

# Machine Learning

## Convolutional Neural Network

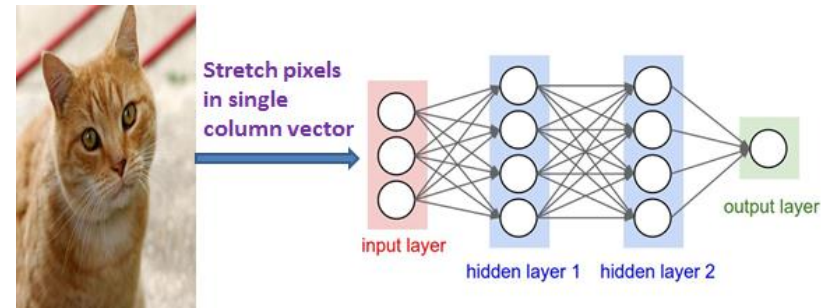
Indian Institute of Information Technology  
Sri City, Chittoor



# This Class

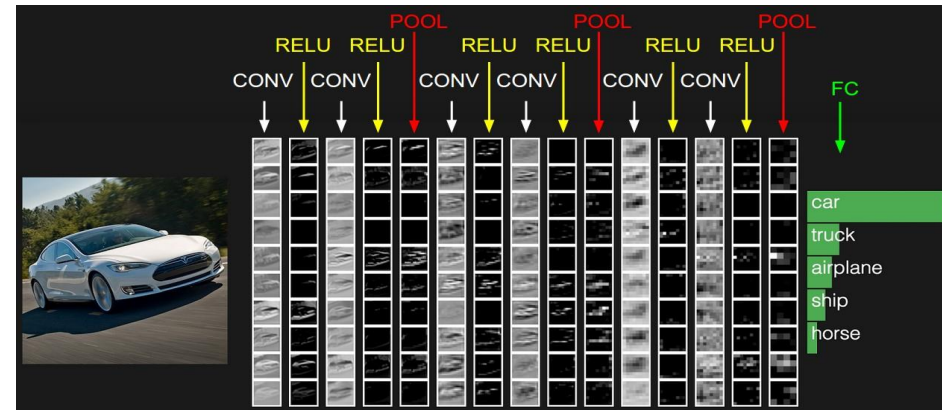
- Neural Network and Image

- Dimensionality
- Local relationship



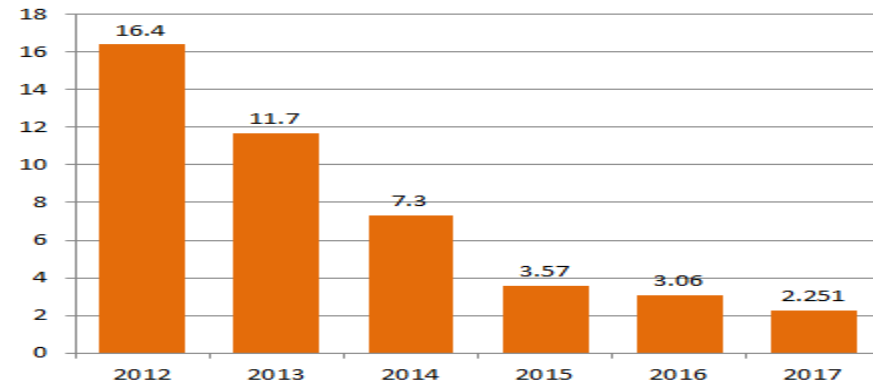
- Convolutional Neural Network (CNN)

- Convolution Layer
- Non-linearity Layer
- Pooling Layer
- Fully Connected Layer
- Classification Layer

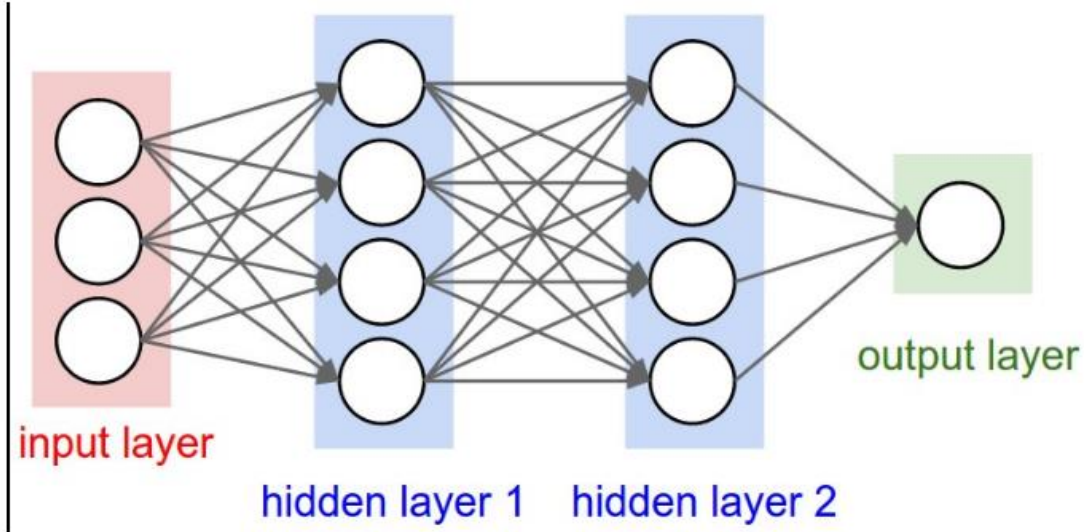
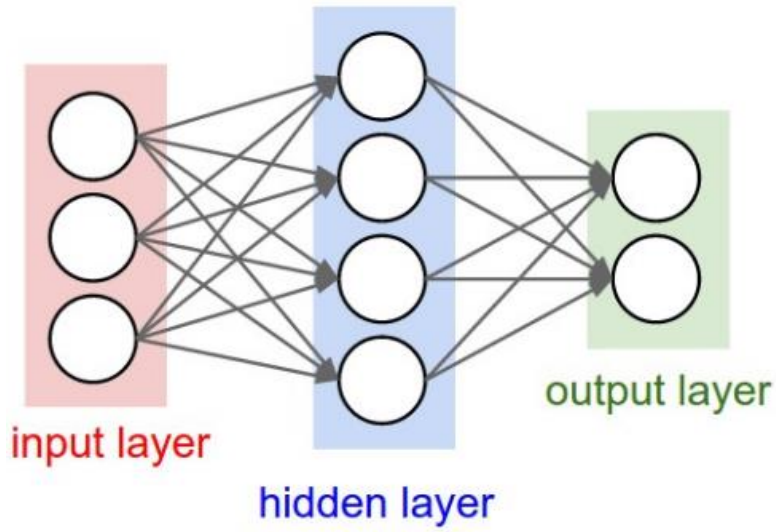


