

Spring 2023



# Distributed Computing

## - Causal Ordering



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# > Distributed Computing?

- How will you design a Distributed Algorithm?



- Learn to Solve using Distributed Algorithms

# > About this Lecture

## What do we learn today?

- This covers a model of distributed computations that every algorithm designer needs to know
  - Causal Ordering in Distributed Systems
  - Total Ordering
  - CO related concepts in of distributed executions

Let us explore these topics → → →

# Recap: Distributed Systems

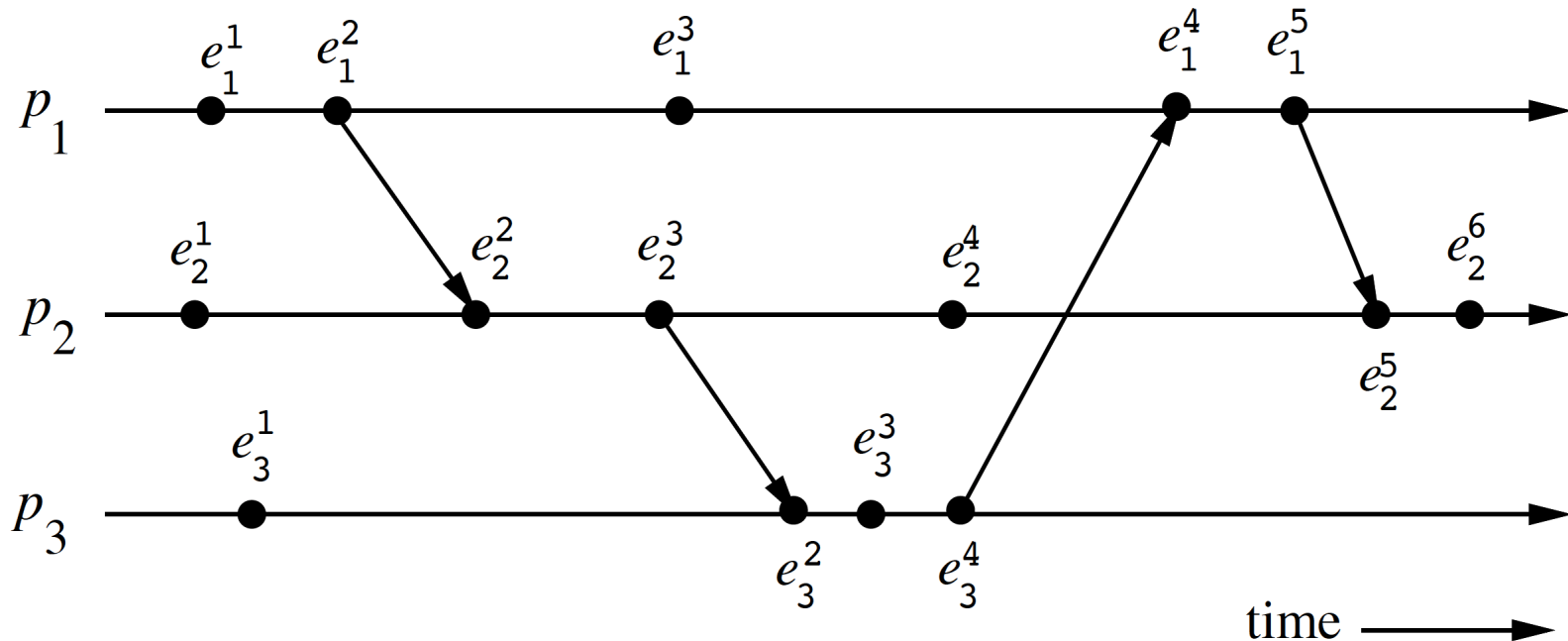
## A Distributed System:

- A collection of independent systems that appears to its users as a single coherent system
- A system in which hardware and software components of networked computers communicate and coordinate their activity only by passing messages
- A computing platform built with many computers that:
  - Operate concurrently
  - Are physically distributed (have their own failure modes)
  - Are linked by a network
  - Have independent clocks

# Recap: Characteristics

- **Concurrent execution of processes:**
  - Non-determinism, Race Conditions, Synchronization, Deadlocks, and so on
- **No global clock**
  - Coordination is done by message exchange
  - No Single Global notion of the correct time
- **No global state**
  - No Process has a knowledge of the current global state of the system
- **Units may fail independently**
  - Network Faults may isolate computers that are still running
  - System Failures may not be immediately known

