Warsaw University of Technology's
Faculty of Mathematics and Information Science

Knowledge Representation and Reasoning

Project number 2:
Deterministic Action With Cost
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1 Introduction

A dynamic system (DS) is viewed as

- a collection of objects, together with their properties, and
- a collection of actions which, while performed, change properties of objects (in consequence, the state of the world).

Let C2 be a class of dynamic systems satisfying the following assumptions:

1. Inertia law
2. Complete information about all actions and fluent.
3. Only Determinism
4. Only sequential actions are allowed.
5. Characterizations of actions:
   - Precondition represented by set of literals(a fluent or its negation);if a precondition does not hold, the action is executed but with empty effect
   - Postcondition (effect of an action) represented by a set of literals.
   - Cost $k \in N$ of an action, actions with empty effects cost 0. Each action has a fixed cost, if it leads to non-empty effects.
6. Effects of an action depends on the state where the action starts.
7. All actions are performed in all states.
8. Partial description of any state of the system are allowed.
9. No constraints are defined.

2 Syntax

2.1 Signature :

A signature is a triplet Υ = (F, Ac, K) where F is a set of fluents; Ac is a set of actions as follows A1,A2,...,An where Ai $\in$ An and i = 1 to n and K is a set of positive integers representing Cost of each action Ai $\in$ Ac as follows K1,k2,...Kn where Ki $\in$ K and i = 1 to n.

2.2 Literal :

A literal is either a fluent f or its negation ¬f.
Notation: for a fluent f ϵ F, we write $\overline{f}$ to denote the literal corresponding to f, i.e., either f or ¬f.

2.3 Statements :

The system and changes occurring within can be described through a sequence of statements defined in the table:

Statement	**Format**	**Description**
Effect Statement	A causes $\overline{f}$ if $\overline{g1}$, ... , $\overline{gk}$	If the action A is performed in any state satisfying $\overline{g1}$, ... , $\overline{gk}$, then in the resulting state $\overline{f}$ holds.
Value Statement	$\overline{f}$ after A1 . . . .An where $\overline{f}$ ∈ F and Ai ∈ Ac, for i = 1, ... ,n	$\overline{f}$ holds after performing the sequence A1 . . . .An of actions in the initial state.
Cost Statement	A costs k where k is a positive integer and k ∈ K	If the action A is performed in any state satisfying $\overline{g1}$, ...,$\overline{gk}$, then change in state results in fixed cost k.

Table 1: Syntax Table

3 Semantics

- A state is a mapping σ : F → {0, 1}. For any f ∈ F, if σ(f) = 1, then we say that f holds in σ and write $\sigma \models$ f. If σ(f) = 0, then we write $\sigma \models$ ¬f and say that f does not hold in σ. Let $\sum$ stand for the set of all states.

- A state transition function is a mapping Ψ : Ac × $\sum \rightarrow \sum$. For any $\sigma \in \sum$, for any A ∈ Ac Ψ(A , σ) is the state resulting from performing the action A in the state σ. Also Ψ(A,σ) will results in same state if the effect of action is empty.

- A cost transition function is a mapping Γ : Ac ×$\sum$ → K.For any $\sigma \in \sum$ and for any A ∈ Ac, Γ(A, σ) is the fixed cost k where k ∈ K, resulting from performing the action A in the state σ. Also Γ(A,σ) will results in 0 cost if there is no change in state.

- A transition function is generalized to the mapping Ψ^* : Ac* × $\sum \rightarrow \sum$ as follows:

1. Ψ^* (ε , σ) = σ,
2. Ψ^* ((A1, . . . , An), σ) = Ψ(An, Ψ^* (A1, . . . , An-1)).

- A cost transition function is generalized to the mapping Γ^* : Ac* $\times$ $\sum \to$ K as follows:

  1. Γ^* (ε , σ) = k,
  2. Γ^* ((A1, . . . , An), σ) = Γ(An, Γ^* (A1, . . . , An-1)).

- Let L be an action language of the class A over the signature Υ = (F, Ac, K). A structure for L is a triplet S = (Ψ, σ_0, Γ) where Ψ is a transition function, Γ is a cost transition function and $\sigma_0 \in \sum$ is the initial state

- Let S = (Ψ, σ_0, Γ) be a structure for L. A statement s is true in S, in symbols S $\models$ s, iff

  1. s is of the form $\overline{f}$ after A1, . . . , An, then Ψ(A1, . . . , An), σ_0) $\models \overline{f}$;
  2. if s is of the form A causes $\overline{f}$ if $\overline{g1}$, . . . , $\overline{gk}$, then for every $\sigma \in \sum$ such that $\sigma \models \overline{gj}$, j = 1, . . . , k, Ψ(A, σ) $\models \overline{f}$.
  3. if s is of the form A costs k, then every $\sigma \in \sum$ such that $\sigma \models \overline{gi}$, i = 1,...,k, Γ(A,σ) $\models$ k.

