



CP303 - CAPSTONE PROJECT II

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Water Scarcity: More Population & Climate Change

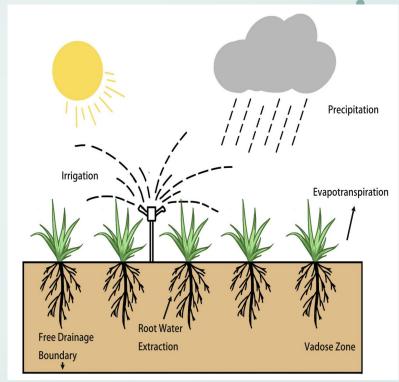
Farming uses about 70% of all freshwater available

Current Scenario:

Open-loop systems: Based on experience, not real-time data (low irrigation efficiency)

Precision Agriculture:

Improve irrigation efficiency Develop smarter, data-driven watering methods Real time feedbacks are considered





Model Predictive Control



Mechanism:

- Collect real-time soil moisture data (sensors)
- Predict future soil moisture levels
- Calculate optimal irrigation actions (water)
- Apply irrigation to the field
- Repeat at next time step

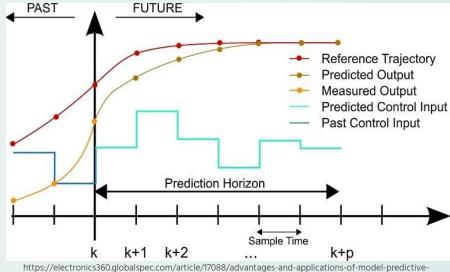
Control Objective:

Minimize the amount of irrigation, while keeping the soil moisture content within target

Challenge:

High computational cost (complex soil-water dynamics)

Model reduction/approx techniques: Neural Networks



control



Objective



To develop a advanced controller which can maintain the soil moisture at desired level.

Design of Model Predictive Controller

Closed loop control

Building data-driven models for predicting soil moisture content

Training Dataset

To generate the sensor data, by solving discretized ODEs (Richard's Equation)

Model Discretization



Soil Water Dynamics



1-dimensional **Richards equation** given by,

$$c(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial z} \left[K(h) \left(\frac{\partial h}{\partial z} + 1 \right) \right] - \alpha(h) \frac{K_c E T_0}{z_r},$$

van Genuchten-Mualem soil hydraulic model given by,

$$c(h) = (\theta_s - \theta_r)\alpha n \left(1 - \frac{1}{n}\right) (-\alpha h)^{n-1} [1 + (-\alpha h)^n]^{\frac{1}{n} - 2},$$

$$K(h) = K_s \left[\left(1 + (-\alpha h)^n\right)^{-\left(1 - \frac{1}{n}\right)} \right]^{\frac{1}{2}} \left[1 - \left[1 - \left[\left(1 + (-\alpha h)^n\right)^{-\left(1 - \frac{1}{n}\right)} \right]^{\frac{n}{n-1}} \right]^{1 - \frac{1}{n}} \right]^2,$$

Soil-water retention equation of van Genuchten

$$\theta(h) = (\theta_s - \theta_r) \left[\frac{1}{1 + (-\alpha h)^n} \right]^{1 - \frac{1}{n}} + \theta_r.$$

where,

h(m) - capillary potential

c (m⁻¹) - the soil capillary capacity

ET₀ (m) - reference evapotranspiration

z_r(m)- rooting depth of crop

z'(m) - vertical axis of the soil

K (h) - soil hydraulic conductivity

K_c - dimensionless crop coefficient

Ks - saturated hydraulic conductivity

 θ s and θ r - saturated and the residual soil moisture α , n - van Genuchten–Mualem parameters(soil specific)



Model Discretization: Finite Difference



Assumptions:

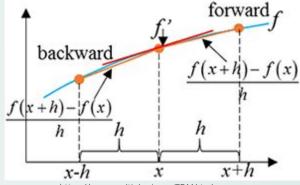
Total depth (z) of soil is 0.5 m, evenly discretized into 26 nodes. Center of each node is used to represent the dynamics of entire node

System Input: Irrigation/Precipitation rate Model states: capillary potential (h) at nodes

Forward Difference:

$$rac{\partial K(h)}{\partial z} = rac{\partial K(h)}{\partial h} \cdot rac{\partial h}{\partial z}$$

$$rac{\partial K(h)}{\partial z} = rac{\partial K(h)}{\partial h} \cdot rac{h(z+\Delta z)}{\Delta h}$$



https://www.multiphysics.us/FDM.html

Boundary Conditions:

Top:
$$\left. \frac{\partial h(t)}{\partial z} \right|_T = -1 - \frac{I(t) + P(t)}{K(h(t))},$$

$$\left. \frac{\partial h(t)}{\partial z} \right|_B = 0,$$



Model Discretization Continued...



After Model Discretization (PDE to ODEs), [time domain]

$$rac{dh_n}{dt} = f(h_{n-1},h_n,h_{n+1})$$

Generalized form of ODEs, (n = 1, 2, 3,, 23, 24)

$$rac{dh_n}{dt} = \left(rac{K(h_n)}{C(h_n)\cdot \Delta z}
ight) \cdot \left(rac{h_{n+1}-2h_n+h_{n-1}}{\Delta z}
ight) + \left(rac{1}{C(h_n)}
ight) \cdot rac{\partial K}{\partial h} \cdot \left(rac{h_{n+1}-h_n}{\Delta z}
ight) \cdot \left(rac{h_{n+1}-h_n}{\Delta z}+1
ight) - rac{C_0}{C(h_n)}$$

Relations from BCs,

$$h(1) = \Delta z \left(1 + rac{I_{\mathrm{val}} + P_t}{K(h(1))}
ight) + h(2)$$

non-linear

$$h(25) = h(26)$$

Non-linear differential term



Open Loop Simulation: MATLAB



1. Non-linear solver: **fsolve**

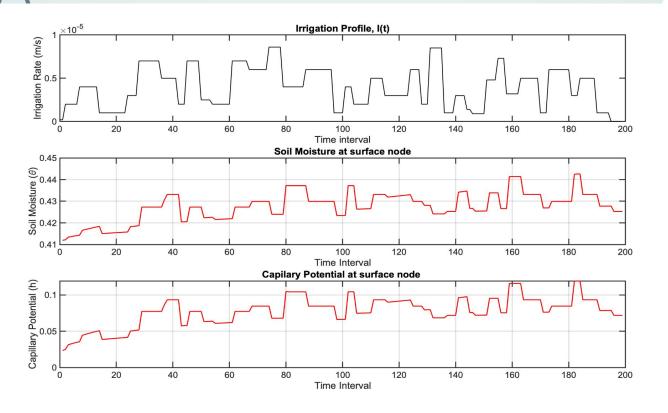
```
fsolve('algebraic_equation',H1_initial);
```

 Ordinary Differential Equations solver: ode45 (variable step solver to approximate the solution of an ODE)

Calculate the soil moisture profile for all 26 nodes

3. Non-linear differential term: Numerical Jacobian

Results





Conclusion & Future Work



In summary, this work focused on:

- Discretizing Richard's equation (PDE) in the z-direction using the finite difference method.
- Obtaining and solving the system of 24 ODEs in MATLAB with boundary conditions.
- Deriving capillary potential (h) and soil moisture profile (θ) at nodes.
- Generating sensor data (training set) for model development.

Future work:

- To develop deep learning based predictive model which can predict the soil moisture content for longer periods of time.
- To integrate the developed model with the advanced control scheme to maintain the soil moisture at desired operating zones.



References



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