# A State-Time diagram - An Example



➔ For Process  $p_1$ :

Second event is a message send event

First and Third events are internal events

Fourth event is a message receive event

# What did you learn so far?

- Goals / Challenges with Distributed Systems
- Message Passing systems
  - Basic Primitive operations
  - 3 types of EVENTS: Internal, send and receive
  - States of a process and Channel
- Distributed Sorting
  - Odd - Even Transposition Sort
  - Sasaki's  $(n-1)$  rounds algorithms
  - Did you try  $(n-2)$  rounds algorithm for distributed sorting on line network?
  - Implementing Discrete Events Simulation

# Causal Ordering



# A Model of Distributed Computations

## Focused Topics:

- Causal Precedence Relation
- Models of Communication Networks
- Causal Ordering
- Global State and Local States
- Cuts of a Distributed System
  - PAST and FUTURE events

# A Model of Distributed Executions

- The execution of a process consists of a sequential execution of its actions.
- The actions are atomic and modeled as three types of events: **internal events**, **message send events**, and **message receive events**
- Let  $e_i^x$  denote the  $x^{\text{th}}$  event at process  $p_i$ .
- For a message  $m$ , let  $\text{send}(m)$  and  $\text{receive}(m)$  denote send and receive events, respectively.
- The occurrence of events changes the states of respective processes and channels.
- Internal event → changes **state of the process**
- Send and Receive events change the **state of the process** that sends / receives the message & the **state of the channel** on which the message is sent / received respectively

# Distributed Executions (contd)

- The events at a process are linearly ordered by their order of occurrence.
- The execution of process  $p_i$  produces a sequence of events  $e_i^1, e_i^2, \dots, e_i^x, e_i^{x+1}, \dots$  and is denoted by  $H_i$  where

$$H_i = (h_i, \rightarrow_i)$$

$h_i$  is the set of events produced by  $p_i$  and binary relation  $\rightarrow_i$  defines a linear order on these events

- **Linear Relation:** Mathematically, the independent variable is multiplied by the slope coefficient, added by a constant, which determines the dependent variable
- Relation  $\rightarrow_i$  expresses causal dependencies among the events of  $p_i$

# A Model of Distributed Executions (contd.)

- The send and the receive events signify the flow of information between processes and establish causal dependency from the sender process to the receiver process
- Define a relation  $\rightarrow_{msg}$  that captures the causal dependency due to message exchanges as follows:

For every message  $m$  that is exchanged between two processes, we have

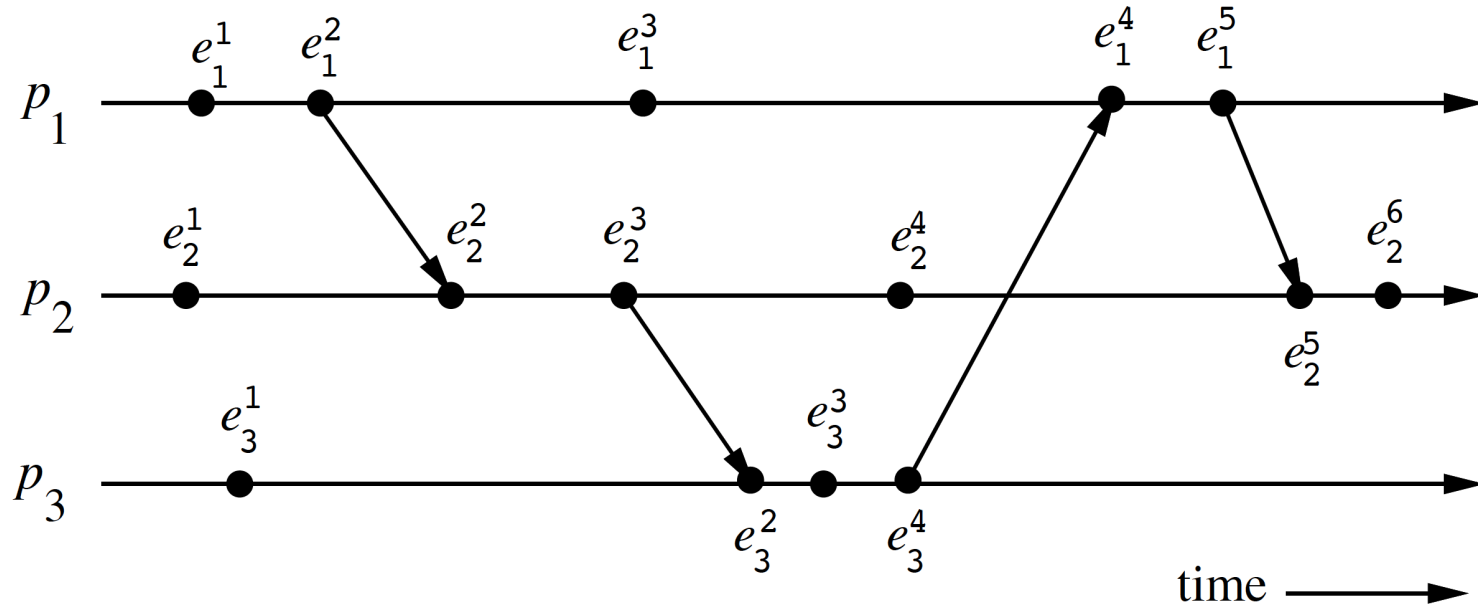
$$send(m) \rightarrow_{msg} receive(m)$$

