Computer Vision
Camera Models and Projective Geometry

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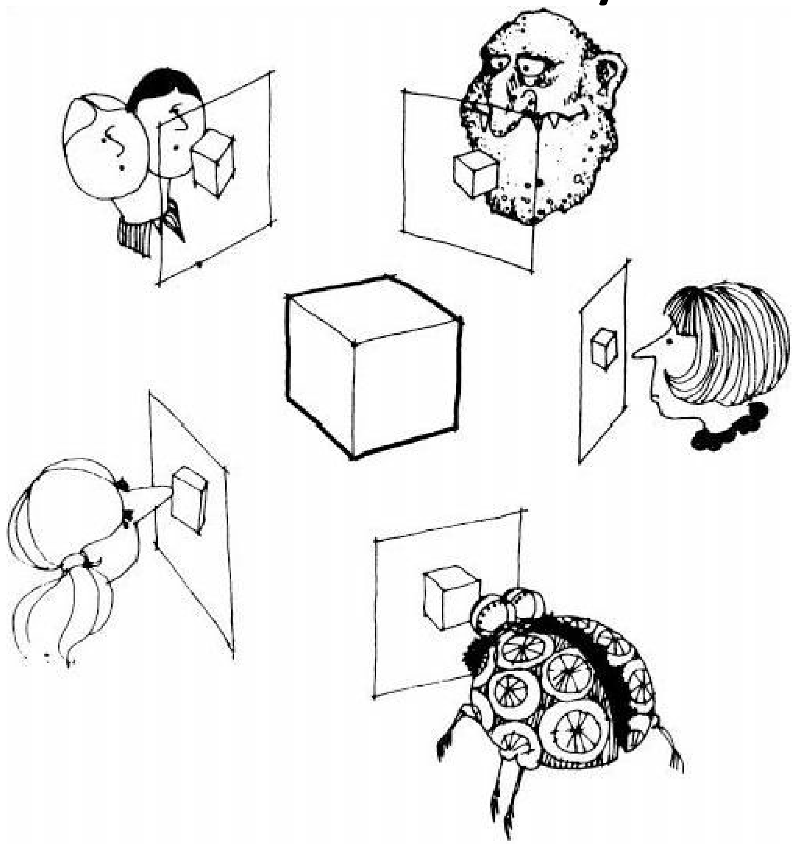
Camera Models and Projective Geometry



Perspective and 3D Geometry

- **Camera models and Projective geometry**
  - What's the mapping between image and world coordinates?
- **Projection Matrix and Camera calibration**
  - What's the projection matrix between scene and image coordinates?
  - How to calibrate the projection matrix?
- **Single view metrology and Camera properties**
  - How can we measure the size of 3D objects in an image?
  - What are the important camera properties?
- **Photo stitching**
  - What's the mapping from two images taken without camera translation?
- **Epipolar Geometry and Stereo Vision**
  - What's the mapping from two images taken with camera translation?
- **Structure from motion**
  - How can we recover 3D points from multiple images?

Next few classes: Single-view Geometry

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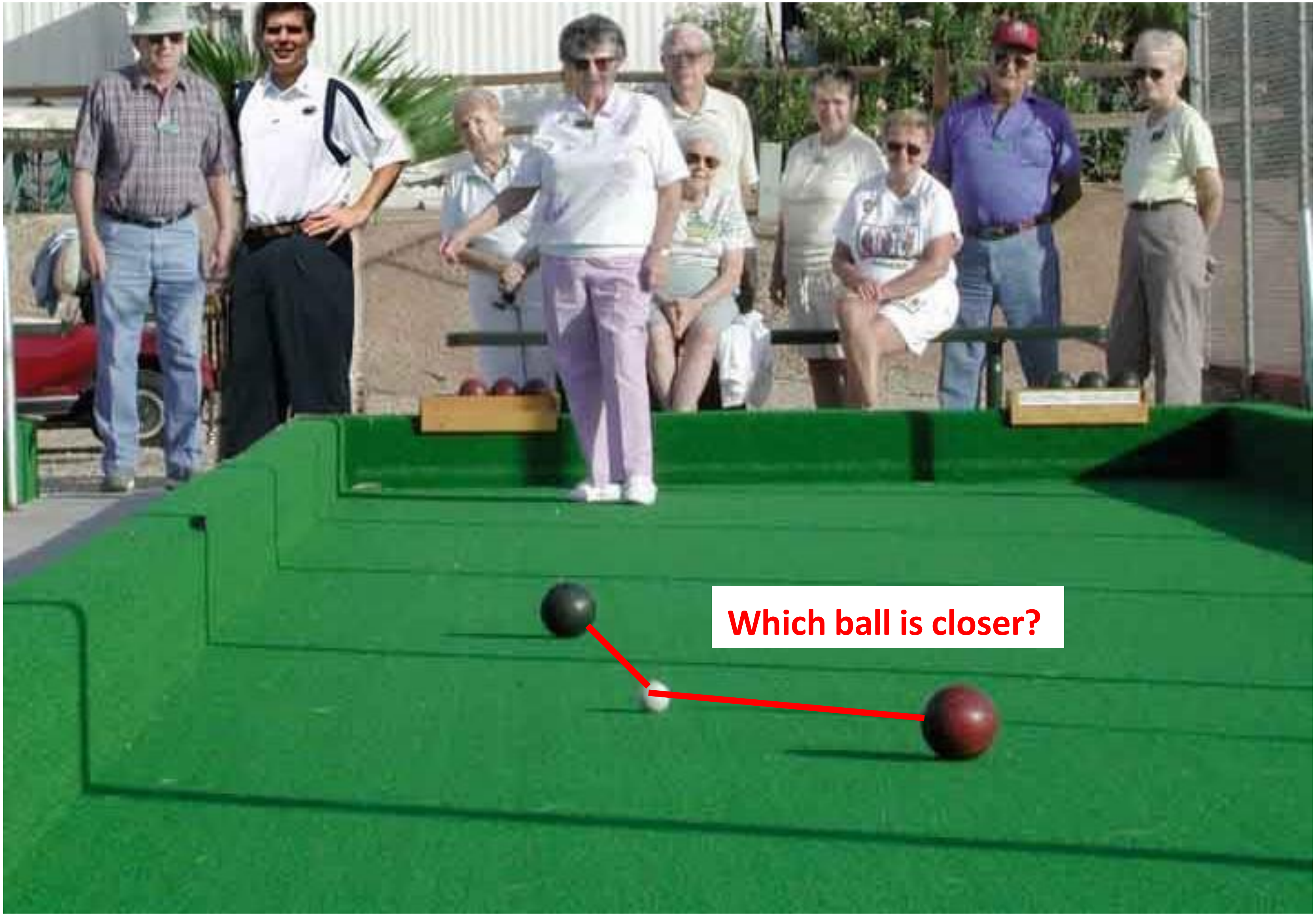
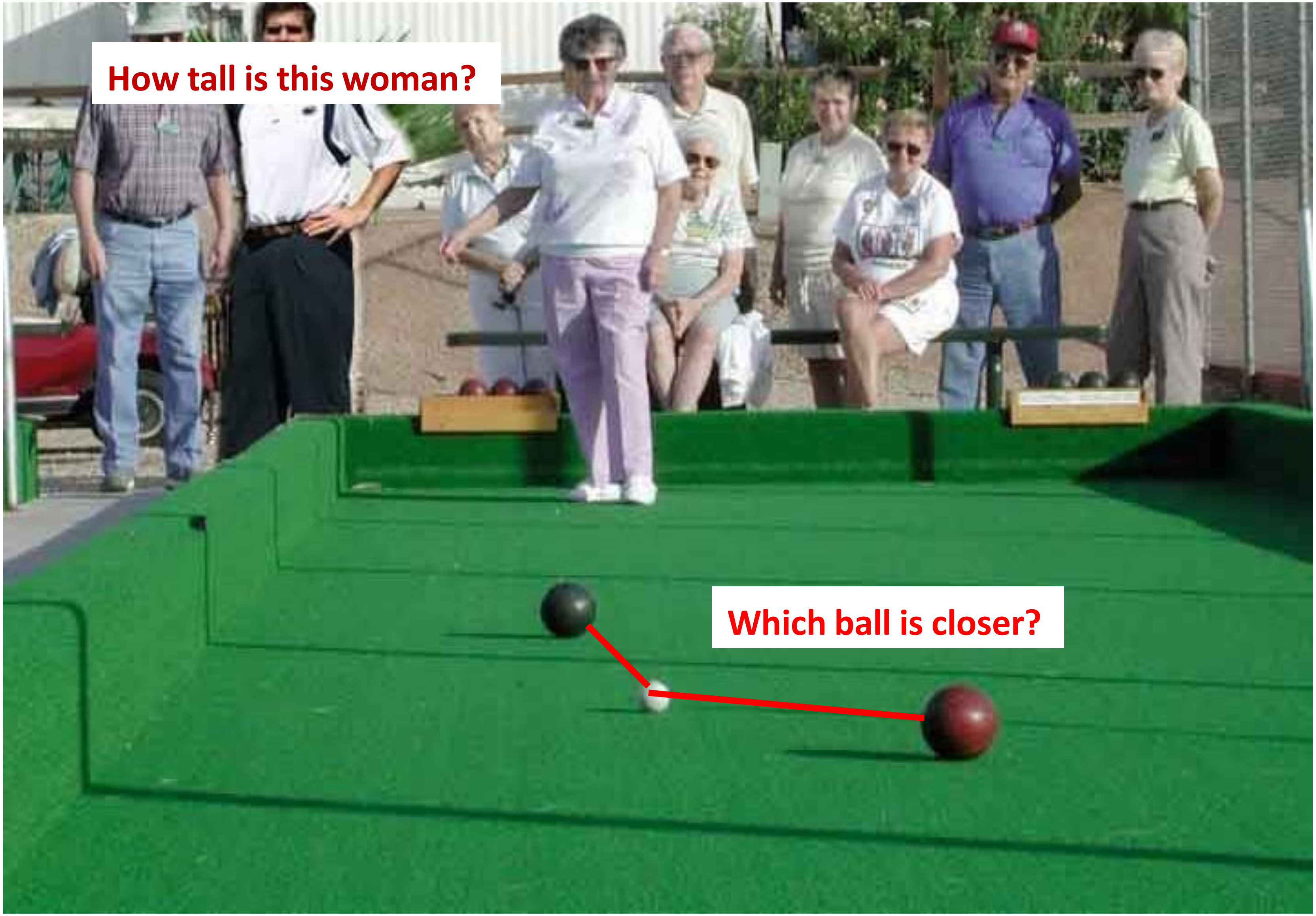


Next few classes: Single-view Geometry



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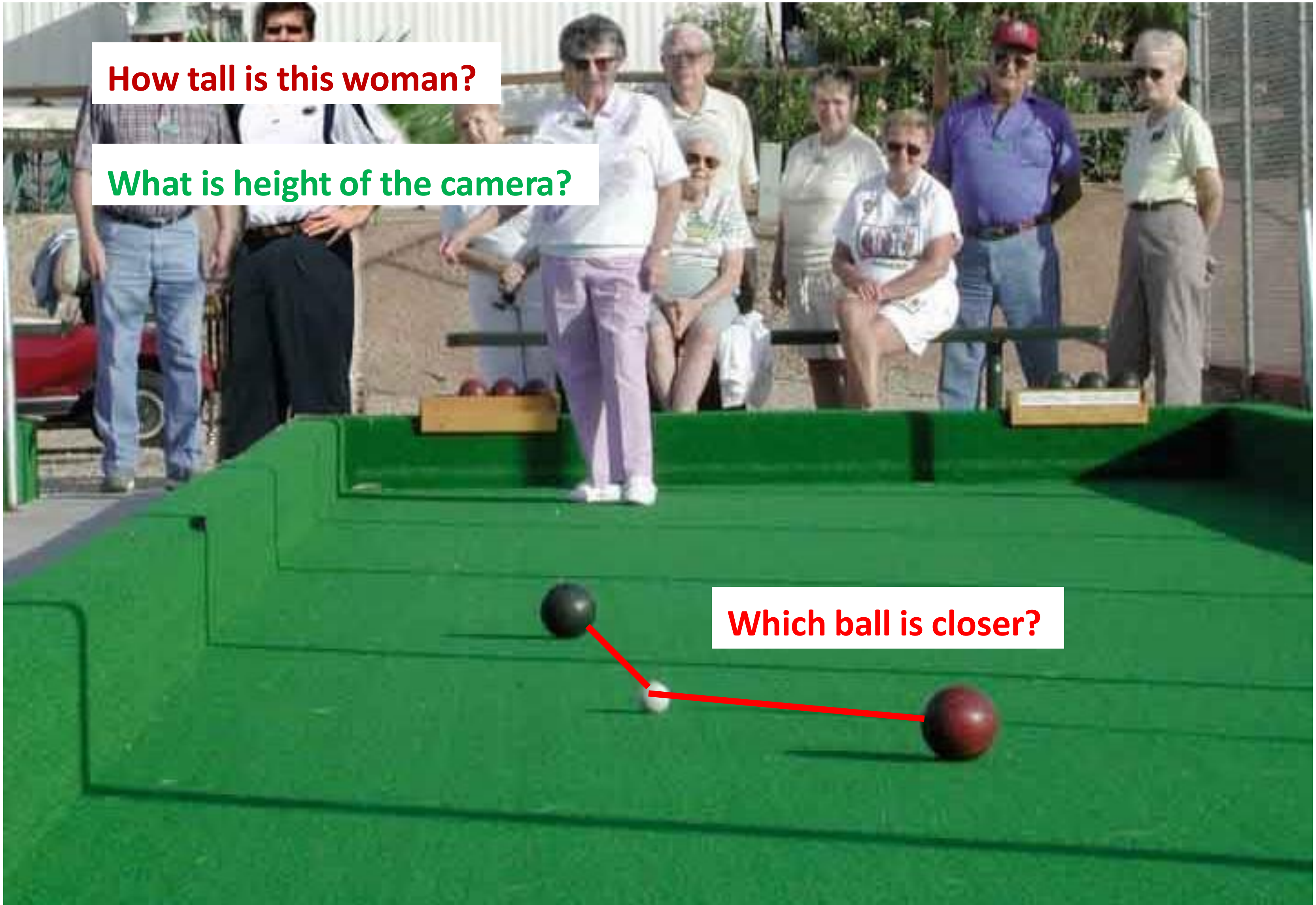

