Computer Vision
Projection Matrix and Camera Calibration

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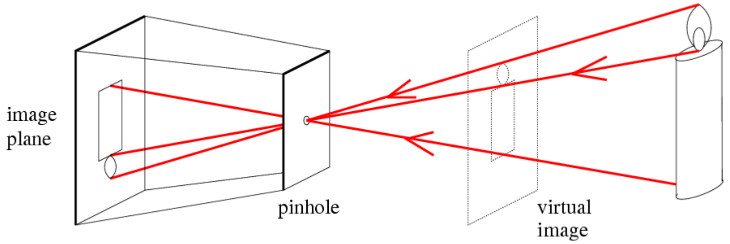
Last Class

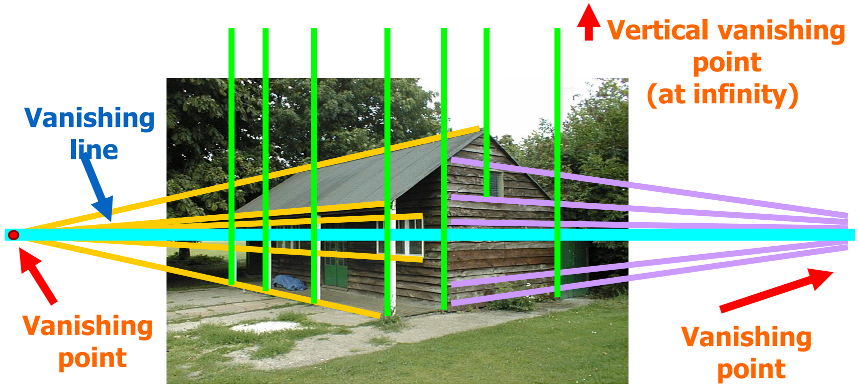
- Pinhole camera model



- Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Vanishing points and vanishing lines



Perspective and 3D Geometry

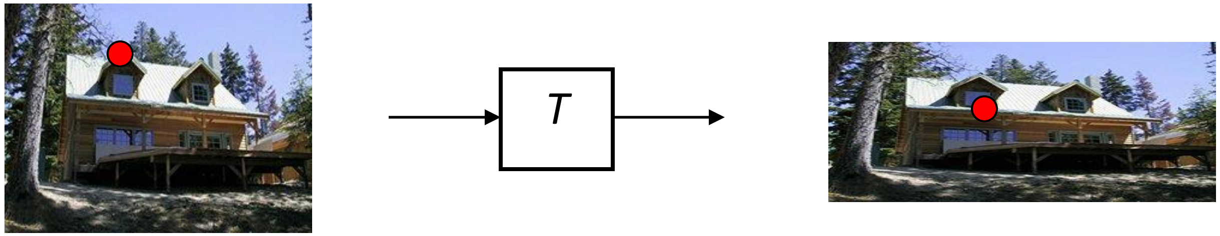
- **Camera models and Projective geometry**
  - What's the mapping between image and world coordiantes?
- **Projection Matrix and Camera calibration**
  - What's the projection matrix between scene and image coordiantes?
  - How to calibrate the projection matrix?
- **Single view metrology and Camera properties**
  - How can we measure the size of 3D objects in an image?
  - What are the important camera properties?
- **Photo stitching**
  - What's the mapping from two images taken without camera translation?
- **Epipolar Geometry and Stereo Vision**
  - What's the mapping from two images taken with camera translation?
- **Structure from motion**
  - How can we recover 3D points from multiple images?

This class

- Common Transformations
- What is the relation between scene point and image point in form of matrix?
- How can we calibrate the camera?

Common Transformations

Parametric (global) warping



p = (x,y) **p'** = (x',y')

Transformation T is a coordinate-changing machine:

$$p' = T(p)$$

For linear transformations, we can represent T as a matrix

$$p' = \mathbf{T}p$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

Common Transformations

original

Transformed

translation

rotation

aspect

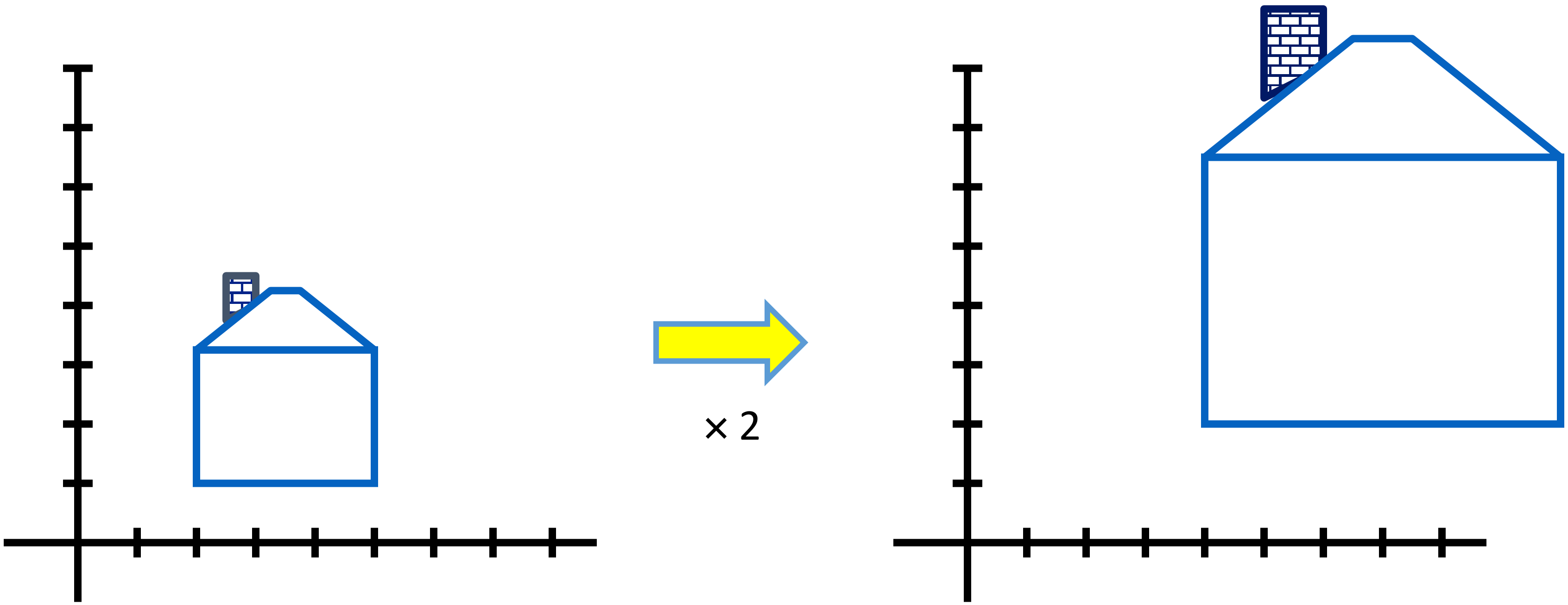
affine

perspective

Slide credit (next few slides):
A. Efros and/or S. Seitz

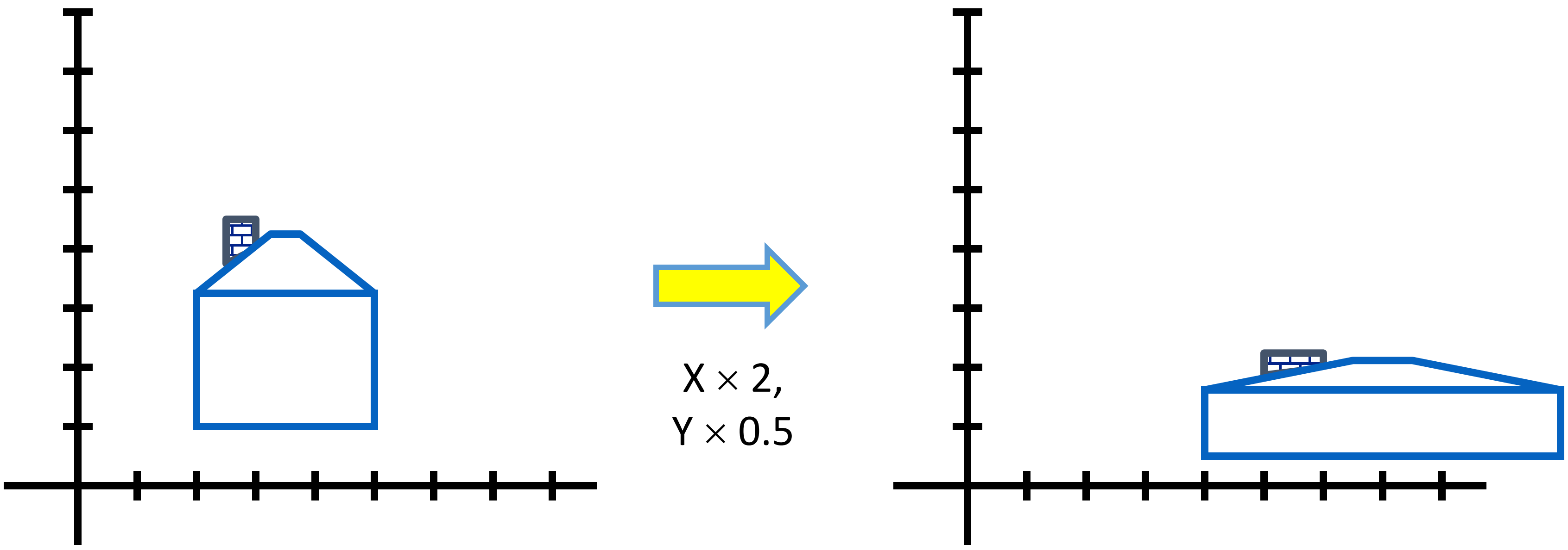
Scaling

- *Scaling* a coordinate means multiplying each of its components by a scalar
- *Uniform scaling* means this scalar is the same for all components:



