Computer Vision

Introduction to Classifiers

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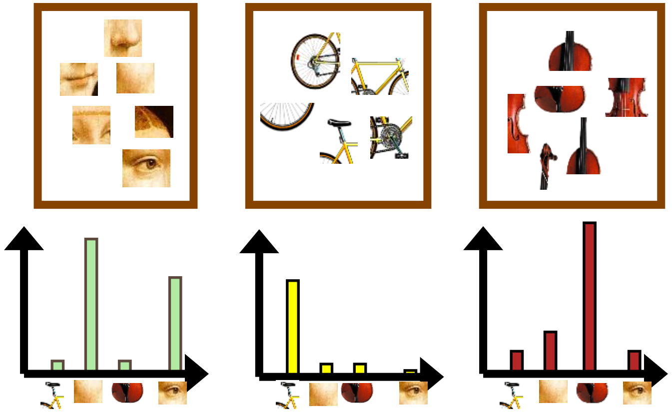
Previous Class

Image features and categorization

Choosing right features

Object, Scene, Action, etc.



Bag-of-visual-words

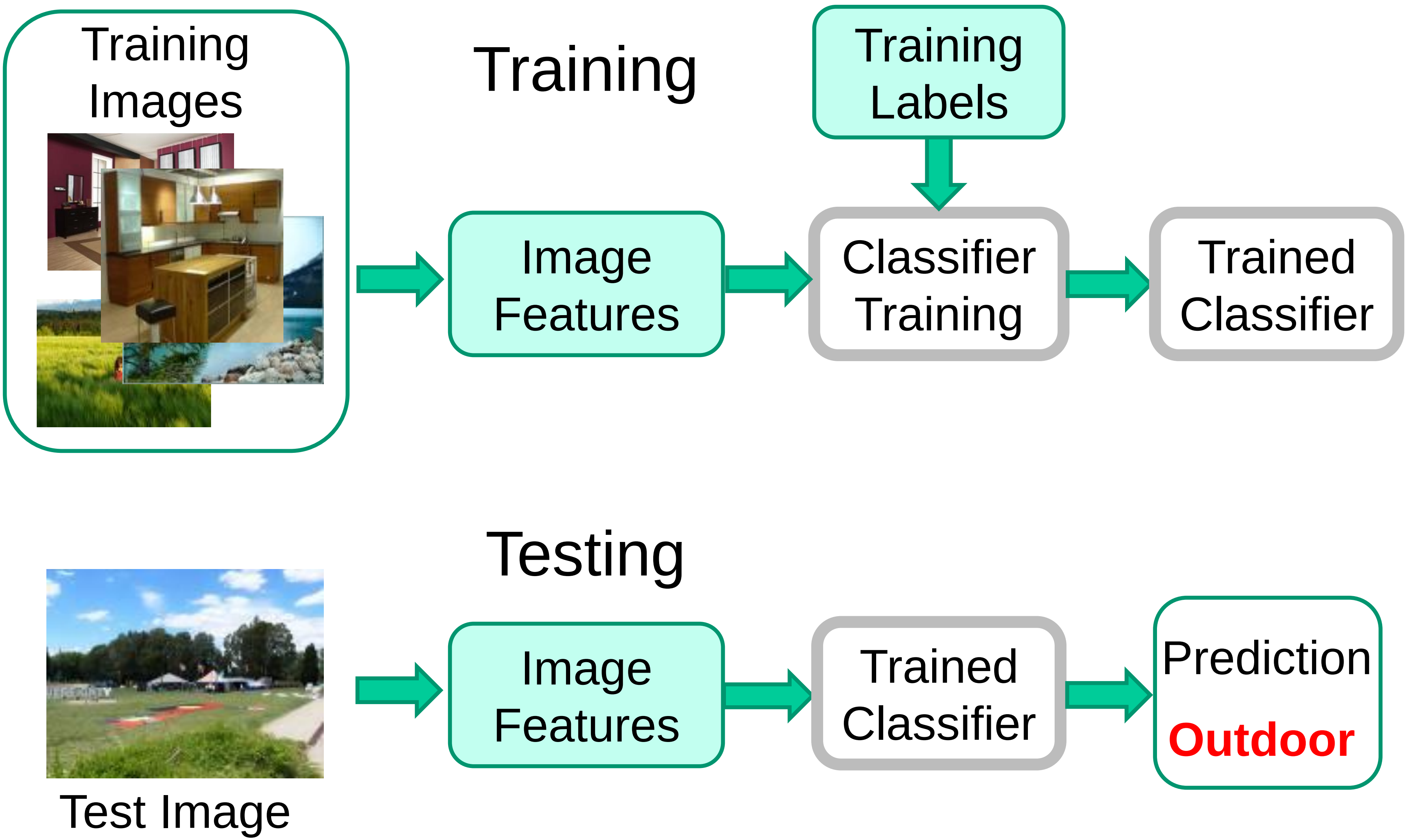
Extract local features

Learn “visual vocabulary”

Quantize features using visual vocabulary

Represent by frequencies of “visual words”

Today’s Class



Today's Class

Nearest Neighbor Classifier

K - Nearest Neighbor Classifier

Linear Classifier

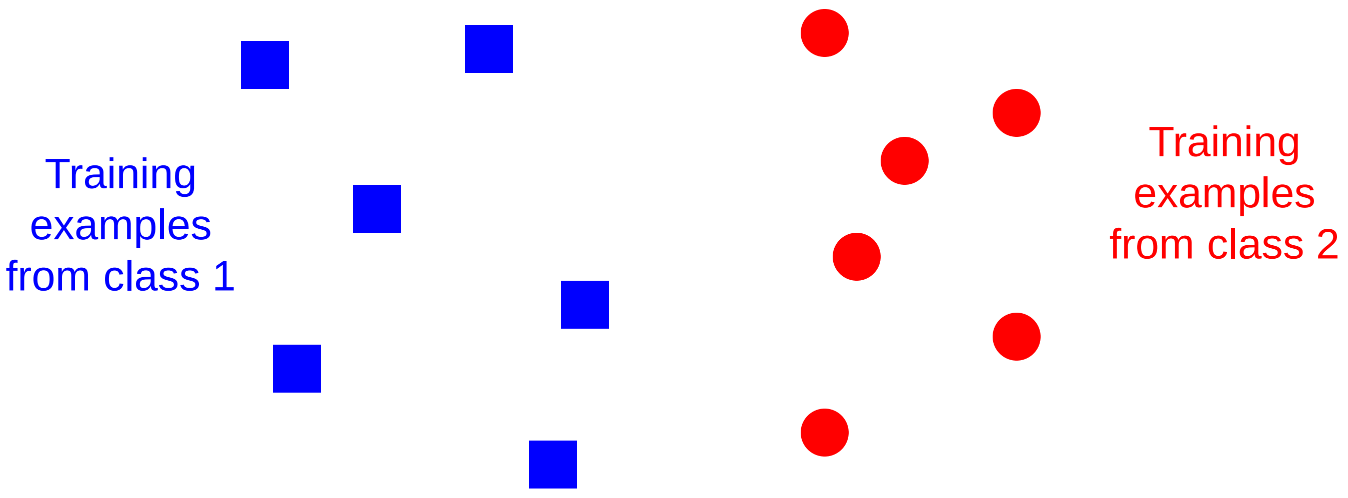
Support Vector Machine

Non-linear SVM

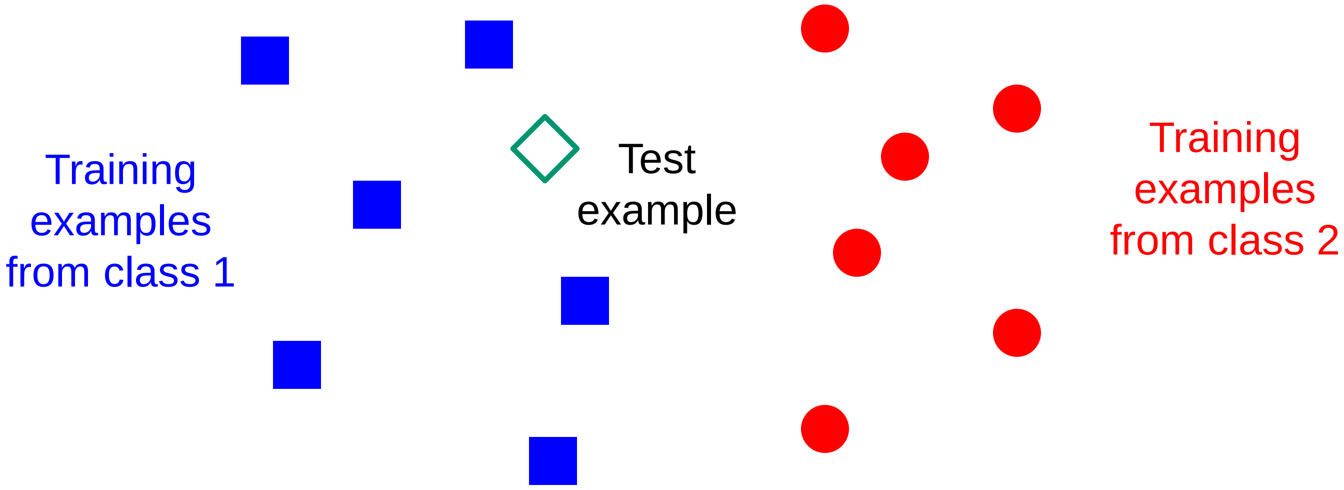
Multi-class SVM

Softmax Classifier

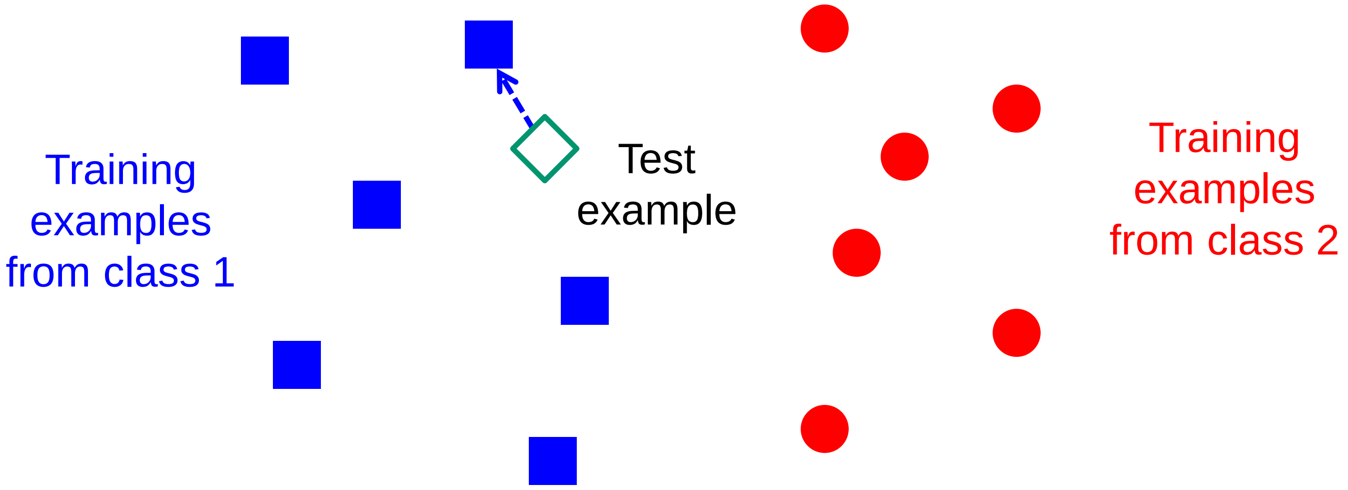
Classifiers: Nearest neighbor



Classifiers: Nearest neighbor



Classifiers: Nearest neighbor



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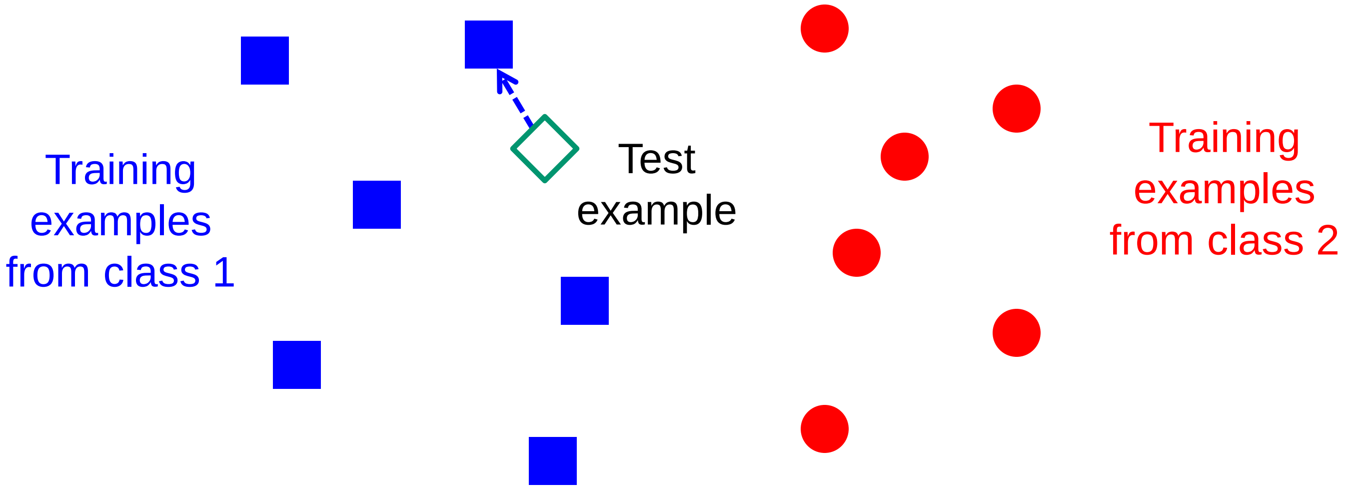


