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## Knowledge Representation and Reasoning

**Project number 2:**  
**Deterministic Action With Cost**  
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# 1 Introduction

A dynamic system (DS) is viewed as

- a collection of objects, together with their properties, and
- a collection of actions which, while performed, change properties of objects (in consequence, the state of the world).

Let  $C2$  be a class of dynamic systems satisfying the following assumptions:

1. Inertia law
2. Complete information about all actions and fluent.
3. Only Determinism
4. Only sequential actions are allowed.
5. Characterizations of actions:
  - Precondition represented by set of literals(a fluent or its negation);if a precondition does not hold, the action is executed but with empty effect
  - Postcondition (effect of an action) represented by a set of literals.
  - Cost  $k \in N$  of an action, actions with empty effects cost 0. Each action has a fixed cost, if it leads to non-empty effects.
6. Effects of an action depends on the state where the action starts.
7. All actions are performed in all states.
8. Partial description of any state of the system are allowed.
9. No constraints are defined.

# 2 Syntax

## 2.1 Signature :

A signature is a triplet  $\Upsilon = (F, Ac, K)$  where  $F$  is a set of fluents;  $Ac$  is a set of actions as follows  $A_1, A_2, \dots, A_n$  where  $A_i \in Ac$  and  $i = 1$  to  $n$  and  $K$  is a set of positive integers representing Cost of each action  $A_i \in Ac$  as follows  $k_1, k_2, \dots, k_n$  where  $k_i \in K$  and  $i = 1$  to  $n$ .

## 2.2 Literal :

A literal is either a fluent  $f$  or its negation  $\neg f$ .

**Notation:** for a fluent  $f \in F$ , we write  $\bar{f}$  to denote the literal corresponding to  $f$ , i.e., either  $f$  or  $\neg f$ .

## 2.3 Statements :

The system and changes occurring within can be described through a sequence of statements defined in the table:

| Statement        | Format   | Description   |
|------------------|--|---|
| Value Statement  | $\bar{f}$ after $A_1 \dots A_n$ where $\bar{f} \in F$ and $A_i \in Ac$ , for $i = 1, \dots, n$                     | $\bar{f}$ holds after performing the sequence $A_1 \dots A_n$ of actions in the initial state.  |
| Abbreviation     | initially $\bar{f}$  | in the initial state $\bar{f}$ holds  |
| Effect Statement | $A_i$ causes $\bar{f}$ if $\bar{g}_1, \dots, \bar{g}_k$  | If the action $A_i$ is performed in any state satisfying $\bar{g}_1, \dots, \bar{g}_k$ , then in the resulting state $\bar{f}$ holds.       |
| Cost Statement   | $A_i$ costs $k_i$ (if $A_i$ causes $\bar{f}$ if $\bar{g}_1, \dots, \bar{g}_n$ ), $k_i \in K$ for $i = 1, \dots, n$ | If the action $A_i$ is performed in any state satisfying $\bar{g}_1, \dots, \bar{g}_k$ , then change in state results in fixed cost $k_i$ . |

Table 1: Syntax Table

## 3 Semantics

- A state is a mapping  $\sigma : F \rightarrow \{0, 1\}$ . For any  $f \in F$ , if  $\sigma(f) = 1$ , then we say that  $f$  holds in  $\sigma$  and write  $\sigma \models f$ . If  $\sigma(f) = 0$ , then we write  $\sigma \models \neg f$  and say that  $f$  does not hold in  $\sigma$ . Let  $\Sigma$  stand for the set of all states.
- A state transition function is a mapping  $\Psi : Ac \times \Sigma \rightarrow \Sigma$ . For any  $\sigma \in \Sigma$ , for any action  $A_i \in Ac$  where  $i = 1, \dots, n$ ,  $\Psi(A_i, \sigma)$  is the state resulting from performing the action  $A_i$  in the state  $\sigma$ . Also  $\Psi(A_i, \sigma)$  will result in same state if the effect of action is empty.
- A cost transition function is a mapping  $\Gamma : Ac \times \Sigma \rightarrow K$ . For any  $\sigma \in \Sigma$  and for any action  $A_i \in Ac$  where  $i = 1, \dots, n$ ,  $\Gamma(A_i, \sigma)$  is the fixed cost  $k_i$  corresponding to the action  $A_i$ , where  $k_i \in K$  and  $i = 1, \dots, n$ , resulting from performing the action  $A_i$  in the state  $\sigma$ . Also  $\Gamma(A_i, \sigma)$  will result in 0 cost if there is no change in state.

