

Computer Vision

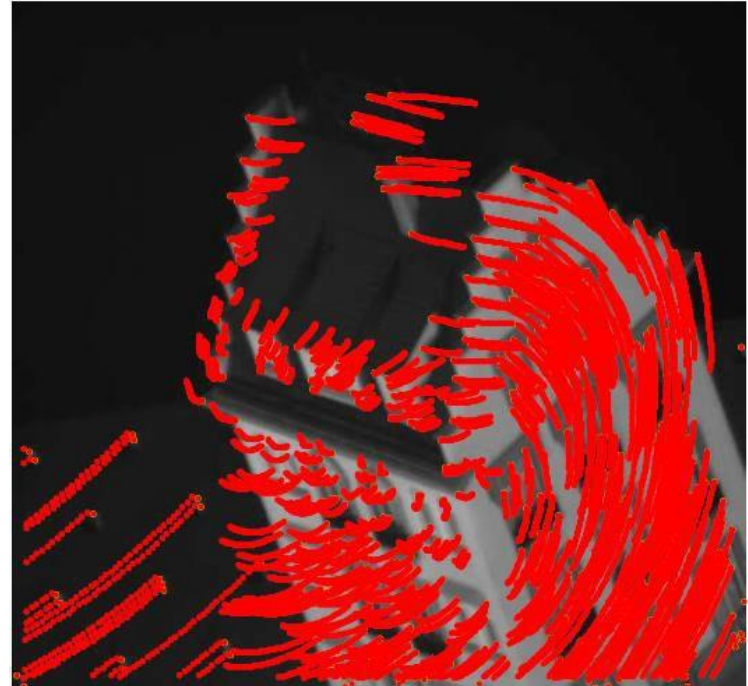
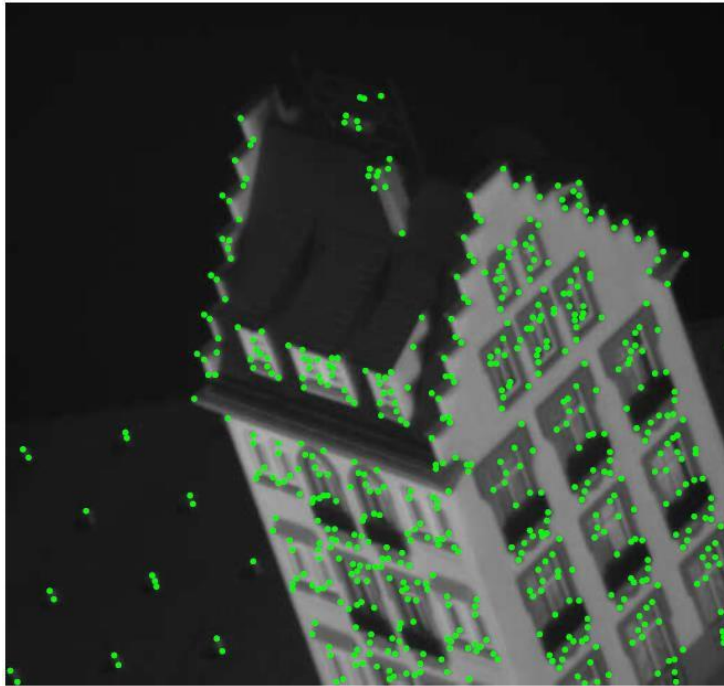
Feature Tracking and Optical Flow

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**Indian Institute of Information Technology
Sri City, Chittoor**



Feature Tracking and Optical Flow



This class: recovering motion

- Feature tracking
 - Extract visual features (corners, textured areas) and “track” them over multiple frames
- Optical flow
 - Recover image motion at each pixel from spatio-temporal image brightness variations

B. Lucas and T. Kanade. [An iterative image registration technique with an application to stereo vision](#). In *Proceedings of the International Joint Conference on Artificial Intelligence*, 1981.

Feature tracking - Challenges

- Figure out which features can be tracked

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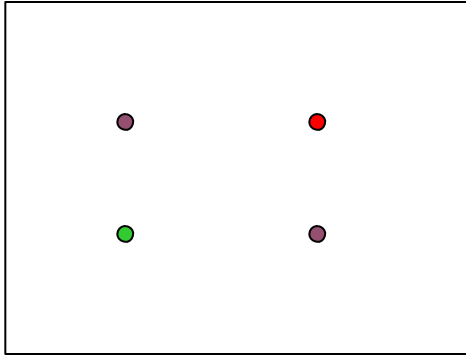
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- Points may appear or disappear: need to be able to add/delete tracked points

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- Figure out which features can be tracked
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- Points may appear or disappear: need to be able to add/delete tracked points
- Drift: small errors can accumulate as appearance model is updated

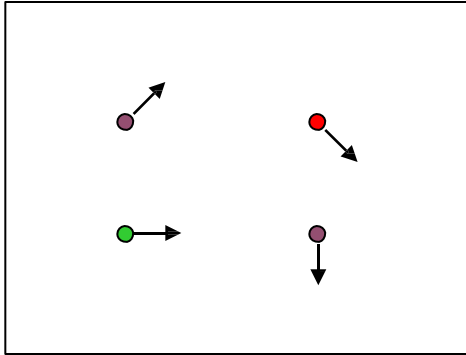
Feature tracking: Lucas-Kanade



$$I(x,y,t)$$

- Given two subsequent frames, estimate the point translation

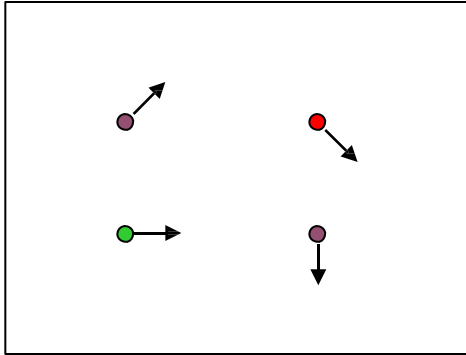
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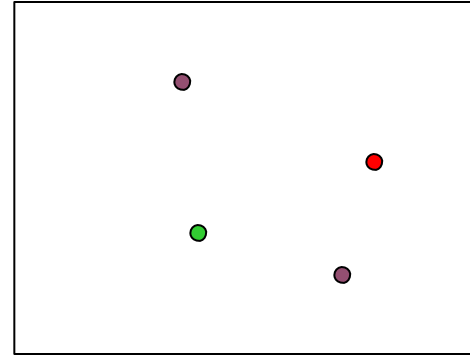
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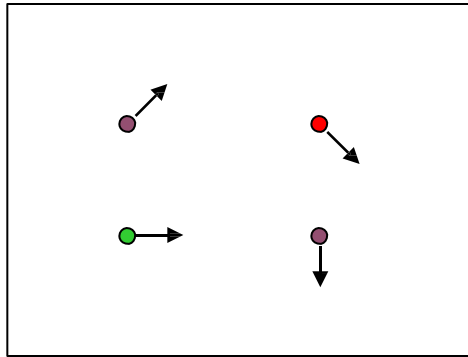
$I(x,y,t)$



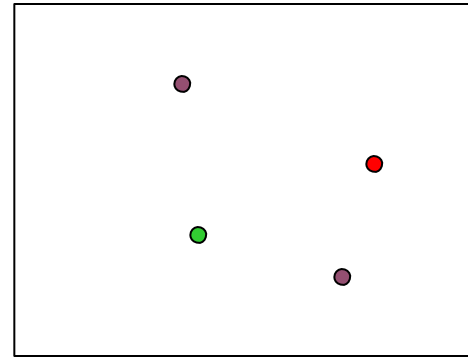
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Feature tracking: Lucas-Kanade



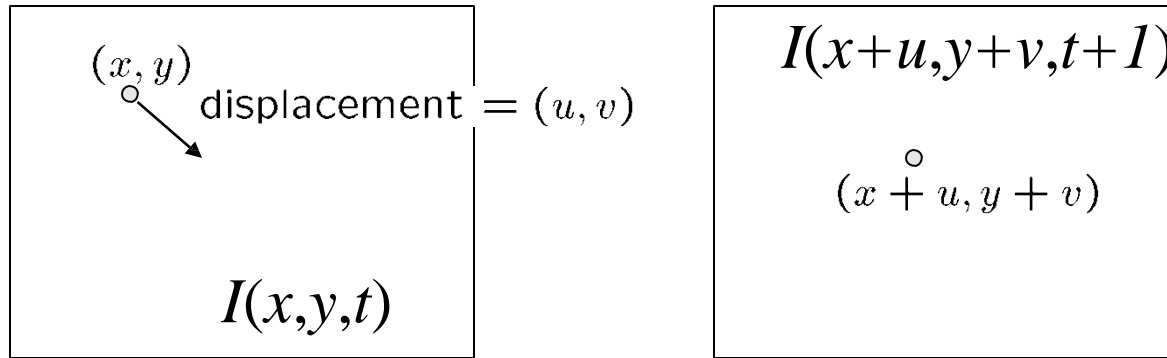
$I(x,y,t)$



$I(x,y,t+1)$

- Given two subsequent frames, estimate the point translation
- Key assumptions of Lucas-Kanade Tracker
 - **Brightness constancy**: projection of the same point looks the same in every frame
 - **Small motion**: points do not move very far
 - **Spatial coherence**: points move like their neighbors

