Performance Analysis of Rigid 3D Pointcloud Registration Algorithms

Registration algorithms are used in different fields and applications, such as 3D object scanning, 3D mapping, 3D localization and ego-motion estimation, human body detection. The registration algorithms can be classified coarsely into rigid and non-rigid approaches. Rigid approaches assume a rigid environment such that the transformation can be modeled using only 6 Degrees of Freedom (DOF). Non-rigid methods on the other hand, can cope with articulated objects or soft bodies that change shape over time.

Most of these employs either a simple Singular Value Decomposition (SVD) or Principal Component Analysis (PCA) based registration, or use a more advance iterative scheme based on the Iterative Closest Point (ICP) algorithm. Recently, many variants on the original ICP approach have been proposed, the most important of which are non-linear ICP, generalized ICP, and non-rigid ICP. The choice for one of these algorithms generally depends on several important characteristics such as accuracy, computational complexity, and convergence rate, each of which depends on the application of interest. Moreover, the characteristics of most registration algorithms heavily depend on the data used, and thus on the environment itself. It is difficult to compare these algorithms objectively. Therefore, in this report we discuss the mathematical foundations that are common to the most widely used 3D registration algorithms, and we compare their strengths and weaknesses in different situations.

REGISTRATION ALGORITHMS

Both rigid and non-rigid registration algorithms can be categorized into pairwise registration algorithms and Multiview registration methods. Pairwise registration algorithms calculate a rigid transformation between two subsequent point clouds while the multi-view registration process takes multiple point clouds into account to correct for the accumulated drift that is introduced by pairwise registration methods.

Here, we discuss five widely used rigid registration algorithms. Each of these methods tries to estimate the optimal rigid transformation that maps a source point cloud on a target point cloud. Both PCA alignment and singular value decomposition are pairwise registration methods based on the covariance matrices and the cross-correlation matrix of the pointclouds, while the ICP algorithm and its variants are based on iteratively minimizing a cost function that is based on an estimate of point correspondences between the pointclouds

A. **Principal Component Analysis: -**
   PCA is often used in classification and compression techniques to project data on a new orthonormal basis in the direction of the largest variance. The direction of the largest variance corresponds to the largest eigenvector of the covariance matrix of the data, whereas the magnitude of this variance is defined by the corresponding eigenvalue. Therefore, if the covariance matrix of two pointclouds differs from the identity matrix, a rough registration can be obtained by simply aligning the eigenvectors of their covariance matrices.

B. **Singular Value Decomposition: -**

PCA based registration simply aligns the directions of the largest variance of each pointcloud and therefore does not minimize the Euclidean distance between corresponding points of the datasets. Consequently, this approach is very sensitive to outliers and only works well if each pointcloud is approximately normally distributed. However, if point correspondences between the two pointclouds are available, a more robust approach would be to directly minimize the sum of the Euclidean distances between these points. This corresponds to a linear least-square problem that can be solved robustly using the SVD method.

C. Iterative Closest Point: -

Whereas the SVD algorithm directly solves the least-square problem, thereby assuming perfect data So, introduced a method that iteratively disregards outliers to improve upon the previous estimate of the rotation and translation parameters. Their method is called 'ICP'. Point correspondences between these pointclouds are defined based on a nearest neighbor approach or a more elaborate scheme using geometrical features or color information.
SVD, is used to obtain an initial estimate of the affine transformation matrix that aligns both pointclouds. After registration, this whole process is repeated by removing outliers and redefining the point correspondences. Two widely used ICP variants are the ICP point-to-point and the ICP point-to-surface algorithms. These approaches only differ in their definition of point correspondences.

ICP point-to-point:

The ICP point-to-point algorithm was originally described in [1] and simply obtains point correspondences by searching for the nearest neighbor target point qi of a point pj in the source pointcloud. The nearest neighbor matching is defined in terms of the Euclidean distance metric.

ICP point-to-surface:

Due to the simplistic definition of point correspondences, the ICP point-to-point algorithm proposed is rather sensitive to outliers. Instead of directly finding the nearest neighbor to a source point pj in the target pointcloud, one could take the local neighborhood of a correspondence candidate qi into account to reduce the algorithm's sensitivity to noise. The ICP point-to-surface algorithm assumes that the point clouds are locally linear, such that the local neighborhood of a point is co-planar. This local surface can then be defined by its normal vector n, which is obtained as the smallest eigenvector of the covariance matrix of the points that surround correspondence candidate qi.

ICP non-linear:

Both the point-to-point and point to-surface ICP approaches defined a differentiable, convex, squared cost function, resulting in a simple linear least-square optimization problem, known as a L2-optimization, that can be solved numerically using SVD. However, L2-optimization is known to be highly sensitive to outliers because the residuals are squared. An approach that solves this problem is known as L1-optimization where the sum of the absolute value of the residuals is minimized instead of the square. However, the L1 cost function is non-differentiable at the origin which makes it difficult to obtain the optimal solution.