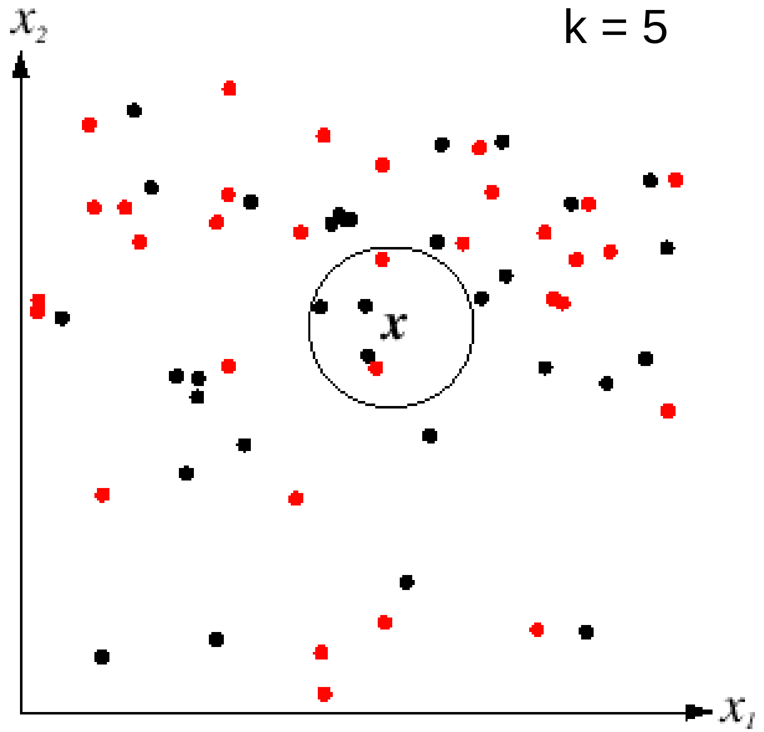
f(**x**) = label of the training example nearest to **x**

All we need is a distance function for our inputs

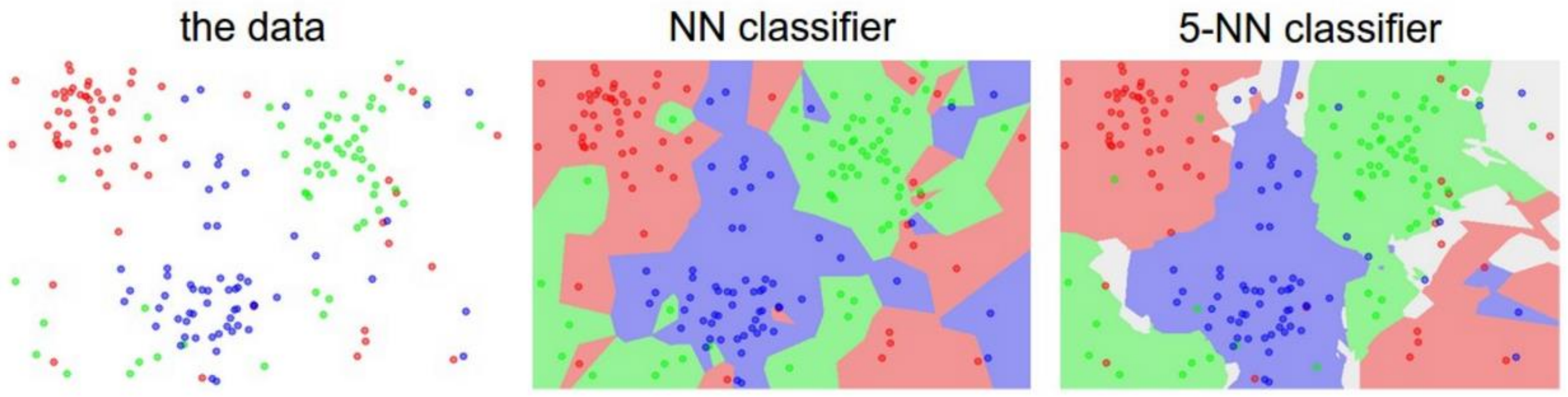
No training required!

K-nearest neighbor classifier

- For a new point, find the k closest points from training data
- Vote for class label with labels of the k points

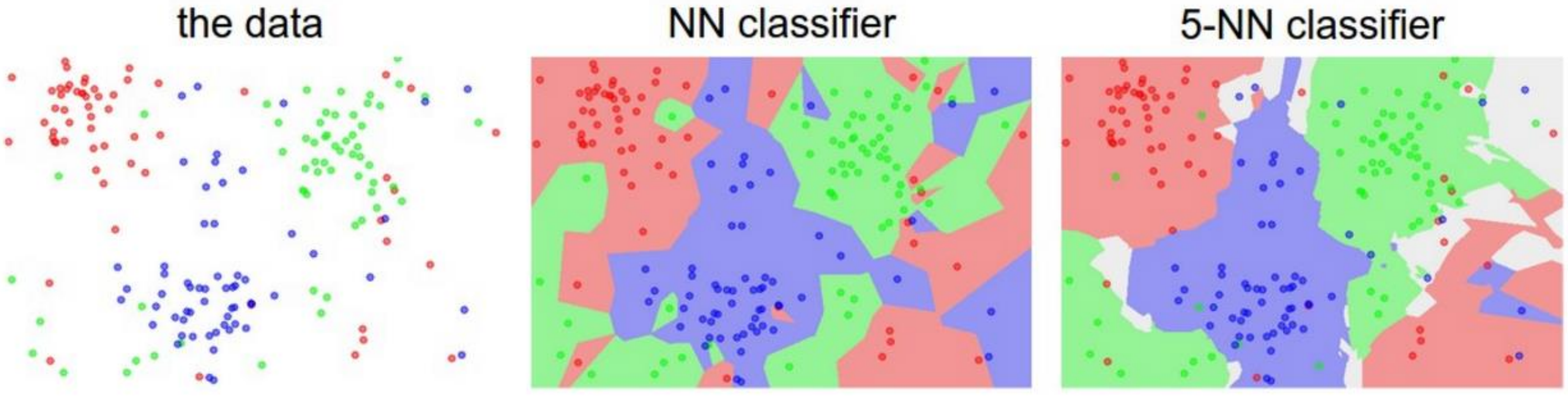


K-nearest neighbor classifier



Credit: cs231n, http://cs231n.github.io/classification/

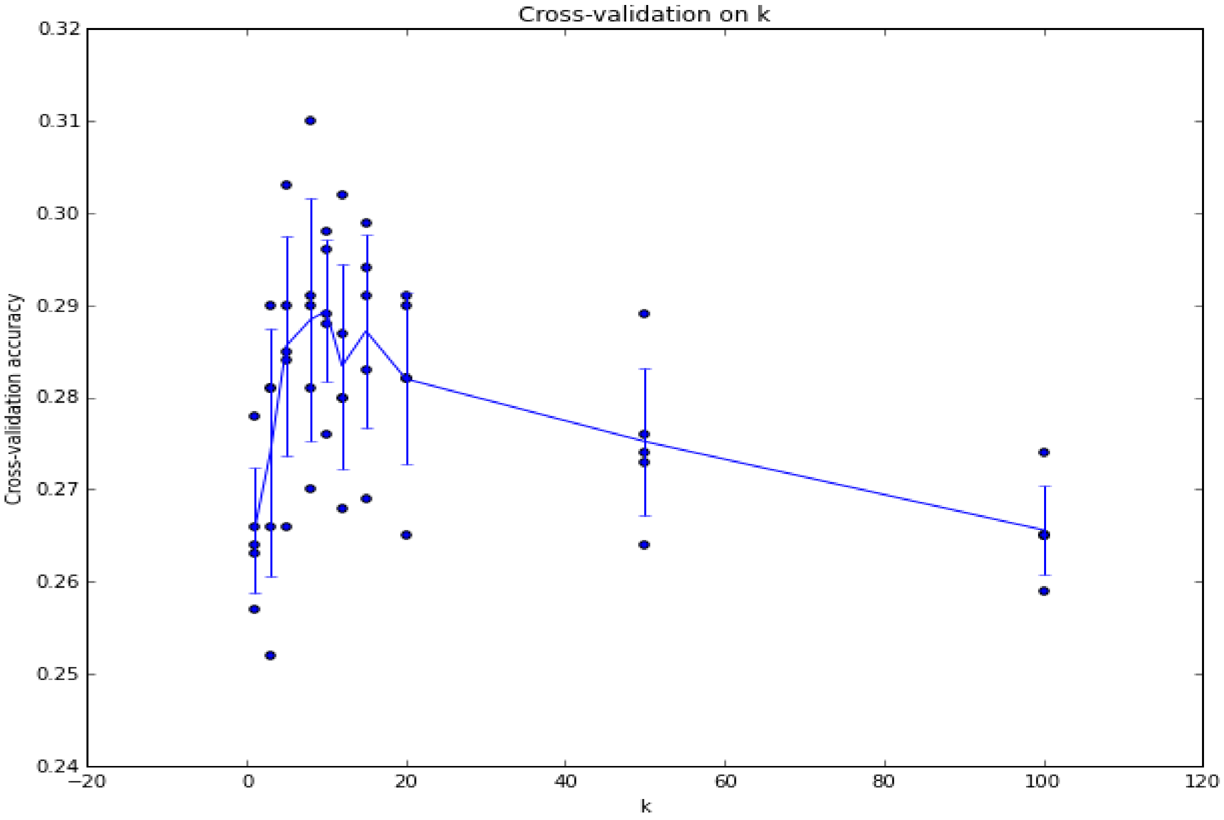
K-nearest neighbor classifier



Which classifier is more robust to *outliers*?

Credit: cs231n, http://cs231n.github.io/classification/

Choice of K in KNN classifier



Example of a 5-fold cross-validation run for the parameter **k.** Note that in this particular case, the cross-validation suggests that a value of about **k** = 7 works best on this particular dataset (corresponding to the peak in the plot).

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Classifiers: K-Nearest neighbor

"Non-parametric" classifier: the entire training set is essentially the model parameters.

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"Non-parametric" classifier: the entire training set is essentially the model parameters.

Pros:

- **Very fast at training** time
- **Flexible**: all it requires is a way to compute similarity or distances between pairs of features. Applies to many different kinds of features.
- Works with **any number of classes**.
- Works well in practice for large datasets (but see cons)

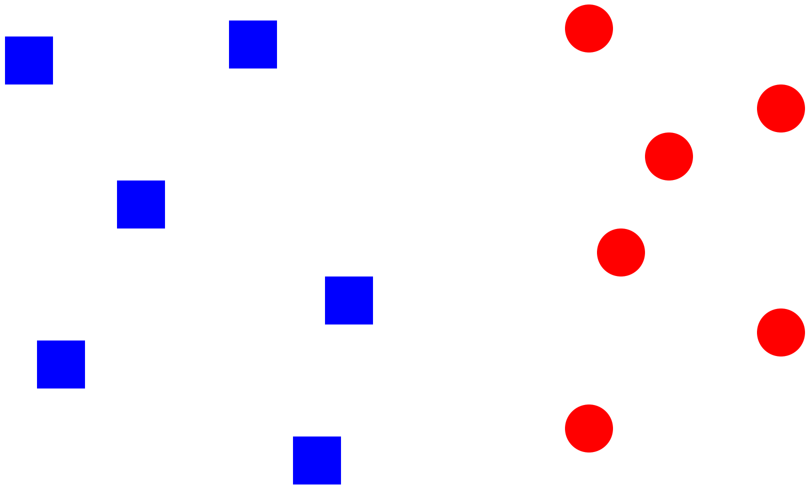
Classifiers: K-Nearest neighbor

"Non-parametric" classifier: the entire training set is essentially the model parameters.

Cons:

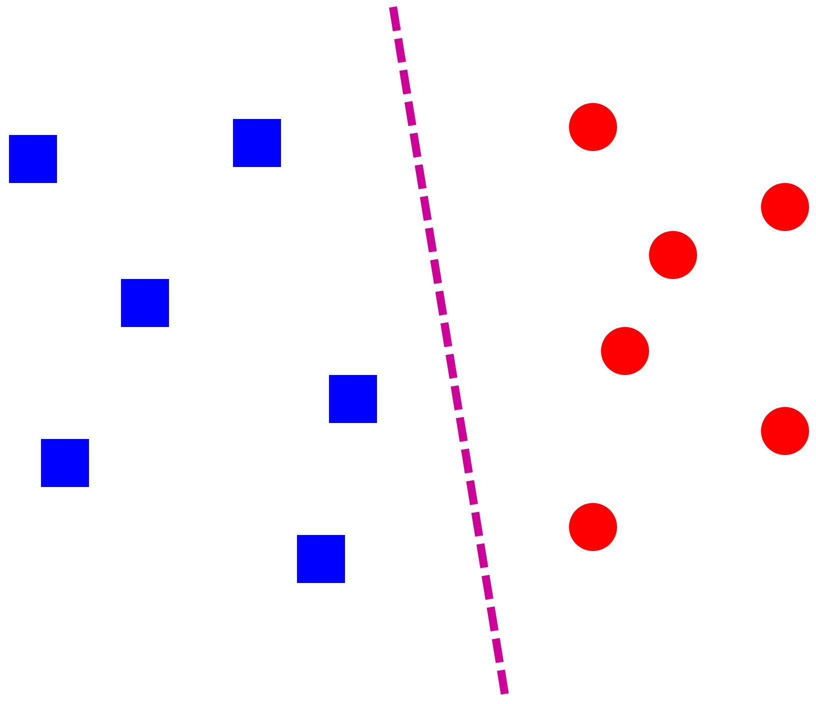
- The classifier must ***remember*** **all of the training data** and store it for future comparisons with the test data.
- This is **space inefficient** because datasets may easily be gigabytes in size.
- **Slow at test time** (need to compute distances between test example and every training example)
- Optimum value of **K is not known**.

