Warsaw University of Technology's
Faculty of Mathematics and Information Science

Knowledge Representation and Reasoning

Project number 2:
Deterministic Action With Cost
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1 Introduction

A dynamic system (DS) is viewed as

- a collection of objects, together with their properties, and
- a collection of actions which, while performed, change properties of objects (in consequence, the state of the world).

Let C2 be a class of dynamic systems satisfying the following assumptions:

1. Inertia law
2. Complete information about all actions and fluent.
3. Only Determinism
4. Only sequential actions are allowed.
5. Characterizations of actions:
   - Precondition represented by set of literals(a fluent or its negation);if a precondition does not hold, the action is executed but with empty effect
   - Postcondition (effect of an action) represented by a set of literals.
   - Cost $k \in N$ of an action, actions with empty effects cost 0. Each action has a fixed cost, if it leads to non-empty effects.
6. Effects of an action depends on the state where the action starts.
7. All actions are performed in all states.
8. Partial description of any state of the system are allowed.
9. No constraints are defined.

2 Syntax

2.1 Signature :

A signature is a pair Υ = (F, Ac, K) where F is a set of fluents; Ac is a set of actions and K is a set of positive integers representing Cost of each action An $\in$ Ac.

2.2 Literal :

A literal is either a fluent f or its negation ¬f.
Notation: for a fluent f ϵ F, we write $\overline{f}$to denote the literal corresponding to f, i.e., either f or ¬f.

2.3 Statements :

The system and changes occurring within can be described through a sequence of statements defined in the table:

Statement	Format	Description
Initial Statement	Initially α holds where α is the initial condition with a set of fluent values.	Initial condition α of the fluent set F_α where F_α = {f1,f2,...,fn} where fi ∈ F and i = 1 to n.
Effect Statement	A causes α if g1, . . . , gk	If the action A is performed in any state satisfying g1, . . . , gk, then in the resulting state α holds.
Value Statement	α after A1 ....An	The condition α always (must) hold after performing the sequence A1 ....An of actions.
Cost Statement	A costs C_β, a numerical cost value where $C_\beta \in$ K	If the action A is performed in any satisfying g1, ...,gk then in the resulting state a cost of C_β is used.C_β is a numerical value in Cost set K

Table 1: Syntax Table

3 Semantics

- A state is a function σ : F → {0, 1}. For any f ∈ F, if σ(f) = 1, then we say that f holds in σ and write $\sigma \vDash$ f. If σ(f) = 0, then we write $\sigma \vDash$ ¬f and say that f does not hold in σ. Let σ stand for the set of all states.

- A transition function is a mapping Υ : (Ac,K) × σ → σ. For any σ ∈ $\sum$, for any A ∈ Ac and for any K_i in K, Υ(A,K_i) , σ) is the state resulting from performing the action A in the state σ.

- In a transition function Υ : (A,K_i) ×σ → σ. For any A_i ∈ Ac there exists a cost value K_i ∈ K where i = 1 to n respectively.

- A transition function is generalized to the mapping Υ^* : (Ac*,K) $\times$ σ $\rightarrow$ σ as follows: Υ^* (ε , σ) = σ, Υ^* (((A1,K_1), . . . , (An,K_n)), σ) = Υ((An,K_n), Υ^* ((A1,K_1) . . . , (An-1,K_{n-1}))).

- Let L be an action language of the class A over the signature Υ = (F, Ac, K). A structure for L is a pair S = (Υ, σ_0) where Υ is a transition function and $\sigma_0 \in \sum$ is the initial state

- Let S = (Υ, σ_0) be a structure for L. A statement s_β is true in S, in symbols S $\vDash$ s_β, iff s_β is of the form f after A1, . . . , An, then Υ(((A1,K_1), . . . , (An,K_n)), σ_0) $\vDash$ f; if s_β is of the form A causes f if g1, . . . , gk and costs K_i, then for every $\sigma \in \sum$ such that $\sigma \vDash$ gj , j = 1, . . . , k, Υ((A,K_i), σ) $\vDash$ f.

Let D be an action domain in the language L over the signature Υ=(F,A_c,K). A structure S = (Υ,σ_0 is a model of D iff
(M1) for every statement $s_{beta} \in$ D, S |= s;
(M2) for every Ai $\in$ Ac there exists $K_i \in$ K where i = 1 to n, for every f,g1,...,gn $\in$ F, and for every $\sigma \in \sum$, if one of the following conditions holds:
(i) D contains an effect statement
A causes $\overline{f}$ for the cost value k if $\overline{g1}$,...,$\overline{gn}$,
where k $\neq$ 0 and σ|= gi for some i = 1,...,n
(ii) D does not contain an effect statement
A causes $\overline{f}$ if g1,...,gn
then σ |$\neq$ gi iff Ψ((A,k),σ) |$\neq$ gi for some i = 1,...,n, where k = 0.

4 Examples

4.1 Example 01

4.1.1 Description

Andrew wants to travel by his car to a place.Travelling costs him 50$ if he uses fuel from the fuel tank of the car.If in case of emergency, Andrew is carrying a bottle of fuel as reserve, which can cost him 50$ for travelling. buying Fuel costs him 100$ if both fuel and reserve are empty. and buying causes both his Fuel and Reserve are filled.

