DEPARTMENT OF PHYSICS
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PH2140 Mathematics on the Computer Assignment 5 31 August 2015

1. Forced damped oscillations and resonance

The equation of motion for a forced oscillator is

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos\omega t,$$

where β is the damping coefficient, ω_0 is the natural frequency of the oscillator and ω is the frequency of the driving force. Correspondingly, $\tau_0 = 2\pi/\omega_0$ is the natural time-period of the oscillator and $\tau = 2\pi/\omega$ is the time-period of the driving force.

- Solve this equation of motion for $x(t)$ subject to the initial conditions $x(0) = \dot{x}(0) = 0$, using $\beta = 0.1$ and $\tau = 1$. Plot the solution for the driving force parameters $A = 1$ and $\tau = 1$.
- Use the "Manipulate" command to study the motion for different values of the damping parameter β. What is the range of values of β for which you observe transient behavior?
- Fixing the value of $\beta = 0.1$, use the "Manipulate" command to study the behavior of the forced oscillator for different value of the driving period τ. What is the value of τ at which you observe resonance?

2. "Humped" potential well

An object of mass m moves in one dimension according to Newton's second law $F = m\ddot{x}$, with a force $F(x) = ax^2 - bx$, where a and b are positive constants.

- Numerically solve for $x(t)$ with initial conditions $x(0) = x_0$, $\dot{x} = 0$, and plot the results for $0 \le t \le t_{\max}$. Choose $t_{\max}$ large enough so that you can observe the $t \to \infty$ behavior. Choose three values for x_0 which satisfy each of these constraints:

  (i) $|x_0| \ll b/2a$

  (ii) x_0 close to $-b/2a$, but a little larger.

  (iii) x_0 close to $b/2a$, but a little smaller.

  You may pick any appropriate numerical values for a, b and m.

- What is special about the value $x_0 = b/2a$? To understand this, solve for the potential energy $U(x) = -\int_0^x F(u)du$.