Linear classifiers – 2 class problem



Find a *linear function* to separate the classes:

Linear classifiers – 2 class problem



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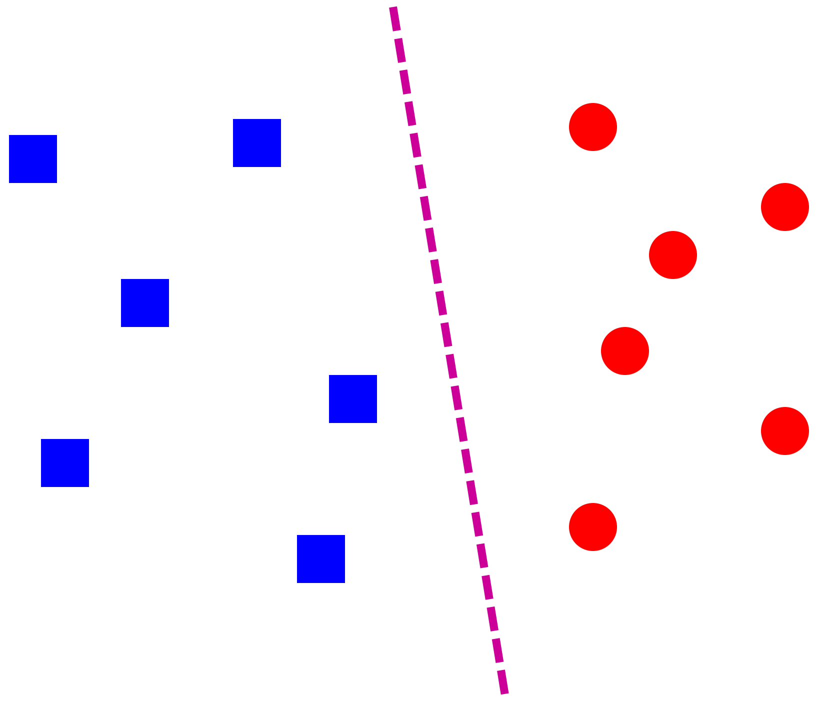
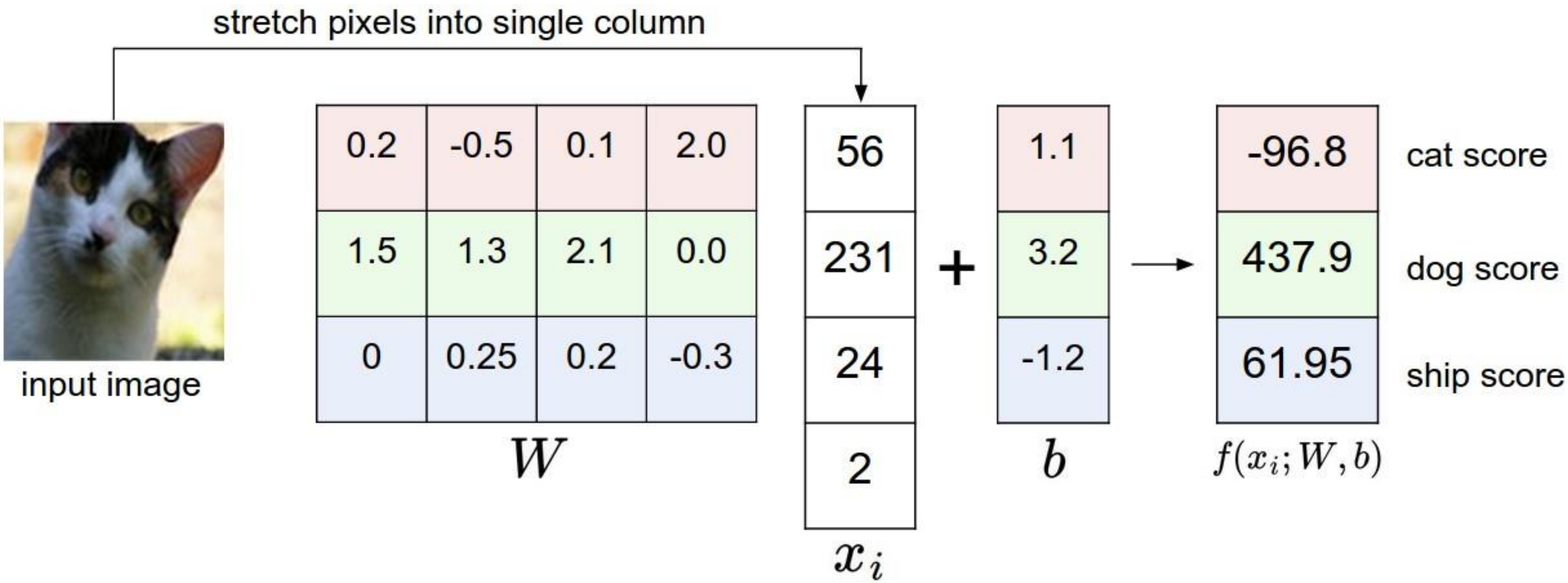


Find a *linear function* to separate the classes:

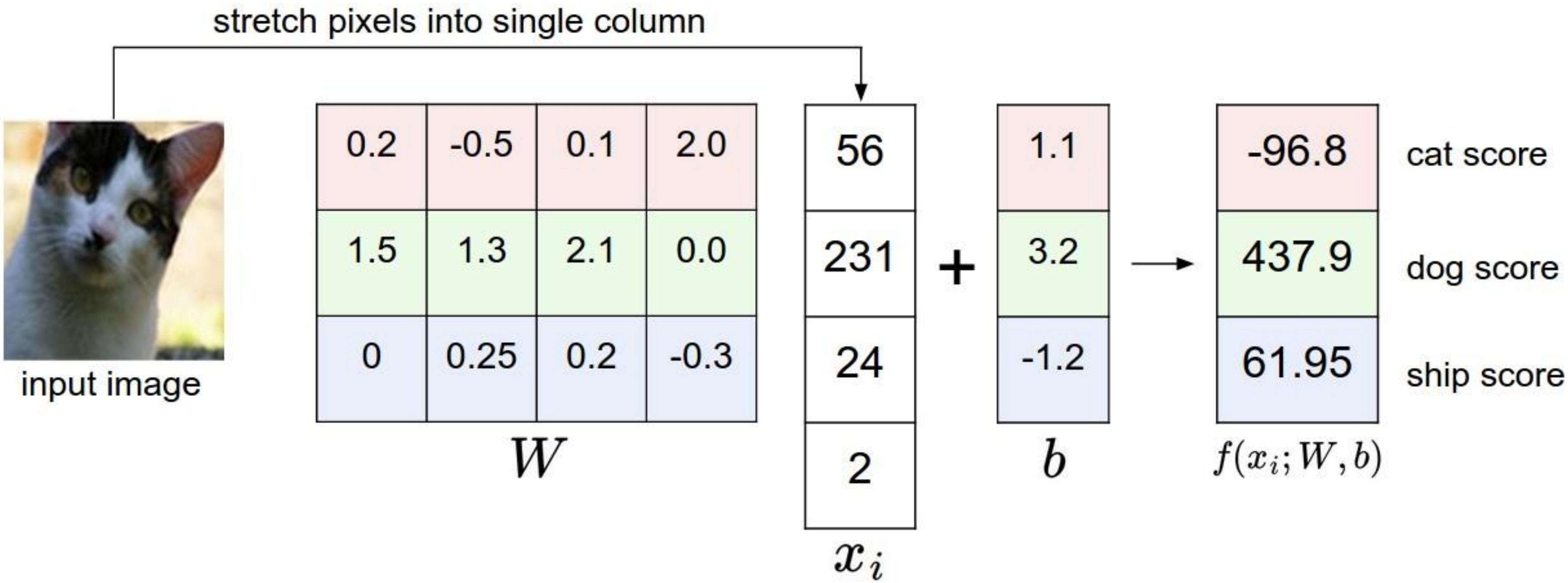
$$f(\mathbf{x}) = \mathrm{sign}(\mathbf{w} \cdot \mathbf{x} + b)$$

Linear classifiers – more than 2 class



Source: cs231n, http://cs231n.github.io/linear-classify/

Linear classifiers – more than 2 class



$$f(x_i, W, b) = Wx_i + b$$

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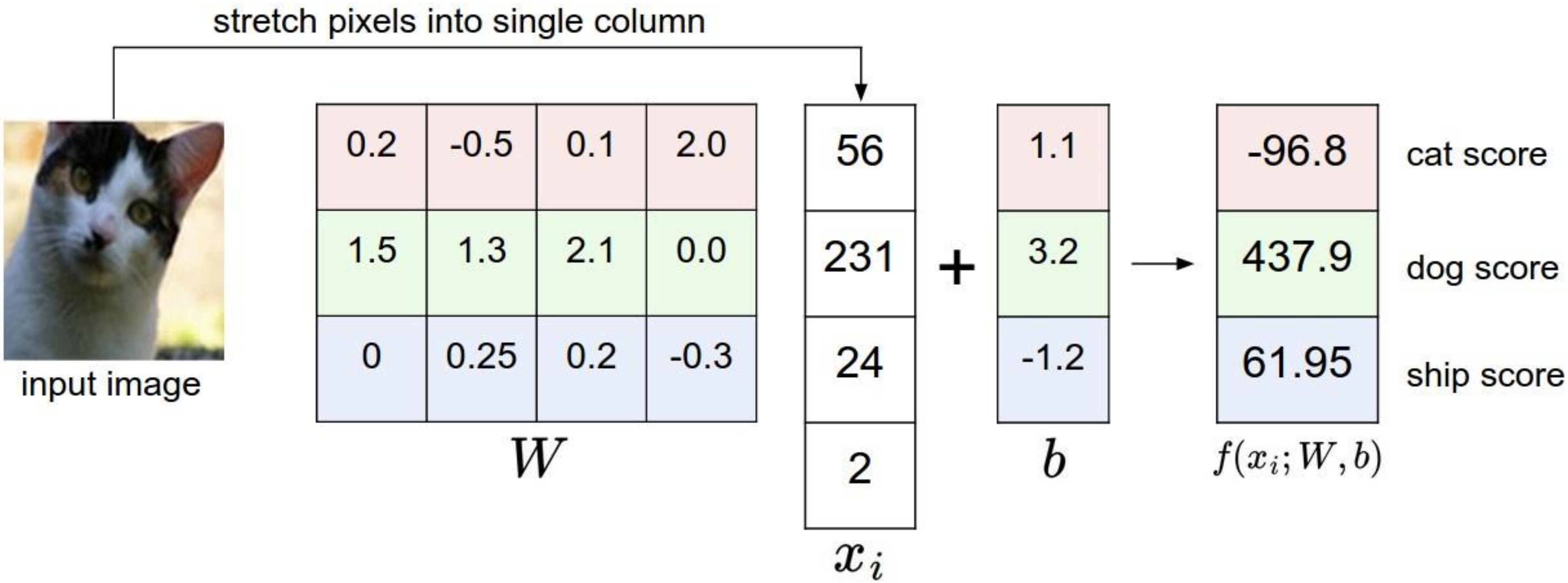
Linear classifiers – more than 2 class



$$f(x_i, W, b) = W x_i + b$$

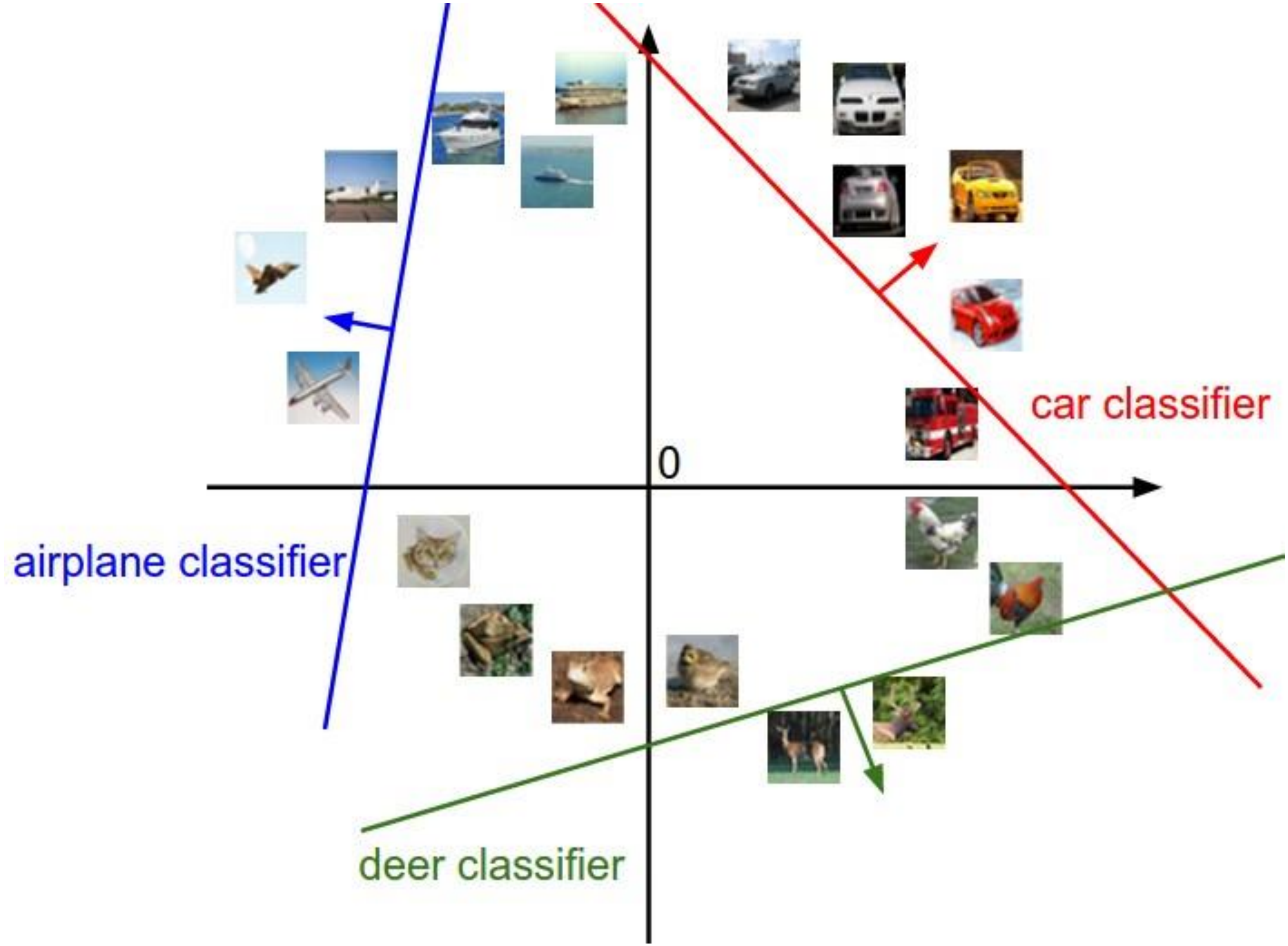
Image x_i has all of its pixels flattened out to a single column vector of shape [D x 1].