Scaling

- *Non-uniform scaling*: different scalars per component:



Scaling

- Scaling operation:

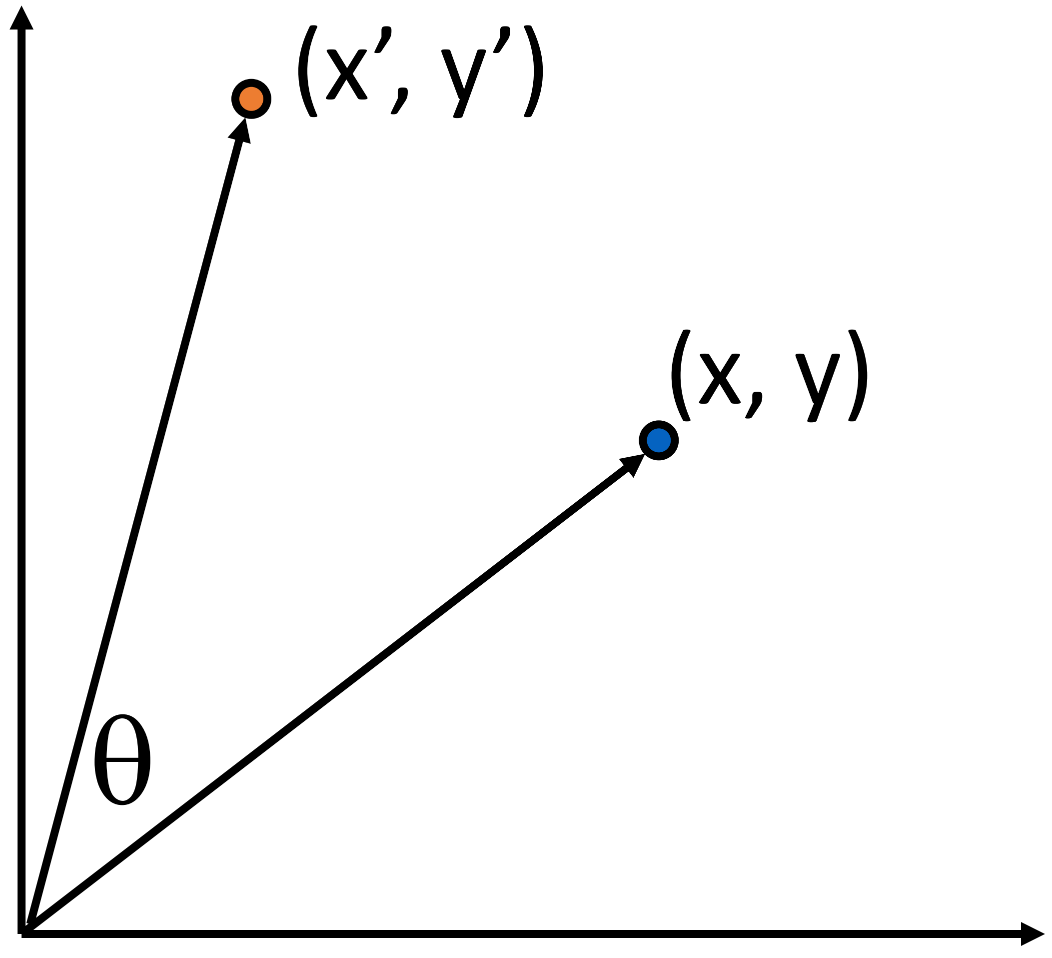
$$x' = ax$$

$$y' = by$$

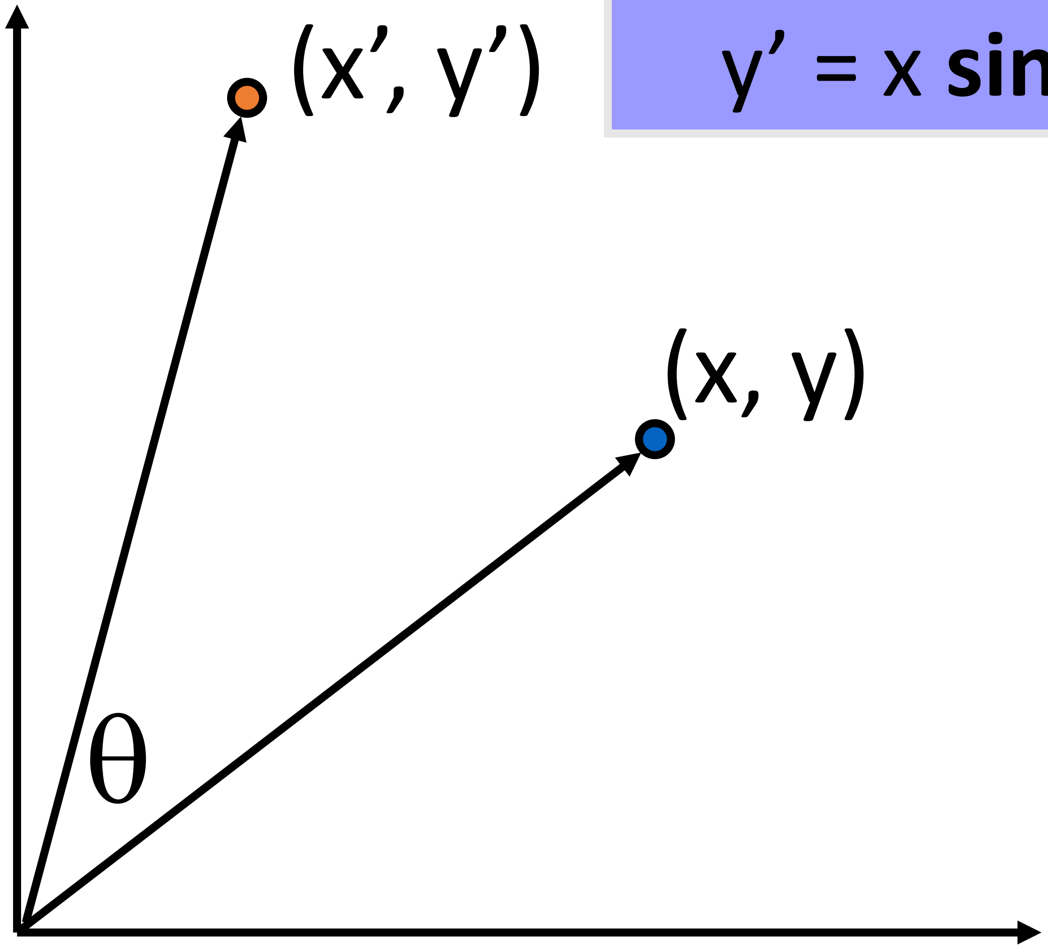
- Or, in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\textit{scaling matrix } S} \begin{bmatrix} x \\ y \end{bmatrix}$$

2-D Rotation

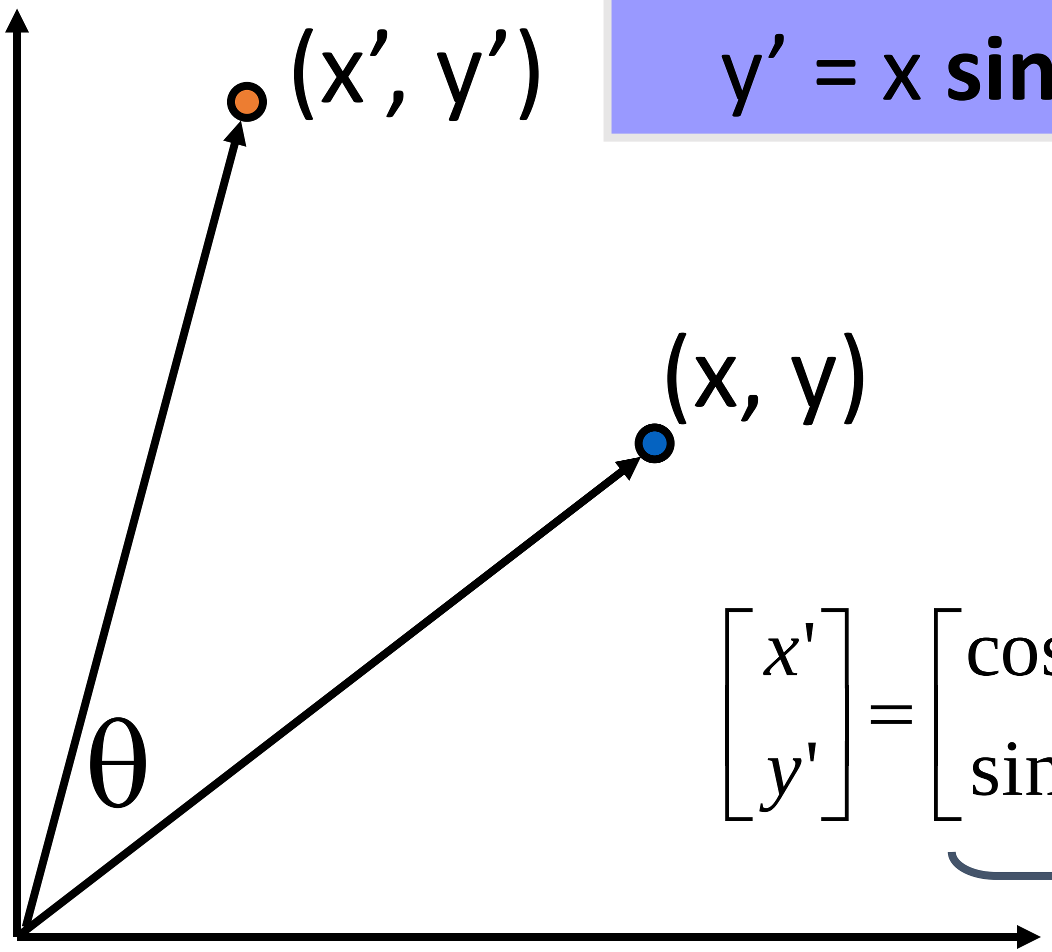


2-D Rotation



$$x' = x\ \mathbf{cos}(\theta) - y\ \mathbf{sin}(\theta)$$
$$y' = x\ \mathbf{sin}(\theta) + y\ \mathbf{cos}(\theta)$$

