

Logistic Regression: Theory & Visualizations

1. Logistic Regression Formula

Logistic regression predicts the probability of a binary outcome using the sigmoid function:

$$P(y = 1 | X) = 1 / (1 + e^{-(b_0 + b_1x_1 + \dots + b_nx_n)})$$

The output is a probability between 0 and 1.

2. Sigmoid Function

The sigmoid function maps any real-valued number to (0, 1):

$$\text{sigmoid}(z) = 1 / (1 + e^{-z})$$

This makes it suitable for modeling probabilities.

3. Gradient Descent in Logistic Regression

To train logistic regression, we use gradient descent to minimize the binary cross-entropy loss:

$$\text{Loss} = -1/m * \sum [y * \log(\hat{y}) + (1 - y) * \log(1 - \hat{y})]$$

Update rule (each epoch):

$$\theta := \theta - \alpha * \text{gradient}$$

Where:

- α is the learning rate
- gradient = derivative of the loss with respect to parameters

4. Decision Boundary

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Logistic regression draws a linear decision boundary between classes in feature space.

This boundary corresponds to the threshold where $P(y=1) = 0.5$.