Matrix **W** (of size [K x D]), and vector **b** (of size [K x 1]) are the **parameters**.

K is the number of classes.

Source: cs231n, http://cs231n.github.io/linear-classify/

Analogy of images as high-dimensional points



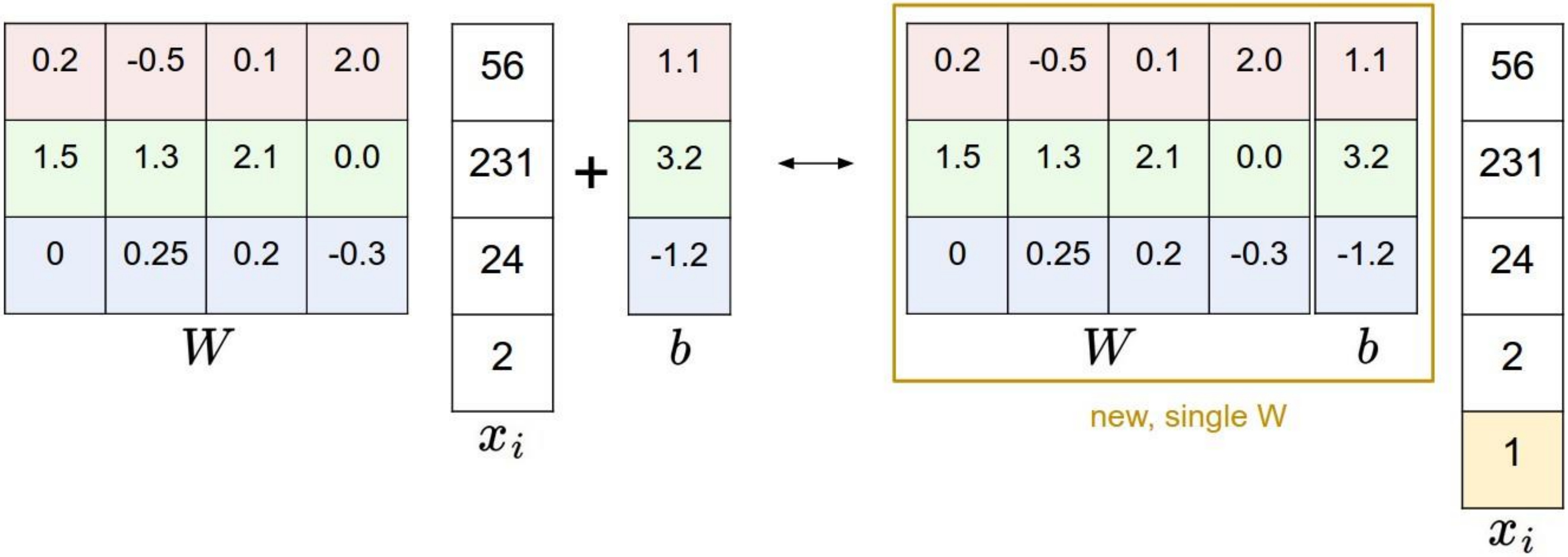
Source: cs231n, http://cs231n.github.io/linear-classify/

Interpretation of linear classifiers as template matching

Example learned weights at the end of learning for CIFAR-10. Note that, for example, the ship template contains a lot of blue pixels as expected. This template will therefore give a high score once it is matched against images of ships on the ocean with an inner product.

Source: cs231n, http://cs231n.github.io/linear-classify/

Bias Trick



Source: cs231n, http://cs231n.github.io/linear-classify/

Linear classifiers

"Parametric" classifier: model defined by a small number of parameters (w, b)

Pros:

- Very fast at test time

Cons:

- Slow at training time: need to estimate the parameters
- Data may not be linearly separable

Nearest neighbor vs. linear classifiers

- NN pros:
  - Simple to implement
  - Decision boundaries not necessarily linear
  - Works for any number of classes
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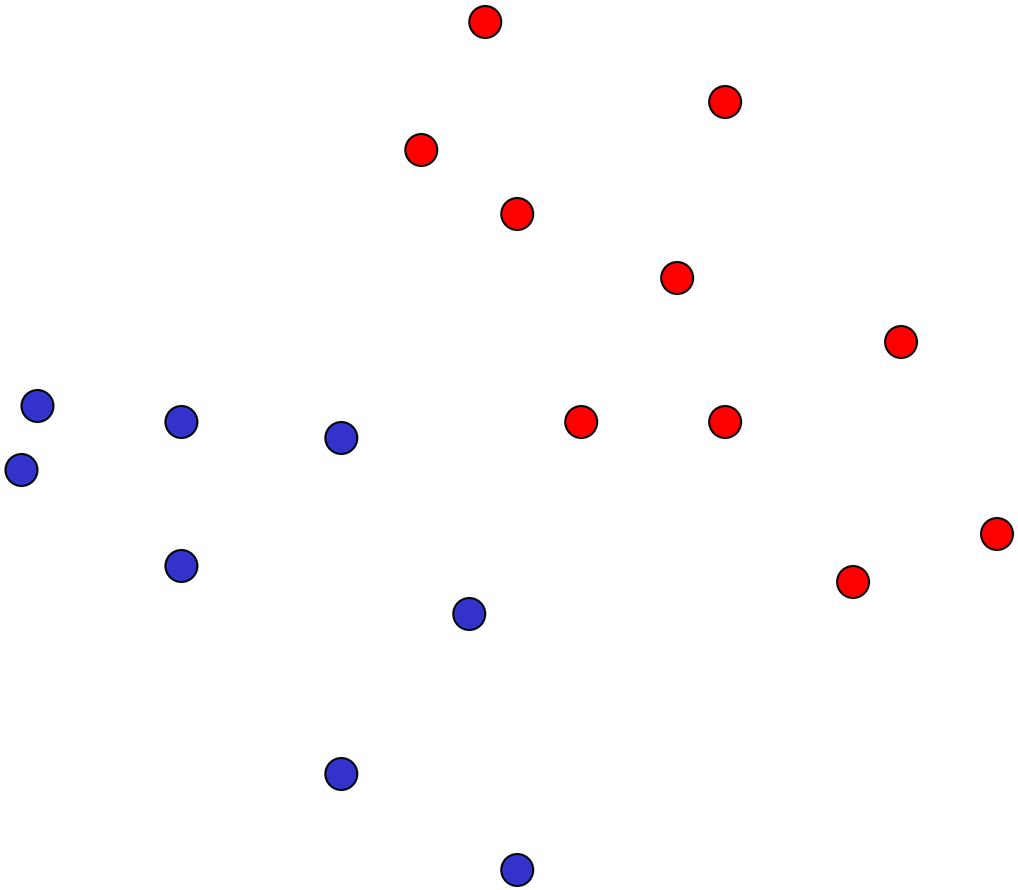
- Linear pros:
  - Low-dimensional *parametric* representation
  - Very fast at test time
- Linear cons:
  - How to train the linear function?
  - What if data is not linearly separable?

Support vector machines

- When the data is linearly separable, there may be more than one separator (hyperplane)



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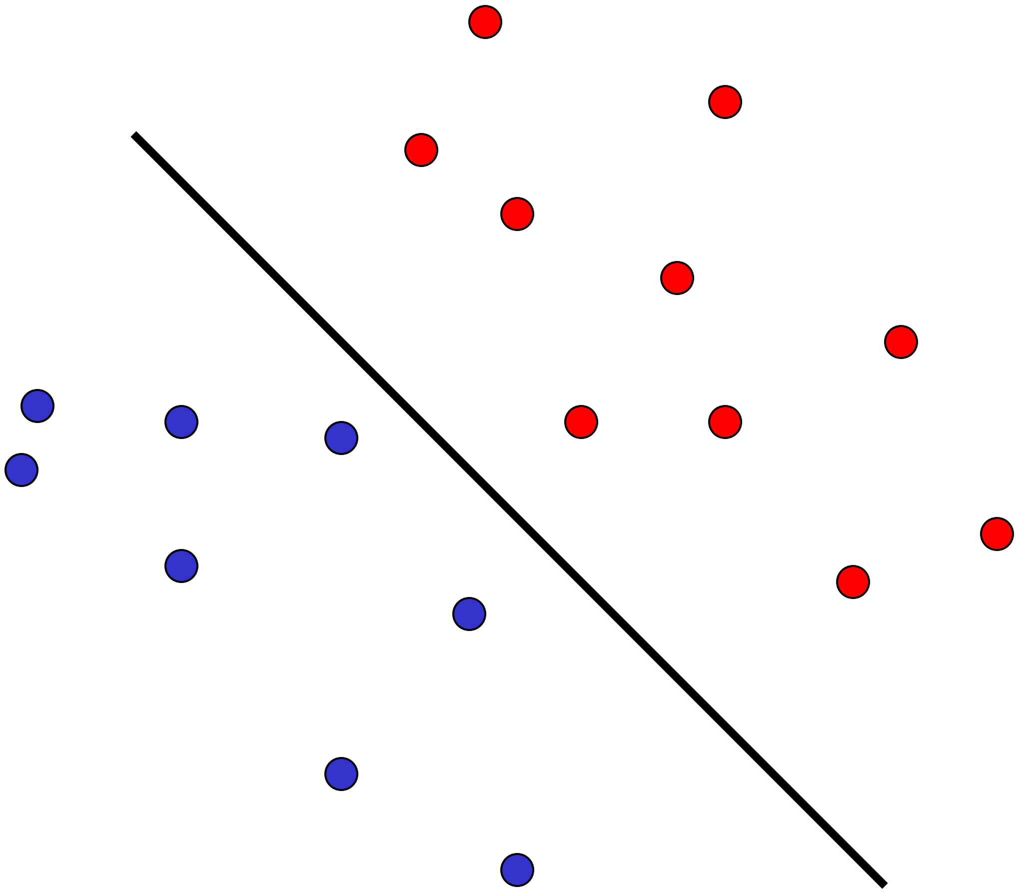

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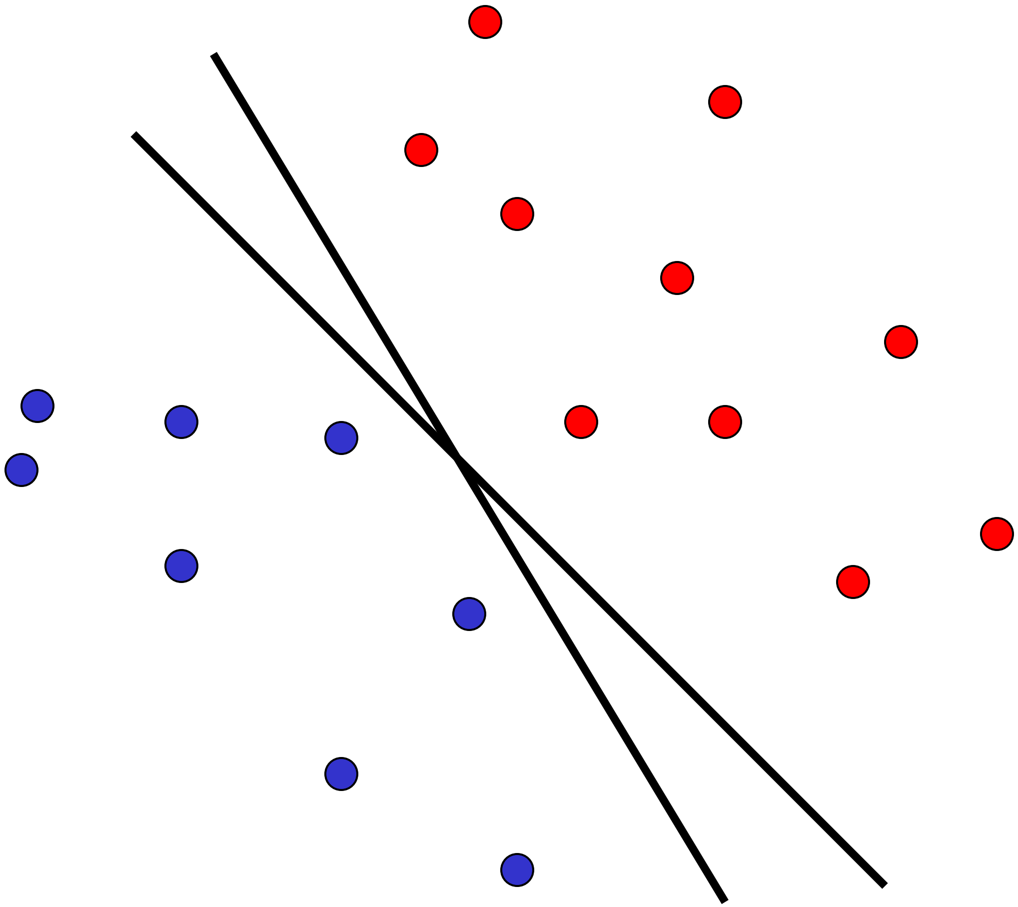