- Relation  $\rightarrow_{msg}$  defines causal dependencies between the pairs of corresponding send and receive events

# A State-Time diagram

- The evolution of a distributed execution is depicted by a space-time diagram
- A horizontal line represents the progress of a specific process
- A dot indicates an event
- A slant arrow indicates a message transfer
- Since an event execution is atomic (indivisible and instantaneous), it is justified to denote it as a dot on a process line

# A State-Time diagram - An Example



→ For Process  $p_1$ :

Second event is a message send event

First and Third events are internal events

Fourth event is a message receive event

# Partial Order / Total Ordering

A relation  $\leq$  is a total order on a set  $S$  (" $\leq$  totally orders  $S$ ") if the following properties hold:

1. Reflexivity:  $a \leq a$  for all  $a$  in  $S$
2. Antisymmetry:  $a \leq b$  and  $b \leq a$  implies  $a = b$
3. Transitivity:  $a \leq b$  and  $b \leq c$  implies  $a \leq c$
4. Comparability (trichotomy law):  
For any  $a, b$  in  $S$ , either  $a \leq b$  or  $b \leq a$ .

First 3 properties  $\rightarrow$  the axioms of a **partial order**

Addition of **trichotomy law** defines a **total order**  $H=(H, \rightarrow)$

# Causal Precedence Relation

- The execution of a distributed app. results in a set of distributed events
- Let  $H = \cup_i h_i$  denote the set of events executed in a distributed computation.
- Define a binary relation  $\rightarrow$  on the set  $H$  that expresses causal dependencies between events in the distributed execution.

$$\forall e_i^x, \forall e_j^y \in H, \quad e_i^x \rightarrow e_j^y \quad \Leftrightarrow \quad \left\{ \begin{array}{l} e_i^x \rightarrow_i e_j^y \quad i.e., (i = j) \wedge (x < y) \\ \text{or} \\ e_i^x \rightarrow_{msg} e_j^y \\ \text{or} \\ \exists e_k^z \in H : e_i^x \rightarrow e_k^z \wedge e_k^z \rightarrow e_j^y \end{array} \right.$$

- The causal precedence relation induces an irreflexive partial order on the events of a distributed computation that is *denoted as*  $H = (H, \rightarrow)$



# Causal Precedence Relation (contd)

- The relation  $\rightarrow$  is as defined by Lamport  
"happens before"

An event  $e_1$  happens before the event  $e_2$  and denoted by  $e_1 \rightarrow e_2$  if the following holds true:

- $e_1$  occurs before  $e_2$  on the same process OR
- $e_1$  is the send message and  $e_2$  is the corresponding receive message OR
- There exists another event  $e'$  such that  $e_1$  happens before  $e'$  and  $e'$  happens before  $e_2$

# Causal Precedence Relation (contd)

- For any two events  $e_i$  and  $e_j$ ,  $e_i \not\rightarrow e_j$  denotes the fact that event  $e_j$  does not directly or transitively dependent on event  $e_i$   
That is, event  $e_i$  **does not causally affect** event  $e_j$
- In this case, event  $e_j$  is not aware of the execution of  $e_i$  or any event executed after  $e_i$  on the same process.

