

CA 202
Mathematical Foundations of Computer Science

Unit-I

Induction and Recursion

1. Define Permutation and Combination.

Permutation: Imagine you have a bag filled with different colored balls, such as red, blue, and green. Permutation means arranging these balls in different orders. For example, if you have three balls and you can arrange them in different ways like red, blue, green or blue, red, green, these are called permutations.

Combination: Now, let's say you have a box of chocolates with different flavors, such as vanilla, strawberry, and chocolate. Combination means selecting a few chocolates from the box without worrying about the order in which you pick them. For example, if you choose two chocolates like vanilla and strawberry, the combination remains the same whether you picked vanilla first or strawberry first.

So, in simple terms, permutation is about arranging things in different orders, while combination is about selecting things without worrying about the order.

2. Explain principals of counting using example.

Let's imagine you have a box of colorful candies. The candies come in three different colors: red, blue, and green. Now, let's say you want to know how many different combinations of candies you can make by picking one candy at a time.

Principle 1: The Multiplication Principle

The multiplication principle tells us that if we have two separate choices to make, and each choice has a certain number of options, we can find the total number of combinations by multiplying the number of options together.

In our example, we have three colors of candies: red, blue, and green. If we want to know how many different combinations we can make by picking one candy at a time, we simply multiply the number of colors (3) by the number of candies we want to pick (1). So, we have 3 choices: red, blue, or green.

Principle 2: The Addition Principle

The addition principle tells us that if we have two separate choices to make, and each choice has a certain number of options, we can find the total number of combinations by adding the number of options together.

Let's say we want to know how many different combinations we can make by picking two candies at a time, without putting the first one back. We still have three colors of candies: red, blue, and green. For the first candy, we have 3 choices. Then, for the

second candy, we have 2 choices because we can't pick the same candy again. To find the total number of combinations, we add the number of choices together: $3 + 2 = 5$. So, we can make 5 different combinations of two candies.

These are the basic principles of counting. Whether we're counting candies, toys, or anything else, we can use these principles to figure out how many different combinations or choices we have.

3. Explain pigeonhole principal.

Imagine you have a bunch of socks in different colors, like red, blue, and green. The pigeonhole principle helps us understand that if we have more things than places to put them, then at least one place must have more than one thing.

Let's say you have four socks: two red socks and two blue socks. You want to put these socks in two drawers, one for red socks and one for blue socks. Since you have four socks and only two drawers, at least one drawer will have more than one sock in it. This is because you have more socks than you have drawers to put them in.

The pigeonhole principle works like this: if you have "pigeons" (the things you're trying to put in places) and "pigeonholes" (the places you're trying to put the things), and you have more pigeons than pigeonholes, then at least one pigeonhole must have more than one pigeon.

This principle is helpful in many different situations. For example, if you have more friends than seats in your car, at least two friends will have to share a seat. Or if you have more apples than pockets in your bag, at least one pocket will have more than one apple.

So, the pigeonhole principle is a way to understand that if you have more things than places to put them, then at least one place must have more than one thing.

4 .What is recursive algorithm explaining it with example.

Imagine you have a big box filled with smaller boxes. Each smaller box can also have more smaller boxes inside. A recursive algorithm is like opening each box, one by one, until you reach the smallest box.

Let's say you want to count the number of candies in all the boxes. Here's how the recursive algorithm works:

1. Start with the big box.
2. Open the big box and count the candies in it.
3. If you find another box inside the big box, repeat steps 1 and 2 for that box. This is where the recursion comes in – you're using the same steps on a smaller version of the problem.
4. Keep opening and counting candies in each smaller box until you reach a box that doesn't have any more boxes inside.
5. Add up the number of candies you counted in each box to get the total number of candies.

Let's see an example:

You start with the big box, which has three smaller boxes inside. Each of these smaller boxes can have more smaller boxes inside or candies directly.

1. You open the big box and find a small box inside.
2. You open the small box and find two candies.
3. You add those two candies to your total count.
4. Then, you go back to the big box and open the second small box.
5. This time, you find another small box inside.
6. You open the new small box and find three candies.
7. You add those three candies to your total count.
8. Now, you go back to the big box and open the third small box.
9. This time, there are no more boxes inside.
10. You count the candies in the third small box and add them to your total count.
11. Finally, you have the total count of candies from all the boxes.

Recursive algorithms are like solving a big problem by breaking it down into smaller, similar problems and solving each of them one by one. It's like opening boxes within boxes until you reach the smallest box.

5 From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

To form a committee with at least 3 men from a group of 7 men and 6 women, let's calculate the number of ways it can be done.

First, let's consider the scenario where exactly 3 men are selected for the committee:

1. Choose 3 men from the group of 7 men. This can be done in "7 choose 3" ways, which is equal to $7! / (3! * (7 - 3)!)$, where "!" represents factorial. Simplifying, we get $7! / (3! * 4!)$.

Next, let's consider the scenario where exactly 4 men are selected for the committee:

1. Choose 4 men from the group of 7 men. This can be done in "7 choose 4" ways, which is equal to $7! / (4! * (7 - 4)!)$, simplifying to $7! / (4! * 3!)$.

Finally, let's consider the scenario where all 5 men are selected for the committee:

1. Choose 5 men from the group of 7 men. This can be done in "7 choose 5" ways, which is equal to $7! / (5! * (7 - 5)!)$, simplifying to $7! / (5! * 2!)$.

To calculate the total number of ways to form the committee, we add up the number of ways from each scenario:

$$\text{Total number of ways} = (7! / (3! * 4!)) + (7! / (4! * 3!)) + (7! / (5! * 2!))$$

Simplifying further, we get:

$$\text{Total number of ways} = (7 * 6 * 5 / (3 * 2 * 1)) + (7 * 6 / (4 * 3 * 2 * 1)) + (7 / (5 * 4 * 3 * 2 * 1))$$

$$\text{Total number of ways} = 35 + 7 + 1$$

$$\text{Total number of ways} = 43$$

Therefore, there are 43 different ways to form the committee with at least 3 men selected from the group of 7 men and 6 women.

6 Solve 5P_2 and 6C_3 and $5!, 8!$

Sure! Let's solve the given expressions:

1. 5P_2 :

The notation " nPm " represents the number of permutations of selecting " m " objects from a set of " n " objects without repetition and considering the order of selection.

$${}^5P_2 = 5! / (5 - 2)!$$

$$= 5! / 3!$$

$$= (5 * 4 * 3!) / 3!$$

$$= 5 * 4$$

$$= 20$$

Therefore, 5P_2 is equal to 20.

2. 6C_3 :

The notation " nCm " represents the number of combinations of selecting " m " objects from a set of " n " objects without repetition and without considering the order of selection.

$$\begin{aligned}
{}^6C_3 &= 6! / (3! * (6 - 3)!) \\
&= 6! / (3! * 3!) \\
&= (6 * 5 * 4!) / (3! * 3!) \\
&= (6 * 5) / (3 * 2) \\
&= 10
\end{aligned}$$

Therefore, 6C_3 is equal to 10.

3. $5!$ (5 factorial):

The factorial of a number "n" is the product of all positive integers from 1 to "n".

$$\begin{aligned}
5! &= 5 * 4 * 3 * 2 * 1 \\
&= 120
\end{aligned}$$

Therefore, 5 factorial is equal to 120.

4. $8!$ (8 factorial):

Similarly, let's calculate 8 factorial.

$$\begin{aligned}
8! &= 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1 \\
&= 40,320
\end{aligned}$$

Therefore, 8 factorial is equal to 40,320.

7 In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together?

To find the number of different ways the letters of the word 'OPTICAL' can be arranged so that the vowels always come together, we can treat the group of vowels (O, I, A) as a single entity.

So, let's consider the group of vowels as one letter. Now, we have the letters 'PTCL' and the vowel group 'OIA'. We need to arrange these six letters.

The total number of ways to arrange these six letters is $6!$, which means arranging six letters in all possible orders.

However, within the vowel group 'OIA', the vowels can be arranged among themselves in $3!$ ways (OIA, OAI, AIO, etc.), since there are three different vowels.

Therefore, the number of arrangements where the vowels always come together is $6! * 3!$.

Simplifying this expression, we get:

$$6! = 6 * 5 * 4 * 3 * 2 * 1 = 720$$

$$3! = 3 * 2 * 1 = 6$$

$$\text{Number of arrangements} = 720 * 6 = 4320$$

So, there are 4320 different ways to arrange the letters of the word 'OPTICAL' such that the vowels always come together.

8 Consider a set having 5 elements a, b, c, d, e. In how many ways 3 elements can be selected (without repetition) out of the total number of elements

To find the number of ways to select 3 elements (without repetition) from a set of 5 elements (a, b, c, d, e), we can use the concept of combinations.

A combination is a selection of items where the order does not matter. We can use the formula for combinations, denoted as " nCr ", to calculate the number of ways.

In this case, we have 5 elements and we want to select 3 elements. So, we can calculate $5C3$, which means finding the number of combinations of selecting 3 elements from a set of 5 elements.

Using the combination formula, we can calculate $5C3$ as follows:

$$5C3 = 5! / (3! * (5 - 3)!)$$

$$\begin{aligned}
&= 5! / (3! * 2!) \\
&= (5 * 4 * 3!) / (3! * 2!) \\
&= 5 * 4 \\
&= 20
\end{aligned}$$

Therefore, there are 20 different ways to select 3 elements (without repetition) from a set of 5 elements (a, b, c, d, e).

9 A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has—a) no girls b) at least one boy and one girl c) at least three girls

Let's solve each part of the question separately:

a) To form a team of 5 members with no girls, we can only choose from the 7 boys. So, we need to calculate the number of ways to select 5 boys from a group of 7 boys.

The number of ways to select 5 boys from 7 boys is given by 7C_5 :

$$\begin{aligned}
{}^7C_5 &= 7! / (5! * (7 - 5)!) \\
&= 7! / (5! * 2!) \\
&= (7 * 6 * 5!) / (5! * 2!) \\
&= 7 * 6 / 2 \\
&= 21
\end{aligned}$$

Therefore, there are 21 different ways to form a team of 5 members with no girls.

b) To form a team of 5 members with at least one boy and one girl, we need to consider two scenarios: (i) selecting 1 girl and 4 boys, and (ii) selecting 2 girls and 3 boys.

(i) Selecting 1 girl and 4 boys:

The number of ways to select 1 girl from 4 girls and 4 boys from 7 boys is given by $4C1 * 7C4$:

$$\begin{aligned}4C1 * 7C4 &= (4! / (1! * (4 - 1)!)) * (7! / (4! * (7 - 4)!)) \\&= (4 * 7 * 6) / (3 * 2) \\&= 4 * 7 * 3 \\&= 84\end{aligned}$$

(ii) Selecting 2 girls and 3 boys:

The number of ways to select 2 girls from 4 girls and 3 boys from 7 boys is given by $4C2 * 7C3$:

$$\begin{aligned}4C2 * 7C3 &= (4! / (2! * (4 - 2)!)) * (7! / (3! * (7 - 3)!)) \\&= (6 * 7 * 5) / (3 * 2) \\&= 10 * 7 \\&= 70\end{aligned}$$

Therefore, the total number of ways to form a team with at least one boy and one girl is $84 + 70 = 154$.

c) To form a team of 5 members with at least three girls, we need to consider two scenarios:

(i) selecting exactly 3 girls and 2 boys, and (ii) selecting all 4 girls and 1 boy.

(i) Selecting exactly 3 girls and 2 boys:

The number of ways to select 3 girls from 4 girls and 2 boys from 7 boys is given by $4C3 * 7C2$:

$$\begin{aligned}4C3 * 7C2 &= (4! / (3! * (4 - 3)!)) * (7! / (2! * (7 - 2)!)) \\&= (4 * 7 * 6) / (2 * 1) \\&= 4 * 7 * 3 \\&= 84\end{aligned}$$

(ii) Selecting all 4 girls and 1 boy:

The number of ways to select all 4 girls from 4 girls and 1 boy from 7 boys is given by ${}^4C_4 * {}^7C_1$:

$$\begin{aligned} {}^4C_4 * {}^7C_1 &= \frac{4!}{(4! * (4 - 4)!)} * \frac{7!}{(1! * (7 - 1)!)} \\ &= \frac{(7 * 4)}{(1)} \\ &= \end{aligned}$$

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Therefore, the total number of ways to form a team with at least three girls is $84 + 28 = 112$.

To summarize:

- a) There are 21 different ways to form a team of 5 members with no girls.
- b) There are 154 different ways to form a team of 5 members with at least one boy and one girl.
- c) There are 112 different ways to form a team of 5 members with at least three girls.

10) In a small village, there are 87 families of which 52 families have at most 2 children. In a rural development programme, 20 families are to be helped chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made

To determine the number of ways the choice can be made for the rural development program, we need to consider the different scenarios based on the number of families with at most 2 children.