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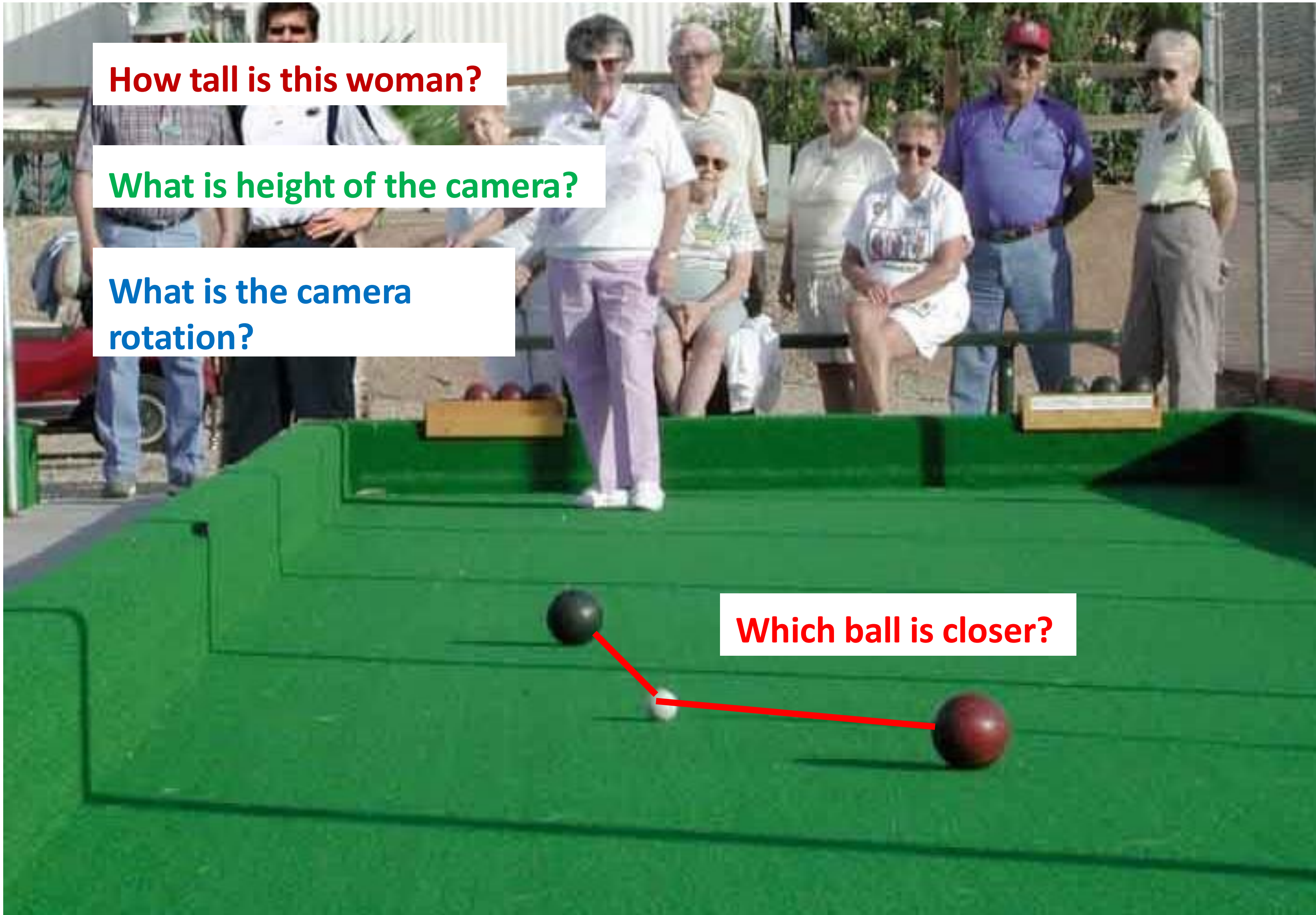

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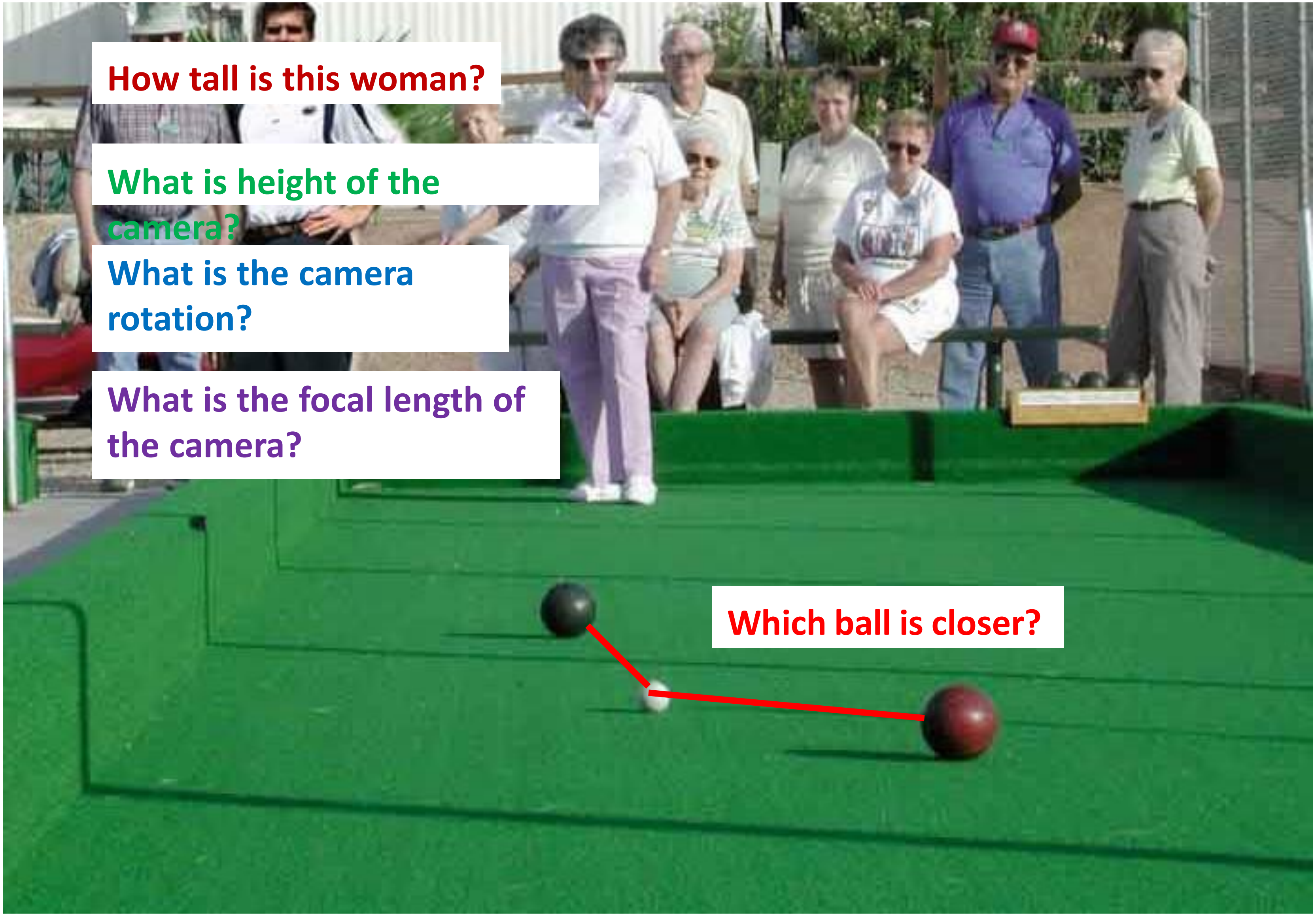
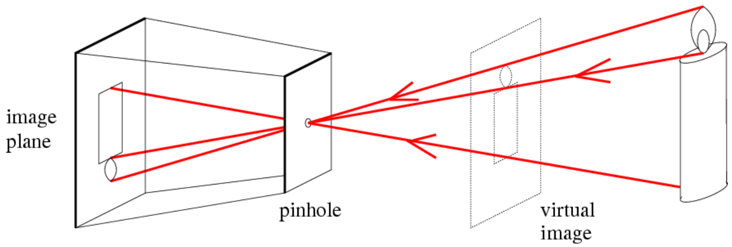


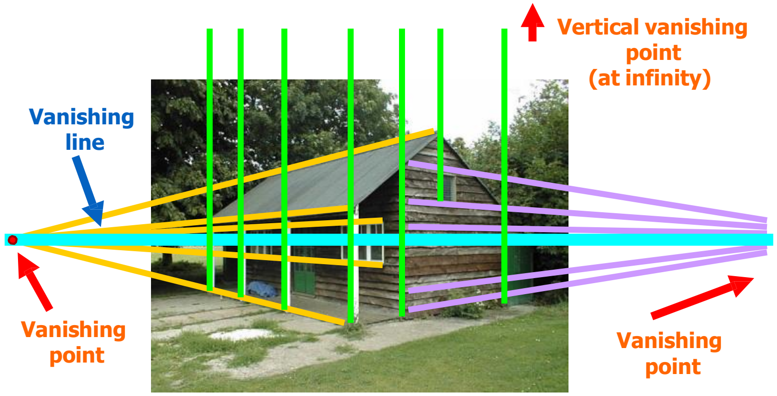
Today's class

Mapping between image and world coordinates

- Pinhole camera model



- Projective geometry
  - Vanishing points and lines



What is an Image?

- Up until now: a function –a 2D pattern of intensity values
- Today: a 2D projection of 3D points

Slide modified from: Bobick

Image formation

Let’s design a camera

–Idea 1: put a piece of film in front of an object

Slide source: Seitz

Image formation

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Image formation

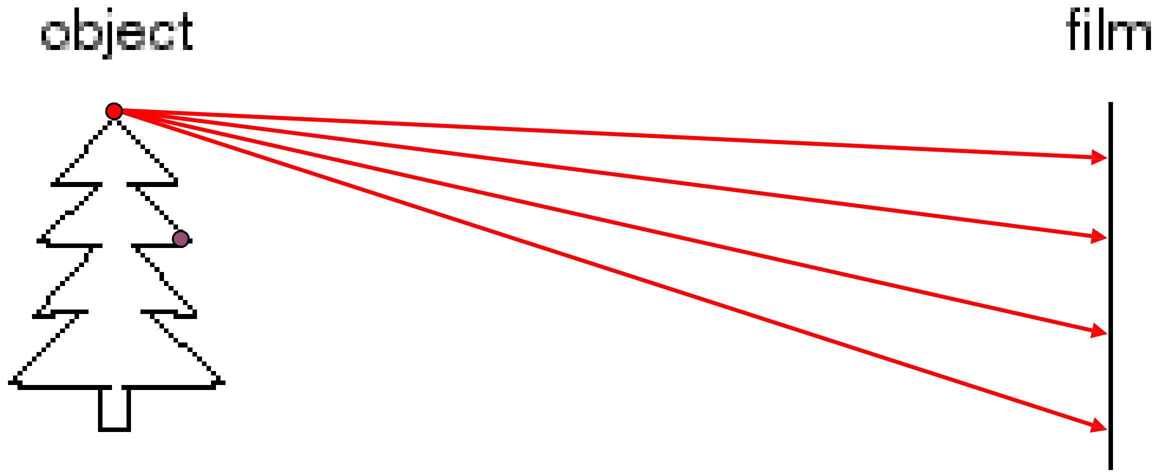

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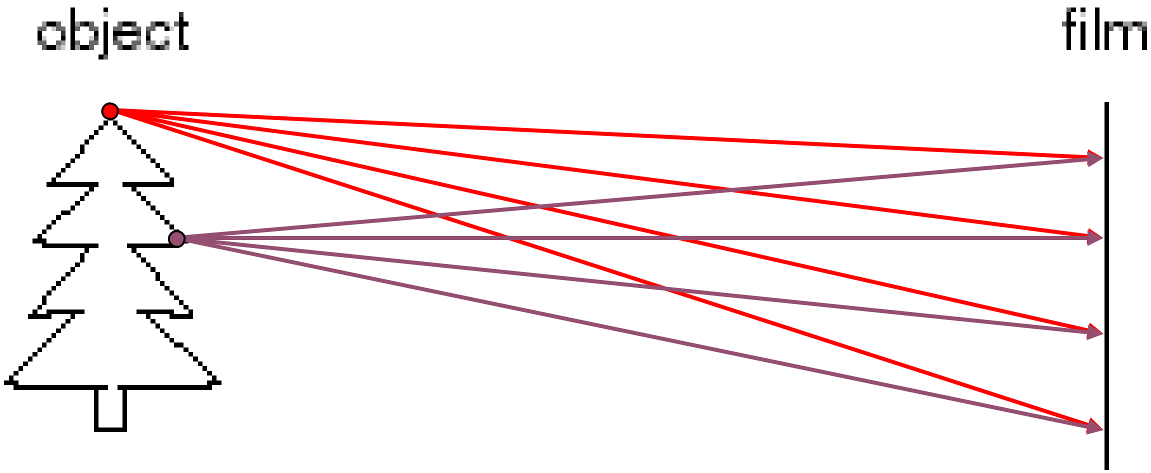
Image formation



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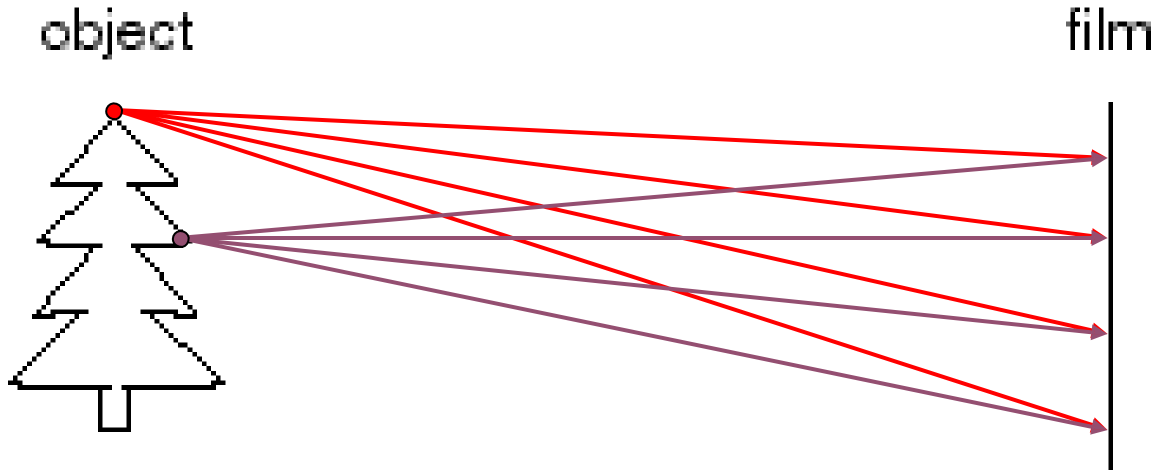