2-D Rotation



$$x' = x\ \mathbf{cos}(\theta) - y\ \mathbf{sin}(\theta)$$
$$y' = x\ \mathbf{sin}(\theta) + y\ \mathbf{cos}(\theta)$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Basic 2D transformations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & \alpha_x \\ \alpha_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Shear

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine

Affine is any combination of translation, scale, rotation, shear

Affine Transformations

Affine transformations are combinations of

- Linear transformations, and
- Translations

Properties of affine transformations:

- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

or

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Projective Transformations

Projective transformations are combos of

- Affine transformations, and
- Projective warps

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

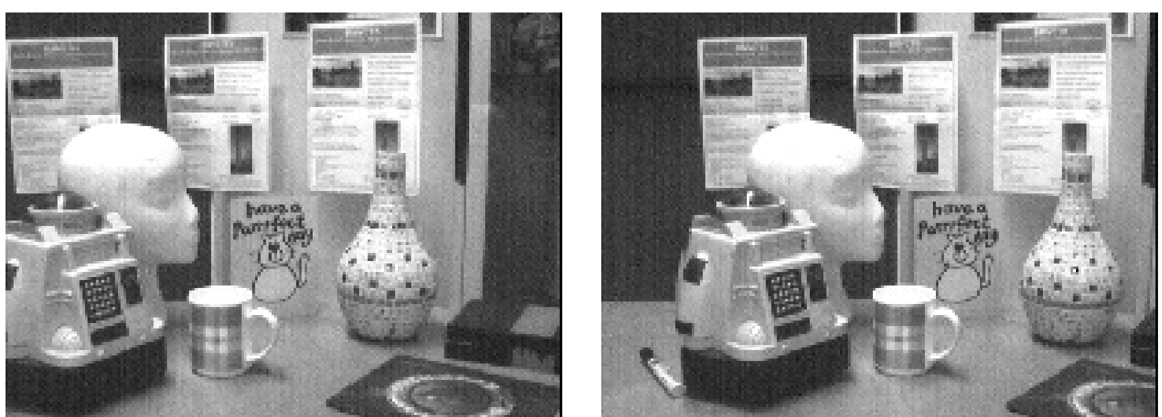
Properties of projective transformations:

- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Projective matrix is defined up to a scale (8 DOF)

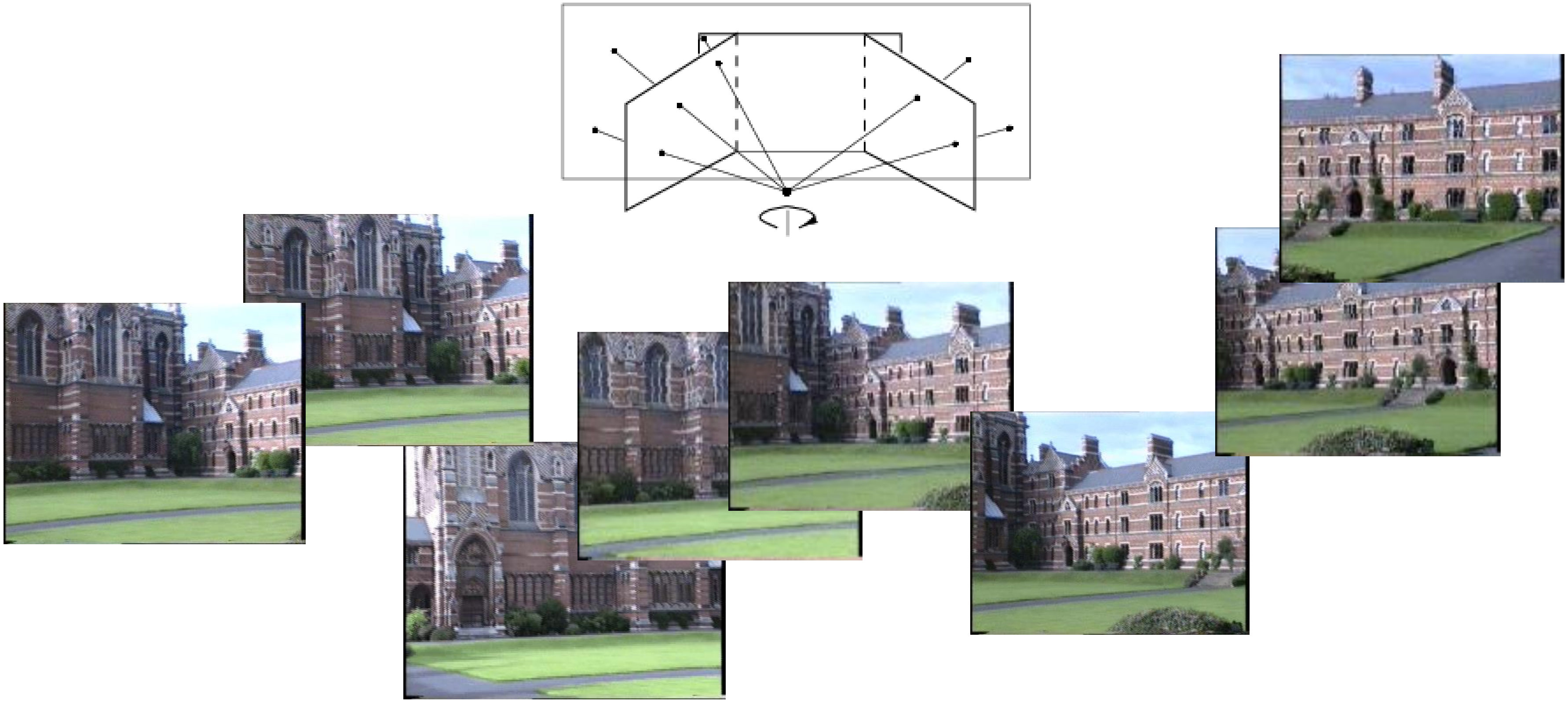
Projective Transformations (homography)

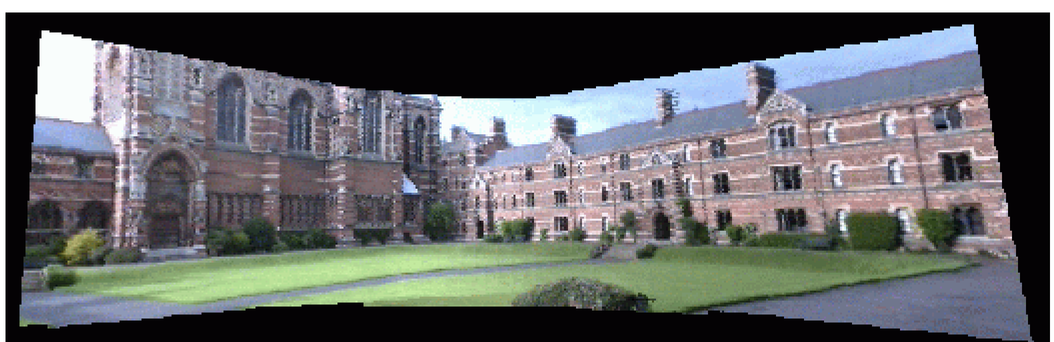
- The transformation between two views of a planar surface