Let's break it down into cases:

Case 1: Selecting 18 families with at most 2 children and 2 families with more than 2 children:

The number of ways to choose 18 families with at most 2 children from the 52 families is given by ${}^{52}C_{18}$.

The number of ways to choose 2 families with more than 2 children from the remaining 35 families is given by ${}^{35}C_2$.

Therefore, the total number of ways for this case is ${}^{52}C_{18} * {}^{35}C_2$.

Case 2: Selecting 19 families with at most 2 children and 1 family with more than 2 children:

The number of ways to choose 19 families with at most 2 children from the 52 families is given by ${}^{52}C_{19}$.

The number of ways to choose 1 family with more than 2 children from the remaining 35 families is given by ${}^{35}C_1$.

Therefore, the total number of ways for this case is ${}^{52}C_{19} * {}^{35}C_1$.

Case 3: Selecting all 20 families with at most 2 children:

The number of ways to choose 20 families with at most 2 children from the 52 families is given by ${}^{52}C_{20}$.

Now, we can calculate the total number of ways by summing up the number of ways from each case:

$$\text{Total number of ways} = ({}^{52}C_{18} * {}^{35}C_2) + ({}^{52}C_{19} * {}^{35}C_1) + ({}^{52}C_{20})$$

We can use combinatorial formulas to calculate the combinations:

$${}^nC_r = n! / (r! * (n - r)!)$$

Calculating the combinations may involve large numbers, so it is recommended to use a calculator or software for the calculations.

Therefore, the total number of ways the choice can be made depends on the values of $({}^{52}C_{18} * {}^{35}C_2)$, $({}^{52}C_{19} * {}^{35}C_1)$, and $({}^{52}C_{20})$, which can be calculated using the formulas above.

**11) How many arrangements can be made, with the letters of the word
CALCULATOR? In how many of these arrangements, vowels occur together?**

To find the number of arrangements that can be made with the letters of the word "CALCULATOR," we need to calculate the total number of arrangements first.

The word "CALCULATOR" has 10 letters. Let's consider each letter individually:

C has 1 occurrence.

A has 2 occurrences.

L has 1 occurrence.

C has 1 occurrence.

U has 1 occurrence.

L has 1 occurrence.

A has 2 occurrences.

T has 1 occurrence.

O has 1 occurrence.

R has 1 occurrence.

To calculate the total number of arrangements, we use the concept of permutations. The number of permutations gives us the number of different arrangements we can make using all the letters.

Using the formula for permutations, we calculate the total number of arrangements as follows:

Total number of arrangements = $10!$

$$\begin{aligned} 10! &= 10 * 9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1 \\ &= 3,628,800 \end{aligned}$$

So, there are 3,628,800 different arrangements that can be made with the letters of the word "CALCULATOR."

Now, let's find the number of arrangements where the vowels occur together.

To calculate this, we can treat the vowels (A, A, O, U) as a single entity. This means we consider these vowels as one letter. So, we have 6 "letters" to arrange: C, L, C, L, T, and (AAOU).

The number of arrangements where the vowels occur together can be found by treating the group of vowels as one letter and calculating the permutations. So, we calculate the number of arrangements of 6 letters, where (AAOU) is considered one letter.

Total number of arrangements with vowels together = $6!$

$$\begin{aligned} 6! &= 6 * 5 * 4 * 3 * 2 * 1 \\ &= 720 \end{aligned}$$

Therefore, there are 720 different arrangements where the vowels (A, A, O, U) occur together in the word "CALCULATOR."

12) Out of 5 consonants and 3 vowels, how many words of 3 consonants and 2 vowels can be formed?

To find out how many words can be formed using 3 consonants and 2 vowels out of a set of 5 consonants and 3 vowels, we can use the concept of combinations.

First, let's consider the number of ways to choose 3 consonants out of 5. This can be calculated using the combination formula, denoted as " nCr ".

The number of ways to select 3 consonants from a set of 5 consonants is given by $5C3$.

Using the combination formula, we calculate 5C_3 as follows:

$$\begin{aligned} {}^5C_3 &= 5! / (3! * (5 - 3)!) \\ &= 5! / (3! * 2!) \\ &= (5 * 4 * 3!) / (3! * 2!) \\ &= (5 * 4) / (2) \\ &= 10 \end{aligned}$$

So, there are 10 different ways to choose 3 consonants from 5 consonants.

Next, let's consider the number of ways to choose 2 vowels out of 3. This can also be calculated using the combination formula.

The number of ways to select 2 vowels from a set of 3 vowels is given by 3C_2 .

Using the combination formula, we calculate 3C_2 as follows:

$$\begin{aligned} {}^3C_2 &= 3! / (2! * (3 - 2)!) \\ &= 3! / (2! * 1!) \\ &= (3 * 2 * 1!) / (2 * 1!) \\ &= (3 * 2) / 2 \\ &= 3 \end{aligned}$$

So, there are 3 different ways to choose 2 vowels from 3 vowels.

To find the total number of words that can be formed using 3 consonants and 2 vowels, we multiply the number of ways to choose the consonants (10) by the number of ways to choose the vowels (3).

$$\begin{aligned} \text{Total number of words} &= 10 * 3 \\ &= 30 \end{aligned}$$

Therefore, there are 30 different words that can be formed using 3 consonants and 2 vowels from a set of 5 consonants and 3 vowels.

13) In a colony, there are 55 members. Every member posts a greeting card to all the members. How many greeting cards were posted by them?

In the colony, there are 55 members, and each member decides to send a greeting card to every other member. We want to find out how many greeting cards were posted in total.

To determine the number of greeting cards posted, we can think of it as each member sending a card to every other member.

Let's break it down step by step:

- First member: They send a greeting card to the remaining 54 members.
- Second member: They also send a greeting card to the remaining 54 members (excluding themselves, as they already received a card from the first member).
- Third member: They send a greeting card to the remaining 54 members as well.
- And so on...

We can see that each member sends a greeting card to the remaining 54 members. Since there are 55 members in total, each member sends 54 cards.

To find the total number of greeting cards posted, we multiply the number of members (55) by the number of cards sent by each member (54):

$$\begin{aligned}\text{Total number of greeting cards} &= 55 * 54 \\ &= 2,970\end{aligned}$$

Therefore, a total of 2,970 greeting cards were posted by the members of the colony.

14. Use mathematical induction to prove $7^n - 2^n$ is divisible by 5.

To prove that the expression $7n - 2n$ is divisible by 5 using mathematical induction, we will follow two steps: the base case and the induction step.

Step 1: Base Case

We start by checking if the statement holds true for the smallest possible value. In this case, we will consider $n = 1$.

Substituting $n = 1$ into the expression $7n - 2n$, we get:

$$7(1) - 2(1) = 7 - 2 = 5$$

Since 5 is divisible by 5, the base case holds true.

Step 2: Induction Step

Now, we assume that the statement holds true for some arbitrary value k , where k is a positive integer. This assumption is known as the induction hypothesis.

Assuming that $7k - 2k$ is divisible by 5, we need to prove that the statement also holds true for $k + 1$, which means proving that $7(k + 1) - 2(k + 1)$ is divisible by 5.

Expanding the expression:

$$7(k + 1) - 2(k + 1) = 7k + 7 - 2k - 2 = 5k + 5$$

We can rewrite $5k + 5$ as $5(k + 1)$, which is clearly divisible by 5.

Since the expression $7(k + 1) - 2(k + 1)$ can be expressed as $5(k + 1)$, and $5(k + 1)$ is divisible by 5, the statement holds true for $k + 1$.

By completing both the base case and induction step, we can conclude that $7n - 2n$ is divisible by 5 for all positive integers n , using mathematical induction.

15 (How many even 4 digit whole numbers are there?)

To find out how many even 4-digit whole numbers there are, we need to consider the possible values for each digit in a 4-digit number.

The first digit cannot be 0, as it would make the number a 3-digit number. So, we have 9 choices for the first digit (from 1 to 9).

The second, third, and fourth digits can be any digit from 0 to 9 (including 0), as they don't affect the number's evenness.

However, for the last digit (the units digit) to make the entire number even, it must be an even number (0, 2, 4, 6, or 8).

Therefore, for the second, third, and fourth digits, we have 10 choices (0 to 9), and for the last digit, we have 5 choices (0, 2, 4, 6, or 8).

To find the total number of even 4-digit numbers, we multiply the number of choices for each digit:

Total number of even 4-digit numbers = 9 (choices for the first digit) * 10 (choices for the second digit) * 10 (choices for the third digit) * 5 (choices for the last digit)

Total number of even 4-digit numbers = $9 * 10 * 10 * 5$

Total number of even 4-digit numbers = 4,500

Therefore, there are 4,500 even 4-digit whole numbers.

16) The code for a safe is of the form PPPQQQQ where P is any number from 0 to 9 and Q represents the letters of the alphabet. How many codes are possible for each of the following cases? Note that the digits and letters of the alphabet can be

repeated.

A family has two children. Given that one of the children is a girl and that she was born on a Monday, what is the probability that both children are girls?

To determine the probability that both children are girls given that one of them is a girl born on a Monday, we can use conditional probability.

Let's break down the problem:

1. Total number of possible outcomes:

Since each child can be either a boy or a girl, there are four possible outcomes for the gender of the two children: BB, BG, GB, GG.

2. Given information:

We know that one of the children is a girl born on a Monday. This information rules out the BB outcome, leaving us with three possible outcomes: BG, GB, GG.

3. Favorable outcomes:

Among the remaining outcomes, BG and GB represent families with one girl and one boy. However, the GG outcome represents a family with two girls, which is the favorable outcome in this case.

4. Calculating the probability:

To calculate the probability, we need to determine the ratio of the favorable outcome (GG) to the total number of possible outcomes (BG, GB, GG).

Probability = (Number of favorable outcomes) / (Number of total outcomes)

In this case, there is only one favorable outcome (GG) out of the three possible outcomes (BG, GB, GG).

Probability = 1 / 3

Therefore, the probability that both children are girls given that one of them is a girl born on a Monday is $\frac{1}{3}$.

Note: The given information about the day of the week the child was born (Monday) is not directly relevant to the probability of the gender of the other child.

17) Five cards are selected at random from a pack of 52 cards with replacement. Find the possible combinations.

To find the possible combinations when five cards are selected at random from a deck of 52 cards with replacement, we can think of it as picking a card, noting its value, and then placing it back into the deck before picking another card.

Let's break down the process:

1. For each card selection, we have 52 options since there are 52 cards in the deck initially.
2. After selecting the first card, we place it back into the deck, so the total number of options remains 52 for the subsequent selections.
3. Since we are selecting five cards in total, we repeat the process of selecting a card and placing it back into the deck five times.

To find the total number of possible combinations, we multiply the number of options for each selection:

Total number of possible combinations = 52 (options for the first selection) * 52 (options for the second selection) * 52 (options for the third selection) * 52 (options for the fourth selection) * 52 (options for the fifth selection)

Total number of possible combinations = 52^5

Using a calculator, we can find that 52^5 is equal to 380,204,032.

Therefore, there are 380,204,032 possible combinations when five cards are selected at random from a deck of 52 cards with replacement.

18) A drawer contains 12 red and 12 blue socks, all unmatched. A person takes socks out at random in the dark. How many socks must he take out to be sure that he has at least two blue socks?

To ensure that the person has at least two blue socks, we need to determine the minimum number of socks they must take out from the drawer.

Let's analyze the worst-case scenario:

1. Initially, the person can draw any sock from the drawer, as there are both red and blue socks available.
2. In the worst-case scenario, the person could keep drawing red socks until they have taken out all 12 red socks.
3. After removing all 12 red socks, the person will be left with only the 12 blue socks in the drawer.
4. Since the person wants to have at least two blue socks, they need to draw one more sock from the drawer.

Therefore, the person must take out a minimum of 12 red socks + 1 additional sock (the 13th sock) to be sure that they have at least two blue socks.

Hence, the person must take out at least 13 socks to guarantee they have at least two blue socks.

19) There are 70 patients admitted in a hospital in which 29 are diagnosed with typhoid, 32 with malaria, and 14 with both typhoid and malaria. Find the number of patients diagnosed with typhoid or malaria or both

To find the number of patients diagnosed with typhoid or malaria or both, we need to consider the total number of patients diagnosed with each disease and account for any overlap.

Let's break it down:

1. Number of patients diagnosed with typhoid: 29
2. Number of patients diagnosed with malaria: 32
3. Number of patients diagnosed with both typhoid and malaria: 14

To calculate the number of patients diagnosed with typhoid or malaria or both, we add the number of patients diagnosed with typhoid to the number of patients diagnosed with malaria. However, we need to subtract the number of patients diagnosed with both diseases to avoid double-counting.

Number of patients diagnosed with typhoid or malaria or both = Number of patients diagnosed with typhoid + Number of patients diagnosed with malaria - Number of patients diagnosed with both typhoid and malaria

Number of patients diagnosed with typhoid or malaria or both = $29 + 32 - 14$

Number of patients diagnosed with typhoid or malaria or both = 57

Therefore, there are 57 patients diagnosed with typhoid or malaria or both in the hospital.

