Computer Vision

Single-View Metrology and Camera Properties

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Perspective and 3D Geometry

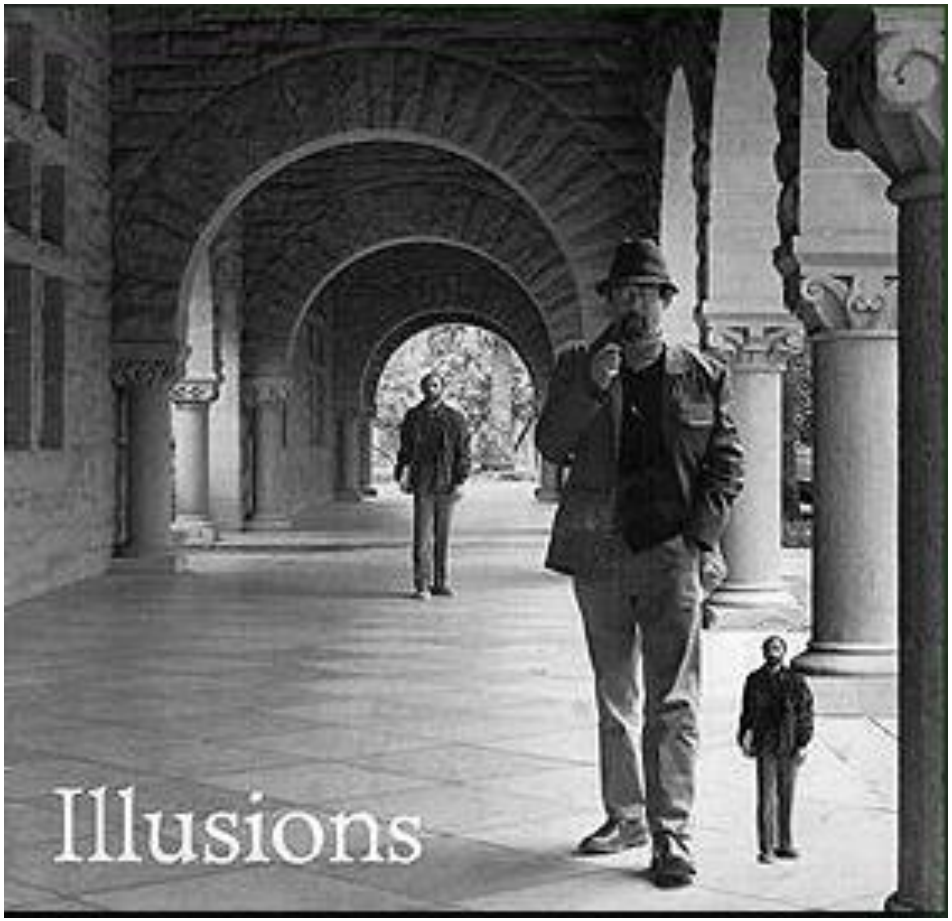
- **Camera models and Projective geometry**
  - What's the mapping between image and world coordiantes?
- **Projection Matrix and Camera calibration**
  - What's the projection matrix between scene and image coordiantes?
  - How to calibrate the projection matrix?
- **Single view metrology and Camera properties**
  - How can we measure the size of 3D objects in an image?
  - What are the important camera properties?
- **Photo stitching**
  - What's the mapping from two images taken without camera translation?
- **Epipolar Geometry and Stereo Vision**
  - What's the mapping from two images taken with camera translation?
- **Structure from motion**
  - How can we recover 3D points from multiple images?

This class

- How can we measure the size of objects in the world from an image?

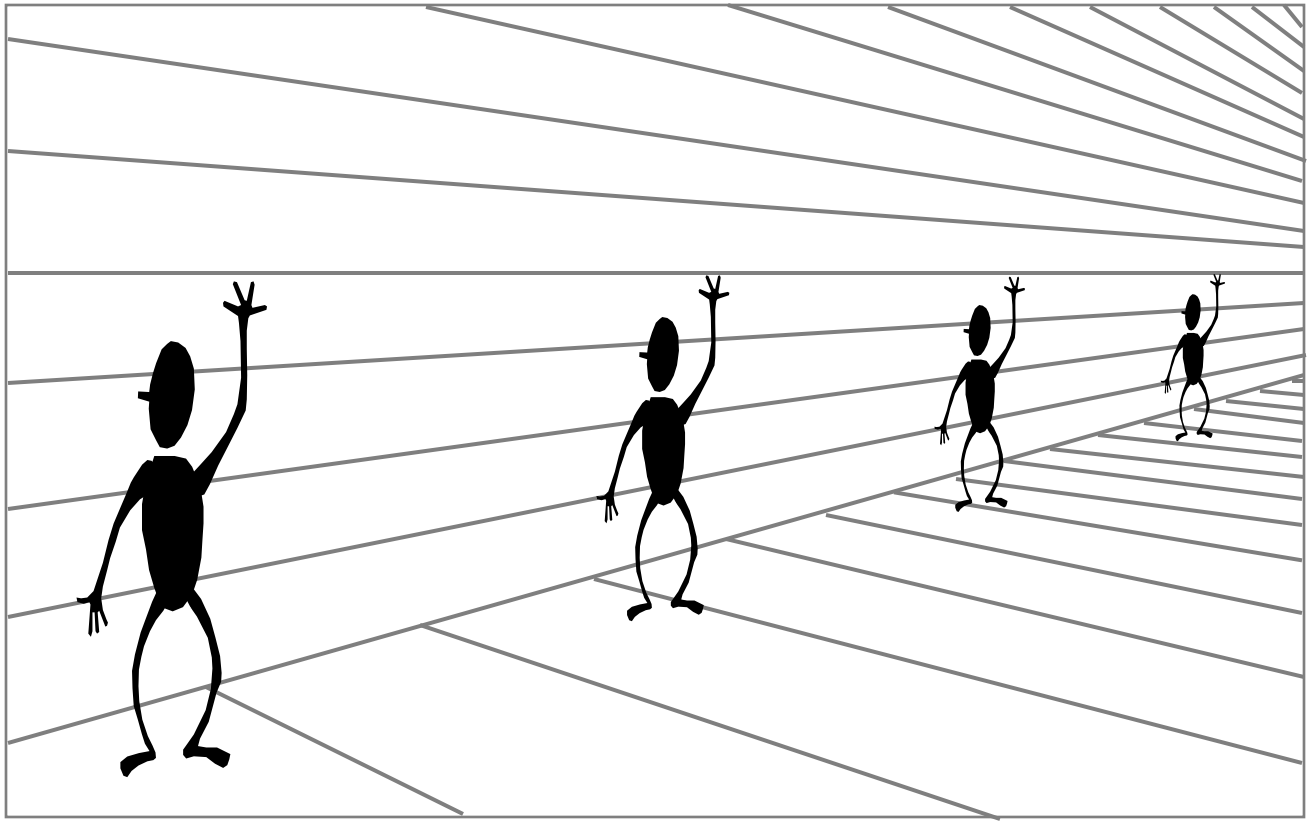
- What about other camera properties:
  - focal length,
  - field of view,
  - depth of field,
  - aperture,
  - f-number

How can we measure the size of 3D objects from an image?

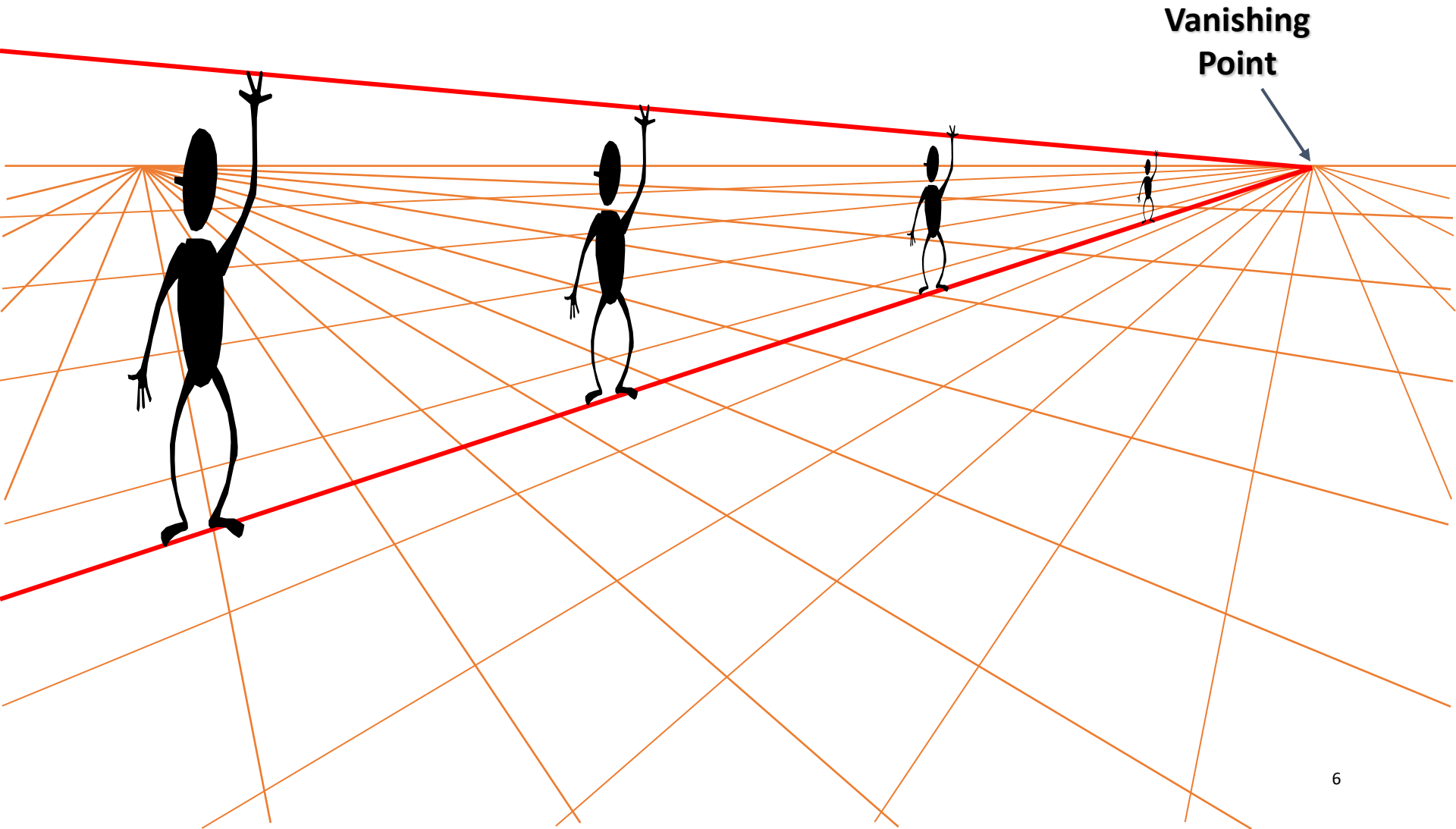


Slide by Steve Seitz

Perspective cues



Comparing heights



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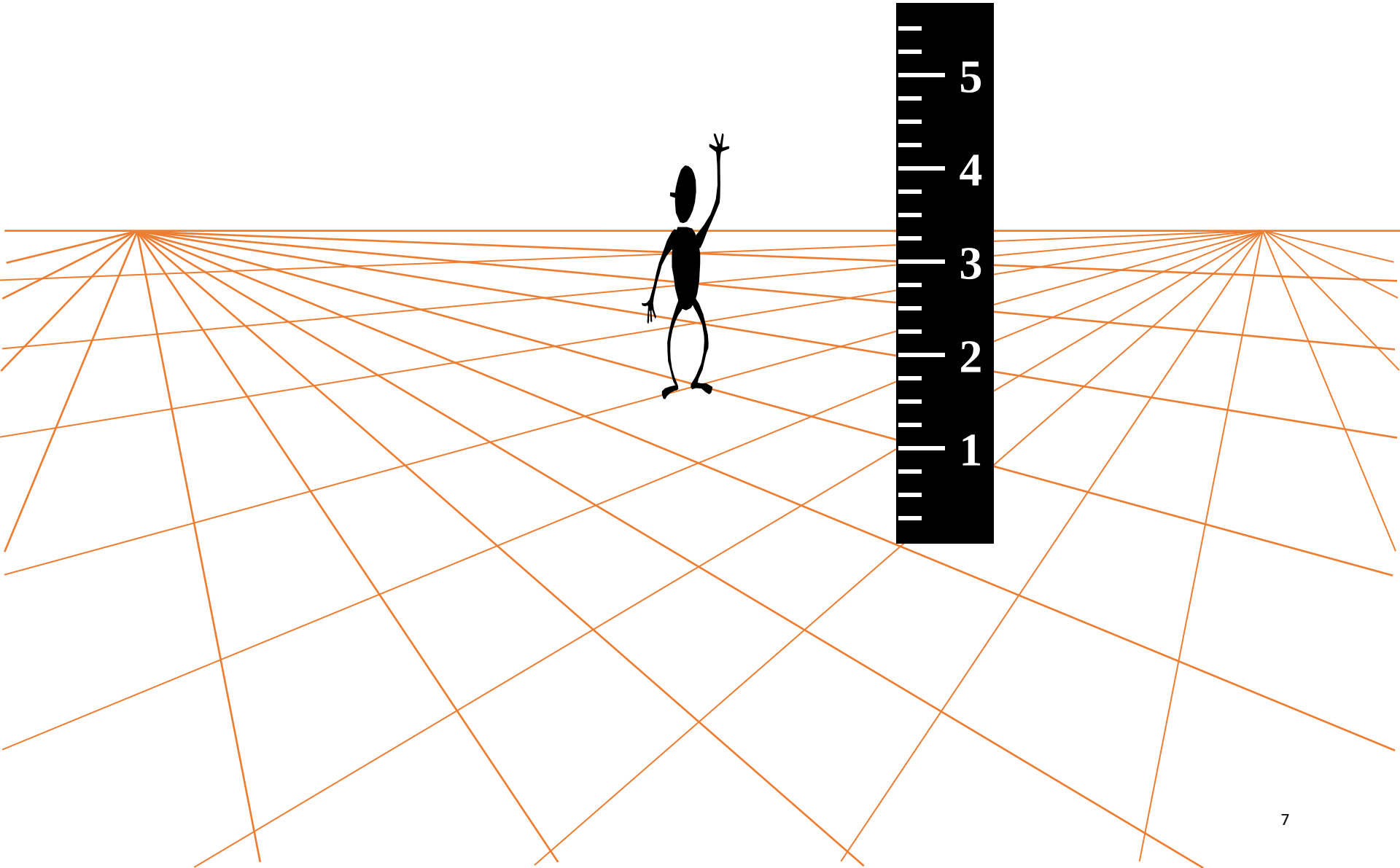
Measuring height



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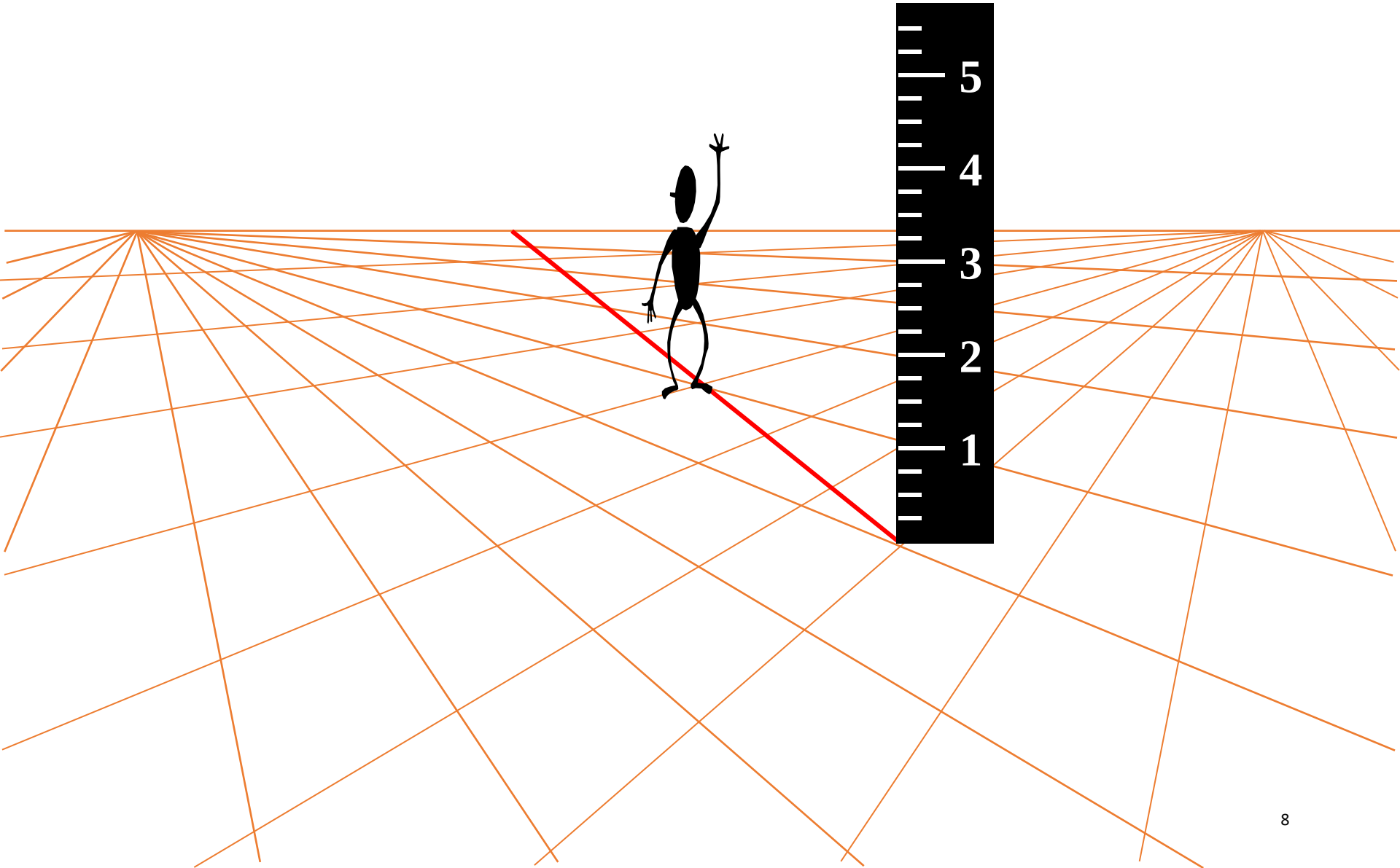

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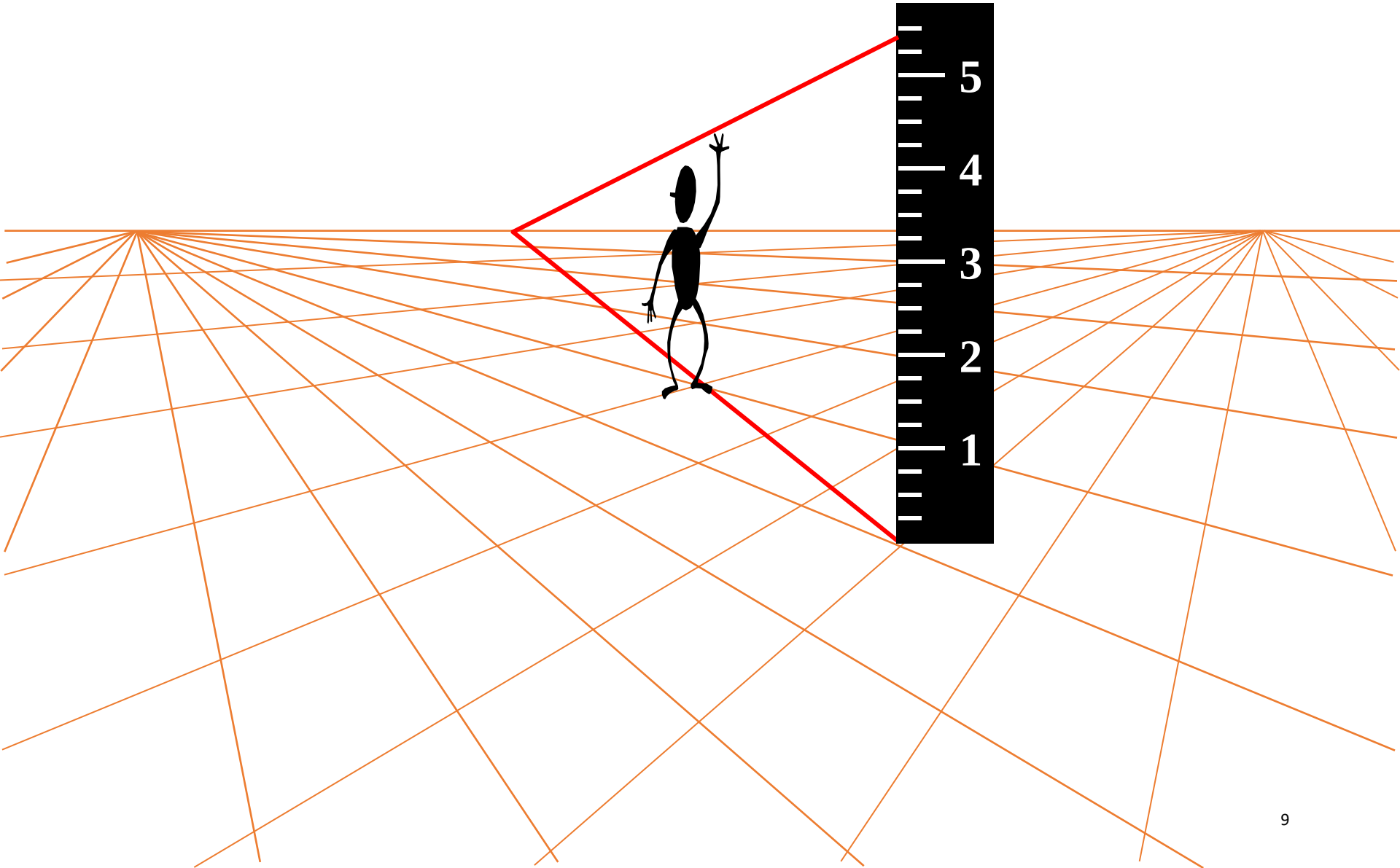

