Computer Vision

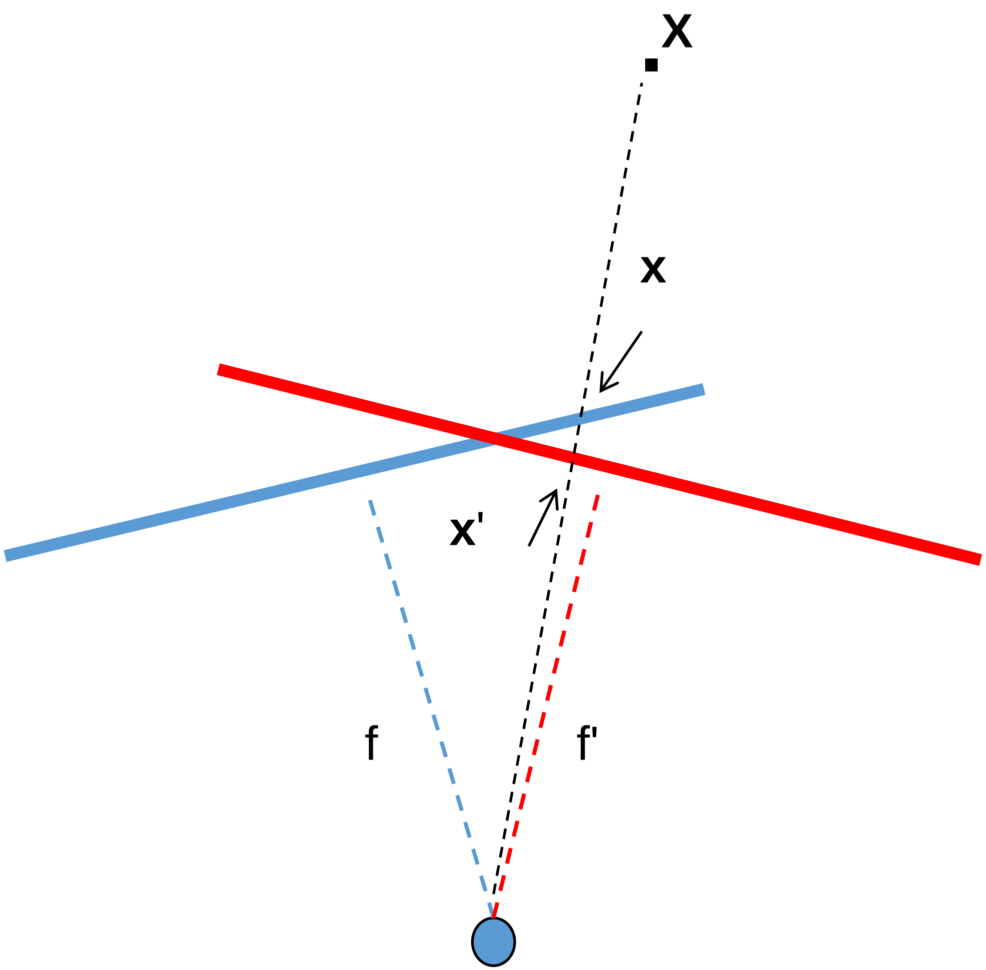
Epipolar Geometry and Stereo Vision

Dr. Mrinmoy Ghorai

Indian Institute of Information Technology
Sri City, Chittoor

Last class: Image Stitching

- Two images with rotation/zoom but no translation



Perspective and 3D Geometry

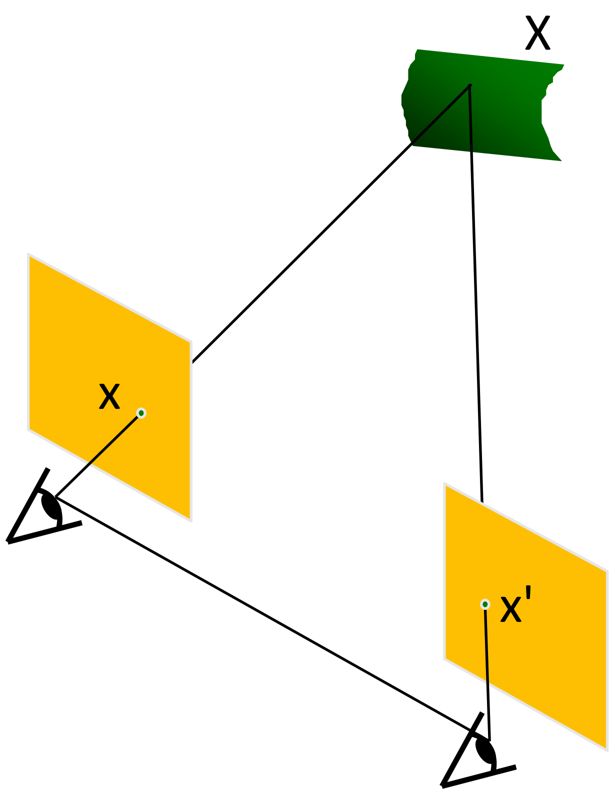
- **Camera models and Projective geometry**
  - What's the mapping between image and world coordiantes?
- **Projection Matrix and Camera calibration**
  - What's the projection matrix between scene and image coordiantes?
  - How to calibrate the projection matrix?
- **Single view metrology and Camera properties**
  - How can we measure the size of 3D objects in an image?
  - What are the important camera properties?
- **Photo stitching**
  - What's the mapping from two images taken without camera translation?
- **Epipolar Geometry and Stereo Vision**
  - What's the mapping from two images taken with camera translation?
- **Structure from motion**
  - How can we recover 3D points from multiple images?

