ICSE QUESTION PAPER

Class X

MATHS

(2017)

(Two and a half hours)

Answers to this Paper must be written on the paper provided separately.

You will not be allowed to write during the first 15 minutes.

This time is to be spent in reading the Question Paper.

The time given at the head of this Paper is the time allowed for writing the answers.

*Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*

All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables are provided.

SECTION A (40 Marks)

*Attempt **all** questions from this Section.*
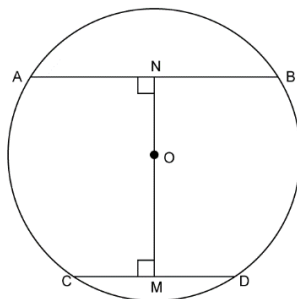
Question 1

(a) If b is the mean proportion between a and c, show that   [3]

$$\frac{a^4 + a^2b^2 + b^4}{b^4 + b^2c^2 + c^4} = \frac{a^2}{c^2}$$

(b) Solve the equation $4x^2 - 5x - 3 = 0$ and give your answer correct to two decimal places.   [3]

(c) AB and CD are two parallel chords of a circle such that AB = 24 cm and CD = 10 cm. If the radius of the circle is 13 cm. find the distance between the two chords.   [4]

Question 2

(a) Evaluate without using trigonometric tables, [3]

$$\sin^2 28° + \sin^2 62° + \tan^2 38° - \cot^2 52° + \frac{1}{4}\sec^2 30°$$

(b) [3]

If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$ and $A^2 - 5B^2 = 5C$. Find matrix C, where C is a 2 by 2 matrix.

(c) Jaya borrowed Rs. 50,000 for 2 years. The rates of interest for two successive years are 12% and 15% respectively. She repays 33,000 at the end of the first year. Find the amount she must pay at the end of the second year to clear her debt. [4]

Question 3

(a) The catalogue price of a computer set is Rs. 42,000. The shopkeeper gives a discount of 10% on the listed price. He further gives an off-season discount of 5% on the discounted price. However, sales tax at 8% is charged on the remaining price after the two successive discounts. Find [3]
(i) the amount of sales tax a customer has to pay
(ii) the total price to be paid by the customer for the computer set.

(b) P(1, -2) is a point on the line segment A(3, -6) and B(x, y) such that AP : PB is equal to 2 : 3. Find the coordinates of B. [3]

(c) The marks of 10 students of a class in an examination arranged in ascending order is as follows: [4]
13, 35, 43, x, x + 4, 55, 61, 71, 80
If the median marks is 48, find the value of x. Hence find the mode of the given data.

Question 4

(a) What must be subtracted from $16x^3 - 8x^2 + 4x + 7$ so that the resulting expression has $2x + 1$ as a factor? [3]

(b) In the given figure ABCD is a rectangle. It consists of a circle and two semi-circles each of which are of radius 5 cm. Find the area of the shaded region. Give your answer correct to three significant figures. [3]
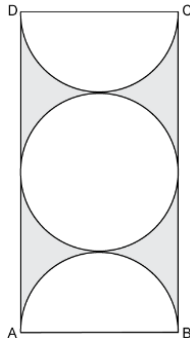
(c) Solve the following inequation and represent the solution set on a number line.

$-8\dfrac{1}{2} < -\dfrac{1}{2} - 4x \le 7\dfrac{1}{2},\ x \in l$ [4]

SECTION B (40 Marks)
Attempt any four questions from this section

Question 5

(a)

Given matrix $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$, find the matrix X if, $X = B^2 - 4B$.

Hence solve for a and b given $X\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$. [3]

(b) How much should a man invest in Rs. 50 shares selling at Rs. 60 to obtain an income of Rs. 450, if the rate of dividend declared is 10%. Also find his yield percent, to the nearest whole number. [3]

(c) Sixteen cards are labeled as a, b, c, ... m, n, o, p. They are put in a box and shuffled. A boy is asked to draw a card from the box. What is the probability that the card drawn is: [4]
(a) a vowel
(b) a consonant
(c) none of the letters of the word median

Question 6

(a) Using a ruler and a compass construct a triangle ABC in which AB = 7 cm, $\angle$CAB = 60º and AC = 5 cm. Construct the locus of [3]
   (i)   points equidistant from AB and AC
   (ii) points equidistant from BA and BC
   Hence construct a circle touching the three sides of the triangle internally.

(b) A conical tent is to accommodate 77 persons. Each person must have 16 m³ of air to breathe. Given the radius of the tent as 7 m, find the height of the tent and also its curved surface area. [3]

(c)
If $\dfrac{7m+2n}{7m-2n}=\dfrac{5}{3}$, use properties of proportion to find
   (i) $m:n$ [4]
   (ii) $\dfrac{m^2+n^2}{m^2-n^2}$

Question 7

(a) A page from a savings bank account passbook is given below: [4]

Date	Particulars	Amount withdrawn (Rs.)	Amount Deposited (Rs.)	Balance (Rs.)
Jan 7, 2016	B/F			3,000.00
Jan 10, 2016	By Cheque		2600.00	5600.00
Feb 8, 2016	To Self	1500.00		4100.00
Apr 6, 2016	By Cheque	2100.00		2000.00
May 4, 2016	By Cash		6500.00	8500.00
May 27, 2016	By Cheque		1500.00	10000.00

   (i) Calculate the interest for the 6 months from January to June 2016, at 6% per annum.
   (ii) If the account is closed on 1st July 2016, find the amount received by the account holder.

