10/10

25/9/23

Tutorial (7)

(1) Prove the following using Mathematical Induction, for each positive integer (n) $6^{n+2} + 7^{2n+1}$ is divisible by 43.

→ $P(n) = 6^{n+2} + 7^{2n+1}$

- Base Case : To show $P(1)$ is true.

$P(1) = 6^{n+2} + 7^{2n+1}$

$= 6^{1+2} + 7^{2+1}$

$= 6^3 + 7^3$

$= 559 = 43 \times 13$

$\therefore$ $P(1)$ is true.

- Inductive Hypothesis : Let $P(K)$ be true.

$P(K) = 6^{K+2} + 7^{2K+1} = 43m$

- Inductive Proof : To show $P(K+1)$ is true.

$= 6^{(K+1)+2} + 7^{2(K+1)+1}$

$= 6^{K+3} + 7^{2K+3}$

$= 6 . 6^{K+2} + 49 . 7^{2K+1}$

$= 6 . 6^{K+2} + 6 . 7^{2K+1} + 43 . 7^{2K+1}$

$= 6 (6^{K+2} + 7^{2K+1}) + 43 . 7^{2K+1}$

$= 6 (43m) + 43 . 7^{2K+1}$ [from Inductive Hypothesis]

$= 43 (6m + 7^{2K+1})$

$P(K+1)$ is divisible by 43.

$\therefore$ $P(K+1)$ is true.

$\forall n \; P(n) = 6^{n+2} + 7^{2n+1}$ is divisible by 43.

(2) Show that $\left(1+\frac{2}{3}\right)^n \geq 1+\frac{2n}{3}$ for all

→ $P(n) = \left(1+\frac{2}{3}\right)^n \geq 1+\frac{2n}{3}$

- Base Case: To show $P(1)$ is true.

$P(1) = \left(1+\frac{2}{3}\right) \geq 1+\frac{2(1)}{3}$

$= \frac{5}{3} \geq \frac{5}{3}$

$\therefore P(1)$ is true.

- Inductive Hypothesis: Let $P(k)$ be

$P(k) = \left(1+\frac{2}{3}\right)^k \geq 1+\frac{2k}{3}$

- Inductive Proof: To show $P(k+1)$ is t

$\left(1+\frac{2}{3}\right)^{k+1} \geq 1+\frac{2(k+1)}{3}$

$\left(1+\frac{2}{3}\right)\left(1+\frac{2}{3}\right)^k \geq \left(1+\frac{2k}{3}\right)+\frac{2}{3}$

$\left(1+\frac{2}{3}\right)\left(1+\frac{2}{3}\right)^k \geq 1+\frac{2k}{3}+\frac{2}{3}$

As we know, $\left(1+\frac{2}{3}\right)^k \geq \left(1+\frac{2k}{3}\right)$

and $\left(1+\frac{2}{3}\right)\left(1+\frac{2}{3}\right)^k \geq \left(\frac{1+2k}{3}\right)+\frac{2}{3}$ (from inductive hypothesis)

$\therefore P(k+1)$ is true.

$\forall n\ P(n) = \left(1+\frac{2}{3}\right)^n \geq 1+\frac{2n}{3}$, $n \geq 1$ is true.

(3) Show by mathematical Induction
$n! \geq 2^{n-1}$ for all $n \geq 1$

→ $P(n) = n! \geq 2^{n-1}$

- Base Case : To show $P(1)$ is true.

$P(1) = 1! \geq 2^{1-1}$
$= 1 \geq 1$

$\therefore$ $P(1)$ is true.

- Inductive Hypothesis : Let $P(k)$ be true.

$P(k) = k! \geq 2^{k-1}$

- Inductive Proof : To show $P(k+1)$ is true.

$(k+1)! \geq 2^{(k+1)-1}$

$k!(k+1) \geq 2 \cdot 2^{k-1}$

As we know, $k! \geq 2^{k-1}$ is true.

$\therefore$ $k!(k+1) \geq 2 \cdot 2^{k-1}$ is true
(from inductive Hypothesis)

$2^{k-1}(k+1) \geq 2^{k}$

$(k+1) \geq 2$

$\because$ $n \geq 1$ $\forall n$

$(k+1) \geq 2$

$\therefore$ $P(k+1)$ is true.

$\forall n$ $P(n) = n! \geq 2^{n-1}$, $n \geq 1$ is true.

(4) Show by Mathematical Induction

$1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$ for $n \geq$

$\rightarrow P(n) = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

- Base Case : To show $P(1)$ is true.

