DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH2140 Mathematics on the Computer Assignment 3 17 August 2015

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(1) Projectile Motion:

A projectile is fired with initial speed v_0 from the top of a tower, at an angle *theta* with respect to the horizontal (x) direction. The tower is at a height $y = h$ above the ground. The equations describing the motion of the projectile are therefore, with $(x, y) = (0, h)$ being the co-ordinates of the top of the tower,

$$x = v_0 t \cos\theta, \quad y = h + v_0 t \sin\theta - \frac{1}{2} g t^2.$$

(i) Set $g = 9.8\, m/s^2$. For $v_0 = 5m/s$, $h = 10m$, and, $\theta = 15°$, make a parametric plot of the trajectory of the projectile (y versus x). Change the values of h and θ and convince yourself that the trajectories look reasonable.

(ii) Solve the equation $y(t) = 0$ for the time when the projectile hits the ground. Use this time to find the range $x(t)$. Make a plot of (a) the range versus θ for a fixed h, and, (b) range versus h for a fixed θ.

(2) Infinite series for π:

The *Madhava-Leibniz* formula for π states that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \ldots + (-1)^n \frac{1}{2n+1},$$

where we have indicated the general form for the n^{th} term in the sum. Evaluate the sum of the first 10 terms in the series. To how many decimal places is the resultant value of Π accurate?

This is infact an extremely slowly convergent series, and you may check that adding the first 50 terms only gives Π accurate to the first decimal place! One approach to improve the convergence properties of the series is to add end correction terms:

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \ldots + (-1)^n \frac{1}{2n+1} + (-1)^{n+1} \frac{1}{a_n}.$$

To what extent is the accuracy improved if we include the following end-correction terms after summing the first 10 terms?

(i) $a_n = 4(n+1)$.

(ii) $a_p = \frac{(2n+2)^2+1}{n+1}$

(iii) $a_n = (n+1)\frac{(2n+2)^2+5}{(n+1)^2+1}$

Here, a_n denotes the correction term to be added after summing the series to n terms.