(b) Use a graph paper for this question (Take 2 cms = 1 unit on both x and y axis) [6]
   (i)   Plot the following points:
        A(0, 4), B(2, 3), C(1, 1) and D(2, 0)
   (ii)   Reflect points B, C, D on the y-axis and write down their coordinates. Name the images as B', C', D' respectively.
   (iii) Join the points A, B, C, D, D', C', B' and A in order, so as to form a closed figure. Write down the equation of the line of symmetry of the figure formed.

Question 8

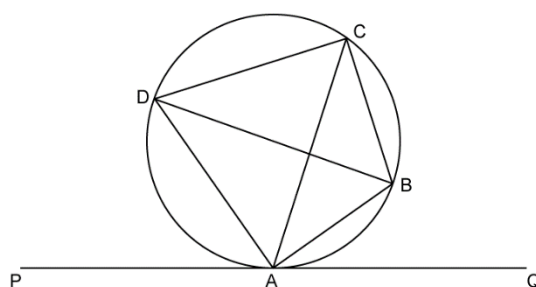(a) Calculate the mean of the following distribution using step deviation method.　　　　[3]

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of students	10	9	25	30	16	10

(b) In the given figure PQ is a tangent to the circle at A, AB and AD are bisectors of $\angle$CAQ and $\angle$PAC. If $\angle$BAQ = 30º, prove that:　　　　[3]
　　(i)　BD is a diameter of the circle
　　(ii) ABC is an isosceles triangle

(c) The printed price of an air conditioner is Rs. 45000/-. The wholesaler allows a discount of 10% to the shopkeeper. The shopkeeper sells the article to the customer at a discount of 5% of the marked price. Sales tax (under VAT) is charged at the rate of 12% at every stage. Find:　　　　[4]
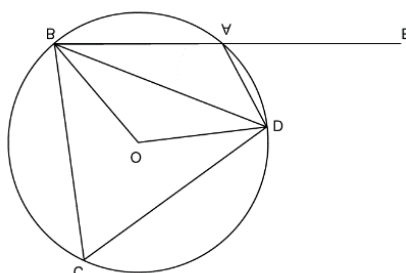　　(i)　VAT paid by the shopkeeper to the government
　　(ii) The total amount paid by the customer inclusive of tax.

Question 9

(a) In the figure given, O is the centre of the circle. $\angle$DAE = 70º, Find giving suitable reasons the measure of:　　　　[3]
　　(i)　　$\angle$BCD
　　(ii)　　$\angle$BOD
　　(iii)　$\angle$OBD

(b) A(-1, 3), B(4, 2) and C(3, -2) are the vertices of a triangle.　　　　[3]
　　(i)　Find the coordinates of the centroid G of the triangle
　　(ii) Find the equation of the line through G and parallel to AC

(c) Prove that [4]

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$$

Question 10

(a) The sum of the ages of Vivek and his younger brother Amit is 47 years. The product of their ages in years is 550. Find their ages. [4]

(b) The daily wages of 80 workers in a project are given below. [6]

Wages (in Rs.)	400-450	450-500	500-550	550-600	600-650	650-700	700-750
No. of Workers	2	6	12	18	24	13	5

Use a graph paper to draw an ogive for the above distribution. (Use a scale of 2 cm = Rs. 50 on x-axis and 2 cm = 10 workers on y-axis). Use your ogive to estimate:
 (i)   the median wage of the workers
  (ii) the lower quartile wage of workers
  (iii) the numbers of workers who earn more than Rs. 625 daily

Question 11

(a) The angles of depression of two ships A and B as observed from the top of a light house 60 m high are 60° and 45° respectively. If the two ships are on the opposite sides of the light house, find the distance between the two ships, Give your answer correct to the nearest whole number. [3]

(b) PQR is a triangle. S is a point on the side QR of ΔPQR such that ∠PSR = ∠QPR. Given QP = 8 cm, PR = 6 cm and SR = 3 cm [3]
  (i)   Prove ΔPQR ~ ΔSPR
  (ii) Find the length of QR and PS
  (iii) $\dfrac{\text{area of } \Delta PQR}{\text{area of } \Delta SPR}$
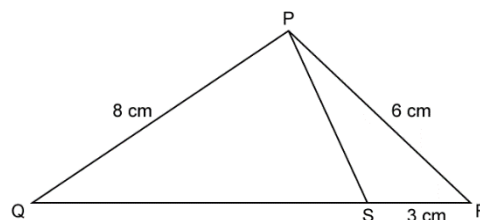
(c) Mr. Richard has a recurring deposit account in a bank for 3 years at 7.5% p. a. simple interest. If he gets Rs. 8325 as interest at the time of maturity, find [4]
  (i) The monthly deposit
  (ii) The maturity value

Class X Mathematics
Board Paper – 2017(Solution)

SECTION A

1.

(a) Cost price of an article = Rs. 3,450

   (i) Marked price of the article = Cost price + 16% of Cost price

$$= 3450 + \frac{16}{100} \times 3450$$

$$= 3450 + 552$$

$$= Rs.\, 4002$$

   (ii) Price paid by the customer = Marked price + Sales Tax

$$= 4002 + \frac{10}{100} \times 4002$$

$$= 4002 + 400.2$$

$$= Rs.\, 4402.20$$

(b) $13x - 5 < 15x + 4 < 7x + 12, \quad x \in R$

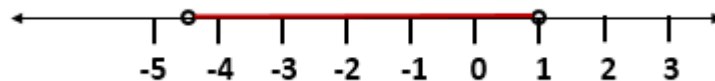   Take $13x - 5 < 15x + 4$ $15x + 4 < 7x + 12$