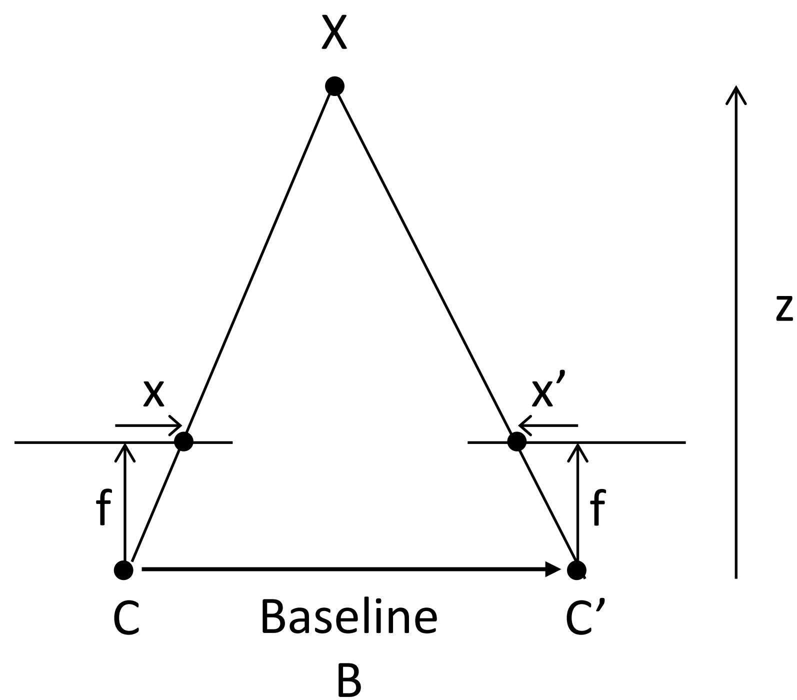
This class: Two-View Geometry

- Epipolar geometry
  - Relates cameras from two positions
- Stereo depth estimation
  - Recover depth from two images

Depth from Stereo

- Goal: Recover depth by finding image coordinate x’ that corresponds to x

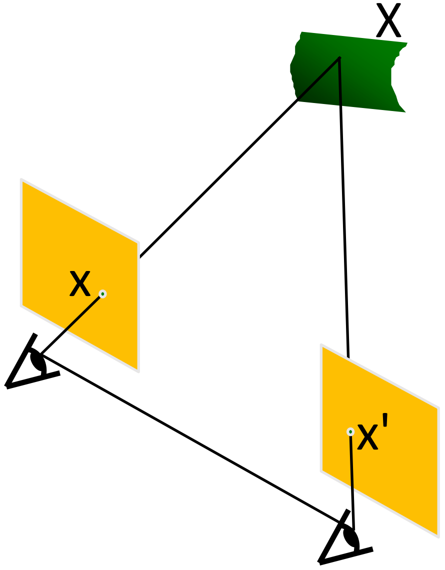




Depth from Stereo

- Goal: Recover depth by finding image coordinate x’ that corresponds to x

- Sub-Problems
  1. **Calibration:**
     What’s the relation of the two cameras?
  2. **Correspondence:**
     Where is the matching point x’?



Correspondence Problem

- Two images taken from cameras with different intrinsic and extrinsic parameters

- How do we match a point in the first image to a point in the second?

- How can we constrain our search?

