DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH2140 Mathematics on the Computer   Assignment 6   8 Sep 2015

Discrete Sine and Cosine Transform

1. Recall that the inner product $\langle f(x), g(x)\rangle$ for a pair of real, square-integrable functions defined over the interval $0 \le x \le 1$ is given by

$$\langle f(x), g(x)\rangle = \int_0^1 dx f(x) g(x).$$

   Using this definition, show that the set of functions $\mathcal{S} \equiv \{\sin(kx), k = 1, 2, 3, \ldots\}$ are mutually orthogonal for $0 \le x \le \pi$. What is the normalization constant that makes this an orthonormal set?

   Similarly, show that the set of functions $\{\mathcal{C} \equiv \cos lt, (0 \le t \le \pi) l = 0, 1, 2, \ldots\}$ is also an orthonormal set. Normalize this set.

2. The normalized sine and cosine functions form an orthonormal basis for the space of square integrable functions. Thus, any square integrable function $f(x)$ can be expressed in terms of the sine (or the cosine) basis, as

$$f(x) = \sum_{k=1}^{\infty} b_k \sin(kx),\ b_k = N \int_0^{\pi} f(x) \sin(kx),$$

   where N is a normalization factor. Obtain the first ten coefficients b_k for the sine and cosine series of the following functions:

   (a) $f(x) = x^2$
   (b) $f(x) = x^3$
   (c) $f(x) = e^{\pm a(x-b)^2}$, for some positive constants a, b.
   (d) The unit step-function:

$$f(x) = \begin{cases} \frac{1}{L}, 0 \le x \le L, \\ 0, \text{otherwise.} \end{cases}$$

3. For each of the above functions, compare the original function with its sine and cosine series, using the Plot command. What can you say about the sine series of an even function and the cosine series of an odd function?