             $13x < 15x + 9$ $15x < 7x + 8$

             $0 < 2x + 9$ $8x < 8$

            $-9 < 2x$ $x < 1$

            $-\dfrac{9}{2} < x$ $x < 1$

$\therefore \quad -\dfrac{9}{2} < x < 1$

i.e. $-4.5 < x < 1$

$\therefore$ Solution set $= \{x : -4.5 < x < 1, \; x \in R\}$

The solution on the number line is as follows:

(c) $\dfrac{\sin 65°}{\cos 25°}+\dfrac{\cos 32°}{\sin 58°}-\sin 28°.\sec 62°+\operatorname{cosec}^2 30°$

$=\dfrac{\sin(90°-25°)}{\cos 25°}+\dfrac{\cos(90°-58°)}{\sin 58°}-\sin 28°\times\dfrac{1}{\cos(90°-28°)}+\dfrac{1}{\sin^2 30}$

$=\dfrac{\cos 25°}{\cos 25°}+\dfrac{\sin 58°}{\sin 58°}-\sin 28°\times\dfrac{1}{\sin 28°}+\left(\dfrac{1}{\left(\dfrac{1}{2}\right)^2}\right)$

$=1+1-1+4$

$=5$

2.

(a) Given : $A=\begin{bmatrix}3 & x \\ 0 & 1\end{bmatrix}$, $B=\begin{bmatrix}9 & 16 \\ 0 & -y\end{bmatrix}$ and $A^2=B$

Now, $A^2=A\times A=\begin{bmatrix}3 & x \\ 0 & 1\end{bmatrix}\times\begin{bmatrix}3 & x \\ 0 & 1\end{bmatrix}=\begin{bmatrix}9 & 3x+x \\ 0 & 1\end{bmatrix}=\begin{bmatrix}9 & 4x \\ 0 & 1\end{bmatrix}$

We have $A^2=B$

Two matrices are equal if each and every corresponding element is equal.

Thus, $\begin{bmatrix}9 & 4x \\ 0 & 1\end{bmatrix}=\begin{bmatrix}9 & 16 \\ 0 & -y\end{bmatrix}$

$\Rightarrow 4x=16$ and $1=-y$

$\Rightarrow x=4$ and $y=-1$

(b) Population after n years $=$ Present population $\times\left(1+\dfrac{r}{100}\right)^n$

Present population $=2,00,000$

After first year, population $=2,00,000\times\left(1+\dfrac{10}{100}\right)^1$

$=2,00,000\times\dfrac{11}{10}$

$=2,20,000$

Population after two years $=2,20,000\times\left(1+\dfrac{15}{100}\right)^1$

$=2,53,000$

Thus, the population after two years is $2,53,000$.

(c) Three vertices of a parallelogram taken in order are A(3, 6), B(5, 10) and C(3, 2)

    (i) We need to find the co-ordinates of D.

       We know that the diagonals of a parallelogram bisect each other.

       Let (x, y) be the co-ordinates of D.

$$\therefore \text{ Mid}-\text{point of diagonal AC} = \left(\frac{3+3}{2}, \frac{6+2}{2}\right) \equiv (3, 4)$$

       And, mid$-$point of diagonal $BD = \left(\dfrac{5+x}{2}, \dfrac{10+y}{2}\right)$

       Thus, we have

$$\frac{5+x}{2} = 3 \quad \text{and} \quad \frac{10+y}{2} = 4$$

$$\Rightarrow 5+x = 6 \quad \text{and} \quad 10+y = 8$$

$$\Rightarrow x = 1 \quad \text{and} \quad y = -2$$

$$\therefore D = (1, -2)$$

   (ii) Length of diagonal $BD = \sqrt{(1-5)^2 + (-2-10)^2}$

$$= \sqrt{(-4)^2 + (-12)^2}$$

$$= \sqrt{16+144}$$

$$= \sqrt{160}$$

$$= 4\sqrt{10}$$

  (iii) $A(3, 6) = (x_1, y_1)$ and $B(5, 10) = (x_2, y_2)$

       Slope of line $AB = m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{10-6}{5-3} = \dfrac{4}{2} = 2$

       $\therefore$ Equation of line AB is given by

$$y - y_1 = m(x - x_1)$$
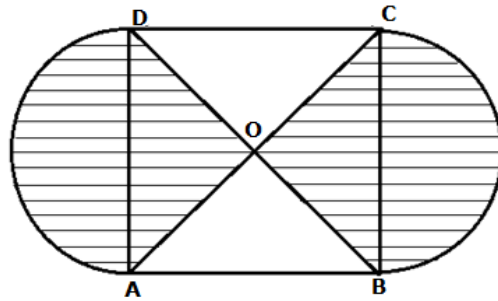
$$\Rightarrow y - 6 = 2(x - 3)$$

$$\Rightarrow y - 6 = 2x - 6$$

$$\Rightarrow 2x - y = 0$$

$$\Rightarrow 2x = y$$

3.

