# DFT - FFT

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## Discrete Fourier Transform (DFT)

Input: time sequence: $x_0, x_1, \dots, x_{N-1}$ $\in \mathbb{C}$

Output: frequency sequence: $X_0, X_1, \dots, X_{N-1}$ $\in \mathbb{C}$

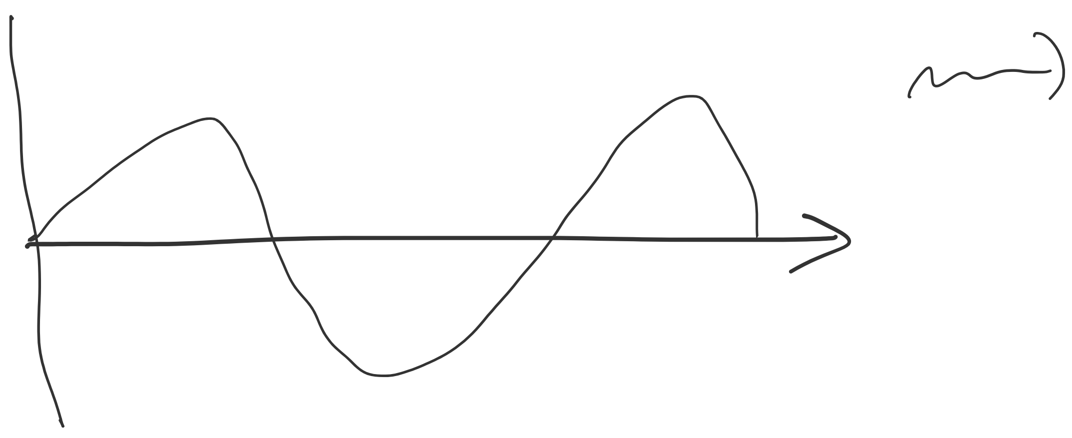
$$\boxed{X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-2\pi i \frac{n \cdot k}{N}}}$$

$O(N^2)$

$\rightsquigarrow O(N \log N)$

(All computations in $\mathbb{C}$)

Time sequence:



Frequencies:

## Cooley - Tukey

$$X_k = \underbrace{\sum_{n=0}^{N/2-1} x_{2n} \cdot e^{-2\pi i \frac{2n k}{N}}}_{\text{even} \quad E_k} + \underbrace{\sum_{n=0}^{N/2-1} x_{2n+1} \cdot e^{-2\pi i \frac{(2n+1) k}{N}}}_{\text{odd} \quad e^{-2\pi i \frac{k}{N}} \cdot O_k}$$

$$\boxed{X_k = E_k + e^{-2\pi i \frac{k}{N}} O_k}$$

Note: $E_{k+\frac{N}{2}} = E_k$ , $O_{k+\frac{N}{2}} = O_k$

$$\boxed{X_{k+\frac{N}{2}} = E_k - e^{-2\pi i \frac{k}{N}} O_k}$$