- ImageNet Challenge

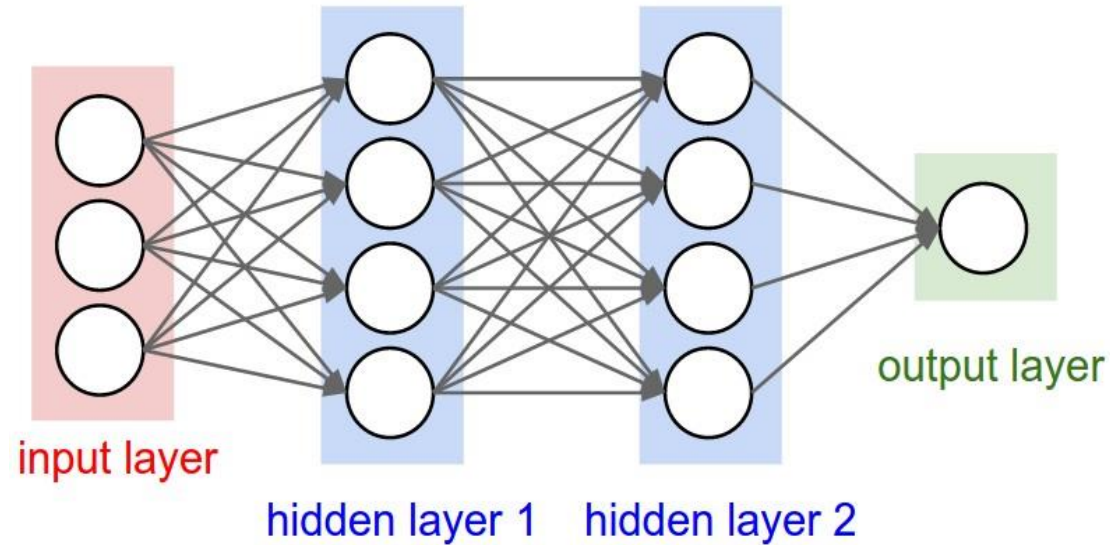
- Progress
- Human Level Performance



# Neural Networks



# Multi-layer Neural Network & Image

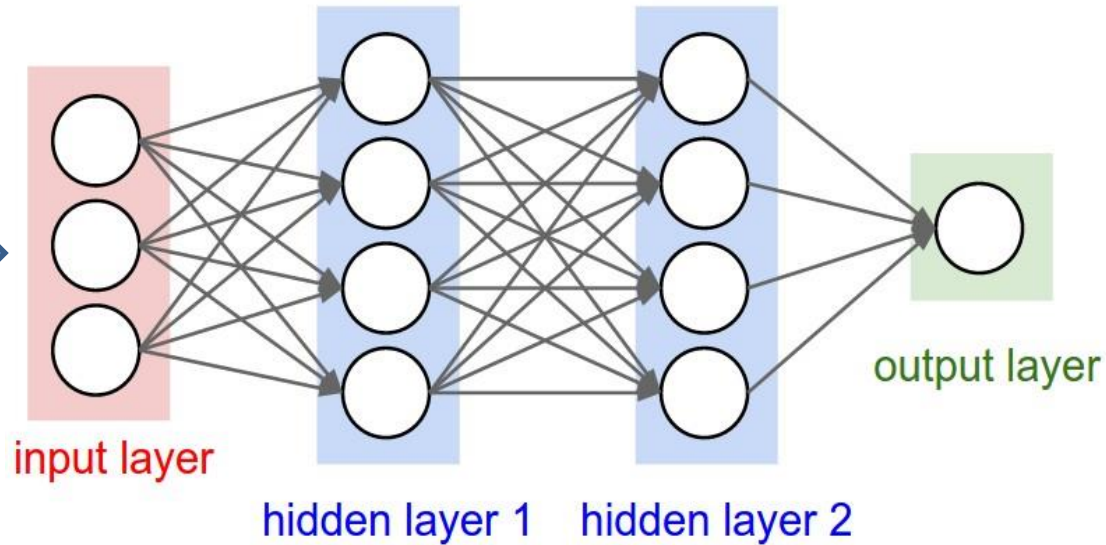


**How to apply NN over Image?**

# Multi-layer Neural Network & Image



Stretch pixels  
in single  
column vector

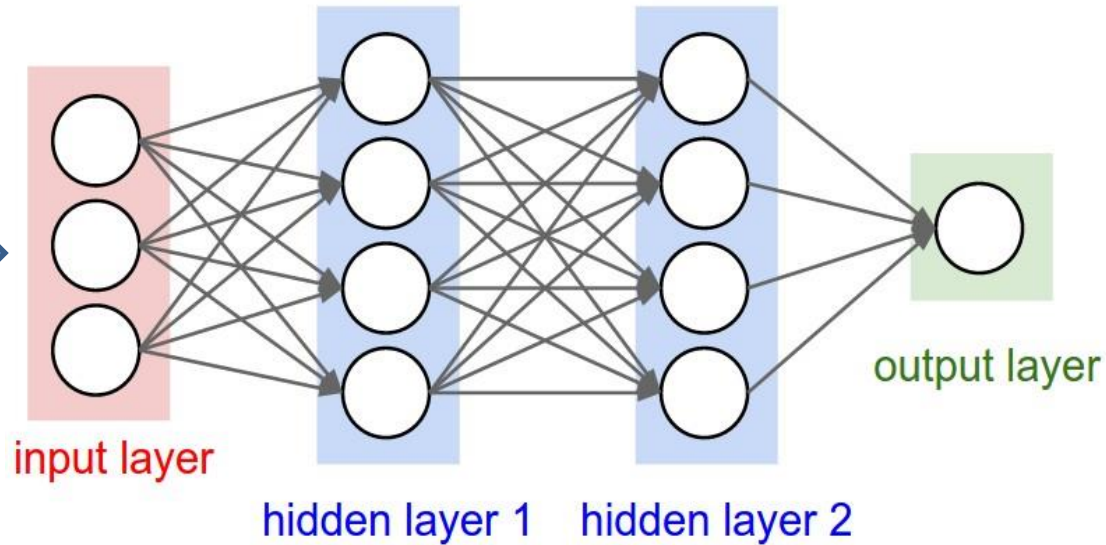




# Multi-layer Neural Network & Image

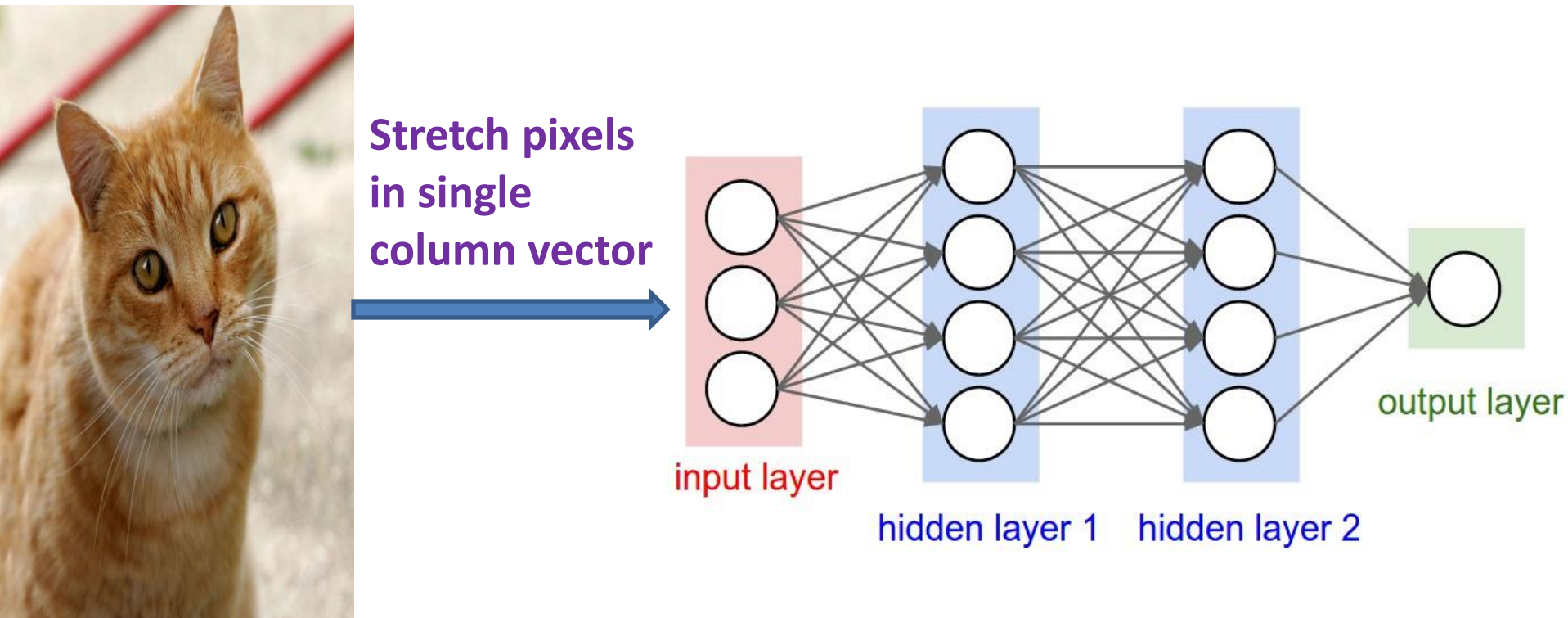


Stretch pixels  
in single  
column vector



## Problems ?

# Multi-layer Neural Network & Image

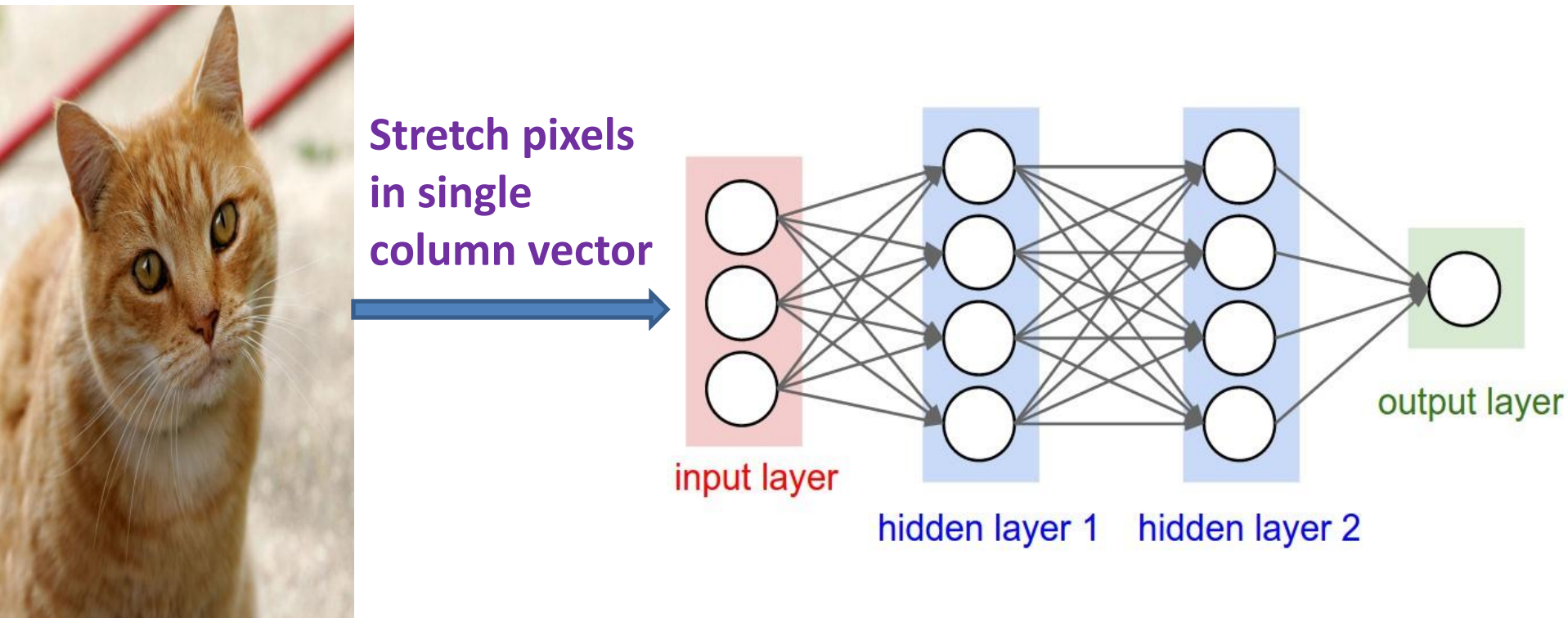


## Problems:

High dimensionality

Local relationship

# Multi-layer Neural Network & Image



**Problems:**

**Solution ?**

**High dimensionality**

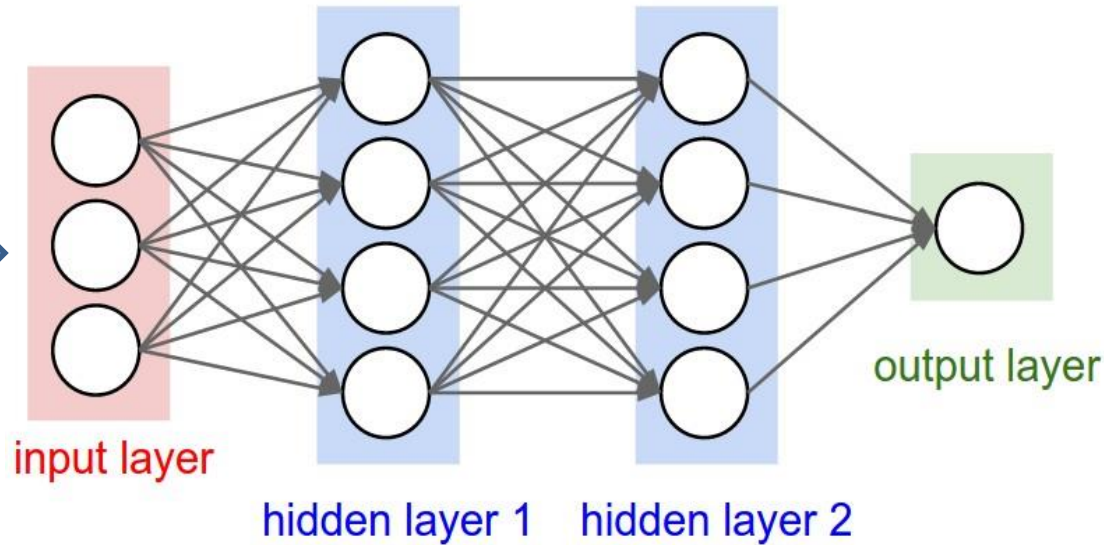
**Local relationship**



# Multi-layer Neural Network & Image



Stretch pixels  
in single  
column vector



## Problems:

High dimensionality

Local relationship

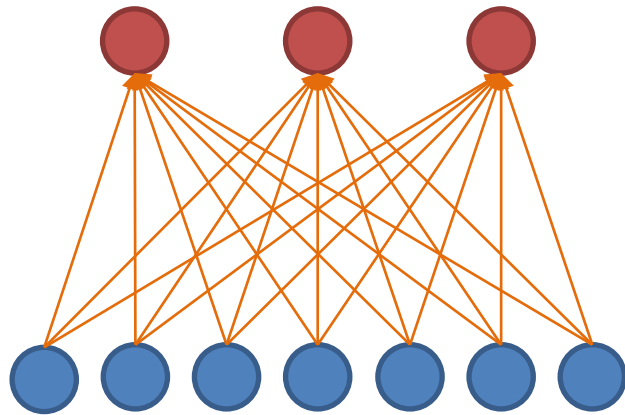
## Solution:

Convolutional Neural Network

# Convolutional Neural Networks

- Also known as  
CNN,  
ConvNet,  
DCN
- CNN = a multi-layer neural network with
  1. Local connectivity
  2. Weight sharing

# CNN: Local Connectivity

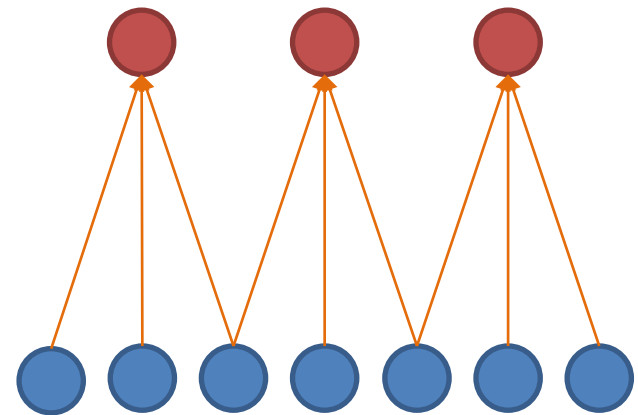


Hidden layer

Input layer

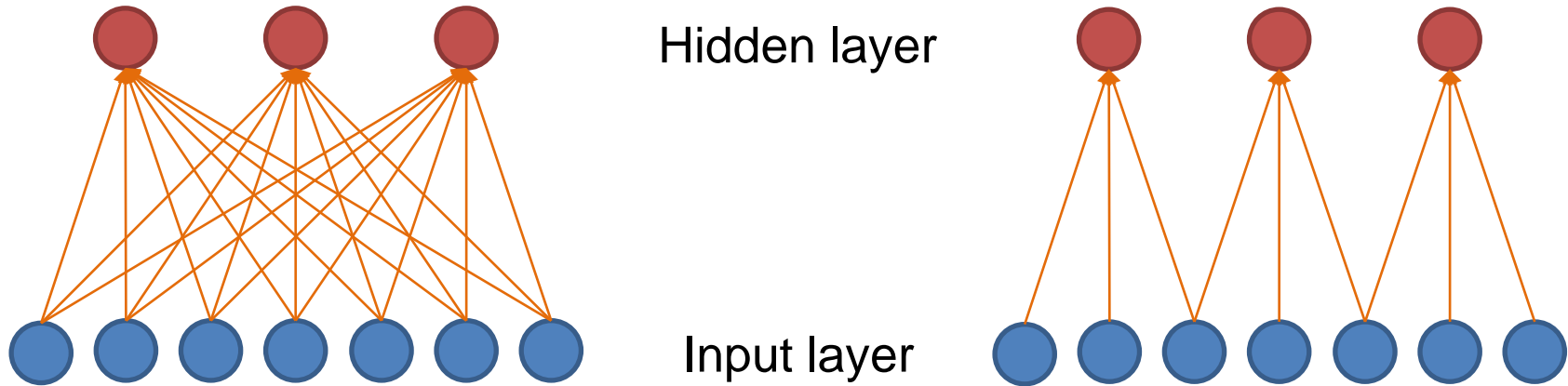
**Global** connectivity

- # input units (neurons): 7
- # hidden units: 3



**Local** connectivity

# CNN: Local Connectivity

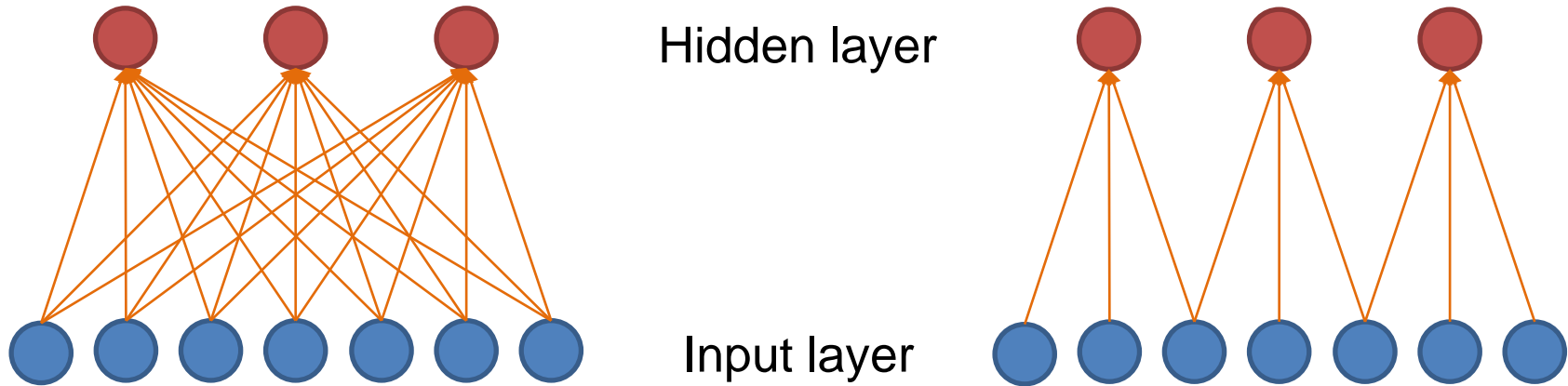


## Global connectivity

- # input units (neurons): 7
- # hidden units: 3
- **Number of parameters**
  - Global connectivity: ?
  - Local connectivity: ?

## Local connectivity

# CNN: Local Connectivity



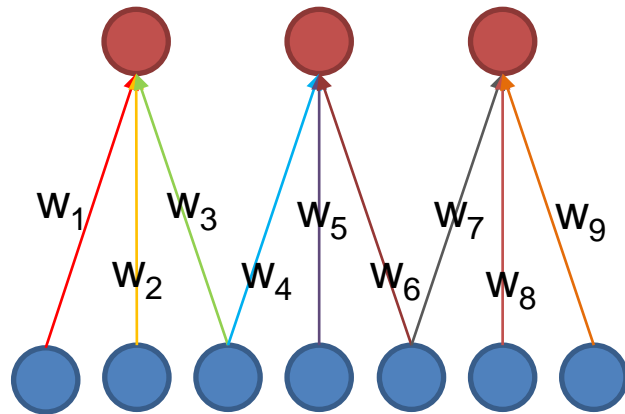
## Global connectivity

- # input units (neurons): 7
- # hidden units: 3
- **Number of parameters**
  - Global connectivity:  **$3 \times 7 = 21$**
  - Local connectivity:  **$3 \times 3 = 9$**

## Local connectivity



# CNN: Weight Sharing

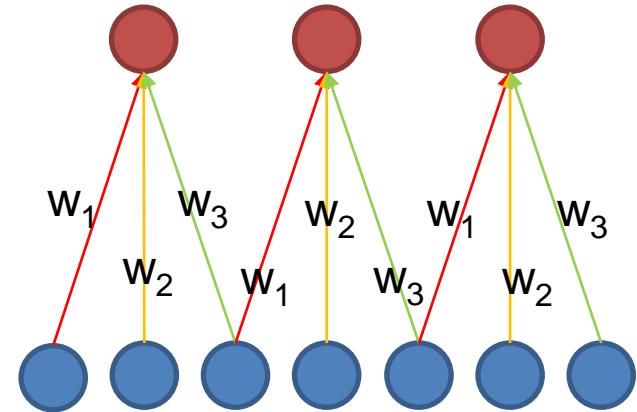


**Without** weight sharing

- # input units (neurons): 7
- # hidden units: 3

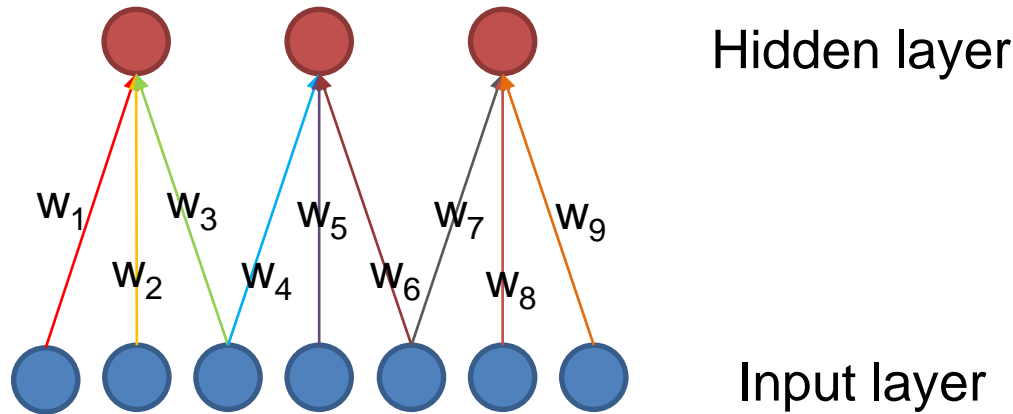
Hidden layer

Input layer

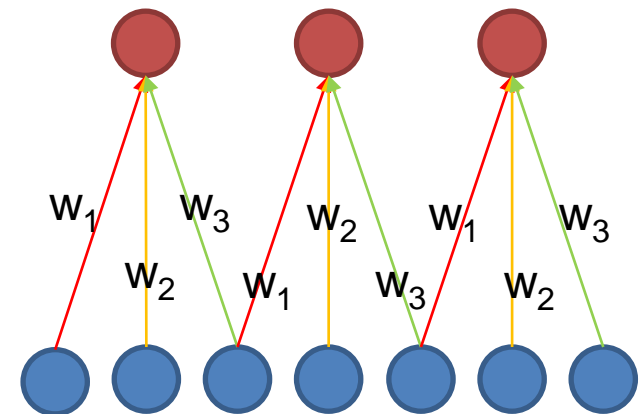


**With** weight sharing

# CNN: Weight Sharing



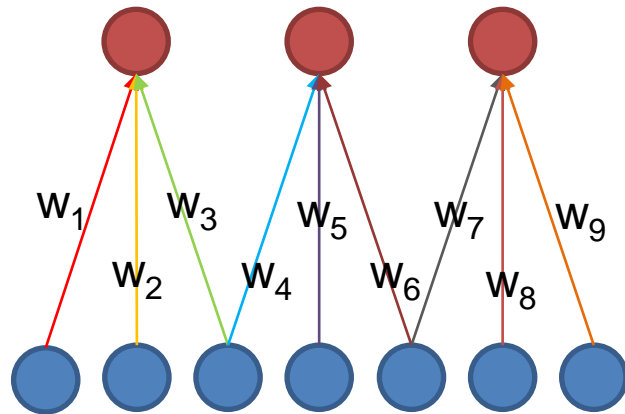
**Without** weight sharing



**With** weight sharing

- # input units (neurons): 7
- # hidden units: 3
- **Number of parameters**
  - Without weight sharing: ?
  - With weight sharing : ?

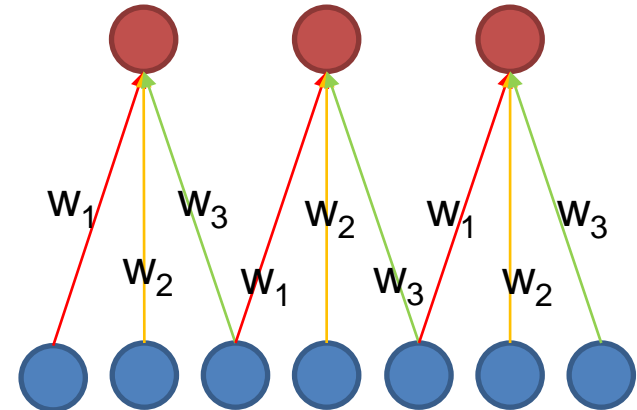
# CNN: Weight Sharing



**Without** weight sharing

Hidden layer

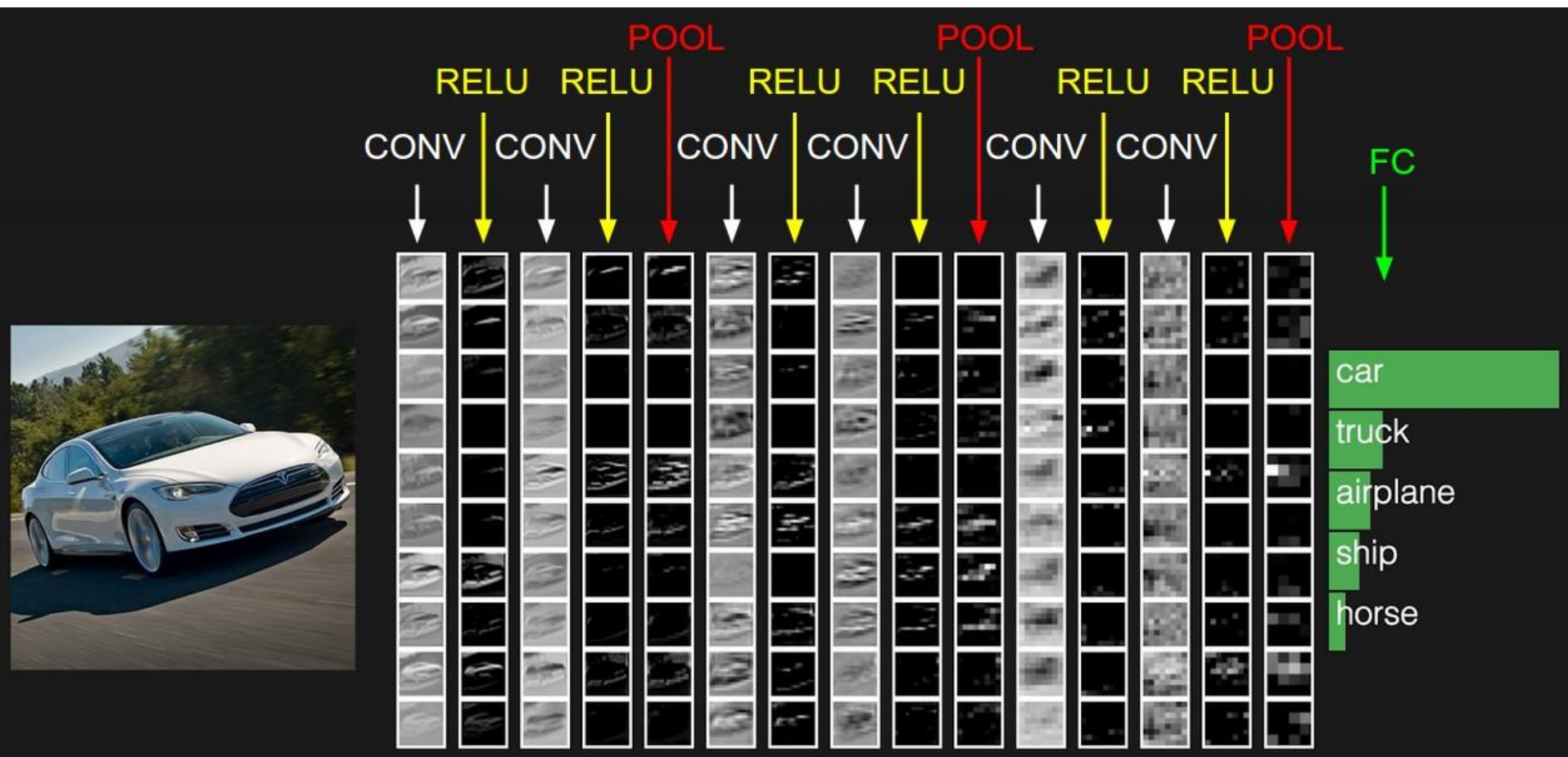
Input layer



**With** weight sharing

- # input units (neurons): 7
- # hidden units: 3
- **Number of parameters**
  - Without weight sharing:  $3 \times 3 = 9$
  - With weight sharing :  $3 \times 1 = 3$