Note the following two rules:

- For any two events  $e_i$  and  $e_j$   
 $e_i \not\rightarrow e_j$  does not imply  $e_j \not\rightarrow e_i$
- For any two events  $e_i$  and  $e_j$   
 $e_i \rightarrow e_j \Rightarrow e_j \not\rightarrow e_i$

# Concurrent Events

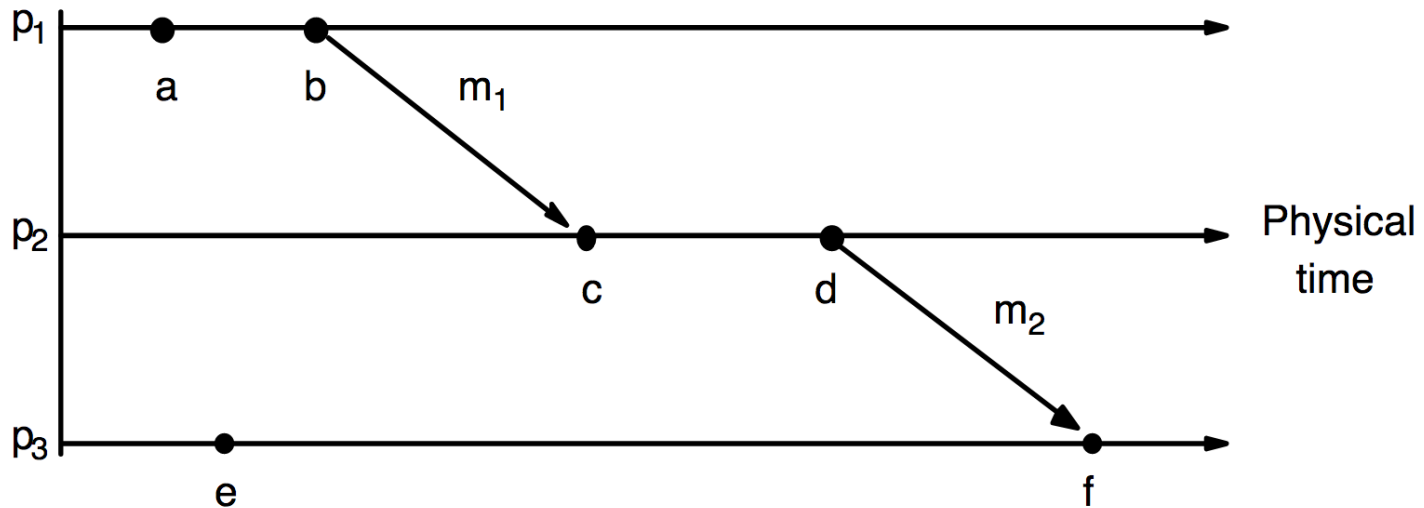
- For any two events  $e_i$  and  $e_j$ :  
if  $e_i \not\rightarrow e_j$  and  $e_j \not\rightarrow e_i$ ,  
then events  $e_i$  and  $e_j$  are said to be concurrent  
(denoted as  $e_i || e_j$ )

Example:

$$e_3^3 || e_2^4 \text{ and } e_2^4 || e_1^5 \text{ but } e_3^3 \text{ not } || e_1^5$$

- The relation  $||$  is not transitive;  
that is,  
 $(e_i || e_j) \wedge (e_j || e_k)$  does not imply  $e_i || e_k$
- For any two events  $e_i$  and  $e_j$  in a distributed execution,  
 $e_i \rightarrow e_j$  OR  $e_j \rightarrow e_i$  OR  $e_i || e_j$

# Concurrency - An Example



$a \rightarrow b$  (at  $p_1$ )  $c \rightarrow d$  (at  $p_2$ )

$b \rightarrow c$  ( $m_1$ )

also  $d \rightarrow f$  ( $m_2$ )

Not all events are related by  $\rightarrow$ , e.g.,  $a \not\rightarrow e$  and  $e \not\rightarrow a$   
they are said to be concurrent; write as  $a \parallel e$

# Logical vs. Physical Concurrency

- Two events are logically concurrent if and only if they do not causally affect each other.
- In physical concurrency: events occur at the same instant in physical time.
- Two+ events may be logically concurrent even though they do not occur at same instant in physical time.
- If processor speed and message delays would have been different, the execution of these events could have very well coincided in physical time.
- Whether a set of logically concurrent events coincide in the physical time or not, does not change the outcome of the computation.
- A set of logically concurrent events may not have occurred at the same instant in physical time, we can assume that these events occurred at the same instant in physical time.