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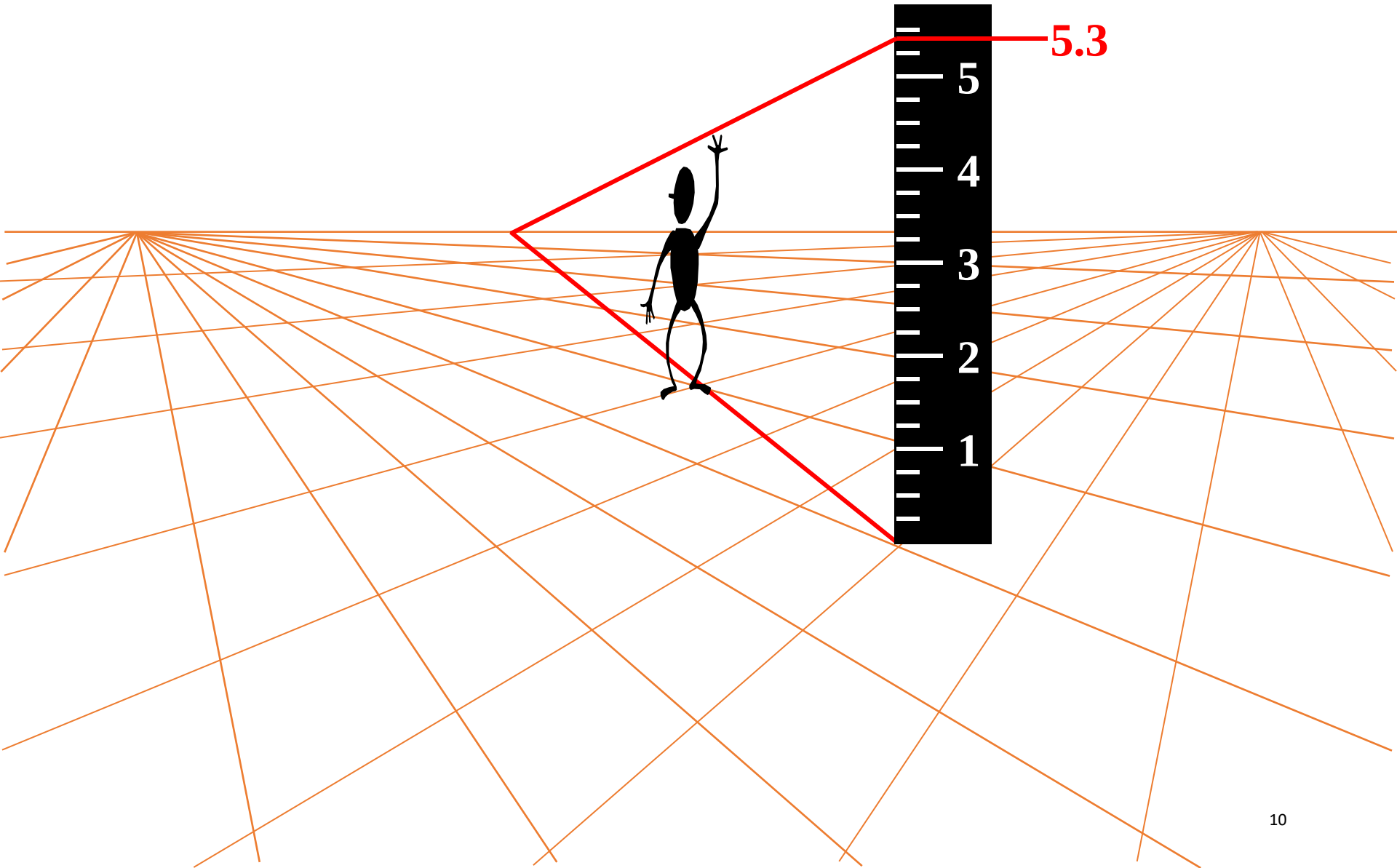

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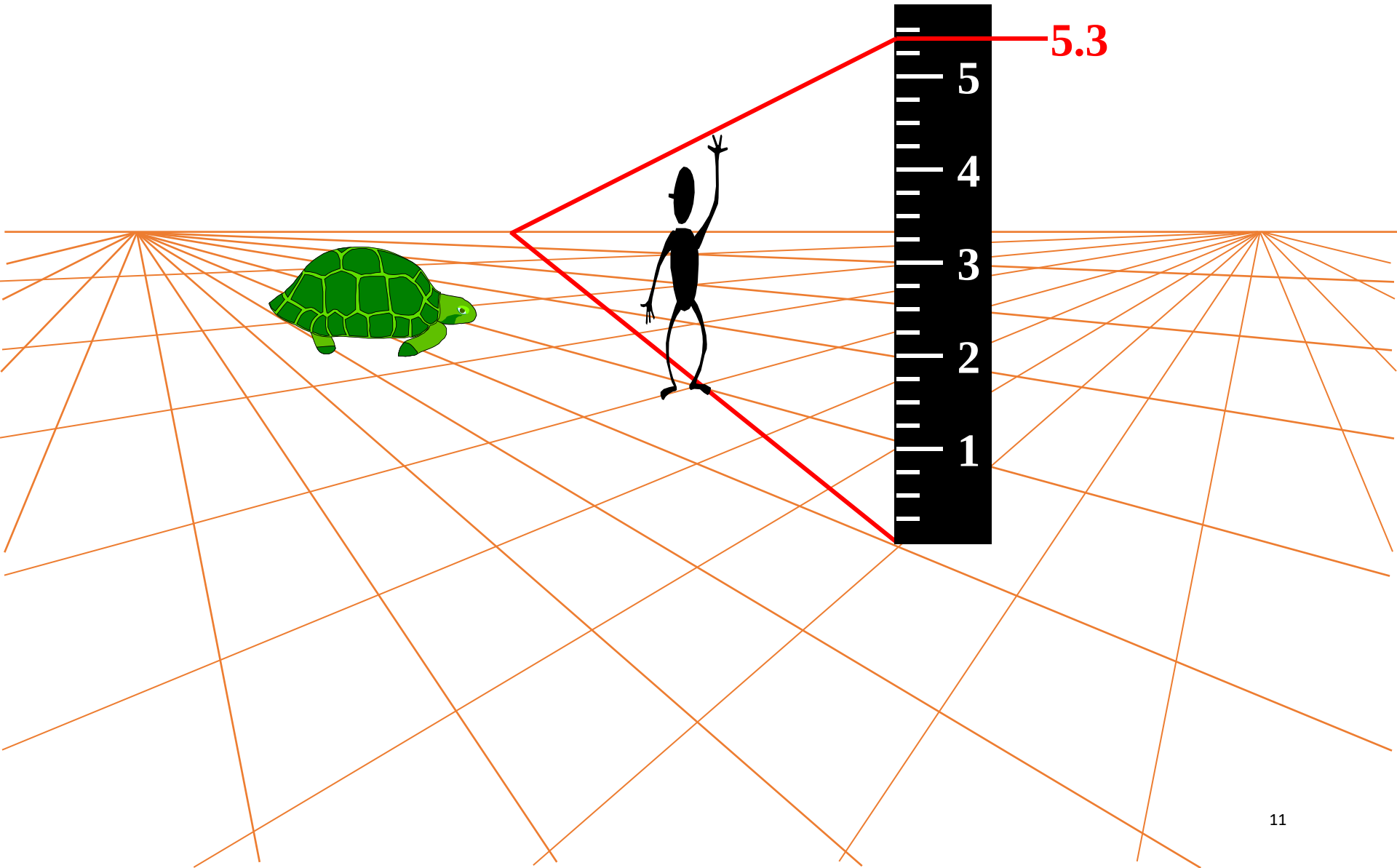

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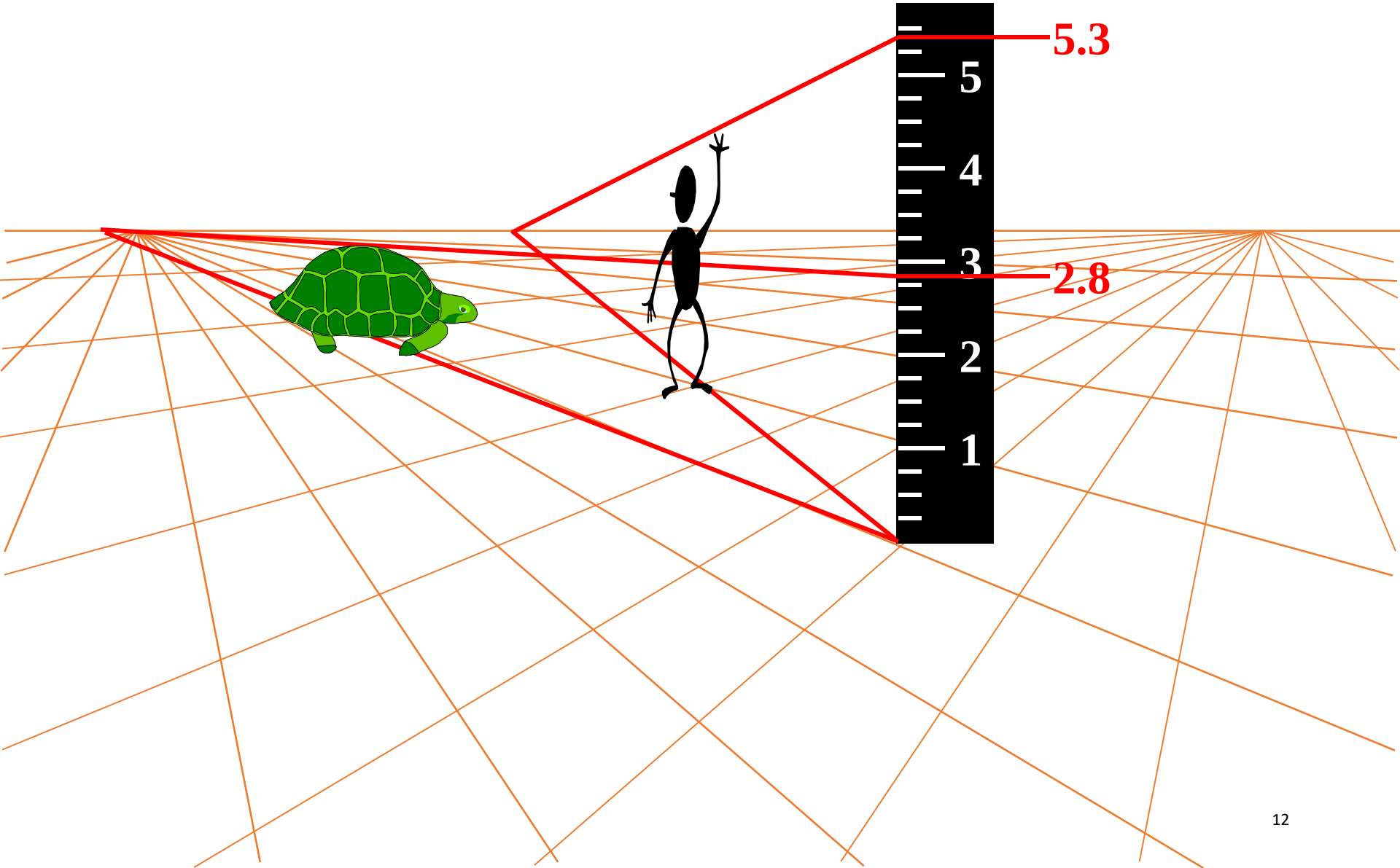

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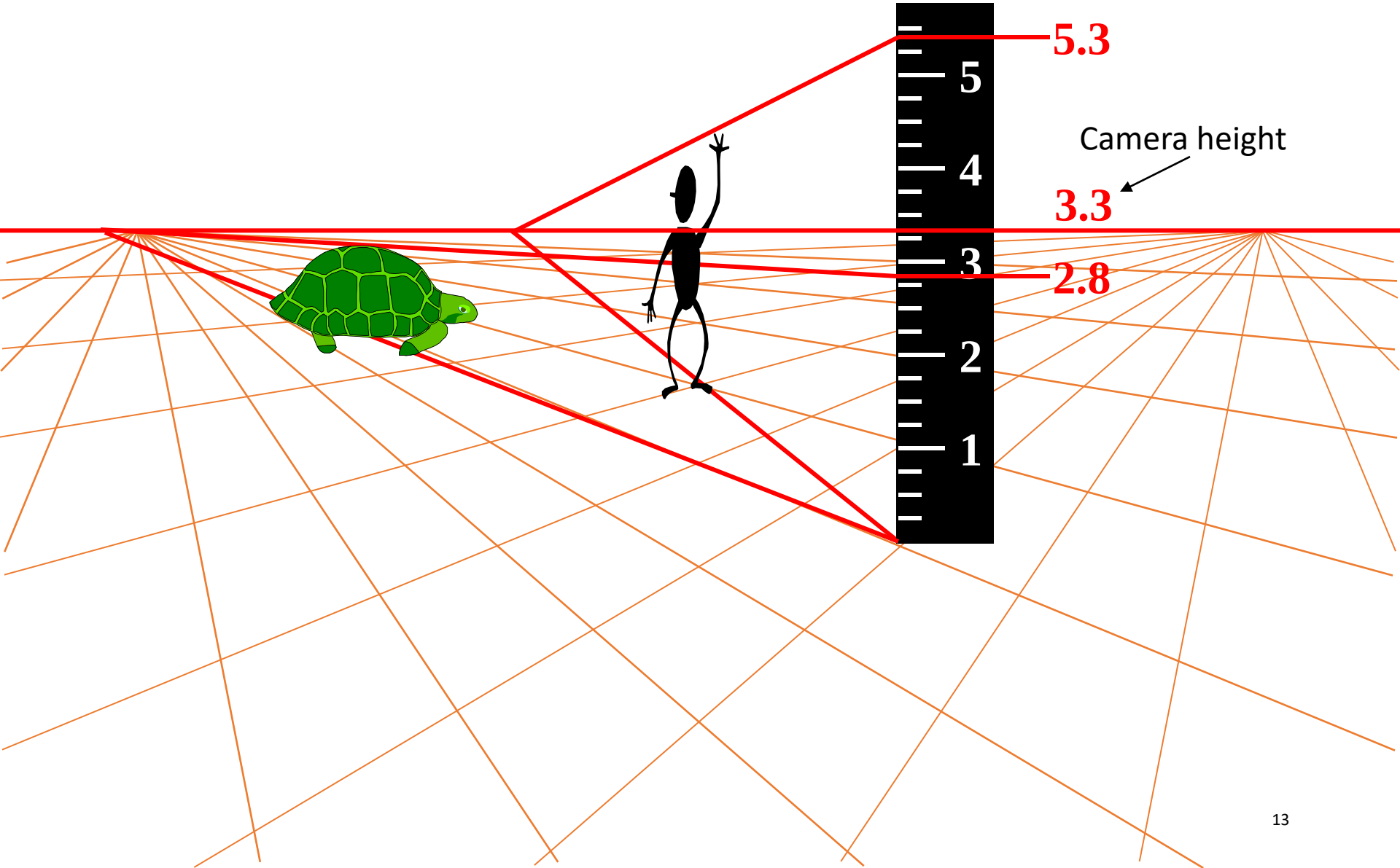
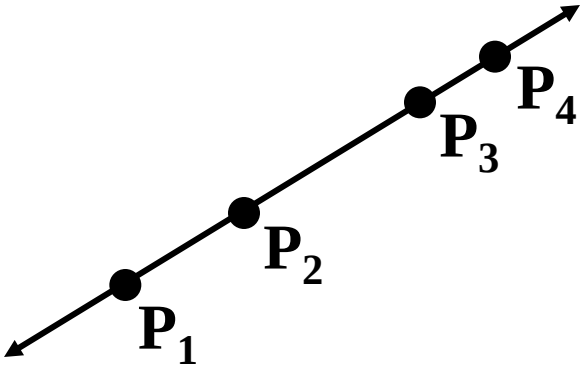


Which is higher – the camera or the man in the parachute?

The cross ratio

- A Projective Invariant
  - Does not change under projective transformations

The cross-ratio of 4 collinear points



$$\frac{\|\mathbf{P}_3 - \mathbf{P}_1\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_3 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_1\|}$$

$$\mathbf{P}_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

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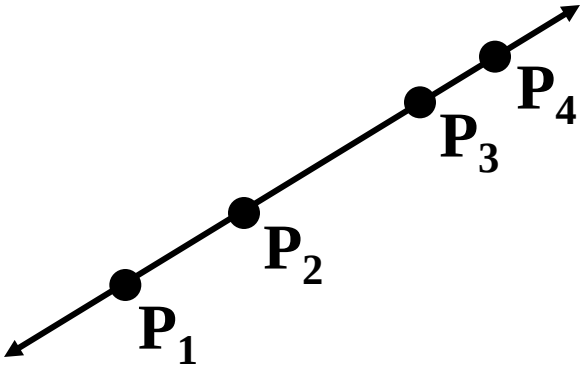

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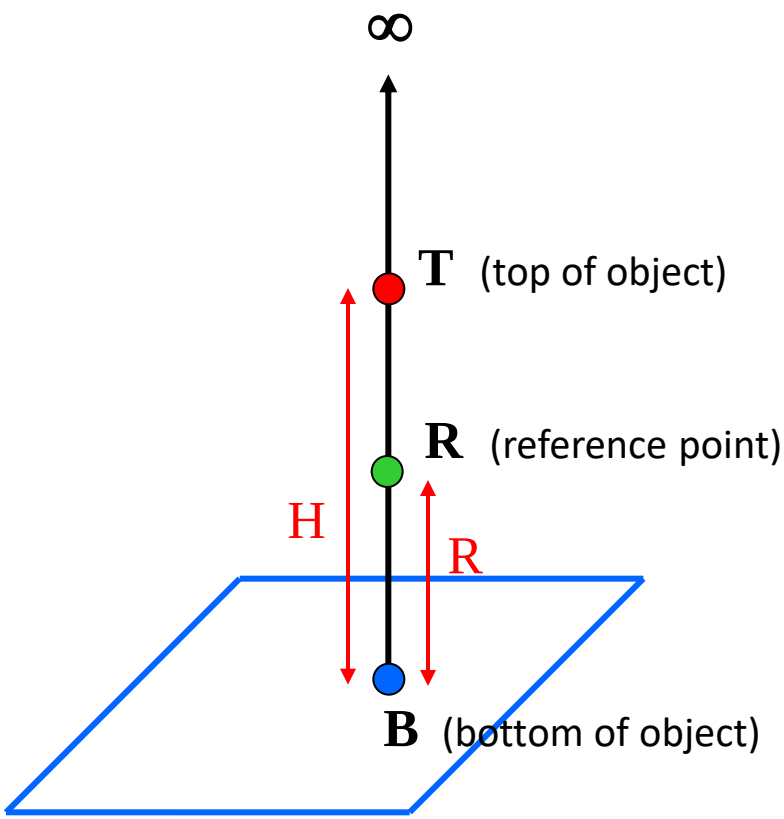
Can permute the point ordering

$$\frac{\|\mathbf{P}_1 - \mathbf{P}_3\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_1 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_3\|}$$

- 4! = 24 different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry

Measuring height



scene points represented as $\mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$ image points as $\mathbf{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