20 Find the number of ways of arranging the letters of the words DANGER, so that no vowel occupies odd place.

To find the number of ways of arranging the letters of the word "DANGER" such that no vowel occupies an odd place, we can break down the problem and consider the positions of the vowels and consonants separately.

Let's analyze the word "DANGER":

1. The word "DANGER" has three vowels: A, E, and the second occurrence of E.
2. The word "DANGER" has four consonants: D, N, G, and R.

Now, let's consider the possible arrangements where no vowel occupies an odd place:

1. We have three vowels to arrange: A, E, and E.
 - Since no vowel can occupy an odd place, we need to place all three vowels in the even positions (2nd, 4th, and 6th positions).
 - There is only one way to arrange the vowels in these even positions.
2. We have four consonants to arrange: D, N, G, and R.
 - The consonants can occupy the remaining odd positions (1st, 3rd, and 5th positions).
 - There are four consonants, so we have four options for the 1st position, three options for the 3rd position, and two options for the 5th position.

To find the total number of arrangements, we multiply the number of arrangements for the vowels and consonants:

Total number of arrangements = Number of arrangements for vowels * Number of arrangements for consonants

Total number of arrangements = $1 * 4 * 3 * 2$

Total number of arrangements = 24

Therefore, there are 24 ways to arrange the letters of the word "DANGER" such that no vowel occupies an odd place.

20) In how many ways can we select 6 people out of 10, of which a particular person is not included?

To calculate the number of ways to select 6 people out of a group of 10, excluding a particular person, we need to consider the remaining 9 people as candidates for selection.

Since we want to choose 6 people, we can use the concept of combinations to find the number of ways.

The formula for combinations is:

$${}^nC_r = n! / (r! * (n - r)!)$$

where n is the total number of items and r is the number of items we want to choose.

In this case, we have 9 candidates to choose from (excluding the particular person) and we want to select 6 people.

Using the combination formula, we can calculate the number of ways:

$${}^9C_6 = 9! / (6! * (9 - 6)!)$$

Calculating the factorials:

$$9! = 9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1$$

$$6! = 6 * 5 * 4 * 3 * 2 * 1$$

$$3! = 3 * 2 * 1$$

Plugging in the values:

$${}^9C_6 = (9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1) / ((6 * 5 * 4 * 3 * 2 * 1) * (3 * 2 * 1))$$

Simplifying:

$${}^9C_6 = (9 * 8 * 7) / (3 * 2 * 1)$$

$${}^9C_6 = 84$$

Therefore, there are 84 ways to select 6 people out of a group of 10, excluding a particular person.

22) Using the principle of mathematical induction, prove that

$$1^2 + 2^2 + 3^2 + \dots + n^2 = (1/6)\{n(n+1)(2n+1)\} \text{ for all } n \in \mathbb{N}.$$

To prove the given equation using the principle of mathematical induction, we will follow two steps:

Step 1: Base Case

Step 2: Inductive Step

Step 1: Base Case

We will start by verifying the equation for the base case, which is when $n = 1$.

When $n = 1$, the equation becomes:

$$1^2 = (1/6)\{1(1+1)(2(1)+1)\}$$

Simplifying both sides:

$$1 = (1/6)(2)(3)$$

$$1 = (1/6)(6)$$

$$1 = 1$$

Since the equation holds true for the base case, we move on to the inductive step.

Step 2: Inductive Step

Assuming the equation holds true for some positive integer k , we will prove that it also holds true for $k + 1$.

Assuming the equation holds for k , we have:

$$1^2 + 2^2 + 3^2 + \dots + k^2 = (1/6)\{k(k + 1)(2k + 1)\}$$

Now, we need to prove that the equation holds for $k + 1$, which means we need to show:

$$1^2 + 2^2 + 3^2 + \dots + k^2 + (k + 1)^2 = (1/6)\{(k + 1)[(k + 1) + 1](2(k + 1) + 1)\}$$

We add $(k + 1)^2$ to both sides of the equation for k :

$$1^2 + 2^2 + 3^2 + \dots + k^2 + (k + 1)^2 = (1/6)\{k(k + 1)(2k + 1)\} + (k + 1)^2$$

Simplifying the right side of the equation:

$$(1/6)\{k(k + 1)(2k + 1)\} + (k + 1)^2 = (1/6)\{(k + 1)[k(2k + 1) + 6(k + 1)]\}$$

Expanding and simplifying further:

$$(1/6)\{(k + 1)[2k^2 + k + 6k + 6]\}$$

$$(1/6)\{(k + 1)[2k^2 + 7k + 6]\}$$

$$(1/6)\{(k + 1)(2k^2 + 4k + 3k + 6)\}$$

$$(1/6)\{(k + 1)(2k(k + 2) + 3(k + 2))\}$$

$$(1/6)\{(k + 1)(2k + 3)(k + 2)\}$$

We can observe that $(k + 1)(2k + 3)(k + 2)$ is the same as $(k + 1)[(k + 1) + 1][2(k + 1) + 1]$.

Therefore, we can conclude that the equation holds true for $k + 1$.

By applying the principle of mathematical induction, we have proven that the equation

$1^2 + 2^2 + 3^2 + \dots + n^2 = (1/6)\{n(n+1)(2n+1)\}$ holds for all positive integers n .

Unit II

Recurrence Relation

1) Consider the recurrence relation $a_1=4$, $a_n=5n+a_{n-1}$. The value of a_{64} .

To find the value of a_{64} using the given recurrence relation $a_1 = 4$ and $a_n = 5n + a_{n-1}$, we can start by recursively calculating the values of a_1 , a_2 , a_3 , and so on until we reach a_{64} .

Let's break down the process step by step:

1. We are given that $a_1 = 4$, which means the first term of the sequence is 4.

2. To find a_2 , we substitute $n = 2$ into the recurrence relation:

$$a_2 = 5(2) + a_1 = 10 + 4 = 14$$

3. To find a_3 , we substitute $n = 3$ into the recurrence relation:

$$a_3 = 5(3) + a_2 = 15 + 14 = 29$$

4. We continue this process, substituting the previous term into the recurrence relation to find the next term.

$$a_4 = 5(4) + a_3 = 20 + 29 = 49$$

$$a_5 = 5(5) + a_4 = 25 + 49 = 74$$

$$a_6 = 5(6) + a_5 = 30 + 74 = 104$$

5. We repeat the process until we reach a_{64} .

$$a_{64} = 5(64) + a_{63}$$

To find a_3 , we need to calculate all the previous terms in a similar manner.

6. Continue this process until we find a_4 .

By following these steps, we can determine the value of a_4 by recursively calculating the terms based on the given recurrence relation.

Note: Since this process involves performing multiple calculations, it may be helpful to use a calculator or a computer program to simplify the task.

2) What is recursion? Explain Tower of Hanoi problem

Recursion is a problem-solving approach where a function calls itself to solve a smaller version of the problem. It breaks down a complex problem into smaller and simpler subproblems, and then combines the results of those subproblems to solve the original problem.

The Tower of Hanoi is a classic example that demonstrates recursion. It involves moving a stack of different-sized disks from one peg to another, using an intermediary peg, while following certain rules:

1. There are three pegs labeled A, B, and C, and a number of disks of different sizes stacked in ascending order on peg A.
2. The objective is to move all the disks from peg A to peg C, using peg B as the intermediary, while following these rules:
 - a) Only one disk can be moved at a time.
 - b) A larger disk cannot be placed on top of a smaller disk.

Let's explain the process of solving the Tower of Hanoi problem step by step:

1. Start with the original configuration where all the disks are stacked on peg A.

2. To move the stack from peg A to peg C:

- a) Move the top (smallest) disk from peg A to peg C directly.
- b) Move the remaining stack of disks (all except the smallest) from peg A to peg B, using peg C as the intermediary.
- c) Move the stack of disks from peg B to peg C, using peg A as the intermediary.

3. The process described in step 2 is done recursively. In other words, to move the stack of disks from one peg to another, we repeat the same steps for the smaller stack of disks until we reach the base case, which is when there is only one disk left to be moved.

4. Once we solve the problem for the smaller stacks, we can combine the solutions to move the entire stack from peg A to peg C.

The key idea behind the recursive solution is to break down the problem into smaller subproblems and solve them recursively, relying on the base case (when there is only one disk) to terminate the recursion.

The Tower of Hanoi problem is a great way to understand recursion because it demonstrates how breaking down a problem into smaller parts and solving them recursively can lead to an efficient solution.

3) Explain divide and conquer algorithm

Divide and conquer is a problem-solving strategy that helps us solve big problems by breaking them down into smaller, more manageable pieces. It's like solving a big puzzle by solving smaller puzzles, and then putting the solutions together to solve the big puzzle.

Here's how divide and conquer works:

1. Divide: First, we divide the big problem into smaller subproblems that are easier to solve. We keep dividing the problem until it becomes small enough to solve easily.

2. Conquer: Once we have divided the problem into smaller subproblems, we solve each subproblem individually. We can use the same divide and conquer strategy to solve these subproblems.

3. Combine: Finally, we combine the solutions of the subproblems to solve the original big problem. We take the solutions from the smaller subproblems and put them together to get the solution to the entire problem.

The key idea behind divide and conquer is that breaking a big problem into smaller subproblems makes it easier to solve. It allows us to focus on solving smaller and simpler parts of the problem, which is usually more manageable.

For example, let's say you have a big pile of toys to organize. Instead of trying to organize the entire pile at once, you can divide the pile into smaller groups of toys. Then, you can organize each group separately, which is easier. Finally, you can combine the organized groups to have the entire pile of toys organized.

Divide and conquer is a powerful strategy because it allows us to solve complex problems by breaking them down into simpler pieces. It is used in many areas, such as computer science, mathematics, and even in everyday life, to tackle challenging tasks.

4) What is Inclusion-Exclusion? Explain applications of Inclusion Exclusion.

Inclusion-Exclusion is a counting principle that helps us count or calculate the number of elements in a set by considering overlapping or non-overlapping conditions. It allows us to account for situations where elements may belong to multiple sets or satisfy multiple conditions.

Let's explain the concept of Inclusion-Exclusion with a simple example:

Imagine you have two sets: Set A, which contains fruits like apples, bananas, and oranges, and Set B, which contains fruits like bananas, oranges, and grapes.

Now, if we want to know the total number of fruits in both sets combined, we might simply count all the fruits from Set A and Set B together. However, this would result in counting the bananas and oranges twice since they appear in both sets.

To avoid double-counting, we can use the principle of Inclusion-Exclusion. Here's how it works:

1. Inclusion: First, we count the number of fruits in Set A and the number of fruits in Set B.
2. Exclusion: Next, we subtract the number of fruits that belong to both Set A and Set B. This ensures that we don't count them twice.

By applying Inclusion-Exclusion, we get the total number of fruits in both sets without double-counting any fruits.

Applications of Inclusion-Exclusion:

1. Counting arrangements: Inclusion-Exclusion is often used to count the number of arrangements or combinations that satisfy certain conditions. For example, counting the number of ways to arrange objects while ensuring that certain objects are always together or apart.
2. Probability: Inclusion-Exclusion is also used in probability theory to calculate the probability of events that have overlapping conditions. It helps in determining the probability of two or more events occurring together or separately.
3. Set operations: Inclusion-Exclusion can be applied to set operations such as union, intersection, and complement. It helps in determining the cardinality or size of sets involved in these operations.

Inclusion-Exclusion is a useful counting principle that allows us to handle situations where elements may overlap or satisfy multiple conditions. It helps us avoid double-counting and provides a systematic way to calculate the total number of elements in a set or solve various counting problems.

- 5) **Determine the solution for the recurrence relation $b_n = 8b_{n-1} - 12b_{n-2}$ with $b_0 = 3$ and $b_1 = 4$.**
Determine the solution of the recurrence relation $F_n = 20F_{n-1} - 25F_{n-2}$ where $F_0 = 4$ and $F_1 = 14$.

Let's solve each recurrence relation step by step:

1. Recurrence relation: $b_n = 8b_{n-1} - 12b_{n-2}$, with $b_0 = 3$ and $b_1 = 4$.

To find the solution to this recurrence relation, we need to compute the values of b_n based on the given formula. Let's start by calculating the first few terms:

$$b_0 = 3 \text{ (given)}$$

$$b_1 = 4 \text{ (given)}$$

Using the recurrence relation, we can find the next terms:

$$b_2 = 8b_1 - 12b_0 = 8(4) - 12(3) = 32 - 36 = -4$$

$$b_3 = 8b_2 - 12b_1 = 8(-4) - 12(4) = -32 - 48 = -80$$

$$b_4 = 8b_3 - 12b_2 = 8(-80) - 12(-4) = -640 + 48 = -592$$

We can continue this process to find more terms, but it seems that the values are becoming negative and may not follow a clear pattern. It's possible that this recurrence relation does not have a simple solution or a repeating pattern.