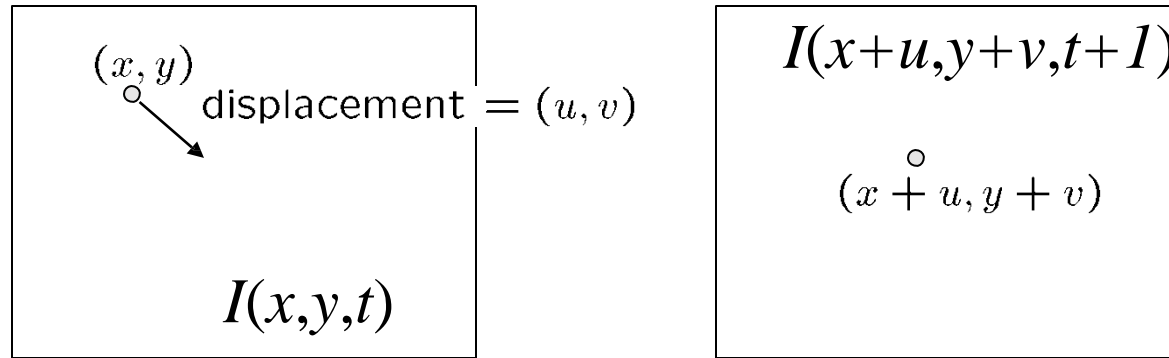
The brightness constancy constraint



- Brightness Constancy Equation:

$$I(x, y, t) = I(x + u, y + v, t + 1)$$

The brightness constancy constraint



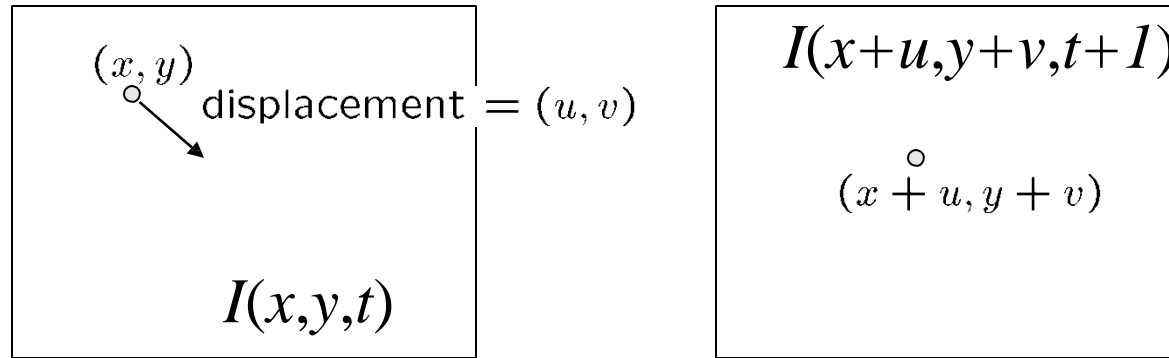
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$$I(x, y, t) = I(x + u, y + v, t + 1)$$

Take Taylor expansion of $I(x+u, y+v, t+1)$ at (x, y, t) to linearize the rightside:

$$I(x + u, y + v, t + 1) \approx I(x, y, t) + I_x \cdot u + I_y \cdot v + I_t$$

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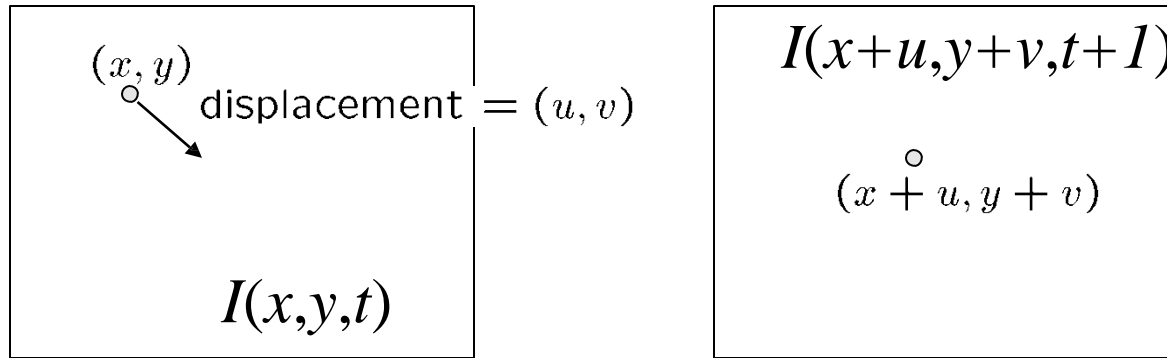
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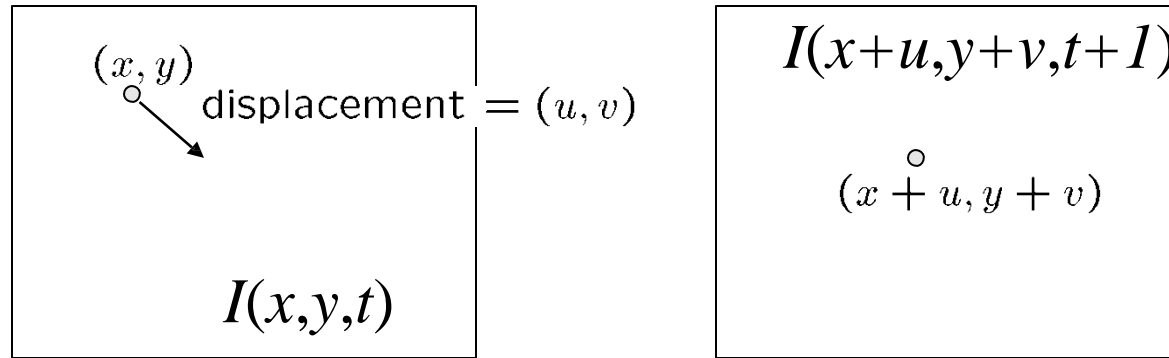
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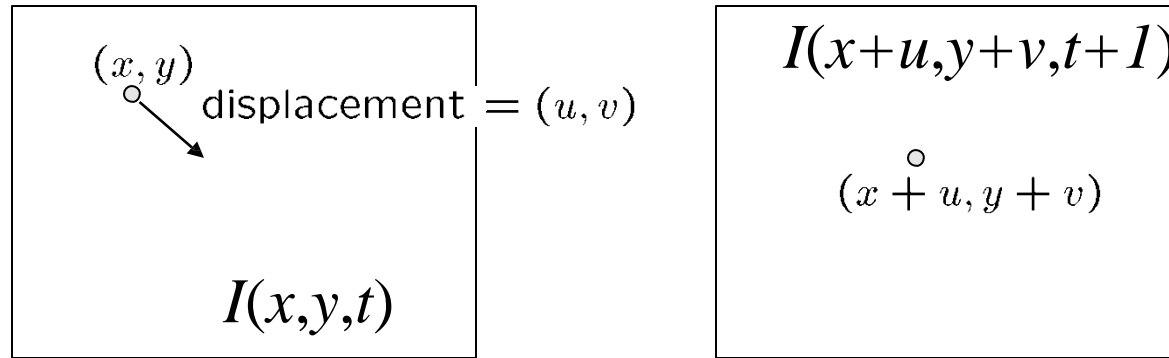
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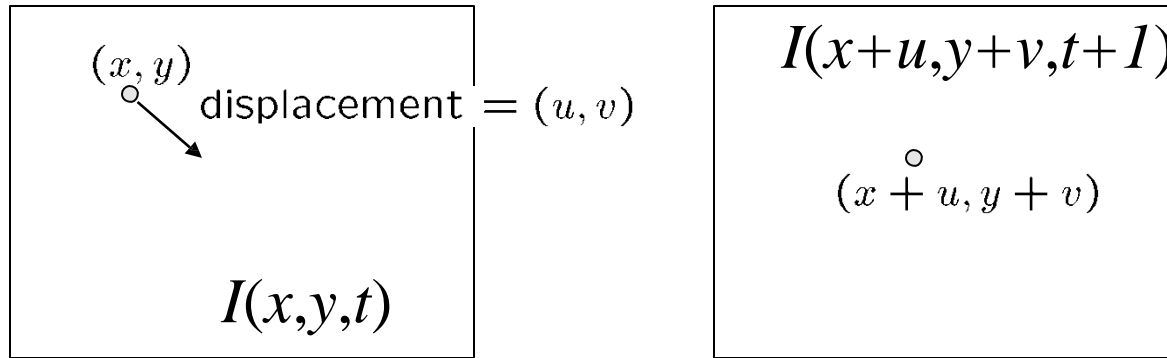
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$$I(x + u, y + v, t + 1) - I(x, y, t) = I_x \cdot u + I_y \cdot v + I_t$$

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$$I(x + u, y + v, t + 1) - I(x, y, t) = I_x \cdot u + I_y \cdot v + I_t$$

So:
$$I_x \cdot u + I_y \cdot v + I_t \approx 0 \quad \rightarrow \quad \nabla I \cdot \begin{bmatrix} u & v \end{bmatrix}^T + I_t = 0$$

The brightness constancy constraint

Can we use this equation to recover image motion (u,v) at each pixel?