# Models of Communication networks

- There are several models of the service provided by communication networks:  
FIFO, Non-FIFO, and causal ordering
- In the **FIFO model**, each channel acts as a first-in first-out message queue and thus, message ordering is preserved by a channel.
- In the **non-FIFO model**, a channel acts like a set in which the sender process adds messages and the receiver process removes messages from it in a random order.

# Causal Ordering

- The “causal ordering” model is based on Lamport’s “happens before” relation
- A system that supports the causal ordering model satisfies the following property:

**CO:** For any two messages  $m_{ij}$  and  $m_{kj}$ ,  
if  $send(m_{ij}) \rightarrow send(m_{kj})$ ,  
then  $receive(m_{ij}) \rightarrow receive(m_{kj})$

- This property ensures that causally related messages destined to the same destination are delivered in an order that is consistent with their causality relation.
- Causally ordered delivery of messages implies FIFO message delivery. (Note that  $CO \subset FIFO \subset Non-FIFO$ .)
- Causal ordering model considerably simplifies the design of distributed algorithms because it provides a built-in synchronization.

# Global State

- A collection of the local states of its components:
  - The processes and the communication channels
- The state of a process is defined by the local contents of processor registers, stacks, local memory, etc
- The state of channel depends the set of messages in transit in the channel
  
- An internal event changes only state of the process
- A send event changes
  - state of the process that sends the message and
  - the state of the channel on which the message is sent.
- Similarly a receive event changes
  - the state of the process that receives the message and
  - the state of the channel on which the message is received



# Global State (contd)

## Notations

- $LS_i^x$  denotes the state of  $p_i$  after occurrence of event  $e_i^x$  and before the event  $e_i^{x+1}$
- $LS_i^0$  denotes the initial state of process  $p_i$
- $LS_i^x$  is a result of the execution of all the events executed by process  $p_i$  till  $e_i^x$
  
- Let  $send(m) \leq LS_i^x$  denote the fact:  
$$\exists y, 1 \leq y \leq x \text{ s.t. } e_i^y = send(m)$$
- Let  $rec(m) (not \leq) LS_i^x$  denote the fact:  
$$\forall y, 1 \leq y \leq x \text{ s.t. } e_i^y \text{ (not equal to) } rec(m)$$

# Global State (contd)

- The global state of a distributed system is a collection of the local states of the processes and the channels.

A global state GS is defined as,

$$GS = \{ \bigcup_i LS_i^{x_i}, \bigcup_{j,k} SC_{jk}^{y_j, z_k} \}$$

- For a global state to be meaningful, the states of all the components of the distributed system must be recorded at the same instant
- Two important situations (Impossible !!):
  - the local clocks at processes were perfectly synchronized
  - there were a global system clock that can be instantaneously read by the processes

# A Consistent Global State

## Basic idea:

- A state should not violate causality - an effect should not be present without its cause
- A message cannot be received if it was not sent.
- Such states are called consistent global states and are meaningful global states.
- Inconsistent global states are not meaningful in the sense that a distributed system can never be in an inconsistent state