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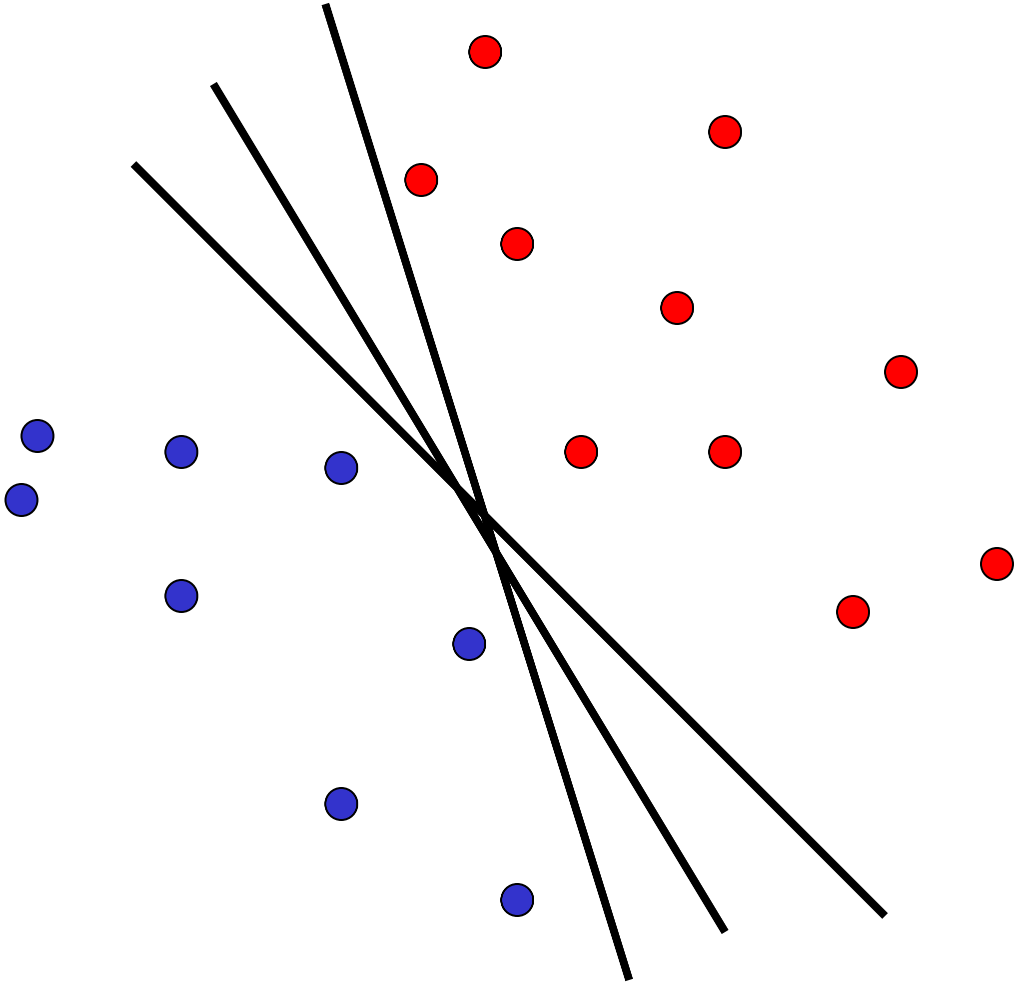

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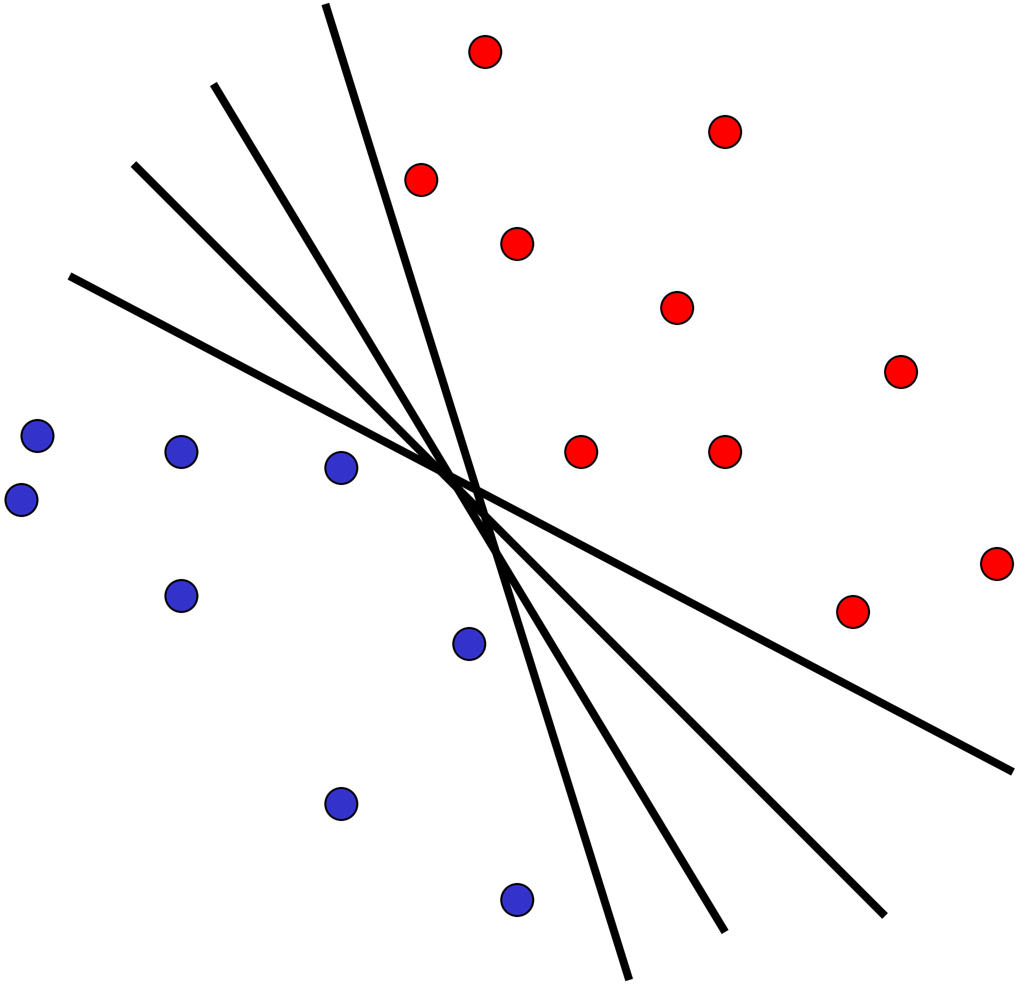

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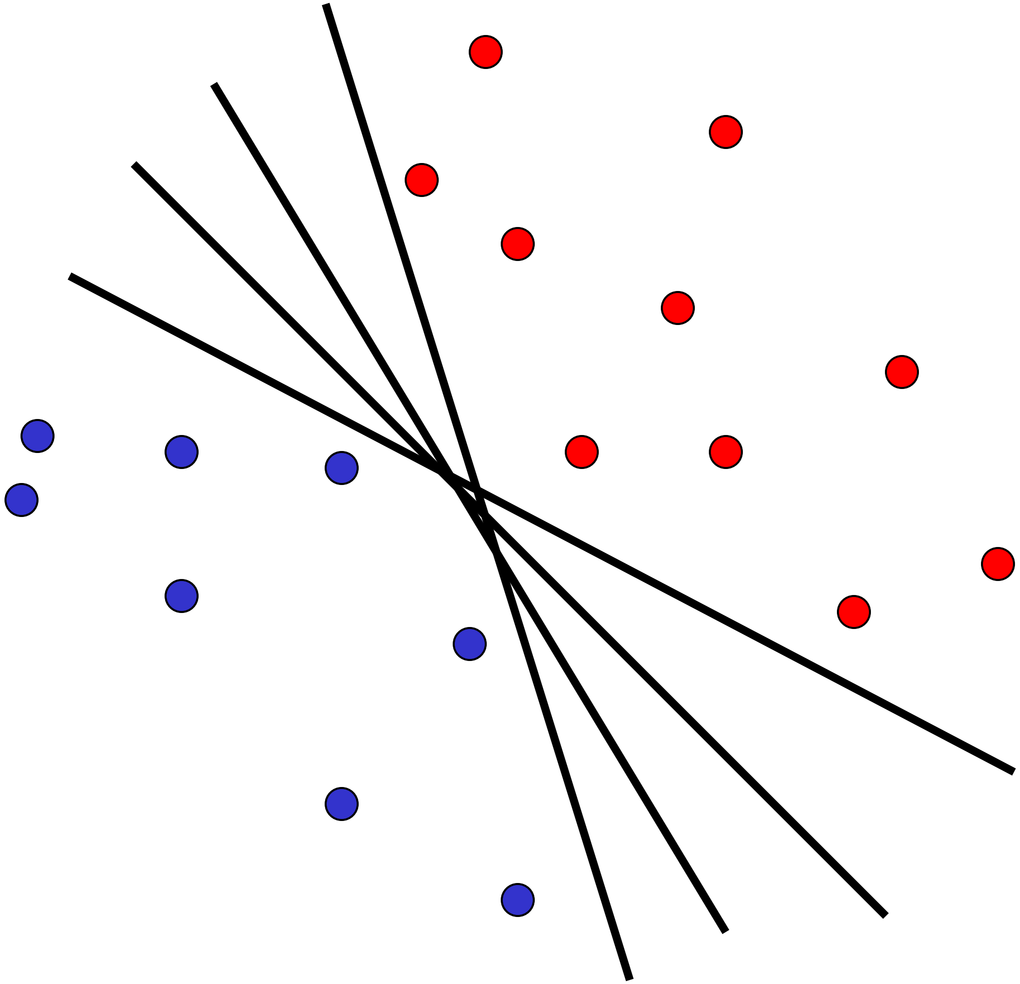
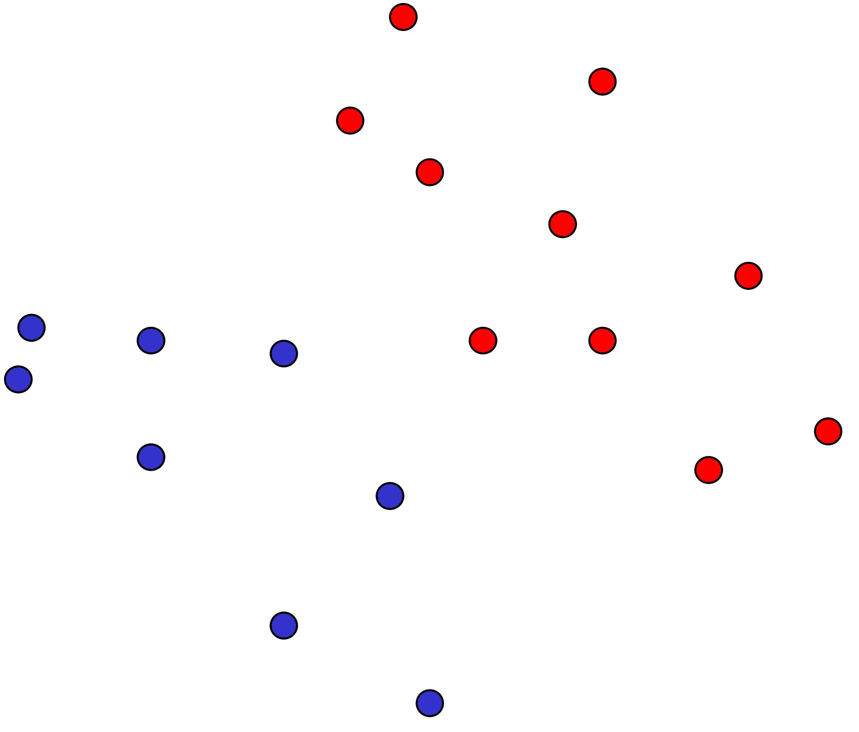


Which separator is best?

Support vector machines

- Find hyperplane that maximizes the *margin* between the positive and negative examples



C. Burges, [A Tutorial on Support Vector Machines for Pattern Recognition](), Data Mining and Knowledge Discovery, 1998

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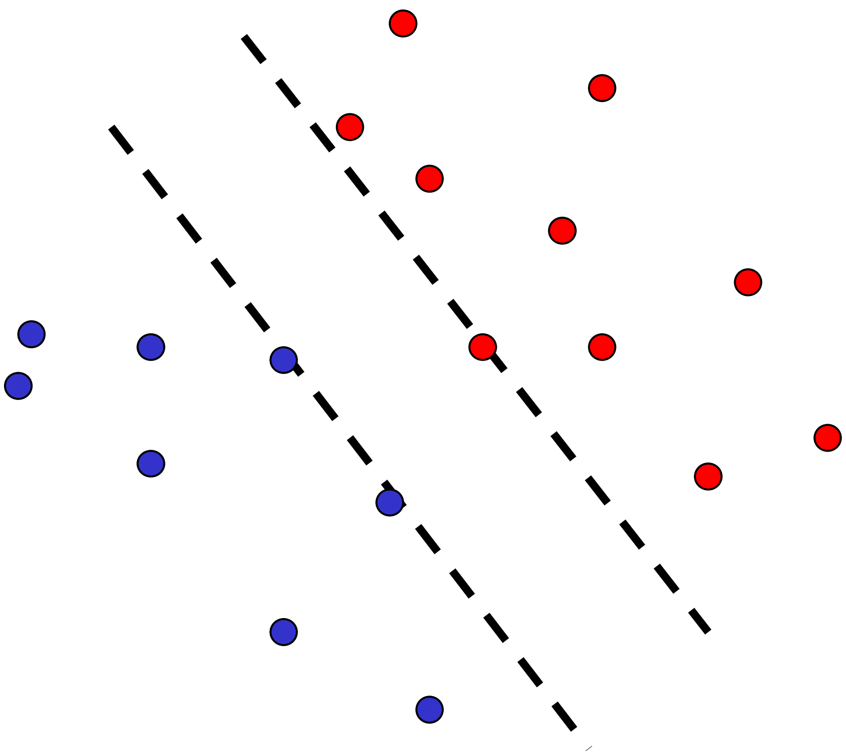

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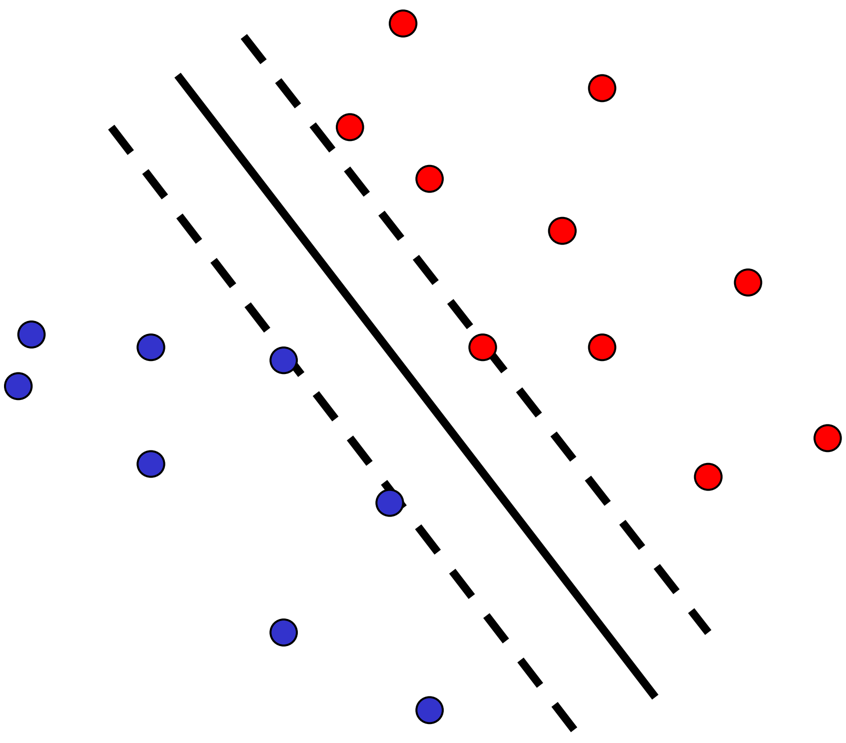