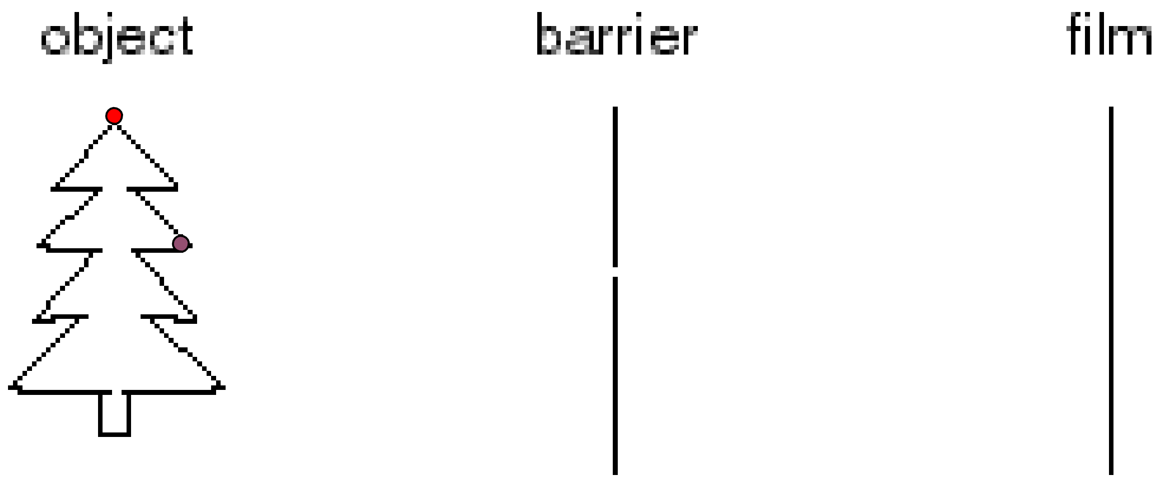
Do we get a reasonable image?

Let's design a camera

–Idea 1: put a piece of film in front of an object

Slide source: Seitz

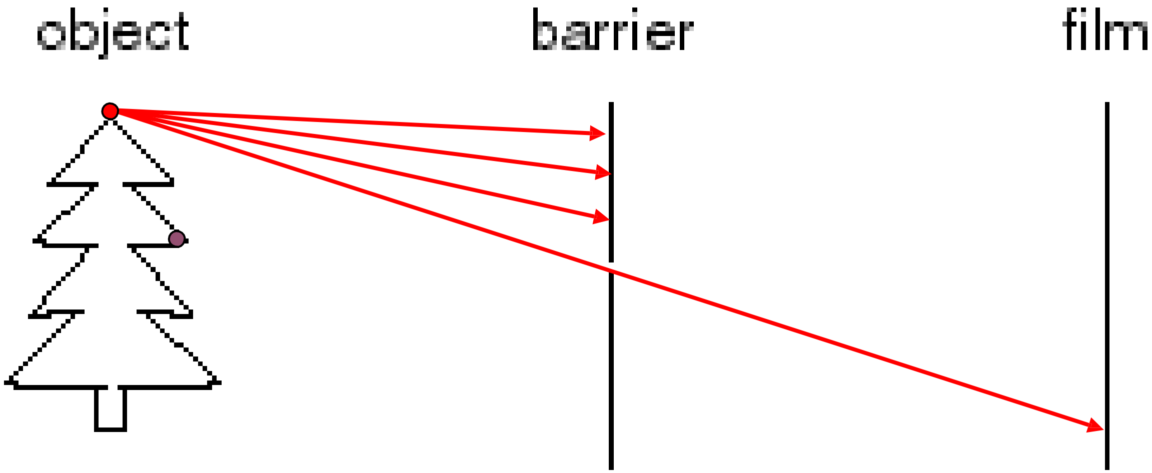
Pinhole camera



Idea 2: add a barrier to block off most of the rays

Slide source: Seitz

Pinhole camera



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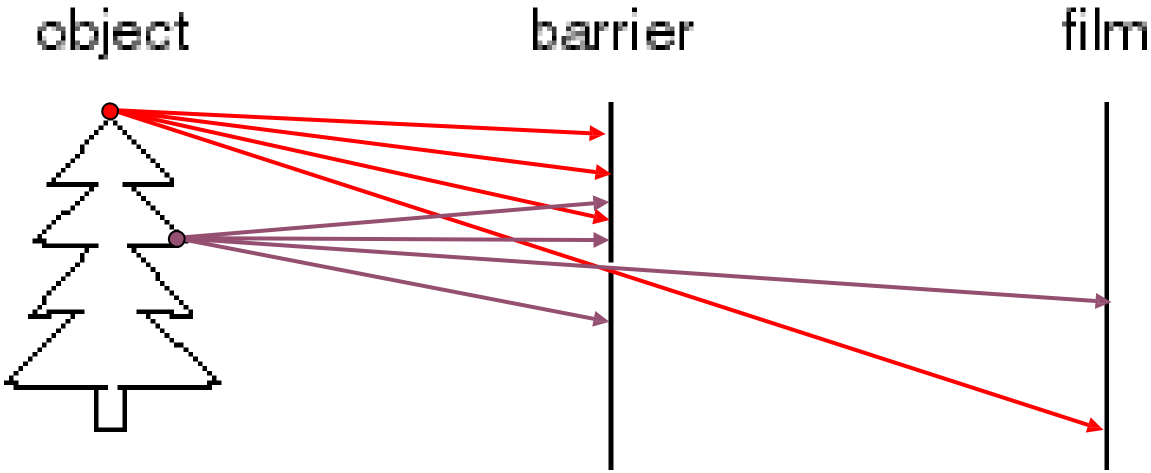

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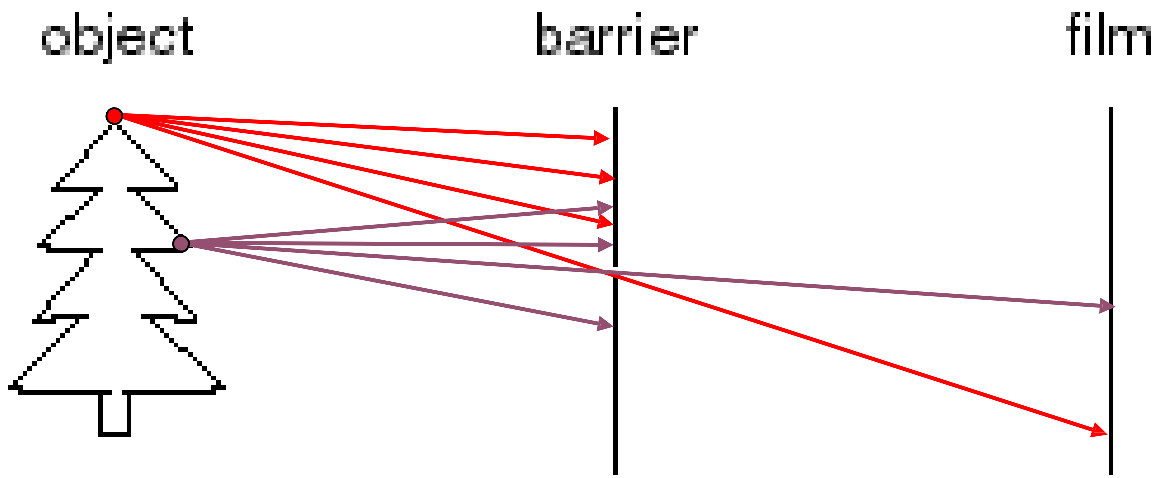
Pinhole camera

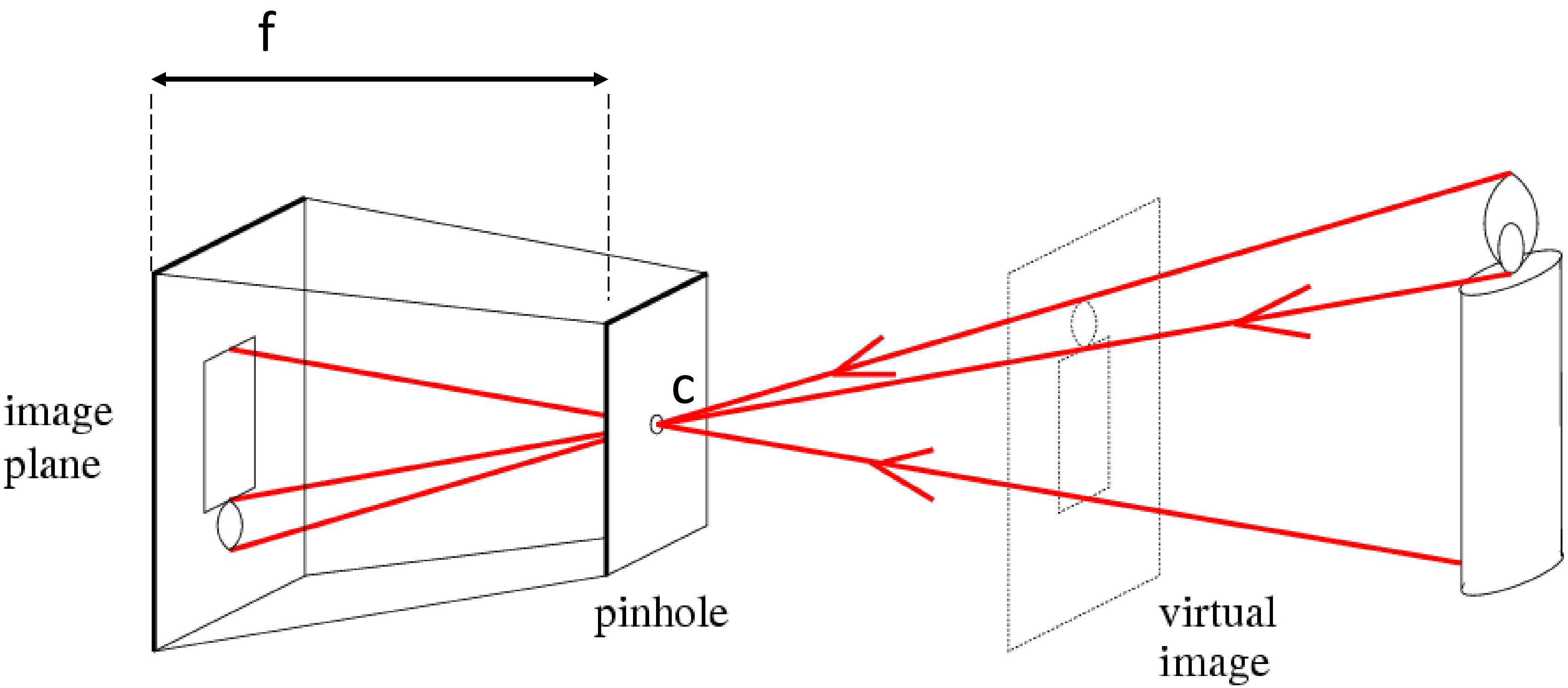


Idea 2: add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the **aperture**

Slide source: Seitz

Pinhole camera



f = focal length

c = center of the camera

Figure from Forsyth

Camera obscura: the pre-camera

- First idea: Mo-Ti, China (470BC to 390BC)
- First built: Alhazen, Iraq/Egypt (965 to 1039AD)

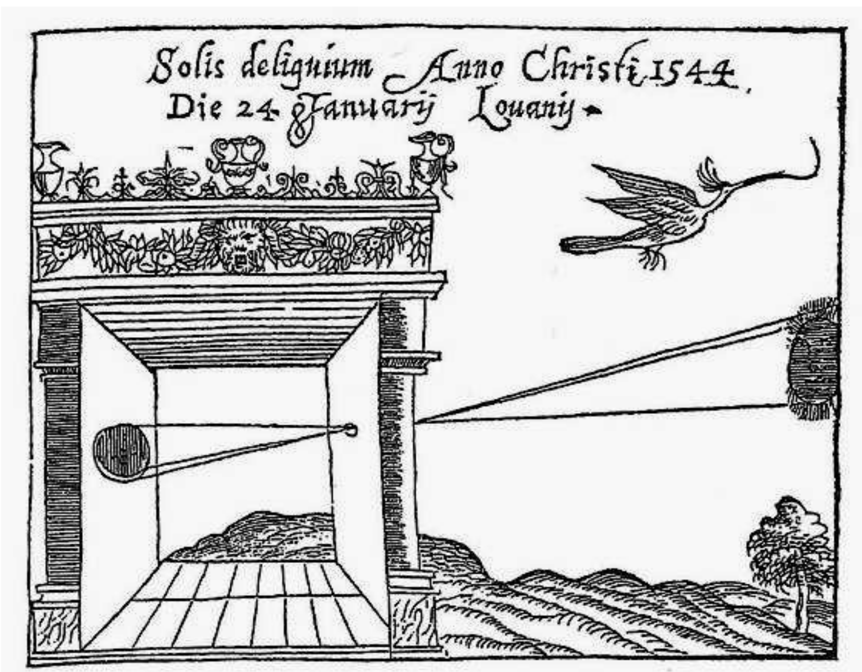


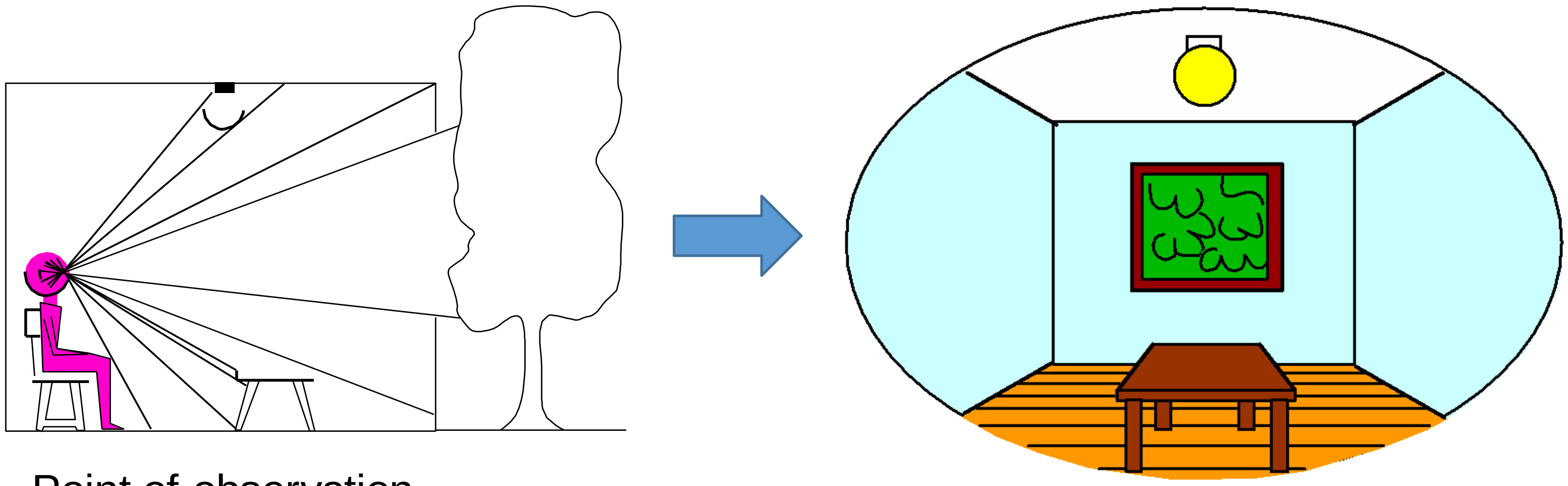
Illustration of Camera Obscura

Freestanding camera obscura at UNC Chapel Hill

Photo by Seth Ilys

Dimensionality Reduction Machine (3D to 2D)