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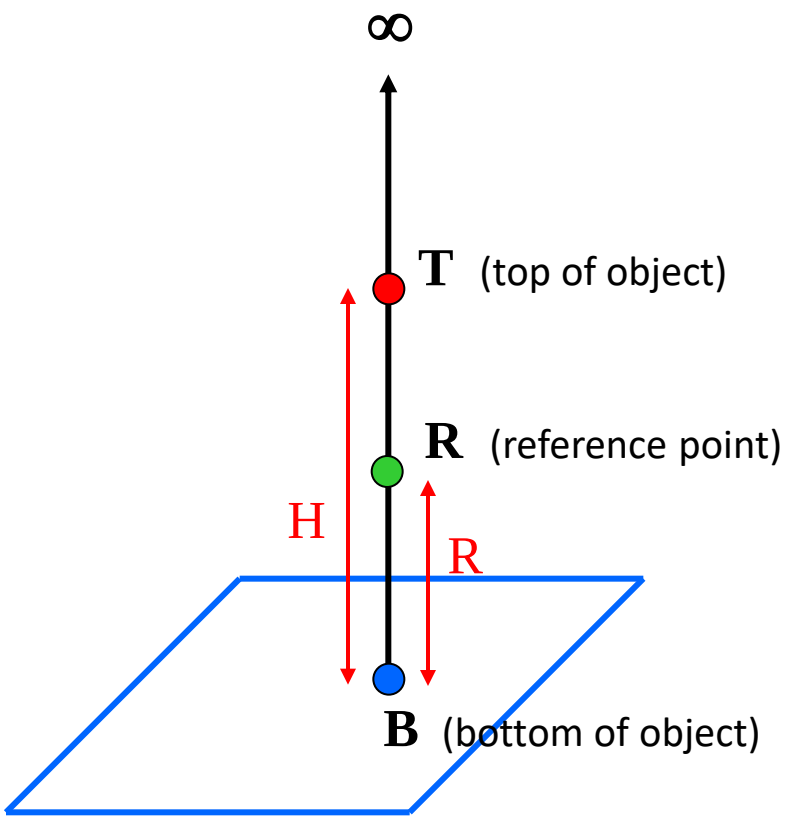

$$\frac{\|\mathbf{B}-\mathbf{T}\|\,\|\infty-\mathbf{R}\|}{\|\mathbf{B}-\mathbf{R}\|\,\|\infty-\mathbf{T}\|}$$

scene cross ratio

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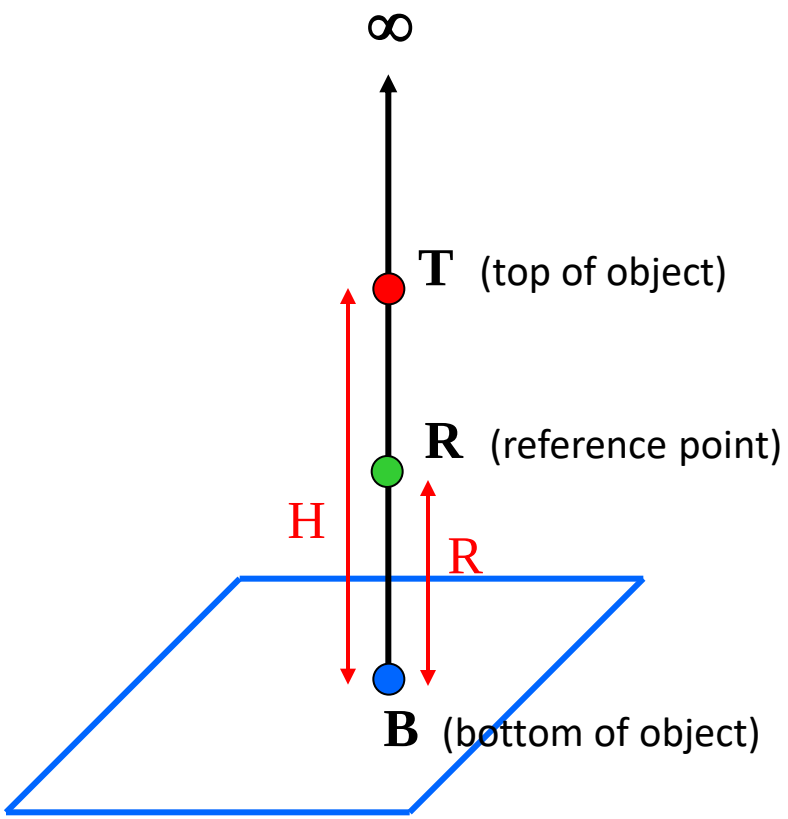

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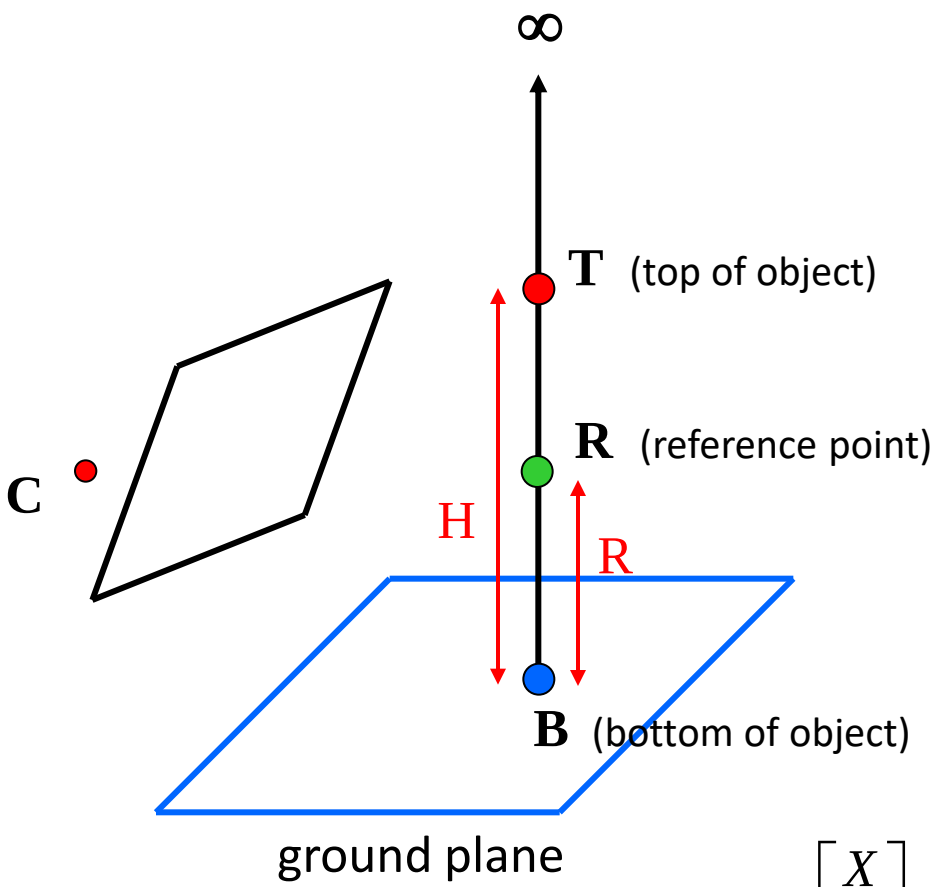

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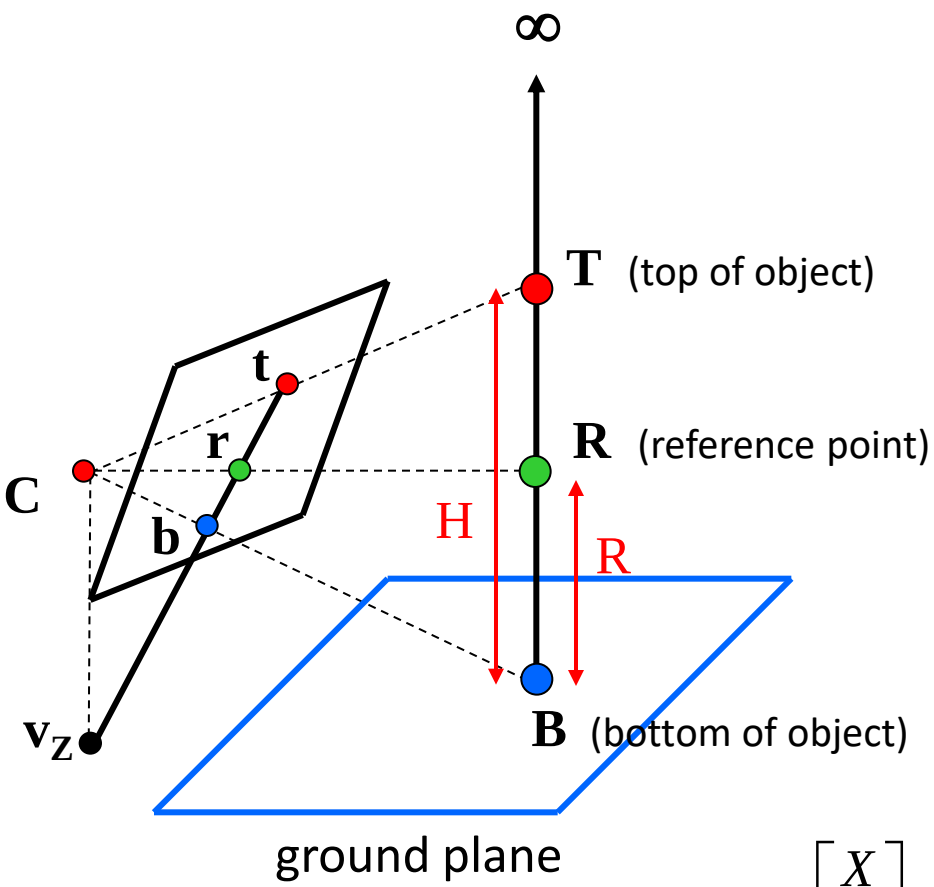

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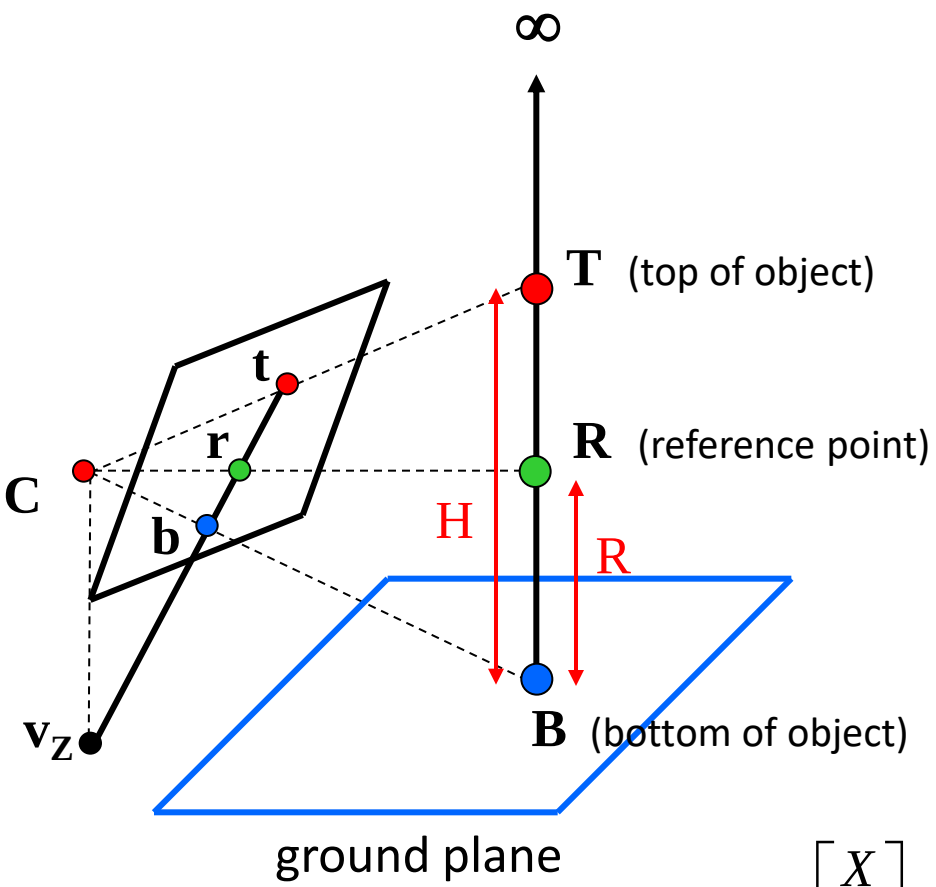

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image cross ratio

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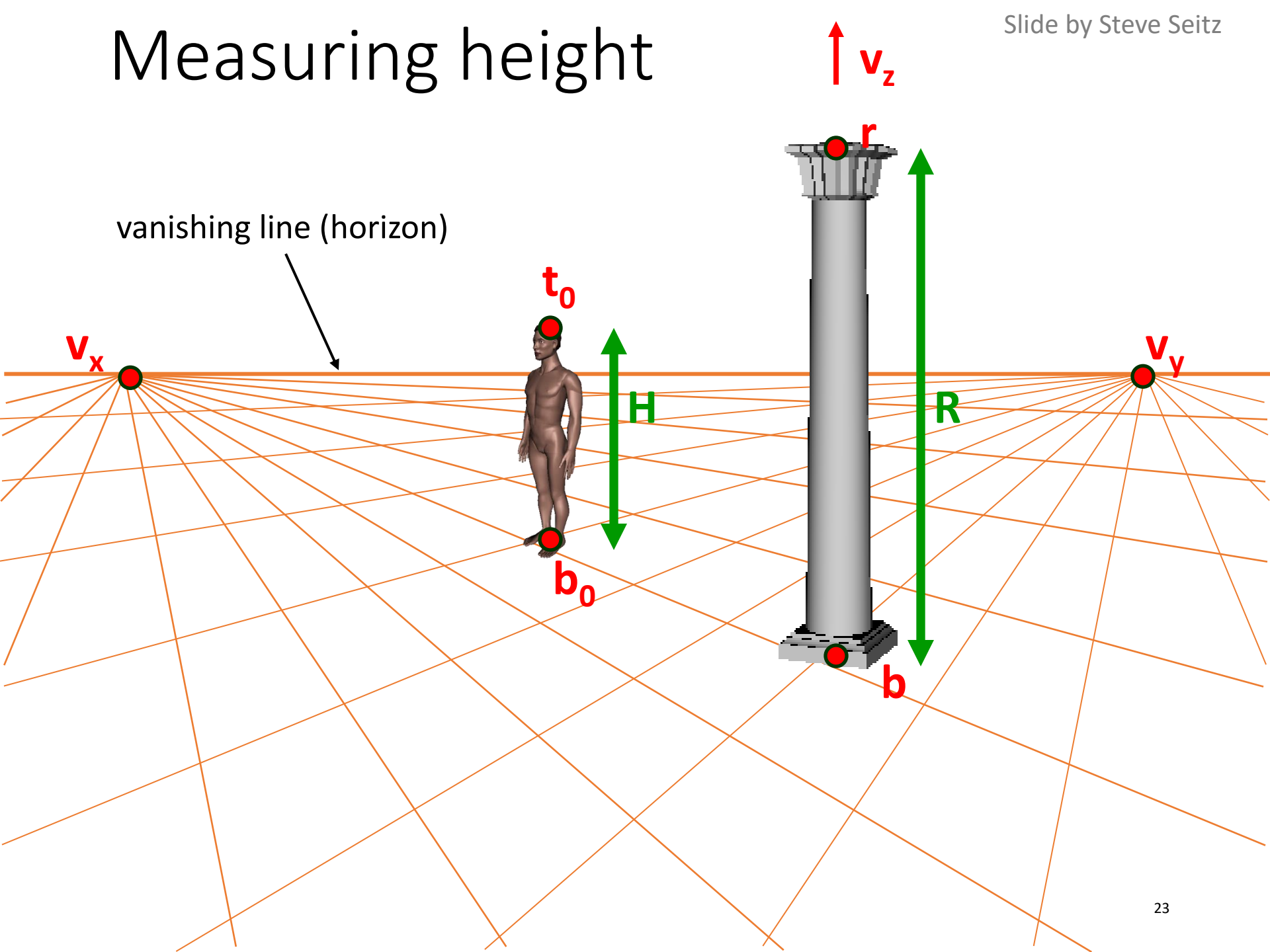
Measuring height

Slide by Steve Seitz



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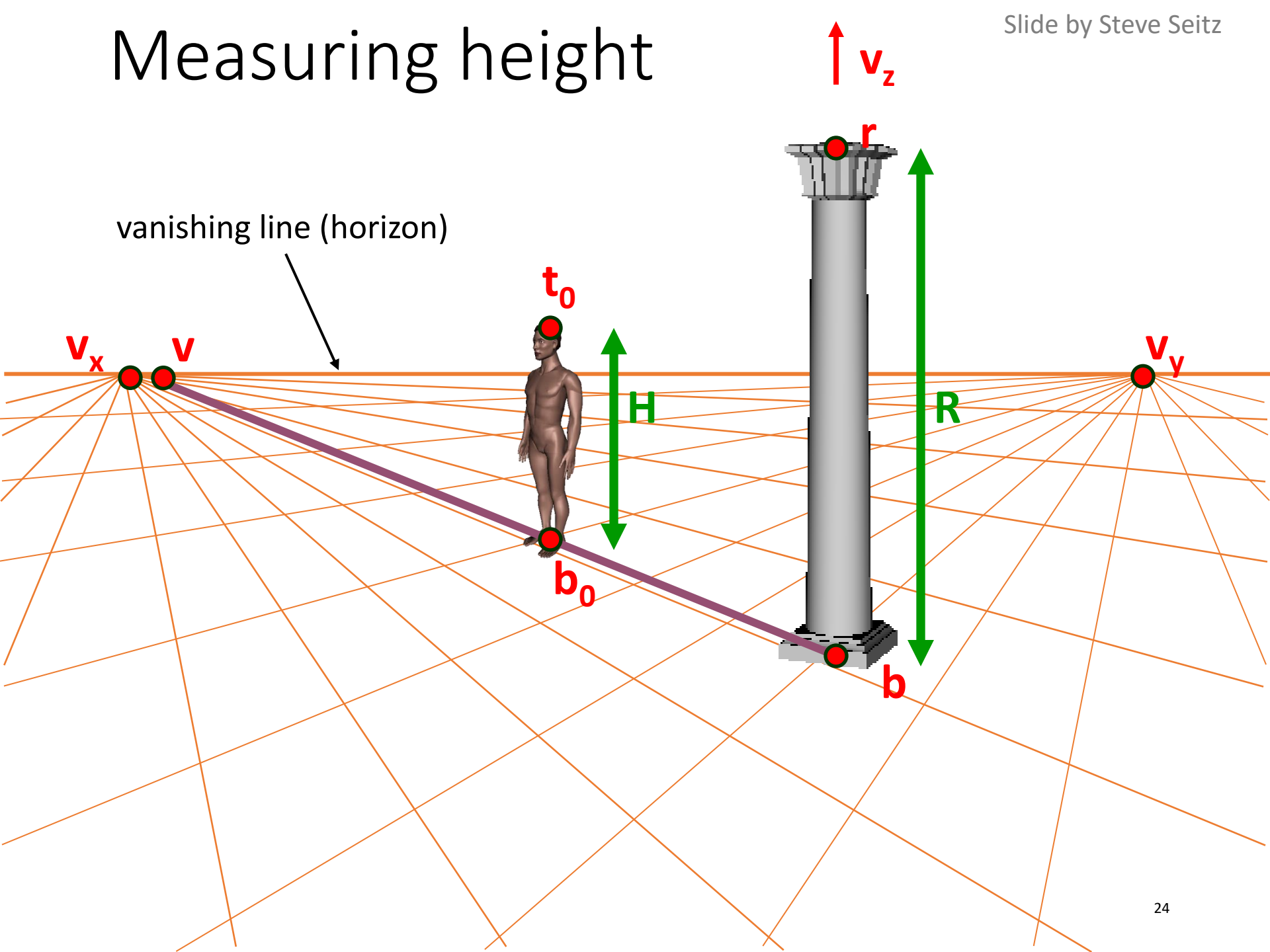

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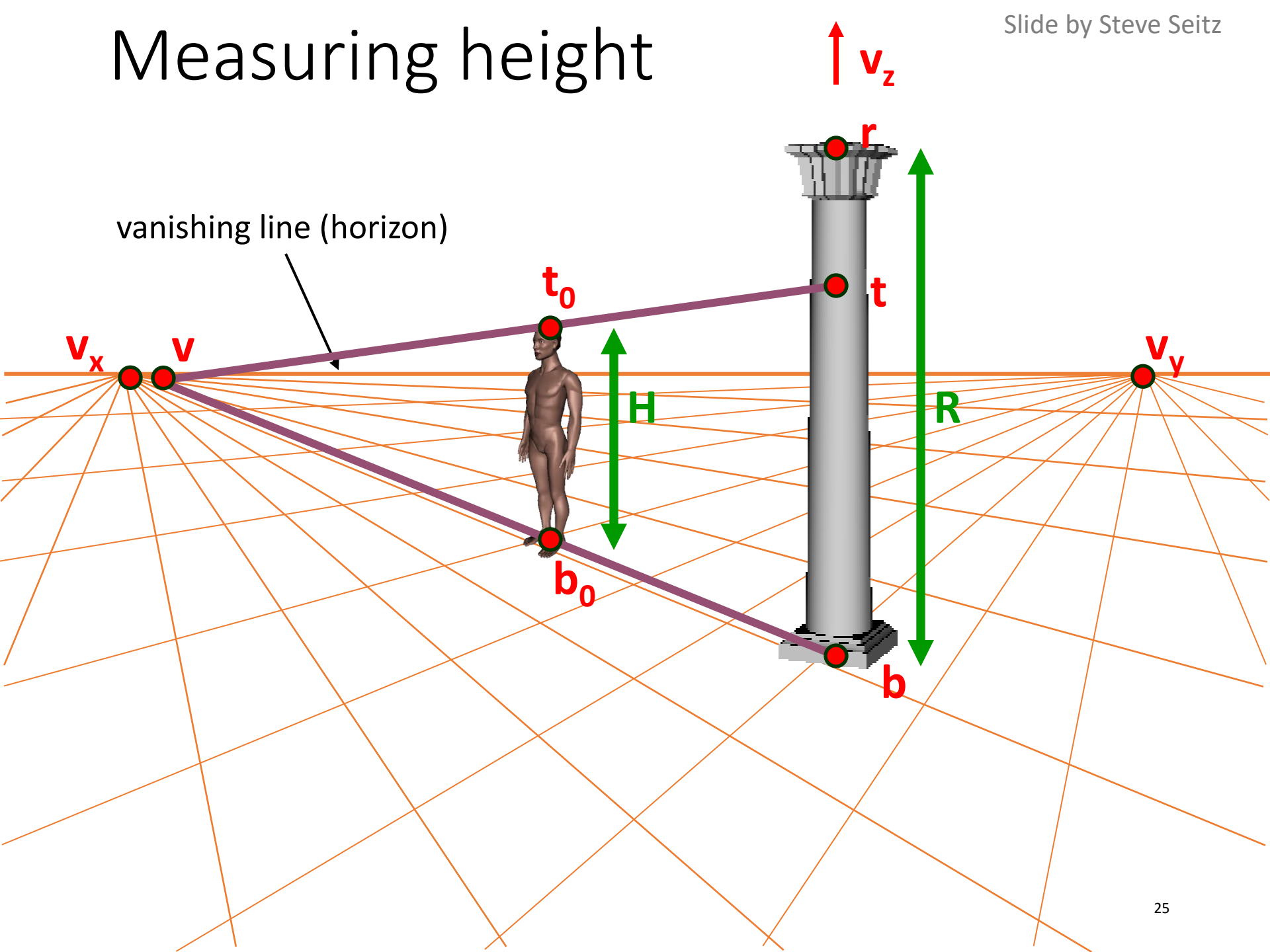

