Sandwich

$a_n \le b_n \le c_n$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = l$

then $\lim_{n\to\infty} b_n = l$

For each $\varepsilon > 0$; $\exists$ a +ve int m

s.t $|s_n - l| \le \epsilon$, $\forall n \ge m$ (limit of convergence)

Geometric series

$|r| < 1 \Rightarrow$ converges

$|r| \ge 1$ diverges ; $r \le -1$ oscillates

$\Sigma \frac{1}{n} \Rightarrow$ diverging

P-test:-

$\sum_{n=1}^{\infty} \frac{1}{n^P} = \frac{1}{1^P} + \frac{1}{2^P} + \cdots$

① converges for $P > 1$

② diverges for $P \le 1$

comparision test

1) Σu_n and Σv_n

(i) Σv_n converges

(ii) $u_n \le v_n$ $\forall n$

then Σu_n also converges

2) Σu_n and Σv_n

(i) Σv_n diverges

(ii) $u_n \ge v_n$ $\forall n$

then Σu_n also diverges

D'Alembert Ratio test

Ratio form :- Σu_n

$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lambda$

$\lambda < 1 \Rightarrow$ converges

$\lambda > 1 \Rightarrow$ diverges

$\lambda = 1 \Rightarrow$ test fail

Leibnitz's Series

(i) Alternating series

(ii) $\lim_{n\to\infty} u_n = 0$

$u_{n+1} < u_n$

then Σu_n converges

Limit form (comp. test)

Σu_n and Σv_n

$\lim_{n\to\infty} \frac{u_n}{v_n} = k \ (\neq 0)$

then Σu_n and Σv_n

converges or diverges

Absolutely convergent

$u_1 + u_2 \cdots + u_n$

$|u_1| + |u_2| + \cdots + |u_n|$

both convergent then

Absolutely

cauchy's root test

$\lim_{n\to\infty} (u_n)^{1/n} = \lambda$

$\lambda < 1 \Rightarrow$ converges

$\lambda > 1 \Rightarrow$ diverges

$\lambda = 1 \Rightarrow$ test fail's

Conditionally

$\Sigma |u_n| \Rightarrow$ divergent

Σu_n is convergent

then conditionally convergent.

$\xi \to$ Eta

ξ contains infinite number of

$\{s_n\}$

$\varepsilon > 0$

$|s_n - \xi| < \varepsilon$ $\forall n$

limit point

Numerical Analysis

Shift operator $E\,(f(x_i)) = f(x_i+h)$ $E^2 f(x_i) = f(x_i+2h)$	Forward difference (Δ) $\Delta f(x_i) = f(x_i+h) - f(x_i)$ $\Delta^2 f(x_i) = f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)$

$\boxed{\Delta = E - 1}$, $\boxed{\nabla = 1 - E^{-1}}$

$\nabla f(x_i) = f(x_i) - f(x_i - h)$

$\nabla^2 f(x_i) = f(x_i) - 2f(x_{i-1}) + f(x_{i-2})$

$\delta f(x_i) = f(x_i + \frac{h}{2}) - f(x_i - h/2)$ central difference

$\boxed{\delta = E^{1/2} - E^{-1/2}}$

Mean
$\mu f(x_i) = \frac{1}{2}[f_{i+1/2} + f_{i-1/2}]$

* $\Delta^n f_i = \nabla^n f_{i+n} = \delta^n f_{i+n/2}$

$\Delta^k P_n(x) = 0 \quad k > n$

$= a_0\, n!\ , \quad k = n$

$\Delta^3 [(1-2x)(1-3x)(1-4x)] \Rightarrow -24 \times 3!$

NFDI :- (for interpolation) (beginning nodal points)

$f(x) = f(x_0) + (x-x_0)\frac{\Delta f_0}{(1!)h} + (x-x_0)(x-x_1)\frac{\Delta^2 f_0}{(2!)h^2} + \cdots$

$(x-x_0)(x-x_1) \cdots (x-x_{n-1})\frac{\Delta^n f_0}{(n!)h^n}$

NBDI :-

$f(x) = f(x_n) + \frac{(x-n)}{1!\,h}\nabla f(x_n) + \frac{(x-n)(x-x_{n-1})}{2!\,h^2}\nabla^2 f(x_n) + \cdots$

$\cdots \frac{(x-n)(x-x_{n-1})\cdots(x-x_0)}{n!\,h^n}\nabla^n f(x_n)$

central diff (book)

$h = \frac{b-a}{N}$

Trapezoidal Rule:

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\{f(x_1) + f(x_2) + \dots f(x_{n-1})\} + f(x_n)\right]$$

Simpson's 1/3rd Rule:

$$I = \int_a^b f(x)\,dx \qquad h = \frac{b-a}{2}$$

$$= \frac{b-a}{6}\left[f(a) + 4\,f\left(\frac{a+b}{2}\right) + f(b)\right]$$

composite Simpson's

$$\int_a^b f(x)\cdot dx \qquad h = \frac{b-a}{2N}$$

$$\frac{h}{3}\left[\left[f(x_0) + 4f(x_1) + f(x_2)\right] + \left(f(x_2) + 4f(x_3) + f(x_4)\right] \dots f(x_{2N-2}) + 4f(x_{2N-1}) + f(x_{2N})\right]$$

Simple Root:

α is simple root of $f(x)=0$ if $f(\alpha) = 0$ and $f'(\alpha) \neq 0$ (or)

$f(x) = (x-\alpha)\cdot g(x)$; $g(\alpha) \neq 0$

Multiple Root

$f(x) = 0$

$f(\alpha) = 0,\ f'(\alpha) = 0 \ \dots\ f^{(m-1)}\ \alpha = 0$

and $f^{m}(\alpha) \neq 0$

$f(x) = (x-\alpha)^m\, g(x)$; $g(\alpha) \neq 0$

Bolzano's theorem (Iterative Method)

$f(3)\cdot f(4) < 0$

n	a_n (-ve)	b_n (+ve)	$x_{n+1} = \frac{a_n + b_n}{2}$	$f(x_{n+1})$
0	3	4	3.5	$f(3.5) = -0.5441$
1	3.5	4		

⋮

Regula falsi, Newton Raphson Method

① $x_0 = 0 \qquad x_1 = 1$

$f_0 = f(x_0) = f(0) = 1$

$f_1 = f(x_1) = f(1) = -1$

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0} = \frac{0-1}{-1-1} = 0.5$$

$f(x_2) = f(0.5) = -0.375$

$f(0.75)$

② $(0, 0.5)$

0.0625   0.125 = 0.5

$f(0.5) = (0.5)^4 + 4(0.5)^3 + 6(0.5)^2 - 0.5 - 3$

0.25

1.5

$\frac{2.5}{6} - \frac{13}{36}\ \frac{25}{12}$

Central Difference Interpolation :-

$$f(x) = f(x_a + kh)$$

$$= \left[1 + k\mu\delta + \frac{k^2}{2!}\delta^2 + \frac{k(k-1)(k+1)}{3!}\mu\delta^3 + \cdots\right] f(x_a)$$

Bolzano's theorem

Iterative Method

$f(x) = 0$ root in (a, b)

$f(a) \cdot f(b) < 0$

Regula - falsi :-

starting (x_0, x_1) in which root lies

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

Newton - Raphson :- (Tangent Method)

$x_0 \rightarrow$ initial approx

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \; ; \quad f'(x_k) \neq 0$$

Error

$\left.\begin{array}{l} P = 1 \\ P = 1 \end{array}\right\}$ Bisection, Regula falsi } Linear rate.

Newton- Raphson } Qudratic Rate