

**DEPARTMENT OF PHYSICS  
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS**

PH2140 Mathematics on the Computer

Assignment 8

21 Sep 2015

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**1.** Consider the matrix representing the  $y$  component of spin 1 angular momentum operator

$$J_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

1. Show that  $J_y$  is Hermitian.
  2. Find the eigenvalues and *normalized* eigenvectors of  $J_y$ .
  3. Show that  $U^\dagger U = I$ , where  $U$  is the matrix formed by taking above vectors as it's columns.
  4. Show that the matrix  $U^\dagger J_y U$  is diagonal. Identify the diagonal elements.
- 2.** Check out the command `PauliMatrix[k]`, and using it, establish the following results for the Pauli matrices  $\sigma^{(k)}$ :

1.  $\text{trace}(\sigma^{(k)} \sigma^{(m)}) = 2\delta_{km}$
2.  $\sum_{k=1}^3 \sigma_{ab}^{(k)} \sigma_{ij}^{(k)} = 2\delta_{aj}\delta_{bi} - \delta_{ab}\delta_{ij}$