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- How many equations and unknowns per pixel?
 - One equation (this is a scalar equation!), two unknowns (u,v)

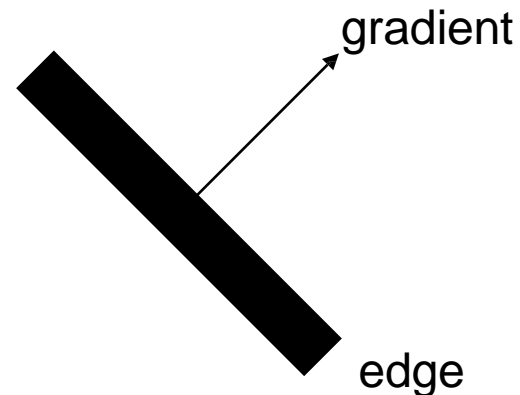
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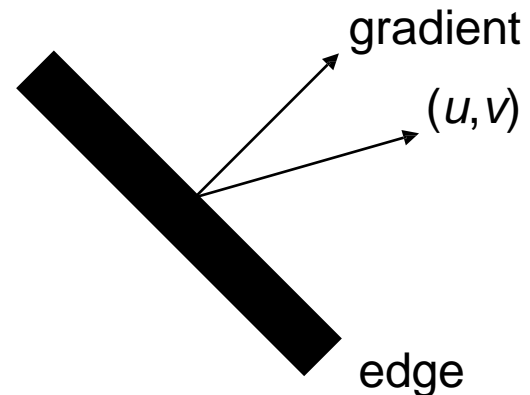
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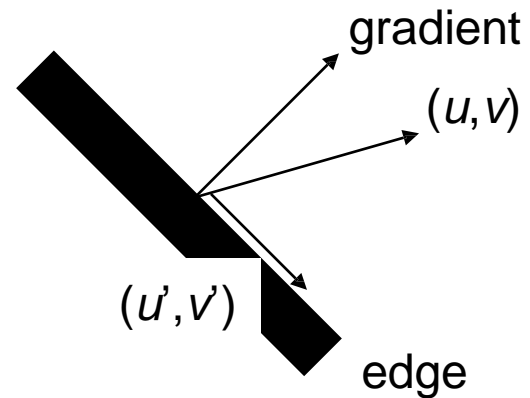
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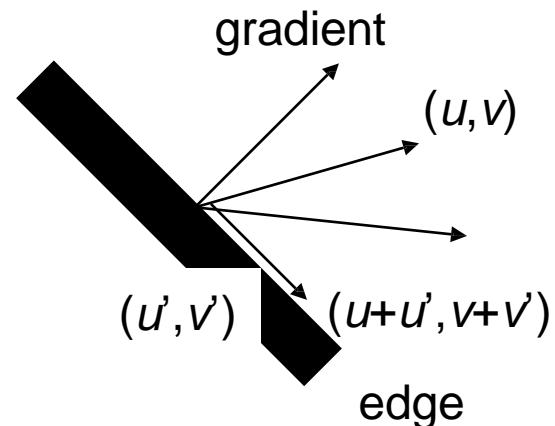
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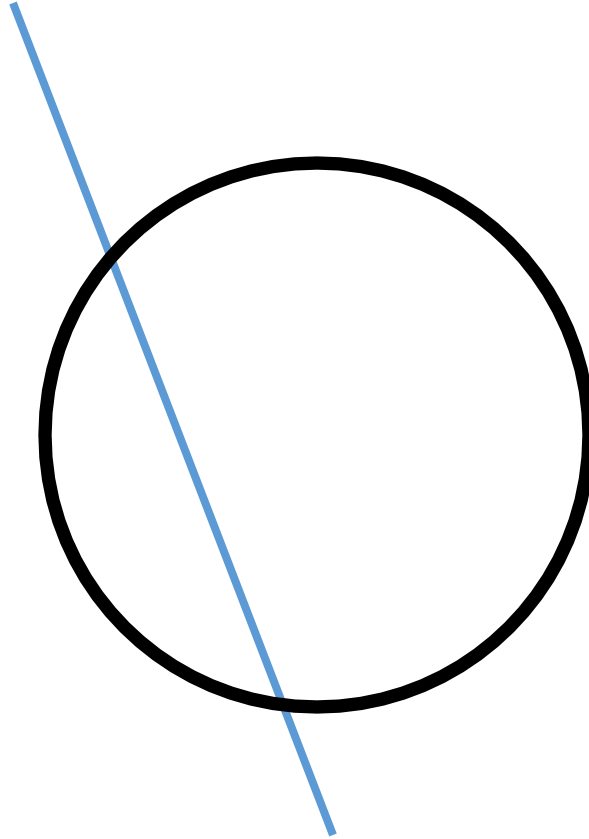
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If (u, v) satisfies the equation, so does $(u+u', v+v')$ if

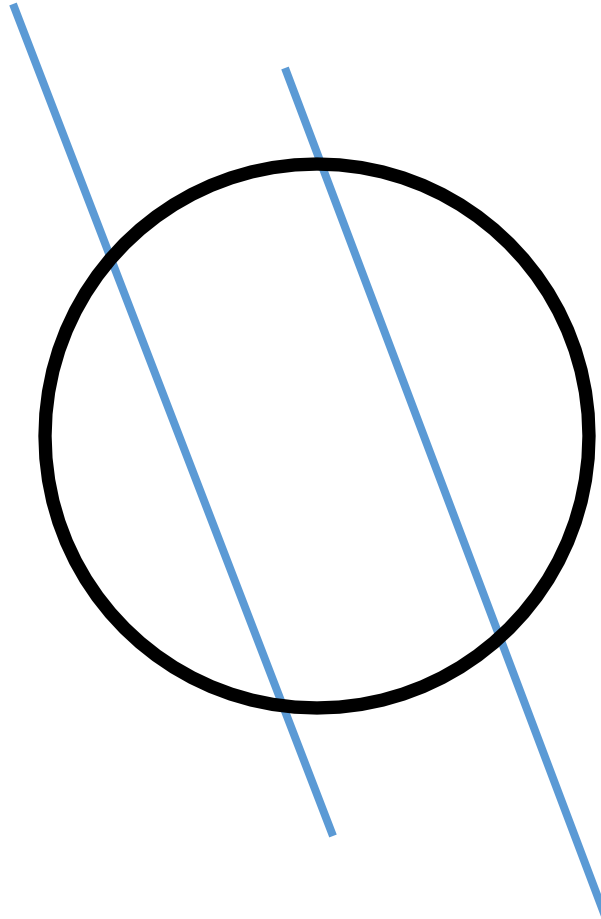
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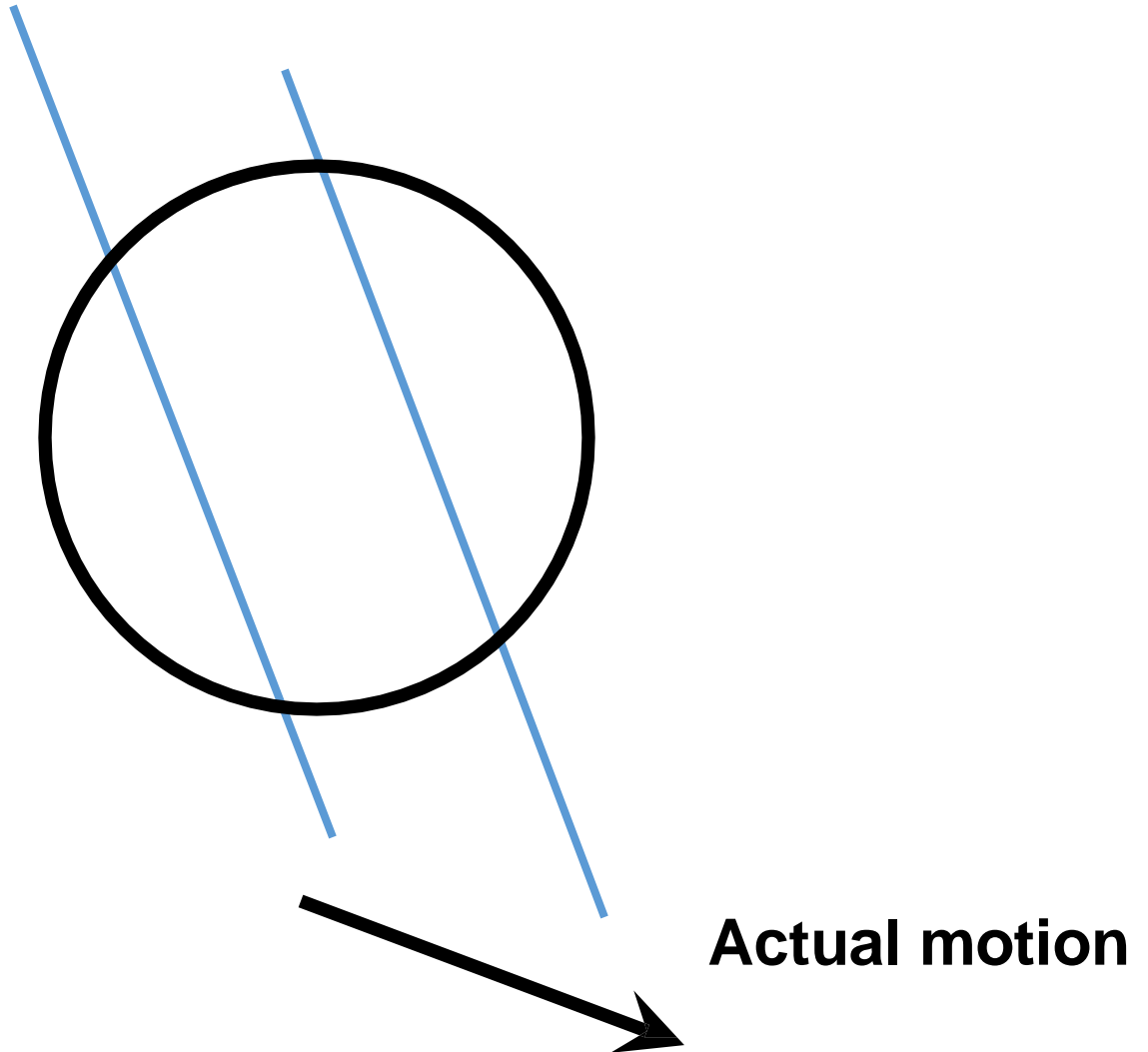
The aperture problem