# What is a Consistent Global State?

## Definition:

→ A global state is a consistent global state iff

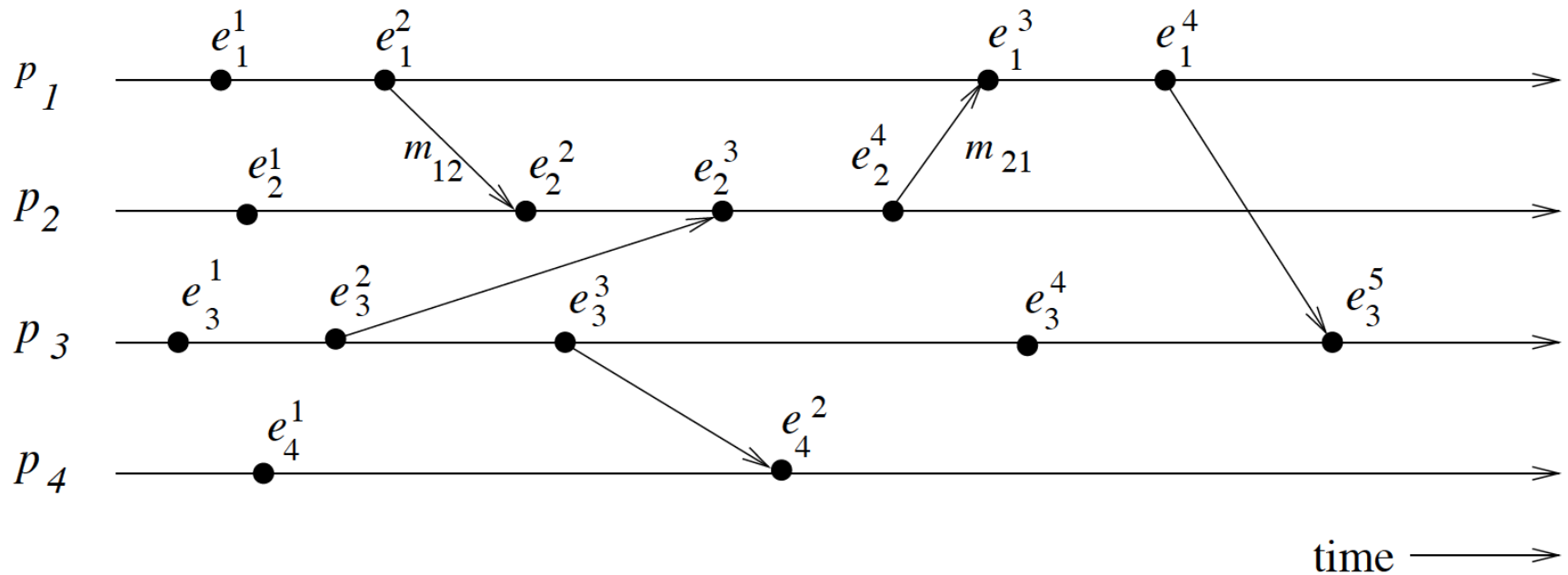
$$\forall m_{ij} : \text{send}(m_{ij}) \not\leq LS_i^{x_i} \Leftrightarrow m_{ij} \notin SC_{ij}^{x_i, y_j} \wedge \text{rec}(m_{ij}) \not\leq LS_j^{y_j}$$

Where the global state is given by

$$GS = \{ \bigcup_i LS_i^{x_i}, \bigcup_{j,k} SC_{jk}^{y_j, z_k} \}$$

→ This implies that the channel state and process state must not include any message that process  $p_i$  sent after executing event

# Consistent Global State - An Example



# Consistent **Global State** - Details

- A global state  $GS1 = \{LS_1^1, LS_2^3, LS_3^3, LS_4^2\}$  is **inconsistent** because
  - the state of  $p_2$  has recorded the receipt of message  $m_{12}$
  - The state of  $p_1$  has not recorded its send
- A global state  $GS2$  consisting of local states  $\{LS_1^2, LS_2^4, LS_3^4, LS_4^2\}$  is **consistent**;
- all the channels are empty except  $C_{21}$  that contains message  $m_{21}$ .