3D world *2D image*



Point of observation

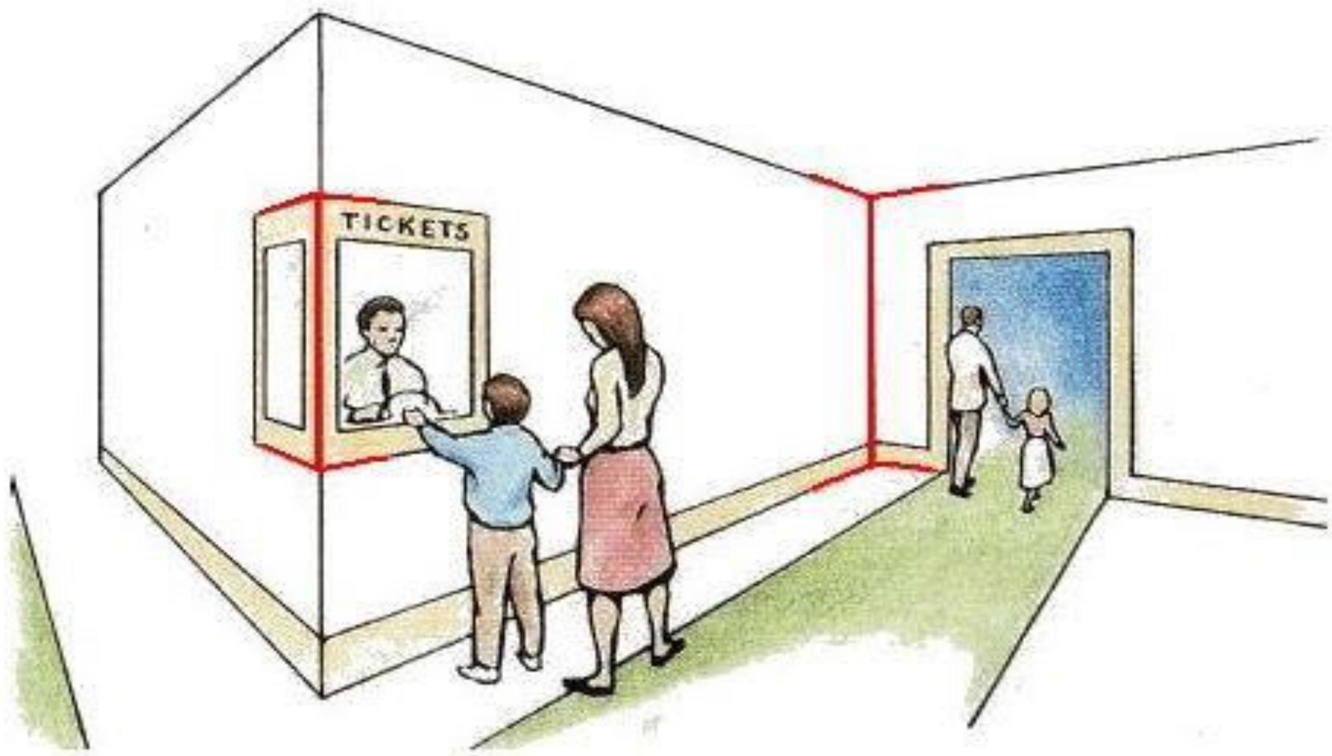
Projection can be tricky…

Slide source: Seitz

Projection can be tricky…

Making of 3D sidewalk art: http://www.youtube.com/watch?v=3SNYtd0Ayt0

Projection can be tricky…



http://www.michaelbach.de/ot/sze_muelue/index.html

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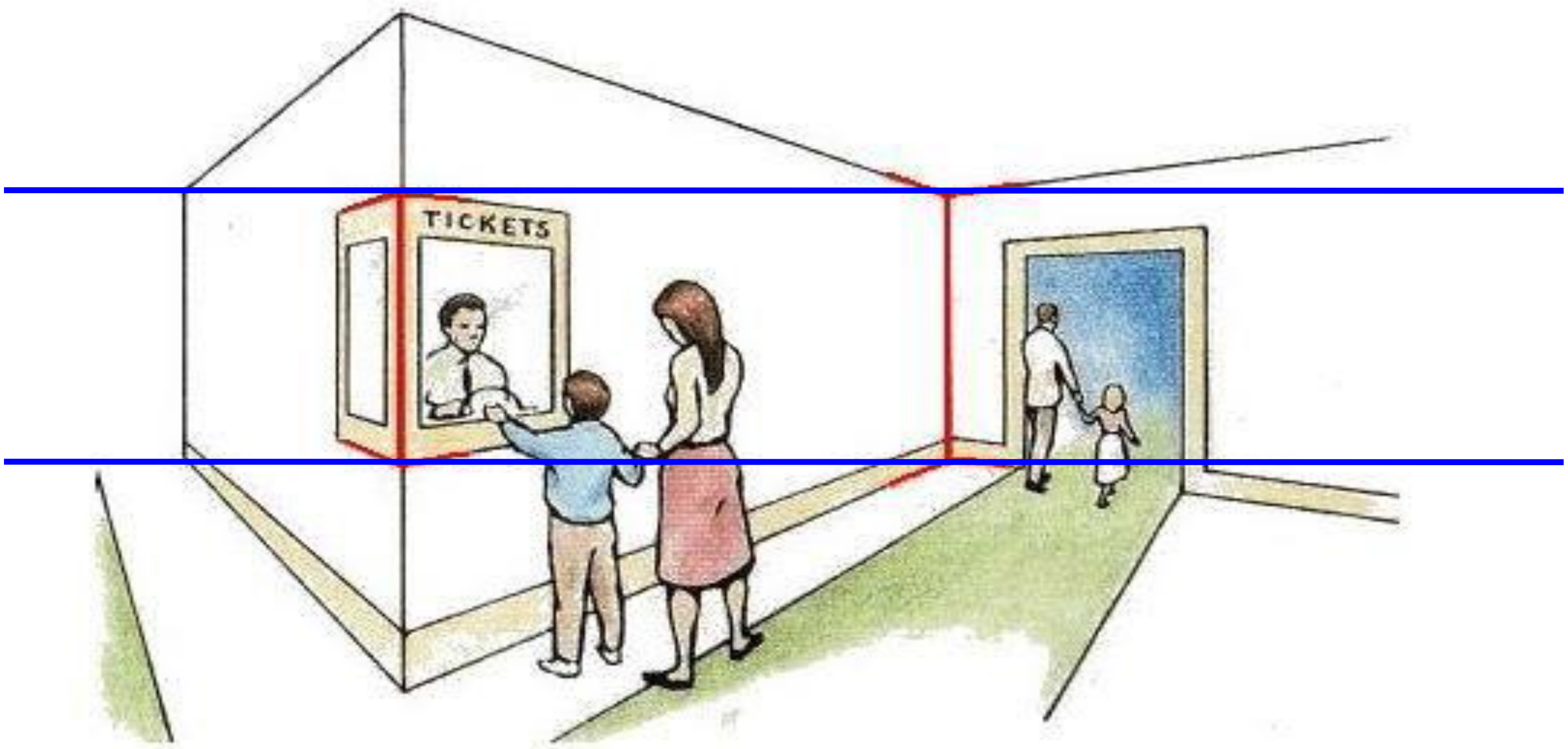

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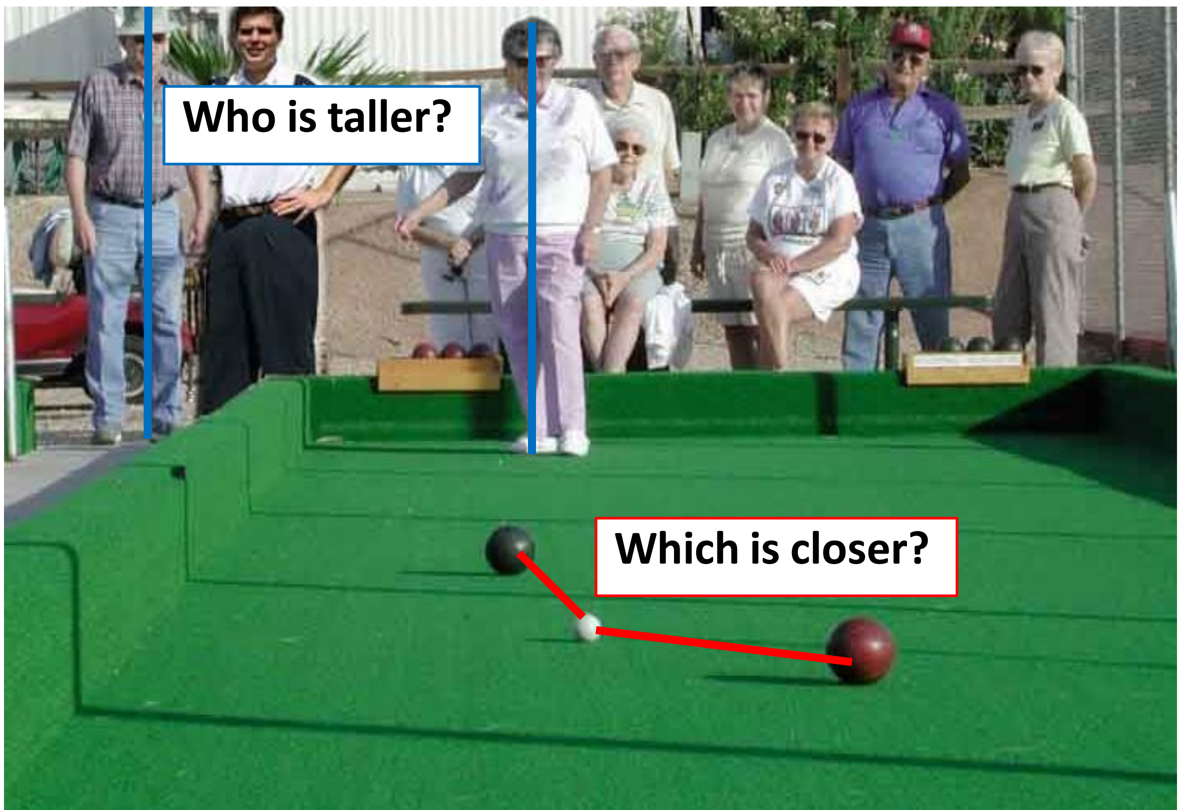
Projective Geometry

What is lost?