2. Recurrence relation: $F_n = 20F_{n-1} - 25F_{n-2}$, with $F_0 = 4$ and $F_1 = 14$.

Similarly, let's find the solution to this recurrence relation by calculating the values of F_n based on the given formula:

$$F_0 = 4 \text{ (given)}$$

$$F_1 = 14 \text{ (given)}$$

Using the recurrence relation, we can find the next terms:

$$F_2 = 20F_1 - 25F_0 = 20(14) - 25(4) = 280 - 100 = 180$$

$$F_3 = 20F_2 - 25F_1 = 20(180) - 25(14) = 3600 - 350 = 3250$$

$$F_4 = 20F_3 - 25F_2 = 20(3250) - 25(180) = 65000 - 4500 = 60500$$

We can continue this process to find more terms, and the values will follow a clear pattern. The recurrence relation has a simple solution that produces a sequence of numbers.

Please note that for more complex recurrence relations, it may require additional techniques or formulas to find the explicit solution or pattern

6) If $16P_{r-1} : 15P_{r-1} = 16 : 7$ then find r .

To find the value of r , we can set up and solve the equation using the given ratio of permutations.

The ratio given is:

$$16P_{r-1} / 15P_{r-1} = 16 / 7$$

To simplify the equation, we can cancel out the common term in both numerator and denominator:

$$16 / 15 = 16 / 7$$

Now we have:

$$1 / 15 = 1 / 7$$

Since the fractions on both sides of the equation are equal, we can conclude that the numerator and denominator must be equal as well:

$$1 = 1$$

This equation is always true, regardless of the value of r . Therefore, there is no specific value of r that satisfies the given ratio.

In other words, there is no solution for r in this case.

7) What is the recurrence relation for the sequence 1, 3, 7, 15, 31, 63...?

The given sequence 1, 3, 7, 15, 31, 63... can be described by a simple recurrence relation.

If we look closely, we can see that each term in the sequence is obtained by doubling the previous term and then subtracting 1.

Let's write it down as a recurrence relation:

To find the next term, we take the previous term, double it, and subtract 1.

For example:

- The second term, 3, is obtained by doubling the first term, 1, and subtracting 1: $2(1) - 1 = 3$.
- The third term, 7, is obtained by doubling the second term, 3, and subtracting 1: $2(3) - 1 = 7$.
- The fourth term, 15, is obtained by doubling the third term, 7, and subtracting 1: $2(7) - 1 = 15$.
- And so on.

So, the recurrence relation for this sequence is:

$$a_n = 2 * a_{n-1} - 1$$

where " a_n " represents the n th term of the sequence.

Using this recurrence relation, we can find any term in the sequence by applying the formula to the previous term.

8) **Explain merge sort in detail.**

Merge sort is a sorting algorithm that helps us arrange a list of numbers or objects in a specific order, such as from smallest to largest. It follows a divide and conquer approach, which means it breaks down the problem into smaller parts and then combines the solutions to get the final sorted list.

Here's how merge sort works:

1. Divide: The first step is to divide the original list into smaller parts. We keep dividing the list until we have individual elements or small groups of elements. This makes it easier to sort them.

2. Sort: Once we have divided the list, we start sorting the individual elements or groups. We compare the elements within each group and arrange them in the desired order. If there are only two elements, we can easily compare and swap them if needed.

3. Merge: After sorting the individual elements or groups, we merge them back together to form a larger sorted list. We compare the elements from each group and select the smallest one. We then move that element to the new sorted list. We repeat this process until all the elements are merged into the final sorted list.

4. Repeat: We repeat the sorting and merging steps recursively until the entire list is sorted. This means we keep dividing, sorting, and merging the smaller parts until we have a completely sorted list.

Merge sort is efficient and reliable because it breaks down the sorting process into smaller and simpler steps. By dividing the list and sorting the smaller parts separately, we can ensure that each part is sorted correctly. Then, by merging the sorted parts together, we obtain a sorted list.

Children can visualize merge sort by imagining a set of cards that need to be sorted. They can divide the cards into smaller piles, sort each pile individually, and then combine the sorted piles to get a final sorted set of cards.

Merge sort is widely used in computer science and is one of the popular sorting algorithms due to its efficiency and effectiveness.

Unit-III

Statistics

1. What do you mean by Population, sample, parameters?

1. Population: In simple terms, a population refers to a large group of people, objects, or things that we want to study or understand. It includes everyone or everything that belongs to a specific group. For example, if we want to study all the students in a school, then the population would be all the students in that school. Population can also refer to all the animals in a forest or all the toys in a toy store.

2. Sample: A sample is a smaller group or subset of the population that we select to study or analyze. It is a representative portion of the larger population. Instead of studying every single person or object in the population, we choose a sample to make it more manageable and practical. For example, if we want to know what students in a school think about a certain topic, we can select a group of students from different grades to form our sample.

3. Parameters: Parameters are specific characteristics or qualities of a population that we want to measure or describe. They give us information about the entire population. For example, if we want to know the average height of all the students in a school, the average height would be a parameter. Parameters can also include things like the total number of objects in a population, the proportion of people who like a particular color, or the average age of all the animals in a zoo.

To summarize:

- Population is the entire group or collection of people, objects, or things we want to study.
- Sample is a smaller group or subset of the population that we choose to study instead of the whole population.
- Parameters are specific characteristics or qualities of the population that we want to measure or describe.

Understanding populations, samples, and parameters helps us gather information and make conclusions about larger groups by studying representative smaller groups.

2) Define mean, median and mode.

1. Mean: The mean is a measure of central tendency that tells us the average value of a set of numbers. To find the mean, we add up all the numbers in a group and then divide the total by the number of values in the group. For example, let's say we have the numbers 2, 4, 6, and 8. To find the mean, we add them up ($2 + 4 + 6 + 8 = 20$) and then divide by 4 (since there are 4 numbers). So, the mean is 20 divided by 4, which is 5. The mean represents the "typical" value in a group of numbers.

2. Median: The median is another measure of central tendency that tells us the middle value in a set of numbers. To find the median, we first arrange the numbers in ascending order (from smallest to largest) and then locate the middle number. If there are an odd number of values, the median is simply the middle number. If there are an even number of values, the median is the average of the two middle numbers. For example, let's say we have the numbers 3, 6, 9, and 12. The middle two numbers are 6 and 9, so to find the median, we take their average. The median in this case is $(6 + 9) / 2 = 7.5$.

3. Mode: The mode is the value that appears most frequently in a set of numbers. In other words, it's the number that occurs the most times. Sometimes a set of numbers may not have a mode, or it can have multiple modes (if there are two or more numbers that appear equally often). For example, let's say we have the numbers 2, 4, 4, 6, and 8. In this case, the number 4 appears twice, which is more than any other number. So, the mode is 4.

Understanding mean, median, and mode helps us describe and understand different aspects of a set of numbers. They are important tools in statistics and help us analyze data.

3) What do you mean by classification of data?

Classification of data refers to organizing or categorizing data into different groups or categories based on their similarities or characteristics. It helps us make sense of large amounts of information and understand it better.

Imagine you have a box filled with different types of fruits, such as apples, oranges, bananas, and grapes. If you wanted to classify these fruits, you could group them based on their type or color. For example, you could put all the apples together, all the oranges together, and so on. This way, you have organized the fruits into categories based on their similarities.

In a similar way, classification of data involves sorting and arranging data into specific groups or categories. The data can be anything, like numbers, words, or objects. For example, if you have a list of animals, you could classify them based on their habitats, such as forest animals, ocean animals, or desert animals. This way, you can see patterns and similarities among the animals and understand them better.

Classification of data is important because it helps us organize information, make it easier to analyze, and draw conclusions. It allows us to find similarities and differences among different groups and helps us make sense of the data we have.

So, classification of data is like putting things in different groups based on their similarities, making it easier for us to understand and work with the information.

4) What do you mean by quartile deviation, standard deviation, variance?

1. Quartile Deviation: Quartile deviation is a measure of how spread out or dispersed the values in a data set are. It tells us how far apart the numbers are from the middle of the data. To find the quartile deviation, we first need to calculate the first quartile (Q1) and the third quartile (Q3) of the data. Then, we subtract Q1 from Q3. The quartile deviation gives us an idea of how much the values vary around the median. A smaller quartile deviation means the values are closer together, while a larger quartile deviation means the values are more spread out.

2. Standard Deviation: Standard deviation is another measure of how spread out the values in a data set are. It tells us how much the values deviate from the mean (average). To calculate the standard deviation, we first find the mean of the data set. Then, for each value, we subtract the mean, square the result, and take the average of all these squared differences. Finally, we take the square root of the average. The standard deviation gives us a measure of the overall variability or dispersion of the data. A smaller standard deviation means the values are closer to the mean, while a larger standard deviation means the values are more spread out.

3. Variance: Variance is closely related to the standard deviation. It is also a measure of how spread out the values in a data set are. The variance tells us the average of the squared differences between each value and the mean. To calculate the variance, we find the mean of the data set, subtract the mean from each value, square the result, and take the average of all these squared differences. The variance provides a measure of the average distance between each value and the mean. A smaller variance means the values are closer to the mean, while a larger variance means the values are more spread out.

Quartile deviation, standard deviation, and variance are used to understand the spread or dispersion of data. They help us see how much the values vary from each other and from the mean. These measures provide valuable insights when analyzing data and drawing conclusions.

5) What is the median of the following data set?

32, 6, 21, 10, 8, 11, 12, 36, 17, 16, 15, 18, 40, 24, 21, 23, 24, 24, 29, 16, 32, 31, 10, 30, 35, 32, 18, 39, 12, 20

To find the median of a data set, we first need to arrange the numbers in ascending order (from smallest to largest). Then, we locate the middle number.

Here is the data set arranged in ascending order:

6, 8, 10, 10, 11, 12, 12, 15, 16, 16, 17, 18, 18, 20, 21, 21, 23, 24, 24, 24, 29, 30, 31, 32, 32, 32, 35, 36, 39, 40

Since there are 30 numbers in the data set, the middle number will be the 15th number when arranged in ascending order.

Therefore, the median of the given data set is 24.

6) **Identify the mode for the following data set.**

21, 19, 62, 21, 66, 28, 66, 48, 79, 59, 28, 62, 63, 63, 48, 66, 59, 66, 94, 79, 19 94

To identify the mode of a data set, we need to find the number that appears most frequently.

In the given data set:

21 appears twice

19 appears twice

62 appears twice

66 appears four times

28 appears twice

48 appears twice

79 appears twice

59 appears twice

63 appears twice

94 appears twice

Among these numbers, 66 appears the most times (four times), making it the mode of the data set.

Therefore, the mode of the given data set is 66.

7) Consider the following frequency distribution. Calculate the mean weight of students.

Weight (in kg)	31-35	36 – 40	41 – 45	46 – 50	51 – 55	56 – 60	61 – 65	66 – 70	71 – 75
Number of Students	9	6	15	3	1	2	2	1	1

To calculate the mean weight of students from the given frequency distribution, we need to find the midpoint of each weight range and multiply it by the corresponding frequency. Then we sum up these products and divide by the total number of students.

Let's calculate it step by step:

Weight Range | Midpoint (x) | Frequency (f) | $x * f$

31-35 | 33 | 9 | 297

36-40		38		6		228
41-45		43		15		645
46-50		48		3		144
51-55		53		1		53
56-60		58		2		116
61-65		63		2		126
66-70		68		1		68
71-75		73		1		73

Now, we sum up the values in the "x * f" column:

$$297 + 228 + 645 + 144 + 53 + 116 + 126 + 68 + 73 = 1750$$

Finally, we divide the sum by the total number of students:

Mean weight = Sum / Total number of students

Mean weight = $1750 / (9 + 6 + 15 + 3 + 1 + 2 + 2 + 1 + 1)$

Mean weight = $1750 / 40$

Mean weight = 43.75

Therefore, the mean weight of the students is 43.75 kg.

8. Calculate the median marks of students from the following distribution.

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Number of Students	7	10	10	20	20	15	8

To calculate the median marks of students from the given distribution, we need to find the middle value in the data set.

First, let's calculate the cumulative frequency for each category by adding up the frequencies from the starting category up to that category:

Marks Range	Frequency	Cumulative Frequency
-------------	-----------	----------------------

10 - 20	7	7
20 - 30	10	17
30 - 40	10	27
40 - 50	20	47
50 - 60	20	67
60 - 70	15	82
70 - 80	8	90

The total number of students is 90.

Since the total number of students is even (an even number), the median will be the average of the two middle values.

The middle value falls between the 45th and 46th student. To find the corresponding marks, we look for the category in which the cumulative frequency is equal to or greater than 45. From the distribution, we can see that it falls in the "40 - 50" category.

