Assignment No 8 - Analysis of circuits using Laplace transforms

Introduction

In this assignment, the focus will be on two powerful capabilities of Python:

- Symbolic algebra
- Analysis of circuits using laplace transforms

We use the sympy module to handle our requirements in solving Modified Nodal Analysis equations.Combining this with the scipy.signal module , we can analyze both highpass and lowpass filters.

Analysis of lowpass filters

The given circuit specified is a lowpass filter.After doing the modified nodal analysis and simplifying the equations , we get:

$$\begin{bmatrix} 0 & 0 & 1 & -1/G \\ \frac{-1}{sR_2C_2} & 1 & 0 & 0 \\ 0 & -G & G & 1 \\ \frac{-1}{R_1} - \frac{1}{R_2} - sC_1 & \frac{1}{R_2} & 0 & sC_1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_p \\ V_m \\ V_o \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{-V_i(s)}{R_1} \end{bmatrix}$$

The block of code to define the lowpass filter:

```
def lowpass(R1,R2,C1,C2,G,Vi):
s=  sympy.symbols("s")
A = sympy.Matrix([[0,0,1,-1/G],[-1/(1+s*R2*C2),1,0,0],
[0,-G,G,1],[-1/R1-1/R2-s*C1,1/R2,0,s*C1]])
b=  sympy.Matrix([0,0,0,-Vi/R1])
V = A.inv()*b
return A,b,V
```

The magnitude response of this circuit is,

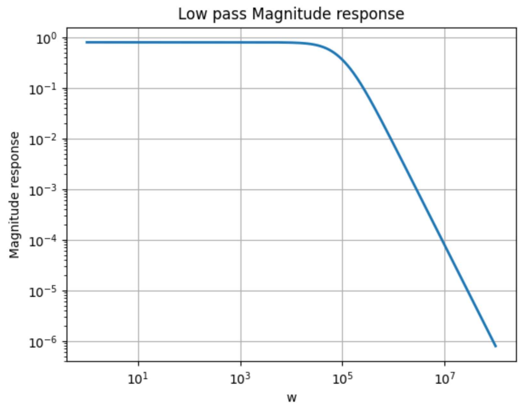

Figure 1: Magnitude response of lowpass filter

Question 1

The unit step response for the lowpass filter:

```
A1,b1,V1 = lowpass(10000,10000,1e-9,1e-9,1.586,1/s)
Vo1 = V1[3]
H1 = symToTransferFn(Vo1)
t,y1 = sp.impulse(H1,None,np.linspace(0,1e-2,10000,dtype=float))
plot_graph(t,y1,"Step response for low pass filter",'t','Vo(t)')
```

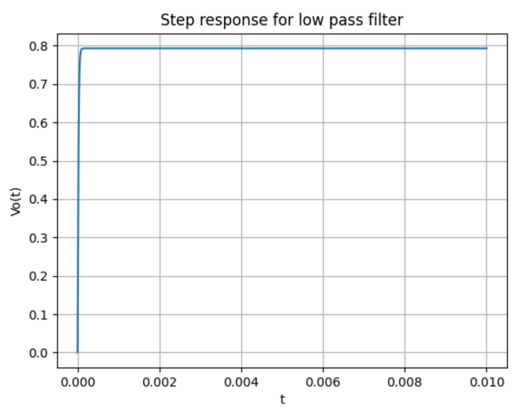

Figure 2: Unit step response of lowpass filter

Question 2

We give a sum of two sinusoids of different sinusoids as the input to the lowpass filter.

$$V_i(t) = [sin(2000\pi t) + cos(2 * 10^6 \pi t)]u_o(t)$$

```
def inp_response(Y,inp=inputs,tlim=1e-2):
t = np.linspace(0,tlim,100000)
Vi=inp(t)
num,den = symToTransferFn(Y)
H = sp.lti(num,den)
t,y,svec = sp.lsim(H,Vi,t)
```

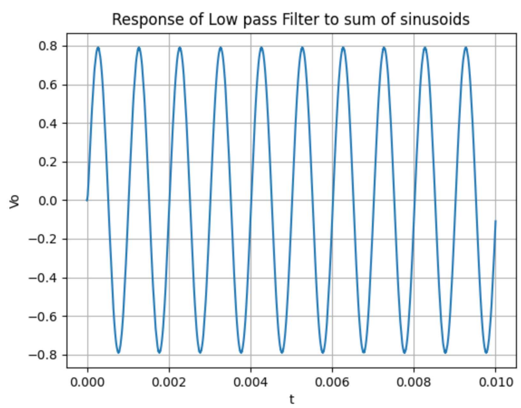

Figure 3: Response of lowpass filter to the sum of sinusoids

Analysis of highpass filters

The given circuit specified is a highpass filter.After doing the modified nodal analysis and simplifying the equations , we get:

$$\begin{bmatrix} 0 & -1 & 0 & -1/G \\ \frac{sC_2R_3}{sC_2R_3+1} & 0 & -1 & 0 \\ 0 & G & -G & 1 \\ -sC_2-\frac{1}{R1}-sC_1 & 0 & sC_2 & \frac{1}{R1} \end{bmatrix} \begin{bmatrix} V_1 \\ V_p \\ V_m \\ V_o \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{-V_i(s)}{R_1} \end{bmatrix}$$

Question 2.2

We plot the response of the highpass filter to the sum of the two sinusoids.