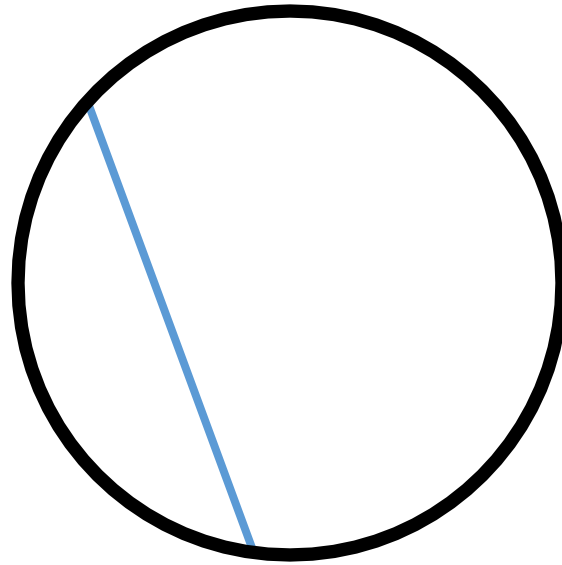
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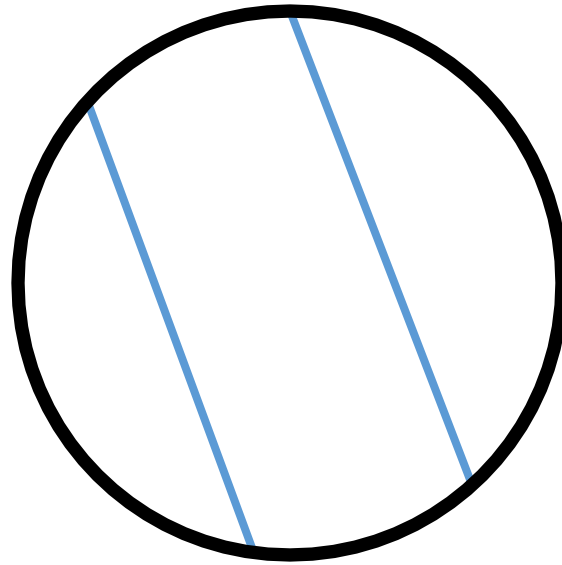
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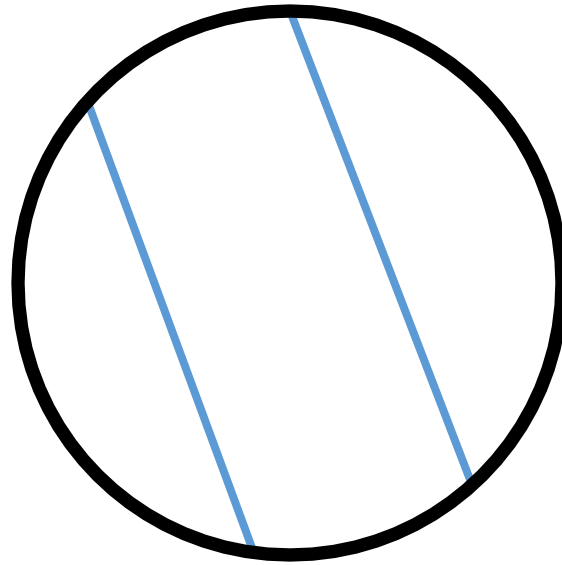
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Perceived motion

Solving the ambiguity...

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 - Assume the pixel's neighbors have the same (u,v)
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Matching patches across images

- Overconstrained linear system

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$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

$A^T A$ $A^T b$

The summations are over all pixels in the $K \times K$ window

Conditions for solvability

Optimal (u, v) satisfies Lucas-Kanade equation

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When is this solvable? i.e., what are good points to track?

- $A^T A$ should be invertible
- $A^T A$ should not be too small due to noise
 - eigenvalues λ_1 and λ_2 of $A^T A$ should not be too small

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Does this remind you of anything?