D. Generalized ICP:

A major disadvantage of the traditional point-to-point ICP algorithm, is that it assumes that the source pointcloud is taken from a known geometric surface instead of being obtained through noisy measurements. However, the point-to-surface ICP algorithm relaxes this constraint by allowing point offsets along the surface, to cope with discretization differences. However, this approach still assumes that the source pointcloud represents a discretized sample set of a known geometric surface model since offsets along the surface are only allowed in the target pointcloud. To solve this, Generalized ICP algorithm is proposed which performs plane-to-plane matching. They introduced a probabilistic interpretation of the minimization process such that structural information from both the source pointcloud and the target pointcloud can be incorporated easily in the optimization algorithm. Moreover, they showed that the traditional point-to-point and point-to-surface ICP algorithms are merely special cases of the Generalized ICP framework.

RESULTS & DISCUSSION

we illustrate the performance difference between a naive PCA based approach, a correspondence based SVD approach, and the ICP point-to-point registration approach. Figure shows the matching error plotted against the number of iterations for the ICP point-to-point algorithm without pre-alignment, and for the ICP point-to-point algorithm (light-gray) where the data has been pre-aligned using the SVD approach. In the latter case, a simple nearest neighbor matching was used to define point correspondences, after which the SVD algorithm was used to solve the least squares problem. This result clearly shows the importance of a rough initial alignment before applying the ICP algorithm.

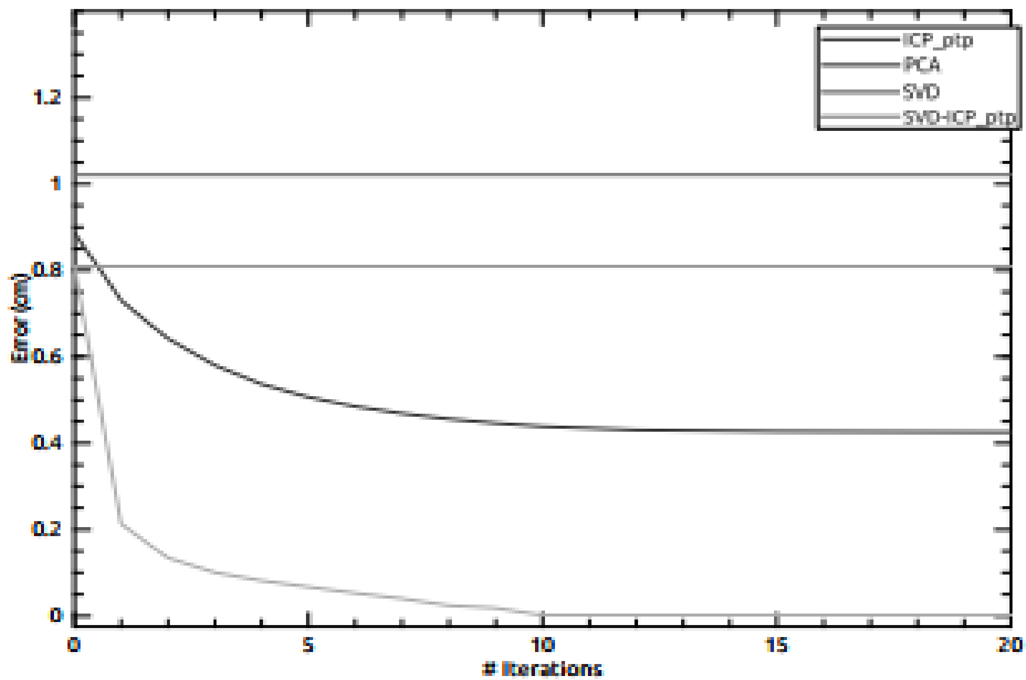

Figure 7. Comparison between PCA, SVD and general point to point ICP

Furthermore, figure shows the results of a single SVD based least-squares iteration, and the results obtained using the PCA based registration approach. The PCA based approach yields the largest matching error, since it does not incorporate correspondence information, such that this method is highly sensitive to outliers. On the other hand, a simple PCA or SVD based approach is extremely computational efficient, whereas the iterative ICP scheme is often too computationally expensive for real-time applications. However, Figure 7 shows that convergence can be reached quickly if a rough initial alignment is available. Finally, it is important to note that result of the variants of ICP such as point-to-plane and plane-to-plane greatly depend on the input data. If the source pointcloud does not contain much noise, while the target pointcloud is mostly smooth and piece-wise planar, the point-to-plane algorithm outperforms. the traditional point-to-point method. On the other hand, if the geometric structures in the scene are mostly quadratic or polynomial, the traditional ICP point-to-point algorithm yields better results. Similarly, if a lot of noise is observed in the source pointcloud, ICP plane-to-plane outperforms ICP point to-plane.