4.1.2 Representation

Fluents: fuel, reserve
Actions: buy, travel

Initially: fuel $\wedge$ reserve
travel causes $\neg$fuel if fuel
travel cost 50
travel causes $\neg$reserve if $\neg$fuel $\vee$ reserve
travel cost 50
buy causes fuel $\wedge$ reserve if $\neg$ fuel $\vee$ $\neg$reserve
buy cost 100

4.1.3 Calculation

$\sum$ = { σ_0,σ_1,σ_2,σ_3 }

σ_0 = { fuel, reserve } σ_1 = { $\neg$fuel, reserve }
σ_2 = { $\neg$fuel, $\neg$reserve } σ_3 = { fuel, $\neg$reserve }
Ψ(buy, σ_0)=σ_0
Ψ(travel, σ_0)=σ_1
Γ(buy, σ_0) = 0
Γ(travel, σ_0) = 50

Ψ(buy, σ_1)=σ_1
Ψ(travel, σ_1)=σ_2
Γ(buy, σ_1) = 0
Γ(travel, σ_1) = 50

Ψ(buy, σ_2)=σ_0
Ψ(travel, σ_2)=σ_2
Γ(buy, σ_2) = 100
Γ(travel, σ_2) = 0

Ψ(buy, σ_3)=σ_3
Ψ(travel, σ_3)=σ_2
Γ(buy, σ_3) = 0
Γ(travel, σ_3) = 50

4.1.4 Graph

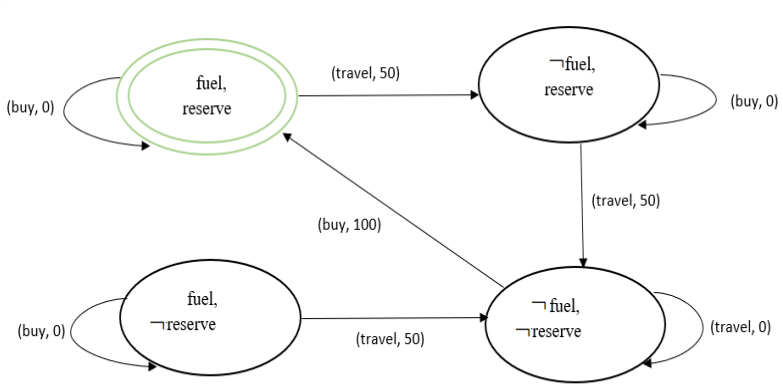

Figure 1: Example 01

4.2 Example 02

4.2.1 Description

John visits a painter to buy a specific painting. The cost of painting is 200$ if its available in the shop. But if painting is not available then John needs to order a new one to be painted and will buy once its available.Order costs 50$ At any time only one copy of painting is available and another one to be ordered once sold.

4.2.2 Representation:

Fluents: available, sold.
Actions: buy, order.
Initially: ¬available ∧ ¬sold
buy causes (sold ∧ ¬available) if available
buy costs 200$
order causes available if ¬available

order Cost: 50$

4.2.3 Calculation:

$\sum = \{ \sigma_0, \sigma_1, \sigma_2, \sigma_3\}$
$\sigma_0 = \{ \neg$available, $\neg$sold$\}$
$\sigma_1 = \{$ available, $\neg$sold$\}$
$\sigma_2 = \{ \neg$available, sold$\}$
$\sigma_3 = \{$ available, sold$\}$

Ψ (buy, $\sigma 0) = \sigma 0$
Ψ (order, $\sigma 0) = \sigma 1$
Γ(buy, $\sigma_0) = 0$
Γ(order, $\sigma_0) = 50$

Ψ (buy $\sigma 1) = \sigma 2$
Ψ (order, $\sigma 1) = \sigma 1$
Γ(buy, $\sigma_1) = 200$
Γ(order, $\sigma_1) = 0$

Ψ (buy, $\sigma 2) = \sigma 2$
Ψ (order, $\sigma 2) = \sigma 1$
Γ(buy, $\sigma_2) = 0$
Γ(order, $\sigma_2) = 50$

Ψ (buy, $\sigma 3) = \sigma 2$
Ψ (order, $\sigma 3) = \sigma 3$
Γ(buy, $\sigma_3) = 200$
Γ(order, $\sigma_3) = 0$