- Length



Length is not preserved

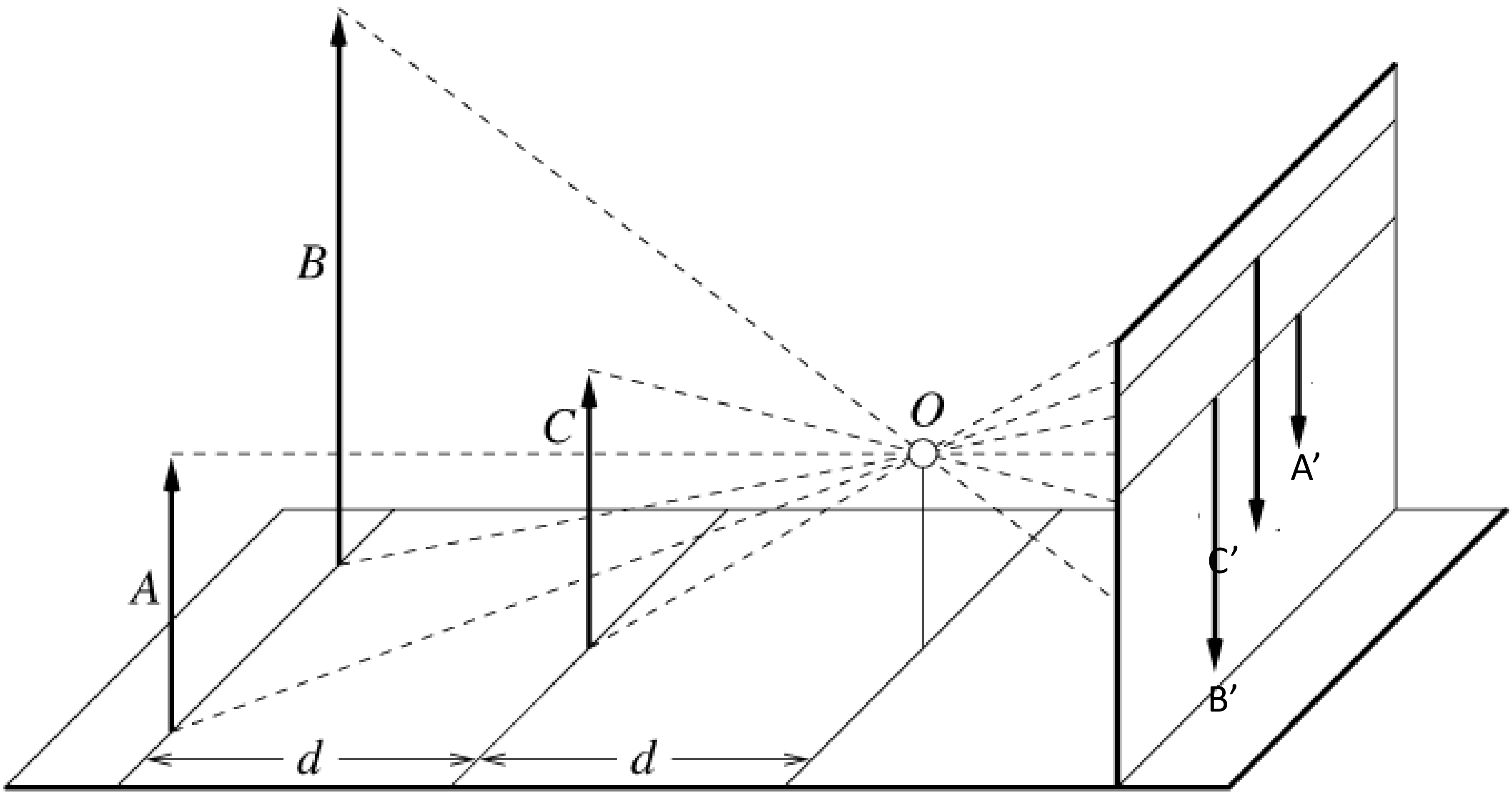
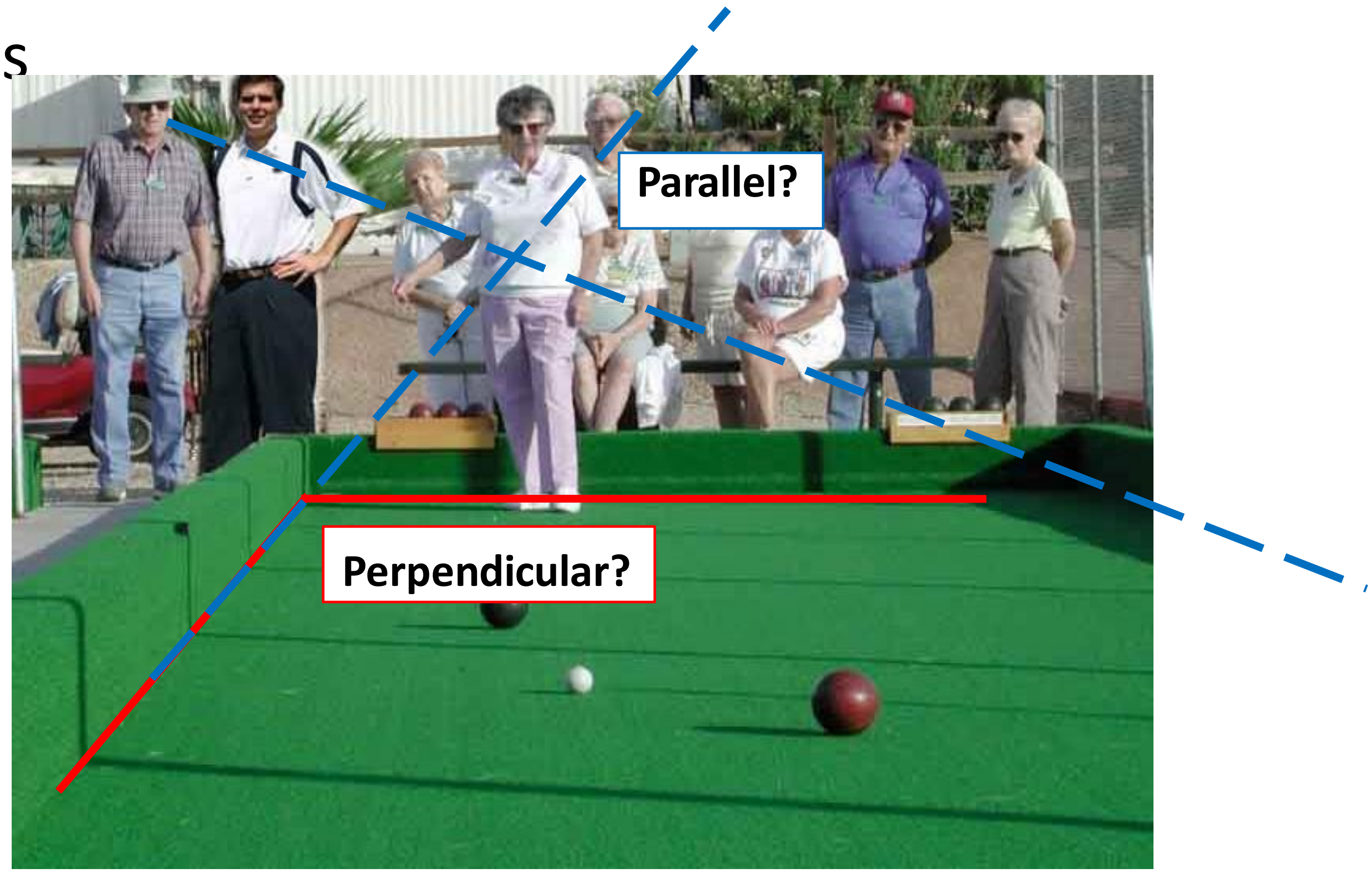


Figure by David Forsyth

Projective Geometry

What is lost?

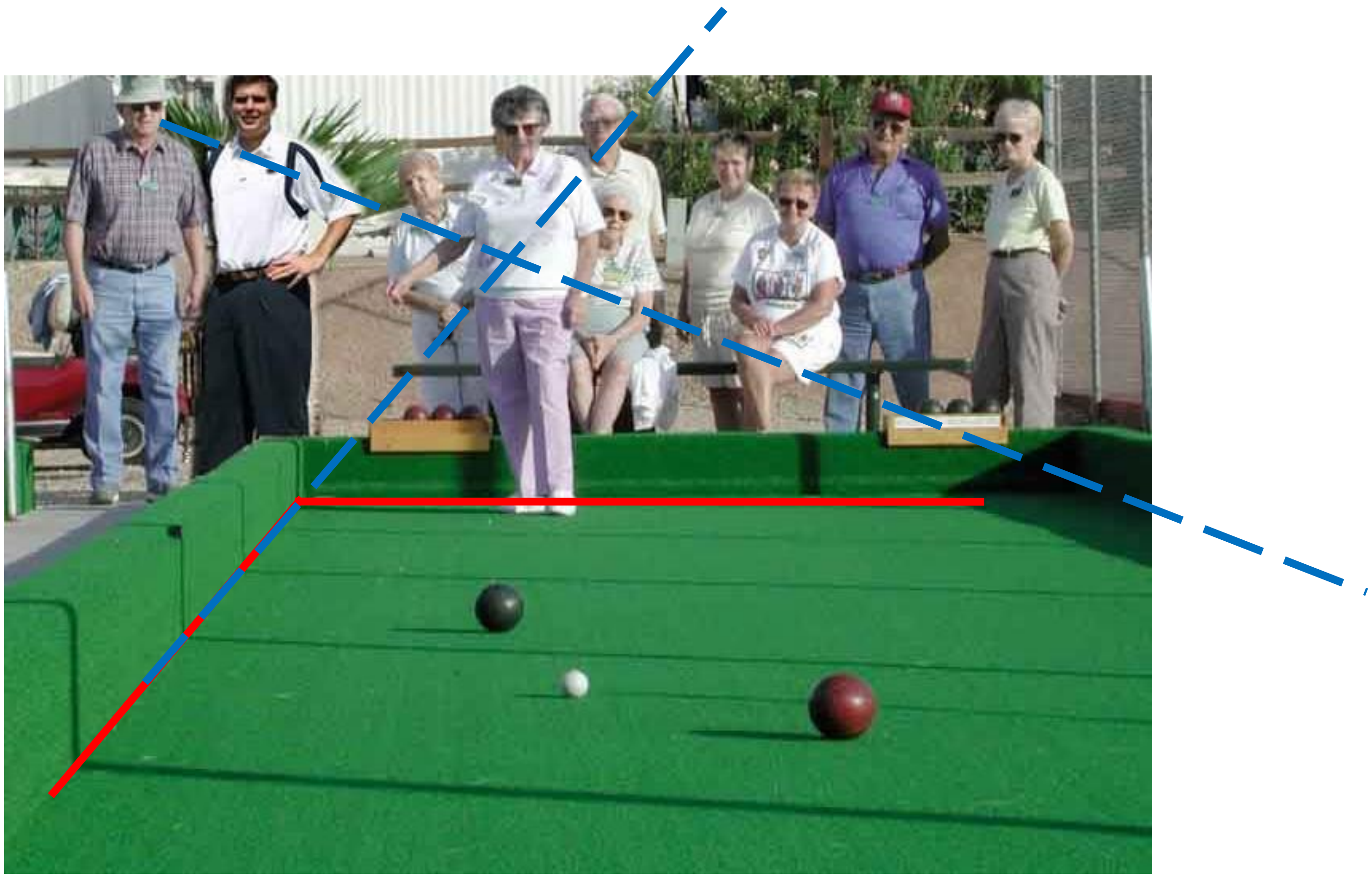
- Length
- Angles



Projective Geometry

What is preserved?

- Straight lines are still straight

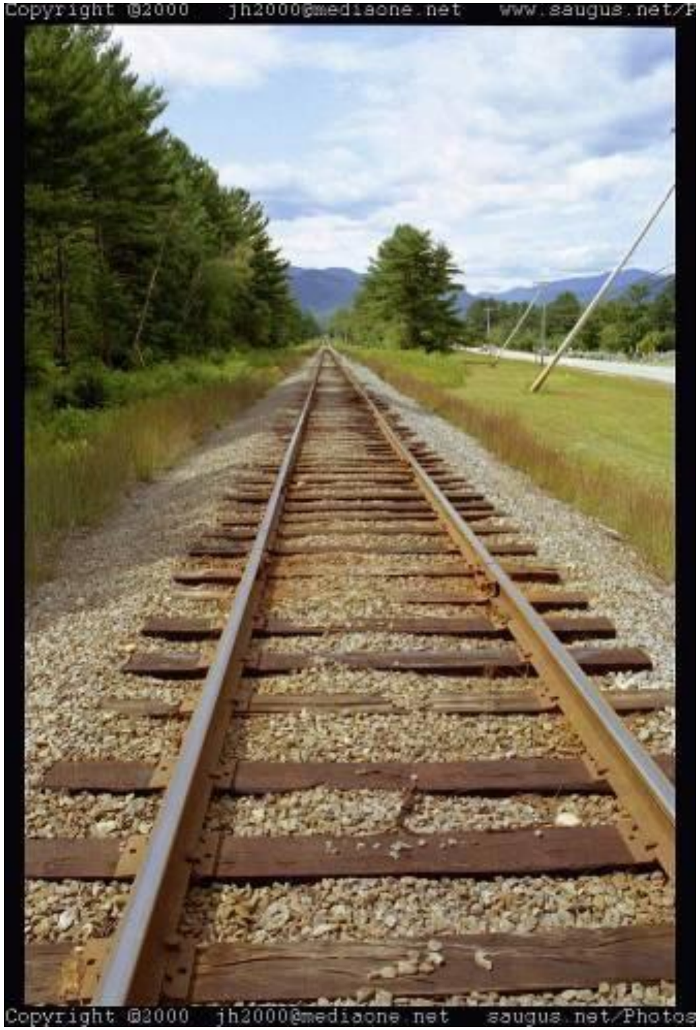


Projection properties

- Many-to-one: Any points along same ray map to same point in image
- Points → points
- Lines → lines (collinearity is preserved)
  - But line through focal point projects to a point
- Planes → planes (or half-planes)
  - But plane through focal point projects to line

Vanishing points and lines

Parallel lines in the world intersect in the image at a "vanishing point"