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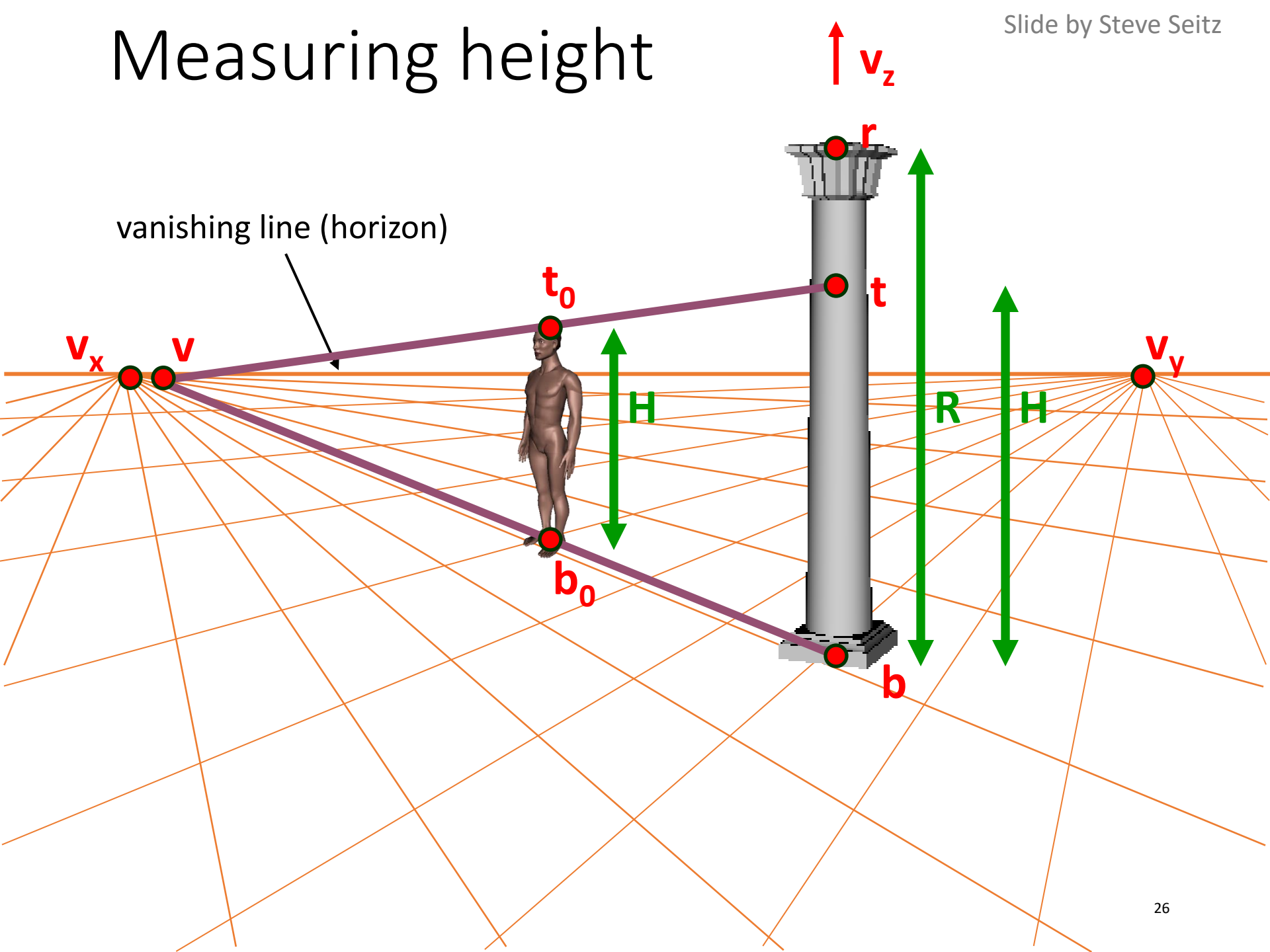

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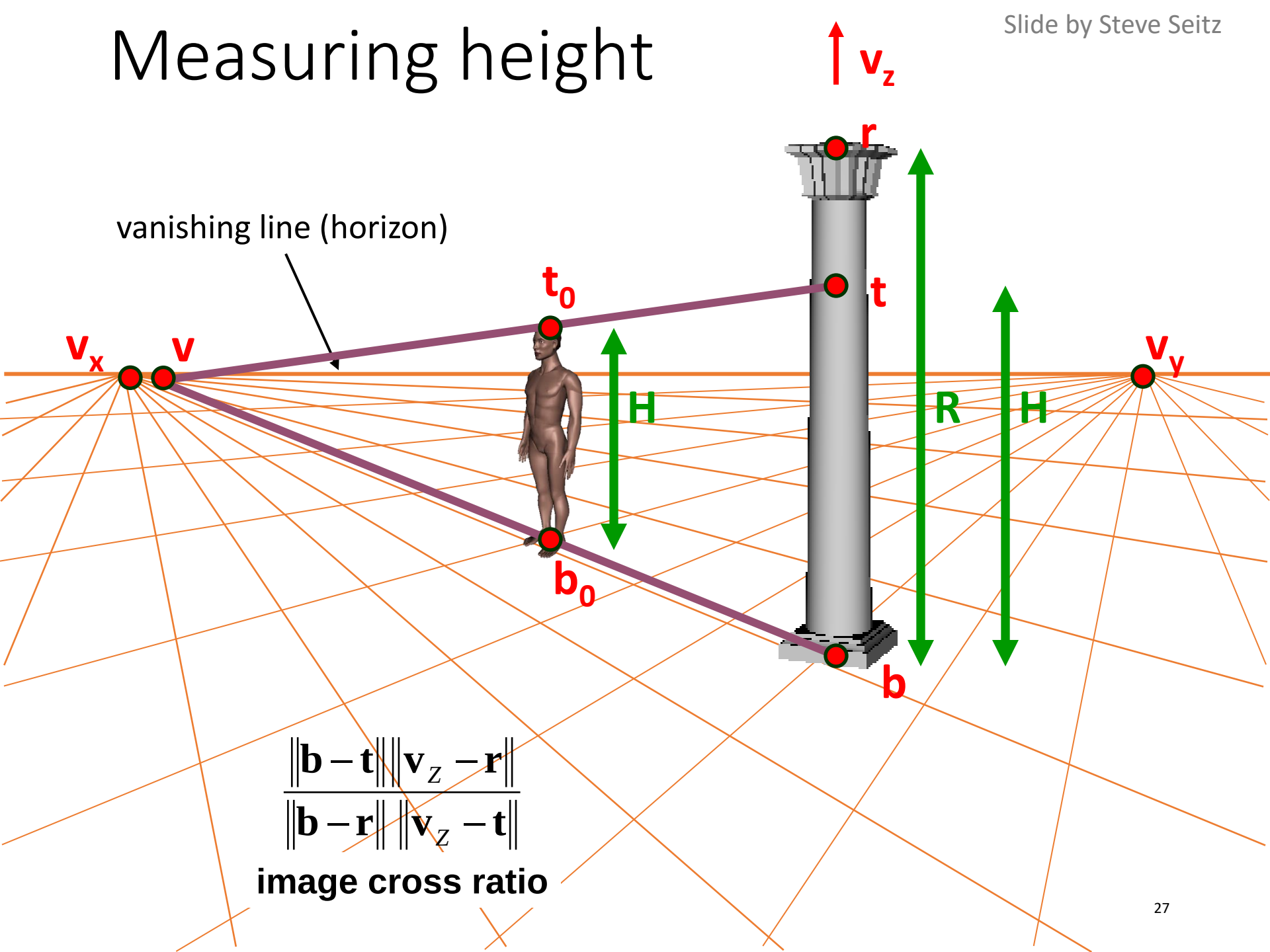

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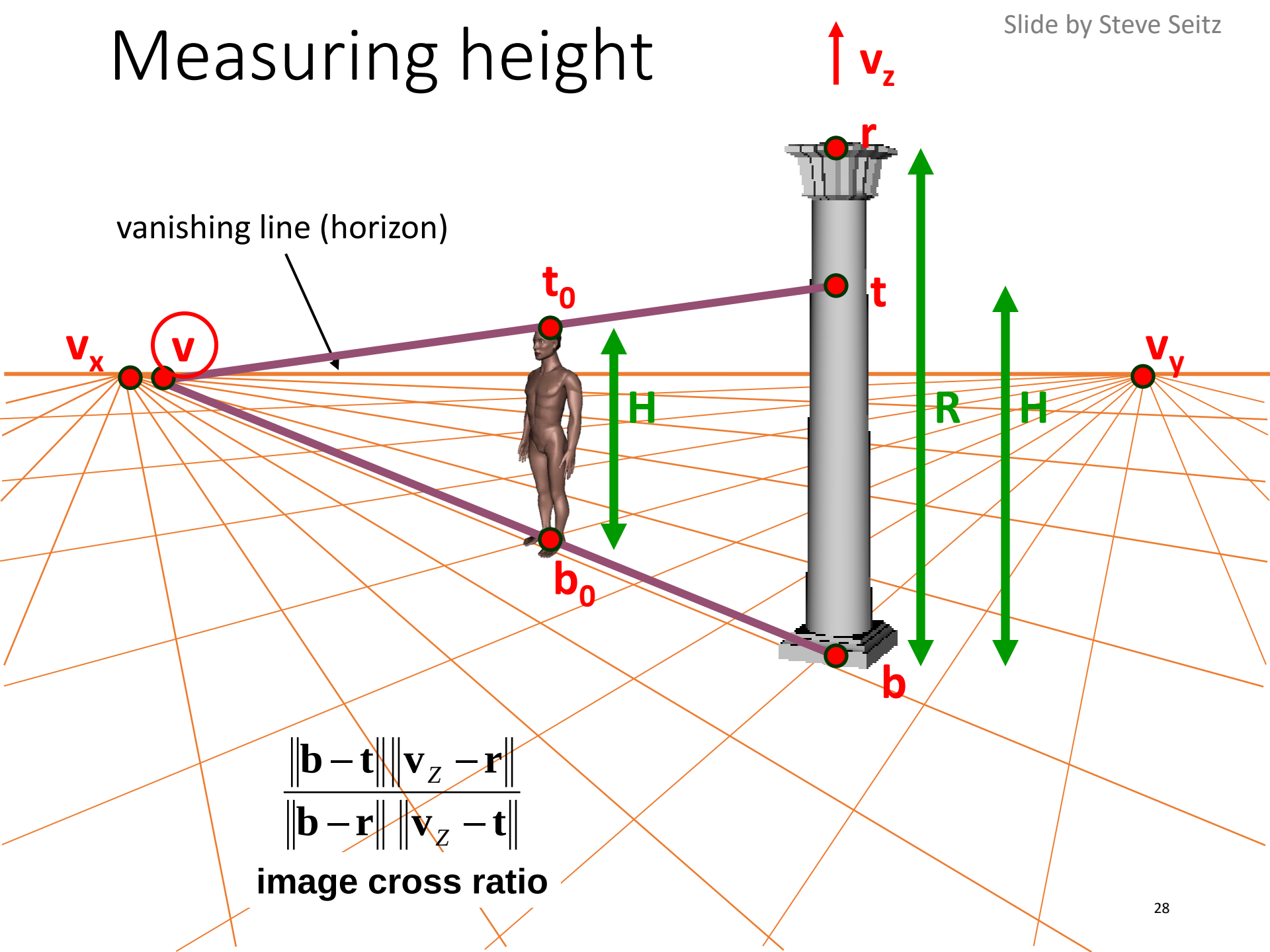

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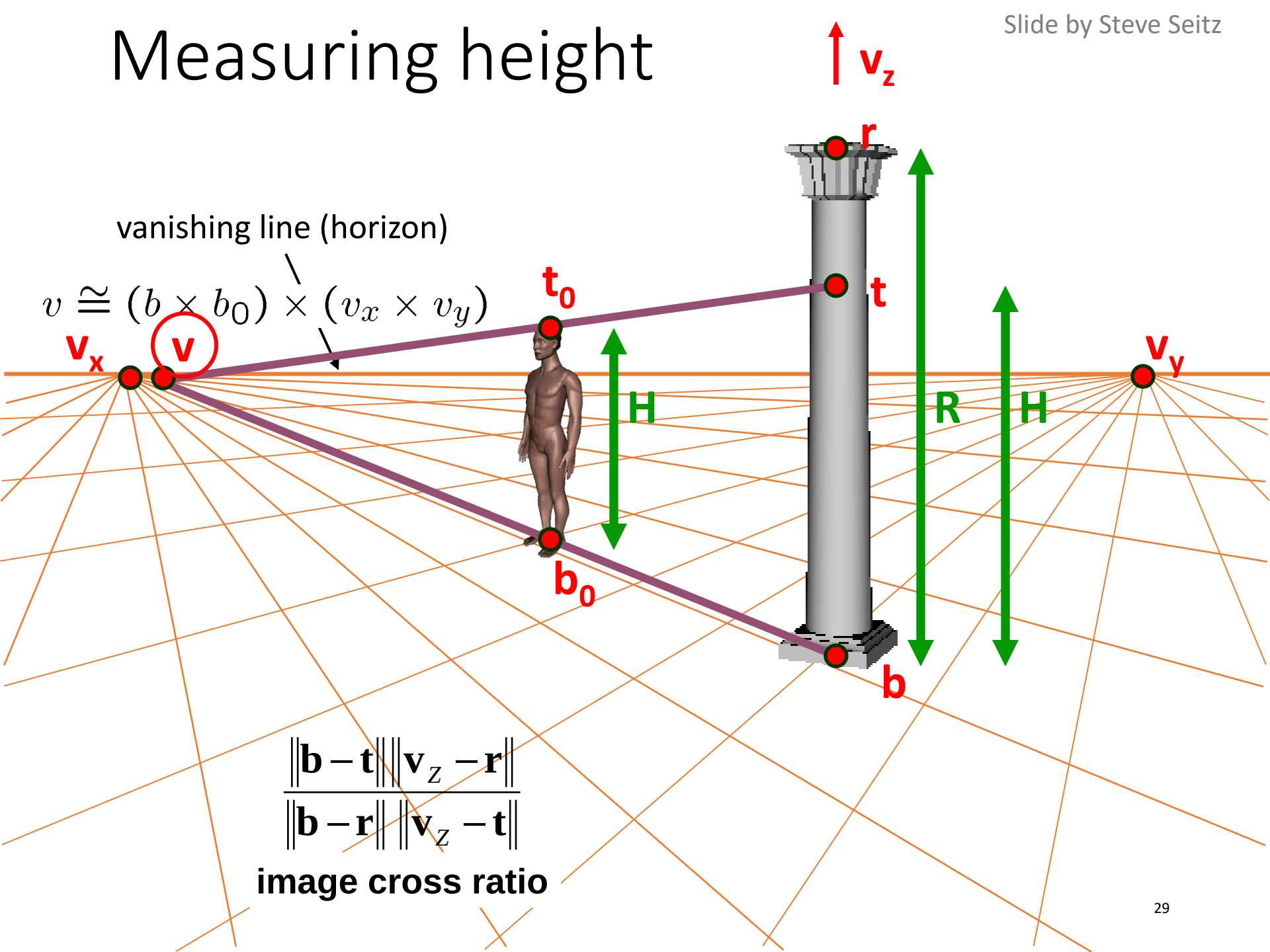

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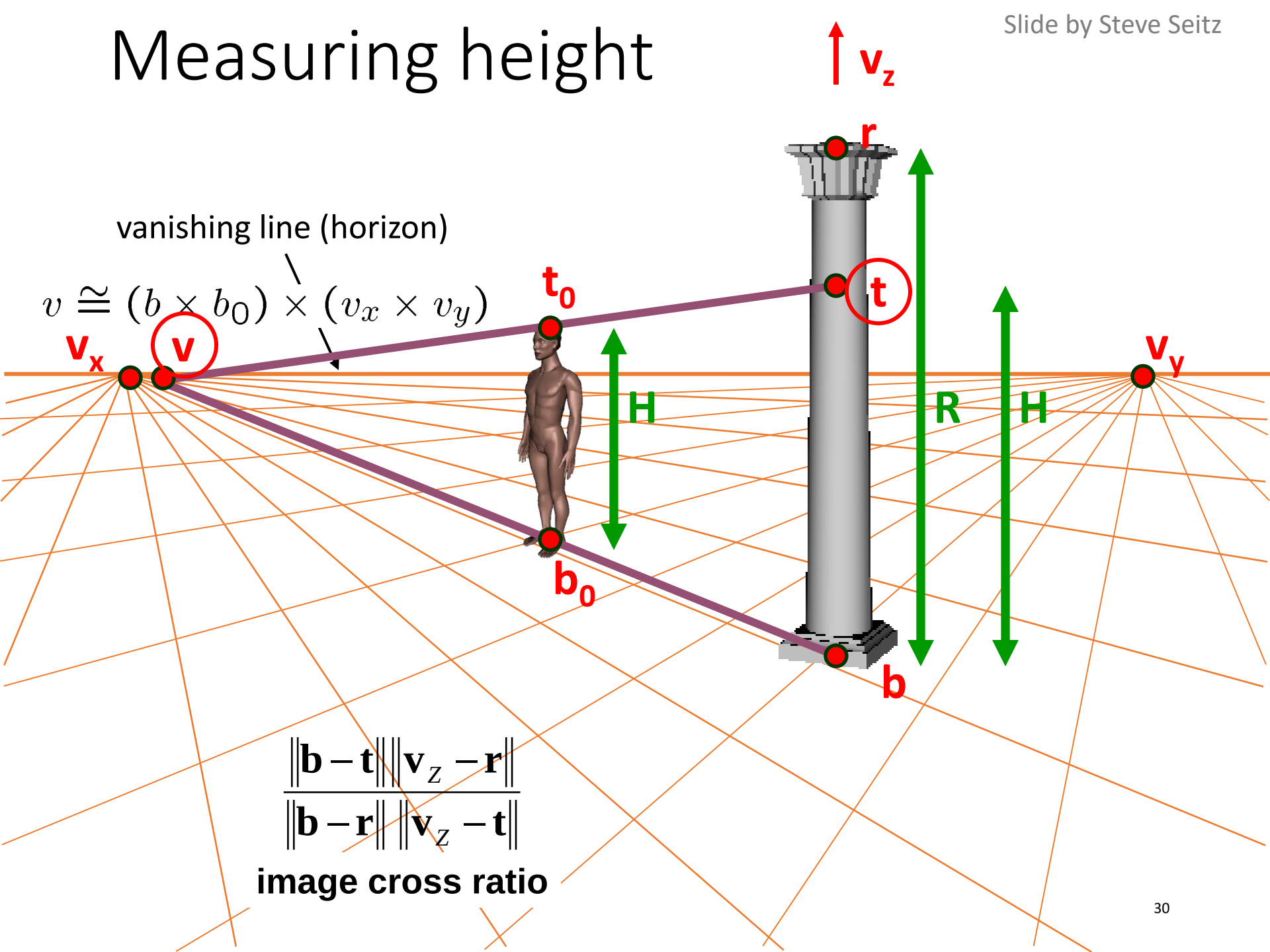