Let D be an action domain in the language L over the signature Υ=(F,Ac,K). A structure S = (Ψ,σ_0, Γ) is a model of D iff
(M1) for every statement s $\in$ D, S $\models$ s;
(M2) for every A $\in$ Ac for every f,g1,...,gn $\in$ F, for every k $\in$ K and for every $\sigma \in \sum$, if one of the following conditions holds:
(i) D contains an effect statement and a cost statement as follows:

- **A causes** $\overline{f}$ **if** $\overline{g1}$,...,$\overline{gk}$,$\sigma \not\models \overline{gi}$ for some i = 1,...,k
- **A costs k,** and $\sigma \not\models \overline{gi}$ for some i = 1,...,k.

(ii) D does not contain an effect statement but contains a 0 cost statement, as follows:

- **A causes** $\overline{f}$ **if** $\overline{g1}$,...,$\overline{gk}$ then $\sigma \not\models$ gi iff Ψ(A,σ) $\models$ f.
- **A costs 0**, and $\sigma \models$ f iff Γ(A,σ) = 0.

4 Examples

4.1 Example 01

4.1.1 Description

Andrew wants to travel by his car to a place.Travelling costs him 50$ if he uses fuel from the fuel tank of the car.If in case of emergency, Andrew is carrying a bottle of fuel as reserve, which can cost him 50$ for travelling. buying Fuel costs him 100$ if both fuel and reserve are empty. and buying causes both his Fuel and Reserve are filled.

4.1.2 Representation

Initially we have:

1. Fuel
2. Reserve

Travel causes ¬fuel if fuel Travel causes ¬reserve if ¬fuel ∨ reserve
Buy causes fuel, reserve if ¬ fuel ∨ reserve

4.1.3 Calculation

$\sum = \{ \sigma_0, \sigma_1, \sigma_2 \}$

$\sigma_0 = \{$ fuel, reserve $\}$ $\quad \sigma_1 = \{$ ¬fuel, reserve $\}$
$\sigma_2 = \{$ ¬fuel, ¬reserve $\}$

$\Psi(\text{Buy},\sigma_0)=\sigma_0 \quad \Psi(\text{Travel},\sigma_0)=\sigma_1$
$\Psi(\text{Buy},\sigma_1)=\sigma_1 \quad \Psi(\text{Travel},\sigma_1)=\sigma_2$
$\Psi(\text{Buy},\sigma_2)=\sigma_1 \quad \Psi(\text{Travel},\sigma_2)=\sigma_2$

4.1.4 Graph

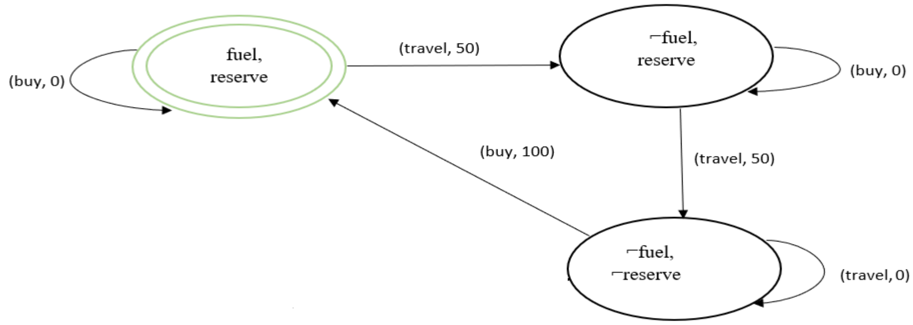

Figure 1: Example 01

4.2 Example 02

4.2.1 Description

John visits a painter to buy a specific painting. The cost of painting is 200\$ if its available in the shop. But if painting is not available then John needs to order a new one to be painted and will buy once its available. At any time only one copy of painting is available and another one to be ordered once sold.

4.2.2 Representation:

Fluents: available, sold.
Actions: BUY, ORDER.
BUY costs 200\$ **if** available
BUY costs 0\$ **if** ¬available
Order Cost: 0 \$
initially: ¬available ∧ ¬sold
BUY causes sold if available

ORDER causes available if ¬available
BUY causes ¬available

4.2.3 Calculation:

$\sum = \{ \sigma 0, \sigma 1, \sigma 2, \sigma 3\}$
$\sigma 0 = \{ \neg\text{available}, \neg\text{sold}\}$
$\sigma 1 = \{ \neg\text{available}, \text{sold}\}$
$\sigma 2 = \{ \text{available}, \neg\text{sold}\}$
$\sigma 3 = \{ \text{available}, \text{sold}\}$
Ψ (BUY, $\sigma 0$) = $\sigma 0$
Ψ (ORDER, $\sigma 0$) = $\sigma 1$
Ψ (BUY, $\sigma 1$) = $\sigma 2$
Ψ (ORDER, $\sigma 1$) = $\sigma 1$
Ψ (BUY, $\sigma 2$) = $\sigma 2$
Ψ (ORDER, $\sigma 2$) = $\sigma 1$
Ψ (BUY, $\sigma 3$) = $\sigma 2$
Ψ (ORDER, $\sigma 3$) = $\sigma 3$