Criteria for Harris corner detector

Low-texture region



$$\sum \nabla I (\nabla I)^T$$

- gradients have small magnitude
- small λ_1 , small λ_2

Edge



$$\sum \nabla I (\nabla I)^T$$

- gradients very large or very small
- large λ_1 , small λ_2

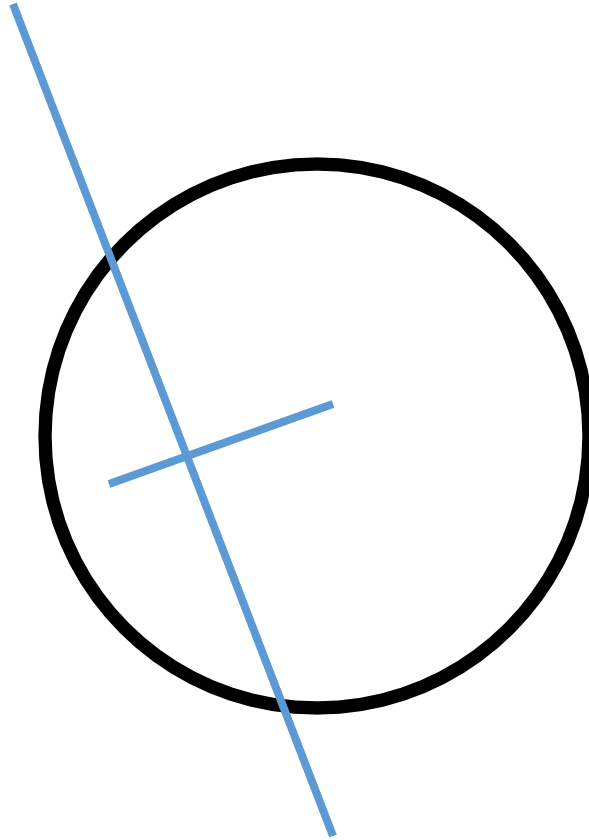
High-texture region



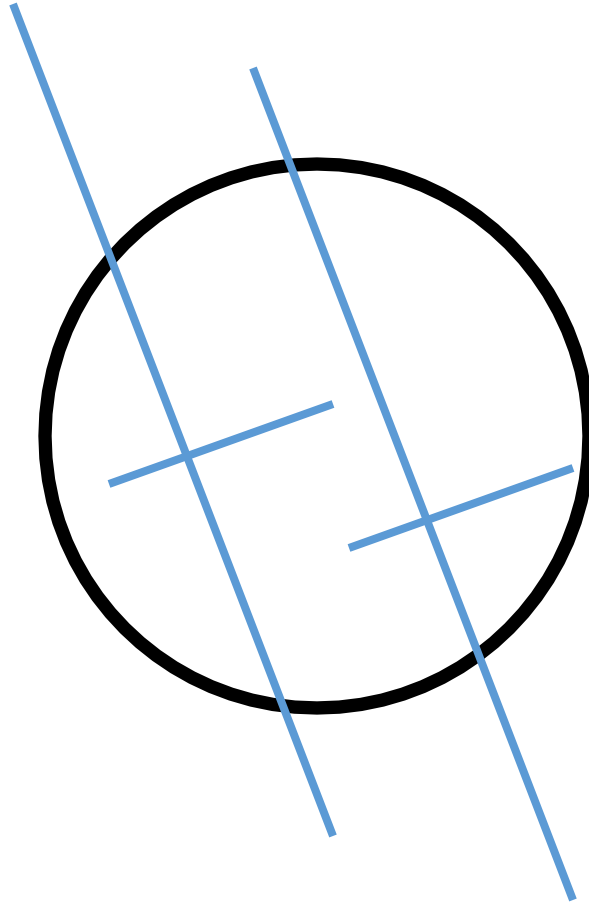
$$\sum \nabla I (\nabla I)^T$$

- gradients are different, large magnitudes
- large λ_1 , large λ_2

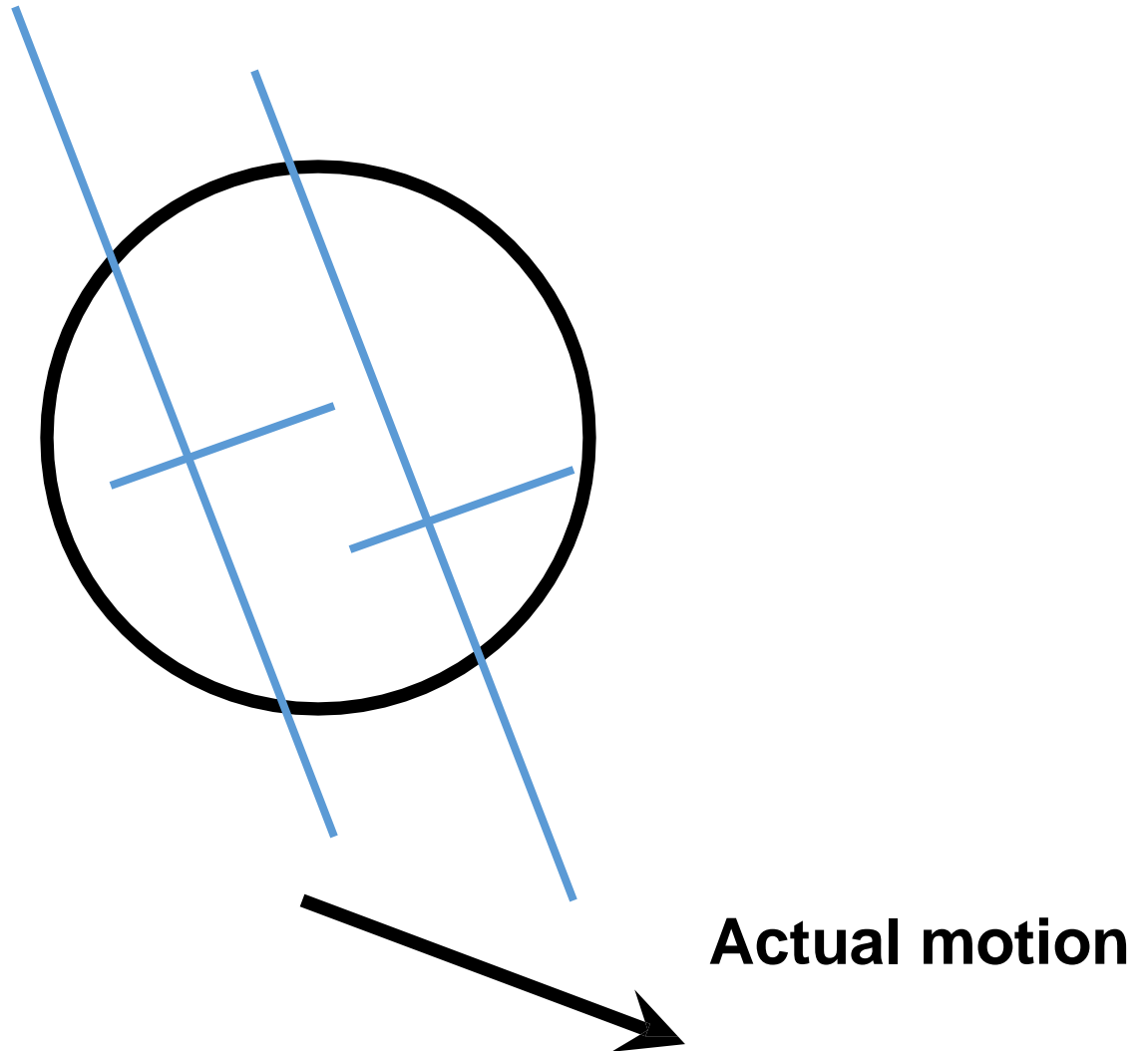
The aperture problem resolved



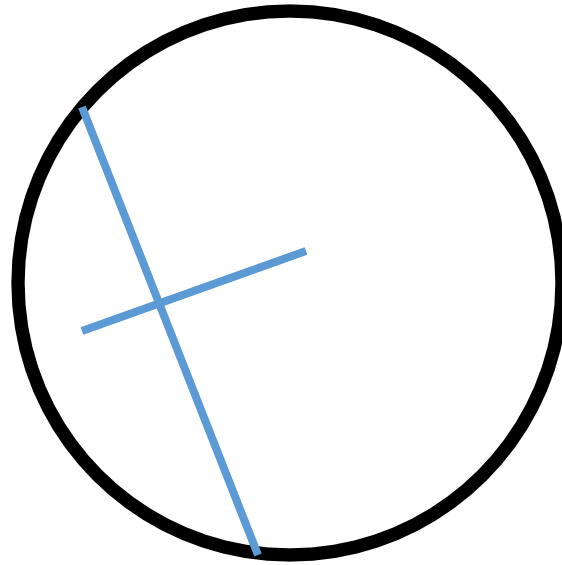
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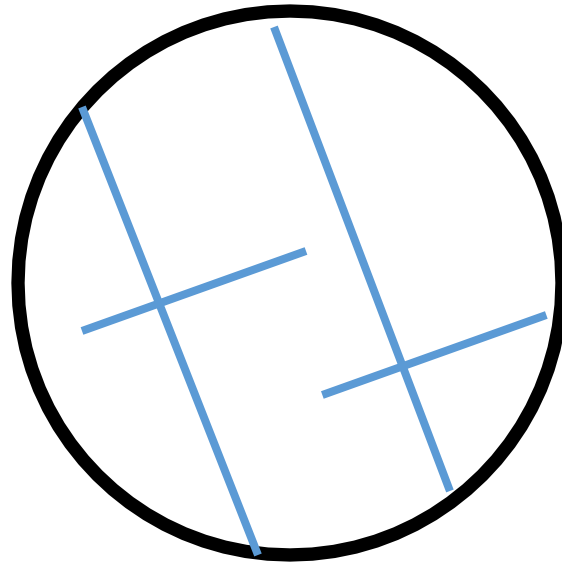
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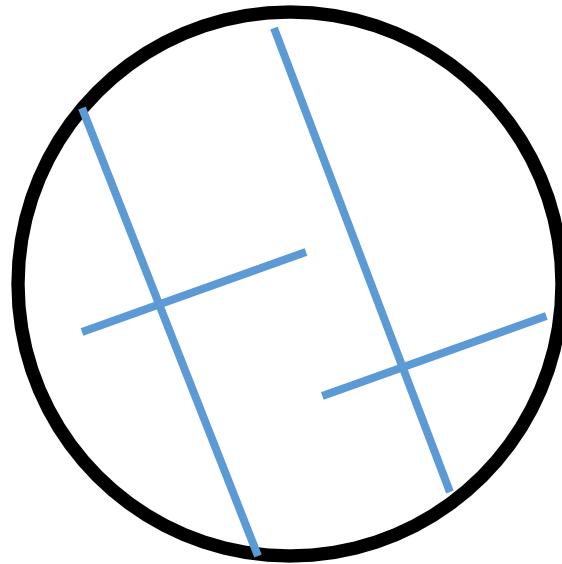
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Perceived motion

Dealing with larger movements: Iterative refinement

Original (x,y) position



1. Initialize $(x', y') = (x, y)$

$$I_t = I(x', y', t+1) - I(x, y, t)$$

Dealing with larger movements: Iterative refinement

Original (x,y) position



1. Initialize $(x', y') = (x, y)$

$$I_t = I(x', y', t+1) - I(x, y, t)$$

2. Compute (u,v) by

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

2nd moment matrix for feature
patch in first image

displacement

Dealing with larger movements: Iterative refinement

Original (x,y) position



1. Initialize $(x', y') = (x, y)$

2. Compute (u,v) by

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

2nd moment matrix for feature patch in first image

displacement

$$I_t = I(x', y', t+1) - I(x, y, t)$$



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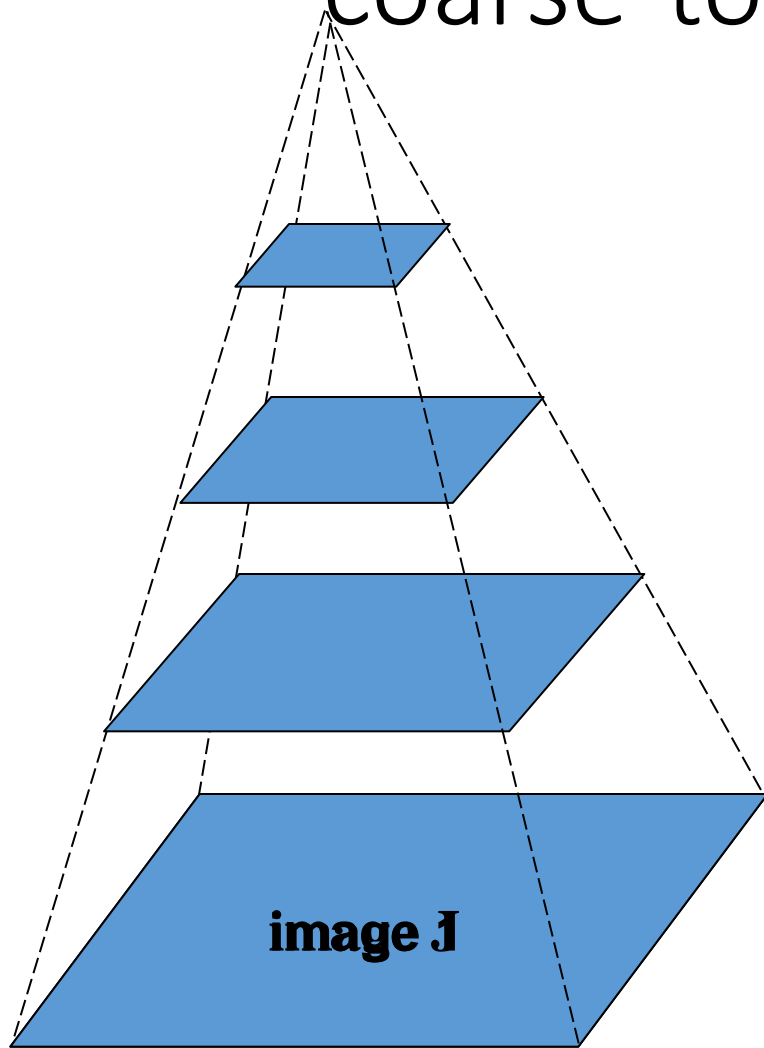
3. Shift window by (u, v) : $x' = x' + u$; $y' = y' + v$;