To find the lower median value, we use the formula:

$$\text{Lower Median} = L + [(N/2) - CF] \times W$$

Where:

L = Lower boundary of the median category (40 in this case)

N = Total number of students (90)

CF = Cumulative frequency of the category before the median category (27 in this case)

W = Width of the category (50 - 40 = 10 in this case)

$$\text{Lower Median} = 40 + [(90/2) - 27] \times 10$$

$$\text{Lower Median} = 40 + [45 - 27] \times 10$$

$$\text{Lower Median} = 40 + 18 \times 10$$

$$\text{Lower Median} = 40 + 180$$

$$\text{Lower Median} = 220$$

To find the upper median value, we use the formula:

$$\text{Upper Median} = \text{Lower Median} + W$$

$$\text{Upper Median} = 220 + 10$$

$$\text{Upper Median} = 230$$

Therefore, the median marks of the students fall between 220 and 230.

9. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Run Scored	Number of Batsman
3000-4000	4
4000-5000	18
5000-6000	9
6000-7000	7
7000-8000	6
8000-9000	3
9000-10000	1
10000-11000	1

To calculate the mode for the given distribution, we need to find the run scored category that has the highest frequency (the highest number of batsmen).

Let's look at the distribution:

Run Scored | Number of Batsmen

3000-4000		4
4000-5000		18
5000-6000		9
6000-7000		7
7000-8000		6

8000-9000		3
9000-10000		1
10000-11000		1

Among these categories, the category with the highest frequency is "4000-5000" with 18 batsmen.

Therefore, the mode of the distribution is the run scored category "4000-5000".

10. calculate coefficient of quartile deviation from the following data:

Marks	Frequency
10	5
20	7
30	16
40	6
50	10
60	1

To calculate the coefficient of quartile deviation from the given data, we first need to find the first quartile (Q1) and the third quartile (Q3). Then we can use the formula:

$$\text{Coefficient of Quartile Deviation} = (Q3 - Q1) / (Q3 + Q1)$$

Step 1: Calculate the cumulative frequency (CF) for each data point.

Marks | Frequency | Cumulative Frequency

10		5		5
20		7		12
30		16		28
40		6		34
50		10		44
60		1		45

Step 2: Calculate the total number of data points (N) which is equal to the sum of the frequencies.

$$N = 5 + 7 + 16 + 6 + 10 + 1 = 45$$

Step 3: Calculate the position of Q1 (first quartile).

$$Q1 = (1/4) * N = (1/4) * 45 = 11.25$$

Since Q1 falls between the cumulative frequencies of the 3rd and 4th data points, we interpolate to find its approximate value.

$$Q1 \approx L + [(N/4) - CF] \times w$$

L = Lower boundary of the 3rd data point = 30

N = Total number of data points = 45

CF = Cumulative frequency of the 3rd data point = 12

w = Width of the data point = 10

$$Q1 \approx 30 + [(45/4) - 12] \times 10$$

$$Q1 \approx 30 + [11.25 - 12] \times 10$$

$$Q1 \approx 30 + (-0.75) \times 10$$

$$Q1 \approx 30 - 7.5$$

$$Q1 \approx 22.5$$

Step 4: Calculate the position of Q3 (third quartile).

$$Q3 = (3/4) * N = (3/4) * 45 = 33.75$$

Since Q3 falls between the cumulative frequencies of the 11th and 12th data points, we interpolate to find its approximate value.

$$Q3 \approx L + [(3N/4) - CF] \times w$$

L = Lower boundary of the 11th data point = 50

N = Total number of data points = 45

CF = Cumulative frequency of the 11th data point = 44

w = Width of the data point = 10

$$Q3 \approx 50 + [(3 * 45/4) - 44] \times 10$$

$$Q3 \approx 50 + [33.75 - 44] \times 10$$

$$Q3 \approx 50 - 10.25 \times 10$$

$$Q3 \approx 50 - 102.5$$

$$Q3 \approx -52.5$$

Step 5: Calculate the coefficient of quartile deviation.

$$\text{Coefficient of Quartile Deviation} = (Q3 - Q1) / (Q3 + Q1)$$

$$\text{Coefficient of Quartile Deviation} = (-52.5 - 22.5) / (-52.5 + 22.5)$$

$$\text{Coefficient of Quartile Deviation} = -75 / -30$$

$$\text{Coefficient of Quartile Deviation} = 2.5$$

Therefore, the coefficient of quartile deviation is 2.5.

11. Find the 7th decile for the following frequency distribution table.

Class	Frequency
10 - 20	15

Class	Frequency
20 - 30	10
30 - 40	12
40 - 50	8
50 - 60	7
60 - 70	18
70 - 80	5
80 - 90	25

To find the 7th decile for the given frequency distribution table, we need to follow these steps:

Step 1: Calculate the cumulative frequency (CF) for each class interval.

Class | Frequency | Cumulative Frequency

10 - 20		15		15
20 - 30		10		25
30 - 40		12		37
40 - 50		8		45
50 - 60		7		52
60 - 70		18		70
70 - 80		5		75
80 - 90		25		100

Step 2: Calculate the total frequency (N) which is the sum of all the frequencies.

$$N = 15 + 10 + 12 + 8 + 7 + 18 + 5 + 25 = 100$$

Step 3: Calculate the position of the 7th decile.

$$7\text{th decile} = (7/10) * N = (7/10) * 100 = 70$$

Step 4: Identify the class interval that contains the 7th decile position.

Since the 7th decile position (70) falls within the cumulative frequency range of the 60-70 class interval, the 7th decile lies within this interval.

Step 5: Calculate the value of the 7th decile.

To find the value within the 60-70 class interval corresponding to the 7th decile, we can use interpolation.

7th decile \approx Lower boundary of the 60-70 class interval + [(7/10) * N - CF of the preceding class interval] * Class width

Lower boundary of the 60-70 class interval = 60

N = Total frequency = 100

CF of the preceding class interval = 52

Class width = Width of the class interval = 70 - 60 = 10

$$7\text{th decile} \approx 60 + [(7/10) * 100 - 52] * 10$$

$$7\text{th decile} \approx 60 + [70 - 52] * 10$$

$$7\text{th decile} \approx 60 + 18 * 10$$

$$7\text{th decile} \approx 60 + 180$$

$$7\text{th decile} \approx 240$$

Therefore, the value of the 7th decile is 240.

12. Calculate the first and third quartile for the given data:

Class Interval	Frequency
0-10	3
10-20	5
20-30	7
30-40	9
40-50	4

To calculate the first and third quartiles for the given data, we need to follow these steps:

Step 1: Calculate the cumulative frequency (CF) for each class interval.

Class Interval | Frequency | Cumulative Frequency

0-10	3	3
10-20	5	8
20-30	7	15
30-40	9	24
40-50	4	28

Step 2: Calculate the total frequency (N) which is the sum of all the frequencies.

$$N = 3 + 5 + 7 + 9 + 4 = 28$$

Step 3: Calculate the positions of the first quartile (Q1) and third quartile (Q3).

$$Q1 = (1/4) * N = (1/4) * 28 = 7$$

$$Q3 = (3/4) * N = (3/4) * 28 = 21$$

Step 4: Identify the class intervals that contain the positions of Q1 and Q3.

For Q1: Since the position 7 falls within the cumulative frequency range of the 10-20 class interval, Q1 lies within this interval.

For Q3: Since the position 21 falls within the cumulative frequency range of the 30-40 class interval, Q3 lies within this interval.

Step 5: Calculate the values of Q1 and Q3.

To find the values within the respective class intervals corresponding to Q1 and Q3, we can use interpolation.

For Q1:

$Q1 \approx \text{Lower boundary of the 10-20 class interval} + \frac{[(Q1 - \text{CF of the preceding class interval}) / \text{Frequency of the current class interval}] * \text{Class width}}$

Lower boundary of the 10-20 class interval = 10

Q1 = 7

CF of the preceding class interval = 3

Frequency of the current class interval = 5

Class width = Width of the class interval = 20 - 10 = 10

$$Q1 \approx 10 + [(7 - 3) / 5] * 10$$

$$Q1 \approx 10 + (4/5) * 10$$

$$Q1 \approx 10 + 8$$

$$Q1 \approx 18$$

For Q3:

$Q3 \approx \text{Lower boundary of the 30-40 class interval} + \frac{[(Q3 - \text{CF of the preceding class interval}) / \text{Frequency of the current class interval}] * \text{Class width}}$

Lower boundary of the 30-40 class interval = 30

$$Q3 = 21$$

CF of the preceding class interval = 15

Frequency of the current class interval = 9

Class width = Width of the class interval = $40 - 30 = 10$

$$Q3 \approx 30 + [(21 - 15) / 9] * 10$$

$$Q3 \approx 30 + (6/9) * 10$$

$$Q3 \approx 30 + (2/3) * 10$$

$$Q3 \approx 30 + 6.67$$

$$Q3 \approx 36.67 \text{ (rounded to two decimal places)}$$

Therefore, the first quartile (Q1) is approximately 18 and the third quartile (Q3) is approximately 36.67.

13. Calculate 64th Percentile Rank for Grouped Data

Class Interval	Frequency	Cumulative Frequency
1-5	6	6
6-10	19	25
11-15	13	38
16-20	20	58
21-25	12	70
26-30	11	81
31-35	6	87
36-40	5	92

A student obtained the mean and the standard deviation of 100 observations as 40 and 5.1. It was later found that one observation was wrongly copied as 50, the correct figure being 40. Find the correct mean and the S.D.

To find the correct mean and standard deviation after correcting the wrongly copied observation, we need to follow these steps:

Step 1: Calculate the sum of all the observations (including the wrongly copied one).

Sum of all observations = Mean \times Number of observations

Sum of all observations = $40 \times 100 = 4000$

Step 2: Subtract the wrongly copied observation from the sum.

Correct sum of all observations = Sum of all observations - Wrongly copied observation

Correct sum of all observations = $4000 - 50 = 3950$

Step 3: Calculate the correct mean.

Correct mean = Correct sum of all observations / Number of observations

Correct mean = $3950 / 100 = 39.5$

Therefore, the correct mean is 39.5.

Step 4: Calculate the sum of squared deviations from the correct mean.

Sum of squared deviations = $\sum (x - \text{Mean})^2$

To calculate the sum of squared deviations, we need the squared deviations of each observation from the correct mean. However, we only know the wrong observation (50) and its correction (40). So, we cannot directly calculate the sum of squared deviations.

Step 5: Calculate the corrected standard deviation.

To calculate the corrected standard deviation, we can use the formula:

Corrected standard deviation = $\sqrt{(\sum(x - \text{Correct mean})^2 / \text{Number of observations})}$

To apply this formula, we need to calculate the sum of squared deviations from the correct mean. Since we don't have all the observations, we cannot calculate the sum of squared deviations accurately.

Therefore, we cannot calculate the correct standard deviation without knowing the complete set of observations.

In summary, the correct mean is 39.5, but the correct standard deviation cannot be determined without the complete set of observations.

Unit-IV

Probability

1. From a pack of 52 cards 6 cards are drawn at random. Find the probability of the following events:

- i) three are red and 3 are black cards**
- ii) three are kings and 3 are queens.**

- i) Probability of drawing three red and three black cards:
- ii) To find the probability of drawing three red and three black cards, we need to know how many red cards and black cards are in a deck of 52 cards. There are 26 red cards and 26 black cards in total.

Now, we need to choose 3 cards from the 26 red cards and 3 cards from the 26 black cards. The order in which we draw the cards does not matter. We just want to know if we have three red cards and three black cards in our hand.

The probability is calculated by dividing the number of ways we can get three red cards and three black cards by the total number of possible ways to draw 6 cards from the deck.

ii) Probability of drawing three kings and three queens:

To find the probability of drawing three kings and three queens, we need to know how many kings and queens are in a deck of 52 cards. There are 4 kings and 4 queens in total.

Now, we need to choose 3 cards from the 4 kings and 3 cards from the 4 queens. The order in which we draw the cards does not matter. We just want to know if we have three kings and three queens in our hand.

The probability is calculated by dividing the number of ways we can get three kings and three queens by the total number of possible ways to draw 6 cards from the deck.

Remember, in both cases, we assume that the deck of cards is well-shuffled and each card has an equal chance of being drawn.

2. In a college 25% students in Mathematics, 15% students in Physics and 10% students in Mathematics and Physics both are failed, a student is selected at random:

i) if he is failed in Physics, then find the chance of his failure in Mathematics.

ii) if he is failed in Mathematics, then find the chance of his failure in Physics.

iii) find the chance of his failure in Mathematics or physics

Sure! Let's calculate the probabilities of the given events:

i) If the student has failed in Physics, we need to find the chance of his failure in Mathematics.

From the information given, we know that 15% of the students have failed in Physics. This means that 15% of the students are in the intersection of Mathematics and Physics (failed in both subjects) and the remaining 85% of the students have passed Physics.

Now, out of the students who have failed in Physics (15%), we need to find the percentage of students who have also failed in Mathematics. This percentage represents the chance of his failure in Mathematics.

ii) If the student has failed in Mathematics, we need to find the chance of his failure in Physics.