# Convolutional Neural Networks



# Layers used to build ConvNets

**Input Layer** (Input image)

**Convolutional Layer**

**Non-linearity Layer** (such as Sigmoid, Tanh, ReLU, PReLU, ELU, Swish, etc.)

**Pooling Layer** (such as Max Pooling, Average Pooling, etc.)

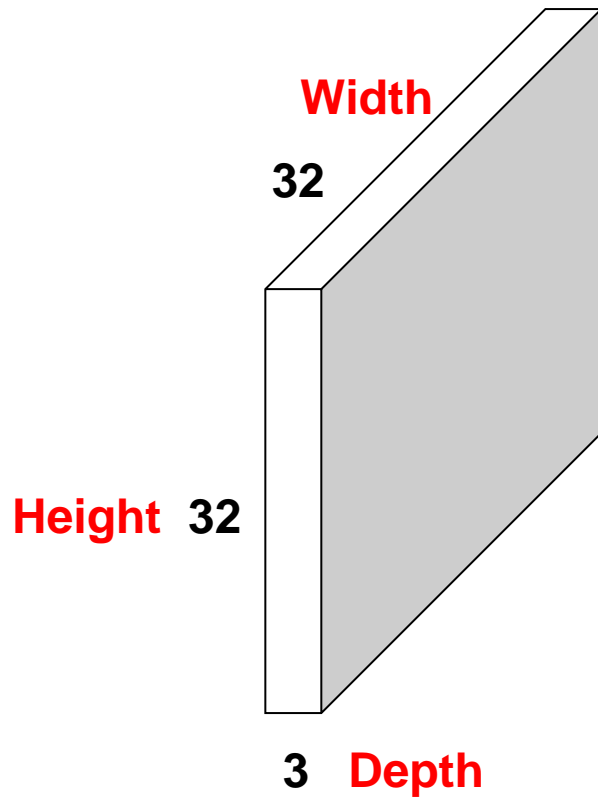
**Fully-Connected Layer**

**Classification Layer** (Softmax, etc.)



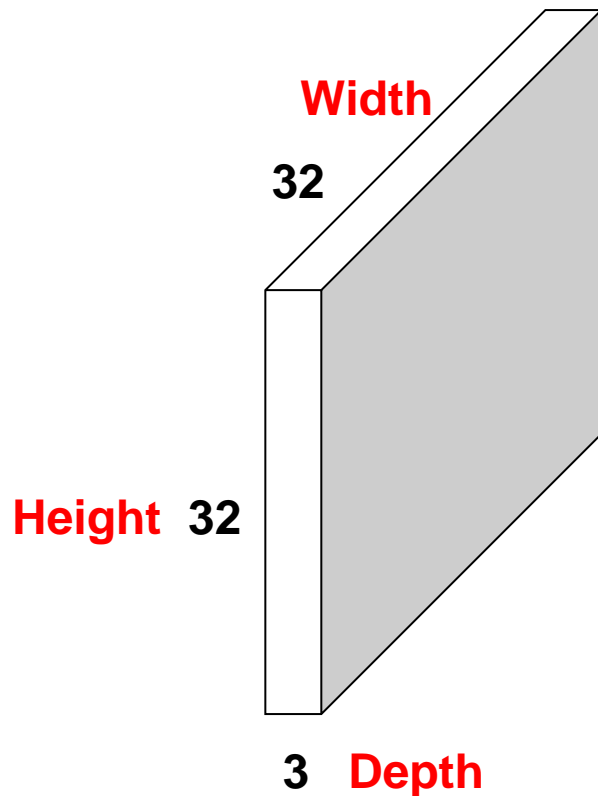
# Convolutional Layer

**$32 \times 32 \times 3$  Image** -> preserve spatial structure



# Convolutional Layer

$32 \times 32 \times 3$  Image



$5 \times 5 \times 3$  Filter



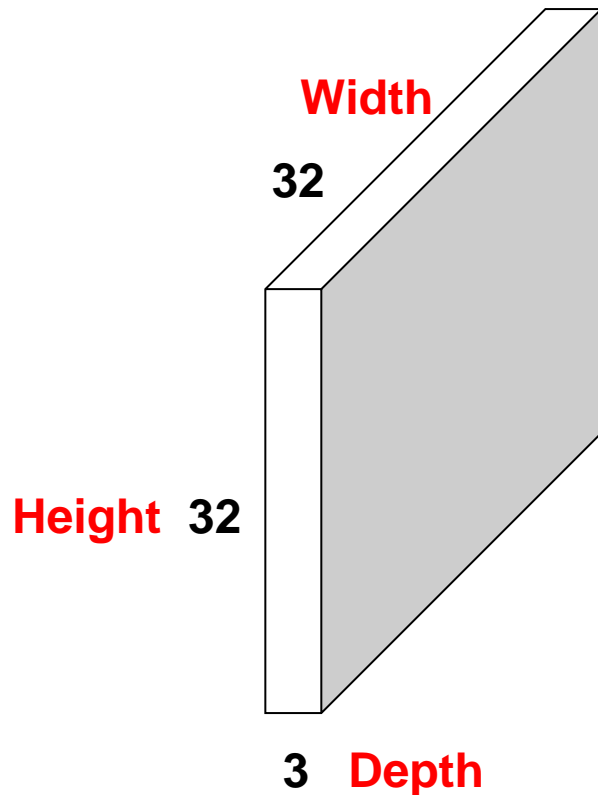
**Convolve** the filter with the image i.e.  
“slide over the image spatially, computing  
dot products”

# Convolutional Layer

## Handling multiple input channels

Filters always extend the full depth of the input volume

$32 \times 32 \times 3$  Image



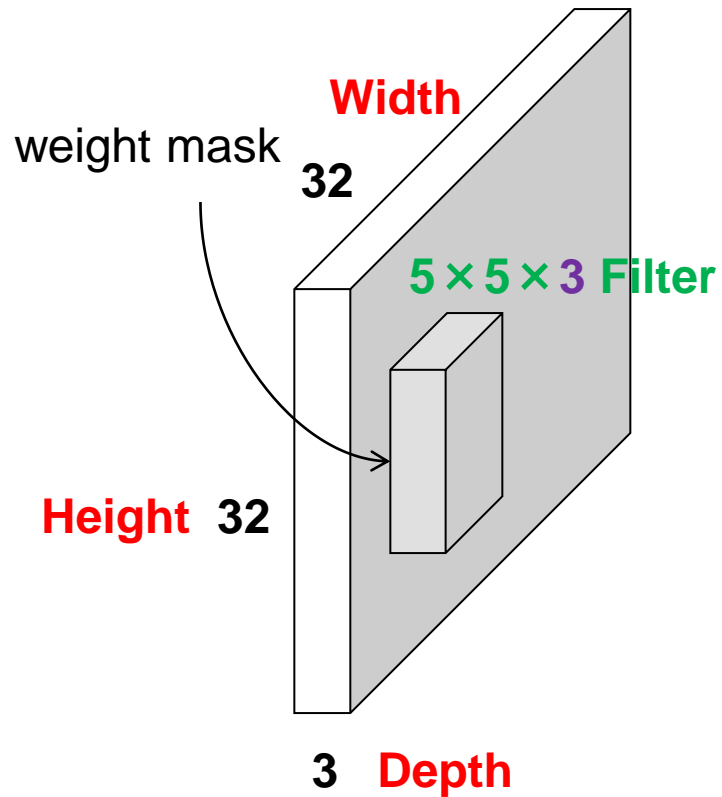
$5 \times 5 \times 3$  Filter



**Convolve** the filter with the image i.e.  
“slide over the image spatially, computing dot products”

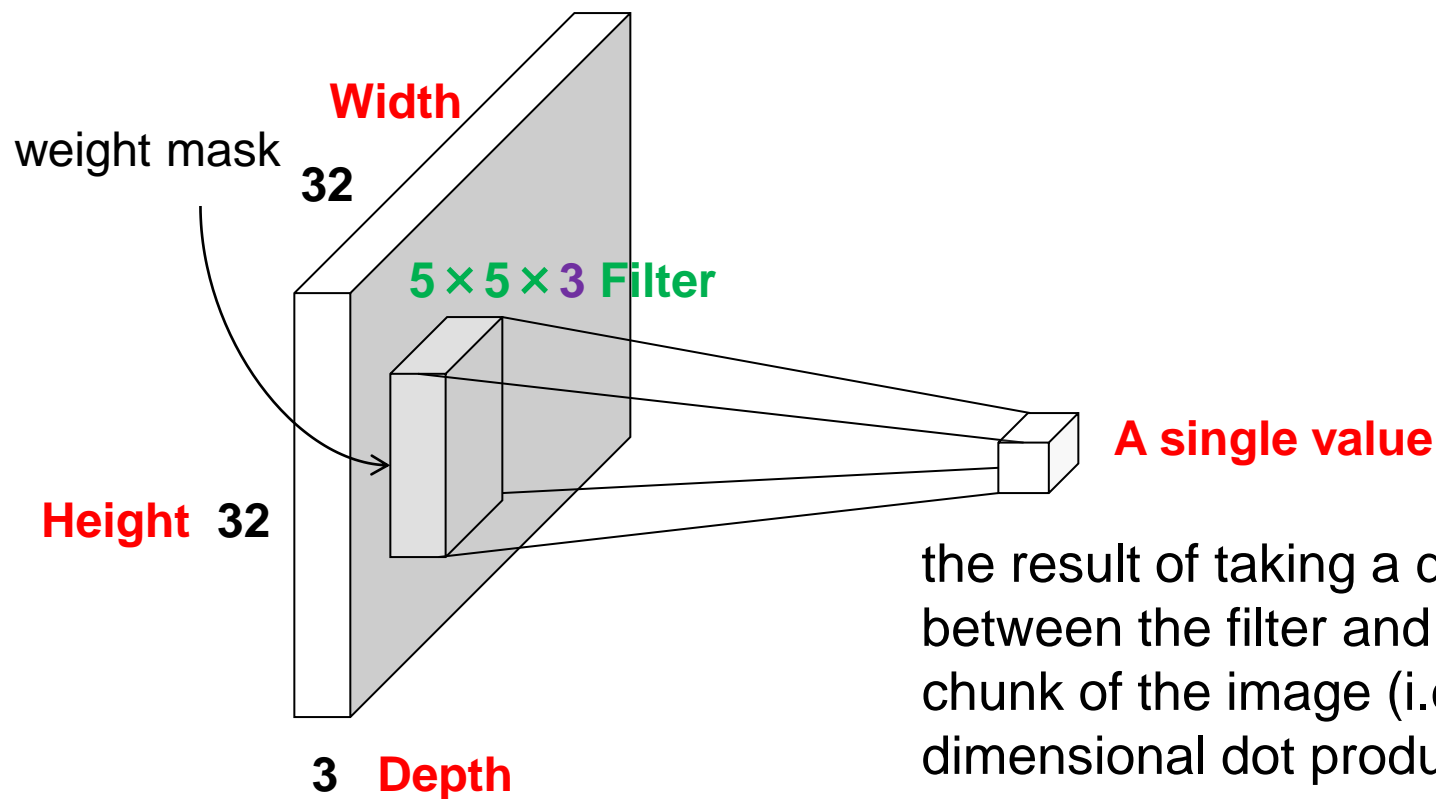
# Convolutional Layer

$32 \times 32 \times 3$  Image



# Convolutional Layer

$32 \times 32 \times 3$  Image

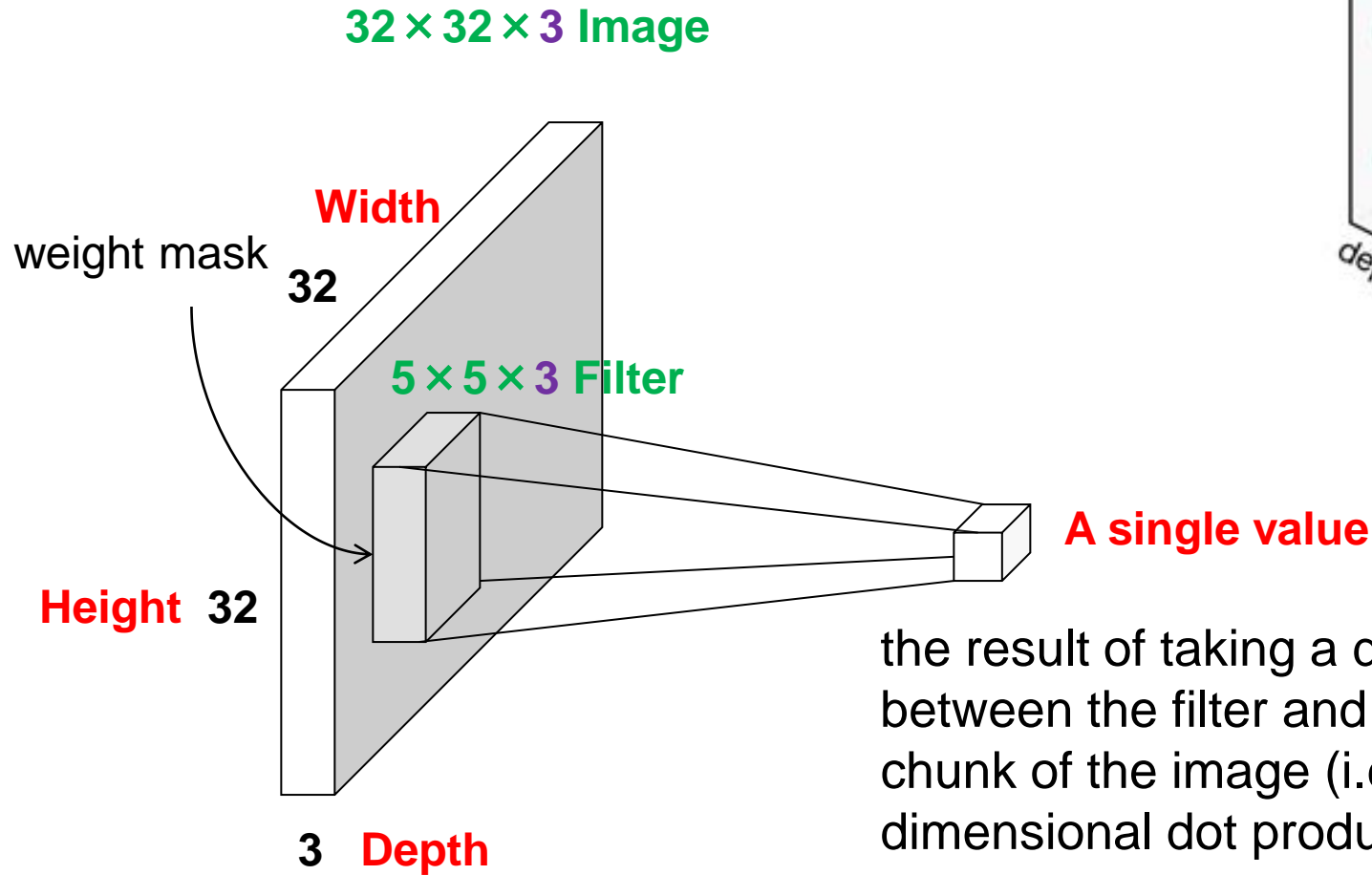


the result of taking a dot product  
between the filter and a small  $5 \times 5 \times 3$   
chunk of the image (i.e.  $5 \times 5 \times 3 = 75$ -  
dimensional dot product + bias)

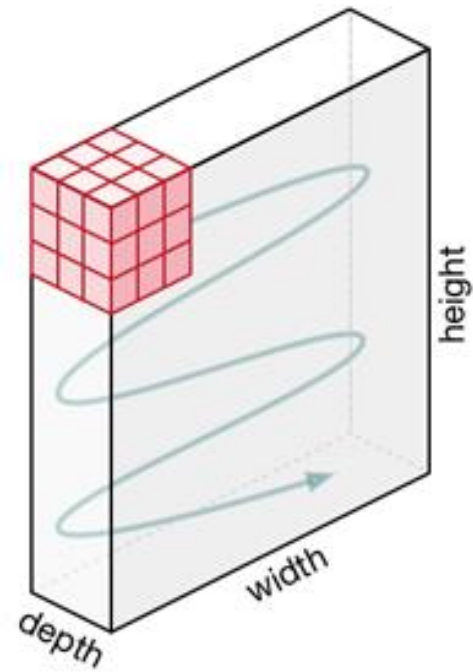
$$\mathbf{w}^T \cdot \mathbf{x} + \mathbf{b}$$