(a)

Area of one semi-circle $= \dfrac{1}{2} \times \pi \times \left(\dfrac{21}{2} \right)^2$

$\Rightarrow$ Area of both semi-circles $= 2 \times \dfrac{1}{2} \times \pi \times \left(\dfrac{21}{2} \right)^2$

Area of one triangle $= \dfrac{1}{2} \times 21 \times \dfrac{21}{2}$

$\Rightarrow$ Area of both triangles $= 2 \times \dfrac{1}{2} \times 21 \times \dfrac{21}{2}$

Area of shaded portion

$= 2 \times \dfrac{1}{2} \times \pi \times \left(\dfrac{21}{2} \right)^2 + 2 \times \dfrac{1}{2} \times 21 \times \dfrac{21}{2}$

$= \dfrac{22}{7} \times \dfrac{441}{4} + \dfrac{441}{2}$

$= \dfrac{693}{2} + \dfrac{441}{2}$

$= \dfrac{1134}{2}$

$= 567 \text{ cm}^2$

(b)

Marks (x)	0	1	2	3	4	5	Total
No. of Students (f)	1	3	6	10	5	5	n = 30
fx	0	3	12	30	20	25	$\Sigma fx = 90$
c.f.	1	4	10	20	25	30	

$$\text{Mean} = \frac{\Sigma fx}{n} = \frac{90}{30} = 3$$

Number of observations $= 30$ (even)

$$\therefore \text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)^{th} \text{observation}}{2}$$

$$= \frac{\left(\frac{30}{2}\right)^{th} \text{observation} + \left(\frac{30}{2}+1\right)^{th} \text{observation}}{2}$$

$$= \frac{15^{th} \text{ observation} + 16^{th} \text{observation}}{2}$$

$$= \frac{3+3}{2}$$

$$= 3$$

Mode $=$ The number (marks) with highest frequency $= 3$

(c) In the given figure, TS $\perp$ SP,

m∠TSR = m∠OSP = 90°

In ∆TSR, m∠TSR + m∠TRS +m∠RTS = 180°

⇒ 90° + 65°+ x = 180°

⇒ x = 180° − 90° − 65°

⇒ x = 25°

Now, y = 2x [Angle subtended at the centre is double that of the angle subtended by the arc at the same centre]

⇒ y = 2 × 25°

∴ y = 50°

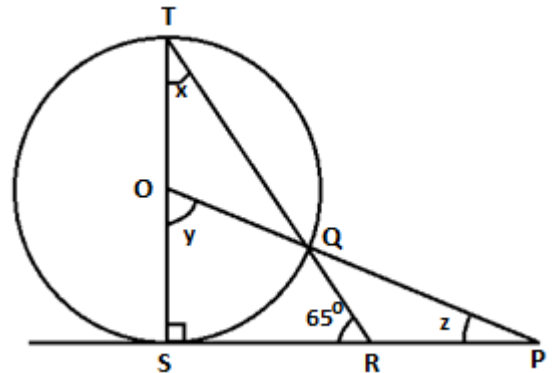In ∆OSP, m∠OSP + m∠SPO + m∠POS = 180°

⇒ 90° + z + 50° = 180°

⇒ z = 180°−140°

∴ z = 40°

4.

(a) Given,

P = Rs. 1000

n = 2 years = 24 months

r = 6%

(i) $\text{Interest} = P \times \dfrac{n(n+1)}{2} \times \dfrac{r}{12 \times 100}$

$= 1000 \times \dfrac{24 \times 25}{2} \times \dfrac{6}{12 \times 100}$

$= 1500$

Thus, the interest earned in 2 years is Rs. 1500.

(ii) $\text{Sum deposited in two years} = 24 \times 1000 = 24{,}000$

Maturity value = Total sum deposited in two years + Interest

= 24,000 + 1,500

= 25,500

Thus, the maturity value is Rs. 25,500.

(b) $(K + 2)x^2 - Kx + 6 = 0$ ....(1)

Substituting x = 3 in equation (1), we get

$(K + 2)(3)^2 - K(3) + 6 = 0$

$\therefore 9(K + 2) - 3K + 6 = 0$

$\therefore 9k + 18 - 3k + 6 = 0$

$\therefore 6k + 24 = 0$

$\therefore K = -4$

Now, substituting K = -4 in equation (1), we get

$(-4 + 2)x^2 - (-4)x + 6 = 0$

$\therefore -2x^2 + 4x + 6 = 0$

$\therefore x^2 - 2x - 3 = 0$

$\therefore x^2 - 3x + x - 3 = 0$

$\therefore x(x - 3) + 1(x - 3) = 0$

$\therefore (x + 1)(x - 3) = 0$

So, the roots are x = -1 and x = 3.

Thus, the other root of the equation is x = -1.

(c) Each interior angle of the regular hexagon $= \dfrac{(2n-4)}{n} \times 90° = \dfrac{(2 \times 6 - 4)}{6} \times 90° = 120°$

Steps of construction:
i. Construct the regular hexagon ABCDEF with each side equal to 5 cm.
ii. Draw the perpendicular bisectors of sides AB and AF and make them intersect each other at point O.
iii. With O as the centre and OA as the radius draw a circle which will pass through all the vertices of the regular hexagon ABCDEF.
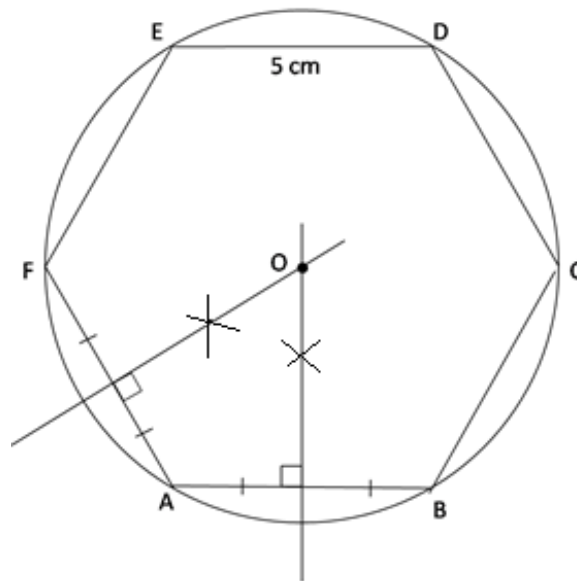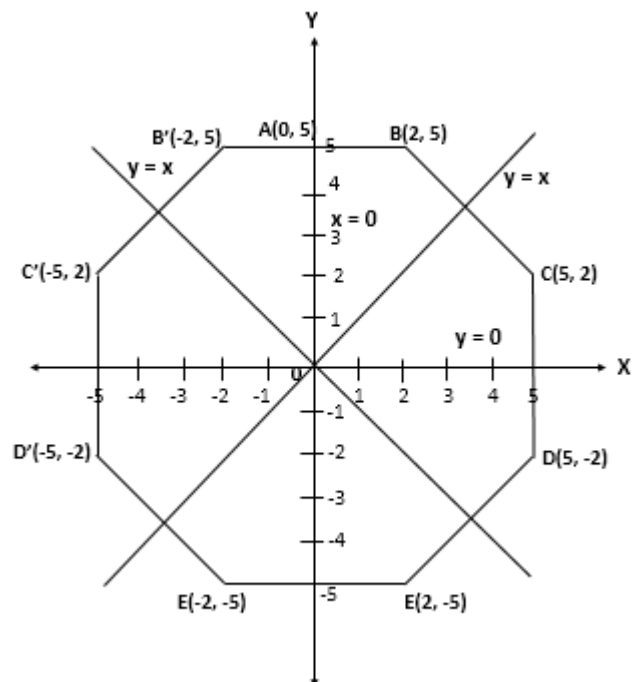