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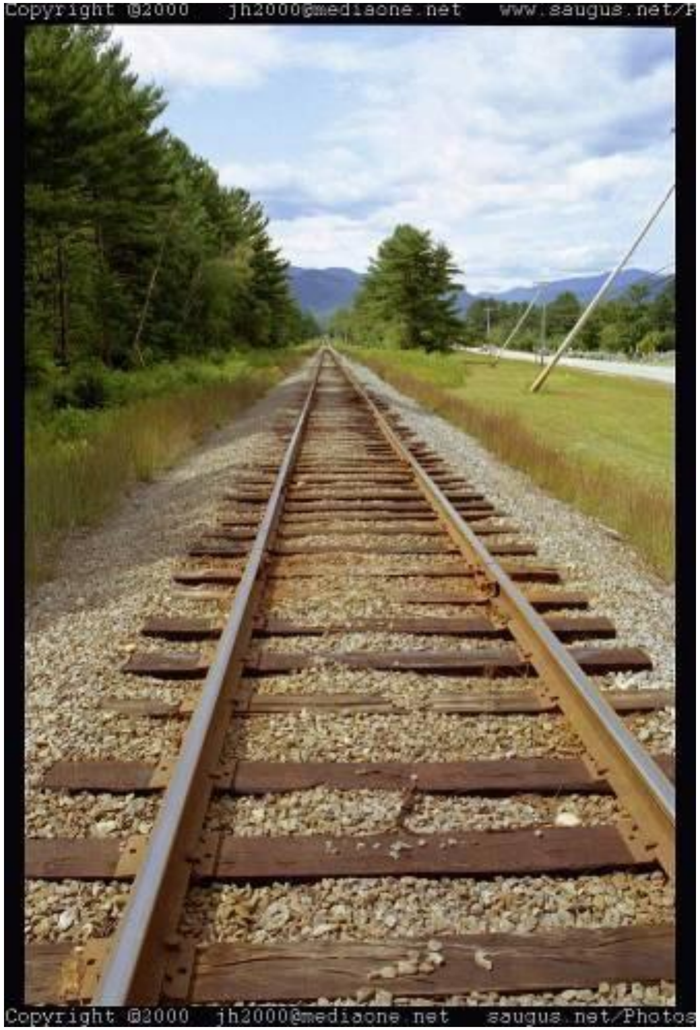
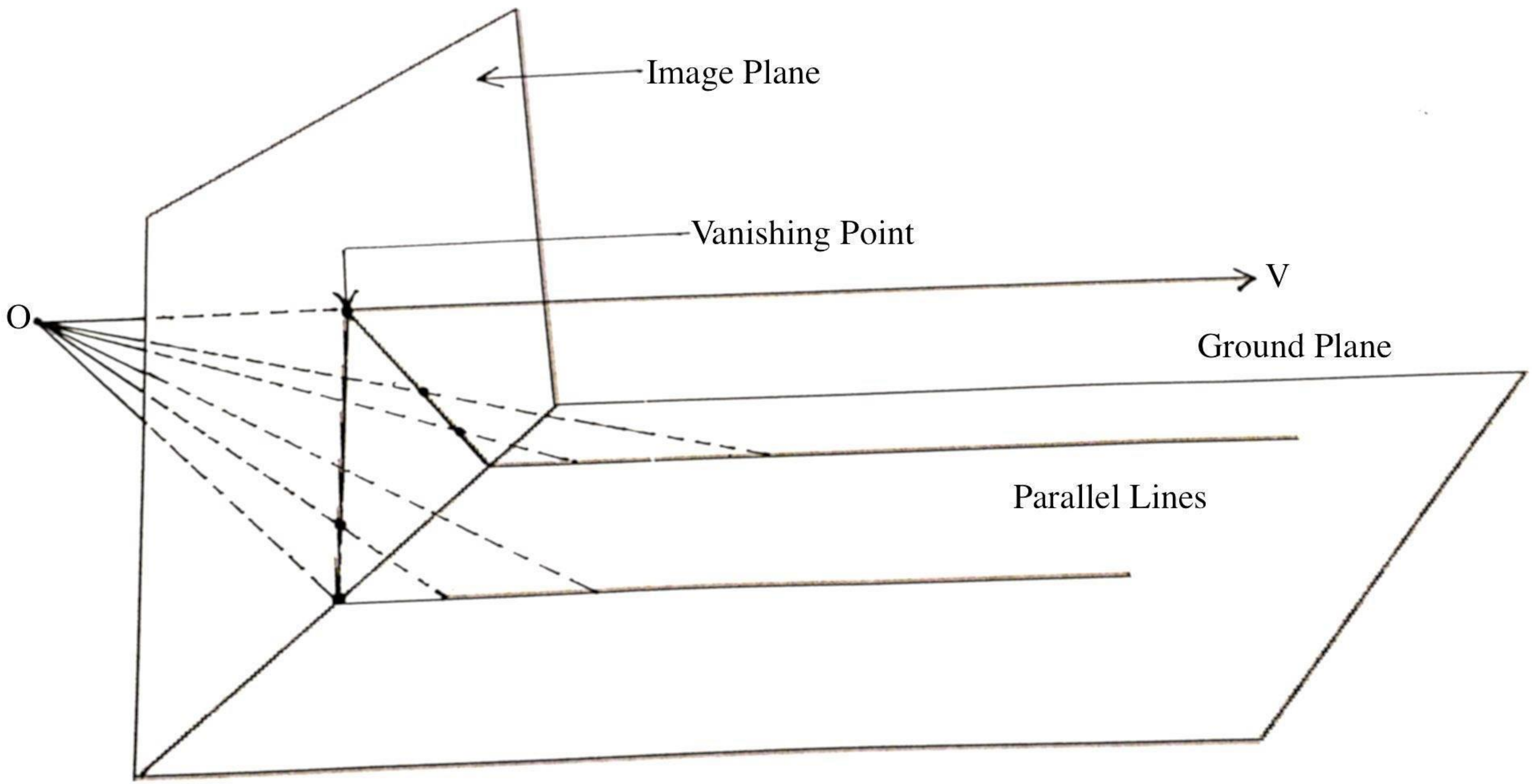


- Each direction in space has its own vanishing point

- But parallel lines to the image plane remain parallel

Vanishing points



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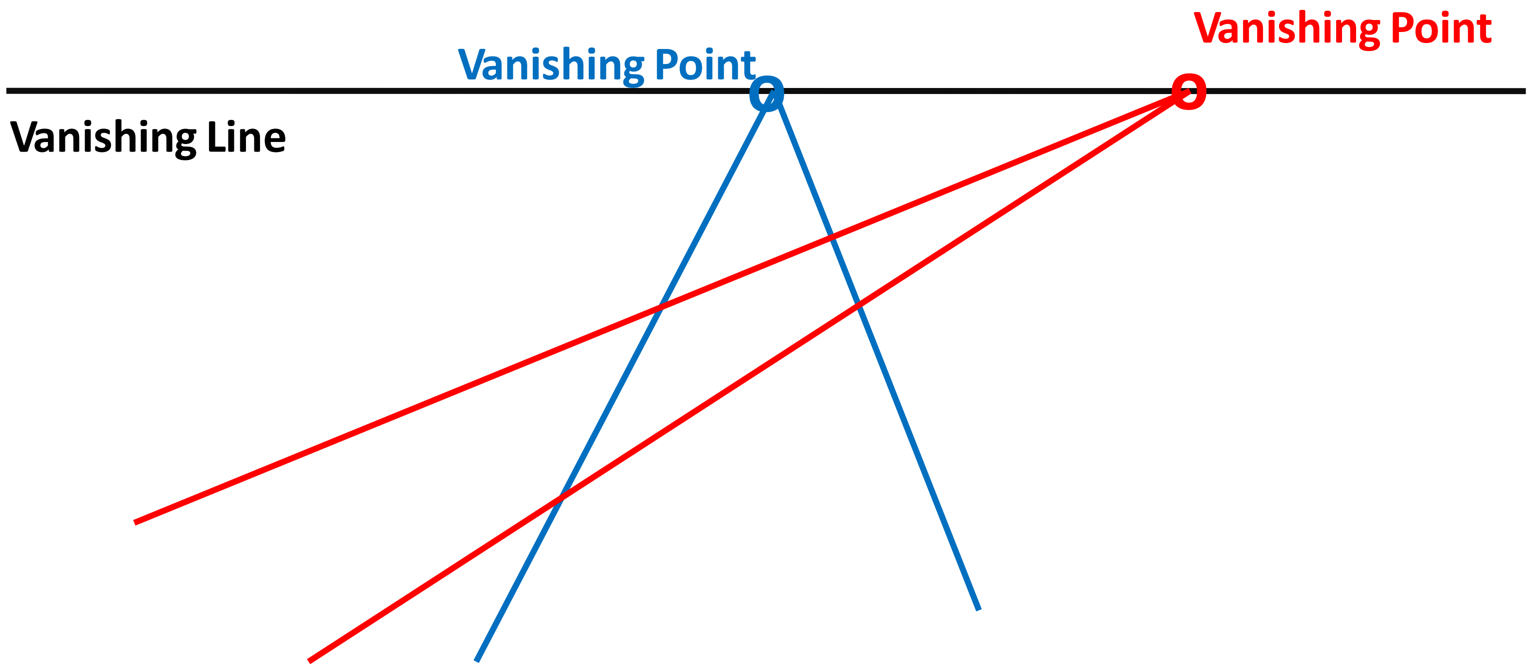

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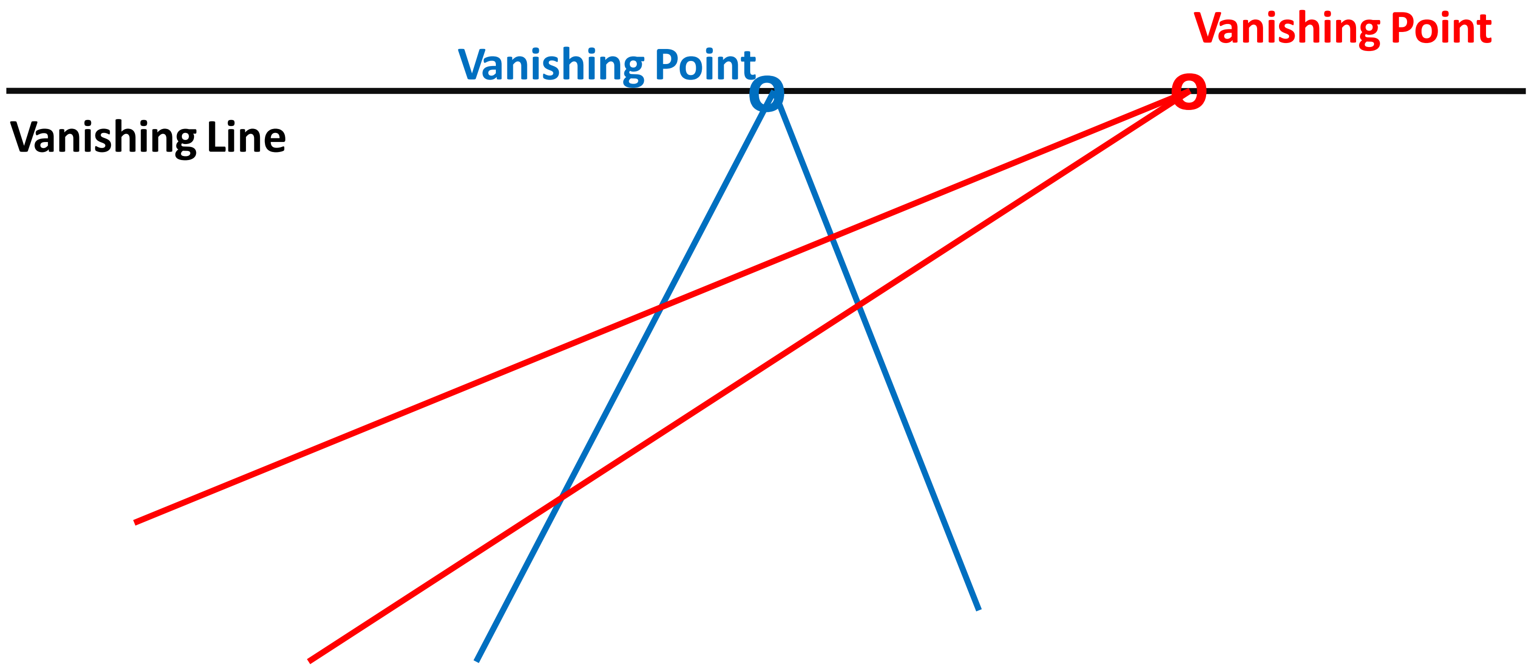
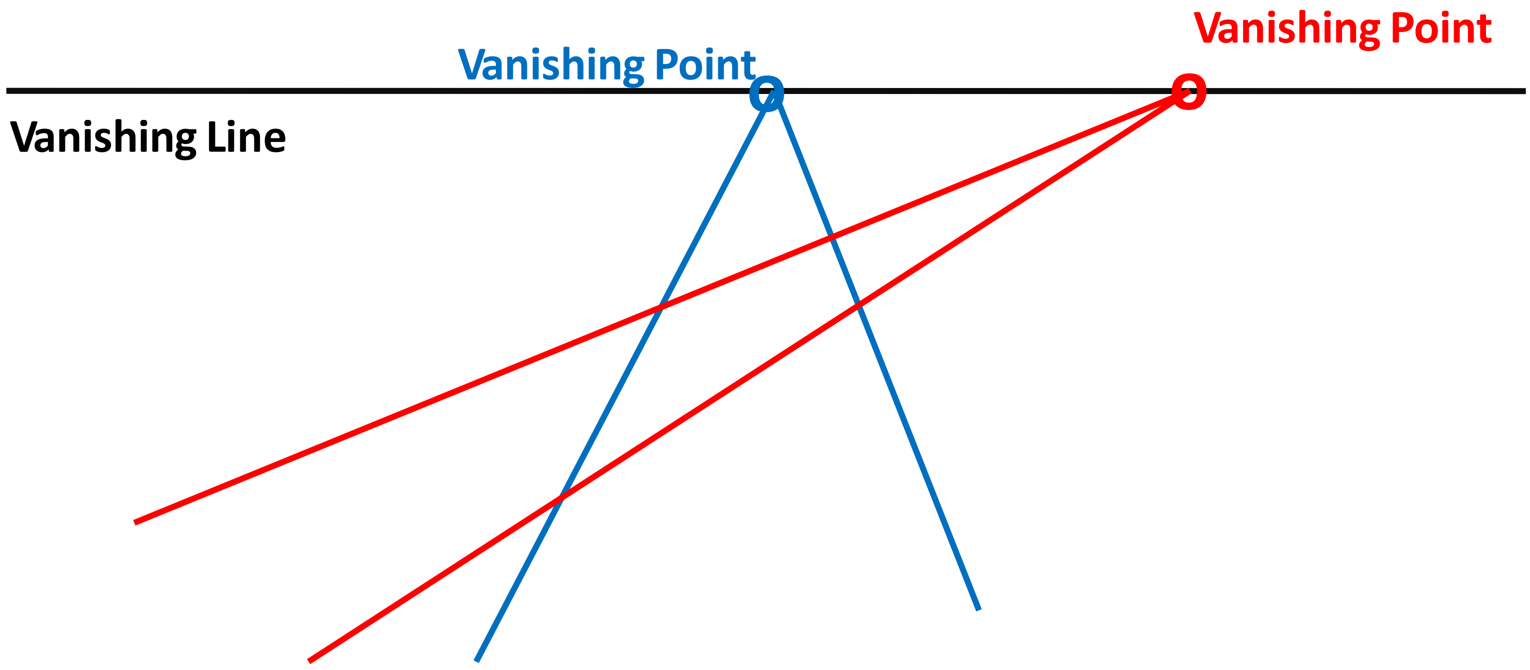


- The projections of parallel 3D lines intersect at a **vanishing point**
- The projection of parallel 3D planes intersect at a **vanishing line**