# Run / Consistent Run

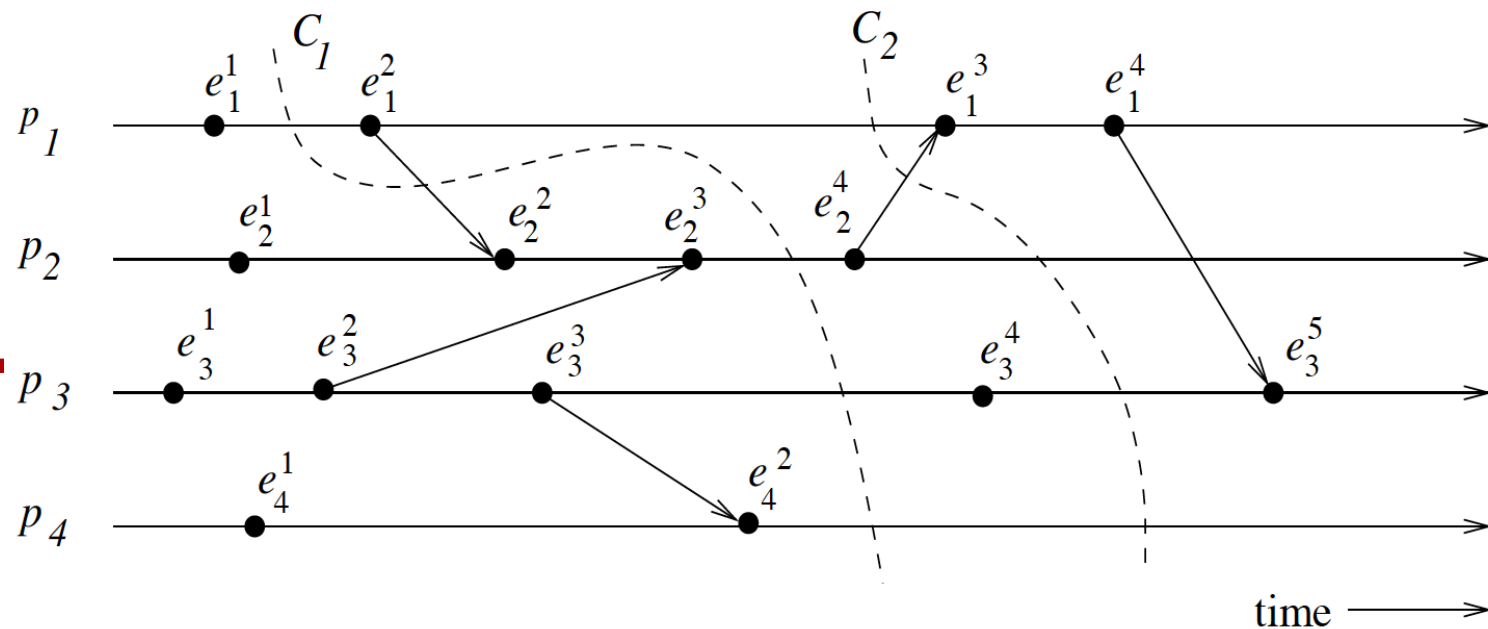
## Run:

- A run is an ordering of the events that satisfies the **happened-before** relation in **one** process

## Consistent Run:

- A consistent run is an ordering of the events that satisfies **all the happened-before** relations