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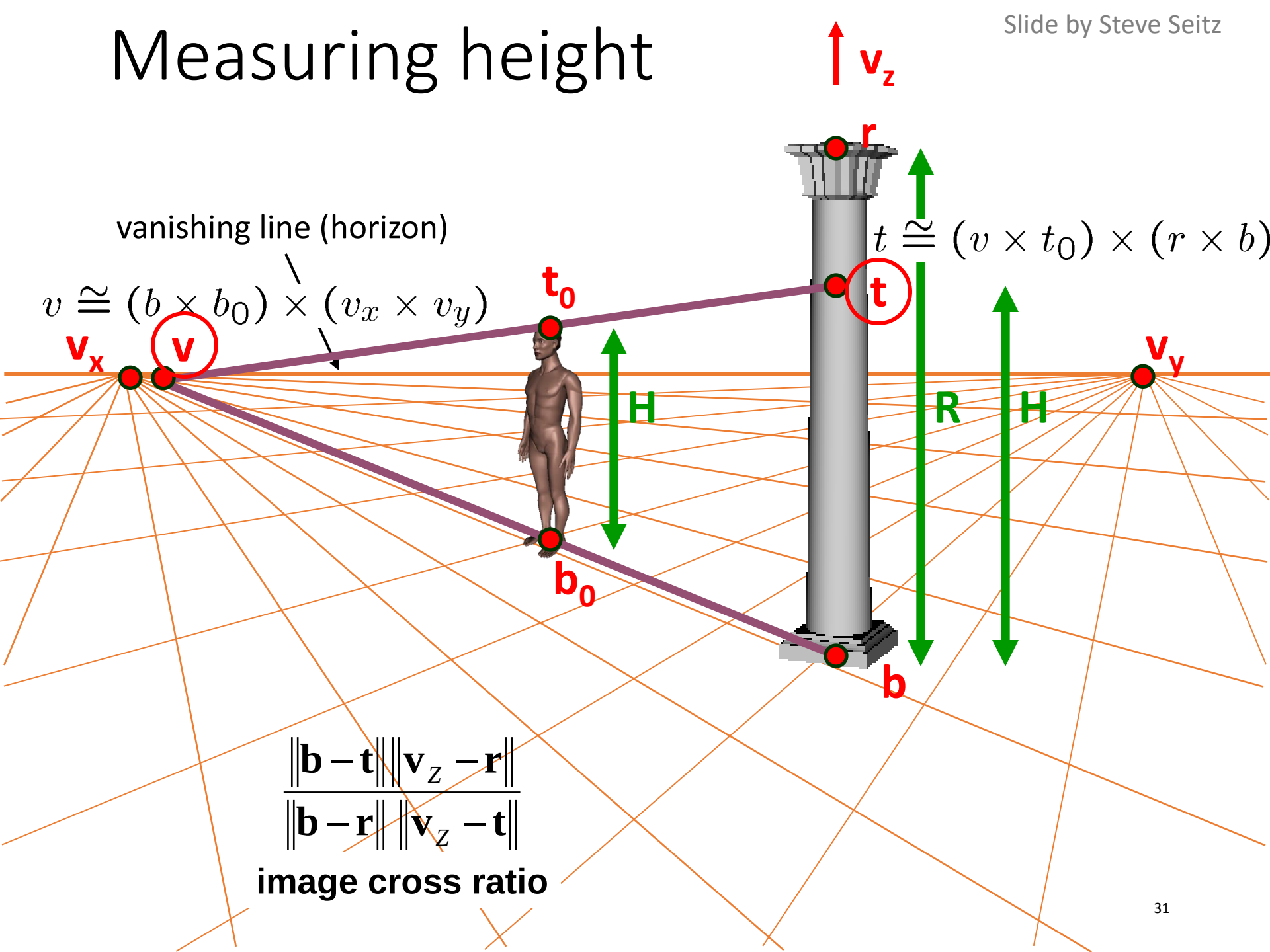

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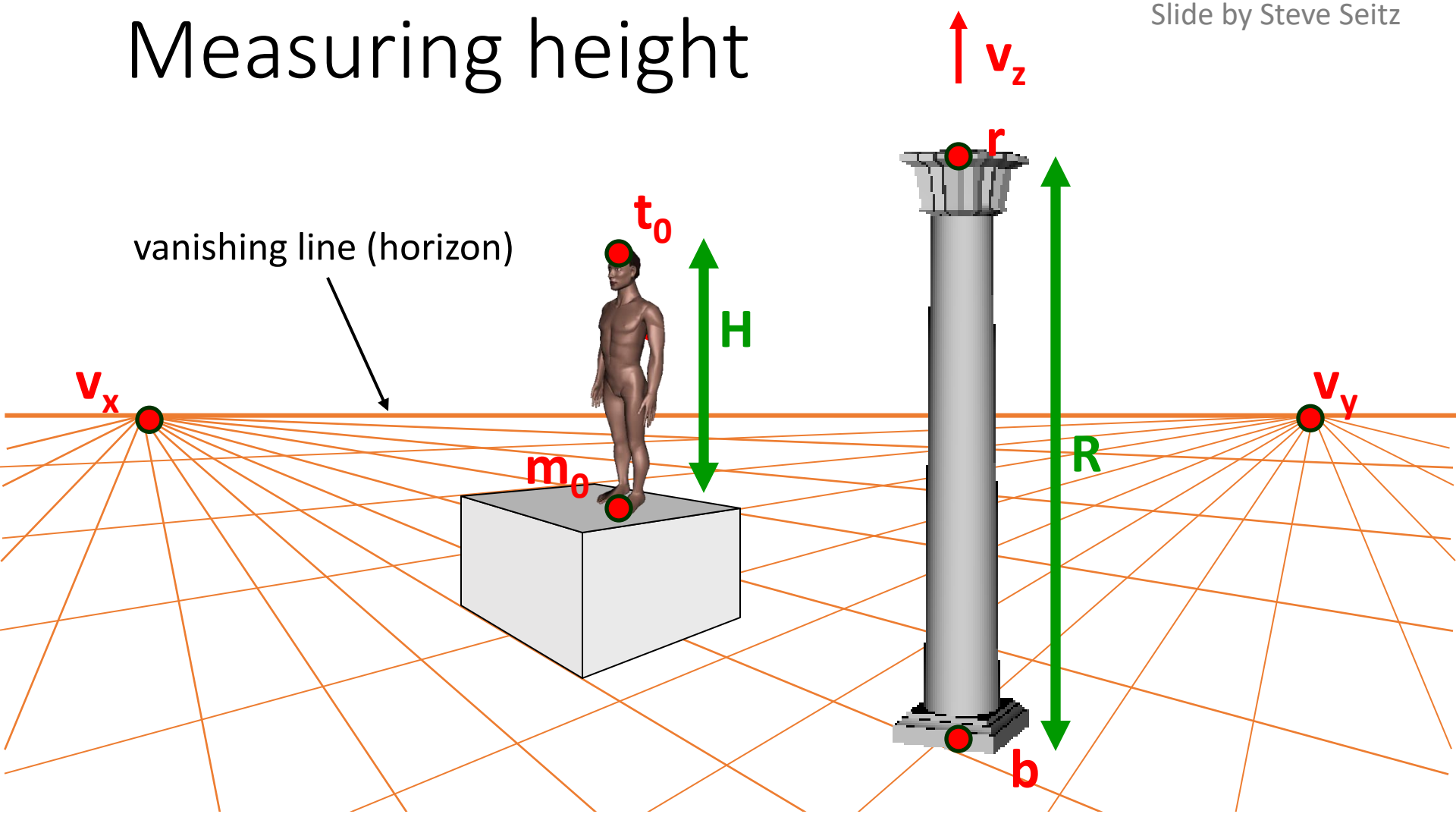
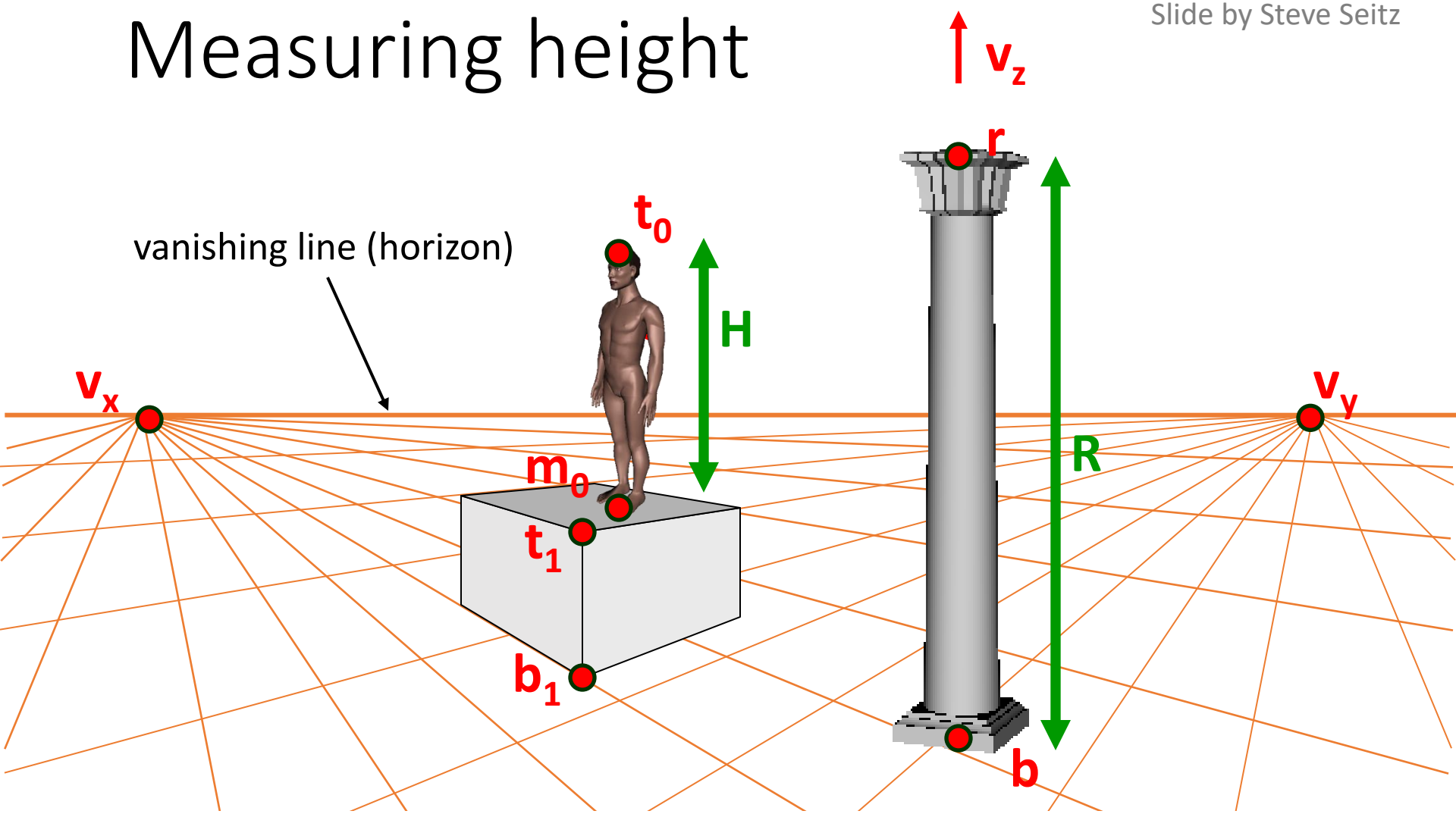


What if the point on the ground plane $\mathbf{b}_0$ is not known?

- Here the guy is standing on the box, height of box is known
- Use one side of the box to help find $\mathbf{b}_0$ as shown above

Measuring height



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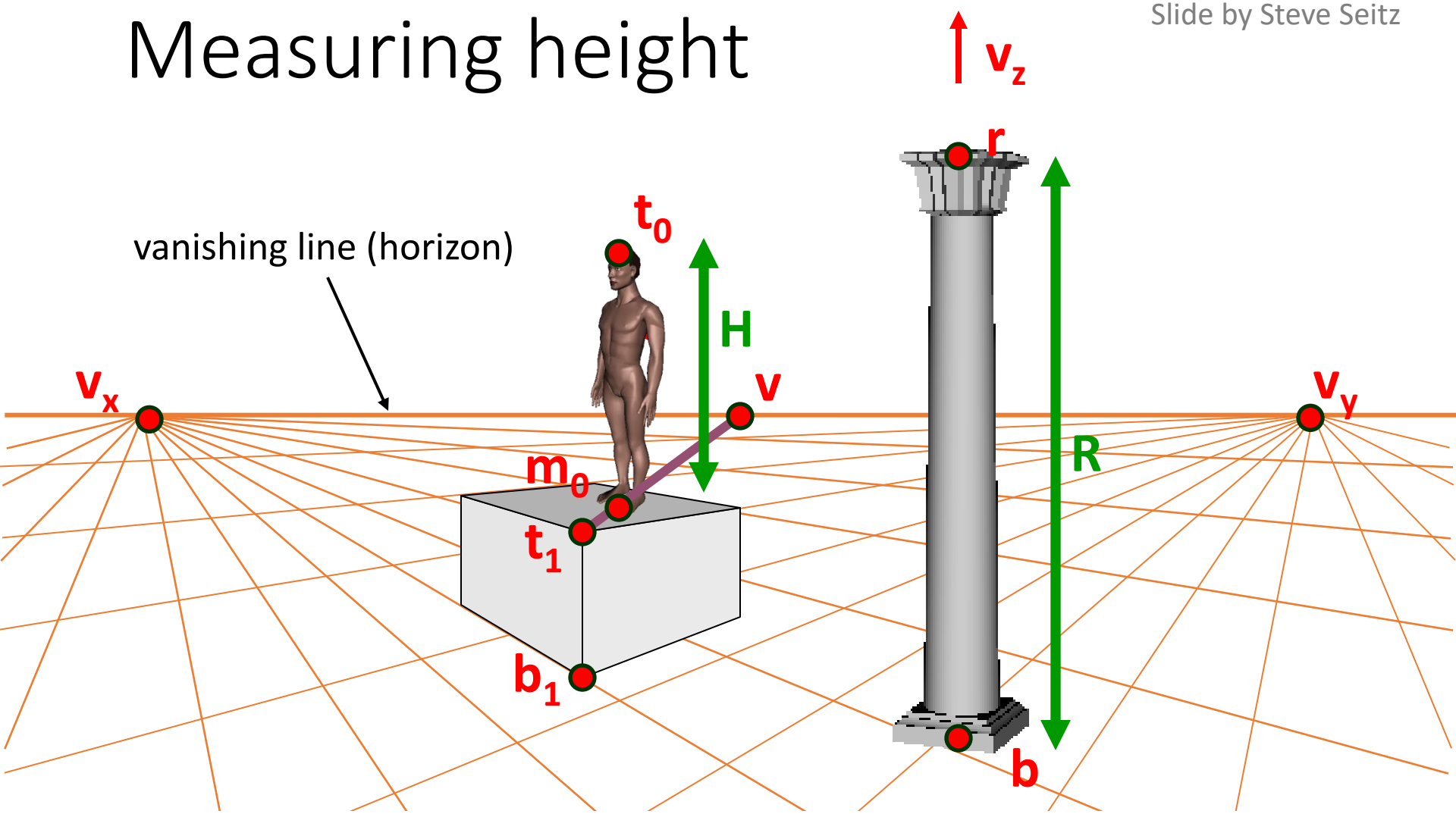

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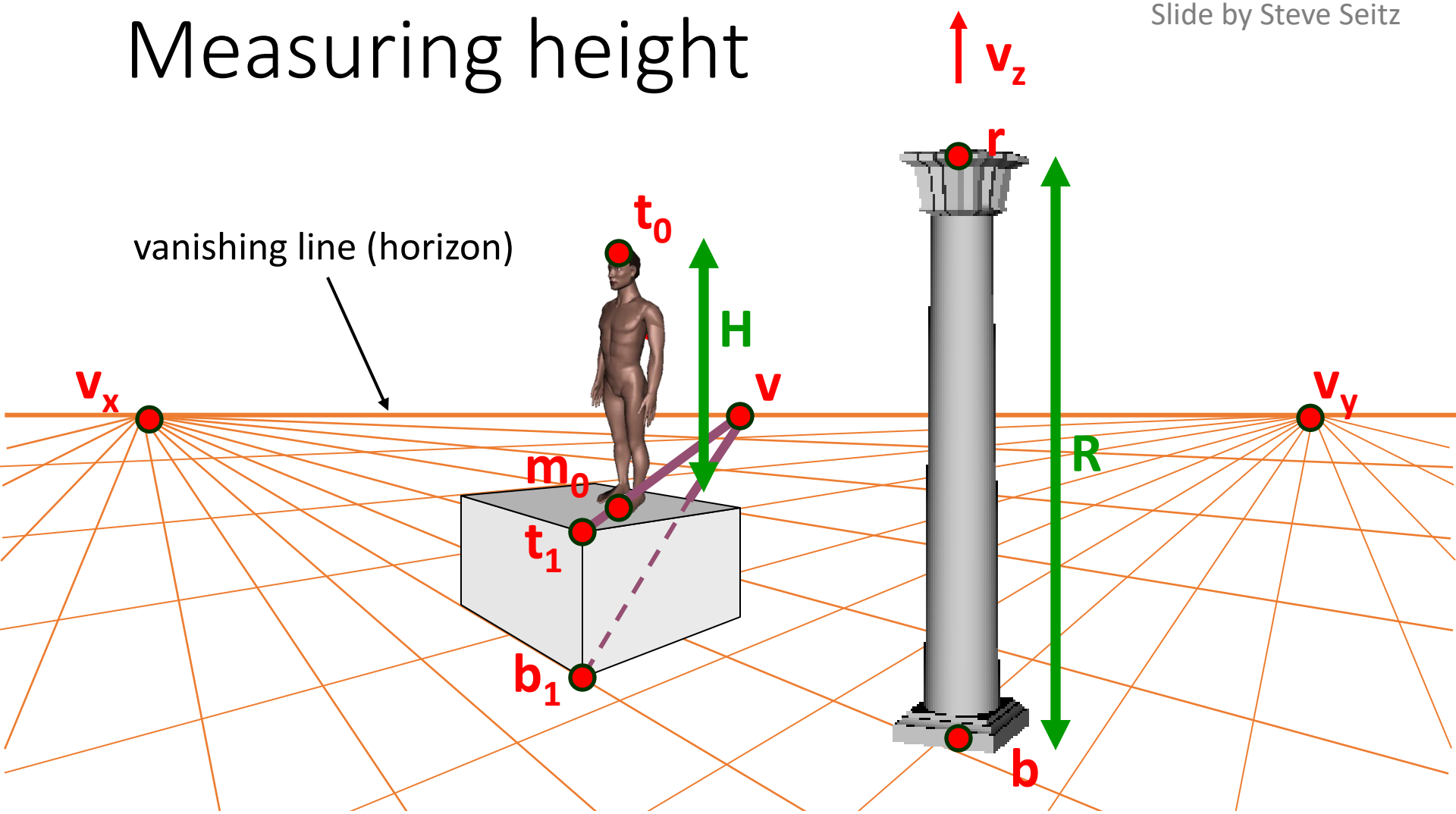

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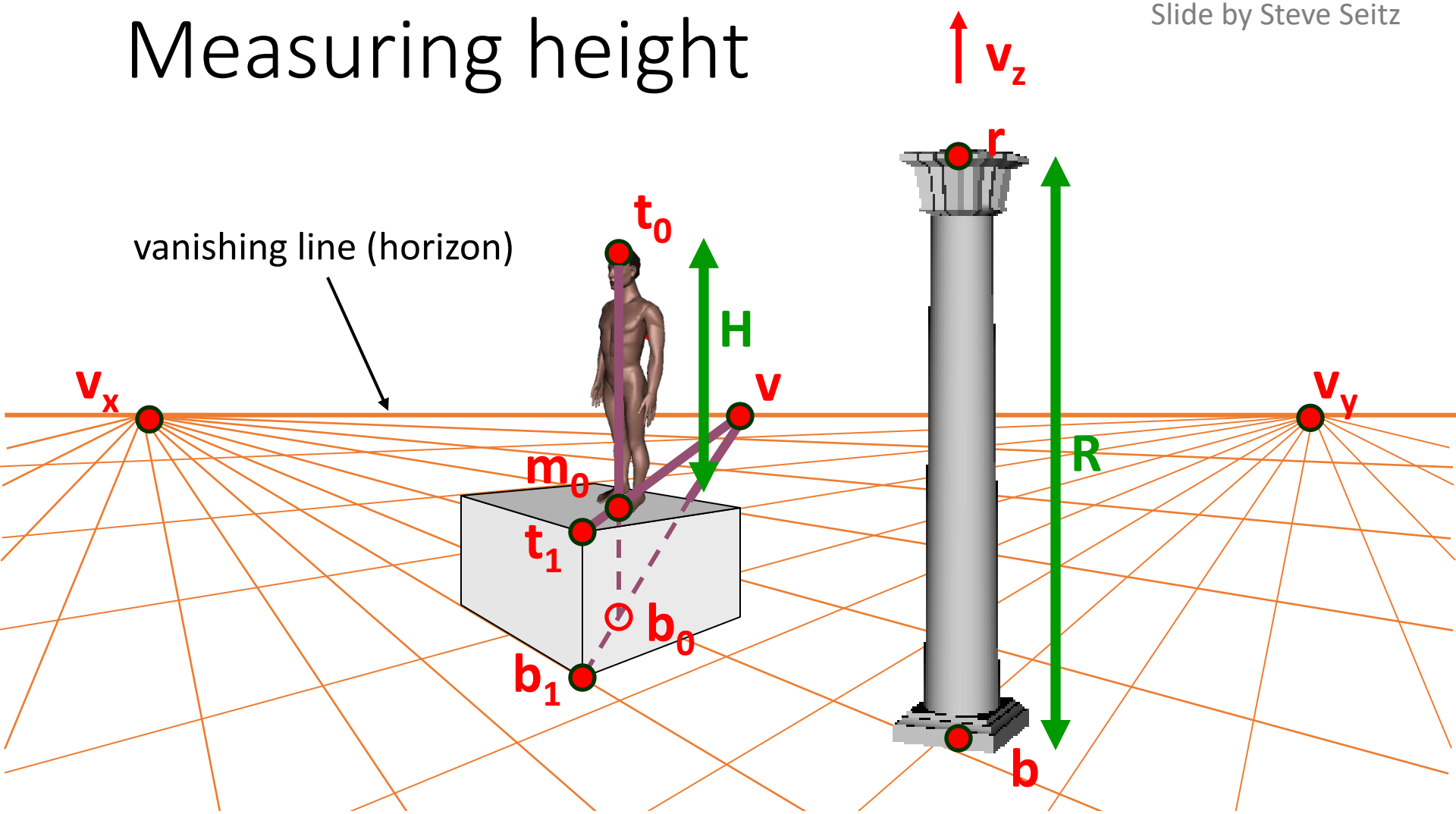