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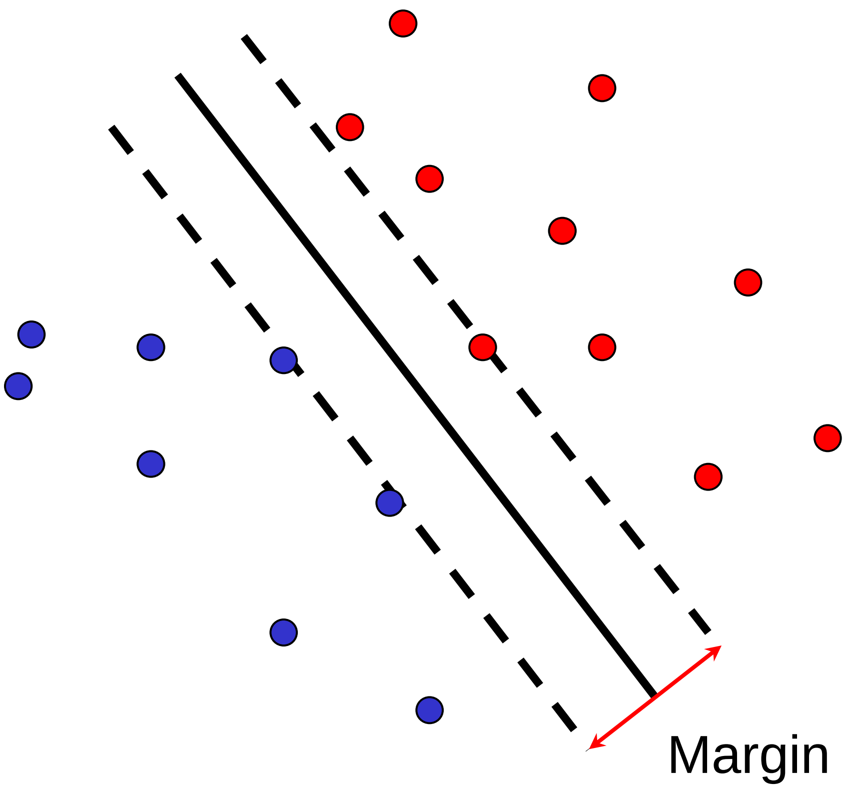

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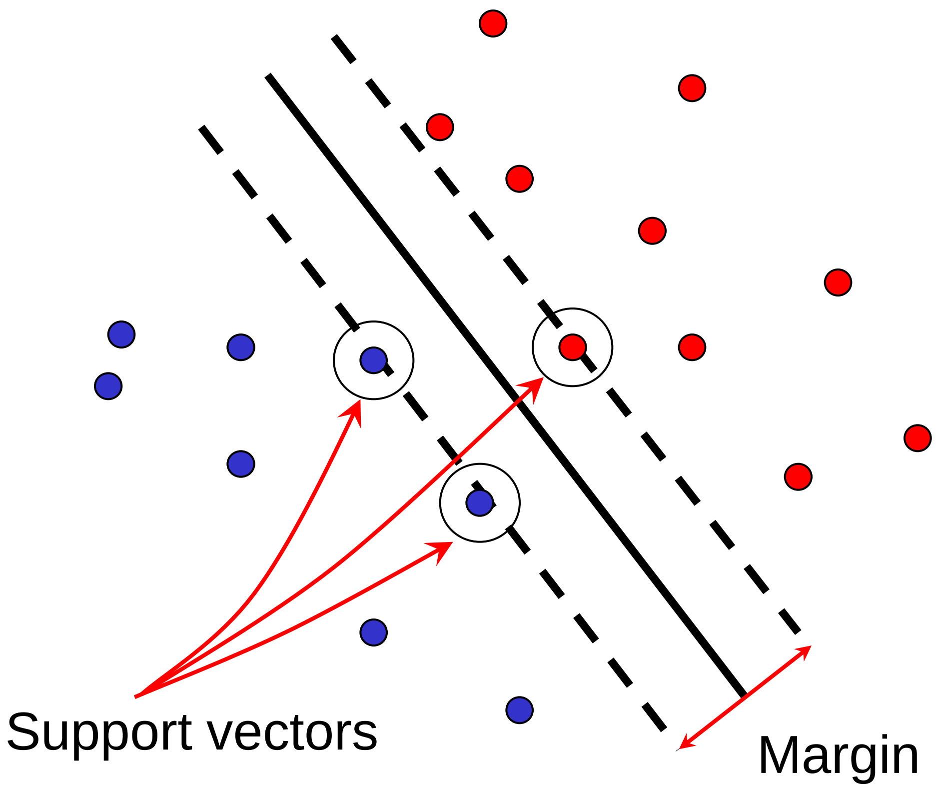

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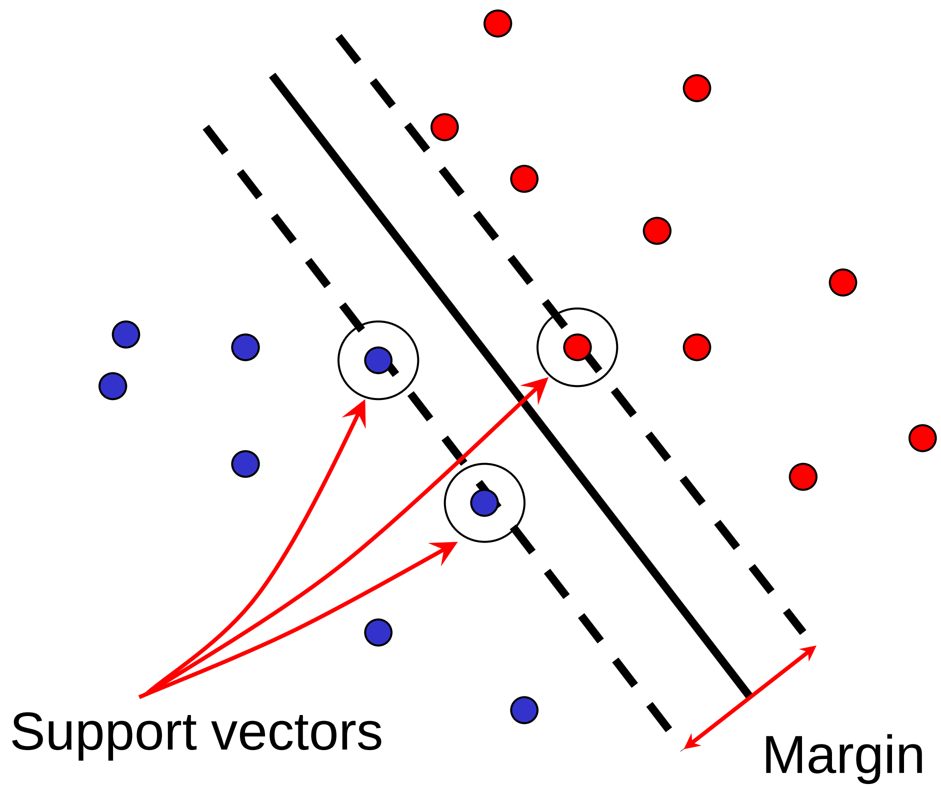

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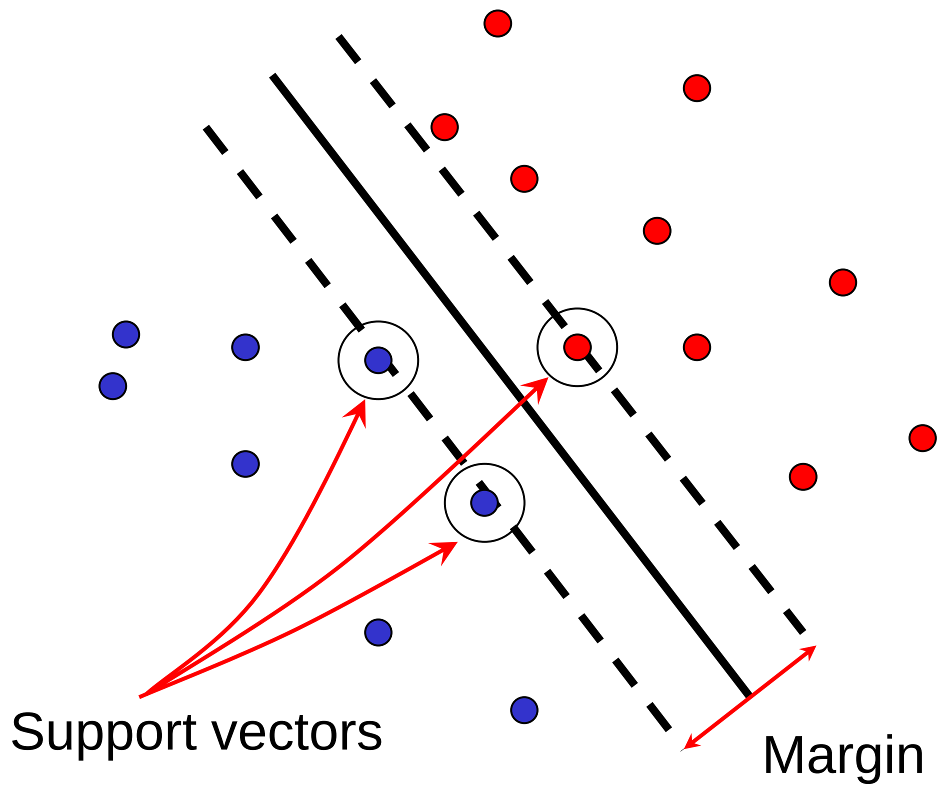

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For support vectors, $\quad \mathbf{x}_i \cdot \mathbf{w} + b = \pm 1$

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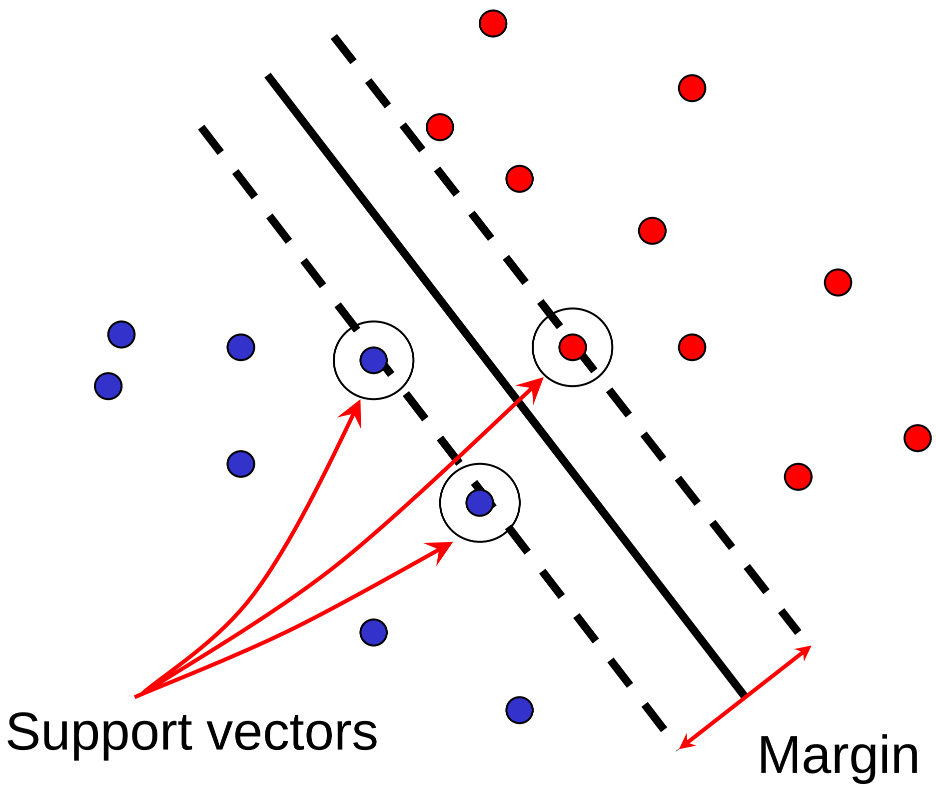

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Distance between point and hyperplane: $\dfrac{|\mathbf{x}_i \cdot \mathbf{w} + b|}{\|\mathbf{w}\|}$