SECTION B

5.
(a)
(i) See the figure.
(ii) Reflection of points on the y-axis will result in the change of the x-coordinate

(iii) Points will be B'(−2, 5), C'(−5, 2), D'(−5, −2), E'(−2, −5).
(iv) The figure BCDEE'D'C'B' is a hexagon.
(v) The lines of symmetry is x-axis or y-axis.

(b) Principal for the month of April = Rs. 0
Principal for the month of May = Rs. 4650
Principal for the month of June = Rs. 4750
Principal for the month of July = Rs. 8950
Total Principal for one month = Rs. 18,350

Time = $\dfrac{1}{12}$ year, Rate = 4%

Interest earned = $\dfrac{18350 \times 1 \times 4}{100 \times 12} = 61.17$

Money received on closing the account on 1st August, 2010
= Last Balance + Interest earned
= Rs. (8950 + 61.17)
= Rs. 9011.17

6.

(a) Given that a, b and c are in continued proportion.

$\Rightarrow \dfrac{a}{b} = \dfrac{b}{c} \Rightarrow b^2 = ac$

L.H.S. $= (a+b+c)(a-b+c)$

$= a(a-b+c) + b(a-b+c) + c(a-b+c)$

$= a^2 - ab + ac + ab - b^2 + bc + ac - bc + c^2$

$= a^2 + ac - b^2 + ac + c^2$

$= a^2 + b^2 - b^2 + b^2 + c^2 \qquad \left[\because b^2 = ac \right]$

$= a^2 + b^2 + c^2$

$= $ R.H.S.

(b)

i. The line intersects the x–axis where, y = 0. Thus, the co-ordinates of A are (4, 0).

ii. Length of AB $= \sqrt{\left(4-(-2)\right)^2 + \left(0-3\right)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$ units

Length of AC $= \sqrt{\left(4-(-2)\right)^2 + \left(0+4\right)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ units

iii. Let Q divides AC in the ratio $m_1 : m_2$. Thus, the co-ordinates of Q are (0, y)

Since $x = \dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$

$\Rightarrow 0 = \dfrac{m_1(-2) + m_2(4)}{m_1 + m_2} \Rightarrow 2m_1 = 4m_2 \Rightarrow m_1 = 2m_2$

$\Rightarrow \dfrac{m_1}{m_2} = \dfrac{2}{1}$

$\therefore$ Required ratio is 2 : 1.

iv. A(4, 0) = A(x_1, y_1) and B(−2, −4) = b(x_2, y_2)

Slope of AC = $\dfrac{-4-0}{-2-4} = \dfrac{-4}{-6} = \dfrac{2}{3}$

∴ Equation of line AC is given by $y - y_1 = m(x - x_1)$

$\Rightarrow y - 0 = \dfrac{2}{3}(x-4)$

$\Rightarrow 3y = 2x - 8$

$\Rightarrow 2x - 3y = 8$

(c) Consider the following distribution:

Class Interval	Frequency (f)	Class mark (x)	fx
0 – 10	8	5	40
10 – 20	5	15	75
20 – 30	12	25	300
30 – 40	35	35	1225
40 – 50	24	45	1080
50 – 60	16	55	880
Total	n = Σf = 100		Σfx = 3600

$\text{Mean} = \dfrac{\Sigma fx}{n} = \dfrac{3600}{100} = 36$

7.

(a) Radius of small sphere = r = 2 cm

Radius of big sphere = R = 4 cm

Volume of small sphere $= \dfrac{4}{3}\pi r^3 = \dfrac{4\pi}{3} \times (2)^3 = \dfrac{32\pi}{3}$ cm^3

Volume of big sphere $= \dfrac{4}{3}\pi R^3 = \dfrac{4\pi}{3} \times (4)^3 = \dfrac{256\pi}{3}$ cm^3

Volume of both the spheres $= \dfrac{32\pi}{3} + \dfrac{256\pi}{3} = \dfrac{288\pi}{3}$ cm^3

We need to find R_1. h = 8 cm (Given)

Volume of the cone $= \dfrac{1}{3}\pi R_1^2 \times (8)$

Volume of the cone = Volume of both the sphere

$\Rightarrow \dfrac{1}{3}\pi R_1^2 \times (8) = \dfrac{288\pi}{3}$

$\Rightarrow R_1^2 \times (8) = 288$

$\Rightarrow R_1^2 = \dfrac{288}{8} \Rightarrow R_1^2 = 36$

$\Rightarrow R_1 = 6$ cm

(b) The given polynomials are $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$.