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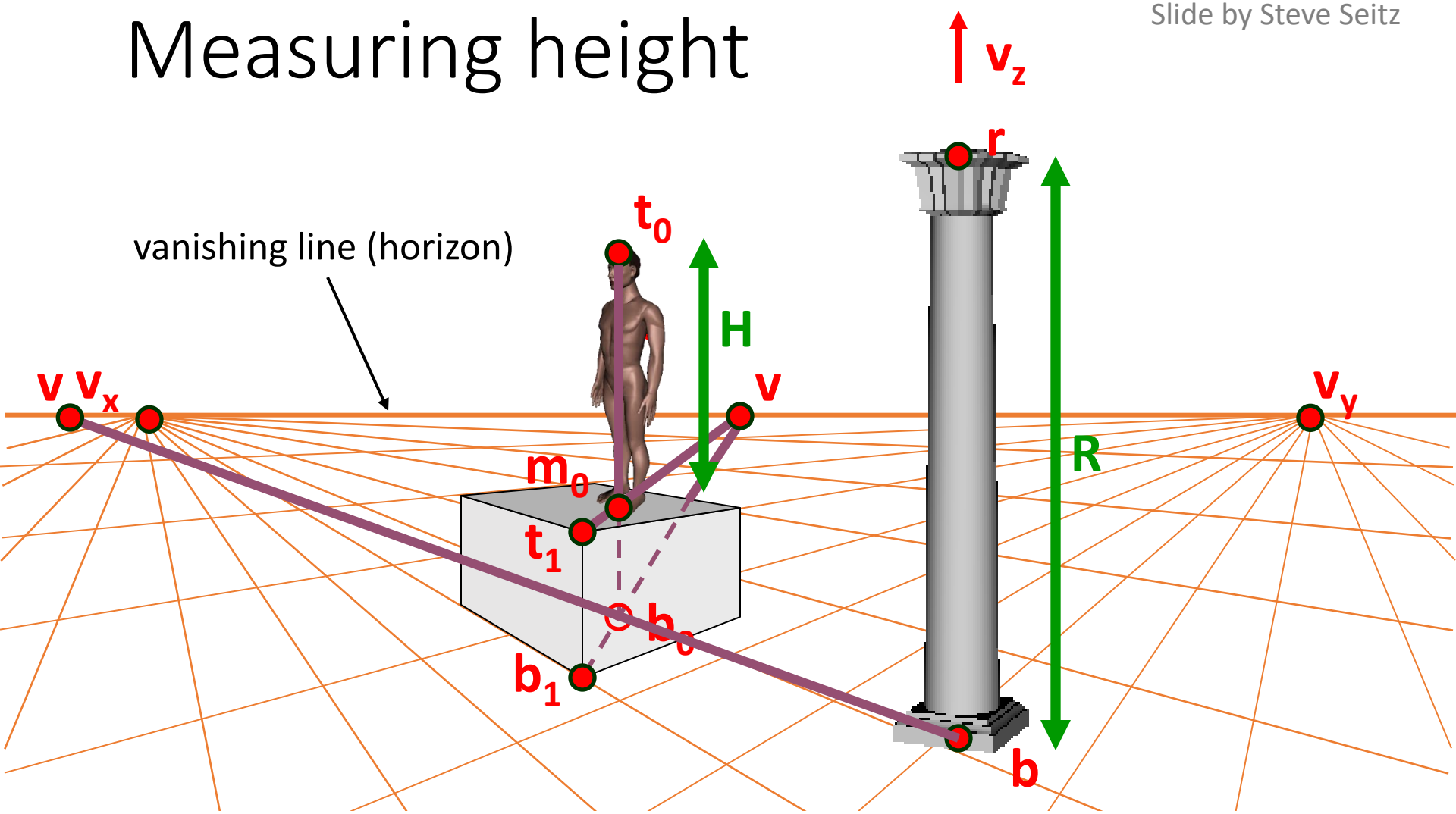

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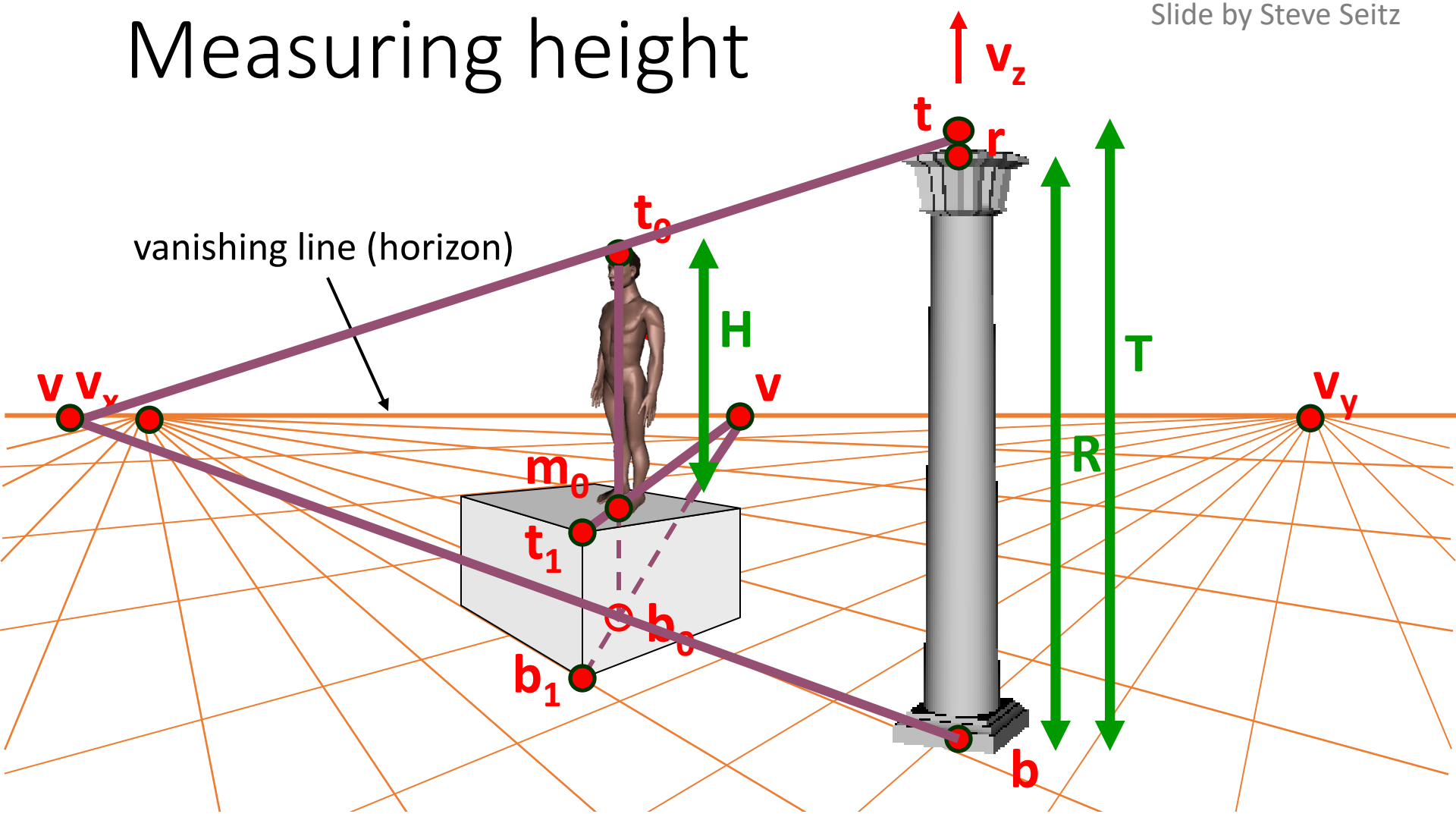

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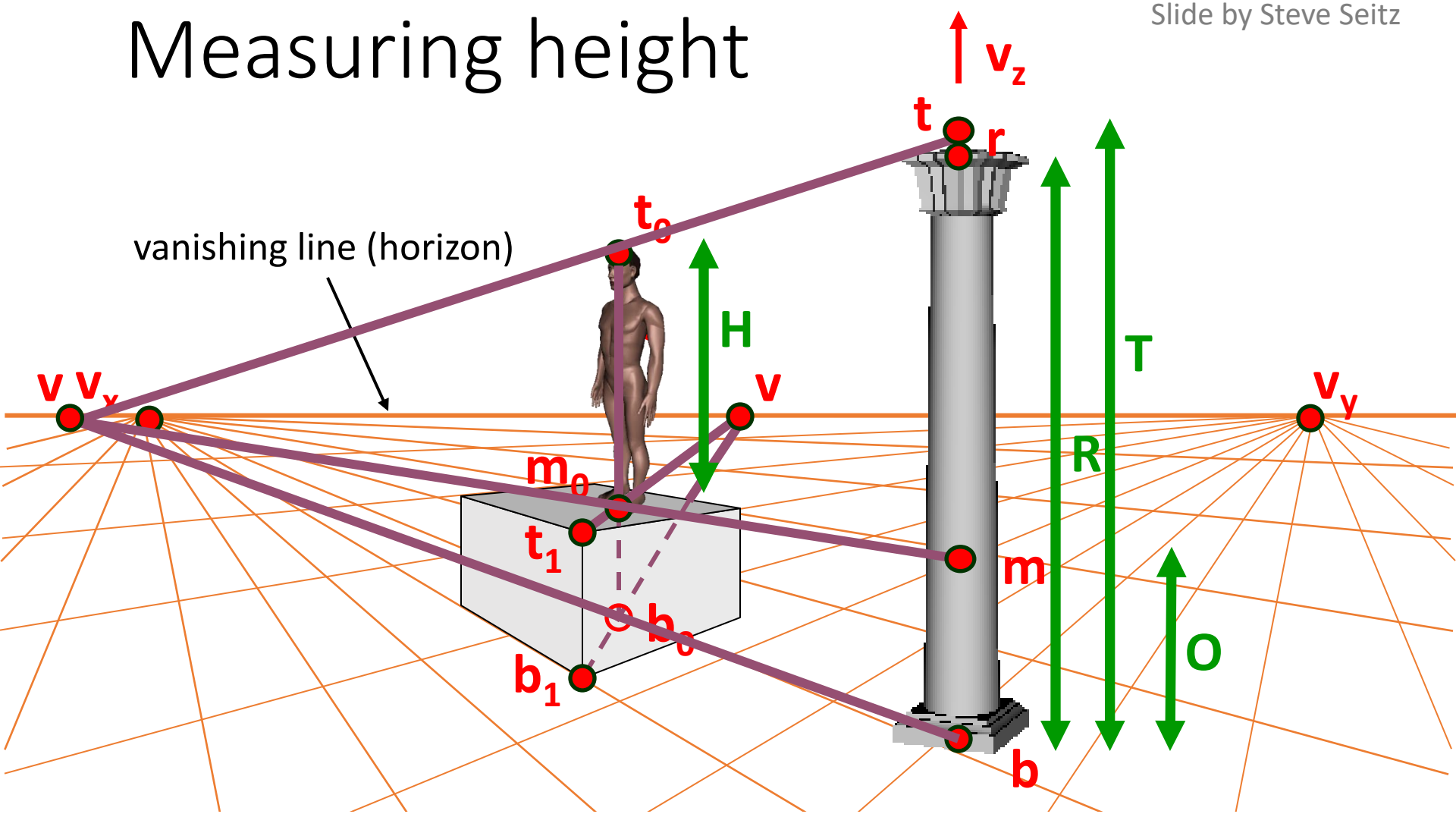

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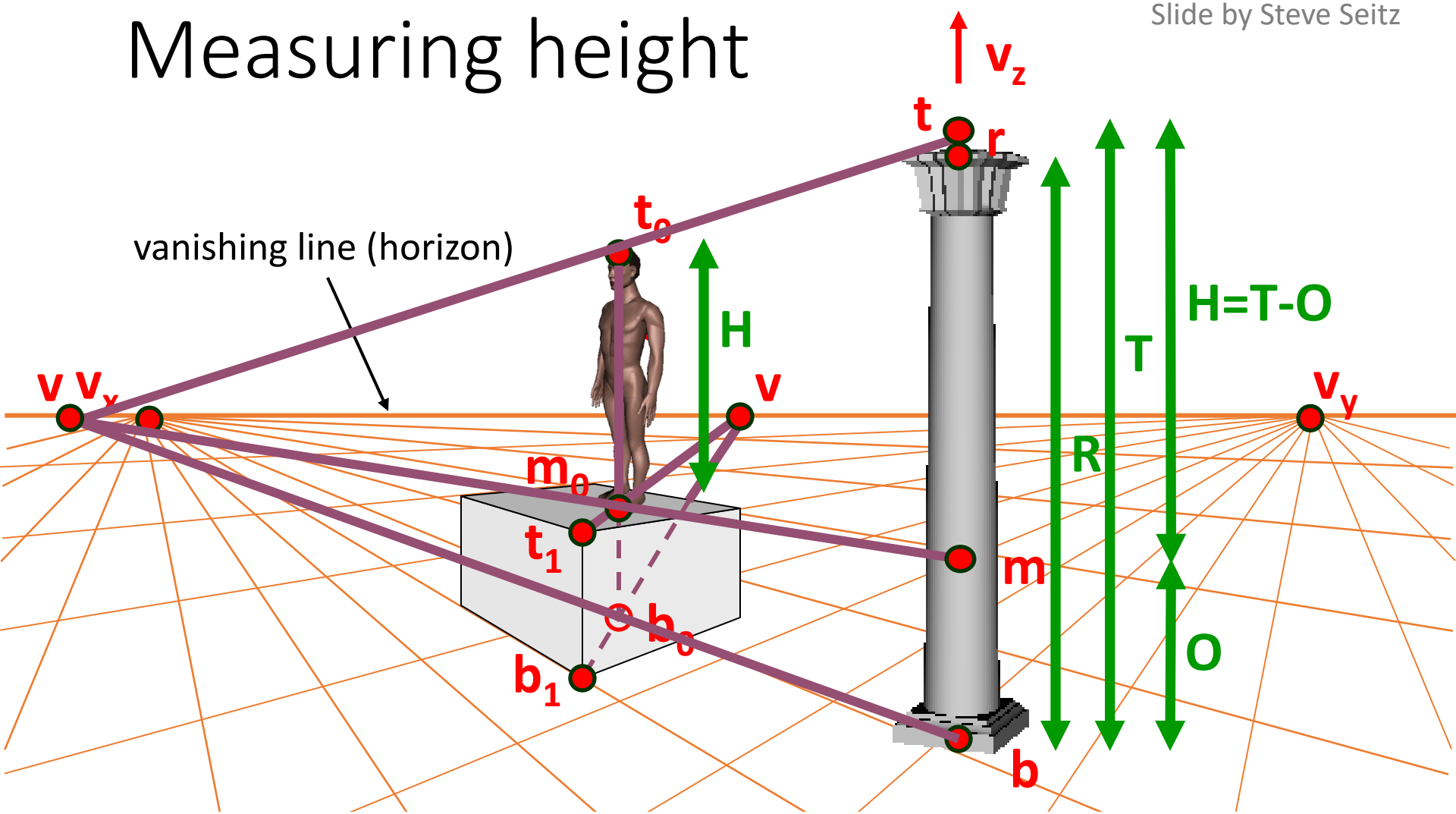

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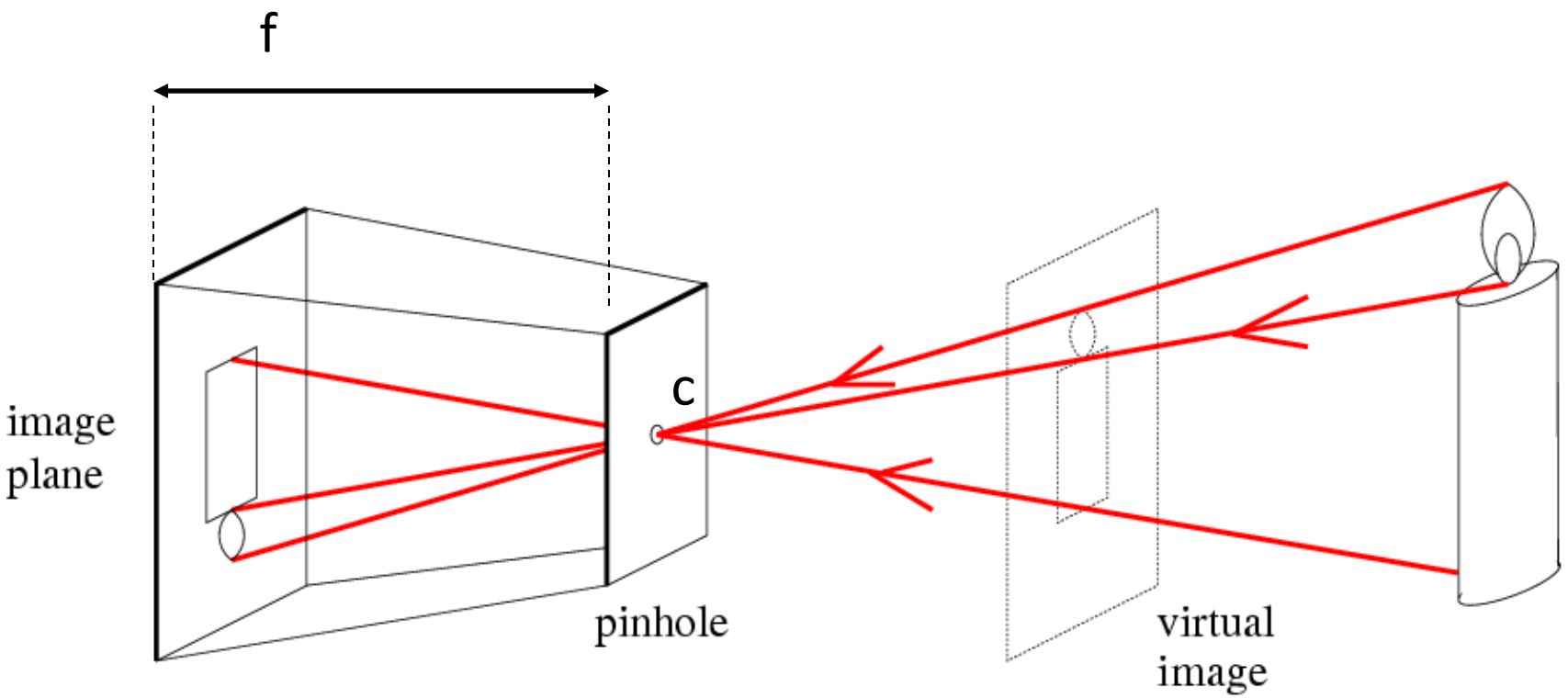
What about focus, aperture, DOF, FOV, etc?

