Q1. $\theta^T Y = w_0 + w_1 x_1 + w_2 x_2 + \cdots + w_n x_n$

$(\theta^T Y)_{new} = w_0' + w_1' x_1 + w_2' x_2 + \cdots + w_n' x_n + w_{n+1}' x_{n+1}$

if $x_{n+1} = x_n$

$= w_0' + w_1' x_1 + w_2' x_2 + \cdots + \underbrace{(w_n' + w_{n+1}')}_{w_n''} x_n$

So,
$w_0 = w_0'$
$w_1 = w_1'$
$w_{n-1} = w_{n-1}'$
$w_n = w_n'' = w_n' + w_{n+1}'$
$w_n' = w_{n+1}$ (due to symmetry)
$w_n' = w_{n+1} = \frac{w_n}{2}$

Q3 $\theta_j := \theta_j - \alpha\left(\frac{1}{m}\sum_{i=1}^{m}\left(h_\theta(x^i) - Y^i\right)x_j^{(i)}\right)$

$j = 1 \ldots n.$

Now for sparse matrix, the multiplication can be done with low computational cost in some of the library

Q4. $c > a > b$

5 (a) MLE

y = no. of times head appears in k tosses

$$f(p) = P(y = k \mid p) = {}^nC_k \, p^k (1-p)^{n-k}$$

$$\frac{df}{dp} = 0 \quad \text{for} \quad f(p) \text{ to be maximum.}$$

$$\Rightarrow ({}^nC_k)\left[k p^{k-1} (1-p)^{n-k} + p^k (n-k)(1-p)^{n-k-1}(-1) \right]$$

$$\Rightarrow \quad p = k/n$$

(b) Prior

$$\Rightarrow \quad f(p) = \begin{cases} 1 & 0 < p < 1 \\ 0 & \text{otherwise} \end{cases}$$

Now,

$$\underset{\text{Posterior}}{f(p)} = \underset{\text{Likelihood}}{{}^nC_k \, p^k (1-p)^{n-k}} \; \underset{\text{Prior}}{f(p)}$$

$$f(p) = \begin{cases} {}^nC_k \, p^k (1-p)^{n-k} & 0 < p < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{mean} = \int_0^1 {}^nC_k \, p^k (1-p)^{n-k} \cdot p \, dp$$

$f(p) \propto$ beta distribution

$$\alpha - 1 = k \Rightarrow \alpha = k+1$$
$$\beta - 1 = n-k \Rightarrow \beta = n-k+1$$

$$\text{mean} = \frac{\alpha}{\alpha+\beta} = \frac{k+1}{k+1+n-k+1} = \frac{k+1}{n+2}$$

(c) mode of β distribution

$$= \frac{\alpha - 1}{\alpha + \beta - 2} = \frac{(k+1) + (-1)}{k+1 + n - k + 1 - 2}$$

$$= \frac{k}{n}.$$

2. (a) false, as we have all the required data.

(b) ~~for~~ A & E

$E = 14\%$ $\quad$ $A = 10\%$

$p_1 = 0.14$ $\quad$ $p_2 = .10$

$n_1 = 1000$ $\quad$ $n_2 = 1000$

$$z = \frac{p_1 - p_2}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$n_1 = n_2$

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} \Rightarrow \frac{p_1 + p_2}{2} = 0.12$$

$$z = \frac{0.14 - 0.10}{\sqrt{\frac{(0.12)(0.88)}{500}}}$$

$$= \frac{4 \times 10^{-2} \times 10 \times \sqrt{5}}{\sqrt{0.12 \times 0.88}} = 2.75$$

for one tailed test p-value = 0.0029
Hence E is better than A with 95%
confidence as

$$0.0029 < 0.05$$

we have written the code for the
calculation for the other templets

B => z = 2.40539 p value = 0.0079 < 0.05
C => 8.5% z = 1.15 p-value = 0.125 > 0.05
D = 12% z = 1.4293 p-value = 0.07 < 0.05

Hence B option is correct.