- The transformation between images from two cameras that share the same center



Application: Panorama stitching

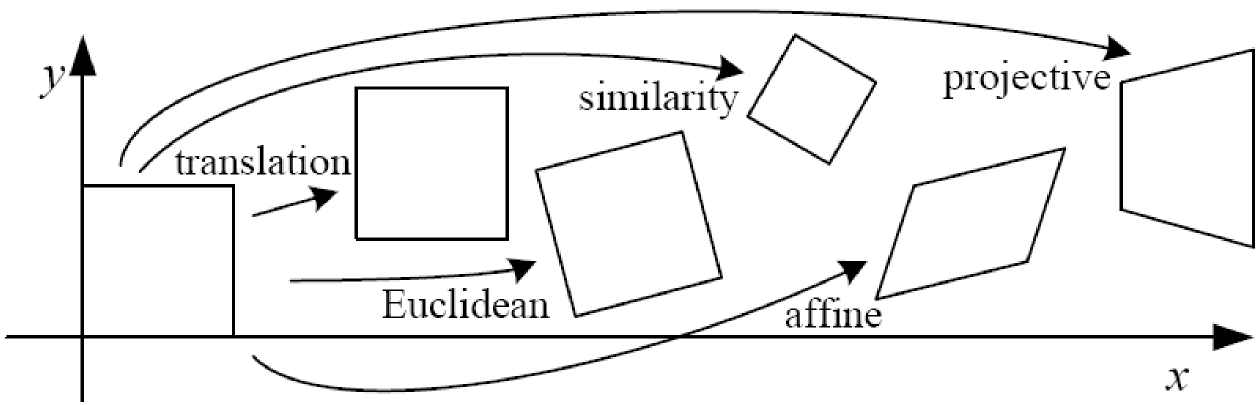




Source: Hartley & Zisserman

Application: document scanning

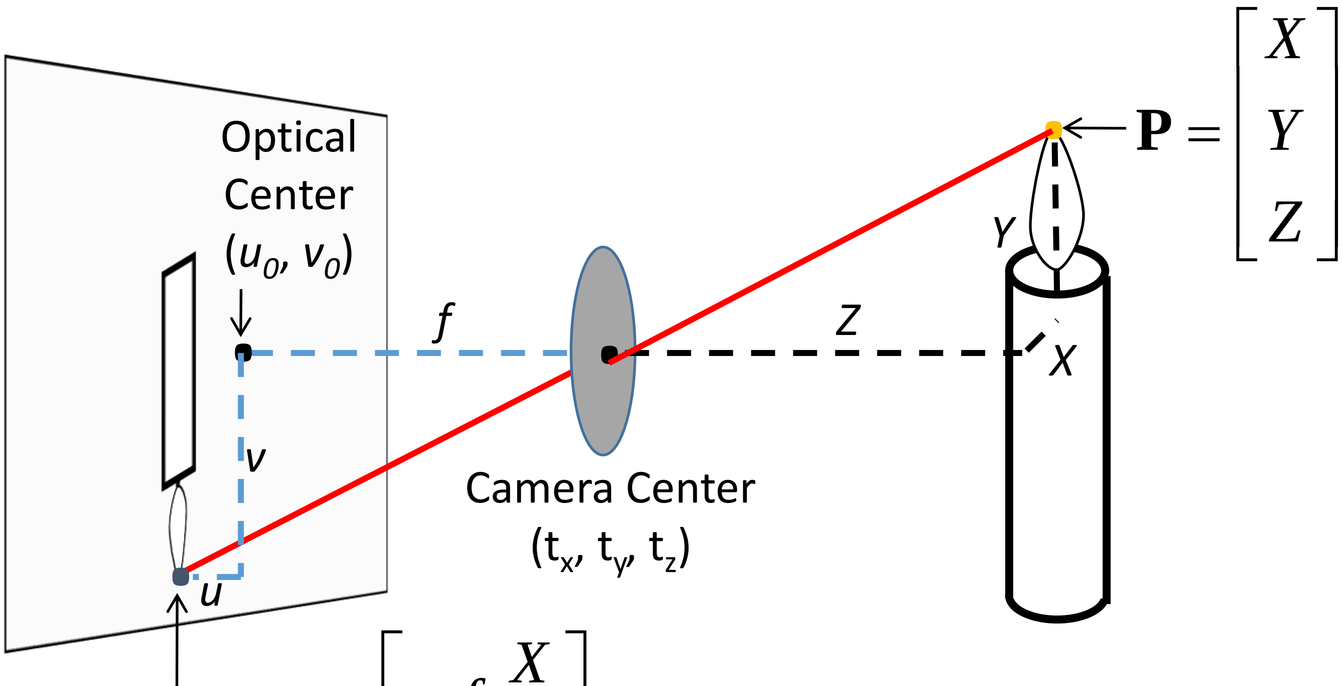
2D image transformations (reference table)



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\left[\begin{array}{c\|c} \boldsymbol{I} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	2	orientation $+\cdots$	
rigid (Euclidean)	$\left[\begin{array}{c\|c} \boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	3	lengths $+\cdots$	
similarity	$\left[\begin{array}{c\|c} s\boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	4	angles $+\cdots$	
affine	$\left[\begin{array}{c} \boldsymbol{A} \end{array}\right]_{2\times 3}$	6	parallelism $+\cdots$	
projective	$\left[\begin{array}{c} \tilde{\boldsymbol{H}} \end{array}\right]_{3\times 3}$	8	straight lines	

What is the relation between scene point and image point in form of matrix?

Projection:
world coordinates → image coordinates



$$\mathbf{p} = \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -f\dfrac{X}{Z} \\ -f\dfrac{Y}{Z} \end{bmatrix}$$

Perspective Projection Matrix

- Projection is a matrix multiplication using homogeneous coordinates

$$\begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \quad \\ \\ \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z}\right)$$

divide by the third coordinate

Perspective Projection Matrix

- Projection is a matrix multiplication using homogeneous coordinates

$$\begin{bmatrix} \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z}\right)$$

divide by the third coordinate

Perspective Projection Matrix

- Projection is a matrix multiplication using homogeneous coordinates

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \qquad \Rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

divide by the third coordinate

Perspective Projection Matrix

- Projection is a matrix multiplication using homogeneous coordinates

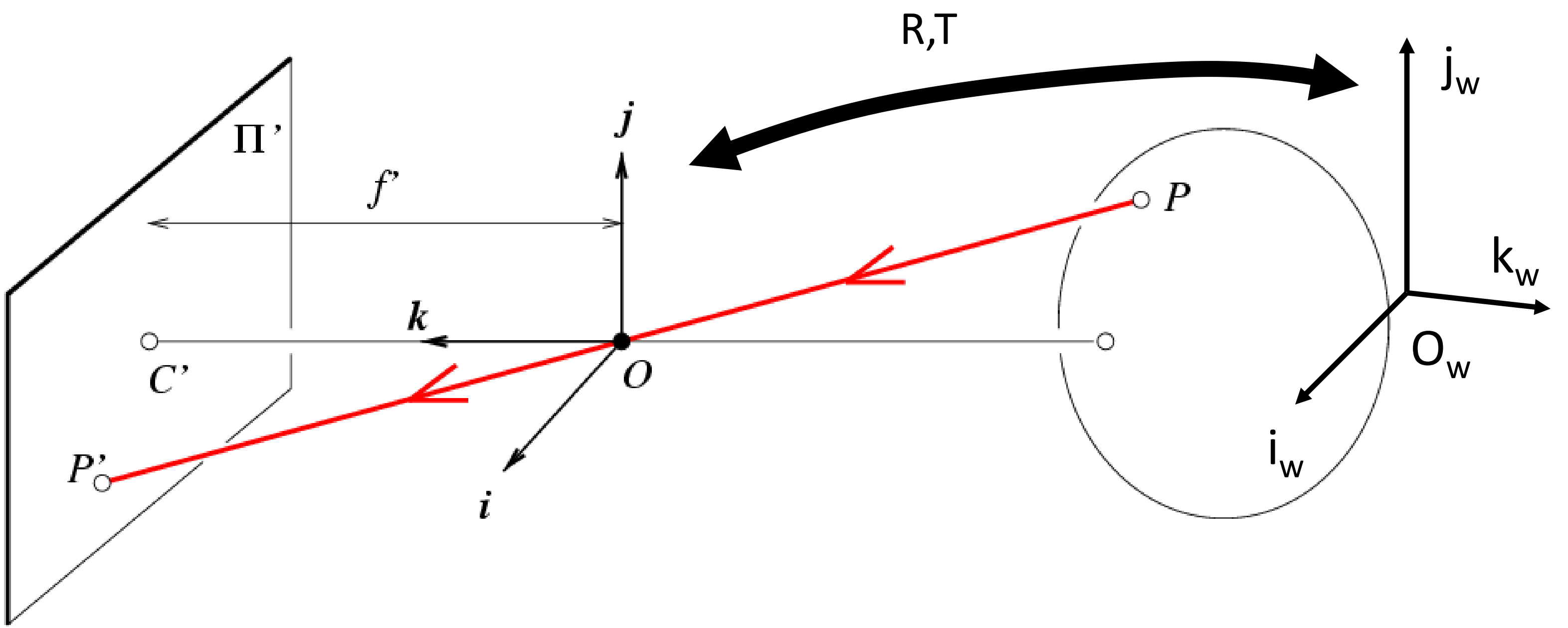
$$
\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
=
\begin{bmatrix} fx \\ fy \\ z \end{bmatrix}
\qquad \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z} \right)
$$

divide by the third coordinate

In practice: lots of coordinate transformations…

$$
\begin{pmatrix} \text{2D} \\ \text{point} \\ \text{(3x1)} \end{pmatrix}
=
\begin{pmatrix} \text{Camera to} \\ \text{pixel coord.} \\ \text{trans. matrix} \\ \text{(3x3)} \end{pmatrix}
\begin{pmatrix} \text{Perspective} \\ \text{projection matrix} \\ \text{(3x3)} \end{pmatrix}
\begin{pmatrix} \text{World to} \\ \text{camera coord.} \\ \text{trans. matrix} \\ \text{(3x4)} \end{pmatrix}
\begin{pmatrix} \text{3D} \\ \text{point} \\ \text{(4x1)} \end{pmatrix}
$$

