$ or lowest pt. in the upper envelope $(m \times 2)$
- consider the pair of lines whose intersection yields the highest / lowest pt. and use the stratergies represented by it.
- this reduces the game to 2×2 and then solve accordingly.