Vanishing points and lines



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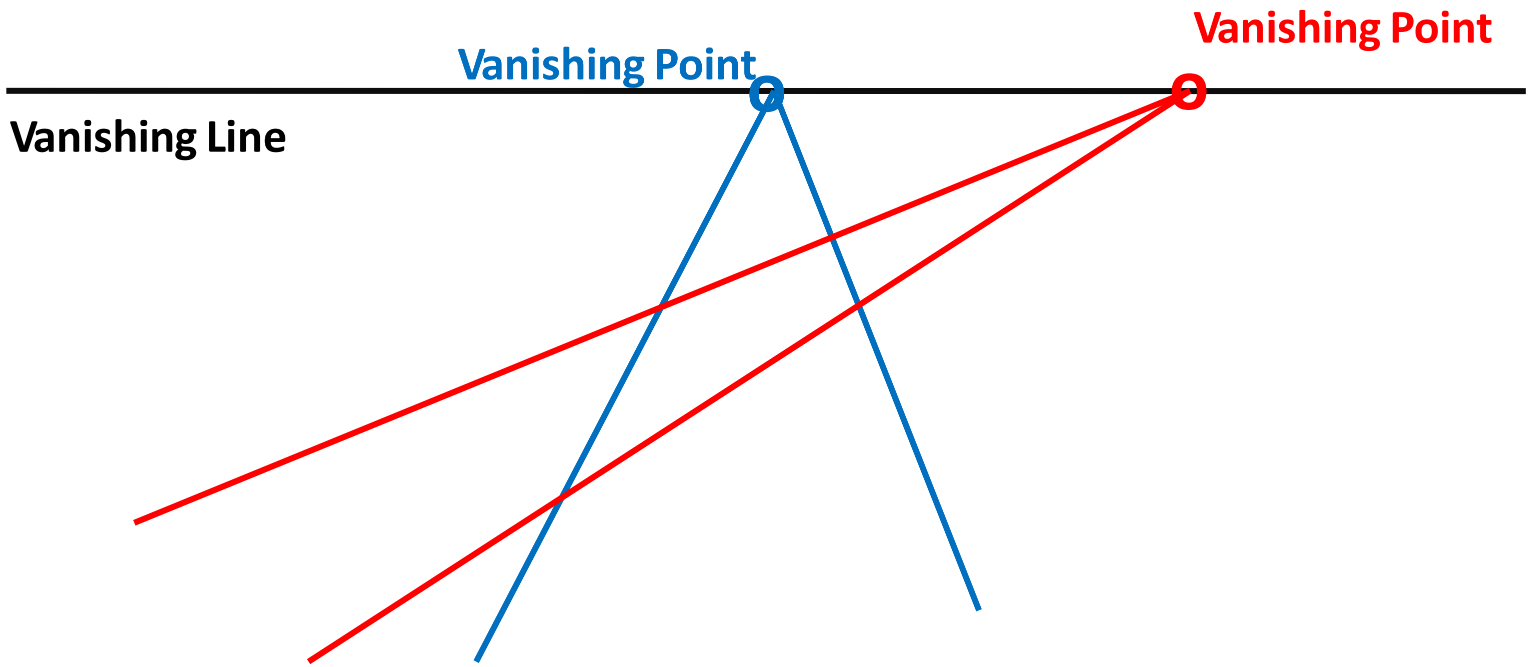
- If a set of parallel 3D lines are also parallel to a particular plane, their vanishing point will lie on the vanishing line of the plane

Vanishing points and lines



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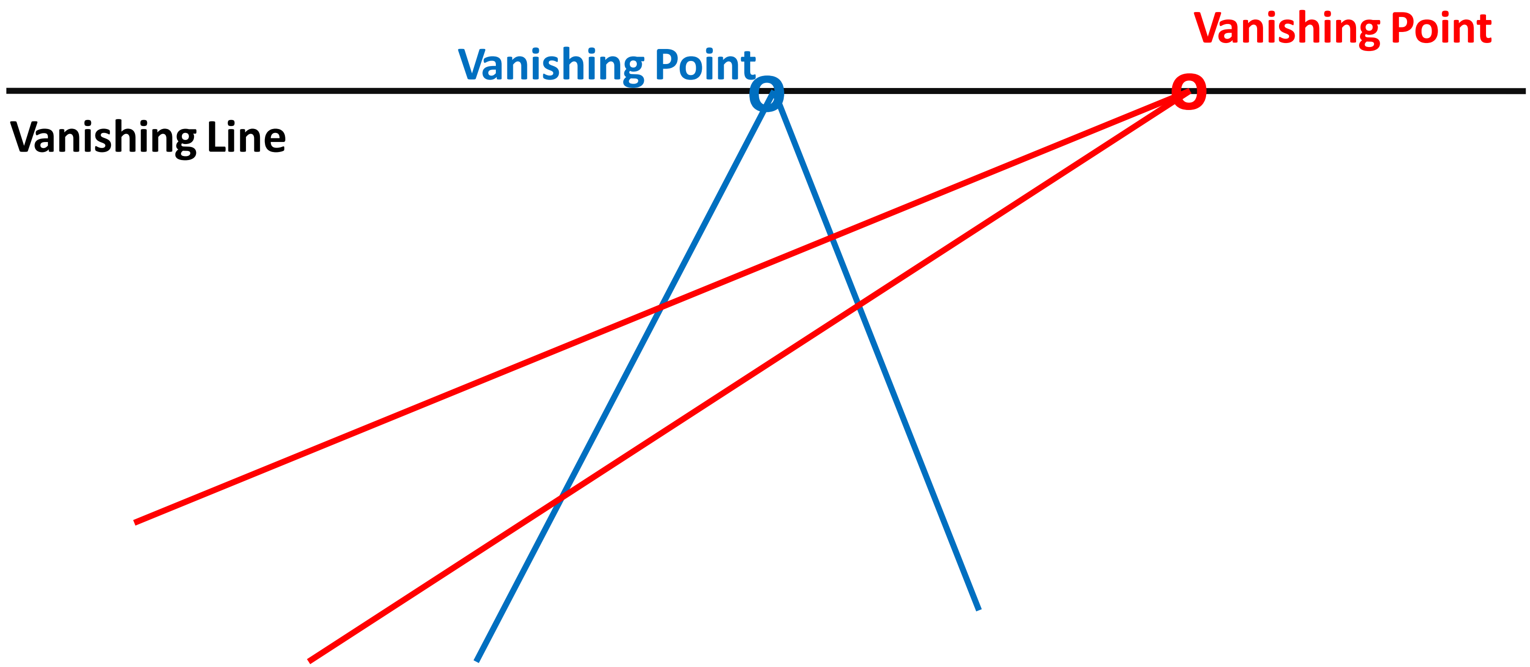
Vanishing points and lines



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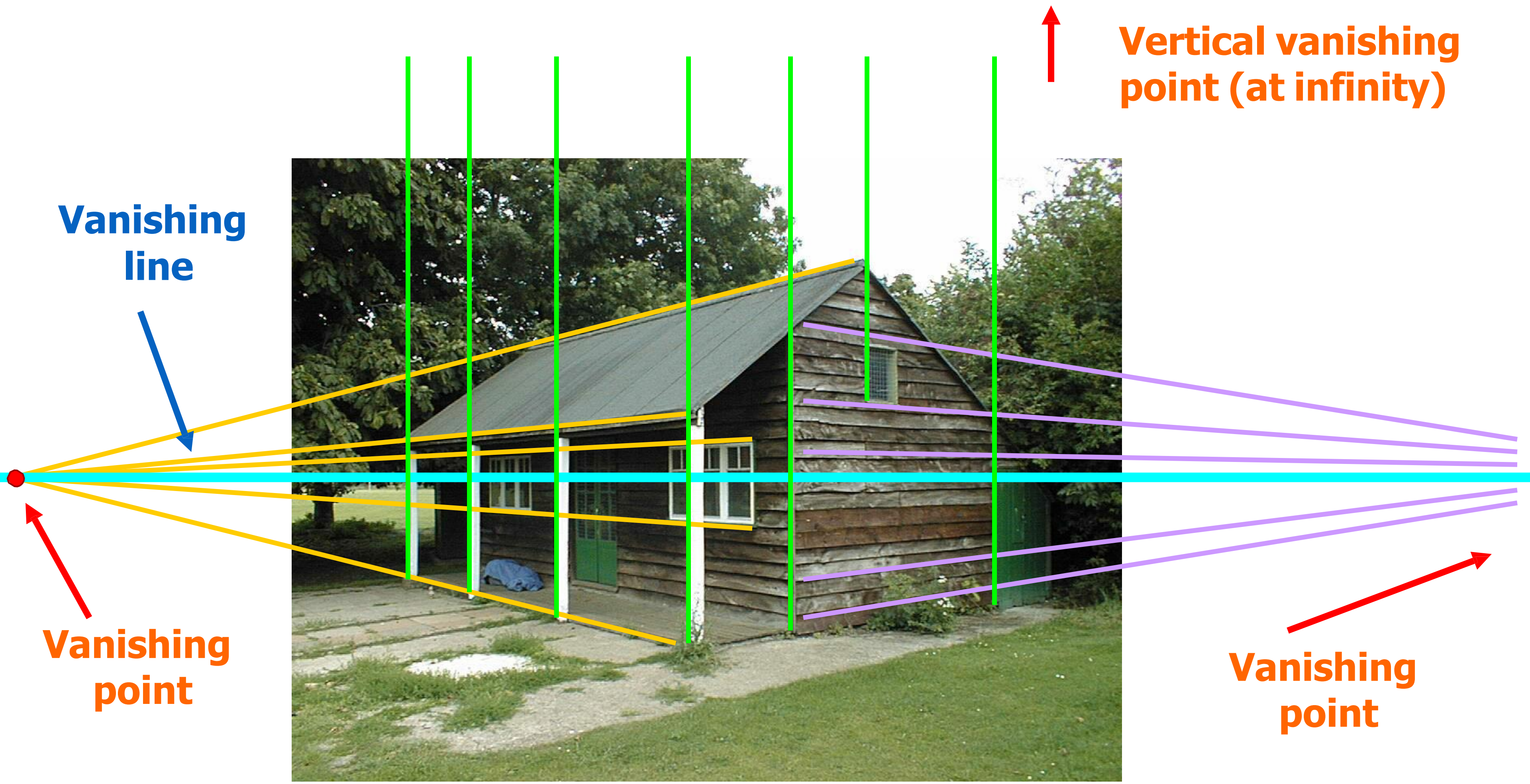
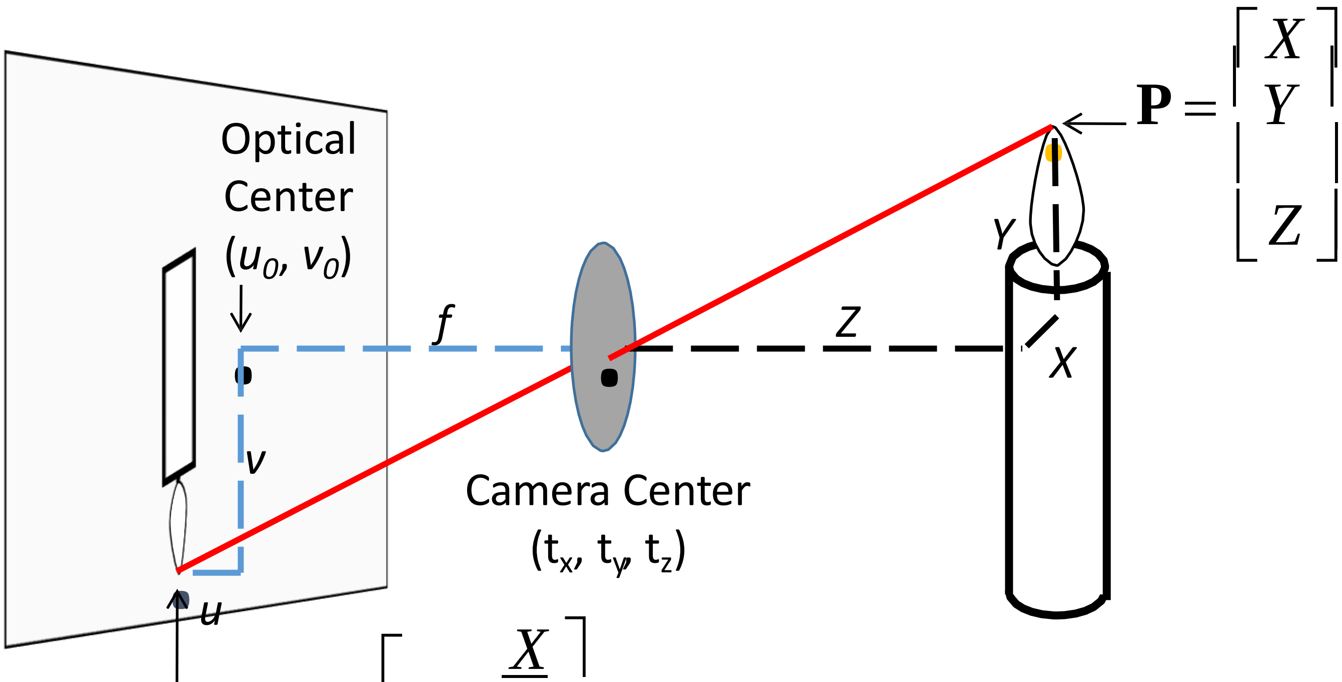
- Not all lines that intersect are parallel
- Vanishing point <-> 3D direction of a line

Vanishing points and lines



Slide from Efros, Photo from Criminisi

Projection:
world coordinates → image coordinates



$$\mathbf{p} = \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -f\dfrac{X}{Z} \\ -f\dfrac{Y}{Z} \end{bmatrix}$$

Homogeneous coordinates

Converting to homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Homogeneous coordinates

Converting to homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting from homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Homogeneous coordinates

Invariant to scaling

$$k\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kw \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{kx}{kw} \\ \frac{ky}{kw} \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \end{bmatrix}$$

Homogeneous Coordinates

Cartesian Coordinates

Homogeneous coordinates

Invariant to scaling

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Homogeneous Coordinates

Cartesian Coordinates

Point in Cartesian is ray in Homogeneous

Basic geometry in homogeneous coordinates

- Append 1 to pixel coordinate to get homogeneous coordinate

$$p = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Basic geometry in homogeneous coordinates

- Append 1 to pixel coordinate to get homogeneous coordinate

$$P = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

- Line equation: $au + bv + c = 0$

$$line = [a\ b\ c]^\top$$

$$line^\top p = 0$$

Basic geometry in homogeneous coordinates

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$$P = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

- Line equation:

$$au + bv + c = 0$$
$$line = [a\ b\ c]^\top$$

$$line^\top p = 0$$

- Line given by cross product of two points

$$line_{ij} = p_i \times p_j$$

Basic geometry in homogeneous coordinates

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- Intersection of two lines given by cross product of the lines