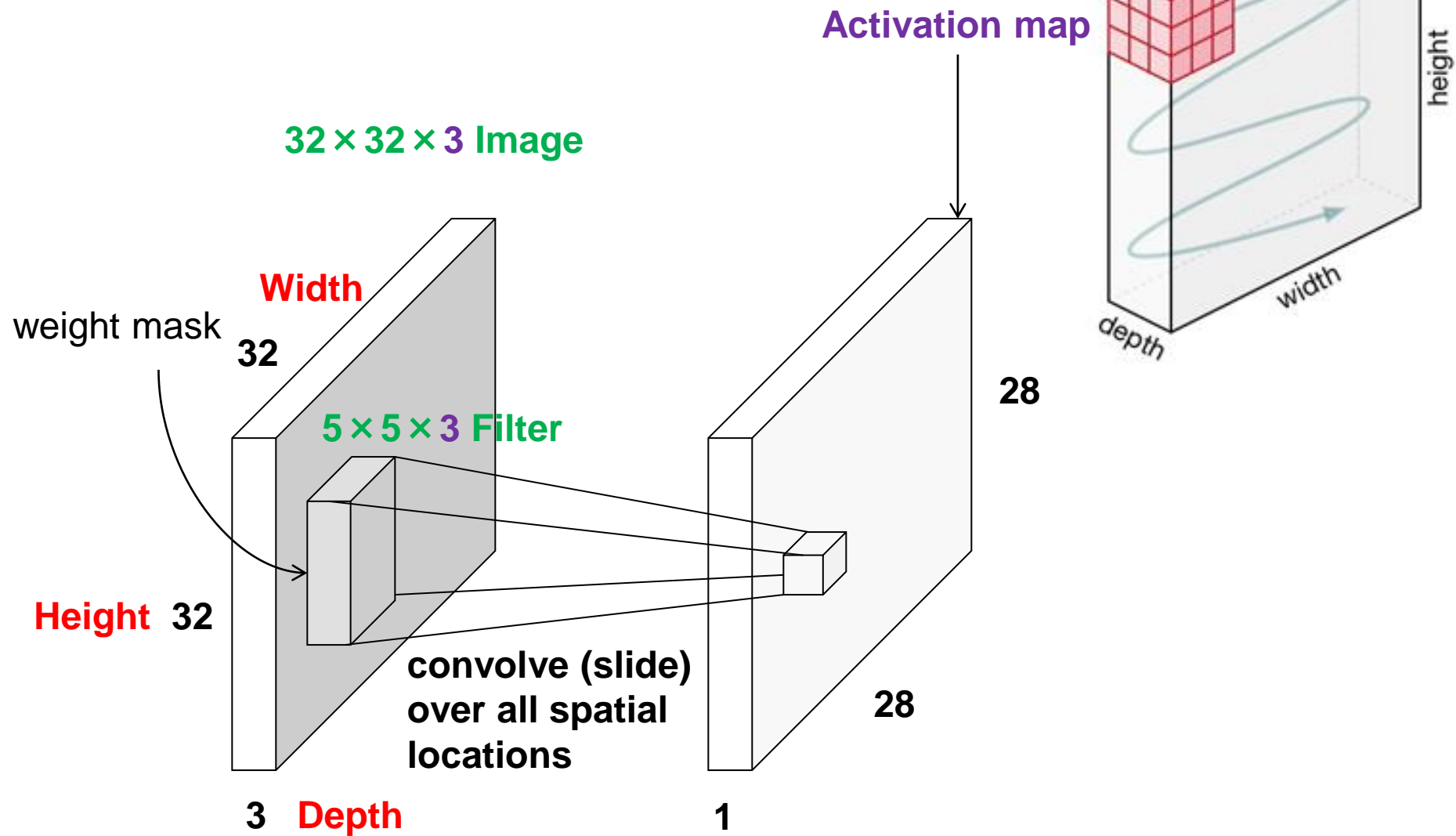
# Convolutional Layer



$$\mathbf{w}^T \cdot \mathbf{x} + \mathbf{b}$$



# Convolutional Layer

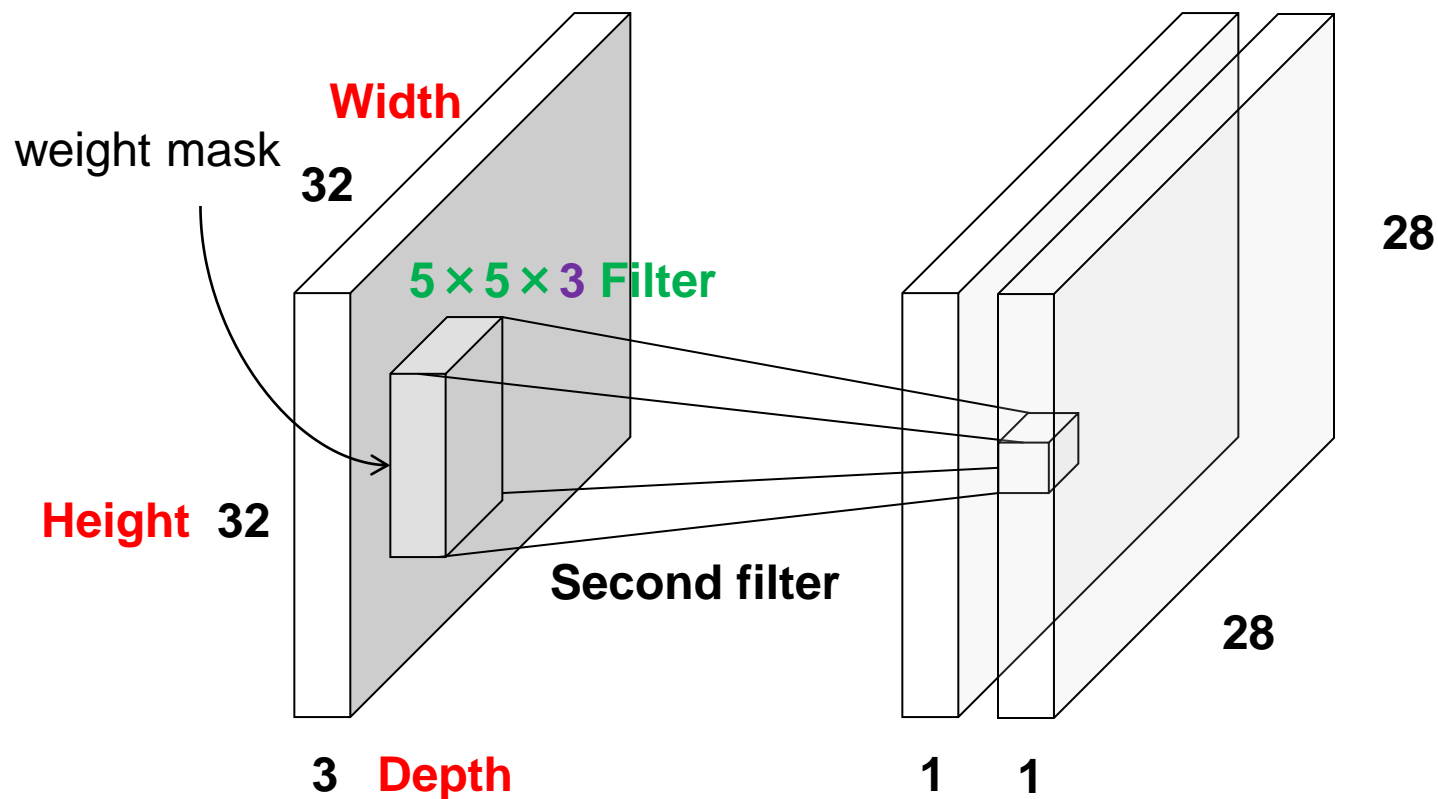


# Convolutional Layer

## Handling multiple output maps

$32 \times 32 \times 3$  Image

Activation maps

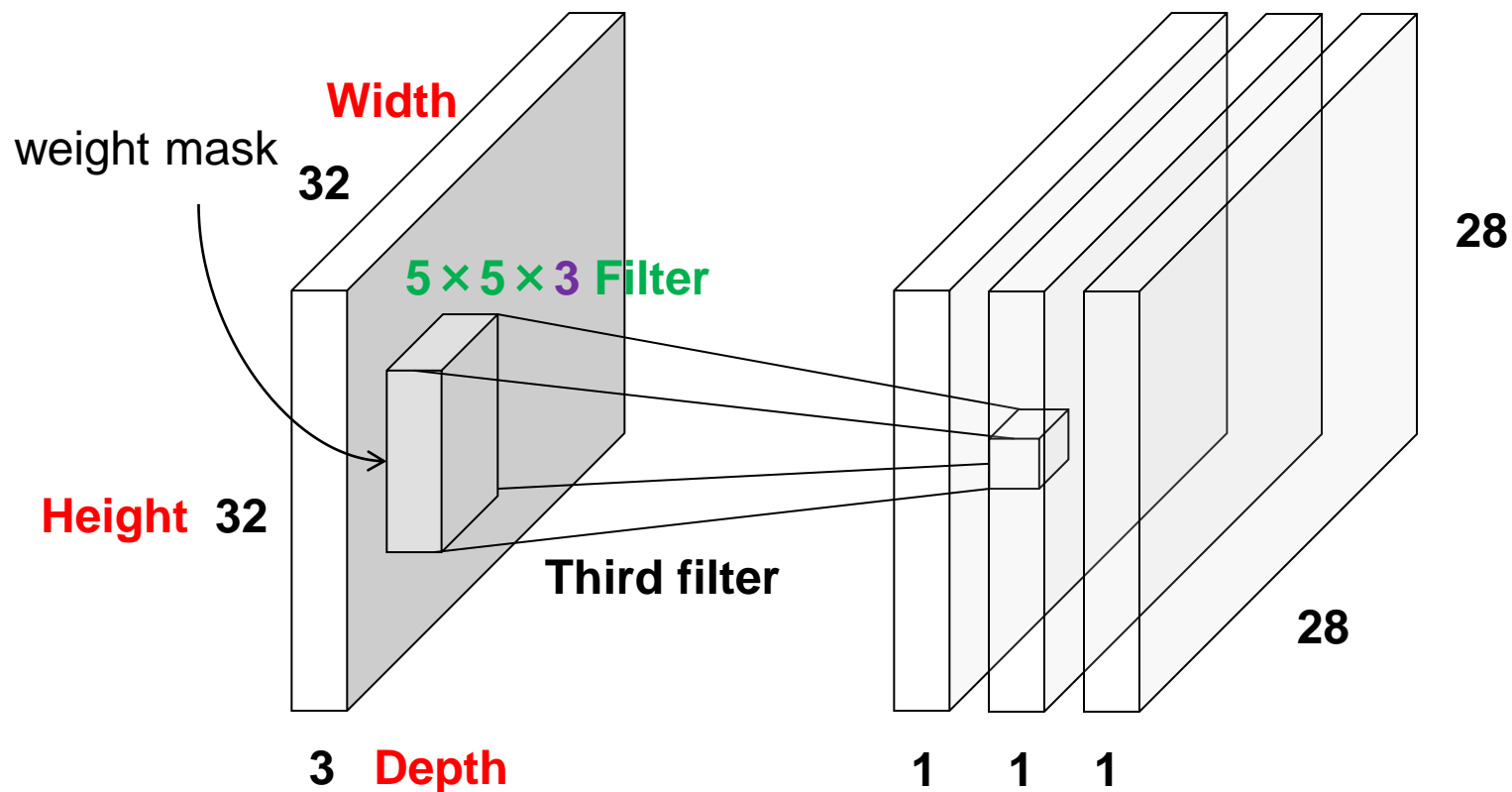


# Convolutional Layer

## Handling multiple output maps

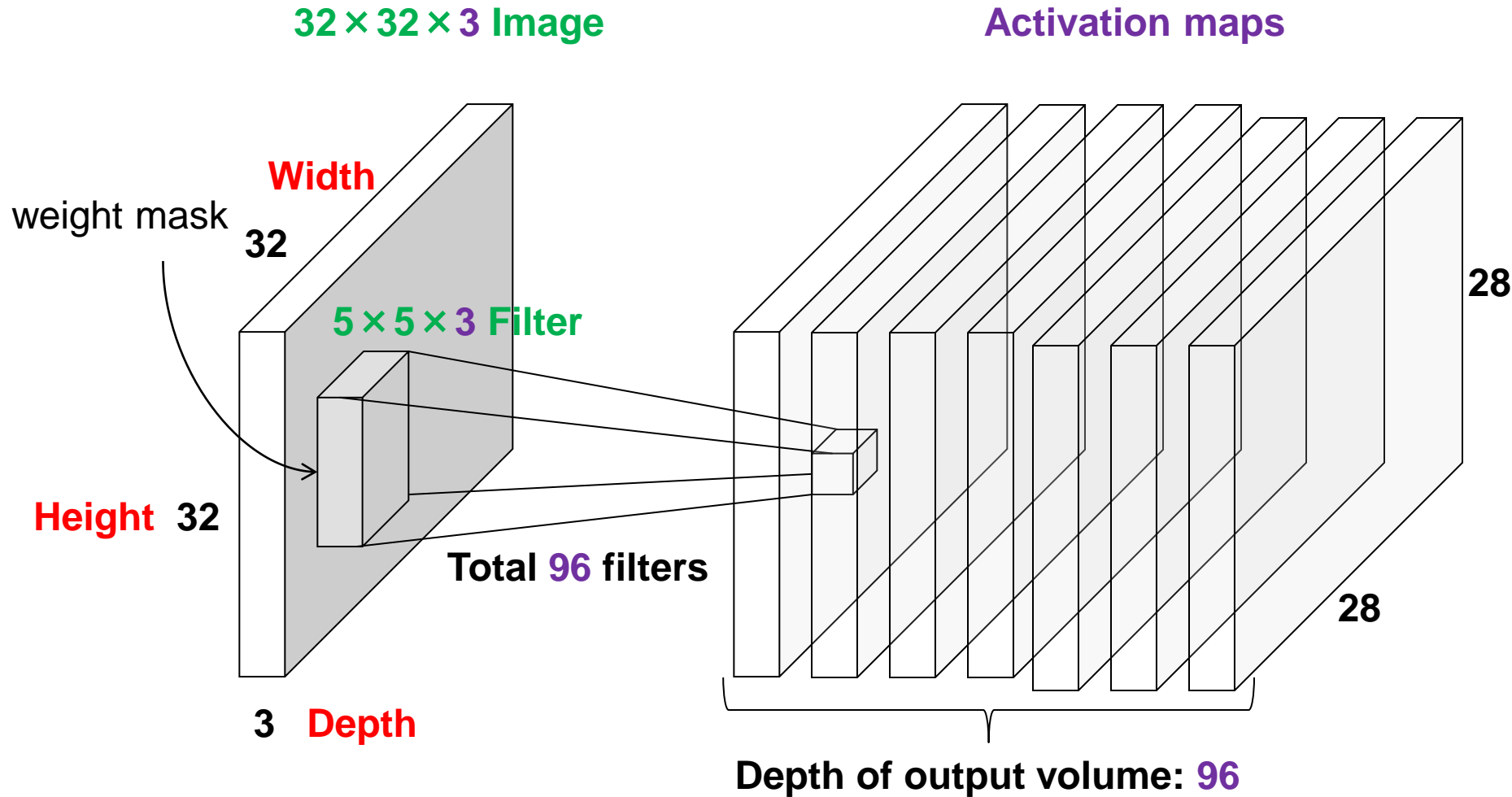
$32 \times 32 \times 3$  Image

Activation maps



# Convolutional Layer

## Handling multiple output maps





Input Volume (+pad 1) (7x7x3)

 $x[:, :, 0]$ 

0	0	0	0	0	0	0
0	0	0	2	0	2	0
0	1	2	2	2	0	0

0	0	2	1	0	0	0
0	0	2	0	2	1	0
0	1	0	2	2	0	0
0	0	0	0	0	0	0

 $x[:, :, 1]$ 

0	0	0	0	0	0	0
0	1	0	0	2	0	0
0	1	0	1	0	1	0

0	0	0	2	1	2	0
0	1	2	1	0	0	0
0	1	2	2	0	2	0
0	0	0	0	0	0	0

 $x[:, :, 2]$ 

0	0	0	0	0	0	0
0	0	2	2	1	0	0
0	0	2	0	0	1	0

0	1	1	1	2	1	0
0	2	0	1	2	2	0
0	2	1	1	0	0	0
0	0	0	0	0	0	0

Filter W0 (3x3x3)

 $w0[:, :, 0]$ 

-1	0	-1
1	-1	1
-1	0	-1

 $w0[:, :, 1]$ 

0	-1	1
1	1	1
-1	1	-1

 $w0[:, :, 2]$ 

0	0	0
-1	1	-1
1	1	-1

Bias b0 (1x1x1)

 $b0[:, :, 0]$ 

1
---

Filter W1 (3x3x3)

 $w1[:, :, 0]$ 

0	1	1
-1	-1	-1
1	-1	1

 $w1[:, :, 1]$ 

-1	0	-1
1	1	1
-1	1	-1

 $w1[:, :, 2]$ 

0	1	0
0	0	-1
1	-1	0

Bias b1 (1x1x1)

 $b1[:, :, 0]$ 

0
---

Output Volume (3x3x2)

 $o[:, :, 0]$ 

-3	-1	0
-1	-8	2
3	0	3

 $o[:, :, 1]$ 

1	4	2
-1	4	5
3	1	3

toggle movement

Input Volume (+pad 1) (7x7x3)

$x[:, :, 0]$

0	0	0	0	0	0	0
0	0	0	2	0	2	0
0	1	2	2	2	0	0
0	0	2	1	0	0	0
0	0	2	0	2	1	0
0	1	0	2	2	0	0
0	0	0	0	0	0	0

$x[:, :, 1]$

0	0	0	0	0	0	0
0	1	0	0	2	0	0
0	1	0	1	0	1	0
0	0	0	2	1	2	0
0	1	2	1	0	0	0
0	1	2	2	0	2	0
0	0	0	0	0	0	0

$x[:, :, 2]$

0	0	0	0	0	0	0
0	0	2	2	1	0	0
0	0	2	0	0	1	0
0	1	1	1	2	1	0
0	2	0	1	2	2	0
0	2	1	1	0	0	0
0	0	0	0	0	0	0

Filter W0 (3x3x3)

$w0[:, :, 0]$

-1	0	-1
1	-1	1
-1	0	-1

$w0[:, :, 1]$

0	-1	1
1	1	1
-1	1	-1

$w0[:, :, 2]$

0	0	0
-1	1	-1
1	1	-1

Bias b0 (1x1x1)