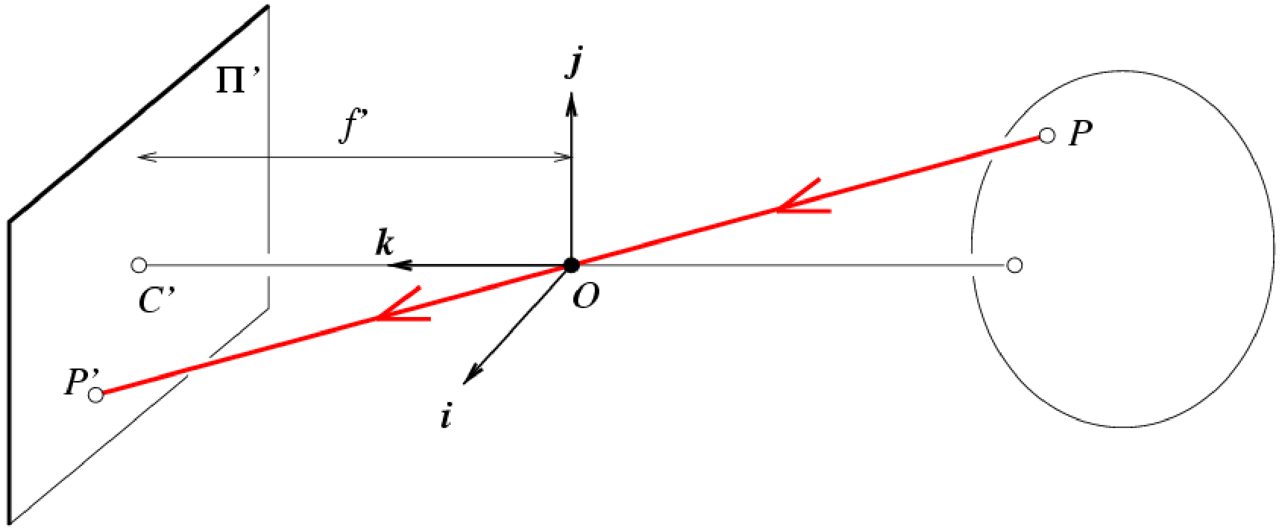
Projection matrix



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

x: Image Coordinates: (u,v,1)
K: Intrinsic Matrix (3x3)
R: Rotation (3x3)
t: Translation (3x1)
X: World Coordinates: (X,Y,Z,1)

Projection matrix under assumptions



Intrinsic Assumptions

- Unit aspect ratio
- No skew
- Optical center at (0,0)
  (i.e., at image origin)

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\left[\mathbf{I} \quad \mathbf{0}\right]\mathbf{X} \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \boxed{\begin{matrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{matrix}} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

K

Slide Credit: Saverese

Remove assumption: known optical center

Intrinsic Assumptions
- Unit aspect ratio
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \;\Rightarrow\; w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}f & 0 & u_0 & 0\\0 & f & v_0 & 0\\0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Remove assumption: square pixels

Intrinsic Assumptions
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}\alpha & 0 & u_0 & 0\\ 0 & \beta & v_0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Remove assumption: non-skewed pixels

Intrinsic Assumptions

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}\alpha & s & u_0 & 0\\0 & \beta & v_0 & 0\\0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Note: different books use different notation for parameters

Remove assumption: Camera translation

Intrinsic Assumptions

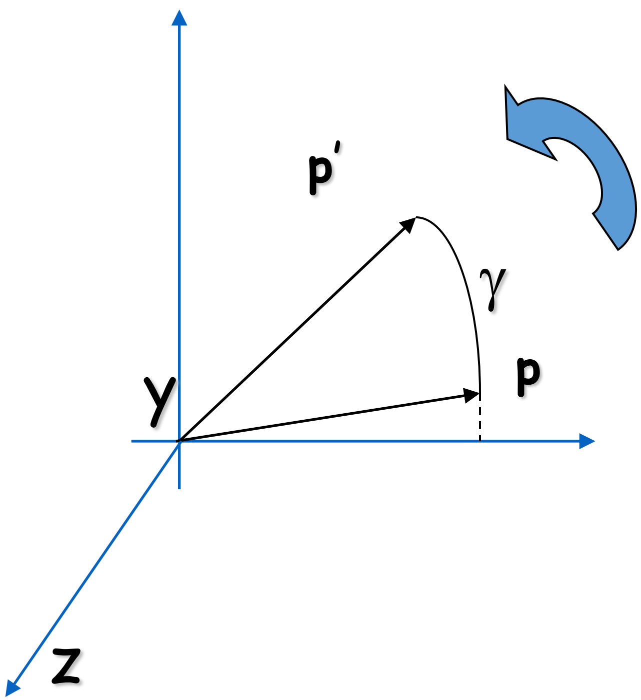
Extrinsic Assumptions
- No rotation

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{t}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}\alpha & s & u_0\\0 & \beta & v_0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & t_x\\0 & 1 & 0 & t_y\\0 & 0 & 1 & t_z\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Slide Credit: Saverese

3D Rotation of Points

Rotation around the coordinate axes, counter-clockwise:



$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Remove all assumption: Camera rotation also

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Remove all assumption: Camera rotation also

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}\alpha & s & u_0\\0 & \beta & v_0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}r_{11} & r_{12} & r_{13} & t_x\\r_{21} & r_{22} & r_{23} & t_y\\r_{31} & r_{32} & r_{33} & t_z\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Aspect Ratio

$$w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}f & s & u_0\\0 & \alpha f & v_0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}r_{11} & r_{12} & r_{13} & t_x\\r_{21} & r_{22} & r_{23} & t_y\\r_{31} & r_{32} & r_{33} & t_z\end{bmatrix}\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$$

Degrees of freedom

$$\mathbf{x} = \mathbf{K}\left[\mathbf{R} \quad \mathbf{t}\right]\mathbf{X}$$

5 **6**

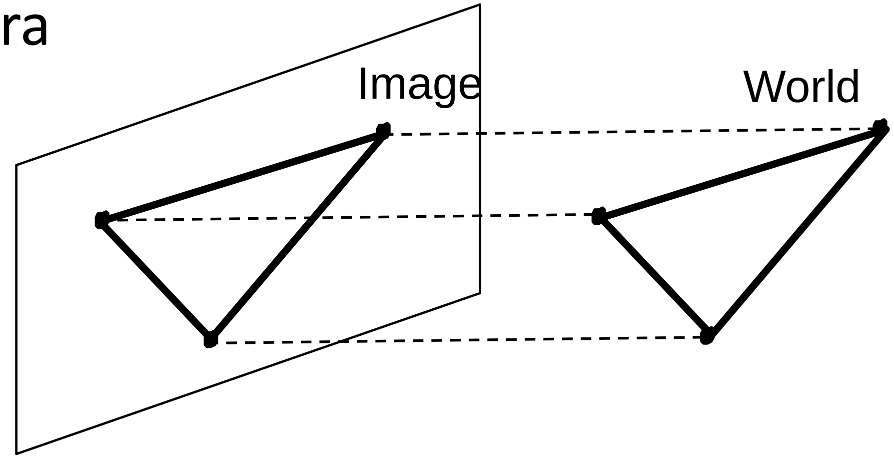
$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Aspect Ratio

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & s & u_0 \\ 0 & \alpha f & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Orthographic Projection

- Object dimensions are small compared to distance to camera



- Good approximation for telephoto optics
- Also called "parallel projection": (x, y, z) → (x, y)
- What's the projection matrix?

$$\begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Orthographic Projection

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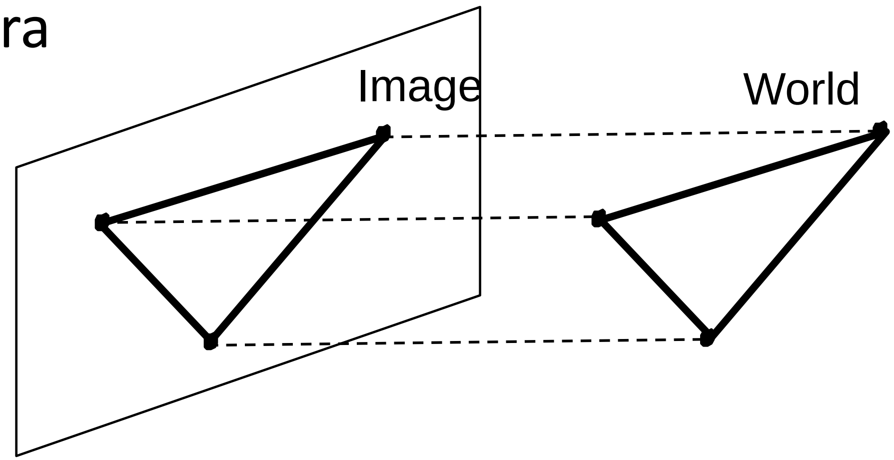

- Good approximation for telephoto optics
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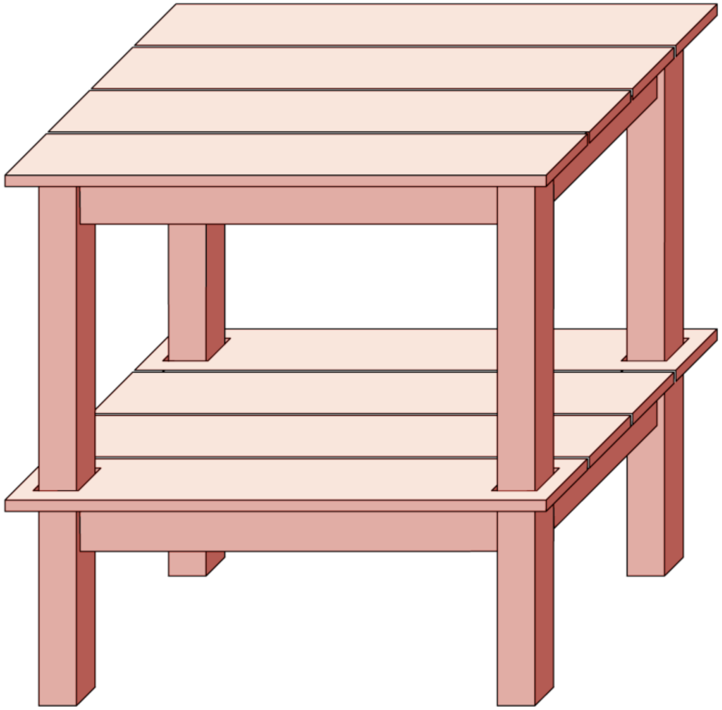
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Orthographic Projection



- What happens to parallel lines?
- What happens to angles?
- What happens to distances?