4. Recalculate I_t

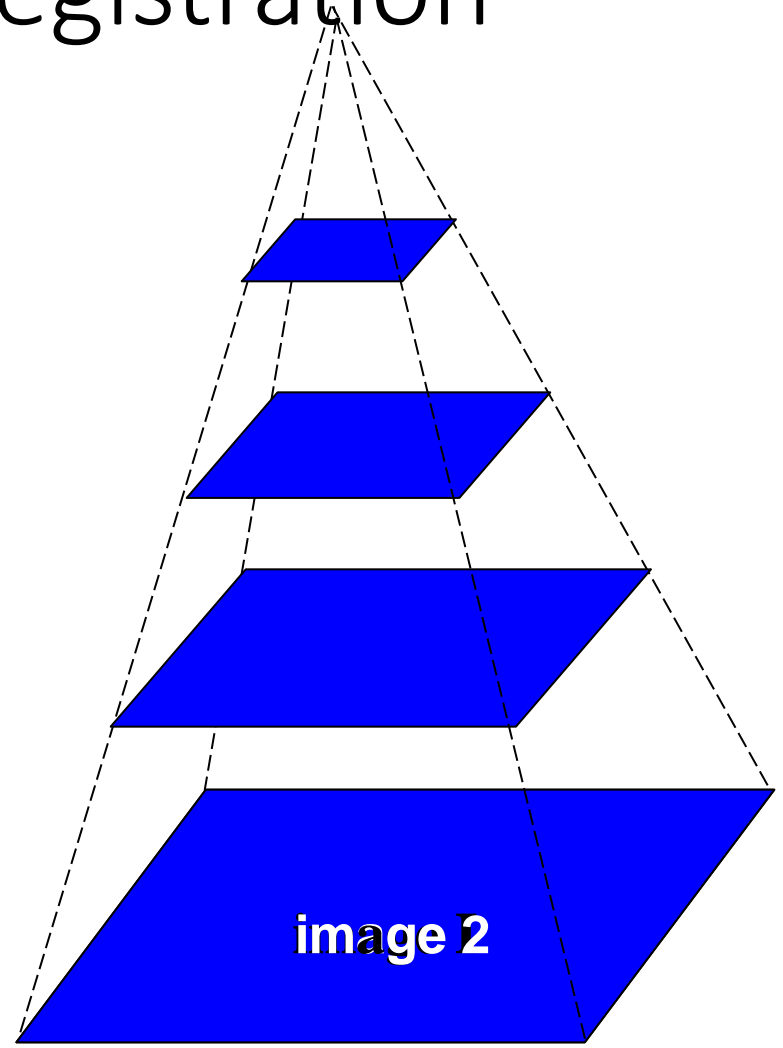
5. Repeat steps 2-4 until small change

- Use interpolation for subpixel values

Dealing with larger movements: coarse-to-fine registration

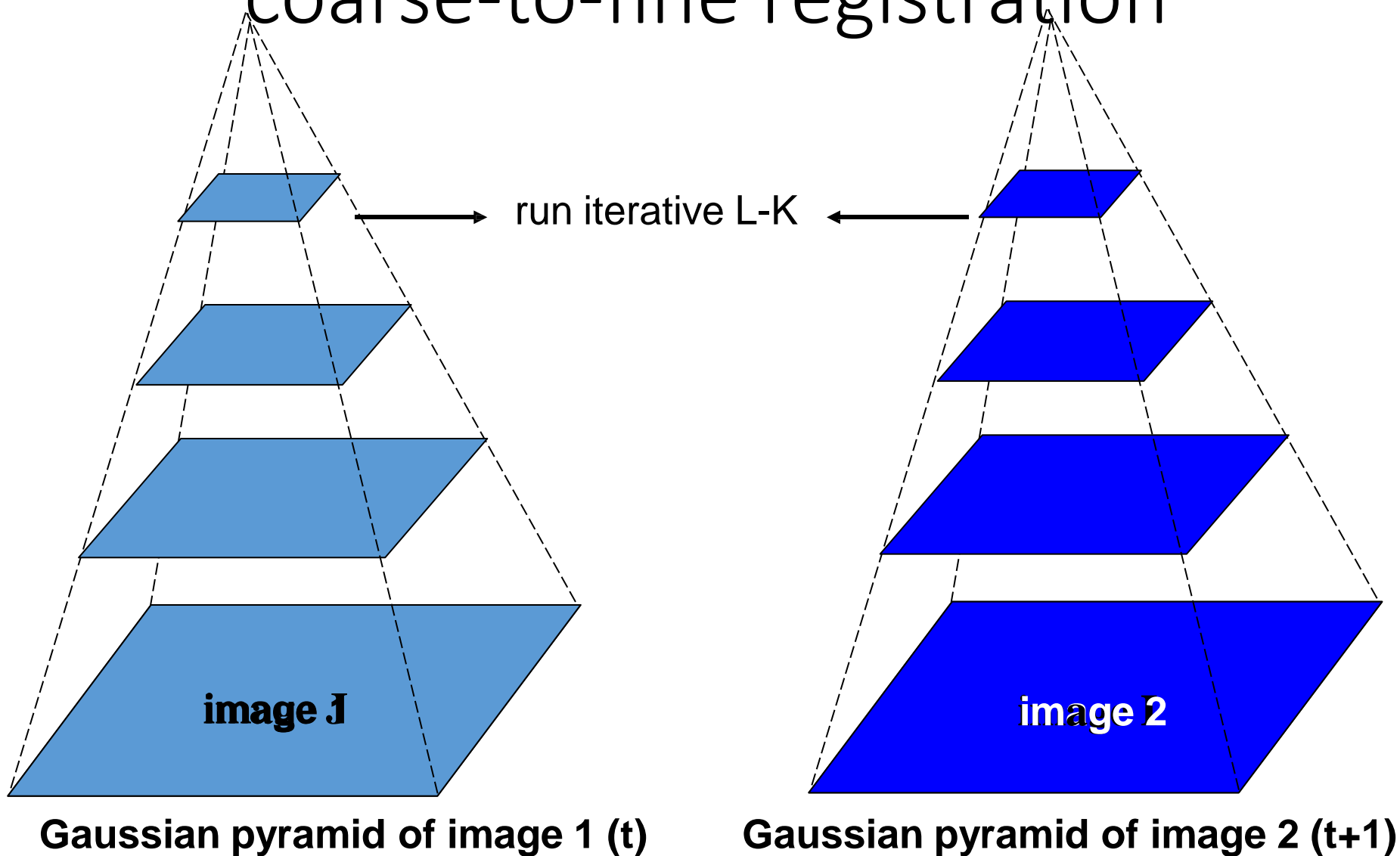


Gaussian pyramid of image 1 (t)

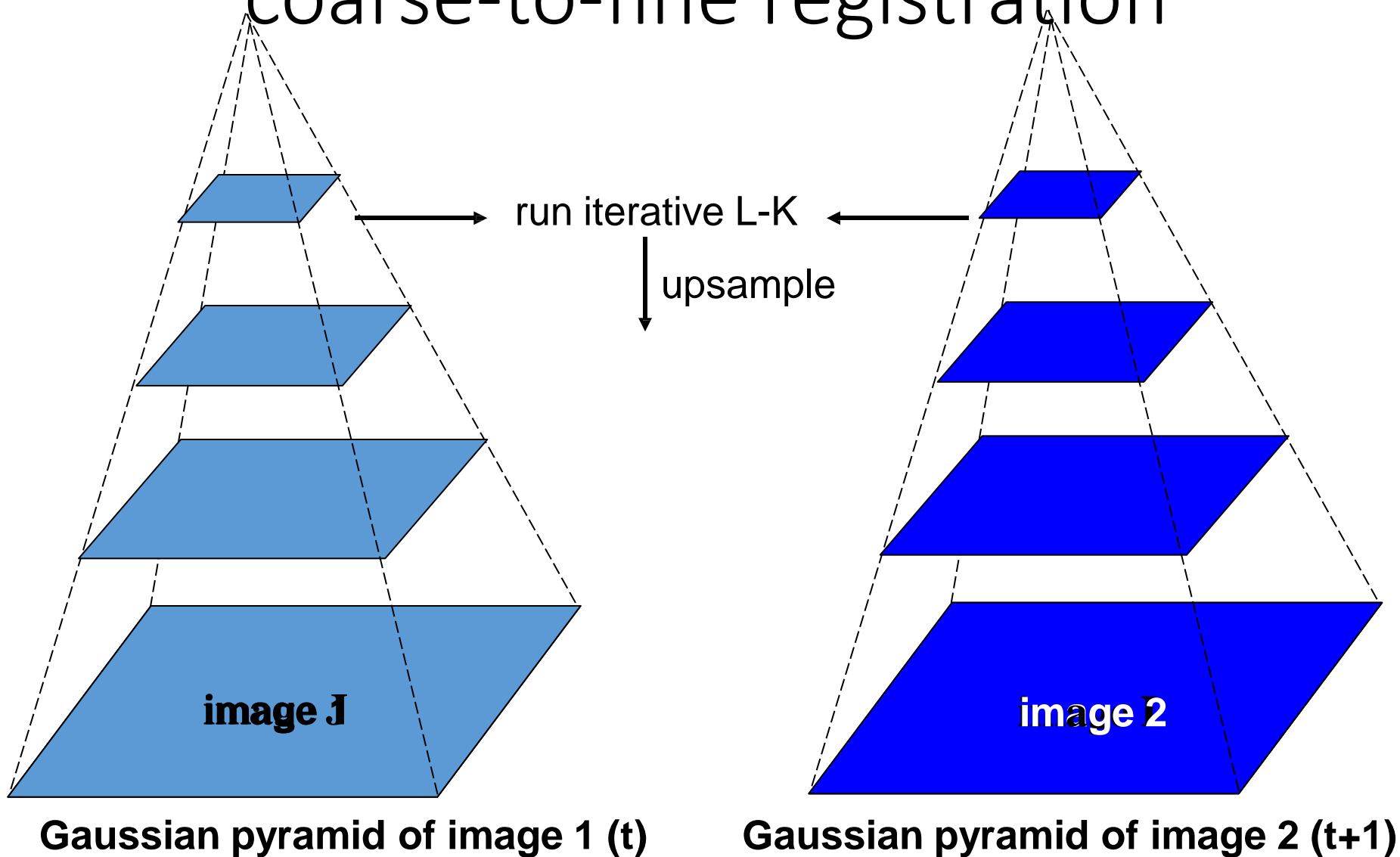


Gaussian pyramid of image 2 (t+1)