- A transition function is generalized to the mapping  $\Psi^* : Ac^* \times \Sigma \rightarrow \Sigma$  as follows:
  1.  $\Psi^* (\varepsilon, \sigma) = \sigma$ ,
  2.  $\Psi^* ((A1, \dots, An), \sigma) = \Psi(An, \Psi^* (A1, \dots, An-1))$ .
- Let L be an action language of the class A over the signature  $\Upsilon = (F, Ac, K)$ . A structure for L is a triplet  $S = (\Psi, \sigma_0, \Gamma)$  where  $\Psi$  is a state transition function,  $\Gamma$  is a cost transition function and  $\sigma_0 \in \Sigma$  is the initial state
- Let  $S = (\Psi, \sigma_0, \Gamma)$  be a structure for L. A statement s is true in S, in symbols  $S \models s$ , iff
  1. s is of the form  $\bar{f}$  after  $A1, \dots, An$ , then  $\Psi((A1, \dots, An), \sigma_0) \models \bar{f}$ ;
  2. if s is of the form  $Ai$  causes  $\bar{f}$  if  $\bar{g1}, \dots, \bar{gk}$ , then for every  $\sigma \in \Sigma$  such that  $\sigma \models \bar{gj}$ ,  $j = 1, \dots, k$ ,  $\Psi(Ai, \sigma) \models \bar{f}$ .
  3. if s is of the form  $A1, \dots, An$  costs  $k1, \dots, kn$  respectively where  $ki \in K$ ,  $Ai \in Ac$  and  $i = 1, \dots, n$ , then every  $\sigma \in \Sigma$  such that  $\sigma \models \bar{gi}$ ,  $i = 1, \dots, k$ ,  $\Gamma(Ai, \sigma) \models ki$ .

Let D be an action domain in the language L over the signature  $\Upsilon = (F, Ac, K)$ . A structure  $S = (\Psi, \sigma_0, \Gamma)$  is a model of D iff

- (M1) for every statement  $s \in D$ ,  $S \models s$ ;  
 (M2) for every  $Ai \in Ac$  for every  $f, g1, \dots, gn \in F$ , for every  $ki \in K$  and for every  $\sigma \in \Sigma$ , if one of the following conditions holds:  
 (i) D contains an effect statement and a cost statement as follows:

- **Ai causes  $\bar{f}$  if  $\bar{g1}, \dots, \bar{gk}$ ,  $\sigma \not\models \bar{gj}$  for some  $j = 1, \dots, k$**
- if  $\Psi(Ai, \sigma) \not\models \sigma$  then  $\Gamma(Ai, \sigma) = ki$ .

(ii) D does not contain an effect statement but contains a 0 cost statement, as follows:

- **Ai causes  $\bar{f}$  if  $\bar{g1}, \dots, \bar{gk}$  then  $\sigma \models f$  iff  $\Psi(Ai, \sigma) \models f$ .**
- if  $\Psi(Ai, \sigma) = \sigma$  then  $\Gamma(Ai, \sigma) = 0$ .