How can we calibrate the camera?

How to calibrate the camera?

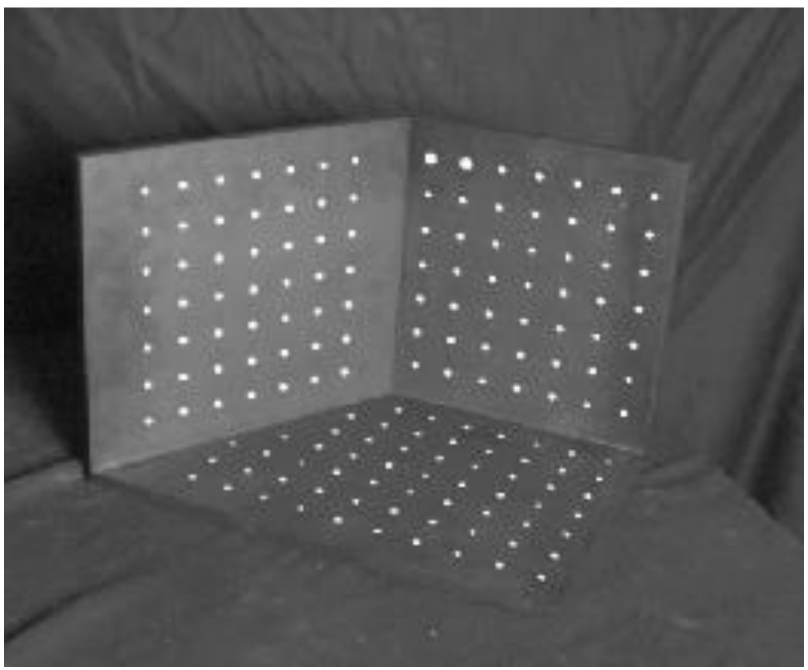
$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\begin{bmatrix} wu \\ wv \\ w \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Calibrating the Camera

Method 1: Use an object (calibration grid) with known geometry

- Correspond image points to 3d points
- Get least squares solution (or non-linear solution)



$$\begin{bmatrix} wu \\ wv \\ w \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Calibrating the Camera

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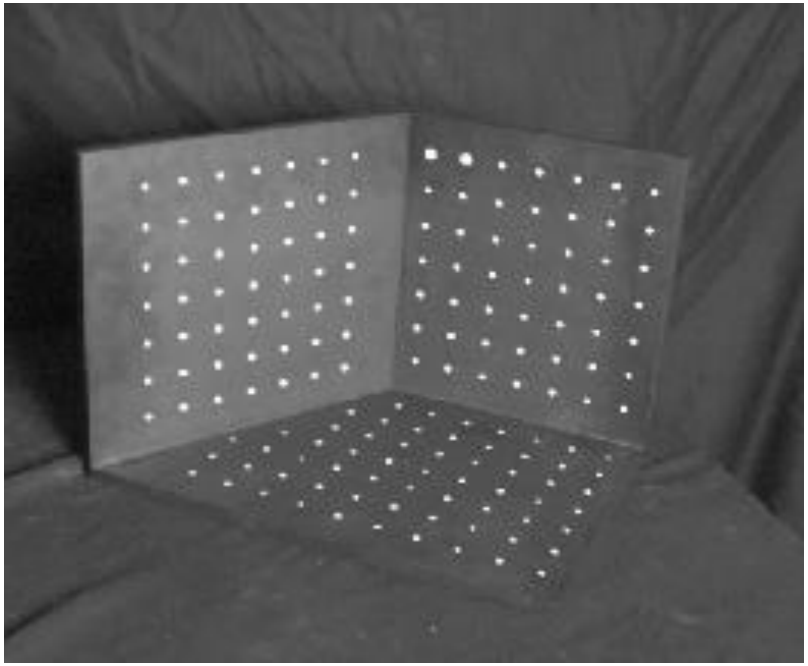
Known 3d locations



$$\begin{bmatrix} wu \\ wv \\ w \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

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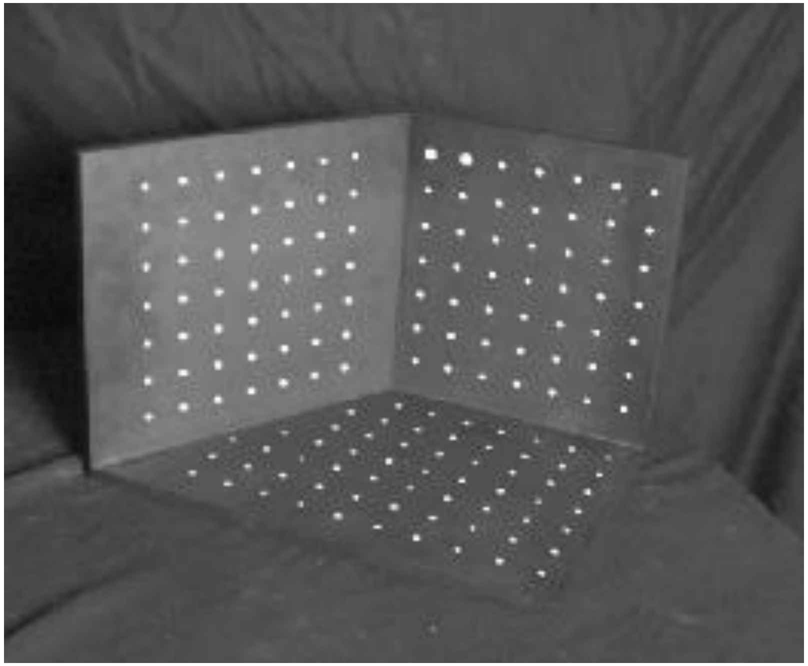

Known 2d image coordinates

Known 3d locations

$$\begin{bmatrix} wu \\ wv \\ w \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Calibrating the Camera

Method 1: Use an object (calibration grid) with known geometry

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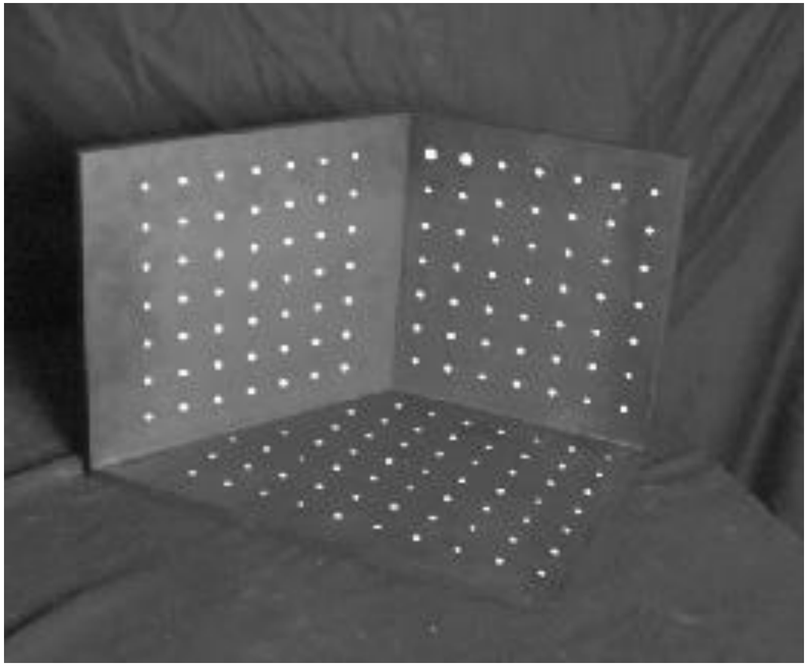

Known 2d image coordinates

Known 3d locations

$$\begin{bmatrix} wu \\ wv \\ w \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Unknown Camera Parameters

Unknown Camera Parameters

Known 2d image coords

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Known 3d locations

Unknown Camera Parameters

Known 2d image coords

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Known 3d locations

$$s = m_{31}X + m_{32}Y + m_{33}Z + m_{34}$$

Unknown Camera Parameters

Known 2d image coords

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Known 3d locations

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$$su = m_{31}uX + m_{32}uY + m_{33}uZ + m_{34}u = m_{11}X + m_{12}Y + m_{13}Z + m_{14}$$

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$$m_{11}X + m_{12}Y + m_{13}Z + m_{14} - m_{31}uX - m_{32}uY - m_{33}uZ - m_{34}u = 0$$
$$m_{21}X + m_{22}Y + m_{23}Z + m_{24} - m_{31}vX - m_{32}vY - m_{33}vZ - m_{34}v = 0$$

Unknown Camera Parameters

Known 2d image coords

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$$m_{21}X + m_{22}Y + m_{23}Z + m_{24} - m_{31}vX - m_{32}vY - m_{33}vZ - m_{34}v = 0$$

- Homogeneous linear system. Solve for m's entries using linear least squares

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\ & & & & & & \vdots & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n & -u_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n & -v_n \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Can we factorize M back to K [R | T]?