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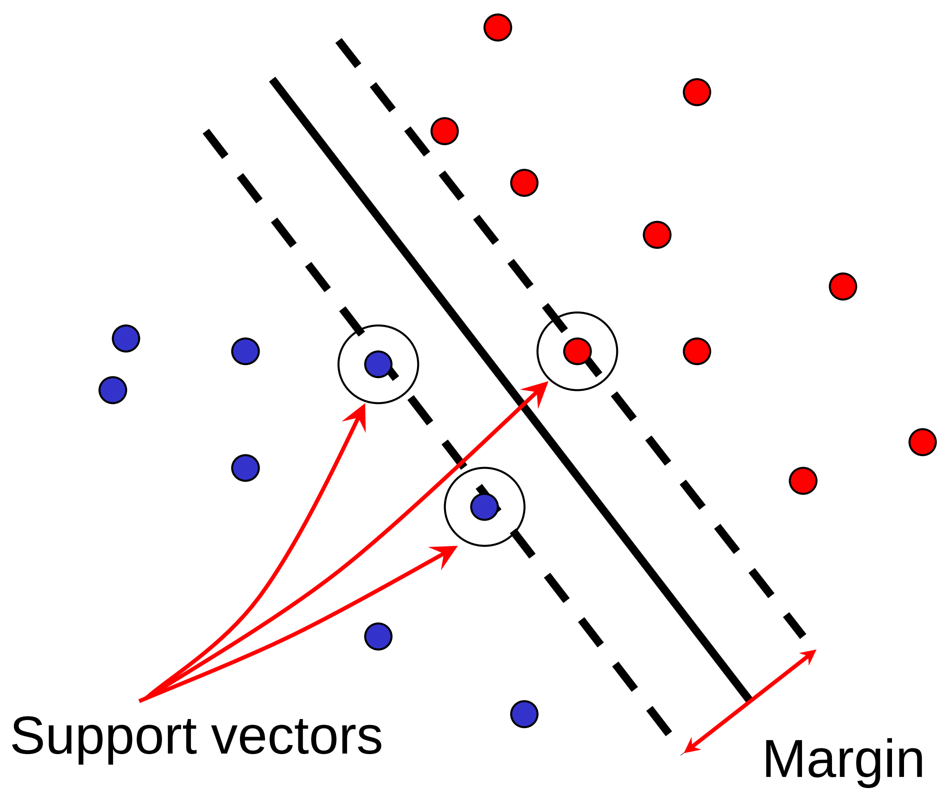

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For support vectors, $\quad \mathbf{x}_i \cdot \mathbf{w} + b = \pm 1$

Distance between point and hyperplane: $\quad \dfrac{|\mathbf{x}_i \cdot \mathbf{w} + b|}{\|\mathbf{w}\|}$

Therefore, the margin is $2 / \|\mathbf{w}\|$

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Finding the maximum margin hyperplane

1. Maximize margin $2 / \|\mathbf{w}\|$

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Finding the maximum margin hyperplane

1. Maximize margin $2 / \|\mathbf{w}\|$
2. Correctly classify all training data:

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Quadratic optimization problem:

$$\min_{\mathbf{w}, b} \frac{1}{2} \|\mathbf{w}\|^2 \quad \text{subject to} \quad y_i(\mathbf{w} \cdot \mathbf{x}_i + b) \geq 1$$

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Finding the maximum margin hyperplane

- Solution: $\mathbf{w} = \sum_i \alpha_i y_i \mathbf{x}_i$

learned weight

Support vector

- The weights α_i are non-zero only at support vectors.

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Finding the maximum margin hyperplane

- Solution: $\mathbf{w} = \sum_i \alpha_i y_i \mathbf{x}_i$

  $b = y_i - \mathbf{w} \cdot \mathbf{x}_i$ (for any support vector)

$$\mathbf{w} \cdot \mathbf{x} + b = \sum_i \alpha_i y_i \mathbf{x}_i \cdot \mathbf{x} + b$$

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$$\mathbf{w} \cdot \mathbf{x} + b = \sum_i \alpha_i y_i \mathbf{x}_i \cdot \mathbf{x} + b$$

- Classification function:

$$f(x) = \text{sign}\,(\mathbf{w} \cdot \mathbf{x} + \text{b})$$

$$= \text{sign}\left(\sum_i \alpha_i y_i \boxed{\mathbf{x}_i \cdot \mathbf{x}} + b\right)$$

If f(x) < 0, classify as negative,

if f(x) > 0, classify as positive

Dot product only!

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

SVM parameter learning

- Separable data: $$\underbrace{\min_{\mathbf{w},b} \frac{1}{2}\left\|\mathbf{w}\right\|^2}_{\text{Maximize margin}} \quad \underbrace{\text{subject to} \quad y_i(\mathbf{w}\cdot\mathbf{x}_i + b) \geq 1}_{\text{Classify training data correctly}}$$

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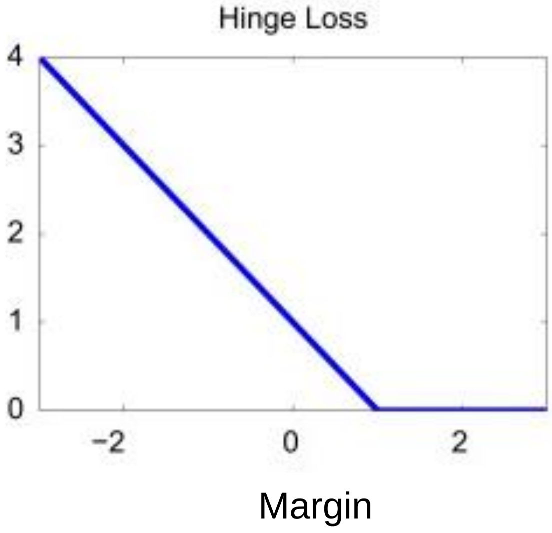
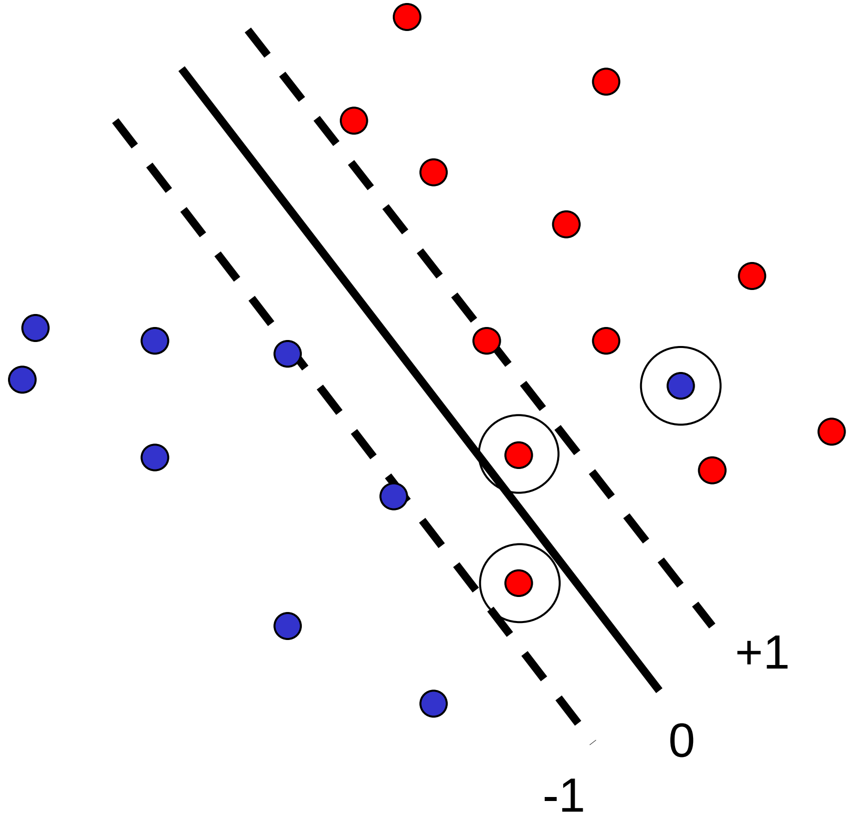
- Non-separable data:

$$\min_{\mathbf{w},b} \underbrace{\frac{1}{2}\|\mathbf{w}\|^2}_{\text{Maximize margin}} + C \underbrace{\sum_{i=1}^{n} \max(0, 1 - y_i(\mathbf{w}\cdot\mathbf{x}_i + b))}_{\text{Minimize classification mistakes}}$$

SVM parameter learning

$$\min_{\mathbf{w},b} \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{n} \max(0, 1 - y_i(\mathbf{w}\cdot\mathbf{x}_i + b))$$





Demo: http://cs.stanford.edu/people/karpathy/svmjs/demo

Nonlinear SVMs

- General idea: the original input space can always be mapped to some higher-dimensional feature space where the training set is separable

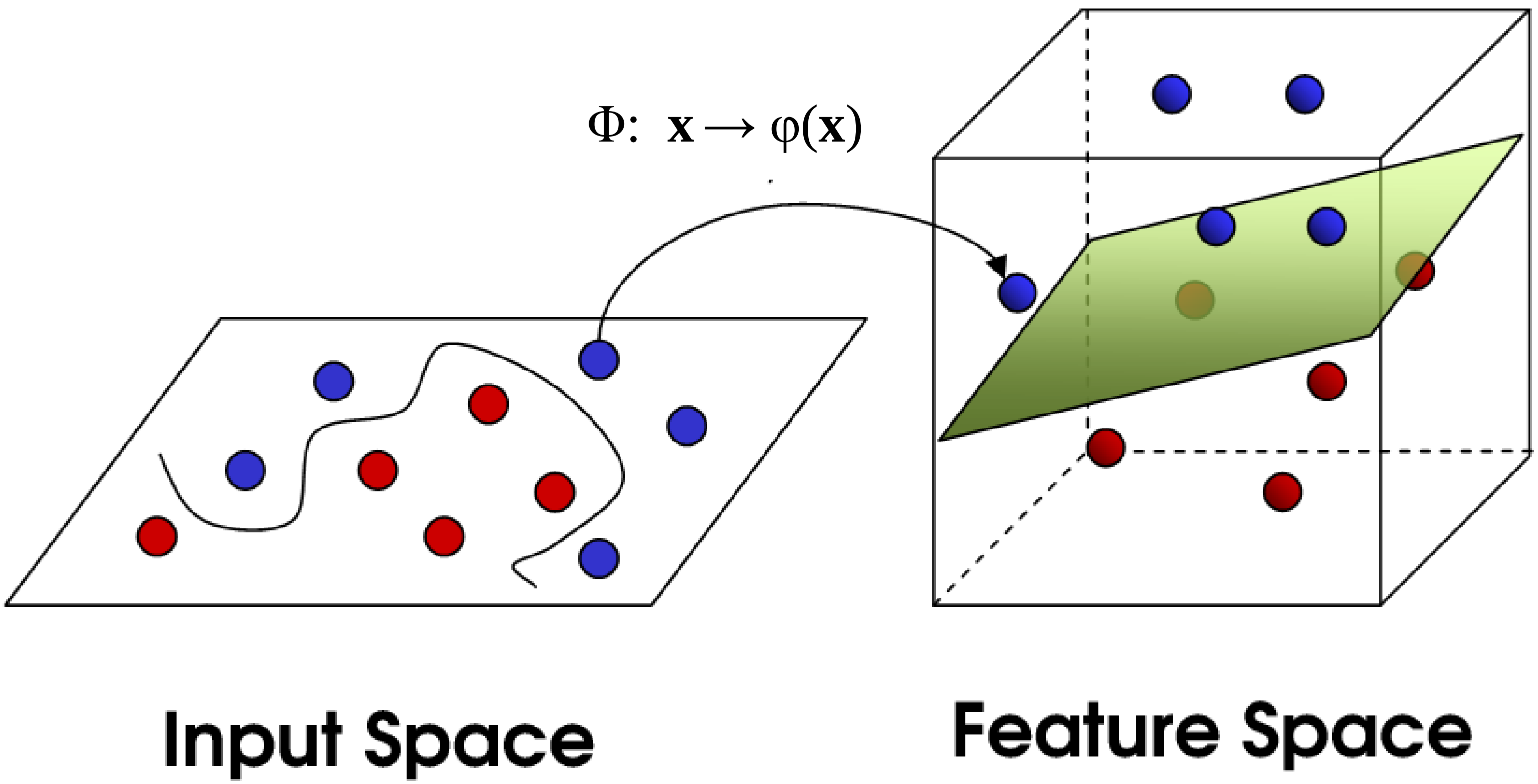
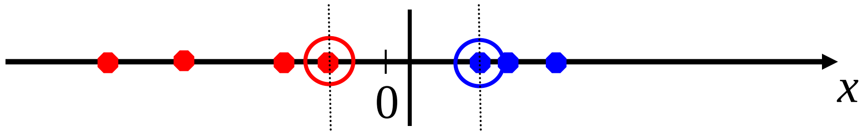


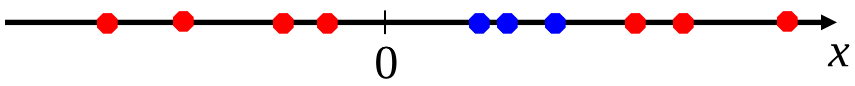
Image source

Nonlinear SVMs

- Linearly separable dataset in 1D:



- Non-linearly separable dataset in 1D:



Slide credit: Andrew Moore

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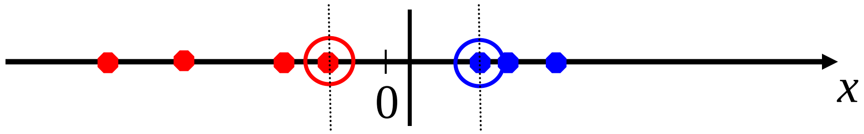
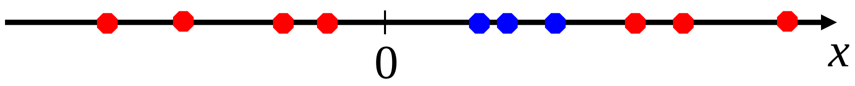
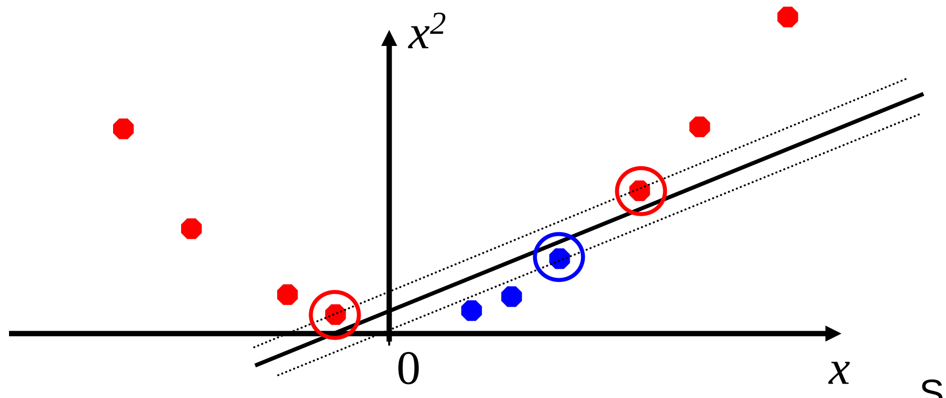