## 4 Query

- State query:  
necessary  $\sigma$  after  $A_1, \dots, A_i$  on  $\sum$   
possibly  $\sigma$  after  $A_1, \dots, A_i$  on  $\sum$

The first statement says that state  $\sigma$  always occurs after performing every action in specific state on  $\sum$

The second statement says that state  $\sigma$  may occurs after performing every action in specific state on  $\sum$

When the option from  $\sum$  is omitted, these queries refer to the initial state

- Cost query:  
necessary execution costs  $k_i$  after  $A_1, \dots, A_i$  on  $\sum$   
possibly execution costs 0 after  $A_1, \dots, A_i$  on  $\sum$

The first statement says that corresponding  $k_i$  cost will always occur after performing every action in a specific state on  $\sum$ .

The second statement says that 0 cost may occur after performing every action in a specific state on  $\sum$ .

When the option from  $\sum$  is omitted, these queries refer to the initial state.

## 5 Examples

### 5.1 Example 01

#### 5.1.1 Description

Andrew wants to travel by his car to a place. Travelling costs him 50\$ when there is fuel in car tank. Travelling costs him 50\$ when there is fuel in reserve. When there is no fuel in any of it, Andrew can buy fuel. Fuel costs him 40\$

### 5.1.2 Representation

Fluents:  $F = \{\text{fuel}, \text{reserve}\}$

Actions:  $Ac = \{\text{buy}, \text{travel}\}$

Costs:  $K = \{40, 50\}$

Initially: fuel;

Initially: reserve;

travel causes  $\neg\text{fuel}$  if fuel, reserve;

travel causes  $\neg\text{reserve}$  if  $\neg\text{fuel}$ , reserve;

travel causes  $\neg\text{fuel}$  if fuel,  $\neg\text{reserve}$

travel costs 50;

buy causes fuel if  $\neg\text{fuel}, \text{reserve}$ ;

buy causes fuel if  $\neg\text{fuel}, \neg\text{reserve}$ ;

buy causes reserve if fuel,  $\neg\text{reserve}$ ;

buy costs 40;

### 5.1.3 Calculation

$\Sigma = \{ \sigma_0, \sigma_1, \sigma_2, \sigma_3 \}$

$\sigma_0 = \{ \text{fuel}, \text{reserve} \}$        $\sigma_1 = \{ \neg\text{fuel}, \text{reserve} \}$

$\sigma_2 = \{ \neg\text{fuel}, \neg\text{reserve} \}$        $\sigma_3 = \{ \text{fuel}, \neg\text{reserve} \}$

$\Psi(\text{buy}, \sigma_0) = \sigma_0$

$\Psi(\text{travel}, \sigma_0) = \sigma_1$

$\Gamma(\text{buy}, \sigma_0) = 0$

$\Gamma(\text{travel}, \sigma_0) = 50$

$\Psi(\text{buy}, \sigma_1) = \sigma_0$

$\Psi(\text{travel}, \sigma_1) = \sigma_2$

$\Gamma(\text{buy}, \sigma_1) = 40$

$\Gamma(\text{travel}, \sigma_1) = 50$

$\Psi(\text{buy}, \sigma_2) = \sigma_3$

$\Psi(\text{travel}, \sigma_2) = \sigma_2$

$\Gamma(\text{buy}, \sigma_2) = 40$

$\Gamma(\text{travel}, \sigma_2) = 0$

$\Psi(\text{buy}, \sigma_3) = \sigma_0$

$\Psi(\text{travel}, \sigma_3) = \sigma_2$   
 $\Gamma(\text{buy}, \sigma_3) = 40$   
 $\Gamma(\text{travel}, \sigma_3) = 50$