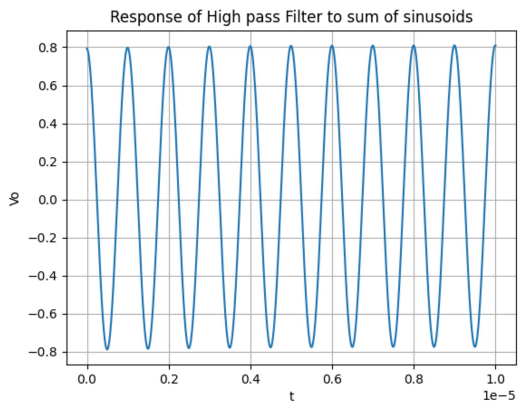

Figure 4: Response of highpass filter to the sum of sinusoids

Question 3

The magnitude bode plot of the highpass filter looks like:

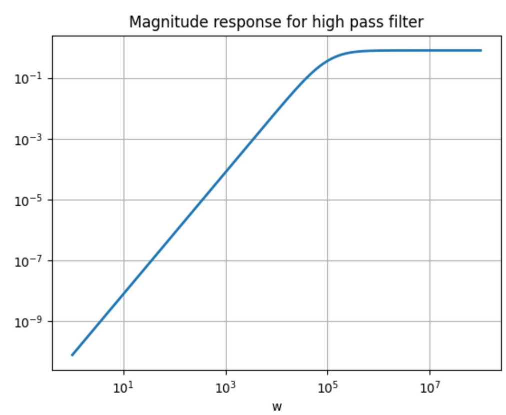

Figure 5: Magnitude response of highpass filter

Question 4

The high frequency damped sinusoid used here is:

$$V_i(t) = cos(10^7t).exp(-3 * 10^3t).u_o(t)$$

The low frequency damped sinusoid used here is:

$$V_i(t) = cos(10^3t).exp(-10t).u_o(t)$$

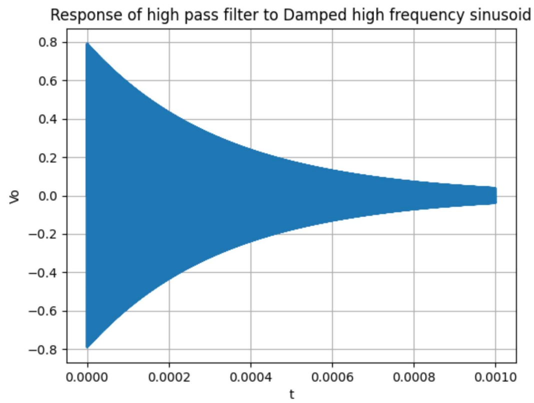

Figure 6: Output of high frequency damped sinusoid to highpass filter

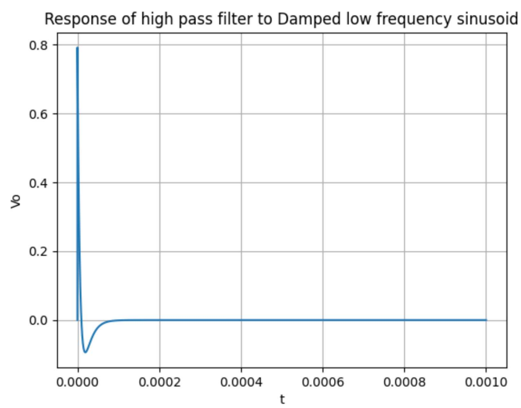

Figure 7: Output of low frequency damped sinusoid to lowpass filter

The high pass filter responds by quickly attenuating the low-frequency input. This is because the input frequency is below the cutoff frequency, so the output goes to 0 very fast.

Question 5

The unit step response, as expected is high at t=0 when there is an abrupt change in the input. Since there is no other change at large time values outside the neighbourhood of 0, the Fourier transform of the unit step has high values near 0 frequency, which the high pass filter attenuates.

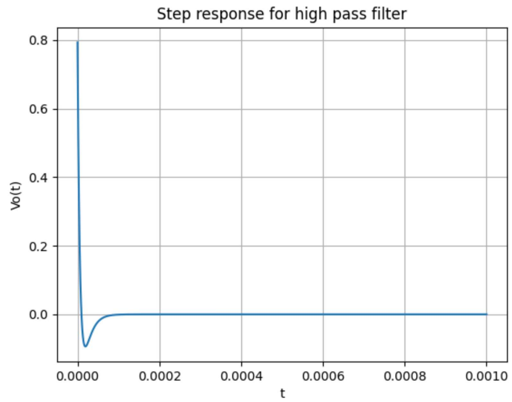

Figure 8: Step response of highpass filter

Conclusion

In conclusion, the sympy module has allowed us to analyse quite complicated circuits by analytically solving their node equations. We then interpreted the solutions by plotting time domain responses using the signals toolbox. Thus, sympy combined with the scipy.signal module is a very useful toolbox for analyzing complicated systems like the active filters in this assignment.