Previous: Pinhole camera

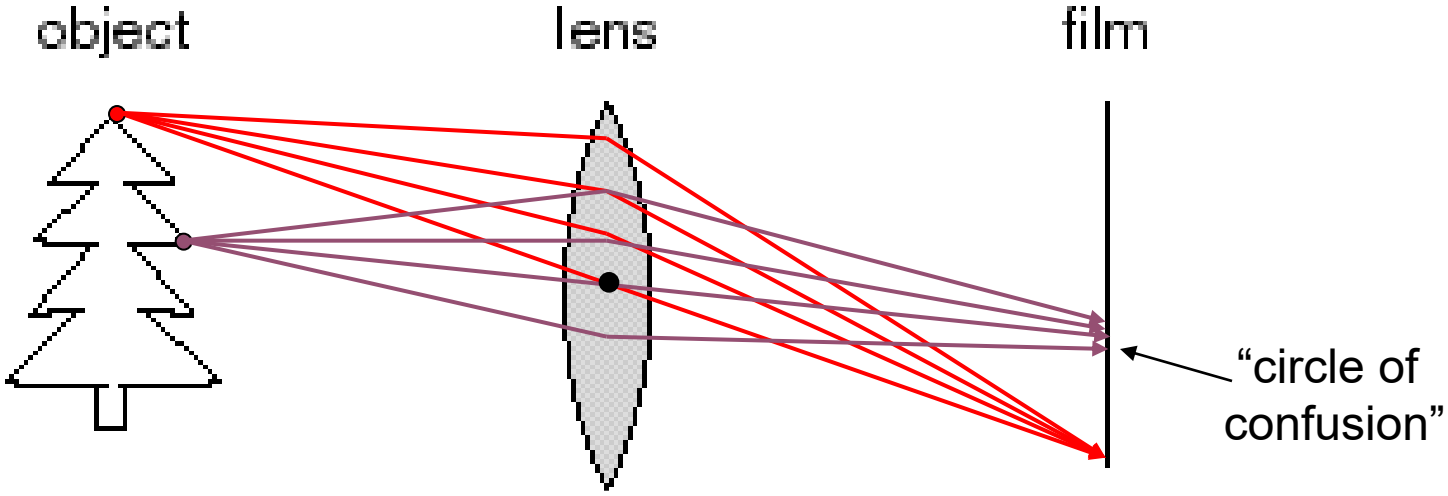


f = focal length

c = center of the camera

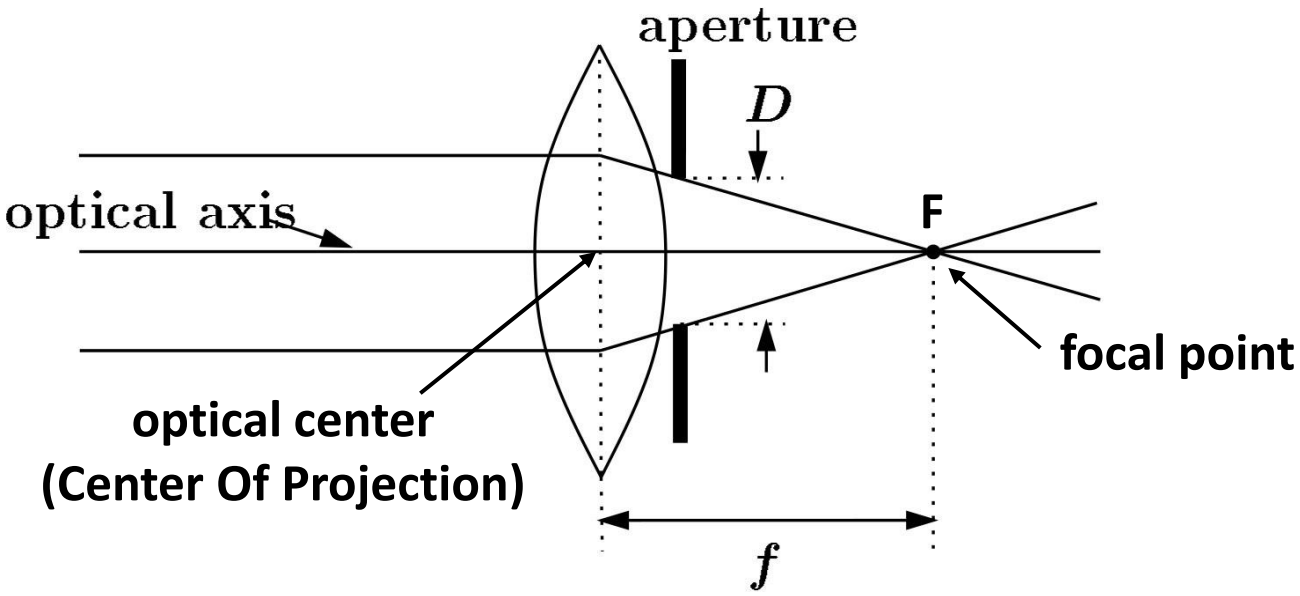
Figure from Forsyth

Adding a lens



- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
  - Other points project to a “circle of confusion” in the image
  - Changing the shape of the lens changes this distance

Lenses

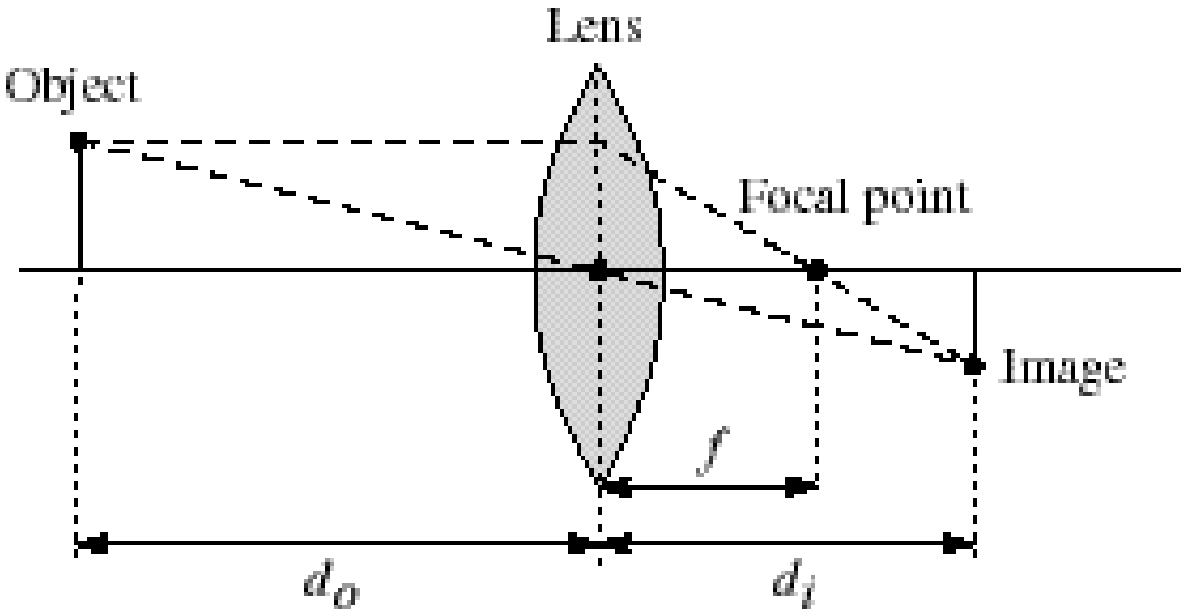


A lens focuses parallel rays onto a single focal point

- Focal point at a distance *f* beyond the plane of the lens
  - *f* is a function of the shape and index of refraction of the lens
- Aperture of diameter D restricts the range of rays
- Real cameras use many lenses together (to correct for abnormalities)

Slide source: Seitz

Thin lenses



- Thin lens equation: $$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$
  - Any object point satisfying this equation is in focus

F-number in Camera

The f-number *N* or f# is given by:

$$N = \frac{f}{D}$$

where ***f*** is the focal length, and ***D*** is the diameter of the entrance pupil (*effective aperture*).

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It is the **reciprocal** of the **relative aperture**.
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It is the **reciprocal** of the **relative aperture**.
The f-number is commonly indicated using the format f/N, where N is the f-number.

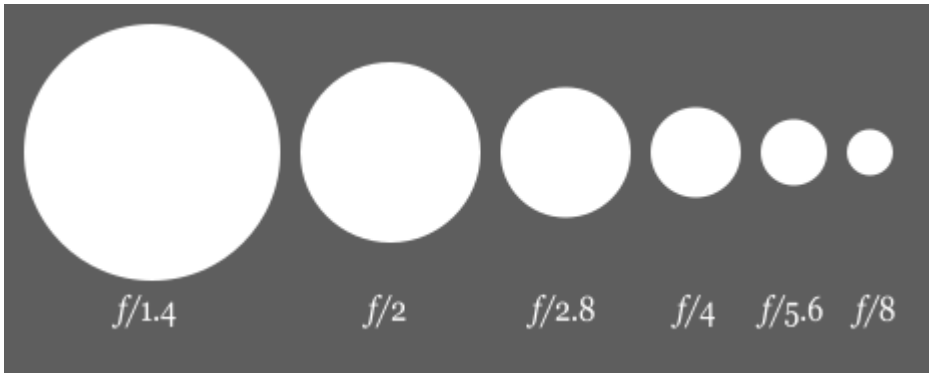
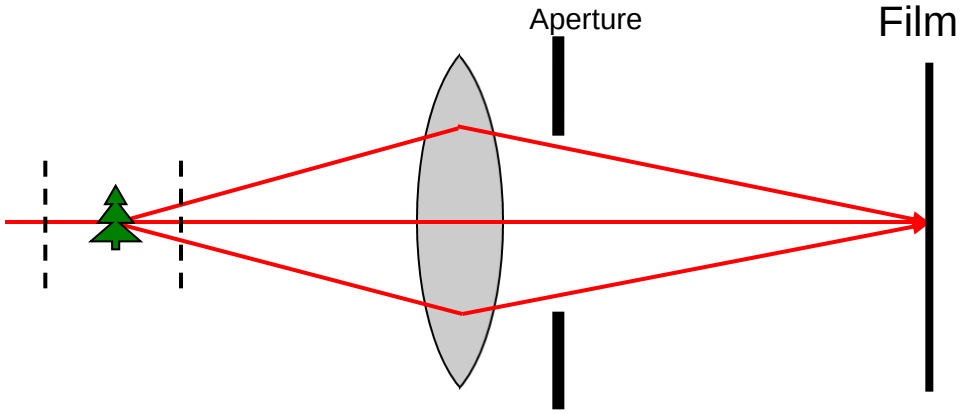


Diagram of decreasing apertures, i.e. increasing f-numbers.
Each aperture has half the light-gathering area of the previous one.

Image from Wikipedia

Depth of field

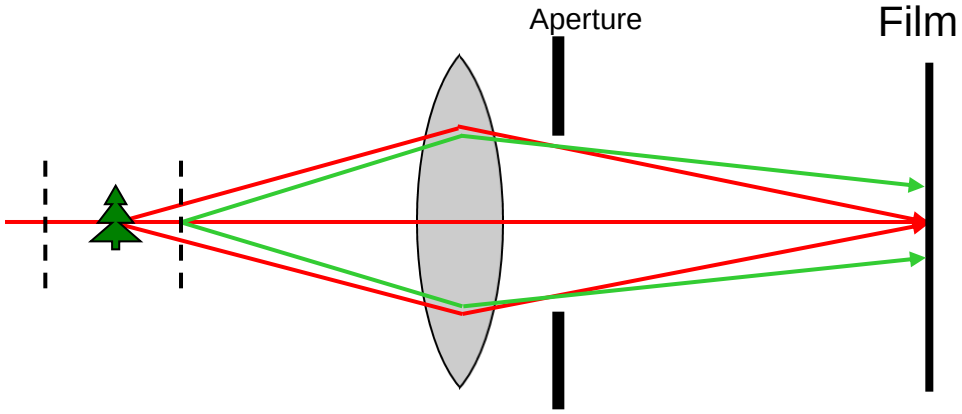


f / 5.6

- Changing the aperture size affects depth of field
  - A smaller aperture increases the range in which the object is approximately in focus

Flower images from Wikipedia http://en.wikipedia.org/wiki/Depth_of_field

Depth of field



f / 5.6

- Changing the aperture size affects depth of field
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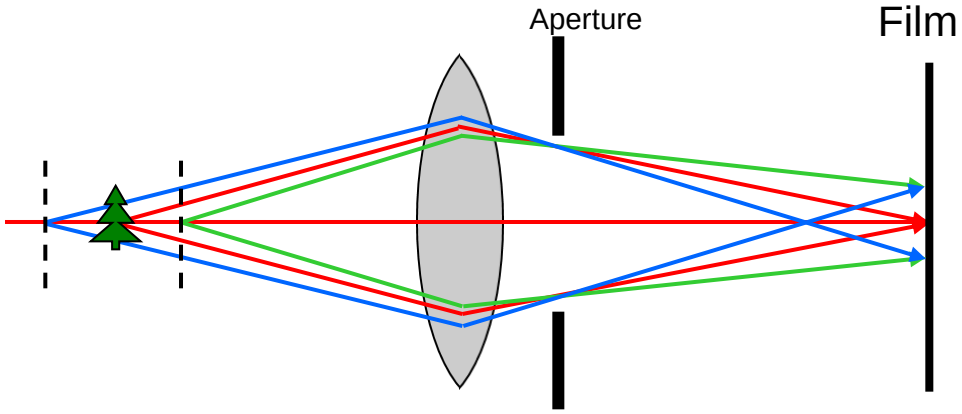
Flower images from Wikipedia http://en.wikipedia.org/wiki/Depth_of_field

Depth of field

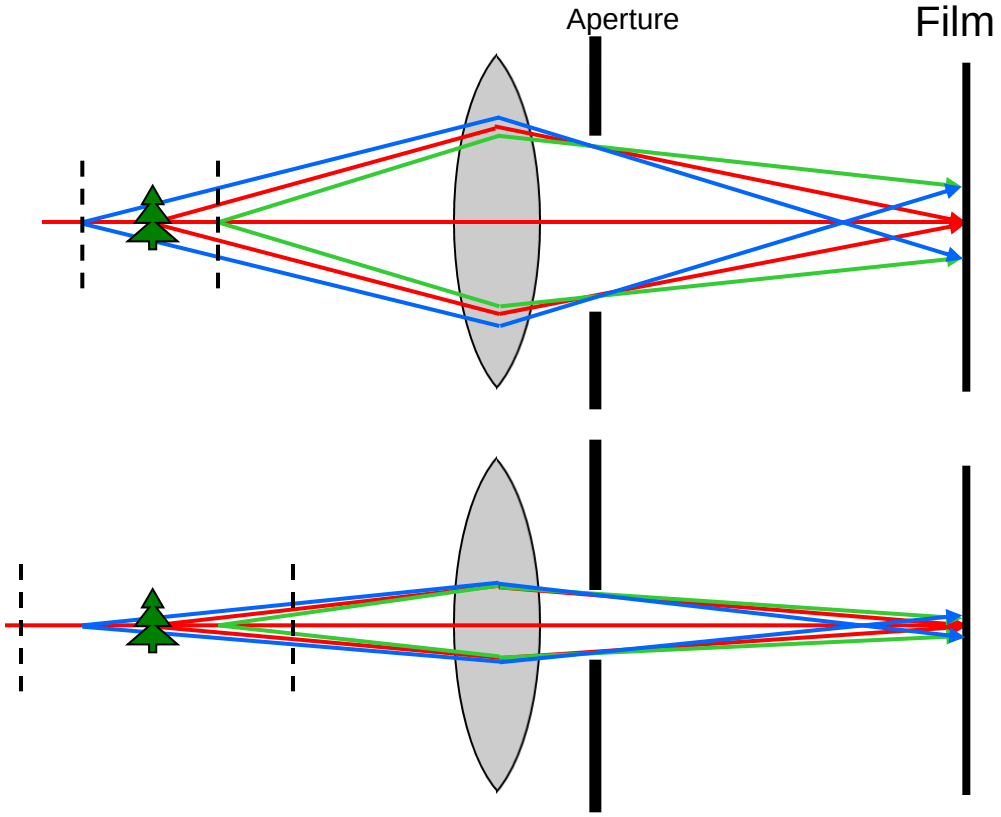


f / 5.6

- Changing the aperture size affects depth of field
  - A smaller aperture increases the range in which the object is approximately in focus

Flower images from Wikipedia http://en.wikipedia.org/wiki/Depth_of_field

Depth of field



f / 5.6

f / 32

- Changing the aperture size affects depth of field
  - A smaller aperture increases the range in which the object is approximately in focus

Flower images from Wikipedia http://en.wikipedia.org/wiki/Depth_of_field

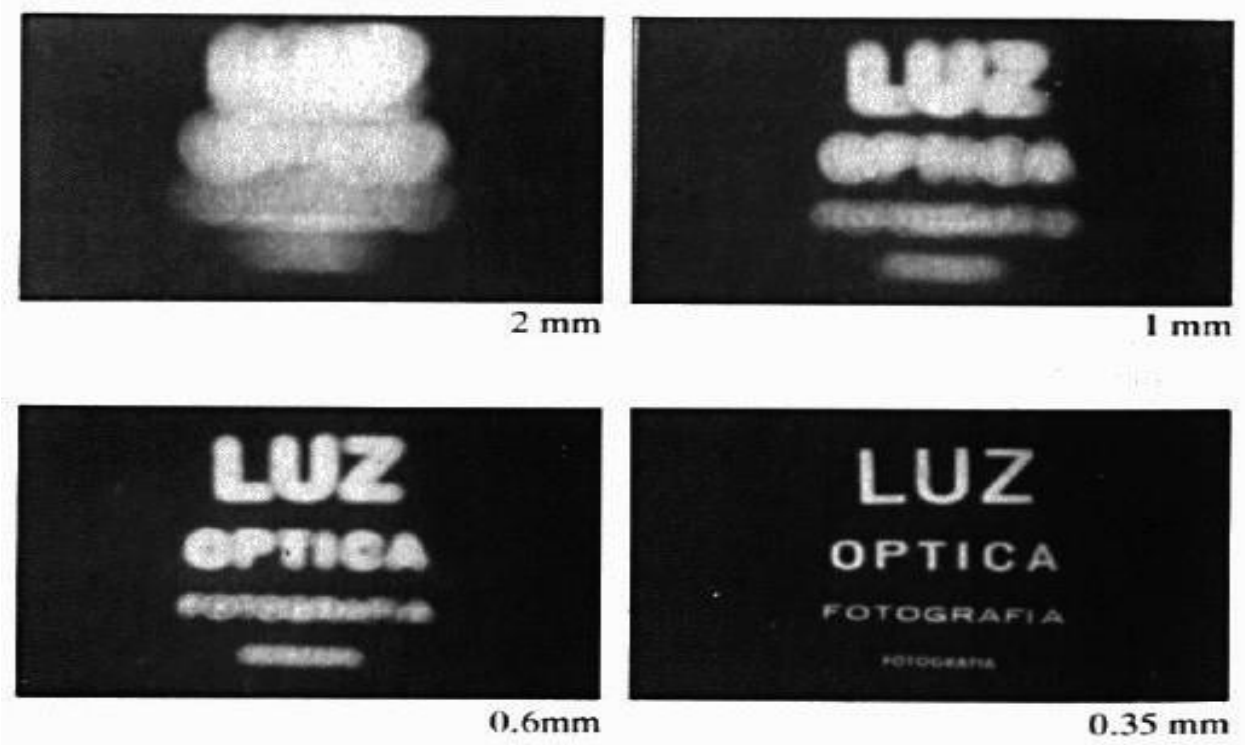
Slide from Efros

Varying the aperture

Large aperture = small DOF

Small aperture = large DOF

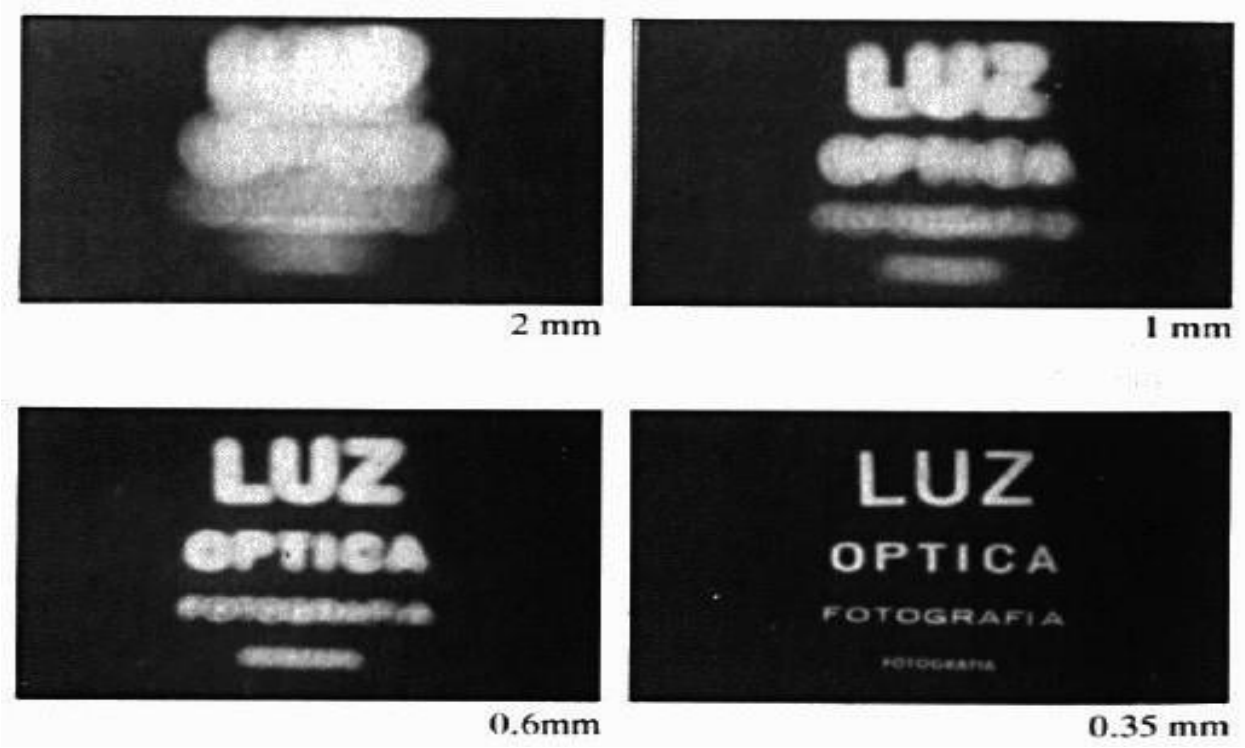
Shrinking the aperture



- Why not make the aperture as small as possible?