4.2.4 Graph

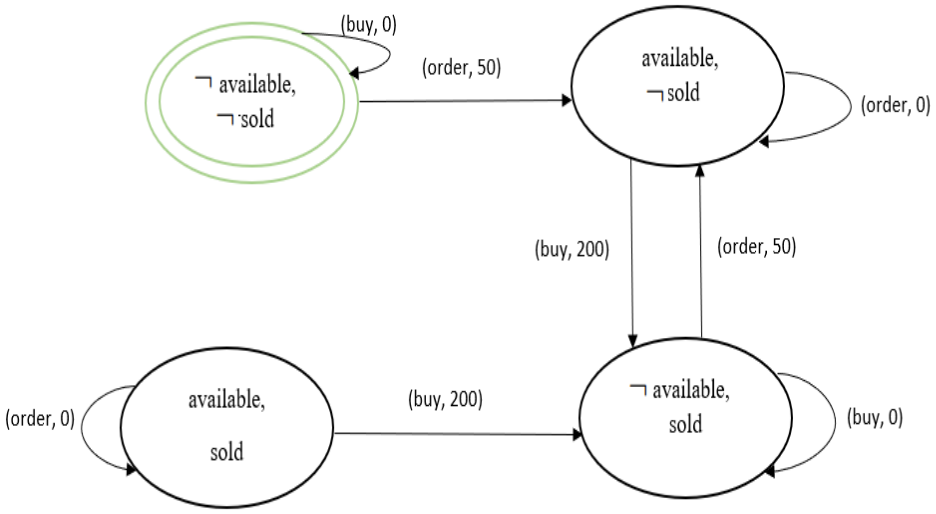

Figure 2: Example 02

4.3 Example 03

4.3.1 Description

There is a man. He can cook, eat, and play. Cooking makes food cooked. he can eat food if it is cooked. After eating he feels not hungry, and food is not cooked again. He can play. Playing makes him hungry. He just can play if he is not hungry. He just cooks when there is no food is cooked. Initially, he is hungry, and no food is cooked. In terms of energy, eating costs 5, cooking costs 15, playing costs 20.

4.3.2 Representation in language

Fluents: cooked, hungry.
Actions: cook, eat, play.

initially $\neg$cooked $\wedge$ hungry
cook causes cooked if $\neg$cooked
cook cost 15
eat causes ($\neg$cooked $\wedge$ $\neg$hungry) if cooked
eat cost 5
play causes hungry if $\neg$hungry
play cost 20

4.3.3 Calculation

$\sum = \{\sigma_0, \sigma_1, \sigma_2, \sigma_3\}$

$\sigma_0 = \{\neg$cooked, hungry$\}$
$\sigma_1 = \{$cooked, hungry$\}$
$\sigma_2 = \{\neg$cooked, $\neg$hungry$\}$
$\sigma_3 = \{$cooked, $\neg$hungry$\}$

$\Psi(\text{eat}, \sigma_0) = \sigma_0$
$\Psi(\text{cook}, \sigma_0) = \sigma_1$
$\Psi(\text{play}, \sigma_0) = \sigma_0$
$\Gamma(\text{eat}, \sigma_0) = 0$
$\Gamma(\text{cook}, \sigma_0) = 15$
$\Gamma(\text{play}, \sigma_0) = 0$

$\Psi(\text{eat}, \sigma_1) = \sigma_2$
$\Psi(\text{cook}, \sigma_1) = \sigma_1$
$\Psi(\text{play}, \sigma_1) = \sigma_1$
$\Gamma(\text{eat}, \sigma_1) = 5$
$\Gamma(\text{cook}, \sigma_1) = 0$
$\Gamma(\text{play}, \sigma_1) = 0$

$\Psi(\text{eat}, \sigma_2) = \sigma_2$
$\Psi(\text{cook}, \sigma_2) = \sigma_3$
$\Psi(\text{play}, \sigma_2) = \sigma_1$
$\Gamma(\text{eat}, \sigma_2) = 0$
$\Gamma(\text{cook}, \sigma_2) = 15$
$\Gamma(\text{play}, \sigma_2) = 20$

$\Psi(\text{eat}, \sigma_3) = \sigma_2$
$\Psi(\text{cook}, \sigma_3) = \sigma_3$
$\Psi(\text{play}, \sigma_3) = \sigma_1$
$\Gamma(\text{eat}, \sigma_3) = 5$
$\Gamma(\text{cook}, \sigma_3) = 0$
$\Gamma(\text{play}, \sigma_3) = 20$

4.3.4 Graph

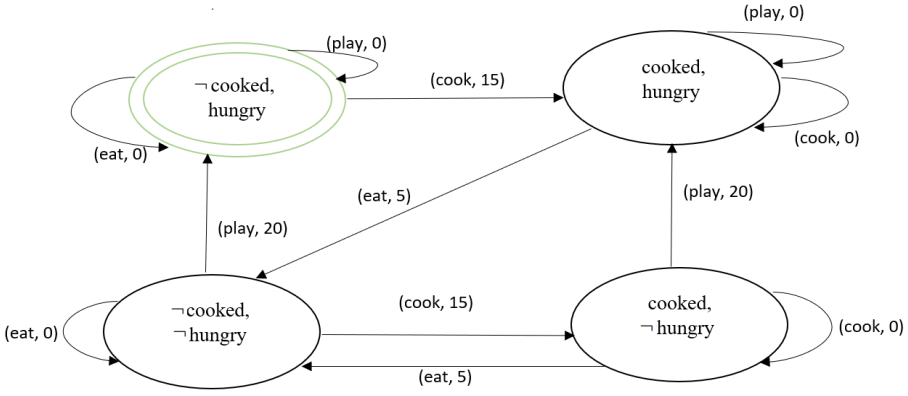

Figure 3: Example 03

5 Appendix

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