#### 5.1.4 Graph

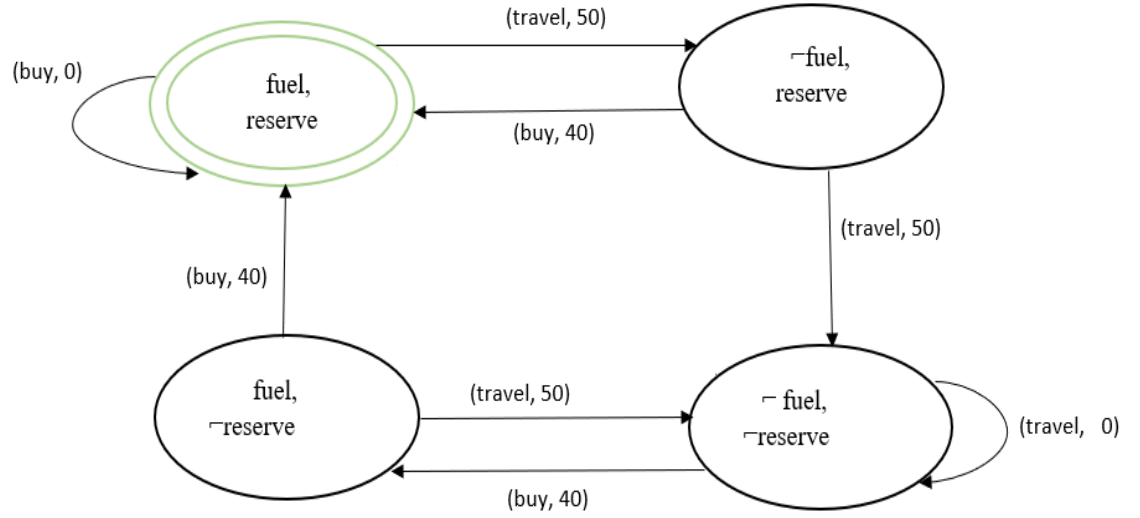


Figure 1: Example 01

#### 5.1.5 Queries

possibly  $\sigma_0$  after buy on  $\sigma_0$   
 necessary  $\sigma_0$  after buy on  $\sigma_1$   
 possibly  $\sigma_2$  after travel on  $\sigma_2$   
 necessary  $\sigma_1$  after travel on  $\sigma_0$

possibly execution costs 0 after buy on  $\sigma_0$   
 necessary execution costs 40 after buy on  $\sigma_2$   
 possibly execution costs 0 after travel on  $\sigma_2$   
 necessary execution costs 50 after travel on  $\sigma_0$



## 5.2 Example 02

### 5.2.1 Description

John visits a painter to buy a specific painting. The cost of painting is 200\$ if its available in the shop. But if painting is not available then John needs to order a new one to be painted and will buy once its available. Order costs 50\$ At any time only one copy of painting is available and another one to be ordered once sold.

### 5.2.2 Representation:

Fluents:  $F = \{\text{available, sold}\}$   
Actions:  $Ac = \{\text{buy, order}\}$   
Costs:  $K = \{200, 50\}$   
Initially:  $\neg\text{available}$ ;  
Initially:  $\neg\text{sold}$ ;  
buy causes sold if available;  
buy causes  $\neg\text{available}$ ;  
buy costs 200\$;  
order causes available if  $\neg\text{available}$ ;  
order costs 50\$;

### 5.2.3 Calculation:

$\Sigma = \{ \sigma_0, \sigma_1, \sigma_2, \sigma_3 \}$   
 $\sigma_0 = \{ \neg\text{available}, \neg\text{sold} \}$   
 $\sigma_1 = \{ \text{available}, \neg\text{sold} \}$   
 $\sigma_2 = \{ \neg\text{available}, \text{sold} \}$   
 $\sigma_3 = \{ \text{available}, \text{sold} \}$

$\Psi(\text{buy}, \sigma_0) = \sigma_0$   
 $\Psi(\text{order}, \sigma_0) = \sigma_1$   
 $\Gamma(\text{buy}, \sigma_0) = 0$   
 $\Gamma(\text{order}, \sigma_0) = 50$