4.2.4 Graph

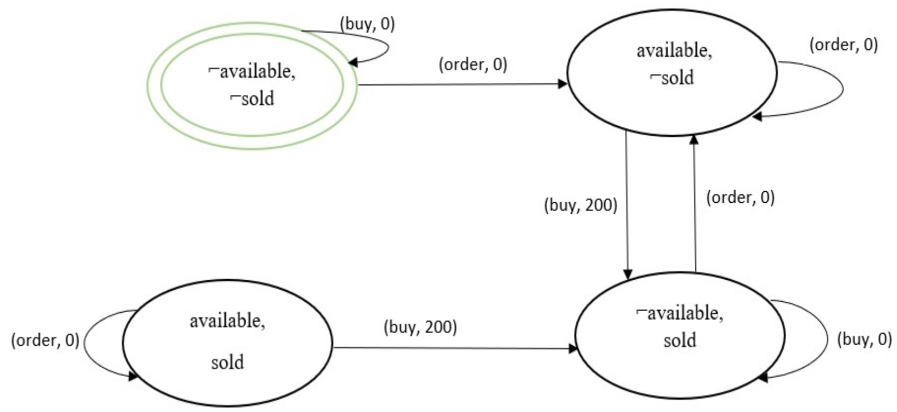

Figure 2: Example 02

4.3 Example 03

4.3.1 Description

There is a man. He can cook, eat, and play. Cooking makes food cooked. he can eat food if it is cooked. After eating he feels not hungry, and food is not cooked again. He can play. Playing makes him hungry. He just can play if he is not hungry. He just cooks when there is no food is cooked. Initially, he is hungry, and no food is cooked. In terms of energy, eating costs 5, cooking costs 15, playing costs 20.

4.3.2 Representation in language

Fluents: cooked, hungry.
Actions: cook, eat, play.

eat cost 5
cook cost 15
play cost 20
initially $\neg$cooked $\wedge$ hungry
cook causes cooked if $\neg$cooked
eat causes ($\neg$cooked $\wedge$ $\neg$hungry) if cooked
play causes hungry if $\neg$hungry

4.3.3 Calculation

$\sum = \{\sigma_0 ,\sigma_1, \sigma_2, \sigma_3\}$

$\sigma_0 = \{\neg\text{cooked, hungry}\}$
$\sigma_1 = \{\text{cooked, hungry}\}$
$\sigma_2 = \{\neg\text{cooked, }\neg\text{hungry}\}$
$\sigma_3 = \{\text{cooked, }\neg\text{hungry}\}$

$\psi(\text{eat}, \sigma_0) = \sigma_0$
$\psi(\text{cook}, \sigma_0) = \sigma_1$
$\psi(\text{play}, \sigma_0) = \sigma_0$

$\psi(\text{eat}, \sigma_1) = \sigma_2$
$\psi(\text{cook}, \sigma_1) = \sigma_1$
$\psi(\text{play}, \sigma_1) = \sigma_1$

$\psi(\text{eat}, \sigma_2) = \sigma_2$
$\psi(\text{cook}, \sigma_2) = \sigma_3$
$\psi(\text{play}, \sigma_2) = \sigma_1$

$\psi(\text{eat}, \sigma_3) = \sigma_2$
$\psi(\text{cook}, \sigma_3) = \sigma_3$
$\psi(\text{play}, \sigma_3) = \sigma_1$

4.3.4 Graph

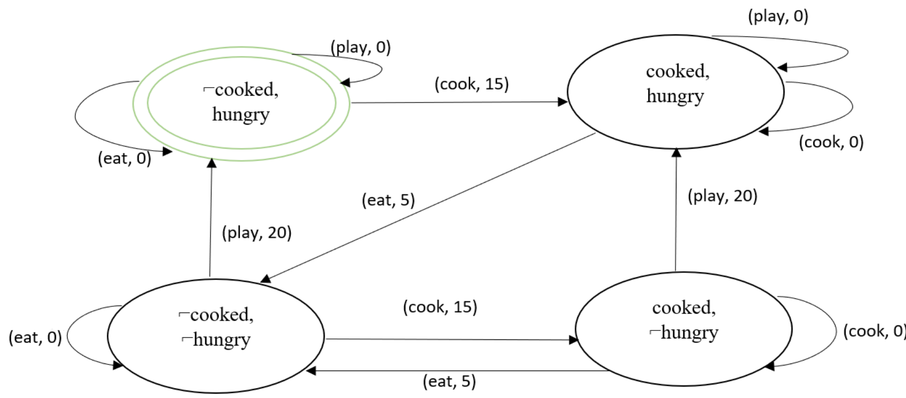

Figure 3: Example 03

5 Appendix

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