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- Three points lies on the same line

$$p_k^\top (p_i \times p_j) = 0$$

Basic geometry in homogeneous coordinates

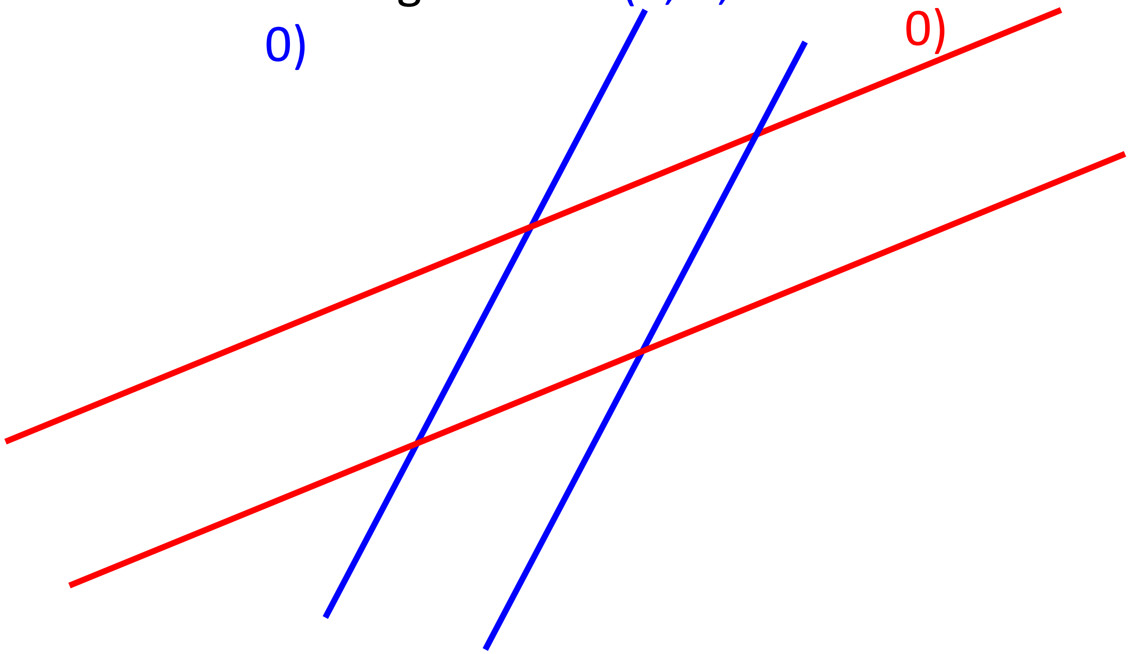
- Append 1 to pixel coordinate to get homogeneous coordinate $p = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$
- Line equation: $au + bv + c = 0$
  $line = [a\ b\ c]^\top$ $line^\top p = 0$
- Line given by cross product of two points
  $$line_{ij} = p_i \times p_j$$
- Intersection of two lines given by cross product of the lines
  $$q_{ij} = line_i \times line_j$$
- Three points lies on the same line
  $$p_k^\top (p_i \times p_j) = 0$$
- Three lines intersect at the same point
  $$line_k^\top (line_i \times line_j) = 0$$

Another problem solved by homogeneous coordinates

Intersection of parallel lines

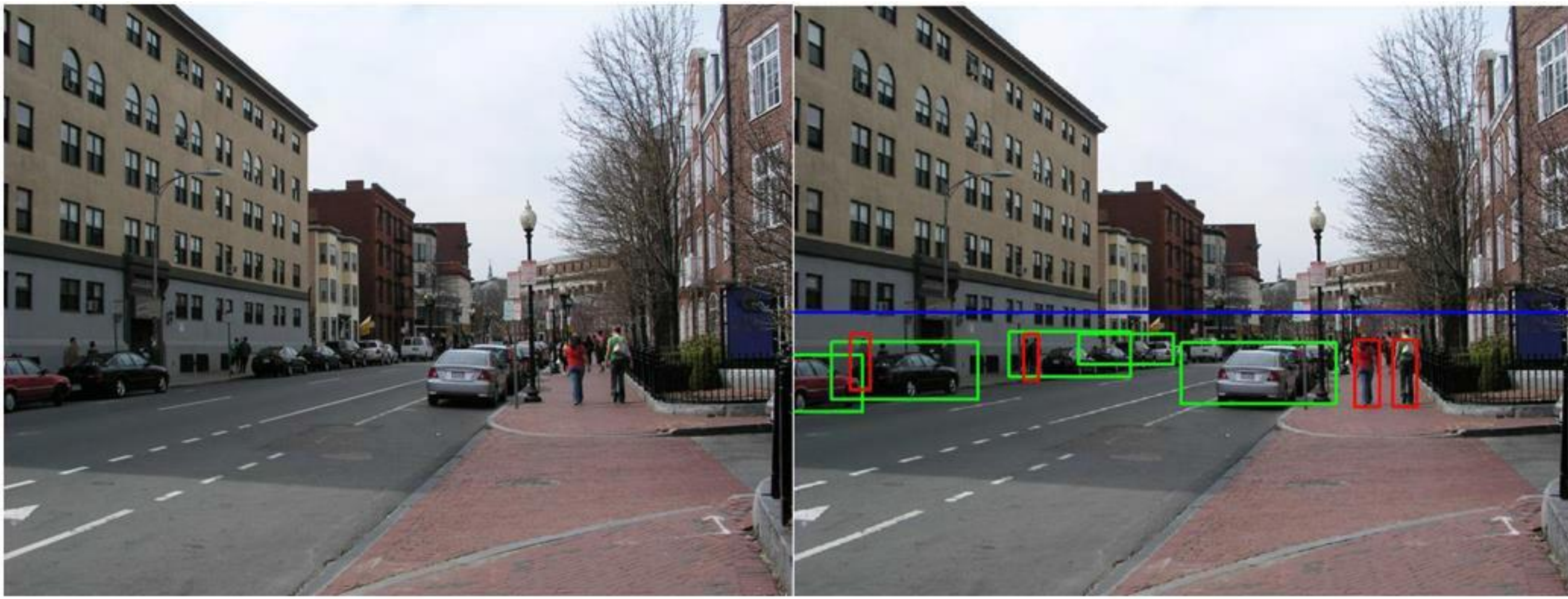
Cartesian: (Inf, Inf)
Homogeneous: (1, 1, 0)

Cartesian: (Inf, Inf)
Homogeneous: (1, 2, 0)



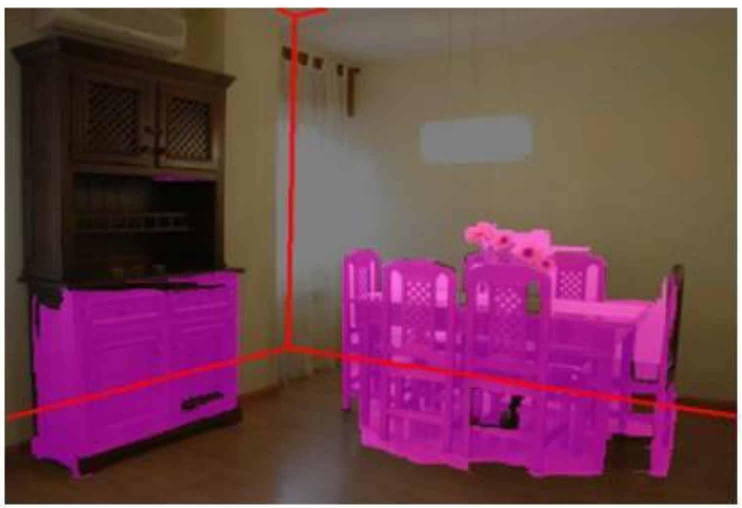
Applications

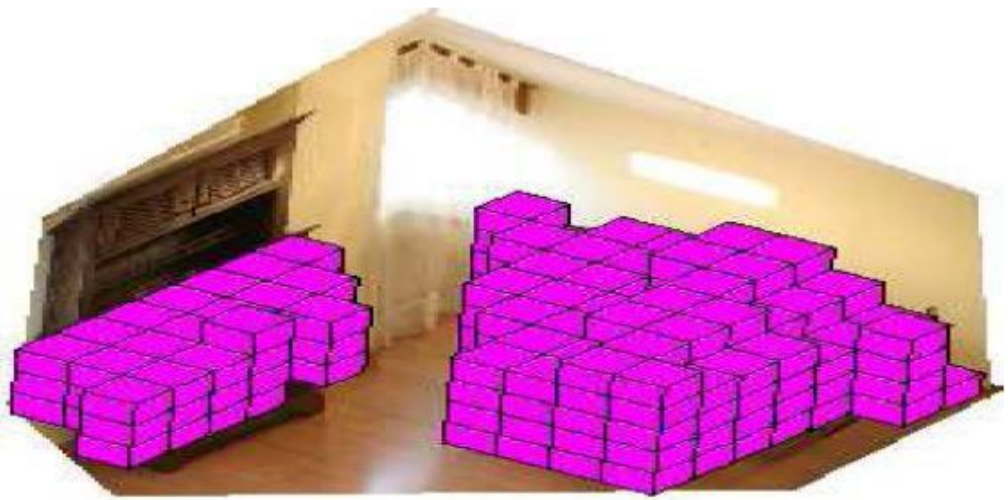
Object Recognition (CVPR 2006)



Applications

Getting spatial layout in indoor scenes (ICCV 2009)





Applications

Inserting synthetic objects into images: http://vimeo.com/28962540

Applications

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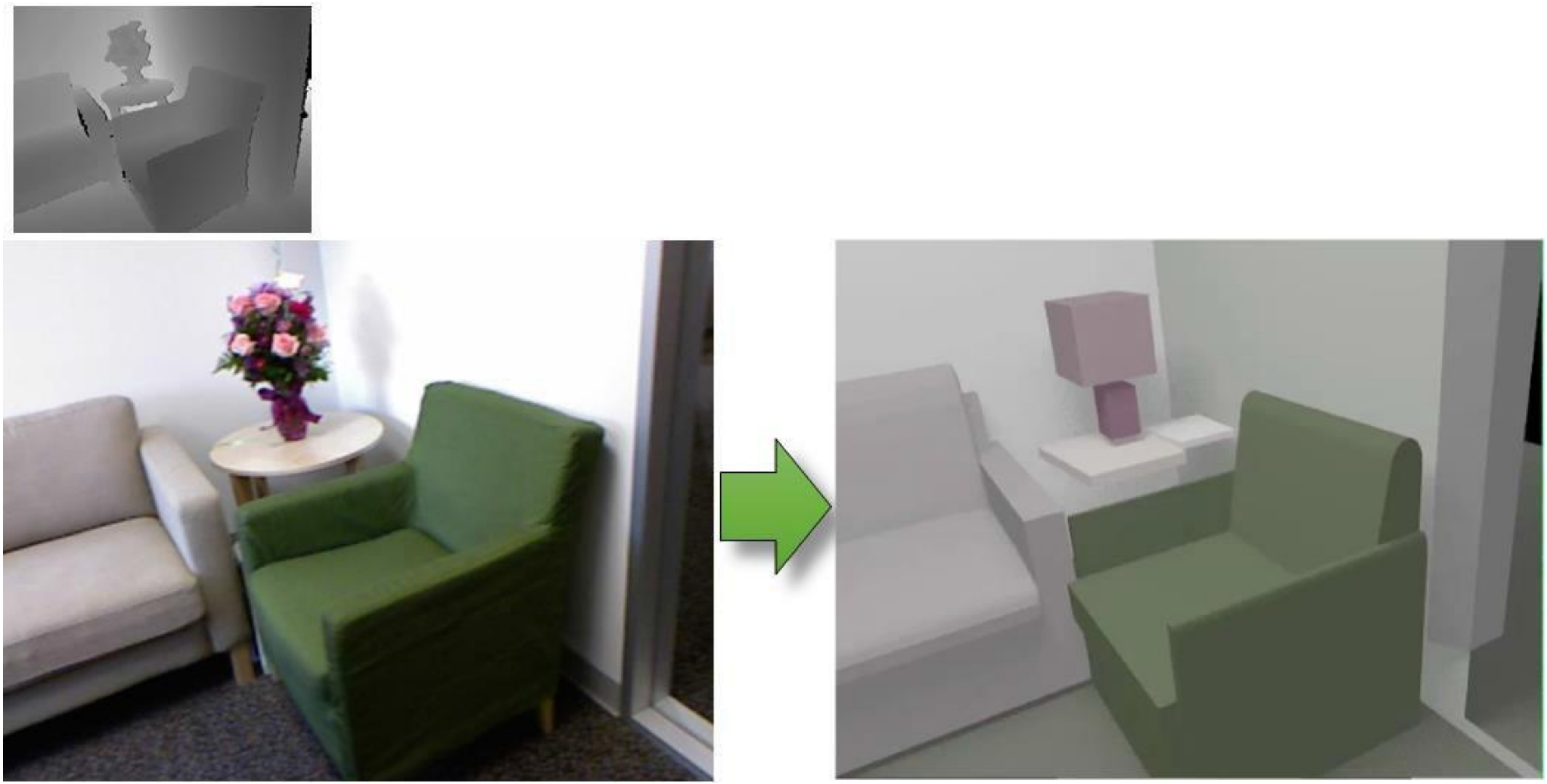
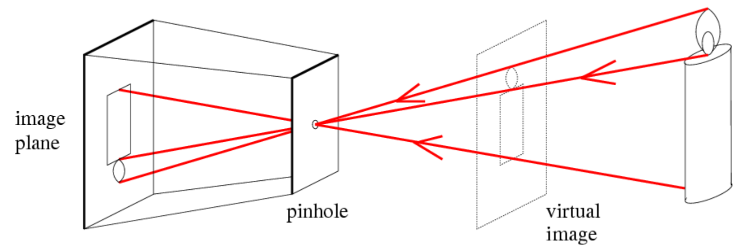
Applications

Creating detailed and complete 3D scene models from a single view



Things to remember

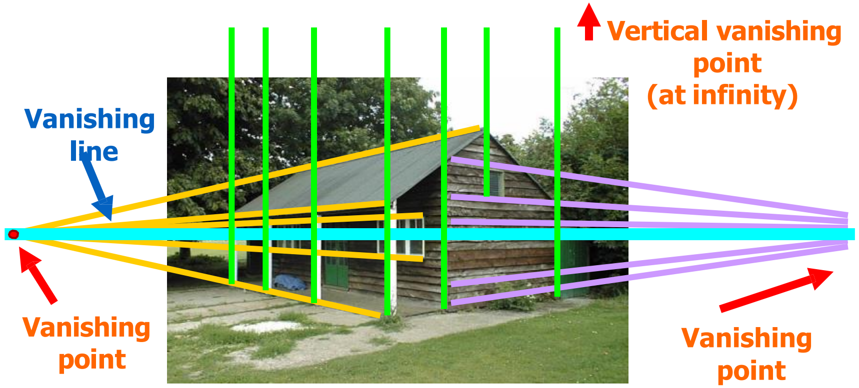
- Pinhole camera model



- Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Vanishing points and vanishing lines



Acknowledgements

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  - Forsyth
  - Steve Seitz
  - Noah Snavely
  - J.B. Huang
  - Derek Hoiem
  - J. Hays
  - J. Johnson
  - R. Girshick
  - S. Lazebnik
  - K. Grauman
  - Antonio Torralba
  - Rob Fergus
  - Leibe
  - And many more ……….

Next class
Projection Matrix and Camera Calibration