$P(1) = \sum_{i=1}^{n} 1 = \frac{1(1+1)}{2} = 1$

$\therefore P(1)$ is true.

- Inductive Hypothesis : Let $P(k)$ be tru

$P(k) = \sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

- Inductive Proof : To show $P(k+1)$ is to

LHS = RHS

$\rightarrow \sum_{i=1}^{k+1} i = \frac{(k+1)(k+1+1)}{2} = \frac{(k+1)(k+2)}{2}$

$\sum_{i=1}^{k} i + k+1 = \frac{k(k+1)}{2} + k+1$

from Inductive Hypothesis, we

$\sum_{i=}^{k} i = \frac{k(k+1)}{2}$

$\therefore \frac{k^2+3k+2}{2} = \frac{(k+1)(k+2)}{2}$ = RHS

$\therefore$ LHS = RHS

so, $P(k+1)$ is true.

$\forall n\, P(n),\ \sum_{i=1}^{n} i = \frac{n(n+1)}{2},\ n \geq 1$

tru

5) Show that $\frac{1}{1.2} + \frac{1}{2.3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$ by mathematical Induction.

→ $P(n) = \sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$

- Base Case : To show $P(1)$ is true.

$P(1) = \sum_{i=1}^{1} \frac{1}{1(1+1)} = \frac{1}{1+1}$

LHS $= \frac{1}{2}$

$\therefore$ LHS = RHS

So, $P(1)$ is true.

RHS $= \frac{1}{2}$

- Inductive Hypothesis : Let $P(K)$ be true.

$P(K) = \sum_{i=1}^{K} \frac{1}{i(i+1)} = \frac{K}{K+1}$

- Inductive Proof : To show $P(K+1)$ is true.

$\sum_{i=1}^{K+1} \frac{1}{i(i+1)} = \frac{K+1}{(K+1)+1}$

RHS $= \frac{K+1}{(K+1)+1}$

$= \frac{K+1}{K+2}$

LHS $= \sum_{i=1}^{K} \frac{1}{i(i+1)} + \frac{1}{(K+1)(K+2)}$

$= \frac{K}{K+1} + \frac{1}{(K+1)(K+2)}$

from Inductive Hypothesis, we have —

$= \frac{K(K+2)+1}{(K+1)(K+2)} = \frac{\cancel{(K+1)}(K+1)}{\cancel{(K+1)}(K+2)}$

$= \frac{K+1}{K+2}$

$\therefore$ LHS = RHS

So, $P(K+1)$ is true.

$\forall n \; P(n) = \sum_{i=1}^{n} i = \frac{n}{n+1}$, $n \in N$ is true.

(6) Prove $1.1! + 2.2! + 3.3! + \cdots + n.n! =$, for $n \geq 1$, by mathematical Indu

$\rightarrow$ $P(n) = \sum_{i=1}^{n} i.i! = (n+1)! - 1$

- Base Case: To show P(1) is true.

$P(1) = \sum_{i=1}^{1} i.i! = (1+1)! - 1$

LHS = 1

RHS = 2! − 1 = 1

$\therefore$ LHS = RHS

So, P(1) is true.

- Inductive Hypothesis: Let P(K) be

$P(K) = \sum_{i=1}^{K} i.i! = (K+1)! - 1$

- Inductive Proof: To show P(K+1) is true

$\sum_{i=1}^{K+1} i.i! = (K+2)! - 1$

LHS $\Rightarrow$ $\sum_{i=1}^{K+1} i.i!$

$\rightarrow \sum_{i=1}^{K} i.i! + (K+1)(K+1)!$

$\rightarrow (K+1)! - 1 + (K+1)(K+1)!$

[from Inductive Hypothesis], we hav

$\rightarrow (K+1)!(K+2) - 1$

$\rightarrow (K+2)! - 1$

RHS $\rightarrow (K+2)! - 1$

$\therefore$ LHS = RHS

So, P(K+1) is true.

$\forall n\ P(n) = \sum_{i=1}^{n} i.i! = (n+1)! - 1, n \geq$.

is tr

FOR EDUCATIONAL USE

7) Show that $1+3+5+7+\ldots+(2n-1) = n^2$ by induction

$$p(n) = \sum_{i=1}^{n} (2i-1) = n^2$$

- Base Case: To show $P(1)$ is true.

$$P(1) = \sum_{i=1}^{1} (2-1) = (1)^2$$

LHS = 1

RHS = $(1)^2 = 1$

$\therefore$ LHS = RHS

So, $P(1)$ is true.