- Non-linearly separable dataset in 1D:



- We can map the data to a *higher-dimensional space*:



Slide credit: Andrew Moore

The kernel trick

- General idea: the original input space can always be mapped to some higher-dimensional feature space where the training set is separable

- **The kernel trick:** instead of explicitly computing the lifting transformation $\varphi(\mathbf{x})$, define a kernel function K such that

$$K(\mathbf{x},\mathbf{y}) = \varphi(\mathbf{x}) \cdot \varphi(\mathbf{y})$$

The kernel trick

- Linear SVM decision function:

$$\mathbf{w}\cdot\mathbf{x}+b=\sum_i \alpha_i y_i \mathbf{x}_i \cdot \mathbf{x} + b$$

learned weight

Support vector

C. Burges, [A Tutorial on Support Vector Machines for Pattern Recognition](), Data Mining and Knowledge Discovery, 1998

The kernel trick

- Linear SVM decision function:

$$\mathbf{w} \cdot \mathbf{x} + b = \sum_i \alpha_i y_i \mathbf{x}_i \cdot \mathbf{x} + b$$

- Kernel SVM decision function:

$$\sum_i \alpha_i y_i \varphi(\mathbf{x}_i) \cdot \varphi(\mathbf{x}) + b = \sum_i \alpha_i y_i K(\mathbf{x}_i, \mathbf{x}) + b$$

- This gives a nonlinear decision boundary in the original feature space

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Example

2-dimensional vectors x=[x_1 x_2];

let $K(\mathrm{x_i},\mathrm{x_j})=(1 + \mathrm{x_i}^T\mathrm{x_j})^2$

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let $K(x_i,x_j)=(1 + x_i^T x_j)^2$

Need to show that $K(x_i,x_j)= \varphi(x_i)^T \varphi(x_j)$:

Example

2-dimensional vectors $\mathrm{x}=[x_1\ \ x_2]$;

let $K(\mathrm{x_i},\mathrm{x_j})=(1 + \mathrm{x_i}^T\mathrm{x_j})^2$

Need to show that $K(\mathrm{x_i},\mathrm{x_j}) = \varphi(\mathrm{x_i})^T\varphi(\mathrm{x_j})$:

$$K(\mathrm{x_i},\mathrm{x_j})=(1 + \mathrm{x_i}^T\mathrm{x_j})^2,$$

$$= 1+ x_{i1}^2x_{j1}^2 + 2\, x_{i1}x_{j1}\, x_{i2}x_{j2} + x_{i2}^2x_{j2}^2 + 2x_{i1}x_{j1} + 2x_{i2}x_{j2}$$

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$= [1\ \ x_{i1}^2\ \ \sqrt{2}\, x_{i1}x_{i2}\ \ x_{i2}^2\ \ \sqrt{2}x_{i1}\ \ \sqrt{2}x_{i2}]^T$

$[1\ \ x_{j1}^2\ \ \sqrt{2}\, x_{j1}x_{j2}\ \ x_{j2}^2\ \ \sqrt{2}x_{j1}\ \ \sqrt{2}x_{j2}]$

Example

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$$= [1 \;\; x_{i1}^2 \;\; \sqrt{2}\, x_{i1}x_{i2} \;\; x_{i2}^2 \;\; \sqrt{2}x_{i1} \;\; \sqrt{2}x_{i2}]^T$$

$$[1 \;\; x_{j1}^2 \;\; \sqrt{2}\, x_{j1}x_{j2} \;\; x_{j2}^2 \;\; \sqrt{2}x_{j1} \;\; \sqrt{2}x_{j2}]$$

$$= \varphi(\mathrm{x_i})^T\varphi(\mathrm{x_j}),$$

where $\varphi(\mathrm{x}) = [1 \;\; x_1^2 \;\; \sqrt{2}\, x_1x_2 \;\; x_2^2 \;\; \sqrt{2}x_1 \;\; \sqrt{2}x_2]$

SVMs: Pros and cons

- Pros
  - Kernel-based framework is **very powerful**, flexible
  - Training is convex optimization, **globally optimal solution** can be found
  - SVMs work very well in practice, even with **very small training** sample sizes

- Cons
  - No “direct” **multi-class SVM**, must combine two-class SVMs (e.g., with one-vs-others)
  - **Computation, memory** (esp. for nonlinear SVMs)

Multiclass Support Vector Machine loss

- *i*th example: image x_i and the label y_i
- Score for the *j*th class: $s_j = f(x_i, W)_j$

Source: cs231n, http://cs231n.github.io/linear-classify/

Multiclass Support Vector Machine loss

- i^{th} example: image x_i and the label y_i
- Score for the j^{th} class: $s_j = f(x_i, W)_j$

- The Multiclass SVM loss for the i^{th} example is then formalized as follows:

$$\underbrace{L_i}_{\text{Hinge Loss}} = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + \underbrace{\Delta}_{\text{Margin}})$$

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Problem: W is not necessarily unique

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Multiclass Support Vector Machine loss

- Regularization Penalty:

$$R(W) = \sum_k \sum_l W_{k,l}^2$$

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$$R(W) = \sum_k \sum_l W_{k,l}^2$$

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$$L = \underbrace{\frac{1}{N} \sum_i L_i}_{\text{data loss}} + \underbrace{\lambda R(W)}_{\text{regularization loss}}$$

Source: cs231n, http://cs231n.github.io/linear-classify/

Multiclass Support Vector Machine loss

- Regularization Penalty:

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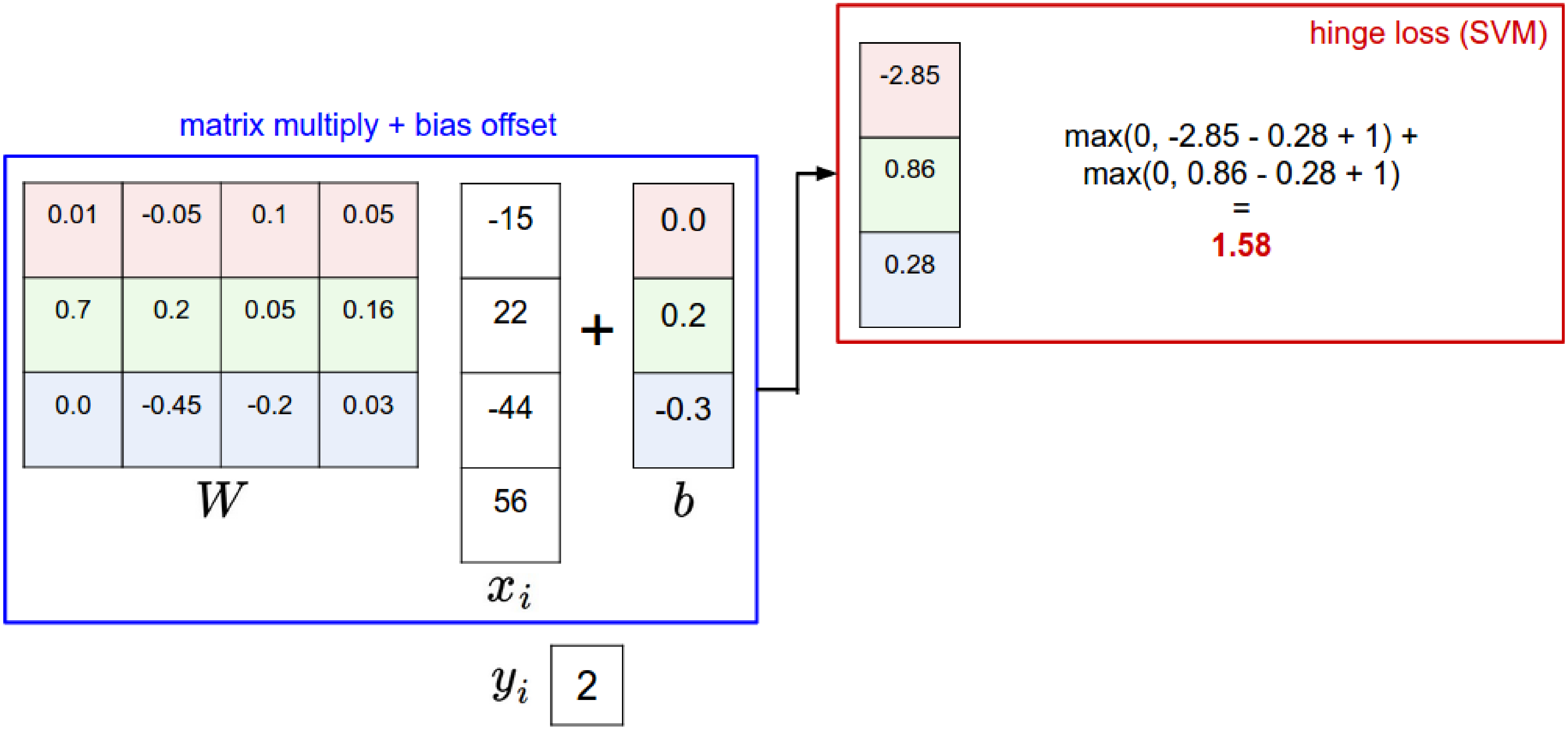
$$\Downarrow$$

$$L = \frac{1}{N} \sum_i \sum_{j \neq y_i} \left[\max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + \Delta) \right] + \lambda \sum_k \sum_l W_{k,l}^2$$

Source: cs231n, http://cs231n.github.io/linear-classify/

Hinge Loss



Source: cs231n, http://cs231n.github.io/linear-classify/

Softmax Classifier

- Interprets the class scores as the unnormalized log probabilities for each class and replace the *hinge loss* with a **cross-entropy loss** that has the form:

$$L_i = -log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right) = -s_{y_i} + log \sum_j e^{s_j}$$

Source: cs231n, http://cs231n.github.io/linear-classify/

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- Softmax Loss:

$$L = \underbrace{\frac{1}{N}\sum_i L_i}_{\text{data loss}} + \underbrace{\lambda R(W)}_{\text{regularization loss}}$$

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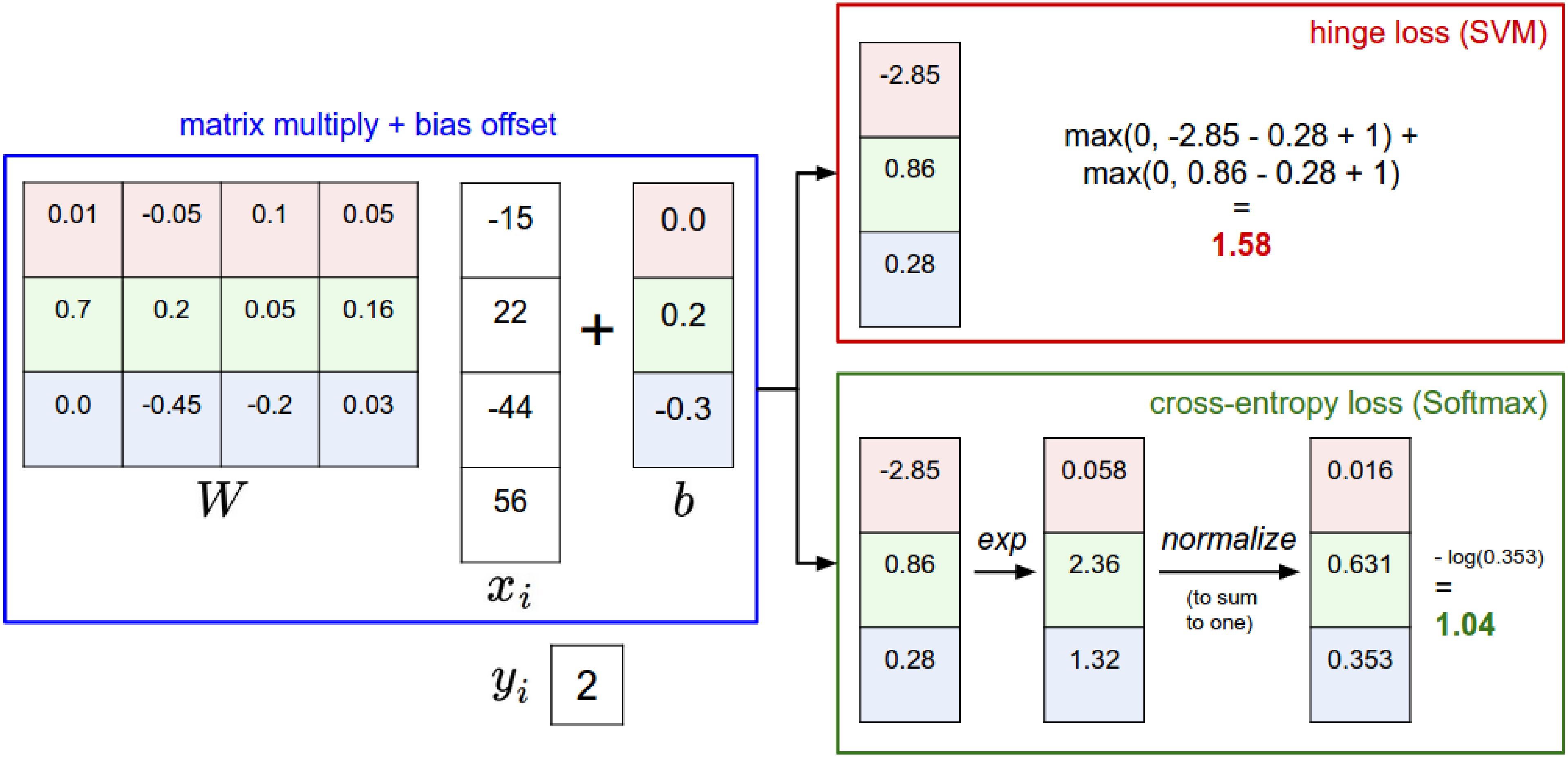
Hinge vs Cross-entropy Loss



Source: cs231n, http://cs231n.github.io/linear-classify/

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- Antonio Torralba
- Rob Fergus
- Leibe
- And many more ………..

Next Lecture

Neural Networks