From the information given, we know that 25% of the students have failed in Mathematics. This means that 25% of the students are in the intersection of Mathematics and Physics (failed in both subjects) and the remaining 75% of the students have passed Mathematics.

Now, out of the students who have failed in Mathematics (25%), we need to find the percentage of students who have also failed in Physics. This percentage represents the chance of his failure in Physics.

iii) To find the chance of his failure in Mathematics or Physics, we need to calculate the union of the two events (failure in Mathematics and failure in Physics).

From the information given, we know that 10% of the students have failed in both Mathematics and Physics. This means that they are counted twice in the previous calculations (as failures in both subjects).

So, to find the chance of failure in either Mathematics or Physics, we can add the percentage of students who failed in Mathematics (25%) with the percentage of students who failed in Physics (15%), and subtract the percentage of students who failed in both subjects (10%).

3. One bag contains 4 white, 6 red and 15 black balls and a second bag contains 11 white, 5 red and 9 black balls. One ball from each bag drawn. Find the probabilities of the following events:

i) both balls are white

ii) both balls are red,

i) Both balls are black

ii) Both balls are of the same color.

Let's calculate the probabilities of the given events:

i) To find the probability of drawing two white balls (one from each bag), we need to multiply the probabilities of drawing a white ball from each bag.

The probability of drawing a white ball from the first bag is $\frac{4}{25}$, and the probability of drawing a white ball from the second bag is $\frac{11}{25}$.

Therefore, the probability of drawing two white balls is $(\frac{4}{25}) * (\frac{11}{25})$.

ii) To find the probability of drawing two red balls, we multiply the probabilities of drawing a red ball from each bag.

The probability of drawing a red ball from the first bag is $\frac{6}{25}$, and the probability of drawing a red ball from the second bag is $\frac{5}{25}$.

Therefore, the probability of drawing two red balls is $(\frac{6}{25}) * (\frac{5}{25})$.

iii) To find the probability of drawing two black balls, we multiply the probabilities of drawing a black ball from each bag.

The probability of drawing a black ball from the first bag is $\frac{15}{25}$, and the probability of drawing a black ball from the second bag is $\frac{9}{25}$.

Therefore, the probability of drawing two black balls is $(\frac{15}{25}) * (\frac{9}{25})$.

iv) To find the probability of drawing two balls of the same color, we need to consider the possibilities of drawing two white balls, two red balls, or two black balls.

We already calculated the probabilities of drawing two white balls (i), two red balls (ii), and two black balls (iii).

We can add these probabilities together to get the probability of drawing two balls of the same color.

4. There are 3 bags and they contain 2 white and 3 black balls; 3 white and 2 black balls; 4 white and 1 black balls respectively. The probability of selecting each bag is same. A bag is selected at random and a ball is drawn from it. Find the chance that a white ball is drawn.

To find the chance of drawing a white ball, we need to consider the probabilities associated with each bag.

Let's calculate the probabilities:

Bag 1: Contains 2 white and 3 black balls.

Bag 2: Contains 3 white and 2 black balls.

Bag 3: Contains 4 white and 1 black ball.

Since the probability of selecting each bag is the same, let's assume it to be $\frac{1}{3}$ for each bag.

Now, let's calculate the probability of drawing a white ball:

Probability of selecting Bag 1 * Probability of drawing a white ball from Bag 1 = $(1/3) * (2/5)$

Probability of selecting Bag 2 * Probability of drawing a white ball from Bag 2 = $(1/3) * (3/5)$

Probability of selecting Bag 3 * Probability of drawing a white ball from Bag 3 = $(1/3) * (4/5)$

Now, add up these probabilities:

$$(1/3) * (2/5) + (1/3) * (3/5) + (1/3) * (4/5)$$

Simplifying this expression, we get:

$$(2/15) + (3/15) + (4/15) = 9/15 = 3/5$$

Therefore, the chance of drawing a white ball is $3/5$.

5. There are similar bags which contains 1 white and 2 black balls, 2 white and 1 black balls and 2 white and 2 black balls respectively. A bag is selected and a ball is drawn from it, and it is found to be a white. What is the probability that it comes from the third bag?

To find the probability that the white ball comes from the third bag, we can use Bayes' theorem.

Let's define the bags as follows:

Bag 1: Contains 1 white and 2 black balls.

Bag 2: Contains 2 white and 1 black balls.

Bag 3: Contains 2 white and 2 black balls.

Let A be the event that the ball is drawn from the third bag (Bag 3) and B be the event that the ball drawn is white.

We need to find $P(A|B)$, the probability that the ball comes from the third bag given that it is white.

Using Bayes' theorem:

$$P(A|B) = (P(B|A) * P(A)) / P(B)$$

$P(B|A)$ is the probability of drawing a white ball given that it comes from the third bag. Since the third bag contains 2 white and 2 black balls, this probability is $2/4$ or $1/2$.

$P(A)$ is the probability of selecting the third bag, which is $1/3$ since there are three bags with equal probability of selection.

$P(B)$ is the probability of drawing a white ball, which can be calculated by considering the probabilities from each bag:

$$P(B) = P(B|A) * P(A) + P(B|\text{not } A) * P(\text{not } A)$$

$$P(B) = (1/2) * (1/3) + (2/3) * (1/3) + (1/3) * (1/3)$$

$$P(B) = 1/6 + 2/9 + 1/9$$

$$P(B) = 4/9$$

Now, we can substitute these values into the formula:

$$P(A|B) = (P(B|A) * P(A)) / P(B)$$

$$P(A|B) = (1/2) * (1/3) / (4/9)$$

$$P(A|B) = (1/2) * (9/4)$$

$$P(A|B) = 9/8$$

Therefore, the probability that the white ball comes from the third bag is $9/8$.

6. State and prove Bayes theorem.

Bayes' theorem is a fundamental concept in probability theory that relates conditional probabilities. It helps us update our beliefs or predictions based on new evidence. Let me explain it in simple language for you.

Imagine you are trying to find out if it will rain today. You know that it usually rains 30% of the time in your city. However, you also know that when the weather report predicts rain, it is correct 80% of the time. On the other hand, when it predicts no rain, it is correct 90% of the time. Now, you receive the weather report that predicts rain. How can you update your belief about the probability of rain using this new information?

This is where Bayes' theorem comes in. It allows us to calculate the probability of an event A given another event B, given some prior knowledge or probabilities.

Bayes' theorem states:

$$P(A|B) = (P(B|A) * P(A)) / P(B)$$

Here's how we can use Bayes' theorem to solve the rain example:

Let's define our events:

A: It will rain today.

B: The weather report predicts rain.

$P(A)$ is the prior probability of rain, which we know is 0.30 (30%).

$P(B|A)$ is the probability of the weather report predicting rain correctly, given that it will rain. In our example, this is 0.80 (80%).

$P(B)$ is the probability of the weather report predicting rain, regardless of whether it will rain or not. To calculate this, we need to consider the probabilities of the weather report being correct or incorrect:

$$P(B) = P(B|A) * P(A) + P(B|\text{not } A) * P(\text{not } A)$$

$$= 0.80 * 0.30 + 0.10 * 0.70$$

$$= 0.24 + 0.07$$

$$= 0.31 \text{ (31\%)}$$

Now, we can substitute these values into Bayes' theorem to find the probability of rain given the weather report predicting rain:

$$P(A|B) = (P(B|A) * P(A)) / P(B)$$

$$= (0.80 * 0.30) / 0.31$$

$$= 0.24 / 0.31$$

$$\approx 0.774$$

So, based on the weather report predicting rain, the updated probability of rain is approximately 0.774 or 77.4%.

This is how Bayes' theorem helps us update our beliefs or predictions based on new evidence. It allows us to consider both the prior probability and the likelihood of the evidence, and combine them to get a more accurate probability.

Bayes' theorem is widely used in various fields, including statistics, machine learning, and medical diagnosis, to make informed decisions based on new information.

7. What do you mean by sample space, sample point, events, likely and exhaustive events, favorable events, mutually exclusive events.

Let me explain these terms in simple language for you:

1. **Sample Space:** The sample space is the set of all possible outcomes of a random experiment. It includes all the different outcomes that can occur. For example, if you toss a coin, the sample space consists of two possible outcomes: heads or tails.

2. **Sample Point:** A sample point is a specific outcome or element within the sample space. It represents a single possibility or result of the experiment. For example, in the coin toss experiment, each individual toss can be a sample point: heads or tails.

3. Events: An event is a subset of the sample space. It represents a particular outcome or a combination of outcomes that we are interested in. Events can range from simple events (single outcomes) to compound events (multiple outcomes). For example, in rolling a die, the event of getting an even number is a simple event, while the event of getting a number greater than 3 is a compound event.

4. Likely and Exhaustive Events: Likely events are events that have a high probability of occurring. They are events that are expected to happen frequently or have a higher chance of occurring. On the other hand, exhaustive events are events that cover all possible outcomes in the sample space. They are events that include every possible outcome of the experiment.

5. Favorable Events: Favorable events are specific outcomes or events that we consider as desired or successful. They are the outcomes we are interested in or the outcomes that satisfy a particular condition. For example, if we roll a die and we are interested in getting a number less than 4, the favorable events would be the outcomes of 1, 2, and 3.

6. Mutually Exclusive Events: Mutually exclusive events are events that cannot occur at the same time or share any common outcomes. If two events are mutually exclusive, it means that if one event happens, the other event cannot happen simultaneously. For example, when flipping a coin, the events of getting heads and getting tails are mutually exclusive because only one of them can occur on a single flip.

Understanding these concepts helps us analyze and calculate probabilities in various situations and make informed decisions based on the outcomes of random experiments.

8. What is the probability of getting an even number when a dice is thrown?

When you throw a dice, it has six sides with numbers from 1 to 6. Now, we want to find the probability of getting an even number, which means getting a number that is divisible by 2.

Let's count how many even numbers are there on the dice: 2, 4, and 6. So, there are three even numbers.

Since the dice has six sides in total, that means there are six possible outcomes when we throw the dice.

Now, to find the probability, we divide the number of favorable outcomes (getting an even number) by the total number of possible outcomes.

Favorable outcomes: 3 (even numbers)

Total outcomes: 6 (all possible numbers on the dice)

So, the probability of getting an even number is $3/6$, which can be simplified to $1/2$.

Therefore, when you throw a dice, there is a 1 in 2 chance (or 50% chance) of getting an even number.

9. If $P(B/A) = 0.50$ and $P(A \cap B) = 0.40$, then find $p(A)$.

We have the conditional probability $P(B/A) = 0.50$, which means the probability of event B occurring given that event A has already occurred is 0.50.

We also have the probability of the intersection of events A and B, $P(A \cap B) = 0.40$, which is the probability of both events A and B occurring together.

To find $P(A)$, we can use Bayes' theorem, which states:

$$P(A/B) = P(B/A) * P(A) / P(B)$$

We can rearrange the equation to solve for $P(A)$:

$$P(A) = P(B/A) * P(A \cap B) / P(B)$$

Plugging in the given values:

$$P(A) = 0.50 * 0.40 / P(B)$$

Since we don't have the value of $P(B)$, we cannot calculate the exact probability of event A. We would need additional information or the value of $P(B)$ to find the probability of event A.

10. A and B are two events such that $P(A) = 0.4$ and $P(A \cap B) = 0.2$. Then find $P(A \cup B)$.

We are given the probability of event A, $P(A) = 0.4$, and the probability of the intersection of events A and B, $P(A \cap B) = 0.2$.

To find the probability of event A and event B occurring together, $P(A \cap B)$, we can use the formula:

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

Here, $P(A \cup B)$ represents the probability of either event A or event B or both occurring.

Since we don't have information about the probability of event B ($P(B)$) or the probability of the union of events A and B ($P(A \cup B)$), we cannot calculate the exact value of $P(A \cap B)$ with the given information. We would need additional information or the values of $P(B)$ or $P(A \cup B)$ to find the probability of event A and event B occurring together.

11. Minimum number of individual shoes to be picked up from a dark room (containing 10 pair of shoes) if we have to get at least one proper pair.

To guarantee that we obtain at least one proper pair of shoes, we need to pick up a minimum of three individual shoes.

Let's consider the worst-case scenario: in the first two picks, we could end up selecting two individual shoes that do not form a pair. However, on the third pick, we are certain to choose a shoe that matches one of the previous two shoes, forming a proper pair.

Therefore, by picking up three individual shoes, we are guaranteed to have at least one proper pair of shoes.