$b0[:, :, 0]$

1
---

Filter W1 (3x3x3)

$w1[:, :, 0]$

0	1	1
-1	-1	-1
1	-1	1

$w1[:, :, 1]$

-1	0	-1
1	1	1
-1	1	-1

$w1[:, :, 2]$

0	1	0
0	0	-1
1	-1	0

Bias b1 (1x1x1)

$b1[:, :, 0]$

0
---

Output Volume (3x3x2)

$o[:, :, 0]$

-3	-1	0
-1	-8	2
3	0	3

$o[:, :, 1]$

1	4	2
-1	4	5
3	1	3

toggle movement

Input Volume (+pad 1) (7x7x3)

 $x[:, :, 0]$ 

0	0	0	0	0	0	0
0	0	0	2	0	2	0
0	1	2	2	2	0	0
0	0	2	1	0	0	0
0	0	2	0	2	1	0
0	1	0	2	2	0	0
0	0	0	0	0	0	0

 $x[:, :, 1]$ 

0	0	0	0	0	0	0
0	1	0	0	2	0	0
0	1	0	1	0	1	0
0	0	0	2	1	2	0
0	1	2	1	0	0	0
0	1	2	2	0	2	0
0	0	0	0	0	0	0

 $x[:, :, 2]$ 

0	0	0	0	0	0	0
0	0	2	2	1	0	0
0	0	2	0	0	1	0
0	1	1	1	2	1	0
0	2	0	1	2	2	0
0	2	1	1	0	0	0
0	0	0	0	0	0	0

Filter W0 (3x3x3)

 $w0[:, :, 0]$ 

-1	0	-1
1	-1	1
-1	0	-1

 $w0[:, :, 1]$ 

0	-1	1
1	1	1
-1	1	-1

 $w0[:, :, 2]$ 

0	0	0
-1	1	-1
1	1	-1

Bias b0 (1x1x1)

 $b0[:, :, 0]$ 

1
---

Filter W1 (3x3x3)

 $w1[:, :, 0]$ 

0	1	1
-1	-1	-1
1	-1	1

 $w1[:, :, 1]$ 

-1	0	-1
1	1	1
-1	1	-1

 $w1[:, :, 2]$ 

0	1	0
0	0	-1
1	-1	0

Bias b1 (1x1x1)

 $b1[:, :, 0]$ 

0
---

Output Volume (3x3x2)