Let $p(x) = ax^3 + 3x^2 - 9$ and $q(x) = 2x^3 + 4x + a$

Given that $p(x)$ and $q(x)$ leave the same remainder when divided by $(x + 3)$,

Thus by Remainder Theorem, we have

$p(-3) = q(-3)$

$\Rightarrow a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a$

$\Rightarrow -27a + 27 - 9 = -54 - 12 + a$

$\Rightarrow -27a + 18 = -66 + a$

$\Rightarrow -27a - a = -66 - 18$

$\Rightarrow -28a = -84$

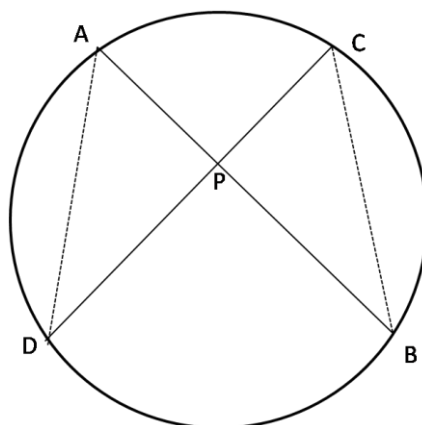$\Rightarrow a = \dfrac{84}{28}$

$\therefore a = 3$

(c) L.H.S. $= \dfrac{\sin\theta}{1 - \cot\theta} + \dfrac{\cos\theta}{1 - \tan\theta}$

$= \dfrac{\sin\theta}{1 - \dfrac{\cos\theta}{\sin\theta}} + \dfrac{\cos\theta}{1 - \dfrac{\sin\theta}{\cos\theta}}$

$= \dfrac{\sin^2\theta}{\sin\theta - \cos\theta} + \dfrac{\cos^2\theta}{\cos\theta - \sin\theta}$

$= \dfrac{\sin^2\theta}{\sin\theta - \cos\theta} - \dfrac{\cos^2\theta}{\sin\theta - \cos\theta}$

$= \dfrac{\sin^2\theta - \cos^2\theta}{\sin\theta - \cos\theta}$

$= \dfrac{(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)}{\sin\theta - \cos\theta}$

$= \sin\theta + \cos\theta$

$= $ R.H.S.

8.

(a) Construction: Join AD and CB.

In ΔAPD and ΔCPB

∠A = ∠C   .....(Angles in the same segment)

∠D = ∠B   .....(Angles in the same segment)

⇒ ΔAPD ∼ ΔCPB   ....(By AA Postulate)

$$\Rightarrow \frac{AP}{CP} = \frac{PD}{PB} \quad ....(\text{Corresponding sides of similar triangles})$$

⇒ AP × PB = CP × PD

(b) Total number of balls = 5 + 6 + 9 = 20

(i) Number of green balls = 9 = Number of favourable cases

$$\therefore P(\text{Green ball}) = \frac{\text{Number of favourable cases}}{\text{Total number of balls}} = \frac{9}{20}$$

(ii) Number of white balls = 5, Number of red balls = 6

Number of favourable cases = 5 + 6 = 11

$$\therefore P(\text{White ball or Red ball}) = \frac{\text{Number of favourable cases}}{\text{Total number of balls}} = \frac{11}{20}$$

(iii) P(Neither green ball nor white ball) = P(Red ball)

$$= \frac{\text{Number of Red balls}}{\text{Total number of balls}}$$

$$= \frac{6}{20}$$

(c)

(i) 100 shares at Rs. 20 premium means:

Nominal value of the share is Rs. 100

Market value of each share = 100 + 20 = Rs. 120

Investment = Rs. 9600

$$\therefore \text{ Number of shares} = \frac{\text{Investment}}{\text{Market Value of each Share}} = \frac{9600}{120} = 80$$

(ii) Sale price of each share = Rs. 160

$\therefore$ The sale proceeds = 80 × 160 = Rs. 12,800

(iii) New investment = Rs. 12,800

Market Value of each share = Rs. 40

$$\therefore \text{ Number of shares} = \frac{\text{Investment}}{\text{Market Value of each share}} = \frac{12800}{40} = 320$$

(iv) Dividend in the 1st investment:

$= \text{Number of shares} \times \text{Rate of dividend} \times \text{N.V. of each share}$

$= 80 \times 8\% \times 100$

$= 80 \times \dfrac{8}{100} \times 100$

$= \text{Rs. } 640$

Dividend in the 2nd investment

$= \text{Number of shares} \times \text{Rate of dividend} \times \text{N.V. of each share}$
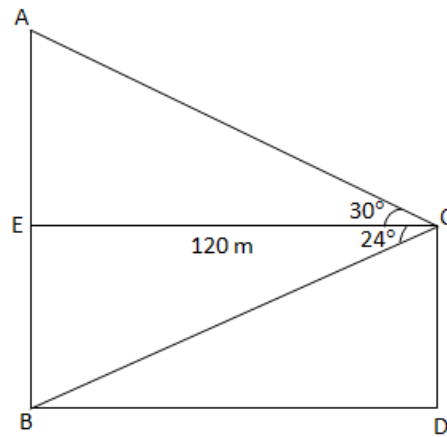
$= 320 \times 10\% \times 50$

$= 320 \times \dfrac{10}{100} \times 50$

$= \text{Rs. } 1600$

Thus, change in two dividends = 1600 – 640 = Rs. 960

9.