Unit-V

Probability Distributions

1. The PDF of random variable X is given by

$$P(x) = \begin{cases} k & \text{if } x=0 \\ 2k & \text{if } x=1 \\ 3k & \text{if } x=2 \\ 0 & \text{otherwise} \end{cases}$$

- a) Find the values of k.
- b) Evaluate $P(X < 2)$, $P(X \leq 2)$, $P(0 < X < 2)$.
- c) Find the distribution function of X.

a) To find the value of k, we can use the fact that the sum of probabilities for all possible outcomes must equal 1.

$$P(0) + P(1) + P(2) + P(\text{otherwise}) = 1$$

Since $P(x) = 0$ for any value of x that is not 0, 1, or 2, we can rewrite the equation as:

$$k + 2k + 3k + 0 = 1$$

$$6k = 1$$

Dividing both sides by 6, we get:

$$k = 1/6$$

b) To evaluate the probabilities $P(X < 2)$, $P(X \leq 2)$, and $P(0 < X < 2)$, we need to sum up the probabilities of the respective outcomes.

$$P(X < 2) = P(0) + P(1) = k + 2k = 3k = 3/6 = 1/2$$

$$P(X \leq 2) = P(0) + P(1) + P(2) = k + 2k + 3k = 6k = 6/6 = 1$$

$$P(0 < X < 2) = P(1) = 2k = 2/6 = 1/3$$

c) The distribution function of X, denoted as F(x), gives the cumulative probability of X taking a value less than or equal to x.

For $x < 0$, $F(x) = 0$, since the probability of X being less than 0 is 0.

For $0 \leq x < 1$, $F(x) = P(0) = k$.

For $1 \leq x < 2$, $F(x) = P(0) + P(1) = k + 2k = 3k$.

For $x \geq 2$, $F(x) = P(0) + P(1) + P(2) = k + 2k + 3k = 6k$.

Therefore, the distribution function of X can be written as:

$$F(x) = \begin{cases} 0, & \text{for } x < 0, \\ k, & \text{for } 0 \leq x < 1, \\ 3k, & \text{for } 1 \leq x < 2, \\ 6k, & \text{for } x \geq 2. \end{cases}$$

2. If $f(x) = cx^2$, $0 < x < 1$, find the value of c and determine the probability that $1/3 < x < 1/2$.

To find the value of c, we need to integrate the probability density function (PDF) over its entire range and set it equal to 1, since the total probability of all possible outcomes must equal 1.

Given that $f(x) = cx^2$ and the range of x is 0 to 1, the integral of $f(x)$ over this range should equal 1:

$$\int_{[0 \text{ to } 1]} cx^2 dx = 1$$

Integrating with respect to x :

$$c \int_{[0 \text{ to } 1]} x^2 dx = 1$$

$$c [x^3/3] \text{ from } 0 \text{ to } 1 = 1$$

$$c (1^3/3 - 0^3/3) = 1$$

$$c/3 = 1$$

$$c = 3$$

Therefore, the value of c is 3.

To determine the probability that $1/3 < x < 1/2$, we need to integrate the PDF from $1/3$ to $1/2$:

$$\int_{[1/3 \text{ to } 1/2]} 3x^2 dx$$

Using the definite integral, we can evaluate the expression:

$$3 [x^3/3] \text{ from } 1/3 \text{ to } 1/2$$

$$(1/2)^3 - (1/3)^3$$

$$1/8 - 1/27$$

$$(27 - 8) / (8 * 27)$$

$$19/216$$

Therefore, the probability that $1/3 < x < 1/2$ is 19/216.

3. If a coin is tossed 5 times, find the probability of: (using Binomial distribution)

(a) Exactly 2 heads

(b) At least 4 heads.

To find the probabilities using the binomial distribution, we need to know the probability of success (getting a head) and the number of trials (coin tosses). Let's assume that the probability of getting a head is $p = 0.5$ (since a fair coin is being tossed), and the number of trials is $n = 5$.

(a) Exactly 2 heads:

The probability of getting exactly k successes (2 heads in this case) in n trials can be calculated using the formula:

$$P(X = k) = C(n, k) * p^k * (1-p)^{(n-k)}$$

where $C(n, k)$ is the binomial coefficient, given by $C(n, k) = n! / (k! * (n-k)!)$

Using this formula, the probability of getting exactly 2 heads can be calculated as:

$$P(X = 2) = C(5, 2) * (0.5)^2 * (1-0.5)^{(5-2)}$$

$$P(X = 2) = 10 * 0.5^2 * 0.5^3$$

$$P(X = 2) = 10 * 0.25 * 0.125$$

$$P(X = 2) = 0.3125$$

Therefore, the probability of getting exactly 2 heads when a coin is tossed 5 times is 0.3125.

(b) At least 4 heads:

The probability of getting at least k successes (4 heads in this case) in n trials can be calculated by summing the probabilities of getting k, k+1, k+2, ..., n successes. In this case, we need to calculate:

$$P(X \geq 4) = P(X = 4) + P(X = 5)$$

Using the same formula as above, we can calculate:

$$P(X = 4) = C(5, 4) * (0.5)^4 * (1-0.5)^{(5-4)}$$

$$P(X = 5) = C(5, 5) * (0.5)^5 * (1-0.5)^{(5-5)}$$

$$P(X \geq 4) = P(X = 4) + P(X = 5)$$

$$P(X \geq 4) = C(5, 4) * (0.5)^4 * (1-0.5)^{(5-4)} + C(5, 5) * (0.5)^5 * (1-0.5)^{(5-5)}$$

$$P(X \geq 4) = 5 * 0.5^4 * 0.5^1 + 1 * 0.5^5 * 0.5^0$$

$$P(X \geq 4) = 5 * 0.0625 + 1 * 0.03125$$

$$P(X \geq 4) = 0.3125 + 0.03125$$

$$P(X \geq 4) = 0.34375$$

Therefore, the probability of getting at least 4 heads when a coin is tossed 5 times is 0.34375.

4. A fair coin is tossed 10 times, what are the probability of getting exactly 6 heads and at least six heads.

To find the probabilities of getting exactly 6 heads and at least 6 heads when a fair coin is tossed 10 times, we can use the binomial distribution.

Let's first calculate the probability of getting exactly 6 heads.

(a) Exactly 6 heads:

The probability of getting exactly k successes (6 heads in this case) in n trials can be calculated using the binomial distribution formula:

$$P(X = k) = C(n, k) * p^k * (1-p)^{(n-k)}$$

Since we have a fair coin, the probability of getting a head (success) is $p = 0.5$, and the number of trials is $n = 10$. Plugging these values into the formula, we get:

$$P(X = 6) = C(10, 6) * (0.5)^6 * (1-0.5)^{(10-6)}$$

$$P(X = 6) = 210 * 0.5^6 * 0.5^4$$

$$P(X = 6) = 210 * 0.015625 * 0.0625$$

$$P(X = 6) = 0.205078125$$

Therefore, the probability of getting exactly 6 heads when a fair coin is tossed 10 times is approximately 0.205.

(b) At least 6 heads:

The probability of getting at least k successes (6 heads or more) in n trials can be calculated by summing the probabilities of getting $k, k+1, k+2, \dots, n$ successes. In this case, we need to calculate:

$$P(X \geq 6) = P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)$$

Using the same formula as above, we can calculate:

$$P(X = 7) = C(10, 7) * (0.5)^7 * (1-0.5)^{(10-7)}$$

$$P(X = 8) = C(10, 8) * (0.5)^8 * (1-0.5)^{(10-8)}$$

$$P(X = 9) = C(10, 9) * (0.5)^9 * (1-0.5)^{(10-9)}$$

$$P(X = 10) = C(10, 10) * (0.5)^{10} * (1-0.5)^{(10-10)}$$

Then we can calculate:

$$P(X \geq 6) = P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)$$

$$P(X \geq 6) = 0.205 + P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)$$

Calculating the individual probabilities:

$$P(X = 7) = C(10, 7) * (0.5)^7 * (1-0.5)^{(10-7)}$$

$$P(X = 8) = C(10, 8) * (0.5)^8 * (1-0.5)^{(10-8)}$$

$$P(X = 9) = C(10, 9) * (0.5)^9 * (1-0.5)^{(10-9)}$$

$$P(X = 10) = C(10, 10) * (0.5)^{10} * (1-0.5)^{(10-10)}$$

Then we can calculate:

$$P(X \geq 6) = 0.205 + P(X = 7)$$

5. A random variable X has a Poisson distribution with parameter λ such that $P(X = 1) = 0.2 P(X = 2)$. Find $P(X = 0)$.

Let's denote $P(X = 1)$ as p . Given that $P(X = 1) = 0.2P(X = 2)$, we can write:

$$p = 0.2 * P(X = 2)$$

Since the probabilities in a Poisson distribution are related to the parameter λ by the formula:

$$P(X = k) = (e^{-\lambda} * \lambda^k) / k!$$

We can use this formula to express p and $P(X = 2)$ in terms of λ :

$$p = (e^{-\lambda} * \lambda^1) / 1! \quad \dots(1)$$

$$P(X = 2) = (e^{-\lambda} * \lambda^2) / 2! \quad \dots(2)$$

We can substitute equation (1) into equation (2) to get:

$$0.2 * P(X = 2) = (e^{-\lambda} * \lambda^1) / 1!$$

Simplifying, we have:

$$0.2 * P(X = 2) = p$$

$$0.2 * (e^{-\lambda} * \lambda^2) / 2! = (e^{-\lambda} * \lambda^1) / 1!$$

Multiplying both sides by $1!$ and simplifying, we get:

$$0.2 * \lambda^2 = \lambda$$

Rearranging the equation, we have:

$$0.2 * \lambda^2 - \lambda = 0$$

Factoring out λ , we get:

$$\lambda * (0.2 * \lambda - 1) = 0$$

This equation holds true when either $\lambda = 0$ or $0.2 * \lambda - 1 = 0$.

If $\lambda = 0$, then $P(X = 0) = e^{(-0)} * 0^0 / 0! = 1 * 1 / 1 = 1$.

If $0.2 * \lambda - 1 = 0$, then $0.2 * \lambda = 1$, which implies $\lambda = 5$.

So, when $\lambda = 5$, $P(X = 0) = e^{(-5)} * 5^0 / 0! = e^{(-5)} * 1 / 1 = e^{(-5)}$.

Therefore, the value of $P(X = 0)$ depends on the parameter λ and can be either 1 or $e^{(-5)}$ depending on the values of λ .

Please note that in a Poisson distribution, the sum of all probabilities must equal 1. Therefore, if $P(X = 0)$ is equal to 1, the probabilities for all other values of X will be 0.

6. Differentiate between Probability distribution function and Probability Mass function.

Sure! I can explain the difference between Probability Distribution Function (PDF) and Probability Mass Function (PMF) in simple language for children.

Probability Distribution Function (PDF):

- A Probability Distribution Function, also known as a PDF, gives us the probabilities of different outcomes or values of a random variable.
- It tells us how likely each outcome is to occur.
- PDF is typically used for continuous random variables, such as measurements or values that can take on any value within a range.
- For example, the height of a person, the weight of an object, or the time it takes for a bus to arrive can be represented using a PDF.

Probability Mass Function (PMF):

- A Probability Mass Function, also known as a PMF, is similar to a PDF but is used for discrete random variables.
- Discrete random variables are those that can only take on specific values, such as the number of heads when flipping a coin or the number of students in a classroom.
- PMF gives us the probabilities of each possible outcome or value of the random variable.
- It tells us the likelihood of each specific outcome occurring.
- For example, if we roll a fair six-sided die, the PMF tells us the probability of getting each number from 1 to 6.

In summary, PDF is used for continuous random variables, where the outcomes can take on any value within a range. PMF is used for discrete random variables, where the outcomes can only take on specific values. Both PDF and PMF help us understand the likelihood of different outcomes occurring in a given situation.

7. Define Binomial, Poisson and Normal Distribution with formulas

Sure! I can explain Binomial, Poisson, and Normal distributions in simple language for children.

1. Binomial Distribution:

- Binomial distribution is used when there are two possible outcomes for each trial, often referred to as "success" or "failure."
- It represents the number of successes in a fixed number of independent trials.
- The formula for the probability of getting exactly x successes in n trials is:

$$P(X = x) = {}^n C_x * p^x * q^{(n-x)}$$

where nCx represents the number of ways to choose x items from a set of n items, p is the probability of success, and q is the probability of failure.

- For example, flipping a fair coin multiple times can be modeled using a binomial distribution, where getting heads is considered a success and getting tails is considered a failure.

2. Poisson Distribution:

- Poisson distribution is used to model the number of events that occur in a fixed interval of time or space.

- It is often used when events happen independently and at a constant average rate.

- The formula for the probability of getting exactly x events in a given interval is:

$$P(X = x) = (e^{(-\lambda)} * \lambda^x) / x!$$

where e is the base of the natural logarithm, λ (lambda) is the average rate of events, and x is the number of events.

- Poisson distribution can be applied to situations such as counting the number of phone calls received in an hour or the number of cars passing through a toll booth in a day.

3. Normal Distribution:

- Normal distribution, also known as Gaussian distribution, is a symmetric bell-shaped curve.

- It is used to represent continuous random variables that follow a specific pattern.