Slide by Steve Seitz

Shrinking the aperture



- Why not make the aperture as small as possible?
  - Less light gets through
  - Diffraction effects

Slide by Steve Seitz

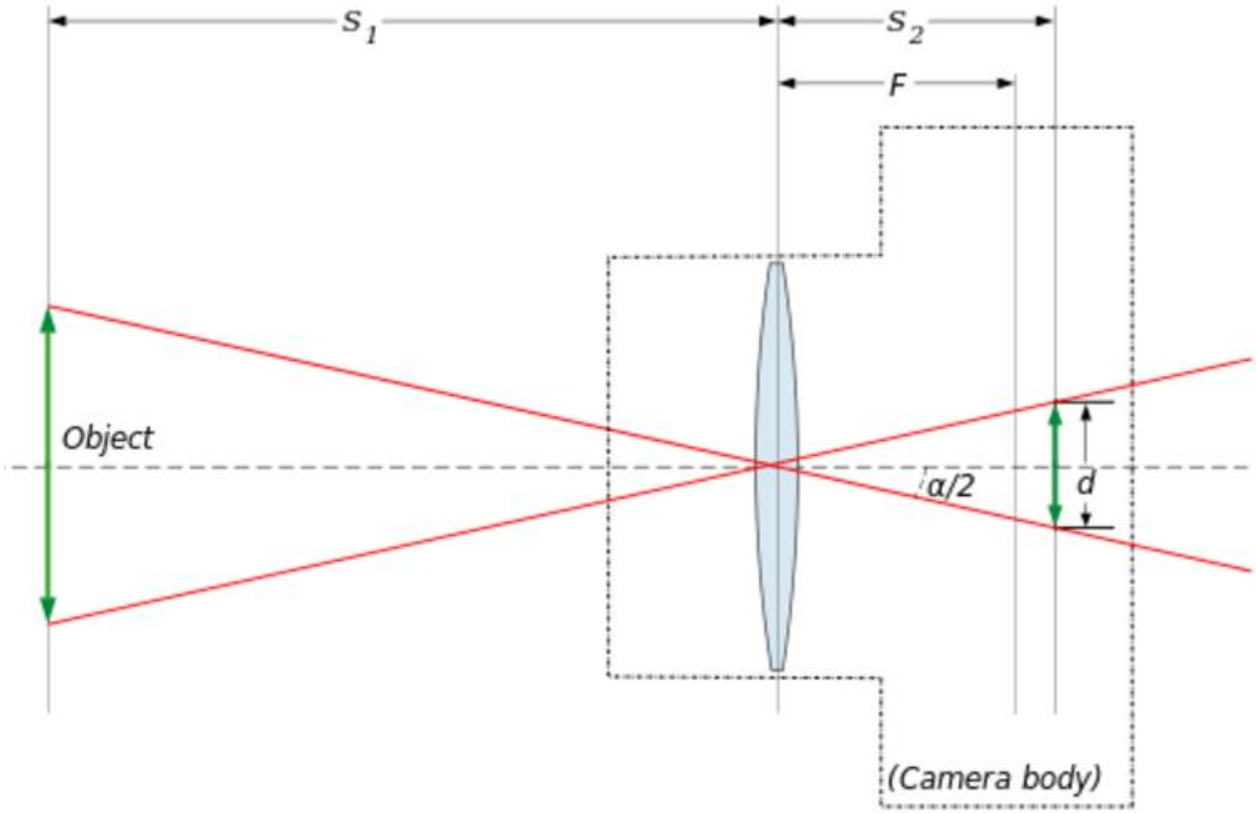
Shrinking the aperture

Slide by Steve Seitz

Field of View

FOV depends on focal length and size of the camera retina

$$\alpha = 2 \arctan \frac{d}{2S_2}$$

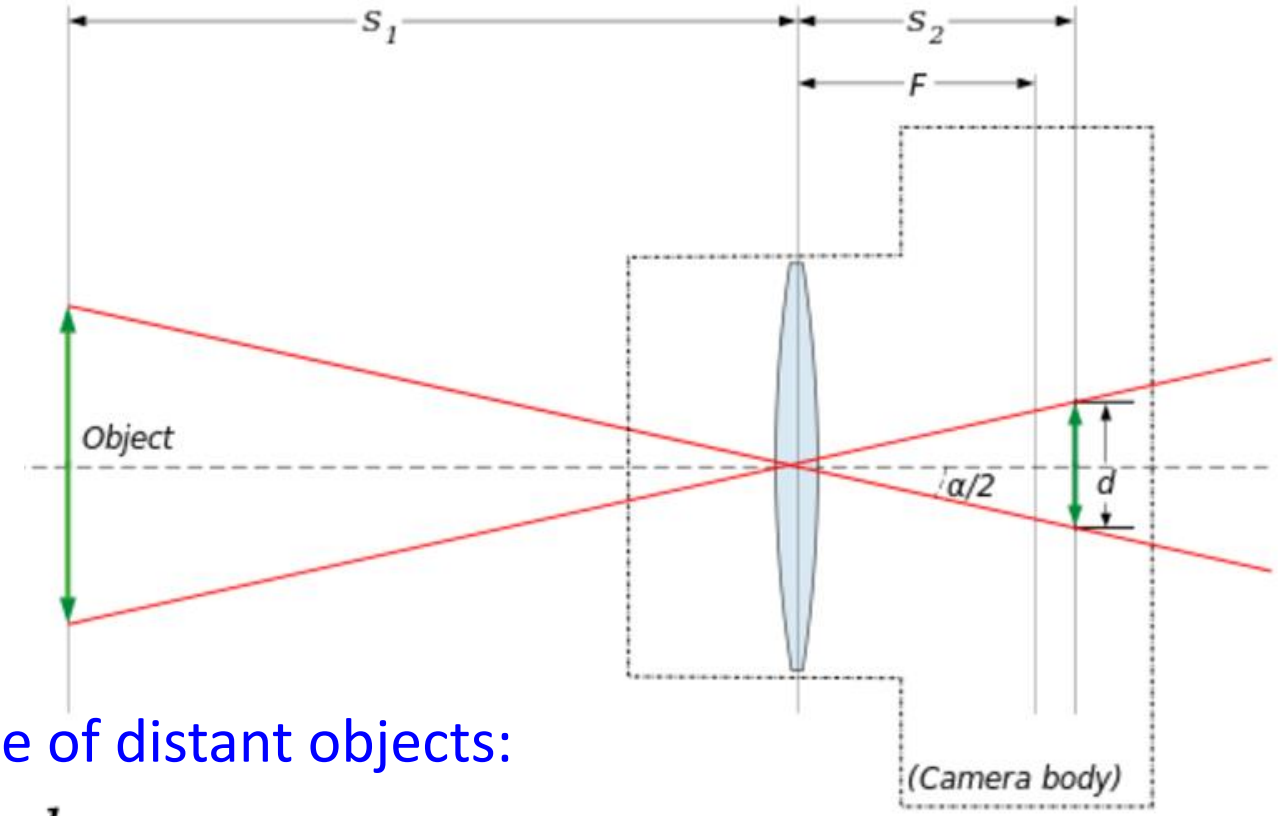


Source - https://en.wikipedia.org/wiki/Angle_of_view

Field of View

FOV depends on focal length and size of the camera retina

$$\alpha = 2 \arctan \frac{d}{2S_2}$$



To project a sharp image of distant objects:

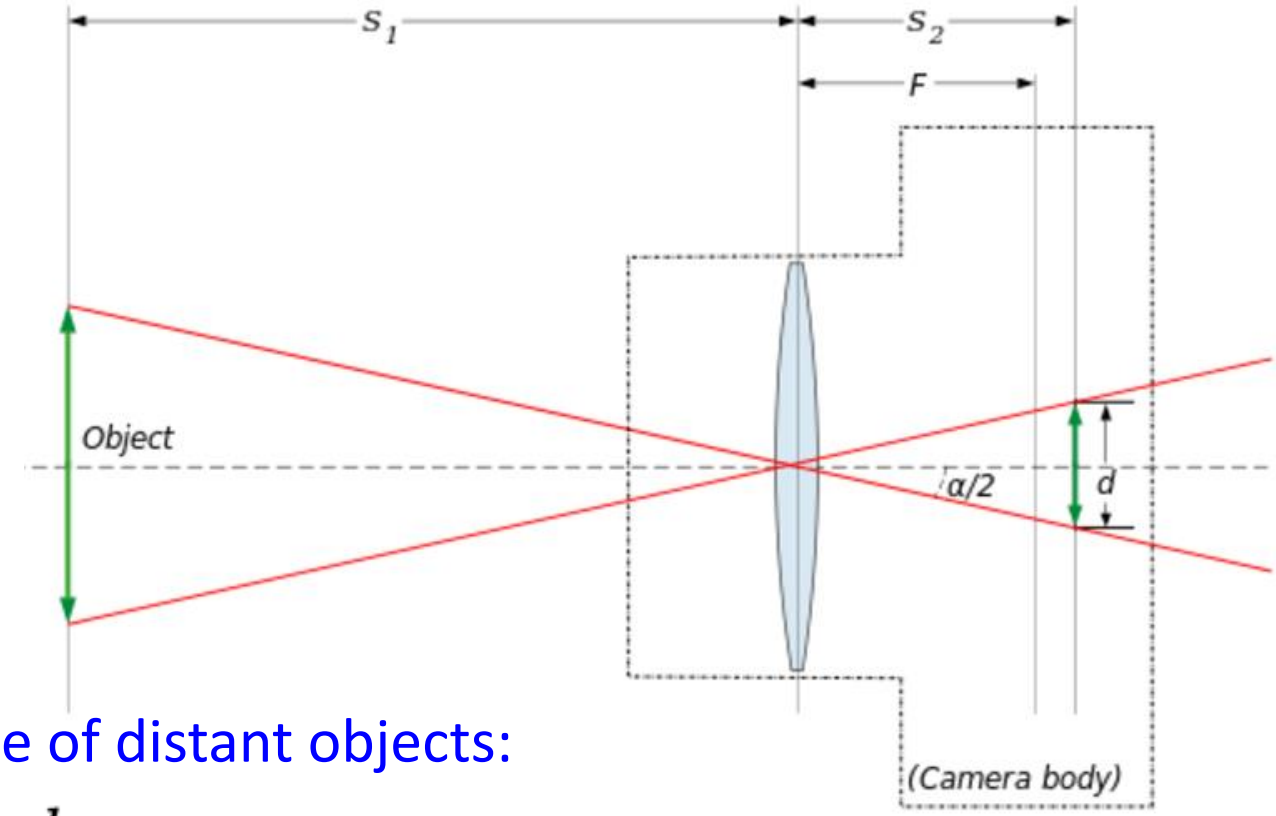
$$\alpha = 2 \arctan \frac{d}{2f} \text{ where } f = F$$

Source - https://en.wikipedia.org/wiki/Angle_of_view

Field of View

FOV depends on focal length and size of the camera retina

$$\alpha = 2 \arctan \frac{d}{2S_2}$$



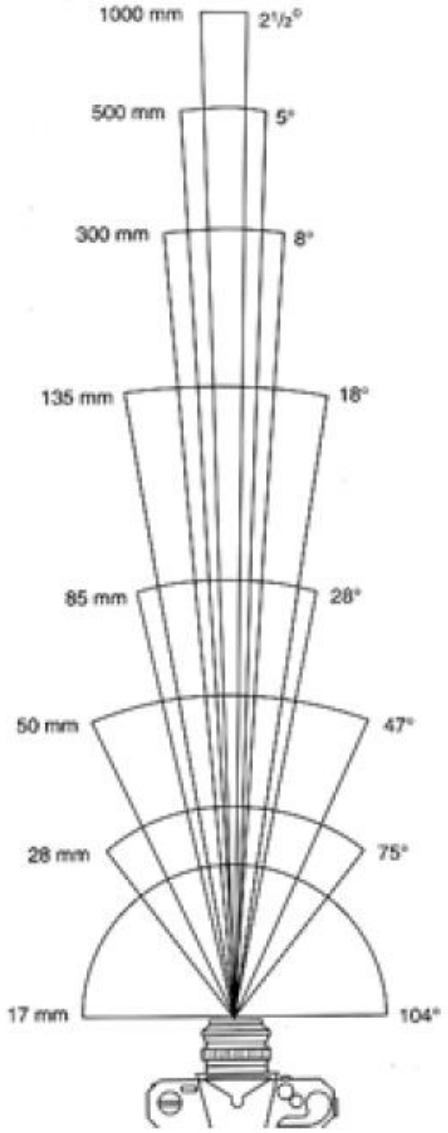
To project a sharp image of distant objects:

$$\alpha = 2 \arctan \frac{d}{2f} \text{ where } f = F$$

Larger focal length = smaller FOV

Source - https://en.wikipedia.org/wiki/Angle_of_view

Field of View



17mm

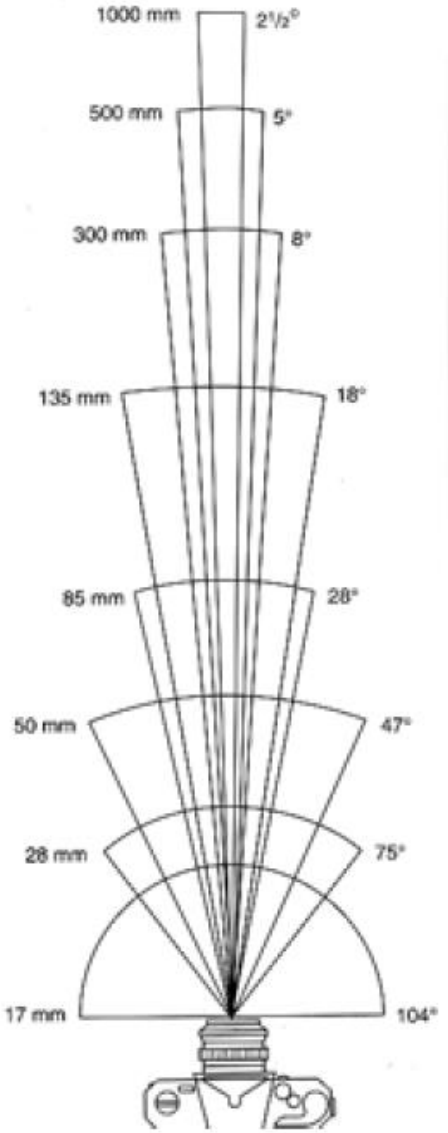
28mm

50mm

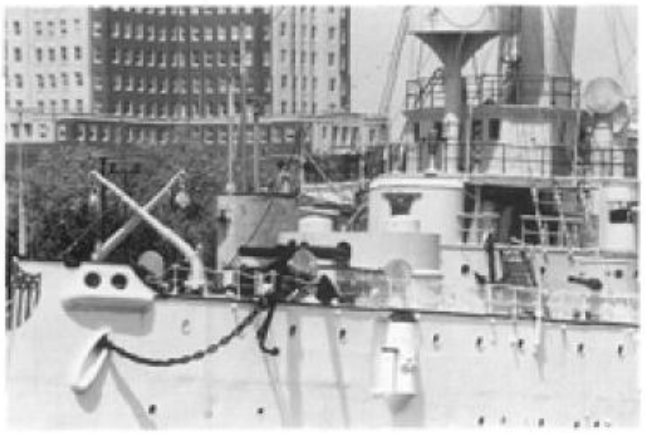
85mm

Slide by A. Efros

Field of View



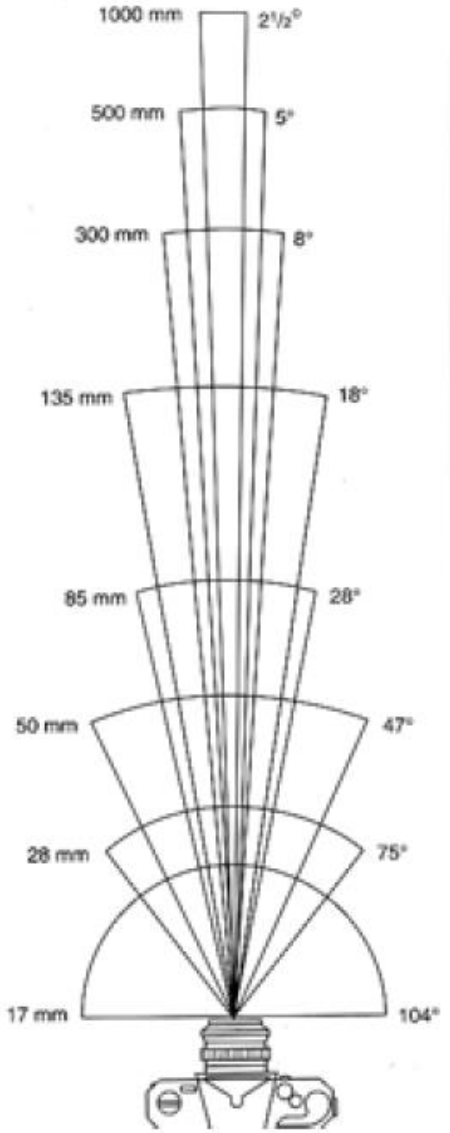
135mm



300mm

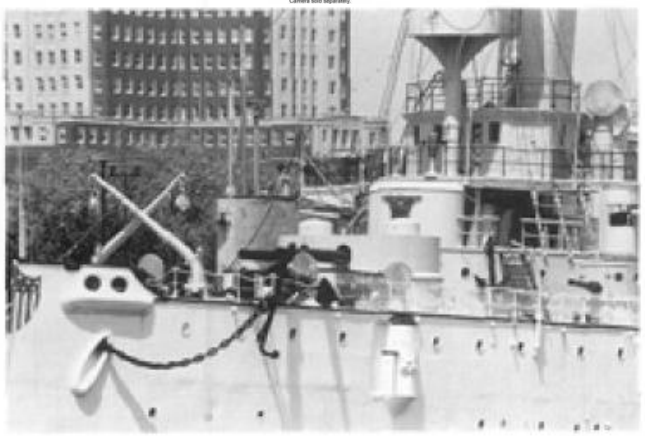
Slide by A. Efros

Field of View



Tripod is must to see

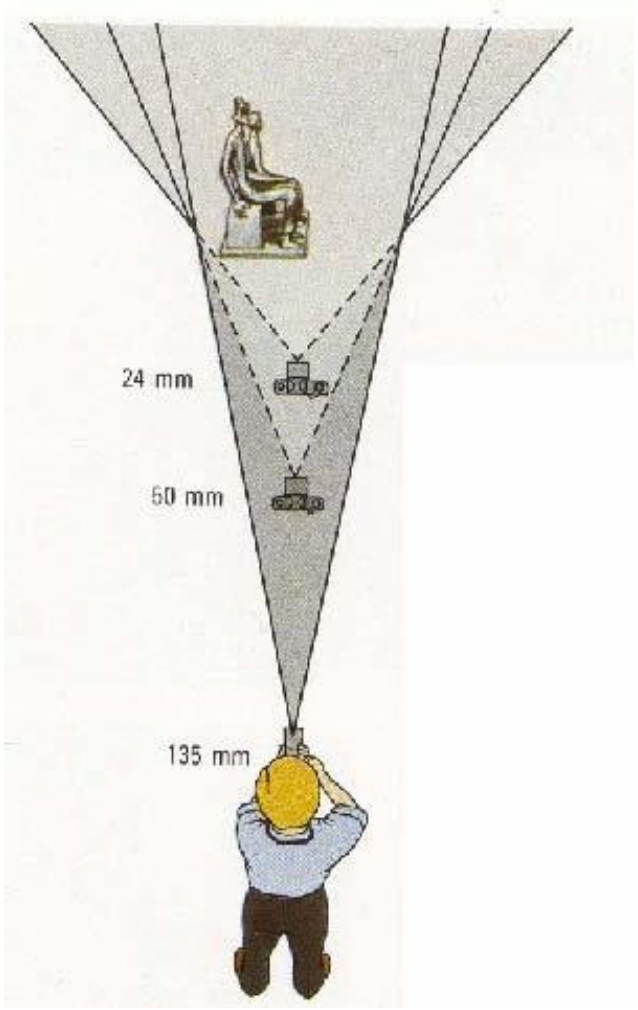
135mm



300mm

Slide by A. Efros

Field of View / Focal Length



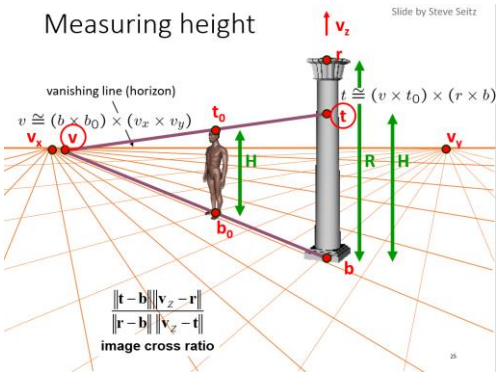
Large FOV, small *f*
Camera close to car

Small FOV, large *f*
Camera far from the car

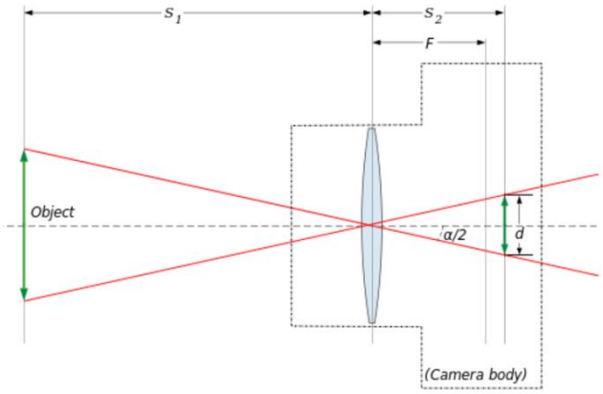
Sources: A. Efros, F. Durand

Things to remember

- Measure the size of objects in the world from an image?
  - Use perspective cues



- Camera properties
  - focal length,
  - field of view,
  - depth of field,
  - aperture,
  - F-number



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  - K. Grauman
  - Antonio Torralba
  - Rob Fergus
  - Leibe
  - And many more …….

Next class

- Image stitching