Prob-11.13

The results using the Gauss-Seidel method without relaxation are as follows:-

```
A =
[[-8  1 -2]
 [ 2 -6 -1]
 [-3 -1  7]]
b = [-20 -38 -34]

Number of iterations =  5

(using gauss-seidel without relaxation)
x = [ 4.00008409  8.00012981 -1.99994542]
-------------------
```

From the output we can see that the Gauss-Seidel takes about 5 iterations to converge to the values for the percent relative error of 5%.

I rearranged the problem to ensure that each of the diagonal elements in the equation was dominant as compared to the other coefficients. Hence this allows the possibility of convergence for the equations.

The results for the relaxation case for Lamb = 1.2 are as follows:-

```
Number of iterations =  11

(using gauss-seidel with relaxation)
x = [ 3.99993812  7.9998072  -2.000134  ]
-------------------
```

In this case we can see that this algorithm takes longer to complete with 11 being the total number of iterations. For values of lamb between 1 and 2, extra weight is placed on the current value. For such values, it is assumed that the new value is traversing towards the correct value but at a slow rate. Hence the added weight used to improve the estimate by pushing it closer to the real value. It is used to accelerate the convergence of an already converging system. However in this case, we see that the real values take longer to converge to the real values using a lambda value of 1.2. It takes longer to converge than expected and hence the choice of a proper value is very problem-specific and needs to determined using various tests.

The real values for the set of equations has also been derived using the Gauss Elimination method. They are as follows:-

```
(using gauss-elim)
x = [ 4.  8. -2.]
```

As we can see, there is a slight difference in the values obtained through both the Gauss-Seidel method.

The true relative error for each of the values can be calculated using the following expression:-

$$\varepsilon_t = \frac{\text{true error}}{\text{true value}} 100\%$$

For the method without relaxation,
et1 = 2.1 x 10^{-3}
et2 = 1.6 x 10^{-3}
et3 = 2.7 x 10^{-3}
The minimum true relative error is 1.6 x 10^{-3}
The average true relative error is 2.133 x 10^{-3}

With relaxation the values are as follows:-
et1 = 1.5 x 10^{-3}
et2 = 2.4 x 10^{-3}
et3 = 6.7 x 10^{-3}
The minimum true relative error is 1.5 x 10^{-3}
The average true relative error is 3.533 x 10^{-3}

We can observe the average true relative error is significantly lower in the Gauss-Seidel method without relaxation even though the true relative error is marginally higher.
Thus we can conclude that in this case the Gauss-Seidel works better without using relaxation for convergence as it converges quickly. Also the true relative error values are better than the values that were obtained for algorithm with relaxation.