$\Psi(\text{buy}, \sigma_1) = \sigma_2$   
 $\Psi(\text{order}, \sigma_1) = \sigma_1$   
 $\Gamma(\text{buy}, \sigma_1) = 200$

$$\Gamma(\text{order}, \sigma_1) = 0$$

$$\Psi(\text{buy}, \sigma_2) = \sigma_2$$

$$\Psi(\text{order}, \sigma_2) = \sigma_1$$

$$\Gamma(\text{buy}, \sigma_2) = 0$$

$$\Gamma(\text{order}, \sigma_2) = 50$$

$$\Psi(\text{buy}, \sigma_3) = \sigma_2$$

$$\Psi(\text{order}, \sigma_3) = \sigma_3$$

$$\Gamma(\text{buy}, \sigma_3) = 200$$

$$\Gamma(\text{order}, \sigma_3) = 0$$

#### 5.2.4 Graph

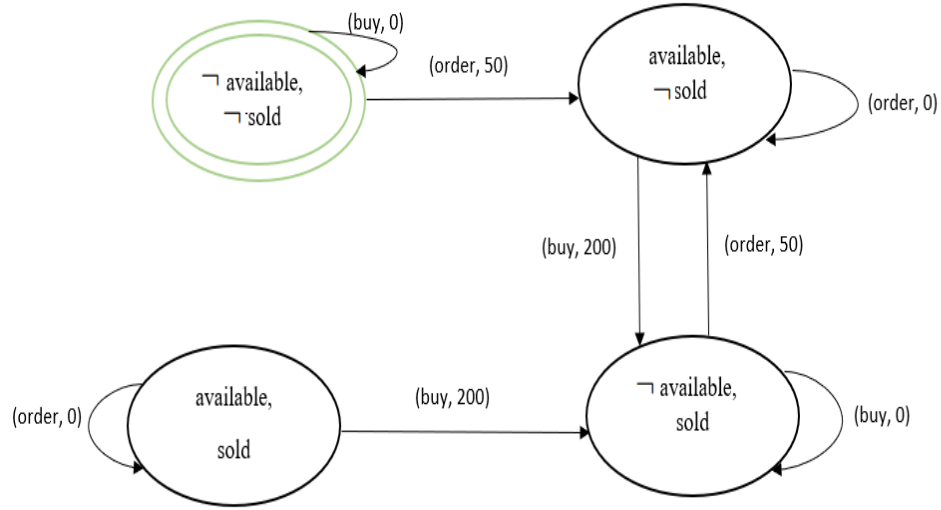


Figure 2: Example 02

#### 5.2.5 Queries

possibly  $\sigma_0$  after buy on  $\sigma_0$

necessary  $\sigma_2$  after buy on  $\sigma_1$

possibly  $\sigma_1$  after order on  $\sigma_1$

necessary  $\sigma_1$  after order on  $\sigma_0$

possibly execution costs 0 after buy on  $\sigma_0$

necessary execution costs 200 after buy on  $\sigma_1$

possibly execution costs 0 after order on  $\sigma_1$

necessary execution costs 50 after order on  $\sigma_0$

### 5.3 Example 03

#### 5.3.1 Description

There is a man. He can cook, eat, and play. Cooking makes food cooked. he can eat food if it is cooked. After eating he feels not hungry, and food is not cooked again. He can play. Playing makes him hungry. He just can play if he is not hungry. He just cooks when there is no food is cooked. Initially, he is hungry, and no food is cooked. In terms of energy, eating costs 5, cooking costs 15, playing costs 20.

#### 5.3.2 Representation in language

Fluents:  $F = \{\text{cooked, hungry}\}$

Actions:  $Ac = \{\text{cook, eat, play}\}$

Costs:  $K = \{15, 5, 20\}$

initially  $\neg\text{cooked}$ ;

initially hungry;

cook causes cooked if  $\neg\text{cooked}$ ;

cook costs 15;

eat causes  $\neg\text{cooked}$  if cooked;

eat causes  $\neg\text{hungry}$  if cooked;

eat costs 5;

play causes hungry if  $\neg\text{hungry}$ ;

play costs 20;