- Inductive Hypothesis: let $P(K)$ is true.

$$P(K) = \sum_{i=1}^{K} (2i-1) = K^2$$

- Inductive Proof: To show $P(K+1)$ is true.

$$P(K+1) = \sum_{i=1}^{K+1} (2i-1) = (K+1)^2$$

LHS $\rightarrow \sum_{i=1}^{K+1} (2i-1)$

$\rightarrow \sum_{i=1}^{K} (2i-1) + (2K+1)$

$\rightarrow K^2 + (2K+1)$ [from inductive hypothesis]

$\rightarrow K^2 + 2K + 1$

$\rightarrow (K+1)^2$

RHS $\rightarrow (K+1)^2$

$\therefore$ LHS = RHS

So, $P(K+1)$ is true.

$\forall n\ P(n) = \sum_{i=1}^{n} (2i-1) = n^2$, $n \in N$ is true.

(8) Show that $1^3 + 2^3 + 3^3 + 4^3 + \ldots + \left[\frac{n(n+1)}{2}\right]^2$

$\rightarrow$ $p(n) = \sum^{n} (i)^3 = \left[\frac{n(n+1)}{2}\right]^2$ To show p...

- Base case: $i = 1$

LHS $\rightarrow \sum_{i=1}^{n} (i)^3$

$\rightarrow (1)^3 \rightarrow 1$

RHS $\rightarrow \frac{1(2)}{2} \Rightarrow 1$

$\therefore$ LHS = RHS

So, $P(1)$ is true.

- Inductive Hypothesis: let $P(K)$ is tr...

$$P(K) = \sum_{i=1}^{K} (i)^3 = \left[\frac{K(K+1)}{2}\right]^2$$

- Inductive Proof: To show $P(K+1)$ is ...

$$P(K+1) = \sum_{i=1}^{K+1} (i)^3 = \left(\frac{(K+1)\left[(K+1)+1\right]}{2}\right)^2$$

LHS $\rightarrow \sum_{i=1}^{K+1} (i)^3$

$\rightarrow \sum_{i=1}^{K} (i)^3 + (K+1)^3$

$\rightarrow \left(\frac{K(K+1)}{2}\right)^2 + (K+1)^3$ [from Ind... Hypothe...

$\rightarrow \frac{K^2(K+1)^2}{4} + (K+1)^3$

$\rightarrow \frac{(K+1)^2(K^2+4K+4)}{4}$

$\rightarrow \frac{(K+1)^2(K+2)^2}{4} \rightarrow \left[\frac{(K+1)(K+2)}{2}\right]^2$

RHS $\rightarrow \left[\frac{(K+1)(K+2)}{2}\right]^2$

$\therefore$ LHS = RHS

So, $P(K+1)$ is true.

$\forall n \; P(n) = \sum_{i=1}^{n} (i)^3 = \left[\frac{n(n+1)}{2}\right]^2, n \in N$ is true.

9) Prove $1.2 + 2.3 + 3.4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

$\rightarrow$ $P(n) = \sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$

- Base case : To show $P(1)$ is true.

$P(1) = \sum_{i=1}^{1} i(i+1) = \frac{1(2)(3)}{3}$

LHS $= 1(1+1) = 2$ $\therefore$ LHS = RHS

RHS $= 2\times\cancel{3}/\cancel{3} = 2$ So, $P(1)$ is true.

- Inductive Hypothesis : let $P(K)$ is true.

$P(K) = \sum_{i=1}^{K} i(i+1) = \frac{K(K+1)(K+2)}{3}$

- Inductive Proof : To show $P(K+1)$ is true.

$P(K+1) = \sum_{i=1}^{K+1} i(i+1) = \frac{(K+1)(K+2)(K+3)}{3}$

LHS $\rightarrow \sum_{i=1}^{K+1} i(i+1)$

$\rightarrow \sum_{i=1}^{K} i(i+1) + (K+1)(K+2)$

$\rightarrow \frac{K(K+1)(K+2)}{3} + (K+1)(K+2)$

$\rightarrow \frac{K\cdot(K+1)(K+2) + 3(K+1)(K+2)}{3}$

$\rightarrow \frac{(K+1)(K+2)(K+3)}{3}$

RHS $\rightarrow \frac{(K+1)(K+2)(K+3)}{3}$

$\therefore$ LHS = RHS

So, $P(K+1)$ is true.