- Yes!
- We can use *QR* factorization

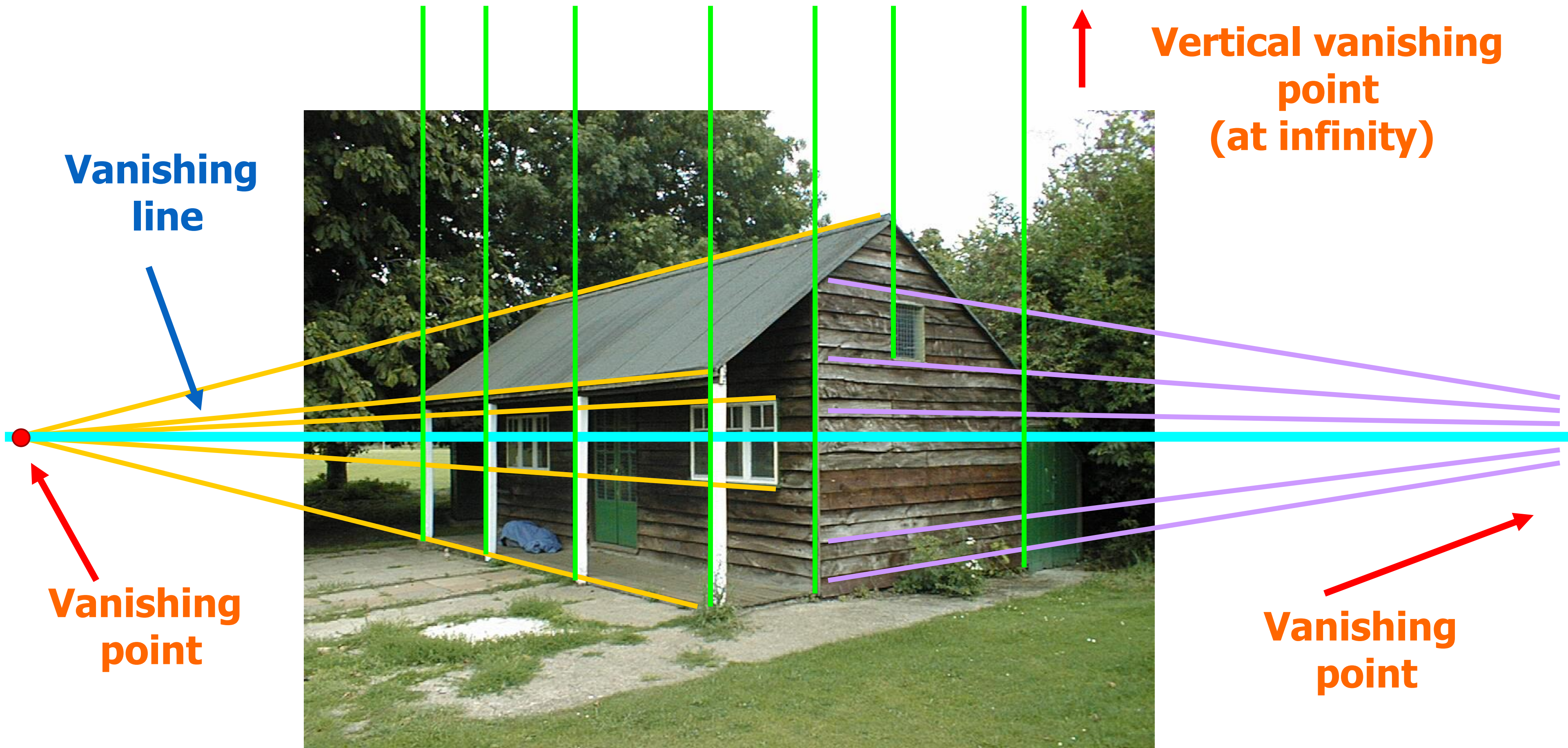
  - *http://ksimek.github.io/2012/08/14/decompose/*
  - *https://en.wikipedia.org/wiki/QR_decomposition*

Slide credit: J. Hays

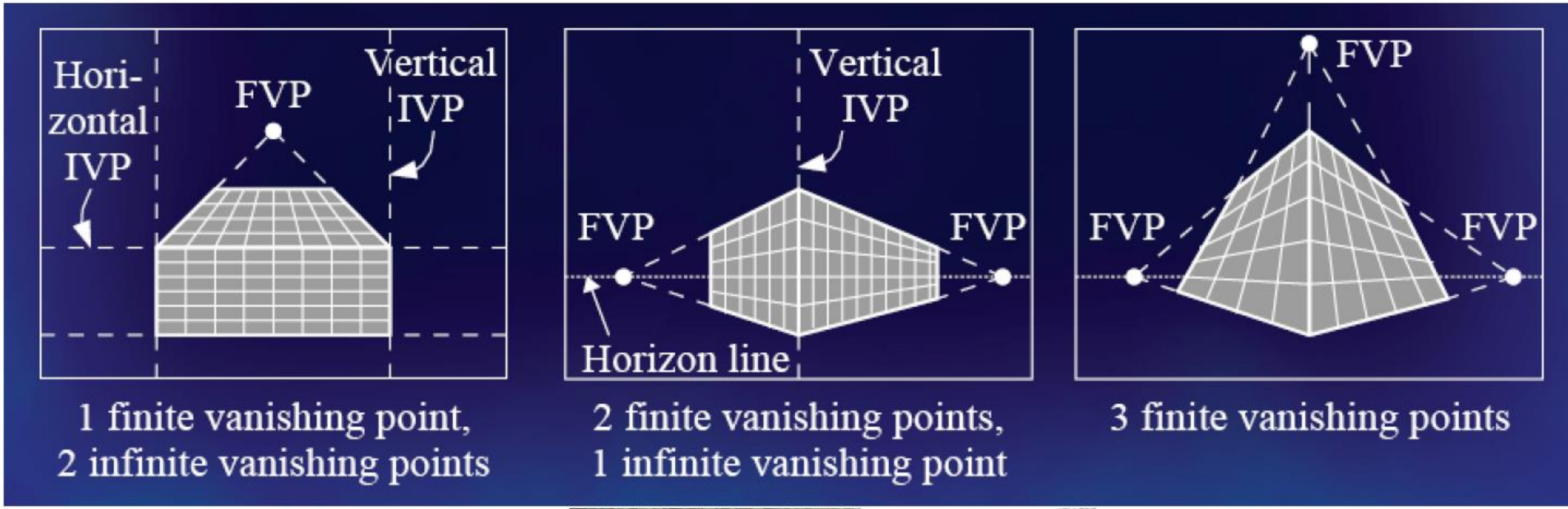
Calibrating the Camera

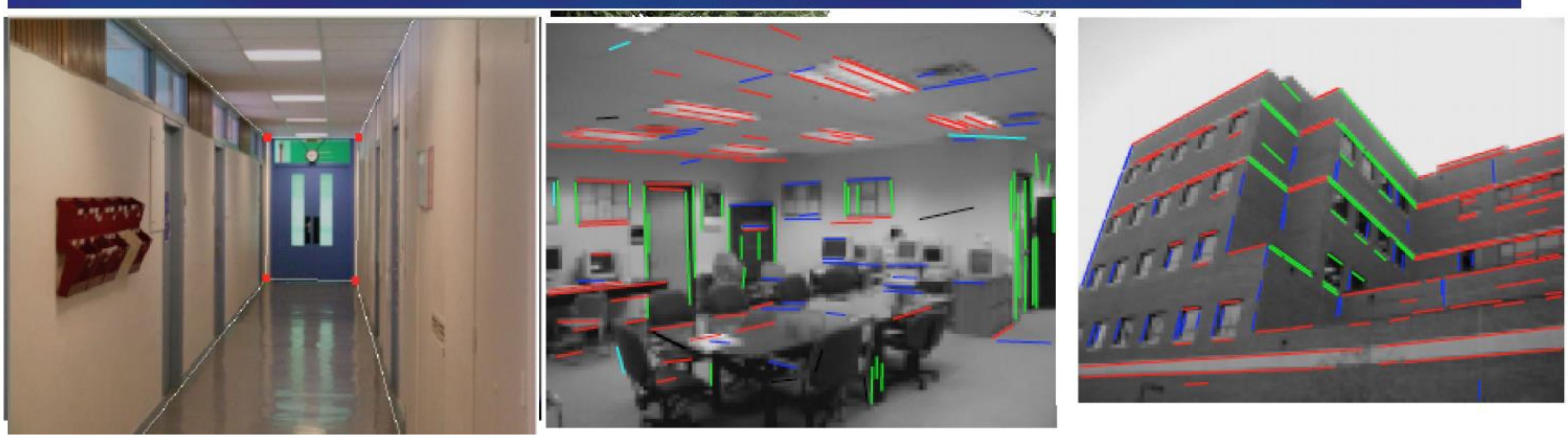
Method 2: Use vanishing points

- Find vanishing points corresponding to orthogonal directions



Calibration from vanishing points





Calibration by orthogonal vanishing points

- Intrinsic camera matrix
  - Use orthogonality as a constraint
  - Model K with only f, u_0, v_0

$$\mathbf{p}_i = \mathbf{KRX}_i$$

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$$\mathbf{X}_i = \mathbf{R}^{-1}\mathbf{K}^{-1}\mathbf{p}_i = \mathbf{R}^{T}\mathbf{K}^{-1}\mathbf{p}_i$$

Calibration by orthogonal vanishing points

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$$\mathbf{X}_i = \mathbf{R}^{-1}\mathbf{K}^{-1}\mathbf{p}_i = \mathbf{R}^{T}\mathbf{K}^{-1}\mathbf{p}_i$$