(a) Consider the following figure:

In ΔAEC,

$$\tan 30° = \frac{AE}{EC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AE}{120}$$

$$\Rightarrow \frac{120}{\sqrt{3}} = AE$$

∴ AE = 69.28 m

In ΔBEC,

$$\tan 24° = \frac{EB}{EC}$$

$$\Rightarrow 0.445 = \frac{EB}{120}$$

∴ EB = 53.427 m

Thus, height of first tower, AB = AE + EB

$$= 69.282 + 53.427$$

$$= 122.709 \text{ m}$$

$$= 122 \text{ m (correct to 3 significant figures)}$$

And, height of second tower, CD = EB = 53.427 m = 53.4 (correct to 3 significant figures)

(b) The cumulative frequency table of the given distribution table is as follows:

Weight in Kg	Number of workers	Cumulative frequency
50-60	4	4
60-70	7	11
70-80	11	22
80-90	14	36
90-100	6	42
100-110	5	47
110-120	3	50

The ogive is as follows:
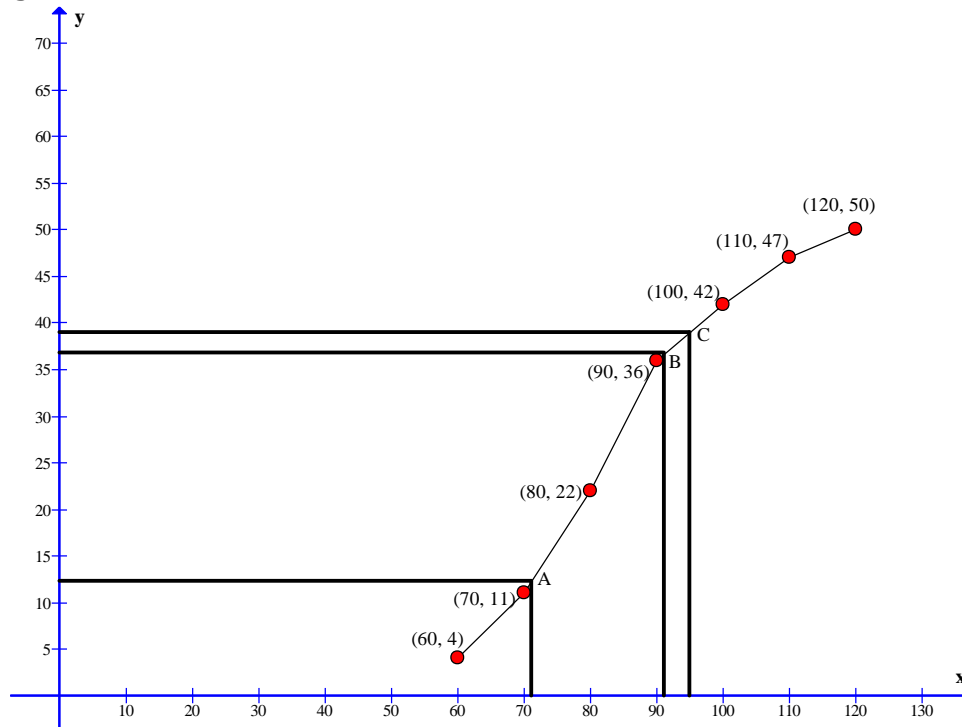
Number of workers = 50

(i) Upper quartile $(Q3) = \left(\dfrac{3 \times 50}{4}\right)^{th}$ term $= (37.5)^{th}$ term $= 92$

Lower quartile $(Q_1) = \left(\dfrac{50}{4}\right)^{th}$ term $= (12.5)^{th}$ term $= 71.1$

(ii) Through mark of 95 on the x − axis, draw a vertical line which meets the graph at point C.

Then through point C, draw a horizontal line which meets the y-axis at the mark of 39.

Thus, number of workers weighing 95 kg and above $= 50 - 39 = 11$

10.

  (a) (i) Selling price of the manufacturer = Rs. 25000

        Marked price of the wholesaler

$$= 25000 + \frac{20}{100} \times 25000$$

$$= 25000 + 5000$$

$$= \text{Rs. } 30,000$$

    (ii) For retailer,

        C.P. = Marked price − Discount

$$= \text{Rs. } 30000 - 10\% \text{ of Rs. } 30000$$

$$= \text{Rs. } 30000 - \frac{10}{100} \times \text{Rs. } 30000$$

$$= \text{Rs. } 30000 - \text{Rs. } 3000$$

$$= \text{Rs. } 27,000$$

        Now, C.P. inclusive of tax = Rs. 27000 + 8% of Rs. 27000

$$= \text{Rs. } 27000 + \frac{8}{100} \times \text{Rs. } 27000$$

$$= \text{Rs. } 27000 + \text{Rs. } 2160$$

$$= \text{Rs. } 29,160$$

    (iii) For wholesaler,

        C.P. = Rs. 25000

        S.P. = Rs. 27000

        Profit = S.P. − C.P. = Rs. (27000 − 25000) = Rs. 2000

        VAT paid by wholesaler $= \dfrac{8}{100} \times \text{Rs. } 2000 = \text{Rs. } 160$

  (b) $AB = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 3 \times 0 + 7 \times 5 & 3 \times 2 + 7 \times 3 \\ 2 \times 0 + 4 \times 5 & 2 \times 2 + 4 \times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + 35 & 6 + 21 \\ 0 + 20 & 4 + 12 \end{bmatrix}$$

$$= \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix}$$

$$5C = 5 \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

$$AB - 5C = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix} = \begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$$

(c)

(i) Consider $\triangle ADE$ and $\triangle ACB$.