$\forall n \; P(n) = \sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$

(10) Show by mathematical induction.

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots + \frac{1}{(2n-1)(2n+1)}$$

$$\rightarrow \quad P(n) = \sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$$

- Base case: To show $P(1)$ is true.

$$P(1) = \sum_{i=1}^{1} \frac{1}{(2i-1)(2i+1)} = \frac{1}{2+1}$$

$$\text{LHS} \Rightarrow \frac{1}{(2-1)(2+1)} \rightarrow \frac{1}{1\times 3} \rightarrow \frac{1}{3}$$

$$\text{RHS} \Rightarrow \frac{1}{3}$$

$\therefore$ LHS = RHS

So, $P(1)$ is true.

- Inductive Hypothesis: let $P(K)$ is

$$P(K) = \sum_{i=1}^{K} \frac{1}{(2i-1)(2i+1)} = \frac{K}{2K+1}$$

- Inductive Proof: To show $P(K+1)$ is true.

$$P(K+1) = \sum_{i=1}^{K+1} \frac{1}{(2i-1)(2i+1)} = \frac{K+1}{2(K+1)+1}$$

$$\text{LHS} \rightarrow \sum_{i=1}^{K+1} \frac{1}{(2i-1)(2i+1)}$$

$$\rightarrow \sum_{i=1}^{K} \frac{1}{(2i-1)(2i+1)} + \frac{1}{(2K+1)(2K+3)}$$

[from Inductive Hypothesis]

$$\rightarrow \frac{K}{2K+1} + \frac{1}{(2K+1)(2K+3)}$$

$$\Rightarrow \frac{2k^2+3k+1}{(2k+1)(2k+3)}$$

$$\rightarrow \frac{(2k+1)(k+1)}{(2k+1)(2k+3)} \rightarrow \frac{k+1}{2k+3}$$

RHS $\rightarrow \frac{k+1}{2k+3}$

$\therefore$ LHS = RHS

So, $P(k+1)$ is true.

$\forall n\,(P(n)) = \sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$ is true.

(11) Prove $1+2+2^2+2^3+\dots+2^n = 2^{n+1}-1$, for $n \geq 0$.

$\rightarrow$ $P(n) = \sum_{i=1}^{n} 2^i = 2^{n+1}-1$

- Base Case : To show $P(0)$ is true.

$P(0) = \sum_{i=1}^{0} 2^0 = 2^{0+1}-1$

LHS $= 2^0 = 1$ $\therefore$ LHS = RHS

RHS $= 2^1 - 1 = 1$ $\rightarrow$ So, $P(0)$ is true.

- Inductive Hypothesis: let $P(k)$ is true.

$P(k) = \sum_{i=1}^{k} 2^i = 2^{k+1}-1$

- Inductive Proof : To show $P(k+1)$ is true.

$P(k+1) = \sum_{i=1}^{k+1} 2^i = 2^{(k+1)+1}-1$

LHS $\Rightarrow \sum_{i=1}^{k+1} 2^i \Rightarrow \sum_{i=1}^{k} 2^i + 2^{k+1}$

$\Rightarrow 2^{k+1} - 1 + 2^{k+1}$ [from inductive Hypothesis]

$\Rightarrow 2 \cdot 2^{k+1} - 1$

$\Rightarrow 2^{(k+1)+1} - 1$

RHS $\Rightarrow 2^{(k+1)+1} - 1$

$\therefore$ LHS = RHS

So, P(k+1) is true.

$\forall n\ P(n) = \sum_{i=1}^{n} 2^i = 2^{n+1} - 1,\ n \geq 0$ is tr

(12) Prove the following using mathemat induction, for any integer (n) –
$11^{n+2} + 12^{2n+1}$ is divisible by 133.

$\rightarrow$ $P(n) = 11^{n+2} + 12^{2n+1}$ is divisible by 13

- Base case: To show P(1) is true.
  $P(1) = 11^3 + 12^3 = 3059 = 133 \times 23$
  $\therefore$ P(1) is true.
- Inductive Hypothesis: let P(K) is tr
  $P(K) = 11^{K+2} + 12^{2K+1} = 133m$
- Inductive Proof: To show P(K+1) is t

$\rightarrow 11^{(K+3)} + 12^{2(K+1)+1}$

$\rightarrow 11^{(K+2)} \cdot 11 + 12^{2K+1} \cdot 12^2 + 11 \cdot 12^{2K+1} - 11 \cdot 12^{2K+1}$

$\rightarrow 11(11^{K+2} + 12^{2K+1}) + 12^{2K+1}(144 - 11)$

$\rightarrow 11(133m) + 12^{2K+1} \cdot 133$ [from inductive Hypothes

$\rightarrow 133(11m + 12^{2K+1})$

P(K+1) is divisible by 133.