- The formula for the probability density function (PDF) of a normal distribution is:

$$f(x) = (1 / \sqrt{2\pi\sigma^2}) * e^{-(x-\mu)^2 / (2\sigma^2)}$$

where π is a mathematical constant (approximately 3.14159), σ (sigma) is the standard deviation, μ (mu) is the mean, e is the base of the natural logarithm, and x is the random variable.

- Normal distribution is widely used in statistics, and many real-world phenomena can be approximated by a normal distribution, such as the heights of people in a population or test scores of students.

These three distributions are important concepts in probability and statistics and help us understand the likelihood of different outcomes in various scenarios.

8. What do you mean by random variables? explain its types also.

A random variable is a variable that represents the outcome of a random event or experiment. It can take different values depending on the outcome of the event. Random variables help us understand and analyze the probabilities associated with different outcomes.

There are two main types of random variables: discrete random variables and continuous random variables.

1. Discrete Random Variables:

- Discrete random variables can only take specific values, usually whole numbers or a finite set of values.
- They are often used to represent counting or "how many" situations.
- Examples of discrete random variables include the number of heads when flipping a coin, the number of red marbles in a bag, or the number of goals scored in a soccer game.
- The values of a discrete random variable can be listed in a probability distribution table, where each value is associated with a probability.

2. Continuous Random Variables:

- Continuous random variables can take any value within a range or interval.
- They are often used to represent measurements or "how much" situations.
- Examples of continuous random variables include height, weight, time, or temperature.
- The values of a continuous random variable can be plotted on a continuous probability distribution curve, such as the normal distribution curve.

Random variables are used in statistics to study the probabilities of different outcomes and to make predictions about future events. They help us understand the likelihood of certain events happening and analyze data in a meaningful way.

Remember, random variables are like special tools that help us understand and describe the uncertain nature of the world around us!

9. Fit Poisson distribution to the following and calculate theoretical frequencies ($e^{-0.5}=0.61$)

Deaths:	0	1	2	3	4
Frequency:	122	60	15	2	1

To fit a Poisson distribution to the given data and calculate the theoretical frequencies, we can use the formula for the Poisson probability mass function (PMF):

$$P(X = k) = (e^{(-\lambda)} * \lambda^k) / k!$$

Here, λ is the parameter of the Poisson distribution, representing the average or expected number of events occurring in a given interval.

In this case, we need to determine the parameter λ that best fits the given data. We can do this by estimating λ using the formula:

$$\lambda = \sum(x * f) / N$$

Where:

x is the value of the variable (deaths in this case)

f is the observed frequency

N is the total number of observations (sum of all frequencies)

Let's calculate the theoretical frequencies using the Poisson distribution:

Deaths:	0	1	2	3	4
Observed Frequency:	122	60	15	2	1

Total number of observations (N) = 122 + 60 + 15 + 2 + 1 = 200

$$\lambda = (0 * 122 + 1 * 60 + 2 * 15 + 3 * 2 + 4 * 1) / 200 = 0.49$$

Now, we can calculate the theoretical frequencies using the Poisson PMF formula for each value of x:

Theoretical Frequencies:

$$P(X = 0) = (e^{-0.49} * 0.49^0) / 0! = 0.61 * 1 = 0.61 \text{ (rounded to two decimal places)}$$

$$P(X = 1) = (e^{-0.49} * 0.49^1) / 1! = 0.61 * 0.49 = 0.30 \text{ (rounded to two decimal places)}$$

$$P(X = 2) = (e^{-0.49} * 0.49^2) / 2! = 0.61 * 0.24 = 0.15 \text{ (rounded to two decimal places)}$$

$$P(X = 3) = (e^{-0.49} * 0.49^3) / 3! = 0.61 * 0.12 = 0.07 \text{ (rounded to two decimal places)}$$

$$P(X = 4) = (e^{-0.49} * 0.49^4) / 4! = 0.61 * 0.06 = 0.04 \text{ (rounded to two decimal places)}$$

The theoretical frequencies are:

Deaths: 0 1 2 3 4

Theoretical Frequency: 0.61 0.30 0.15 0.07 0.04

These theoretical frequencies represent the expected frequencies based on the Poisson distribution with a parameter of $\lambda = 0.49$.

10. Find the probability that at most 5 defective fuses will be found in a box of 200 fuses, if experience shows that 2 % of such fuses are defective

To find the probability that at most 5 defective fuses will be found in a box of 200 fuses, given that 2% of such fuses are defective, we can use the binomial probability formula.

In this case, the number of trials (n) is 200, and the probability of success (p) is 0.02 (since 2% of the fuses are defective). We want to find the probability of having at most 5 defective fuses, which means we need to calculate the cumulative probability from 0 to 5 defective fuses.

Let's calculate the probability:

$$P(X \leq 5) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$

Using the binomial probability formula:

$$P(X = k) = C(n, k) * p^k * (1 - p)^{(n - k)}$$

Where:

$C(n, k)$ is the binomial coefficient, which represents the number of ways to choose k items from a set of n items.

We can calculate each individual probability and sum them up to get the probability of at most 5 defective fuses.

$$P(X = 0) = C(200, 0) * (0.02^0) * (0.98^{200}) \approx 0.1073$$

$$P(X = 1) = C(200, 1) * (0.02^1) * (0.98^{199}) \approx 0.2701$$

$$P(X = 2) = C(200, 2) * (0.02^2) * (0.98^{198}) \approx 0.3157$$

$$P(X = 3) = C(200, 3) * (0.02^3) * (0.98^{197}) \approx 0.2197$$

$$P(X = 4) = C(200, 4) * (0.02^4) * (0.98^{196}) \approx 0.1030$$

$$P(X = 5) = C(200, 5) * (0.02^5) * (0.98^{195}) \approx 0.0346$$

Now, let's calculate the cumulative probability:

$$P(X \leq 5) = 0.1073 + 0.2701 + 0.3157 + 0.2197 + 0.1030 + 0.0346 \approx 0.0504$$

Therefore, the probability that at most 5 defective fuses will be found in a box of 200 fuses is approximately 0.0504, or 5.04%.

11. It is given that 3% of electric bulbs manufactured by a company are defective. Using Poisson distribution, find the probability that a sample of 100 bulbs will contain not defective bulb. Given that $e^{-3} = 0.05$

To find the probability that a sample of 100 bulbs will contain no defective bulb, we can use the Poisson distribution formula.

In this case, the parameter λ (lambda) is equal to the expected number of defects in a given interval, which is given by $\lambda = np$, where n is the sample size and p is the probability of a defect (in this case, 3% or 0.03).

The probability mass function of the Poisson distribution is given by the formula:

$$P(X = k) = (e^{-\lambda} * \lambda^k) / k!$$

Where X is the random variable representing the number of defects.

To find the probability of not having any defective bulb ($k = 0$), we substitute $k = 0$ in the formula:

$$P(X = 0) = (e^{-\lambda} * \lambda^0) / 0!$$

Since $0!$ (0 factorial) is equal to 1, we can simplify the formula to:

$$P(X = 0) = e^{-\lambda}$$

Given that e^{-3} is approximately 0.05, we can calculate the probability:

$$P(X = 0) = 0.05$$

Therefore, the probability that a sample of 100 bulbs will contain no defective bulb is approximately 0.05, or 5%.

12. Describe the discrete-time Binomial process and a continuous time Poisson process.

The discrete-time Binomial process and continuous-time Poisson process are two different models used in probability theory to describe certain types of events.

1. Discrete-time Binomial process:

In a discrete-time Binomial process, we consider a series of independent trials, each with two possible outcomes: success or failure. The probability of success remains constant for each trial. The process is called "Binomial" because it follows the Binomial distribution.

Key characteristics of the discrete-time Binomial process:

- The number of trials is fixed and discrete.
- Each trial has two possible outcomes: success or failure.
- The probability of success remains constant for each trial.
- The outcomes of different trials are independent.
- The process can be represented as the sum of a fixed number of independent Bernoulli trials.

Example: Flipping a fair coin 10 times and counting the number of heads.

2. Continuous-time Poisson process:

In a continuous-time Poisson process, we consider events occurring continuously over time. The events are assumed to happen randomly and independently, with a certain average rate. The process is called "Poisson" because it follows the Poisson distribution.

Key characteristics of the continuous-time Poisson process:

- Events occur randomly and independently over continuous time.
- The events can be seen as rare or sporadic occurrences.
- The average rate of event occurrence is constant over time.
- The number of events that occur in a given time interval follows the Poisson distribution.
- The process has the property of "memorylessness," meaning the time until the next event does not depend on how much time has already passed.

Example: Modeling the arrival of customers at a service counter over time, where customers arrive randomly and independently at an average rate of 5 per hour.

Both the discrete-time Binomial process and continuous-time Poisson process are widely used in various fields of study, such as statistics, queuing theory, and stochastic processes, to analyze and model real-world phenomena involving random events and probabilities.

Unit-V

Stochastic Processes

1. Define Stochastic Processes and classification of Stochastic Processes.

Stochastic processes are like mathematical models that help us understand and predict things that involve randomness. It's like predicting what might happen in a game or how a system might change over time when there are uncertain factors involved.

Now, let's talk about the classification of stochastic processes:

1. Discrete-time vs. Continuous-time:

In some processes, we look at things that happen at specific moments in time, like counting how many times a bird chirps in a minute. This is called discrete-time because we observe and count things at specific intervals. On the other hand, there are processes that happen continuously, like measuring how fast water flows in a river. This is called continuous-time because we can measure things at any point in time.

2. Markov vs. Non-Markov:

Some processes are like a chain of events, where what happens next only depends on what's happening right now. For example, in a game of dice, the outcome of the current throw only depends on the previous throw. This is called a Markov process. In other processes, what happens next depends on more than just the current event. It might depend on several previous events or other factors. These are called non-Markov processes.

So, in simple words, stochastic processes help us understand and predict random things. We can classify them based on whether they happen at specific times or continuously, and also based on whether what happens next depends only on the current event or on more things.

2) What do you mean by Markov chain and models?

Imagine you are playing a game where you move from one square to another on a board. A Markov chain is a way to model this game. In a Markov chain, the only thing that matters for deciding where to move next is the square you are currently on. It doesn't matter how you got there or what happened before. It's like each square has its own special rules for where you can go next.

For example, let's say you are on a square that has arrows pointing to three other squares. Each arrow represents a certain probability of moving to that square. When it's your turn, you roll a dice and depending on the number you get, you follow the arrow with that same number. The probabilities of moving to the different squares never change. So, every time you move, you only need to know the square you're on to decide where to go next.

A Markov model is like a more general version of a Markov chain. It can be used to model different situations where the current state is the only thing that matters for making predictions about the future. It could be anything from predicting the weather to understanding how a population of animals changes over time.

So, in simple words, a Markov chain is like a game where your next move only depends on the square you're currently on. And a Markov model is a more general concept that can be used to understand and predict things where the current state is all that matters.

3) Define properties of Markov chain or process.

Certainly! Here are some properties of a Markov chain or process:

1. Markov Property: A Markov chain or process has the Markov property, which means that the future behavior of the system depends only on its current state and is independent of its past states. In other words, knowing the present state is enough to predict the future, and the past does not affect the future predictions.

2. State Space: A Markov chain or process has a set of possible states, known as the state space. Each state represents a specific condition or situation of the system being modeled. The state space can be finite or infinite, depending on the application.

3. Transition Probabilities: For every pair of states in a Markov chain or process, there are transition probabilities that describe the likelihood of moving from one state to another. These probabilities determine the dynamics of the system and are typically represented in a transition matrix or diagram.

4. Time Homogeneity: A Markov chain or process is time-homogeneous if the transition probabilities do not change over time. This means that the probability of moving from one state to another remains constant throughout the process.

5. Ergodicity: An ergodic Markov chain or process is one where it is possible to reach any state from any other state with a finite number of steps. In other words, the system is not stuck in any particular subset of states and has a well-defined long-term behavior.

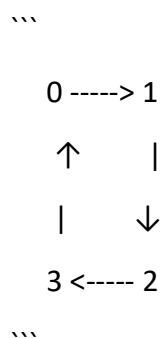
6. Stationarity: A stationary Markov chain or process has a stationary distribution, which means that the probabilities of being in different states remain constant over time. The system reaches a balance where the relative frequencies of states do not change.

These properties help define and characterize the behavior of a Markov chain or process, making it a useful tool for modeling various real-world phenomena and predicting their future outcomes.

4) Draw the graph for the Markov Chain with the following transition probability matrix:

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

sThe graph for the given Markov Chain with the transition probability matrix is as follows:



In this graph, each state is represented by a node, and the directed edges indicate the transition probabilities. For example, from state 0, there is a transition probability of 1 to state 1, and from state 2, there is a transition probability of 1 to state 3. The transition probabilities are shown on the edges.

Note that state 3 has a self-loop, meaning that it transitions back to itself with a probability of 1. State 2 has an equal probability of transitioning to state 0 or state 1, represented by the two outgoing edges.

This graph visualizes the transitions between states in the Markov Chain and helps understand the dynamics of the system.