### 5.3.3 Calculation

$$\Sigma = \{\sigma_0, \sigma_1, \sigma_2, \sigma_3\}$$

$$\sigma_0 = \{\neg\text{cooked}, \text{hungry}\}$$

$$\sigma_1 = \{\text{cooked}, \text{hungry}\}$$

$$\sigma_2 = \{\neg\text{cooked}, \neg\text{hungry}\}$$

$$\sigma_3 = \{\text{cooked}, \neg\text{hungry}\}$$

$$\Psi(\text{eat}, \sigma_0) = \sigma_0$$

$$\Psi(\text{cook}, \sigma_0) = \sigma_1$$

$$\Psi(\text{play}, \sigma_0) = \sigma_0$$

$$\Gamma(\text{eat}, \sigma_0) = 0$$

$$\Gamma(\text{cook}, \sigma_0) = 15$$

$$\Gamma(\text{play}, \sigma_0) = 0$$

$$\Psi(\text{eat}, \sigma_1) = \sigma_2$$

$$\Psi(\text{cook}, \sigma_1) = \sigma_1$$

$$\Psi(\text{play}, \sigma_1) = \sigma_1$$

$$\Gamma(\text{eat}, \sigma_1) = 5$$

$$\Gamma(\text{cook}, \sigma_1) = 0$$

$$\Gamma(\text{play}, \sigma_1) = 0$$

$$\Psi(\text{eat}, \sigma_2) = \sigma_2$$

$$\Psi(\text{cook}, \sigma_2) = \sigma_3$$

$$\Psi(\text{play}, \sigma_2) = \sigma_1$$

$$\Gamma(\text{eat}, \sigma_2) = 0$$

$$\Gamma(\text{cook}, \sigma_2) = 15$$

$$\Gamma(\text{play}, \sigma_2) = 20$$

$$\Psi(\text{eat}, \sigma_3) = \sigma_2$$

$$\Psi(\text{cook}, \sigma_3) = \sigma_3$$

$$\Psi(\text{play}, \sigma_3) = \sigma_1$$

$$\Gamma(\text{eat}, \sigma_3) = 5$$

$$\Gamma(\text{cook}, \sigma_3) = 0$$

$$\Gamma(\text{play}, \sigma_3) = 20$$

### 5.3.4 Graph

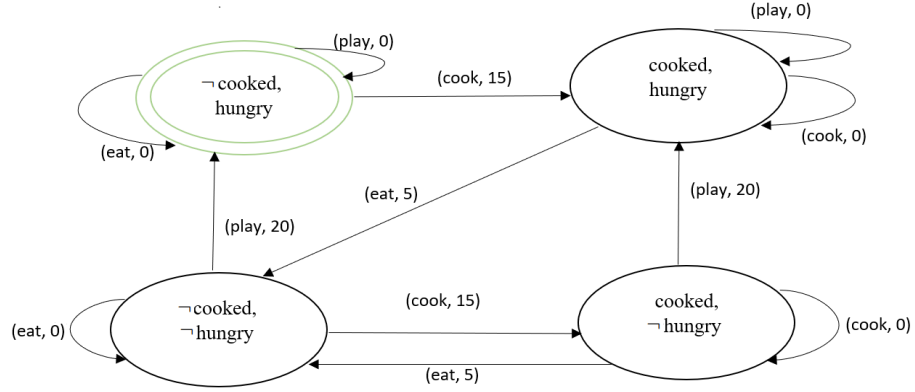


Figure 3: Example 03

### 5.3.5 Queries

possibly  $\sigma_0$  after eat on  $\sigma_0$   
necessary  $\sigma_1$  after cook on  $\sigma_0$   
possibly  $\sigma_0$  after play on  $\sigma_0$   
necessary  $\sigma_3$  after eat on  $\sigma_1$

possibly execution costs 0 after eat on  $\sigma_0$   
necessary execution costs 15 after cook on  $\sigma_0$   
possibly execution costs 20 after play on  $\sigma_2$   
necessary execution costs 5 after eat on  $\sigma_1$

## 6 Appendix

### List of Figures

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