$\therefore$ P(K+1) is true.

$\forall n\ [P(n)] = 11^{n+2} + 12^{2n+1}$ is divisible 13

(13) Prove $\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} > \frac{13}{24}$ for

(14)

$\rightarrow P(n) = \sum_{i=2}^{n} \frac{1}{n+i} > \frac{13}{24}$

- Base Case: To show P(2) is true.

$$P(2) = \sum_{i=2}^{2} \frac{1}{2+i} = \frac{1}{2+1} + \frac{1}{2+2}$$

$$= \frac{7}{12} > \frac{13}{24}$$

$\therefore$ P(2) is true.

- Inductive Hypothesis: Let P(K) be true.

$$\sum_{i=2}^{k} \frac{1}{2+i} > \frac{13}{24}$$

- Inductive Proof: To show P(K+1) is true.

$$\sum_{i=2}^{K+1} \frac{1}{n+i} > \frac{13}{24}$$

$$\sum_{i=2}^{k} \frac{1}{n+i} + \frac{1}{(K+1)+n} > \frac{13}{24}$$

From inductive Hypothesis, we have,

$$\sum_{i=2}^{k} \frac{1}{n+i} > \frac{13}{14}$$

Hence, $\sum_{i=2}^{k} \frac{1}{n+i} + \frac{1}{n+(K+1)} > \frac{13}{24}$ is true.

$\therefore$ P(K+1) is true.

$\forall n\ p(n) = \sum_{i=2}^{n} \frac{1}{n+i} > \frac{13}{24}$, $n \geq 2$ is true.

(14) Show that $2^n \times 2^n - 1$ is divisible by 3 for all $n \geq 1$.

$\rightarrow$ $P(n) = 2^n \times 2^n - 1$

- Base Case : To show P(1) is true.

$P(1) = 2^1 \times 2^1 - 1$

$= 4 - 1 = 3$

$\therefore$ P(1) is divisible by 3

So, P(1) is true.

- Inductive Hypothesis : let P(K) be

$P(K) = 2^K \times 2^K - 1 = 3m$

$2^{2K} - 1 = 3m$

- Inductive Proof : To show P(K+1) is t

$= 2^{(K+1)} \times 2^{(K+1)} - 1$

$= 2.2^K . 2.2^K - 1$

$= 4 . 2^K . 2^K - 1$

$= (3.2^K.2^K + 2^K.2^K) - 1$

$= 3.2^{2K} + 2^{2K} - 1$ (from Induct Hypothesis

$= 3.2^{2K} + 3m$

$= 3(2^{2K} + m)$

$\therefore$ P(K+1) is divisible by 3.

So, P(K+1) is true.

$\forall n \; P(n) = 2^n \times 2^n - 1 ; n \geq 1$ is tr

(15) Prove that $1^2 + 3^2 + 5^2 + 7^2 + \dots + (2n-1)$

$= \frac{n(2n-1)(2n+1)}{3}$ by induction.

$\rightarrow P(n) = \sum_{i=1}^{n} (2i-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

- Base Case: To show $P(1)$ is true.

$P(1) = \sum_{i=1}^{1} (2i-1)^2 = \frac{1(2-1)(2+1)}{3}$

LHS: $\sum_{i=1}^{1} (2-1)^2 = 1$

RHS: $\frac{1 \times 3}{3} = 1$

$\therefore$ LHS = RHS
SO, $P(1)$ is true.

- Inductive Hypothesis: Let $P(K)$ be true.

$P(K) = \sum_{i=1}^{K} (2i-1)^2 = \frac{K(2K-1)(2K+1)}{3}$

- Inductive Proof: To show $P(K+1)$ is true.

$\sum_{i=1}^{K+1} (2i-1)^2 = \frac{(K+1)(2K+1)(2K+3)}{3}$

LHS: $\sum_{i=1}^{K+1} (2i-1)^2 = \sum_{i=1}^{K} (2i-1)^2 + (2(K+1)-1)^2$

$= \frac{K(2K-1)(2K+1)}{3} + (2K+1)^2$ (from inductive Hypothesis)

$= \frac{2K+1}{3}[K(2K-1)+3(2K+1)]$

$= \frac{2K+1}{3}(2K^2+5K+3)$

$= \frac{(2K+1)(K+1)(2K+3)}{3}$

RHS: $\frac{(K+1)(2K+1)(2K+3)}{3}$

$\therefore$ LHS = RHS
SO, $P(K+1)$ is true.