# Cuts of a Distributed Computation

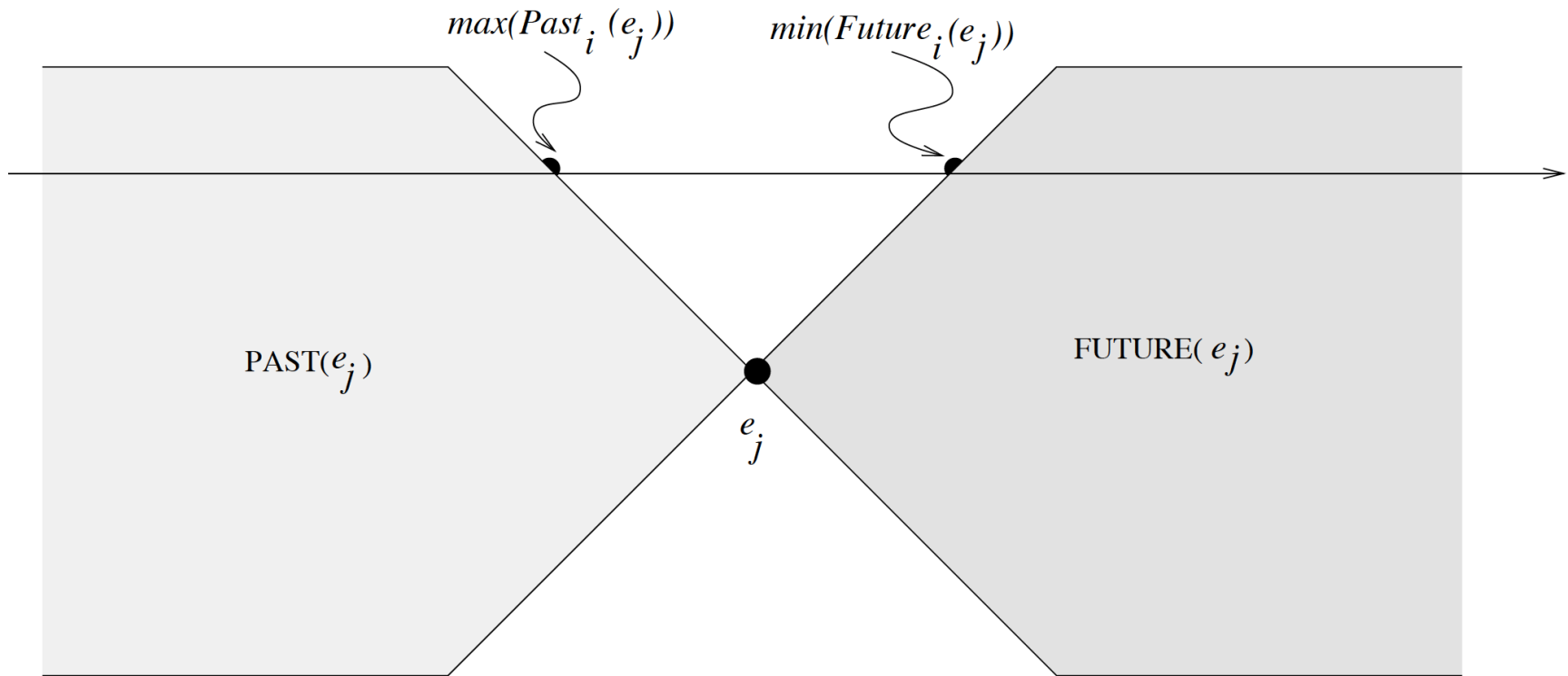




# Cuts of a Distributed Computation

- In a consistent cut, every message received in the PAST of the cut was sent in the PAST of that cut
  - In previous figure, cut C2 is a consistent cut
- All messages that cross the cut from the PAST to the FUTURE are in transit in the corresponding consistent global state.
- A cut is inconsistent if a message crosses the cut from the FUTURE to the PAST
  - In previous figure cut C1 is an inconsistent cut

# Past and Future Cones of an event



# Physical vs Logical clocks?

- Logical Clocks
  - Design and Implementation
- Three Different Ways
  - Scalar Time
  - Vector Time
  - Matrix Time
- Virtual Clocks
  - Time Wrap Mechanism
- Clock Synchronization
  - NTP Synchronization Protocol

# Summary

## → A Model of Distributed Computations

### → Distributed Sorting

#### → Design and Implementation Issues

### → Causal Precedence Relations

### → Global State and Cuts of a DS

### → PAST and FUTURE events

### → What about the ordering of events?

#### → How do we efficiently handle the ordering of events (discrete events)?

#### → Lamport's Logical Clocks ?

#### → Many more to come up ... stay tuned in !!

# Penalties



- Every Student is expected to strictly follow a fair Academic Code of Conduct to avoid penalties
- Penalties is heavy for those who involve in:
  - Copy and Pasting the code
  - Plagiarism (copied from your neighbor or friend - in this case, both will get "0" marks for that specific take home assignments)
  - If the candidate is unable to explain his own solution, it would be considered as a "copied case"!!
  - Any other unfair means of completing the assignments

# Help among Yourselves?

- **Perspective Students** (having CGPA above 8.5 and above)
- **Promising Students** (having CGPA above 6.5 and less than 8.5)
- **Needy Students** (having CGPA less than 6.5)
  - Can the above group help these students? (Your work will also be rewarded)
- You may grow a culture of **collaborative learning** by helping the needy students

# How to reach me?

→ Please leave me an email:

rajendra [DOT] prasath [AT] iiits [DOT] in

→ Visit my homepage @

→ <https://www.iiits.ac.in/people/regular-faculty/dr-rajendra-prasath/>

(OR)

→ <http://rajendra.2power3.com>

# Assistance

- You may post your questions to me at any time
- You may meet me in person on available time or with an appointment
- You may ask for one-to-one meeting

## Best Approach

- You may leave me an email any time  
(email is the best way to reach me faster)





# Questions It's Your Time



Contact Information

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