For vanishing points

$$\mathbf{X}_i^{\ T}\mathbf{X}_j = 0$$

Calibration by orthogonal vanishing points

- Intrinsic camera matrix
  - Use orthogonality as a constraint
  - Model K with only f, u_0, v_0

$$\mathbf{p}_i = \mathbf{KRX}_i$$

For vanishing points

$$\mathbf{X}_i{}^T \mathbf{X}_j = 0$$

$$\mathbf{X}_i = \mathbf{R}^{-1}\mathbf{K}^{-1}\mathbf{p}_i = \mathbf{R}^{T}\mathbf{K}^{-1}\mathbf{p}_i$$

$$\mathbf{p}_i^{T}(\mathbf{K}^{-1})^{T}(\mathbf{R}^{T})^{T}(\mathbf{R}^{T})(\mathbf{K}^{-1})\mathbf{p}_j = \mathbf{0}$$

Calibration by orthogonal vanishing points

- Intrinsic camera matrix
  - Use orthogonality as a constraint
  - Model K with only f, u_0, v_0

For vanishing points

$$\mathbf{p}_i = \mathbf{K}\mathbf{R}\mathbf{X}_i \qquad \mathbf{X}_i^{\,T}\mathbf{X}_j = 0$$

$$\mathbf{X}_i = \mathbf{R}^{-1}\mathbf{K}^{-1}\mathbf{p}_i = \mathbf{R}^T\mathbf{K}^{-1}\mathbf{p}_i$$

$$\mathbf{p}_i^T(\mathbf{K}^{-1})^T(\mathbf{R}^T)^T(\mathbf{R}^T)(\mathbf{K}^{-1})\mathbf{p}_j = \mathbf{0}$$

$$\mathbf{p}_i^T\mathbf{K}^{-T}\mathbf{R}\mathbf{R}^T\mathbf{K}^{-1}\mathbf{p}_j = \mathbf{0}$$

Calibration by orthogonal vanishing points

- Intrinsic camera matrix
  - Use orthogonality as a constraint
  - Model K with only f, u_0, v_0

For vanishing points

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Calibration by orthogonal vanishing points

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  - Use orthogonality as a constraint
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For vanishing points

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$$\mathbf{p}_i^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{p}_j = \mathbf{0}$$

- Similarly

$$\mathbf{p}_i^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{p}_k = \mathbf{0}$$

$$\mathbf{p}_j^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{p}_k = \mathbf{0}$$

Calibration by orthogonal vanishing points

- Intrinsic camera matrix
  - Use orthogonality as a constraint
  - Model K with only f, u_0, v_0

$$\mathbf{p}_i = \mathbf{KRX}_i$$

For vanishing points

$$\mathbf{X}_i{}^T \mathbf{X}_j = 0$$

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$$\mathbf{p}_i^T(\mathbf{K}^{-1})^T(\mathbf{R}^T)^T(\mathbf{R}^T)(\mathbf{K}^{-1})\mathbf{p}_j = \mathbf{0}$$

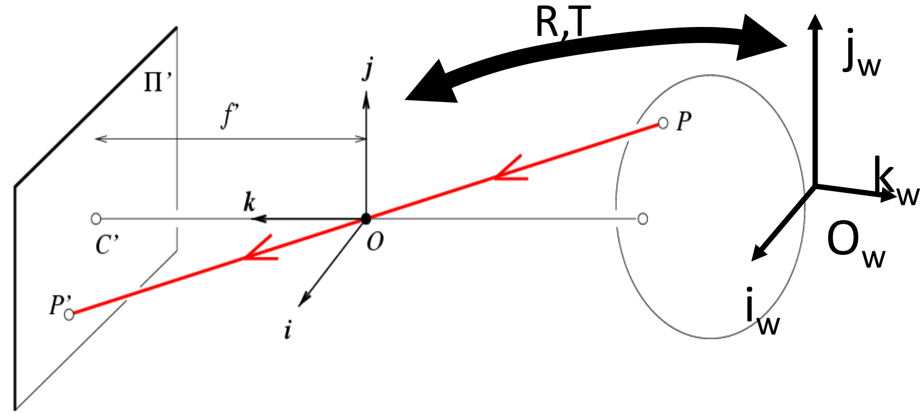
$$\mathbf{p}_i^T\mathbf{K}^{-T}\mathbf{R}\mathbf{R}^T\mathbf{K}^{-1}\mathbf{p}_j = \mathbf{0}$$

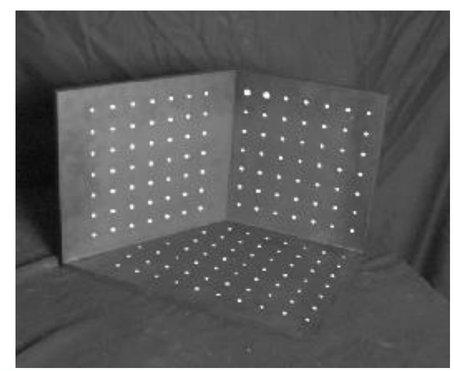
$$\mathbf{p}_i^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{p}_j = \mathbf{0}$$

- What if you don't have three finite vanishing points?
  - Two finite VP: get valid u_0, v_0 closest to image center; solve f
  - One finite VP: u_0, v_0 is at vanishing point; can't solve for f

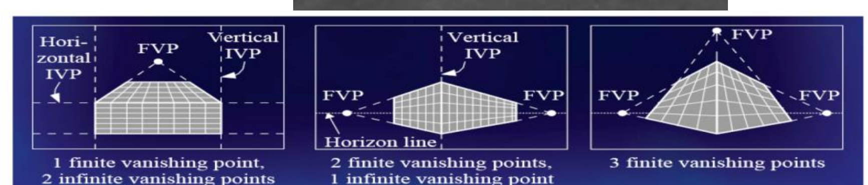
Things to remember

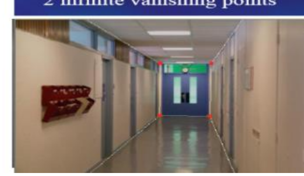
- Projection Matrix?
  - Extrinsic Matrix
    - Camera Rotation
    - Camera Translation
  - Intrinsic Matrix
    - Focal Length
    - Optical Center
    - Aspect Ratio
    - Skewness
  - Degree of Freedom
    - 5- Intrinsic
    - 6- Extrinsic





- Calibrate the camera?
  - Use an object with known geometry
  - Use vanishing points

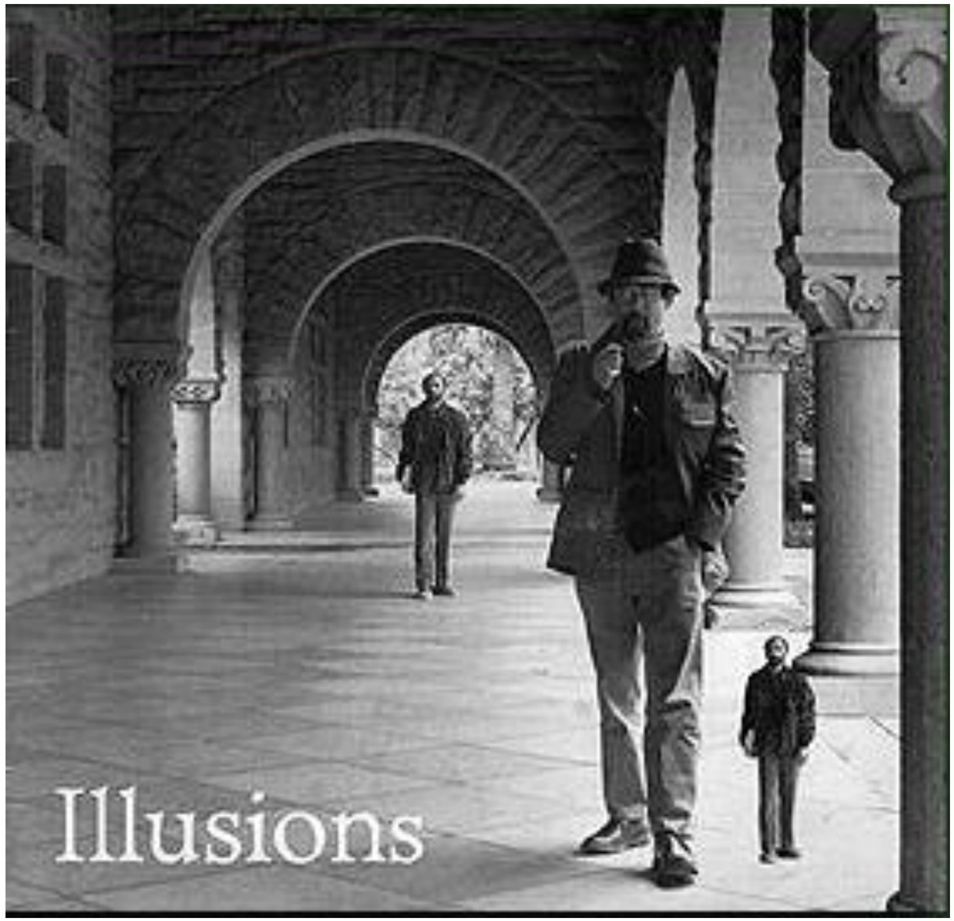




Acknowledgements

- Thanks to the following researchers for making their teaching/research material online
  - Forsyth
  - Steve Seitz
  - Noah Snavely
  - J.B. Huang
  - Derek Hoiem
  - J. Hays
  - J. Johnson
  - R. Girshick
  - S. Lazebnik
  - K. Grauman
  - Antonio Torralba
  - Rob Fergus
  - Leibe
  - And many more ……….

Next Class:
How can we measure the size of 3D objects from an image?



Slide by Steve Seitz