 $o[:, :, 0]$ 

-3	-1	0
-1	-8	2
3	0	3

 $o[:, :, 1]$ 

1	4	2
-1	4	5
3	1	3

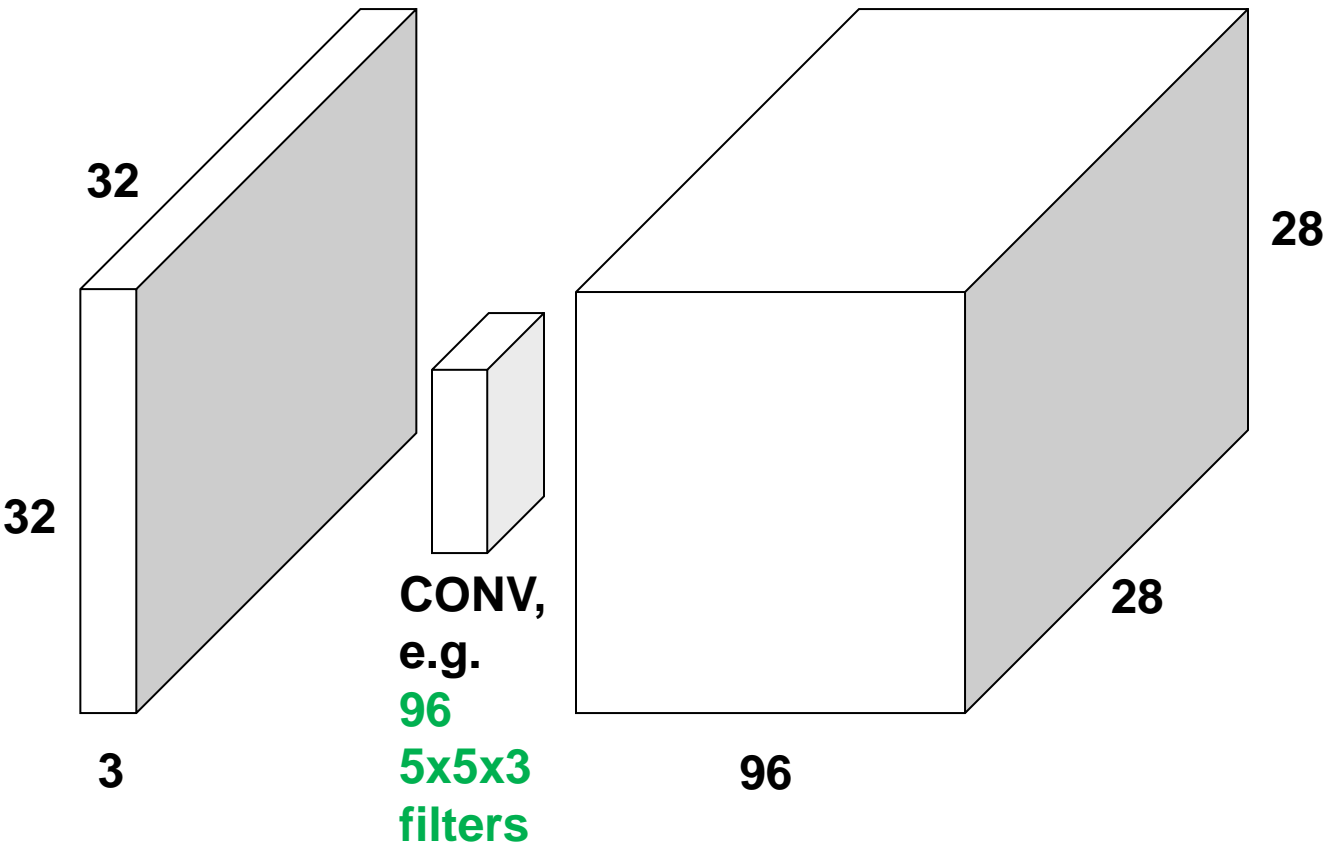
toggle movement

# Convolutional Layer

**Preview:** ConvNet is a sequence of Convolution Layers, interspersed with activation functions

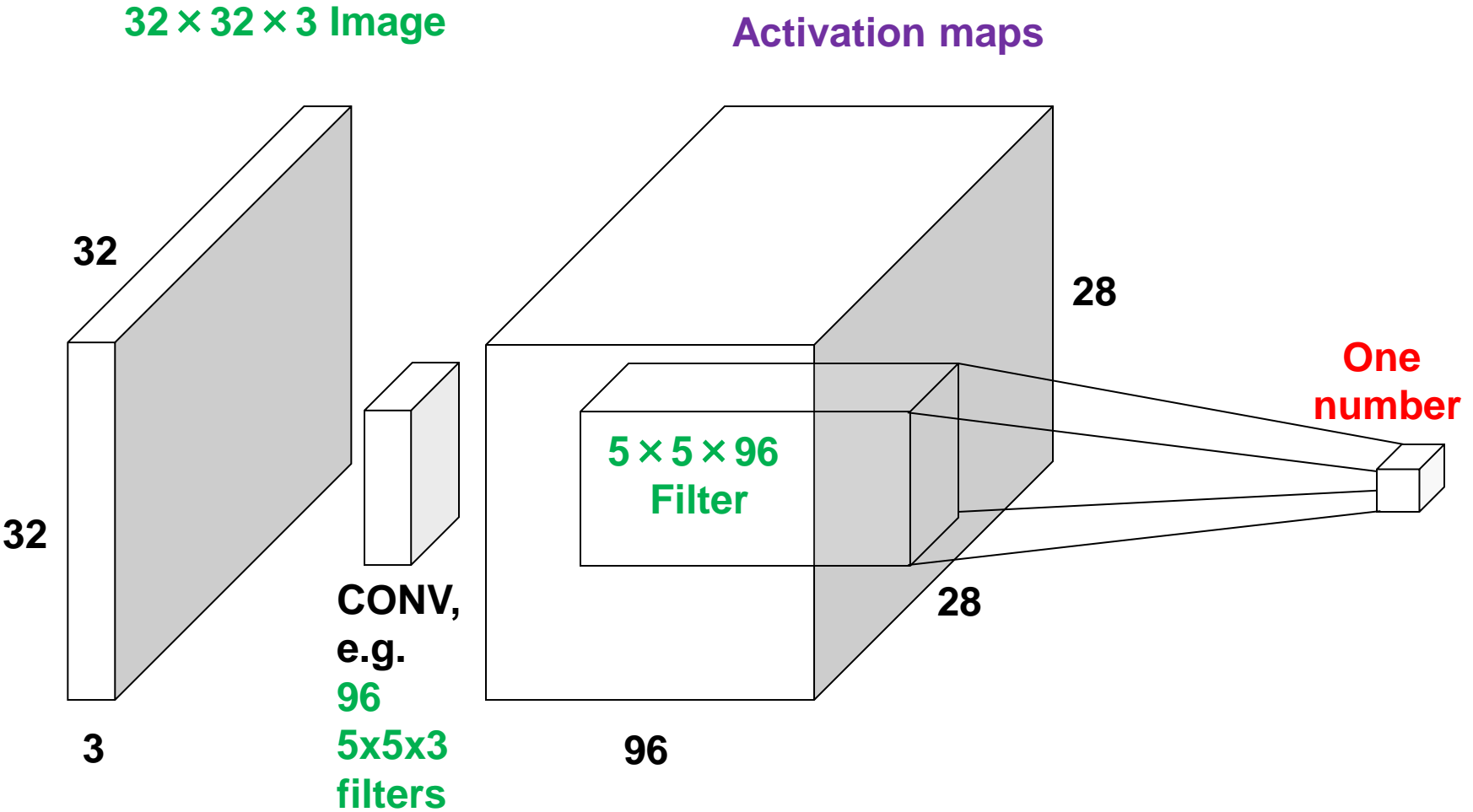
**32 × 32 × 3 Image**

**Activation maps**



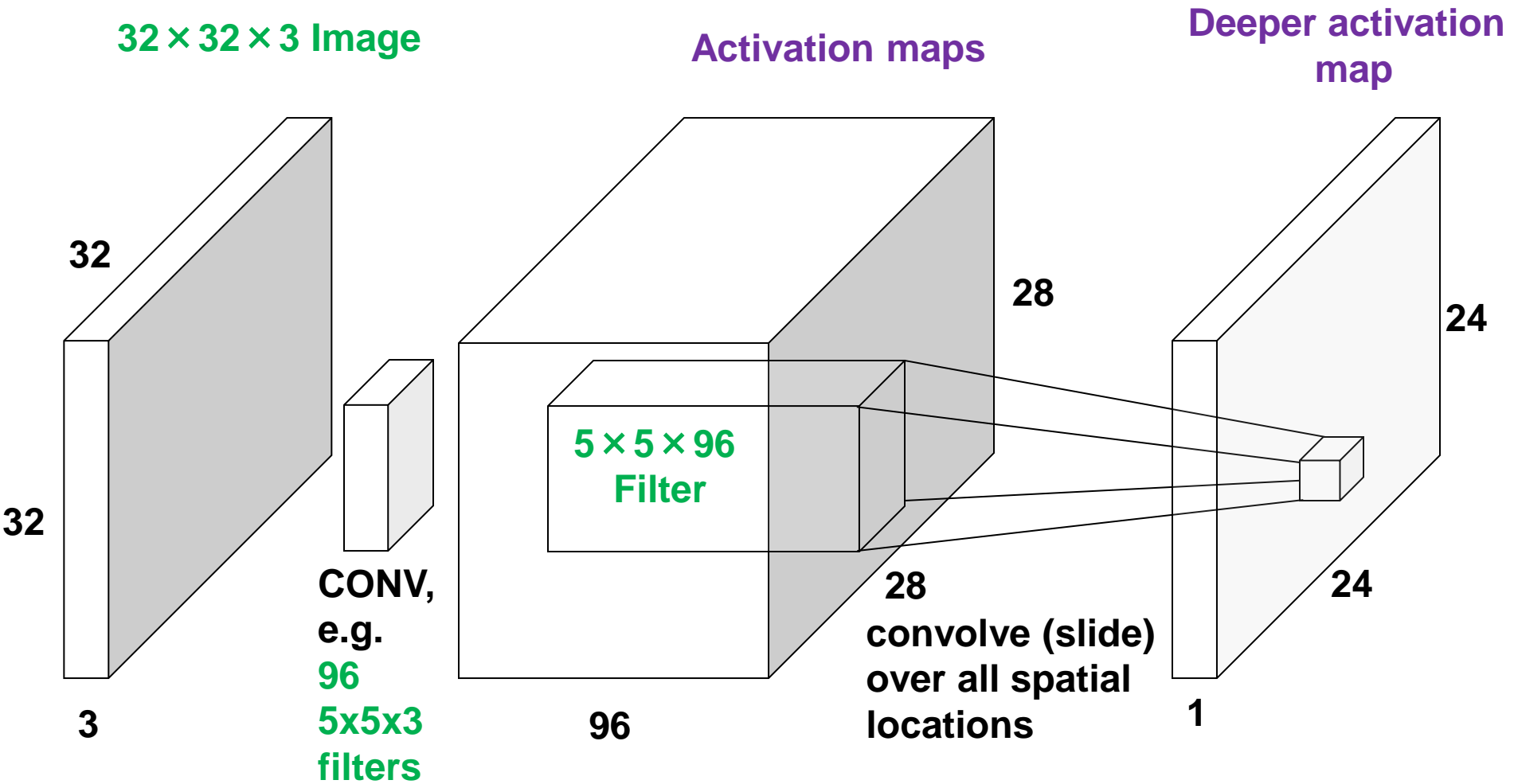
# Convolutional Layer

**Preview:** ConvNet is a sequence of Convolution Layers, interspersed with activation functions



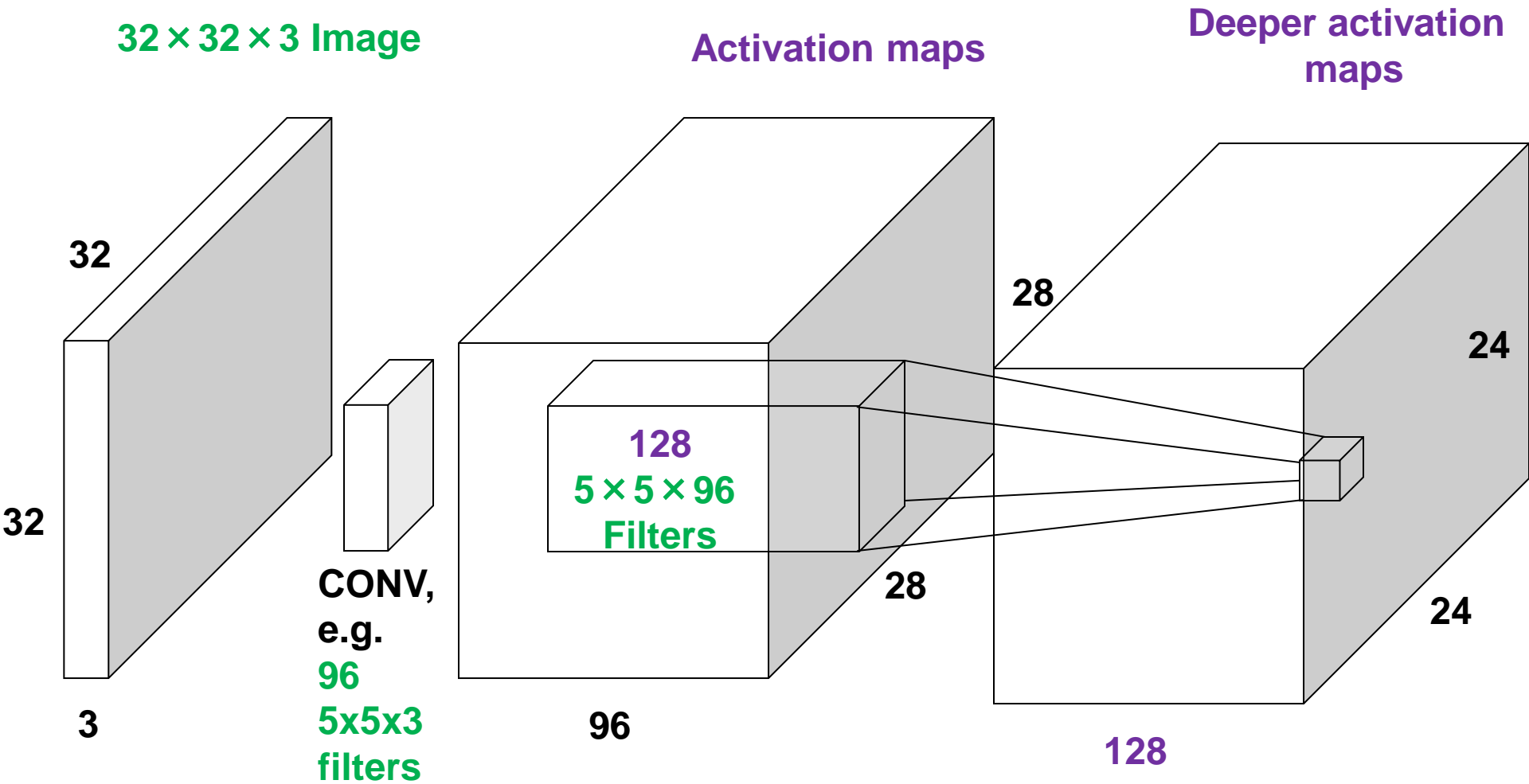
# Convolutional Layer

**Preview:** ConvNet is a sequence of Convolution Layers, interspersed with activation functions



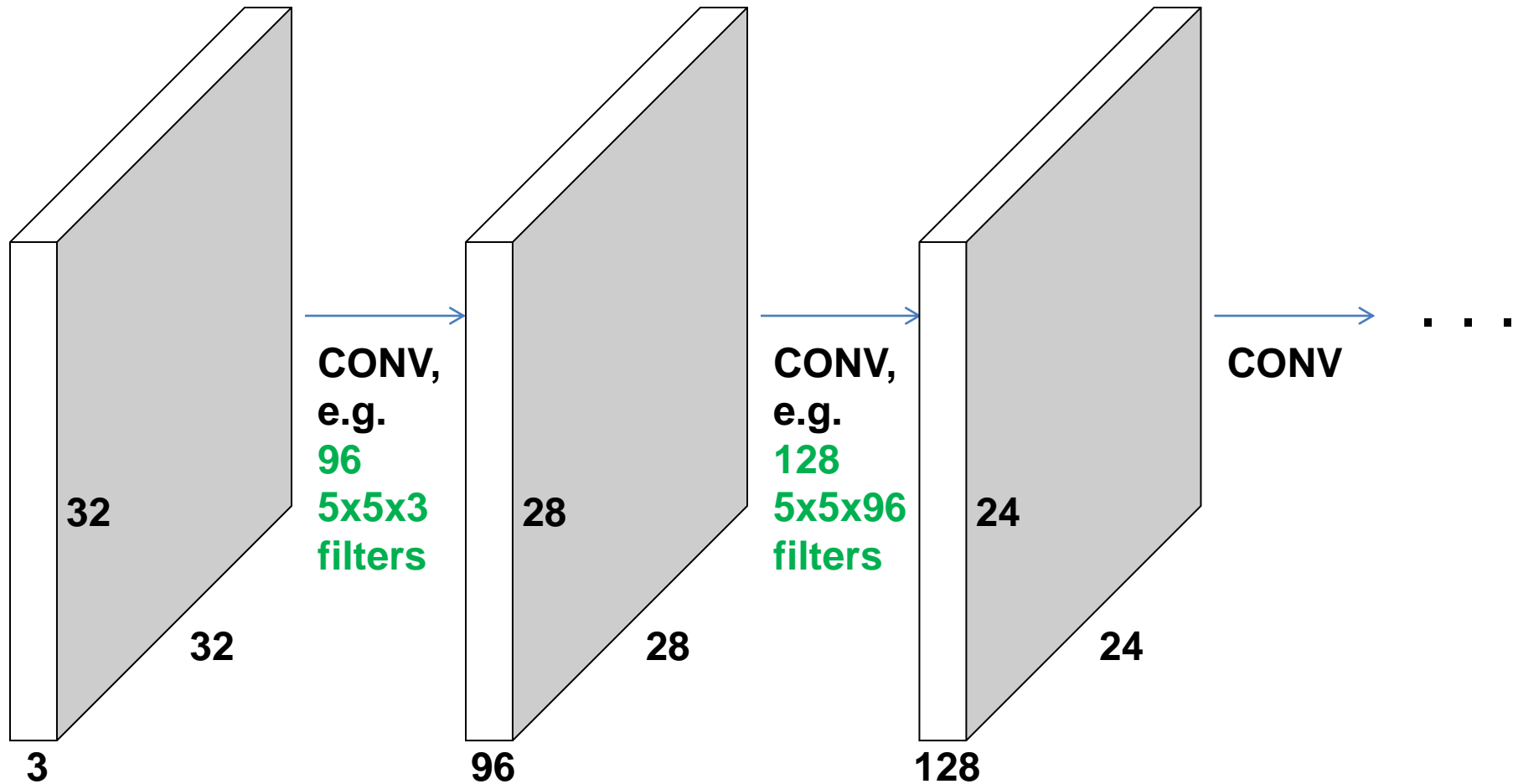
# Convolutional Layer

**Preview:** ConvNet is a sequence of Convolution Layers, interspersed with activation functions



# Multilayer Convolution

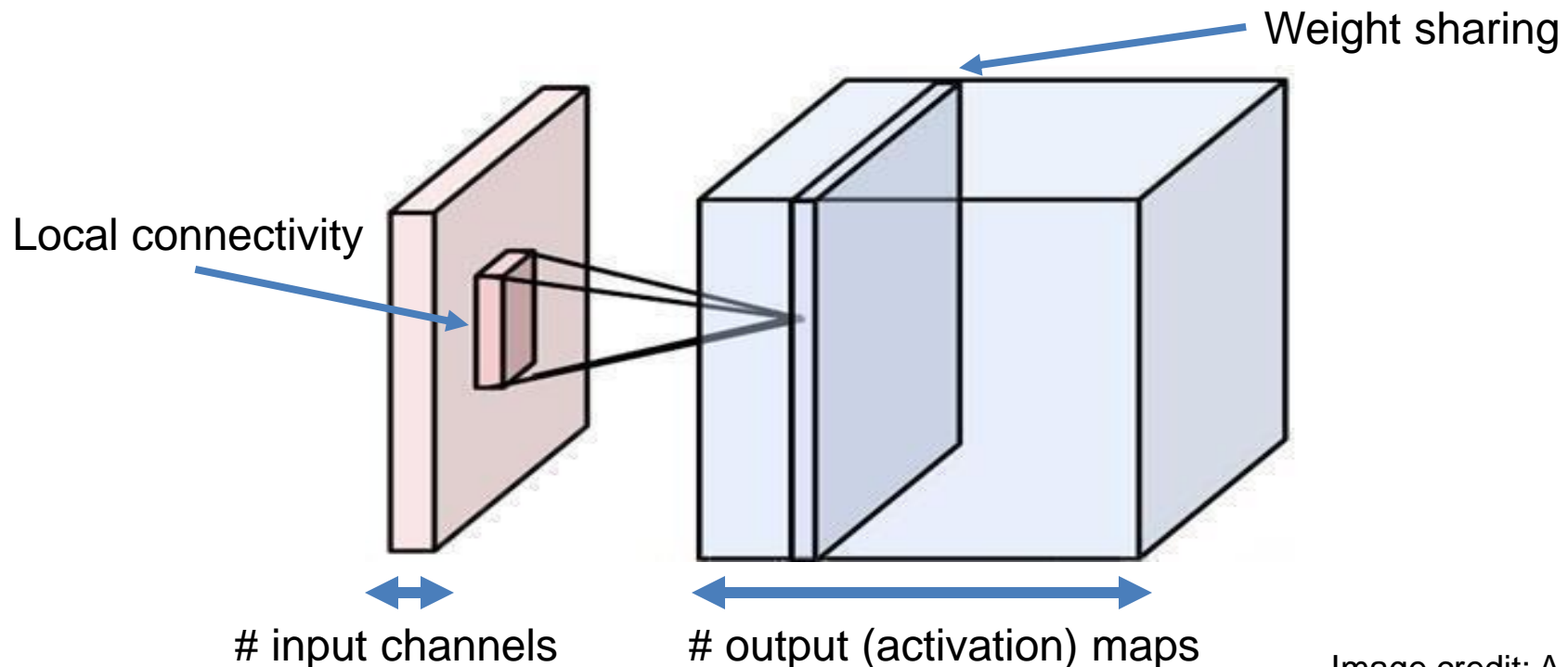
**Preview:** ConvNet is a sequence of Convolution Layers, interspersed with activation functions





# Any Convolution Layer

- Local connectivity
- Weight sharing
- Handling multiple input channels
- Handling multiple output maps



# A closer look at spatial dimensions

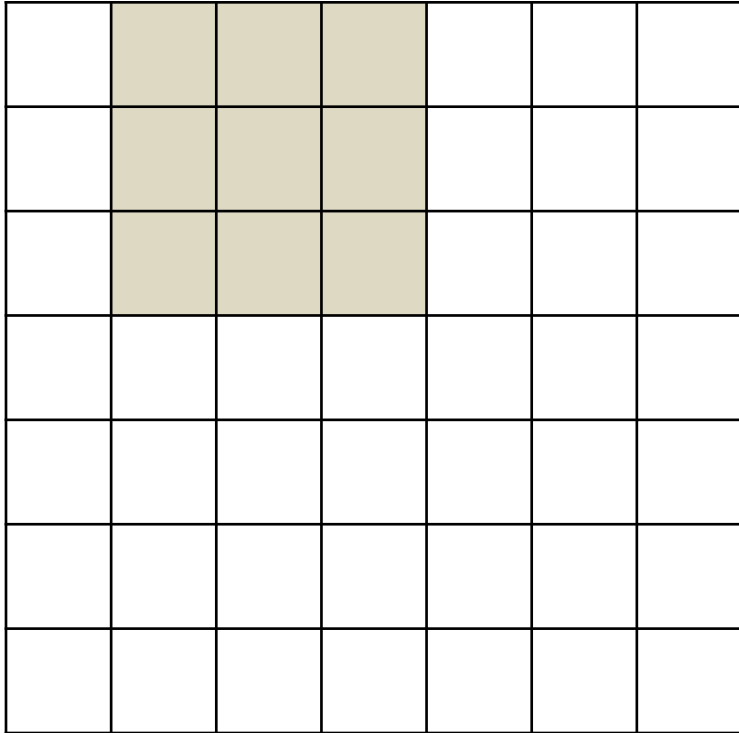
7


7

$7 \times 7$  input (spatially)  
assume  $3 \times 3$  filter

# A closer look at spatial dimensions

7

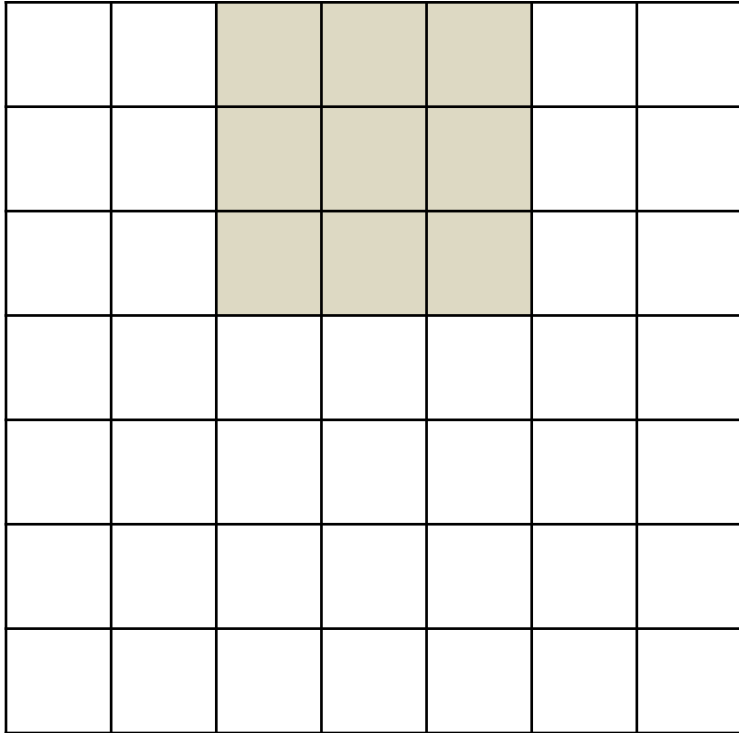


7

$7 \times 7$  input (spatially)  
assume  $3 \times 3$  filter

# A closer look at spatial dimensions

7



7

$7 \times 7$  input (spatially)  
assume  $3 \times 3$  filter

# A closer look at spatial dimensions

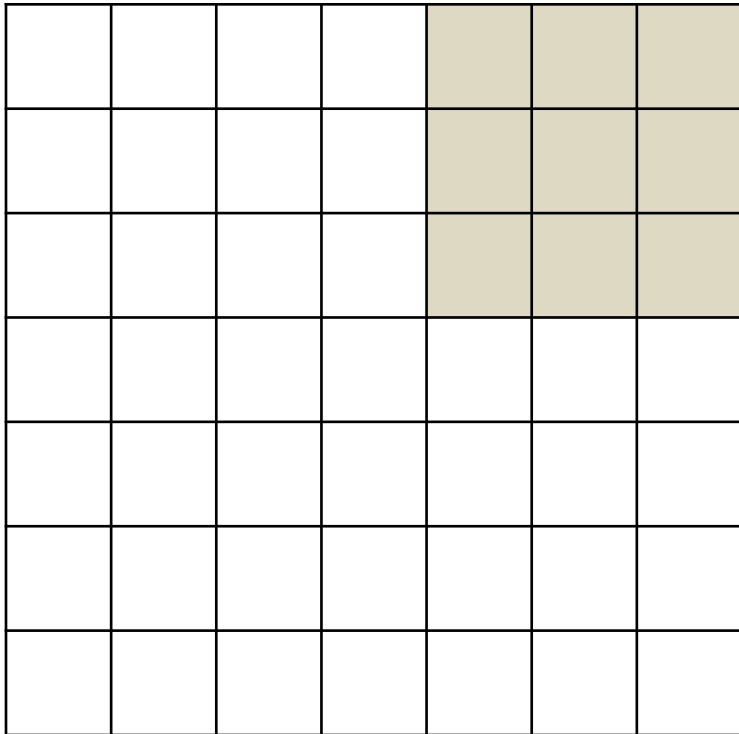
7


7

$7 \times 7$  input (spatially)  
assume  $3 \times 3$  filter

# A closer look at spatial dimensions

7



7

$7 \times 7$  input (spatially)  
assume  $3 \times 3$  filter



**$5 \times 5$  output**

# A closer look at spatial dimensions

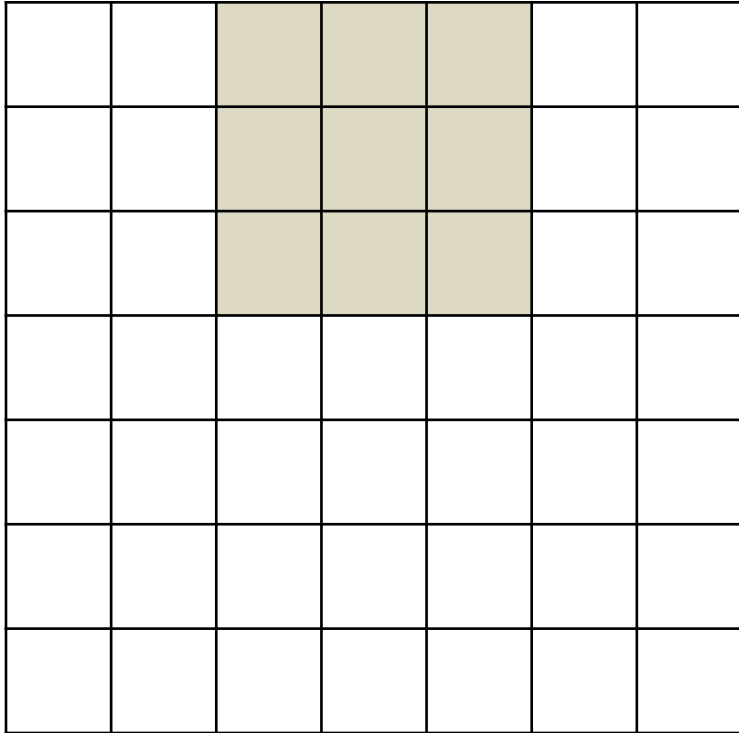
7


7

$7 \times 7$  input (spatially)  
assume  $3 \times 3$  filter  
applied with **stride 2**

# A closer look at spatial dimensions

7



7

$7 \times 7$  input (spatially)  
assume  $3 \times 3$  filter  
applied with **stride 2**

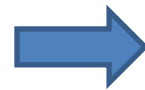


# A closer look at spatial dimensions

7


7

$7 \times 7$  input (spatially)  
assume  $3 \times 3$  filter  
applied with **stride 2**



**$3 \times 3$  output**

# A closer look at spatial dimensions

7


7

$7 \times 7$  input (spatially)  
assume  $3 \times 3$  filter  
applied with **stride 3**

# A closer look at spatial dimensions

7


7

$7 \times 7$  input (spatially)  
assume  $3 \times 3$  filter  
applied with **stride 3**

**doesn't fit!**  
cannot apply  $3 \times 3$  filter on  
 $7 \times 7$  input with stride 3.

# A closer look at spatial dimensions

N

			F			
	F					

Output size

$$(N - F) / \text{stride} + 1$$

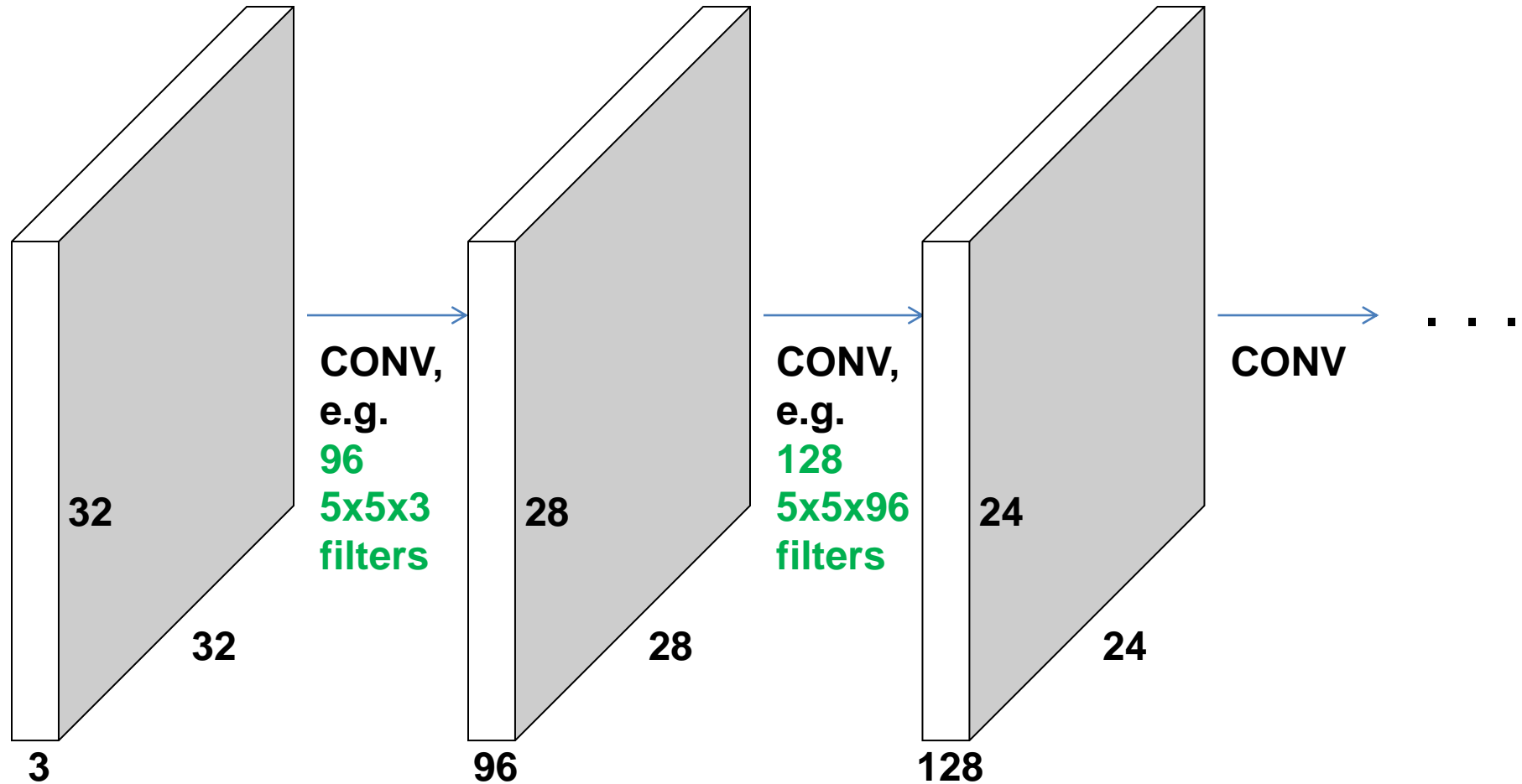
N e.g.  $N = 7, F = 3$

$$\text{stride } 1 \Rightarrow (7 - 3) / 1 + 1 = 5$$

$$\text{stride } 2 \Rightarrow (7 - 3) / 2 + 1 = 3$$

$$\text{stride } 3 \Rightarrow (7 - 3) / 3 + 1 = 2.33$$

# A closer look at spatial dimensions



E.g. 32x32 input convolved repeatedly with 5x5 filters shrinks volumes spatially! (32 -> 28 -> 24 ...). Shrinking too fast is not good, doesn't work well.