$\forall n\ P(n) = \sum_{i=1}^{n} (2i-1)^2 = \frac{n(2n-1)(2n+1)}{3}$, $n \in \mathbb{N}$ is true.

(16) Show that $n^3 + 2n$ is divisible by
for all $n \geq 1$.

→ $P(n) = n^3 + 2n$

- Base Case : To show $P(1)$ is true.

$P(1) = 1^3 + 2 \cdot 1$
$= 3$

$P(1)$ is divisible by 3.
$\therefore$ $P(1)$ is true.

- Inductive Hypothesis : let $P(k)$ be

$P(k) = k^3 + 2k = 3m$

- Inductive proof : To show $P(k+1)$ is

$P(k+1) = (k+1)^3 + 2(k+1)$

$= (k+1)(k^2 + 2k + 1) + 2k + 2$
$= k^3 + 3k^2 + 3k + 1 + 2k + 2$
$= k^3 + 3k^2 + 5k + 3$
$= k^3 + 2k + 3k^2 + 3k + 3$
$= 3m + 3k^2 + 3k + 3$ (from inductive Hypoth
$= 3(m + k^2 + k + 1)$

$\therefore$ $P(k+1)$ is divisible by 3.
So, $P(k+1)$ is true.

$\forall n$ $P(n) = n^3 + 2n$ is divisible
is true.

(17) Show by mathematical Induction –

$$\frac{1^2}{1\cdot 3} + \frac{2^2}{3\cdot 5} + \frac{3^2}{5\cdot 7} \cdots\cdots + \frac{n^2}{(2n-1)(2n+1)} = \frac{n(n+1)}{2(2n+1)}$$

→ $P(n) = \sum_{i=1}^{n} \frac{(i)^2}{(2i-1)(2i+1)} = \frac{n(n+1)}{2(2n+1)}$

- Base Case: To show $P(1)$ is true.

$$P(1) = \sum_{i=1}^{1} \frac{(1)^2}{(2-1)(2+1)} = \frac{1(1+1)}{2(2+1)}$$

$LHS = \frac{1}{1(3)} = \frac{1}{3}$

$RHS = \frac{1\times\cancel{2}}{\cancel{2}(3)} = \frac{1}{3}$

$\therefore$ LHS = RHS

So, $P(1)$ is true.

- Inductive Hypothesis: let $P(K)$ be true.

$$P(K) = \sum_{i=1}^{K} \frac{(i)^2}{(2i-1)(2i+1)} = \frac{K(K+1)}{2(2K+1)}$$

- Inductive Proof: To show $P(K+1)$ is true.

$$\sum_{i=1}^{K+1} \frac{(i)^2}{(2i-1)(2i+1)} = \frac{(K+1)(K+2)}{2(2K+3)}$$

LHS ⇒

→ $\sum_{i=1}^{K} \frac{(i)^2}{(2i-1)(2i+1)} + \frac{(K+1)^2}{(2K+1)(2K+3)}$

→ $\frac{K(K+1)}{2(2K+1)} + \frac{(K+1)^2}{(2K+1)(2K+3)}$

(from inductive Hypothesis)

$\rightarrow \frac{k+1}{2k+1}\left[\frac{k}{2}+\frac{k+1}{2k+3}\right]$

$\rightarrow \frac{k+1}{2k+1}\left[\frac{2k^2+5k+2}{2(2k+3)}\right]$

$\rightarrow \frac{k+1}{2k+1}\left[\frac{(2k^2+4k)+k+2}{2(2k+3)}\right]$

$\rightarrow \frac{k+1}{\cancel{2k+1}}\left[\frac{\cancel{(2k+1)}(k+2)}{2(2k+3)}\right]$

$\rightarrow \frac{k+1}{[illegible]} \quad \frac{(k+2)}{2(2k+3)}$

$\rightarrow \frac{(k+1)(k+2)}{2(2k+3)}$

RHS : $\frac{(k+1)(k+2)}{2(2k+3)}$

$\therefore$ LHS = RHS

So, $P(k+1)$ is true.

$\forall n \; P(n) = \sum_{i=1}^{n} \frac{(i)^2}{(2i-1)(2i+1)} = \frac{n(n+1)}{2(2n+1)}$

$n \in N$, is tr