$\angle A = \angle A \ [\text{Common}]$

$m\angle B = m\angle E = 90°$

Thus by Angle-Angle similarity, triangles, $\triangle ACB \sim \triangle ADE$.

(ii) Since $\triangle ADE \sim \triangle ACB$, their sides are proportional.

$$\Rightarrow \frac{AE}{AB} = \frac{AD}{AC} = \frac{DE}{BC}\ldots(1)$$

In $\triangle ABC$, by Pythagoras Theorem, we have

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow AB^2 + 5^2 = 13^2$$

$$\Rightarrow AB = 12 \text{ cm}$$

From equation (1), we have,

$$\frac{4}{12} = \frac{AD}{13} = \frac{DE}{5}$$

$$\Rightarrow \frac{1}{3} = \frac{AD}{13}$$

$$\Rightarrow AD = \frac{13}{3} \text{ cm}$$

Also $\dfrac{4}{12} = \dfrac{DE}{5}$

$$\Rightarrow DE = \frac{20}{12} = \frac{5}{3} \text{ cm}$$

(iii) We need to find the area of $\triangle ADE$ and quadrilateral BCED.

Area of $\triangle ADE = \dfrac{1}{2} \times AE \times DE = \dfrac{1}{2} \times 4 \times \dfrac{5}{3} = \dfrac{10}{3} \text{ cm}^2$
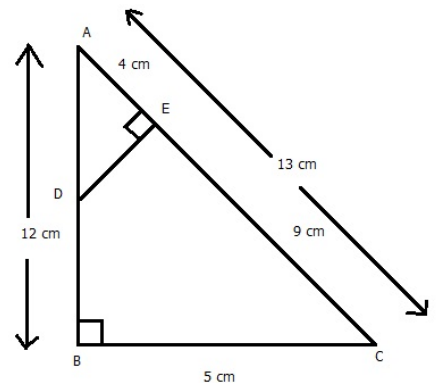
Area of quad.BCED = Area of $\triangle ABC$ – Area of $\triangle ADE$

$$= \frac{1}{2} \times BC \times AB - \frac{10}{3}$$

$$= \frac{1}{2} \times 5 \times 12 - \frac{10}{3}$$

$$= 30 - \frac{10}{3}$$

$$= \frac{80}{3} \text{ cm}^2$$

Thus ratio of areas of $\triangle ADE$ to quadrilateral BCED $= \dfrac{\dfrac{10}{3}}{\dfrac{80}{3}} = \dfrac{1}{8}$

11.

(a) Let the two natural numbers be x and (8 – x). Then, we have

$$\frac{1}{x} - \frac{1}{8-x} = \frac{2}{15}$$

$$\Rightarrow \frac{8-x-x}{x(8-x)} = \frac{2}{15}$$

$$\Rightarrow \frac{8-2x}{x(8-x)} = \frac{2}{15}$$

$$\Rightarrow \frac{4-x}{x(8-x)} = \frac{1}{15}$$

$$\Rightarrow 15(4-x) = x(8-x)$$

$$\Rightarrow 60 - 15x = 8x - x^2$$

$$\Rightarrow x^2 - 15x - 8x + 60 = 0$$

$$\Rightarrow x^2 - 23x + 60 = 0$$

$$\Rightarrow x^2 - 20x - 3x + 60 = 0$$

$$\Rightarrow (x-3)(x-20) = 0$$

$$\Rightarrow (x-3) = 0 \text{ or } (x-20) = 0$$

$$\Rightarrow x = 3 \text{ or } x = 20$$

Since sum of two natural numbers is 8, x cannot be equal to 20

$$\Rightarrow x = 3 \text{ and } 8 - x = 8 - 3 = 5$$

Hence, required natural numbers are 3 and 5.

(b) $\dfrac{x^3 + 12x}{6x^2 + 8} = \dfrac{y^3 + 27y}{9y^2 + 27}$

$$\Rightarrow \frac{x^3 + 12x + 6x^2 + 8}{x^3 + 12x - 6x^2 - 8} = \frac{y^3 + 27y + 9y^2 + 27}{y^3 + 27y - 9y^2 - 27} \quad \text{(Using componendo-dividendo)}$$

$$\Rightarrow \frac{(x+2)^3}{(x-2)^3} = \frac{(y+3)^3}{(y-3)^3}$$

$$\Rightarrow \left(\frac{x+2}{x-2}\right)^3 = \left(\frac{y+3}{y-3}\right)^3$$

$$\Rightarrow \frac{x+2}{x-2} = \frac{y+3}{y-3}$$

$$\Rightarrow \frac{2x}{4} = \frac{2y}{6} \quad \text{(Using componendo-dividendo)}$$

$$\Rightarrow \frac{x}{2} = \frac{y}{3}$$

$$\Rightarrow \frac{x}{y} = \frac{2}{3} \Rightarrow x : y = 2 : 3$$

(c)

1. Draw a line segment AB of length 5.5 cm.

2. Make an angle m∠BAX = 105° using a protractor.

3. Draw an arc AC with radius AC = 6 cm on AX with centre at A.

4. Join BC.

Thus ΔABC is the required triangle.

(i)   Draw BR, the bisector of ∠ABC, which is the locus of points equidistant from BA and BC.

(ii)   Draw MN, the perpendicular bisector of BC, which is the locus of points equidistant from B and C.

(iii)   The angle bisector of ∠ABC and perpendicular bisector of BC meet at point P. Thus, P satisfies the above two loci.

         Length of PC = 4.8 cm