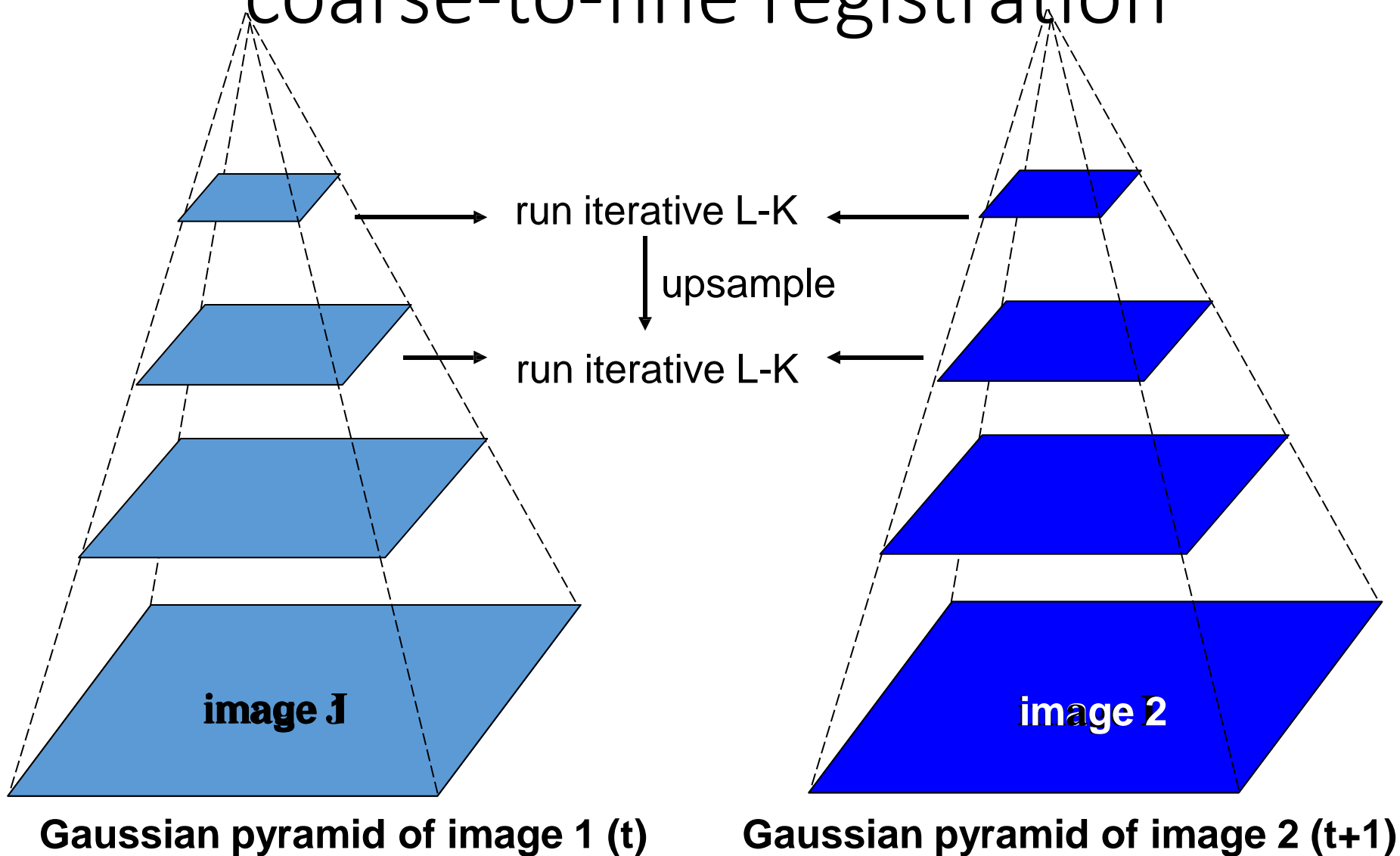
Dealing with larger movements: coarse-to-fine registration



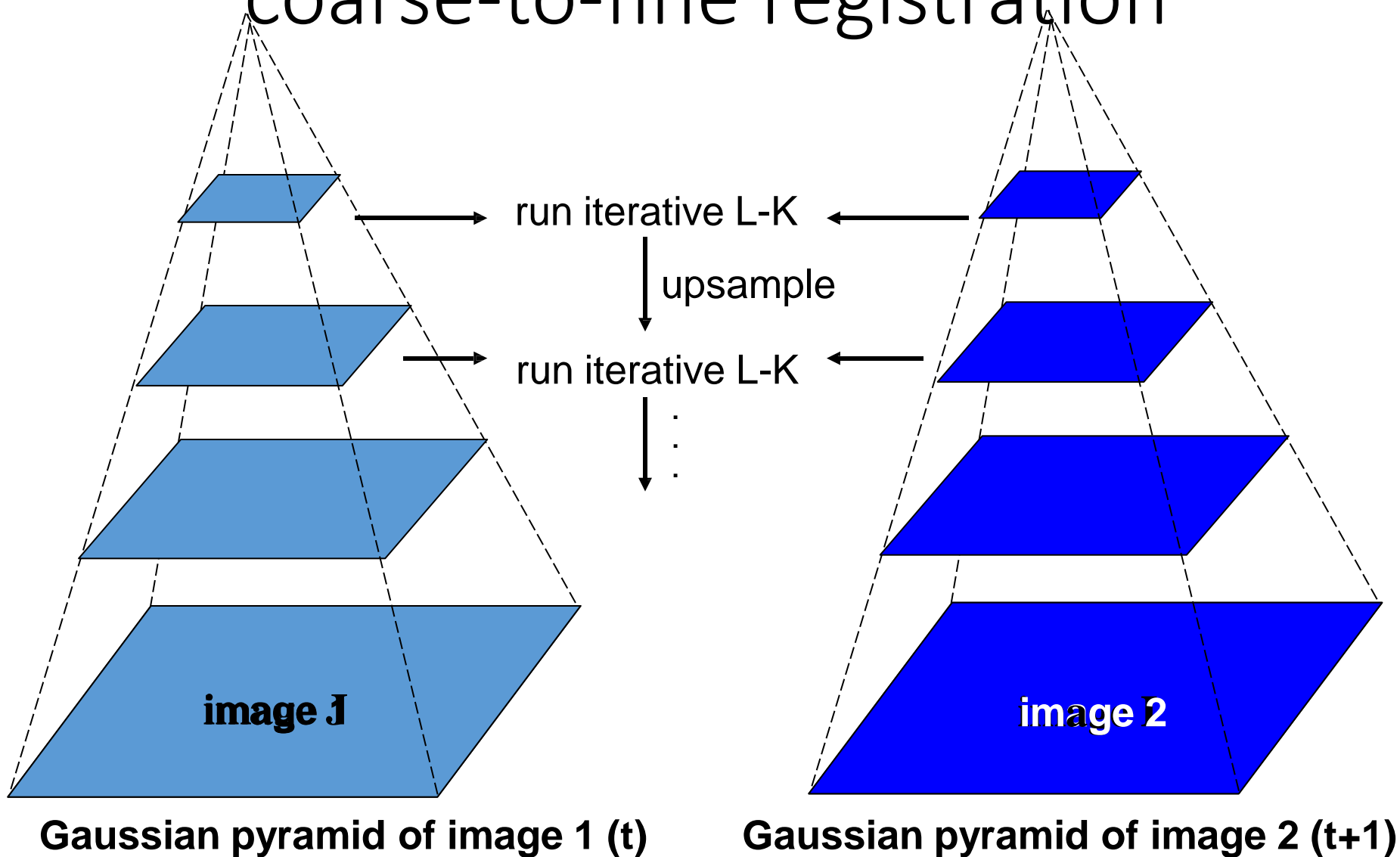
Dealing with larger movements: coarse-to-fine registration



Dealing with larger movements: coarse-to-fine registration



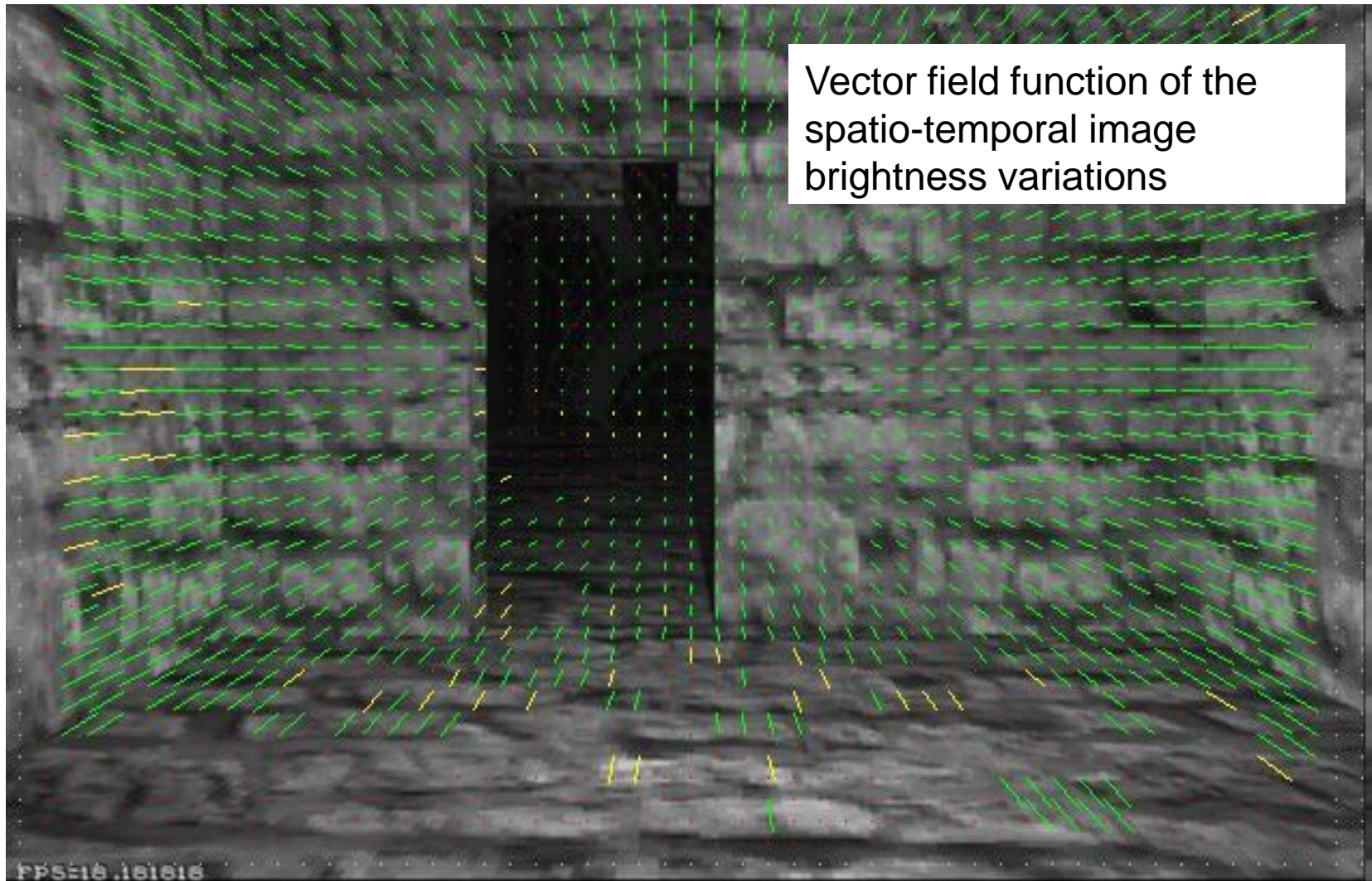
Dealing with larger movements: coarse-to-fine registration