# In practice: common to zero pad

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

e.g. input  $7 \times 7$  (spatially)

$3 \times 3$  filter, applied with **stride 1**  
**pad with 1 pixel** border

**What is the output dimension?**

# In practice: common to zero pad

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

e.g. input  $7 \times 7$  (spatially)

$3 \times 3$  filter, applied with **stride 1**  
**pad with 1 pixel border**

**$7 \times 7$  Output**

# In practice: common to zero pad

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

e.g. **input  $7 \times 7$**  (spatially)

**$3 \times 3$  filter**, applied with **stride 1**  
**pad with 1 pixel** border

**$7 \times 7$  Output**

in general, common to see CONV layers with stride 1, filters of size  $F \times F$ , and zero-padding with  $(F-1)/2$ . (will preserve size spatially)

e.g.

$F = 3 \Rightarrow$  zero pad with 1

$F = 5 \Rightarrow$  zero pad with 2

$F = 7 \Rightarrow$  zero pad with 3

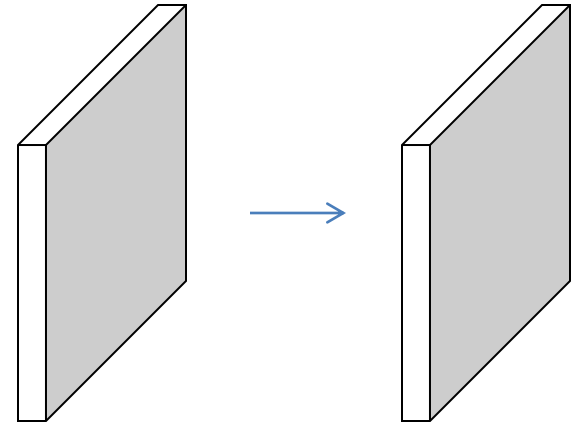


# Example

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

Output volume size: ?



# Example

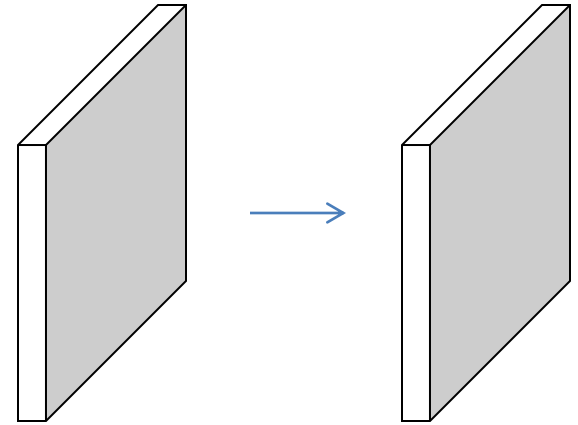
Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

Output volume size:

$(32+2*2-5)/1+1 = 32$  spatially, so

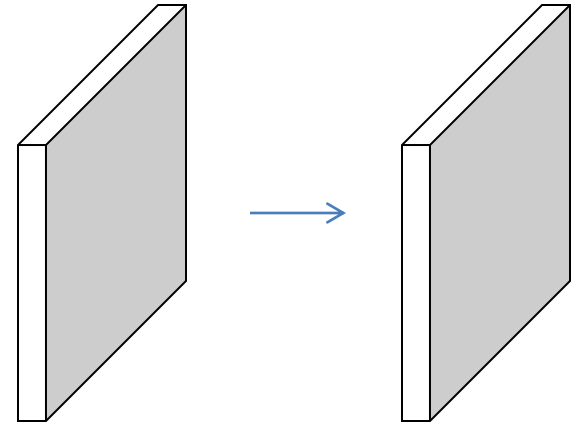
**32x32x10**



# Example

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

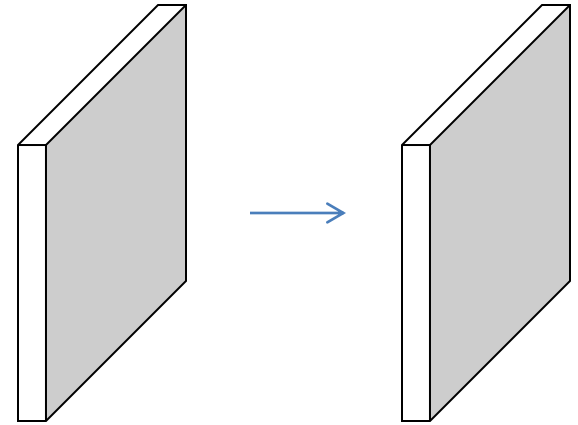


Number of parameters in this layer?

# Example

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2



Number of parameters in this layer?

each filter has

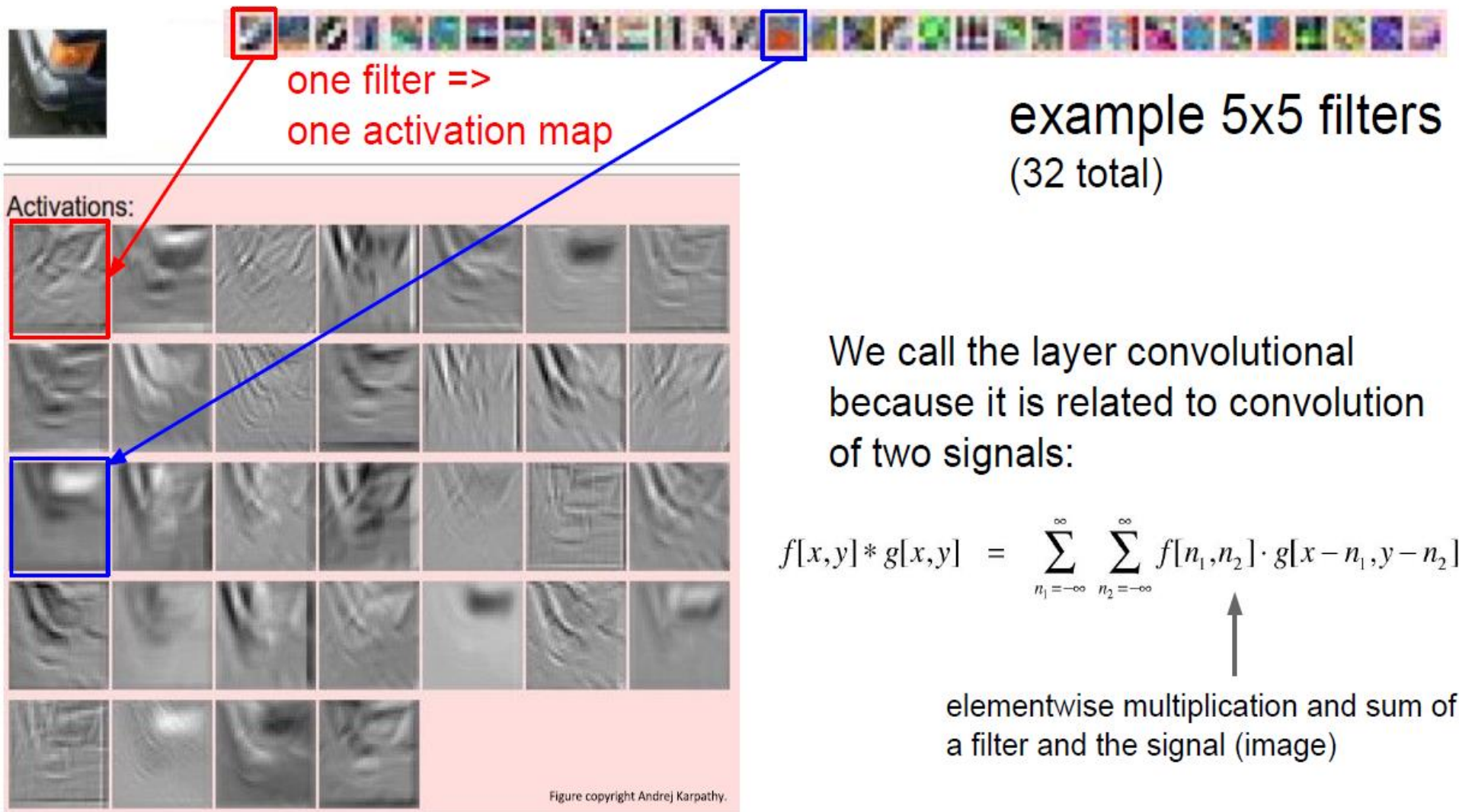
$5*5*3 + 1 = 76$  params (+1 for bias)

$\Rightarrow 76*10 = \mathbf{760}$

**Summary.** To summarize, the Conv Layer:

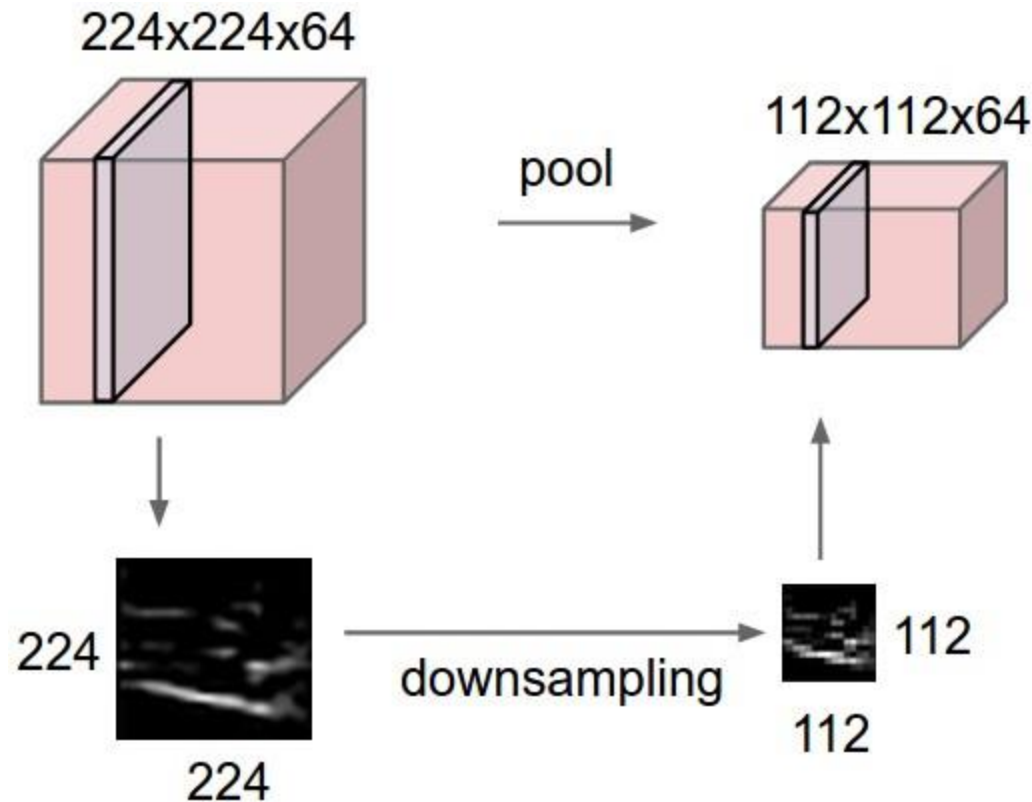
- Accepts a volume of size  $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
  - Number of filters  $K$ ,
  - their spatial extent  $F$ ,
  - the stride  $S$ ,
  - the amount of zero padding  $P$ .
- Produces a volume of size  $W_2 \times H_2 \times D_2$  where:
  - $W_2 = (W_1 - F + 2P)/S + 1$
  - $H_2 = (H_1 - F + 2P)/S + 1$  (i.e. width and height are computed equally by symmetry)
  - $D_2 = K$
- With parameter sharing, it introduces  $F \cdot F \cdot D_1$  weights per filter, for a total of  $(F \cdot F \cdot D_1) \cdot K$  weights and  $K$  biases.

# Convolution as feature extraction

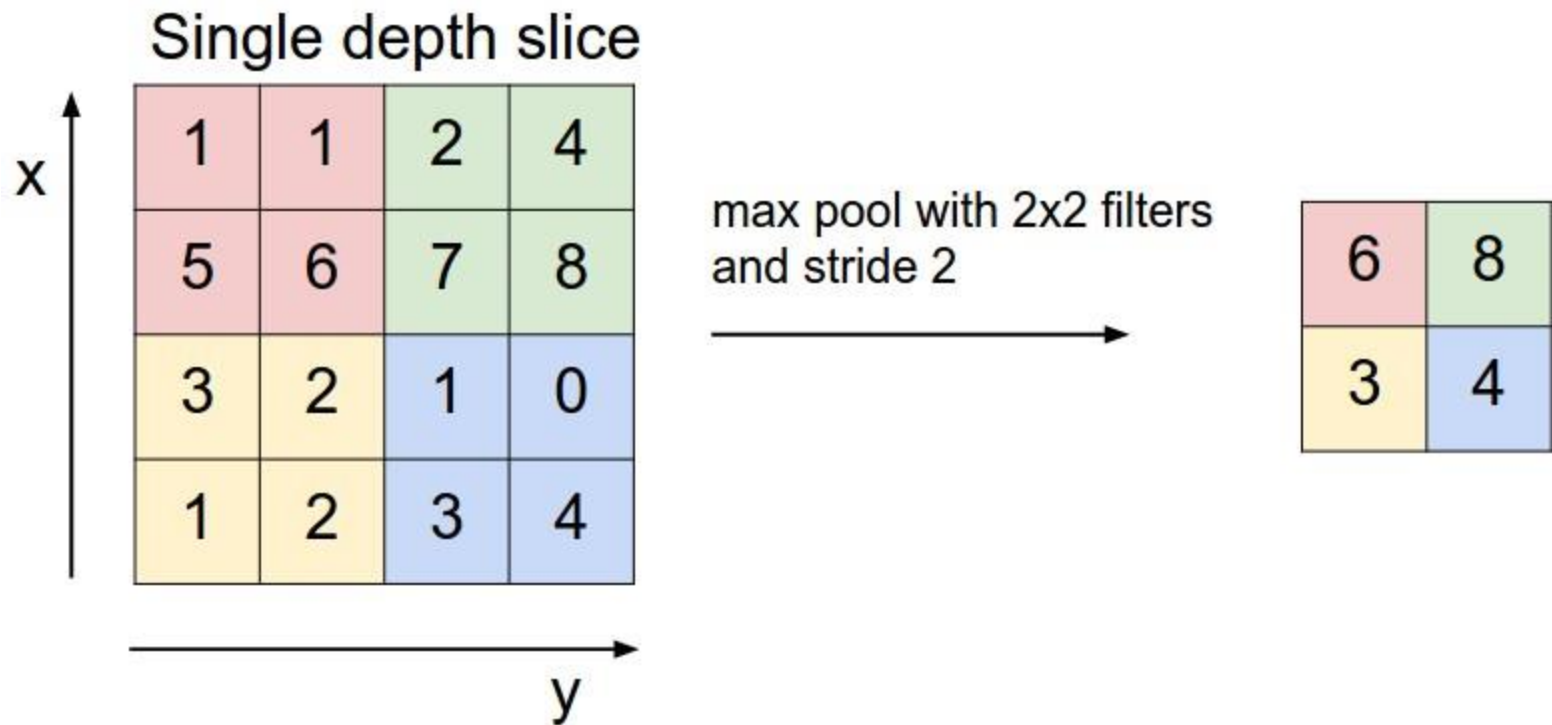


# Pooling Layer

- makes the representations smaller and more manageable
- operates over each activation map independently:



# Max Pooling



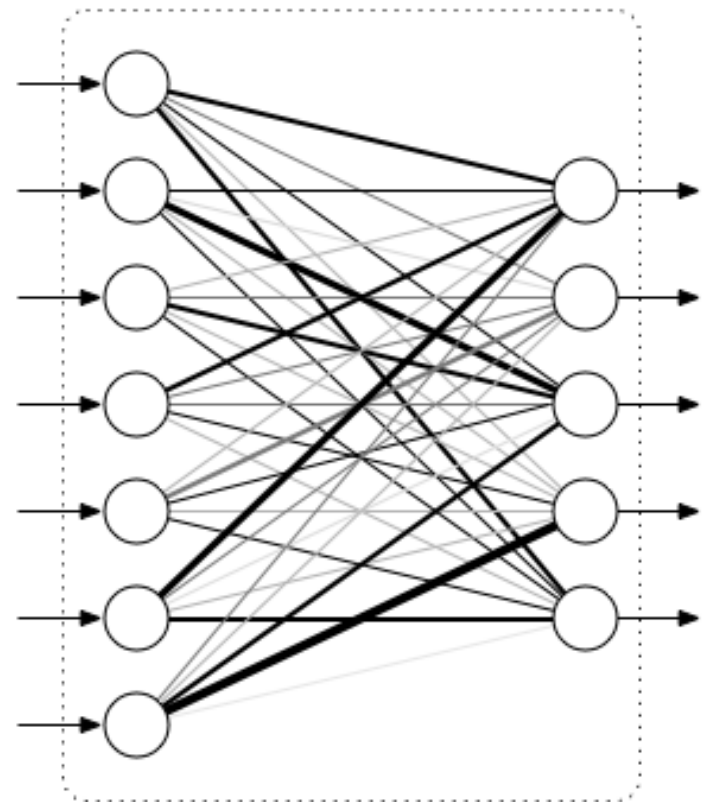


# Pooling Layer

- Accepts a volume of size  $W_1 \times H_1 \times D_1$
- Requires two hyperparameters:
  - their spatial extent  $F$ ,
  - the stride  $S$ ,
- Produces a volume of size  $W_2 \times H_2 \times D_2$  where:
  - $W_2 = (W_1 - F)/S + 1$
  - $H_2 = (H_1 - F)/S + 1$
  - $D_2 = D_1$
- Introduces zero parameters since it computes a fixed function of the input
- Note that it is not common to use zero-padding for Pooling layers

# Fully Connected Layer

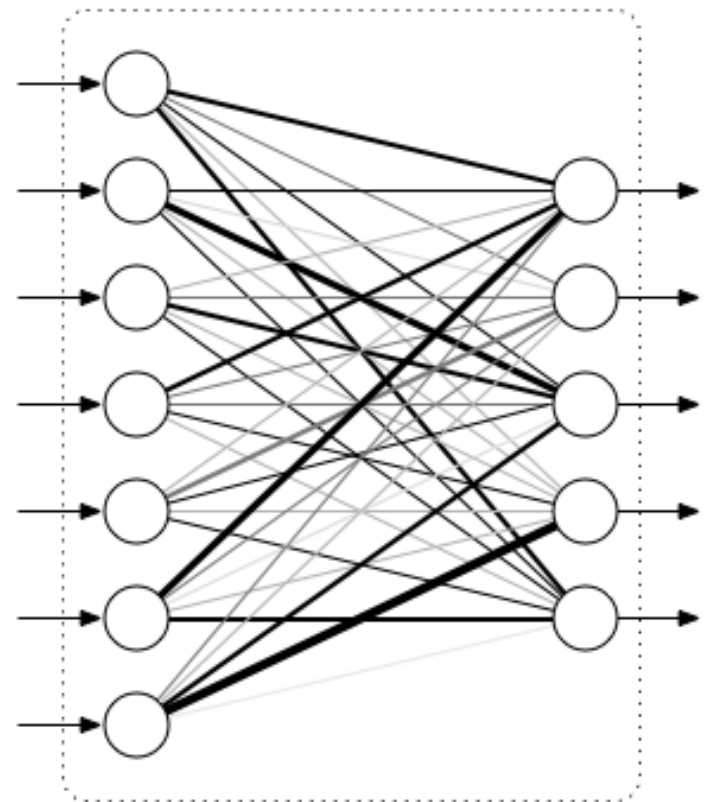
- Connect every neuron in one layer to every neuron in another layer
- Same as the traditional multi-layer perceptron neural network



# Fully Connected Layer

- Connect every neuron in one layer to every neuron in another layer
- Same as the traditional multi-layer perceptron neural network

**No. of Neurons (Last FC)  
= No. of classes**



# Loss/Classification Layer

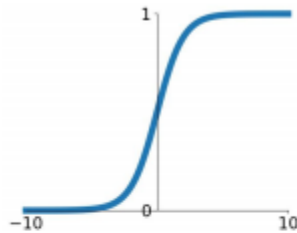
- SVM Classifier (SVM Loss/Hinge Loss/Max-margin Loss)
- Softmax Classifier (Softmax Loss/Cross-entropy Loss)

# Non-linearity Layer

## Activation Functions

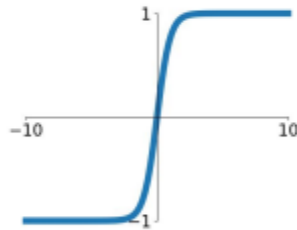
### Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



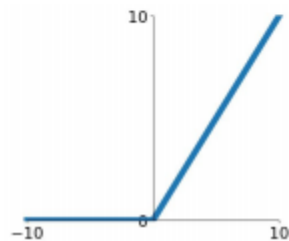
### tanh

$$\tanh(x)$$



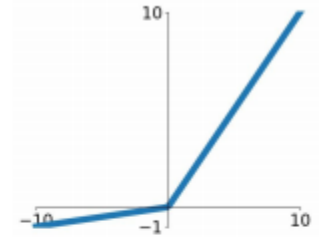
### ReLU

$$\max(0, x)$$



### Leaky ReLU

$$\max(0.1x, x)$$

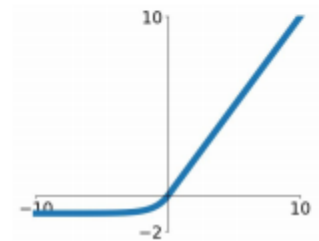


### Maxout

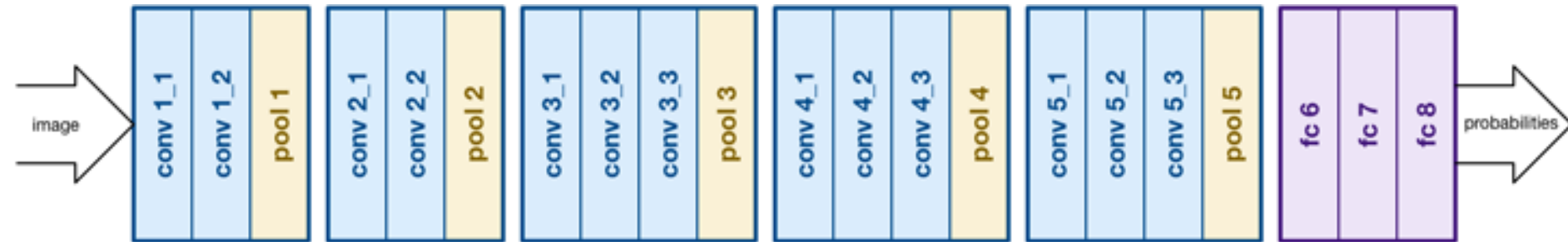
$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

### ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$

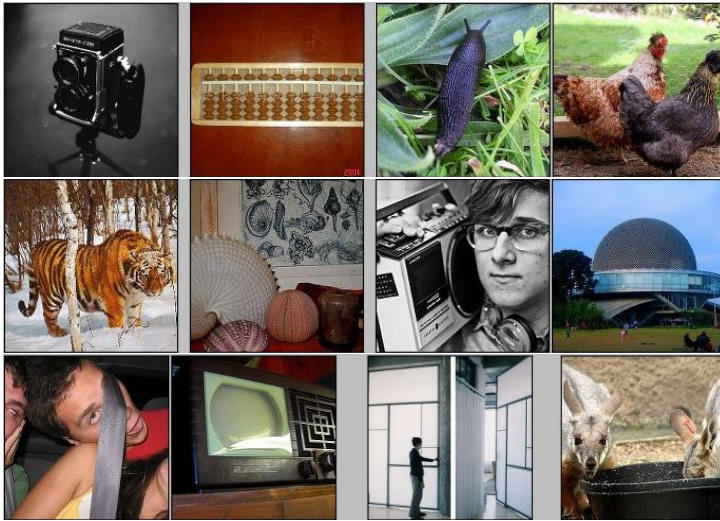


# A typical CNN structure



# ImageNet Challenge

IMAGENET

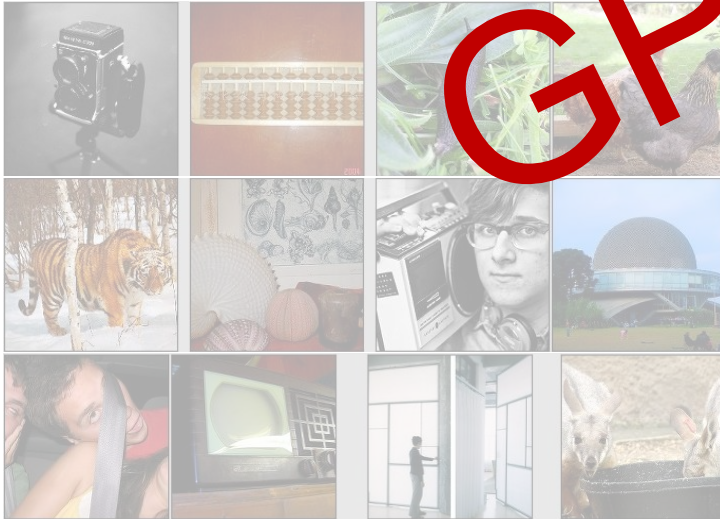


- ~14 million labeled images, 20k classes
- Images gathered from Internet
- Human labels via Amazon MTurk
- Challenge: 1.2 million training images, 1000 classes

[www.image-net.org/challenges/LSVRC/](http://www.image-net.org/challenges/LSVRC/)

# ImageNet Challenge

IMAGENET



GPUS  
+  
Data

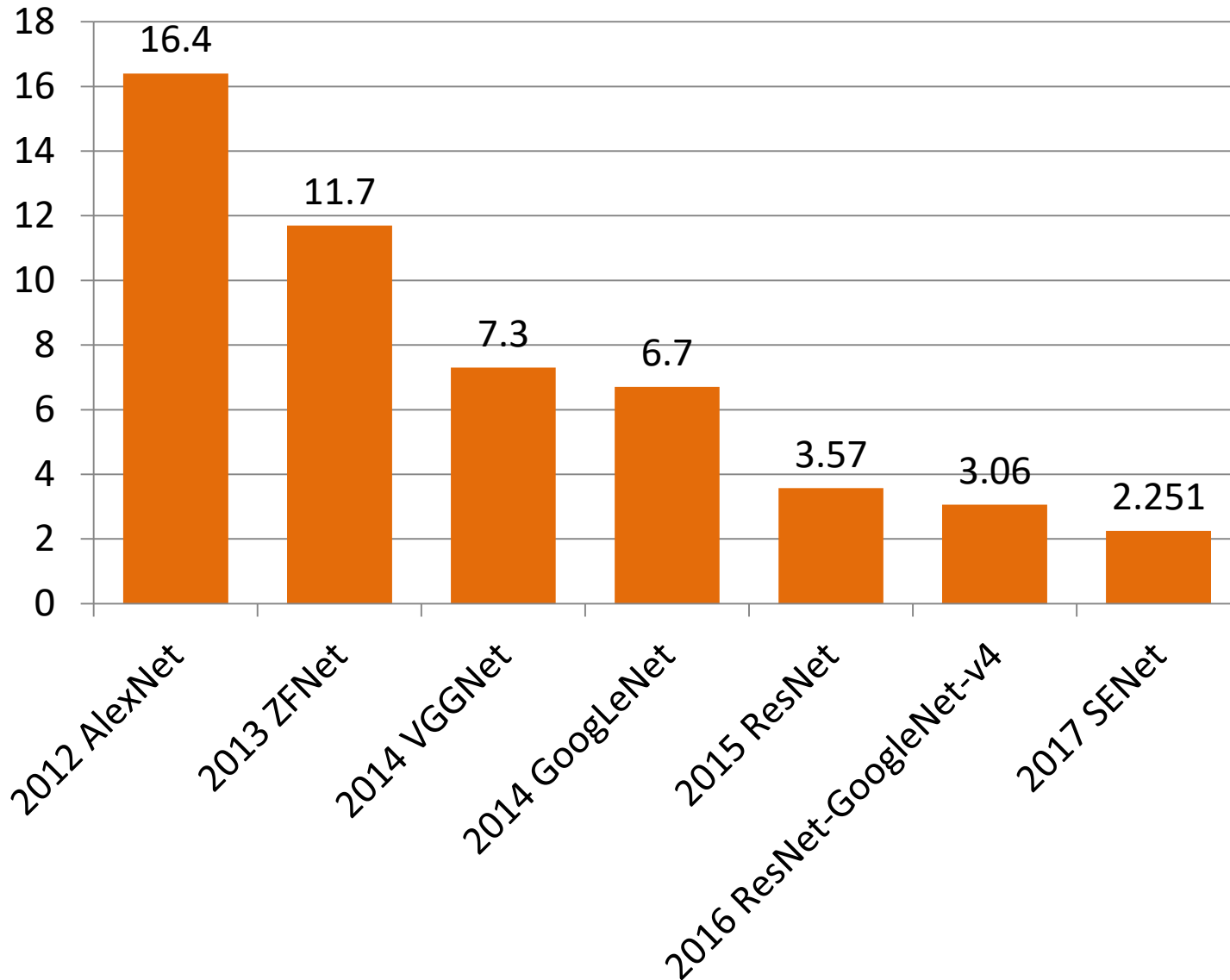
- ~14 million labeled images, 20k classes
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[www.image-net.org/challenges/LSVRC/](http://www.image-net.org/challenges/LSVRC/)



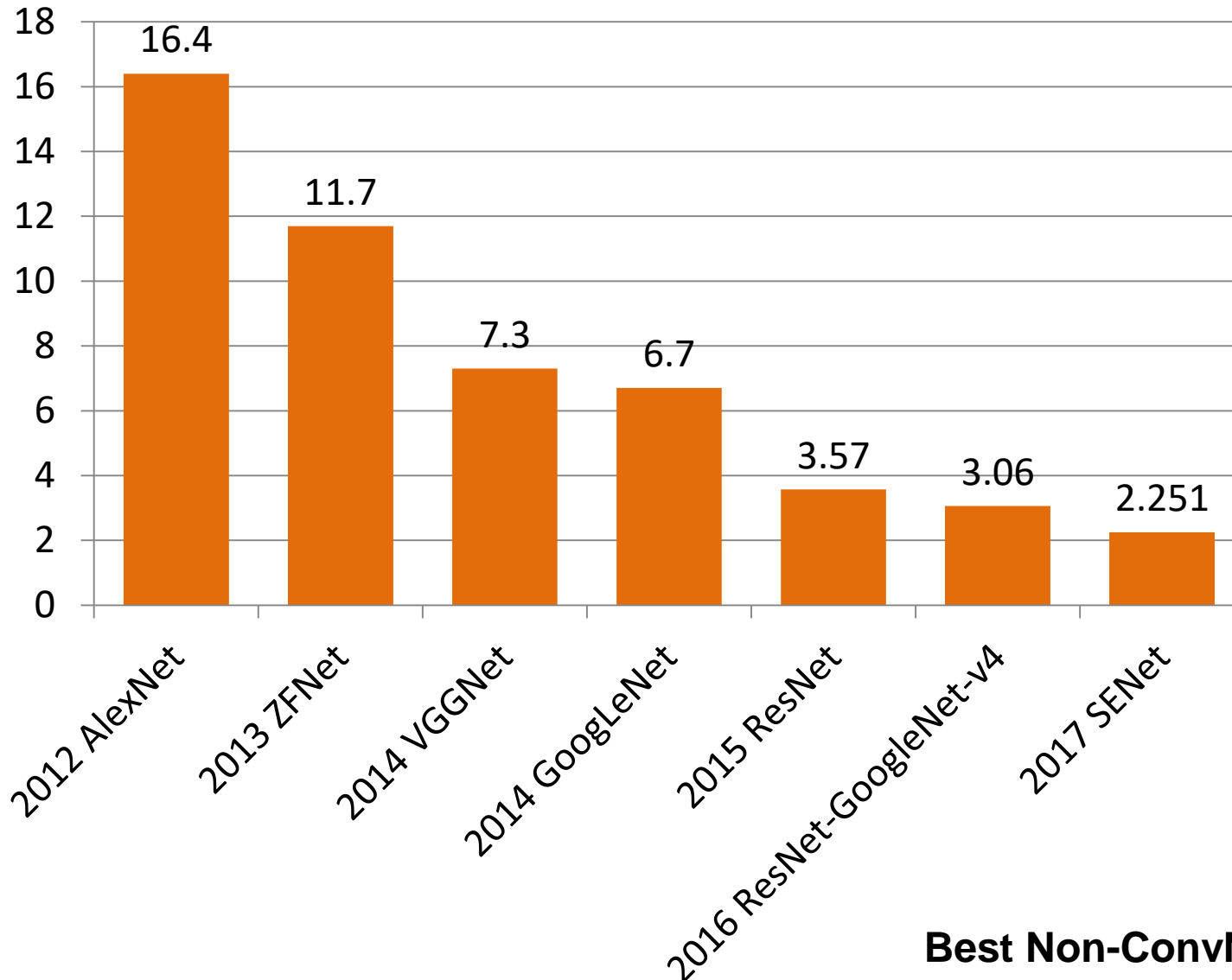
# Progress on ImageNet Challenge

ImageNet Image Classification Top5 Error



# Progress on ImageNet Challenge

ImageNet Image Classification Top5 Error



Best Non-ConvNet in 2012: 26.2%

# Things to remember

- Neural network and Image
  - Neuroscience, Perceptron, Problems due to High Dimensionality and Local Relationship
- Convolutional neural network (CNN)
  - Convolution Layer,
  - Nonlinearity Layer,
  - Pooling Layer,
  - Fully Connected Layer,
  - Loss/Classification Layer
- Progress on ImageNet challenge
  - Latest SENet, Winner 2017

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  - S. Lazebnik
  - K. Grauman
  - R. Zaleski
  - Antonio Torralba
  - Rob Fergus
  - Leibe
  - And many more .....

# Next Class

## Training Aspects of CNN

- Activation Functions
- Dataset Preparation
- Data Preprocessing
- Weight Initialization
- Optimization Methods
- Learning Rate
- Transfer Learning
- Generalization