Summary of Lucas & Kanade tracking

- Find a good point to track
- Use intensity second moment matrix and difference across frames to find displacement
- Iterate and use coarse-to-fine search to deal with larger movements
- When creating long tracks, check appearance of registered patch against appearance of initial patch to find points that have drifted

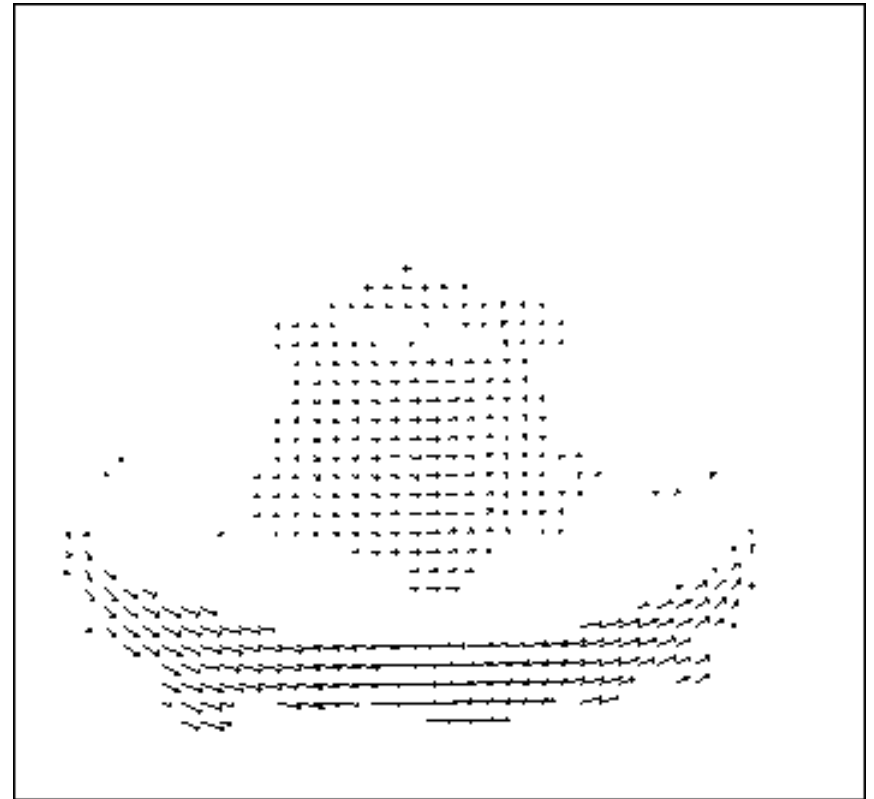
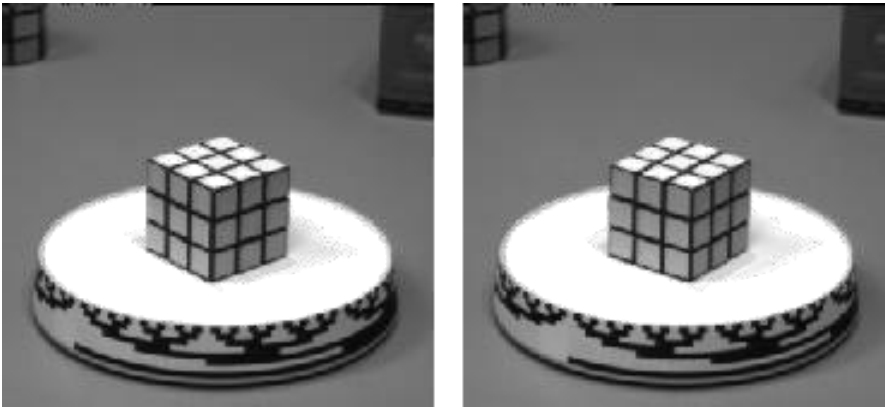
Optical flow



Picture courtesy of Selim Temizer - Learning and Intelligent Systems (LIS) Group, MIT

Motion field

- The motion field is the projection of the 3D scene motion into the image



Optical flow

- Definition: optical flow is the *apparent* motion of brightness patterns in the image

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- Have to be careful: apparent motion can be caused by lighting changes without any actual motion
 - Think of a uniform rotating sphere under fixed lighting vs. a stationary sphere under moving illumination

Lucas-Kanade Optical Flow

- Same as Lucas-Kanade feature tracking, but for each pixel
 - As we saw, works better for textured pixels
- Operations can be done one frame at a time, rather than pixel by pixel
 - Efficient

Errors in Lucas-Kanade

- The motion is large

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 - Possible Fix: Region-based matching
- Brightness constancy does not hold
 - Possible Fix: Gradient constancy

State-of-the-art optical flow

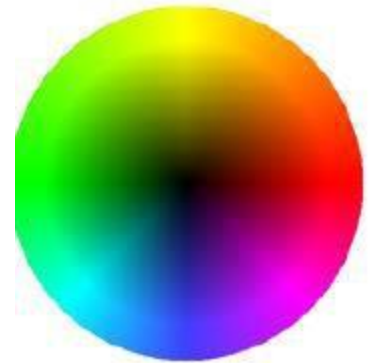
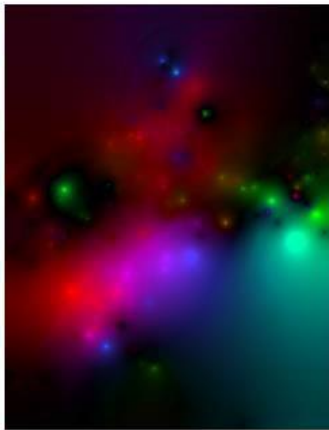
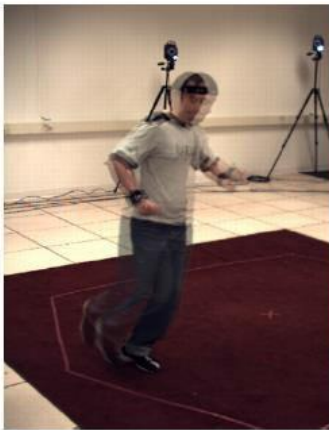
Start with something similar to Lucas-Kanade

+ gradient constancy

+ energy minimization with smoothing term

+ region matching

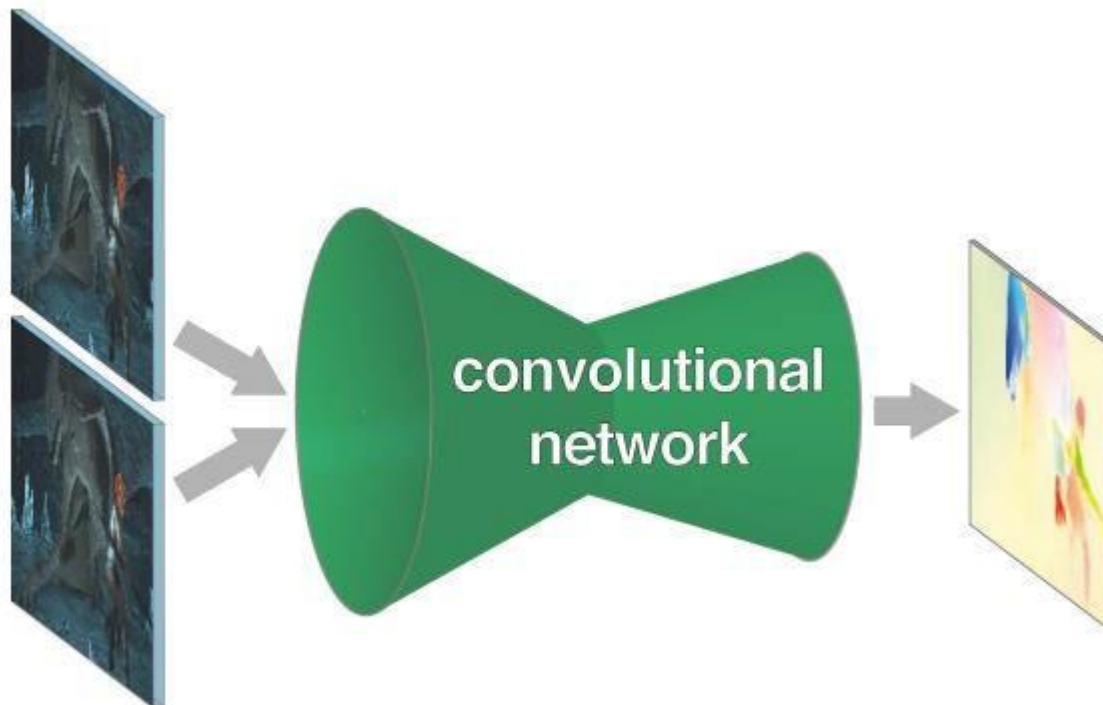
+ keypoint matching (long-range)



Region-based + Pixel-based + Keypoint-based

[Large displacement optical flow](#), Brox et al., CVPR 2009

Recent Trends



[DeepFlow: Large displacement optical flow with deep matching.](#) ICCV 2013

[FlowNet: Learning Optical Flow with Convolutional Networks.](#) ICCV 2015

[Flow fields: Dense correspondence fields for highly accurate large displacement optical flow estimation.](#) ICCV 2015

[A large dataset to train convolutional networks for disparity, optical flow, and scene flow estimation.](#) CVPR 2016

[FlowNet 2.0: Evolution of Optical Flow Estimation with Deep Networks.](#) CVPR 2017

[Optical flow estimation using a spatial pyramid network.](#) CVPR 2017

[Unsupervised Deep Learning for Optical Flow Estimation.](#) AAAI 2017

[Semi-supervised learning for optical flow with generative adversarial networks.](#) NIPS 2017

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