Key idea: Epipolar constraint

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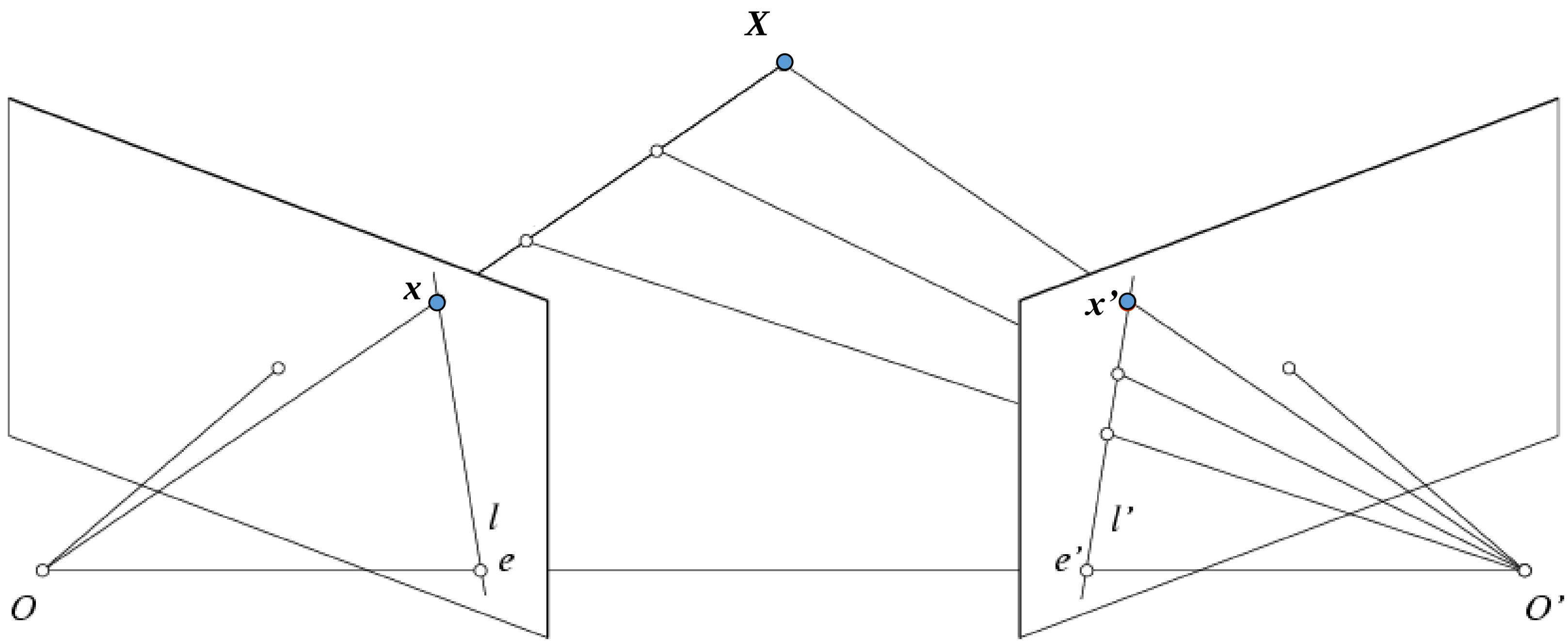
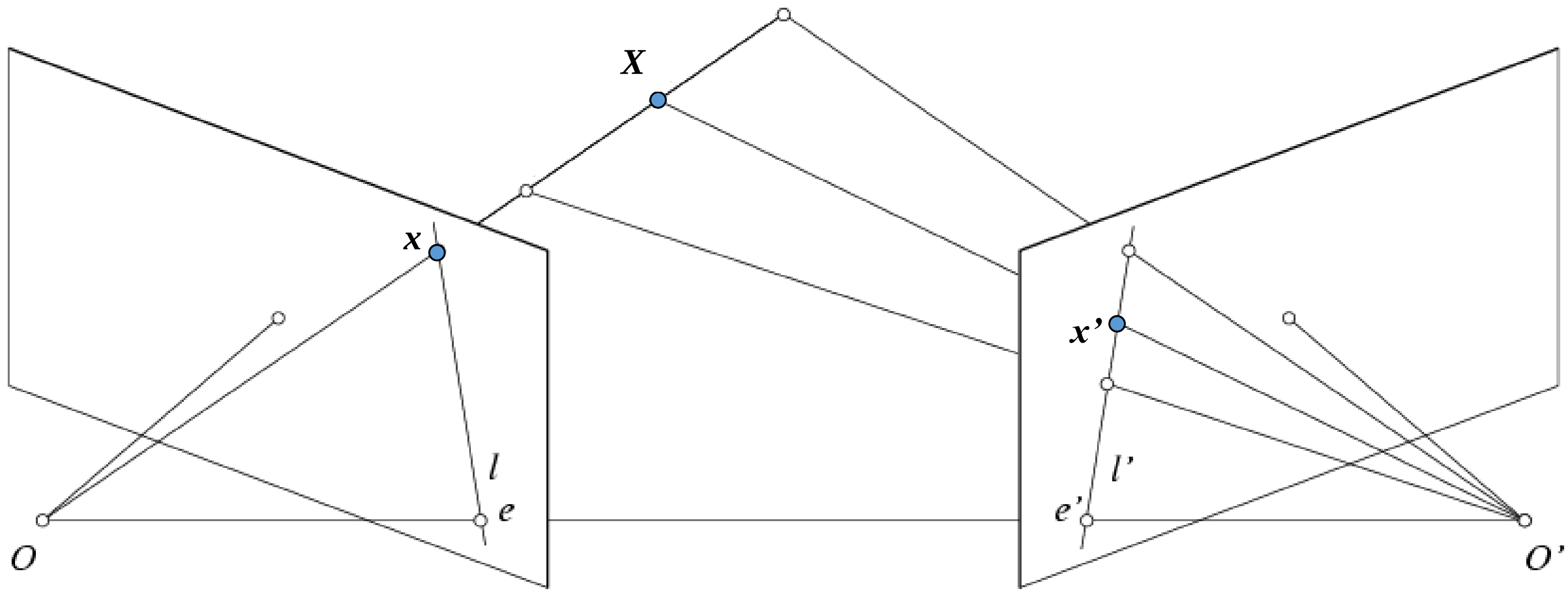
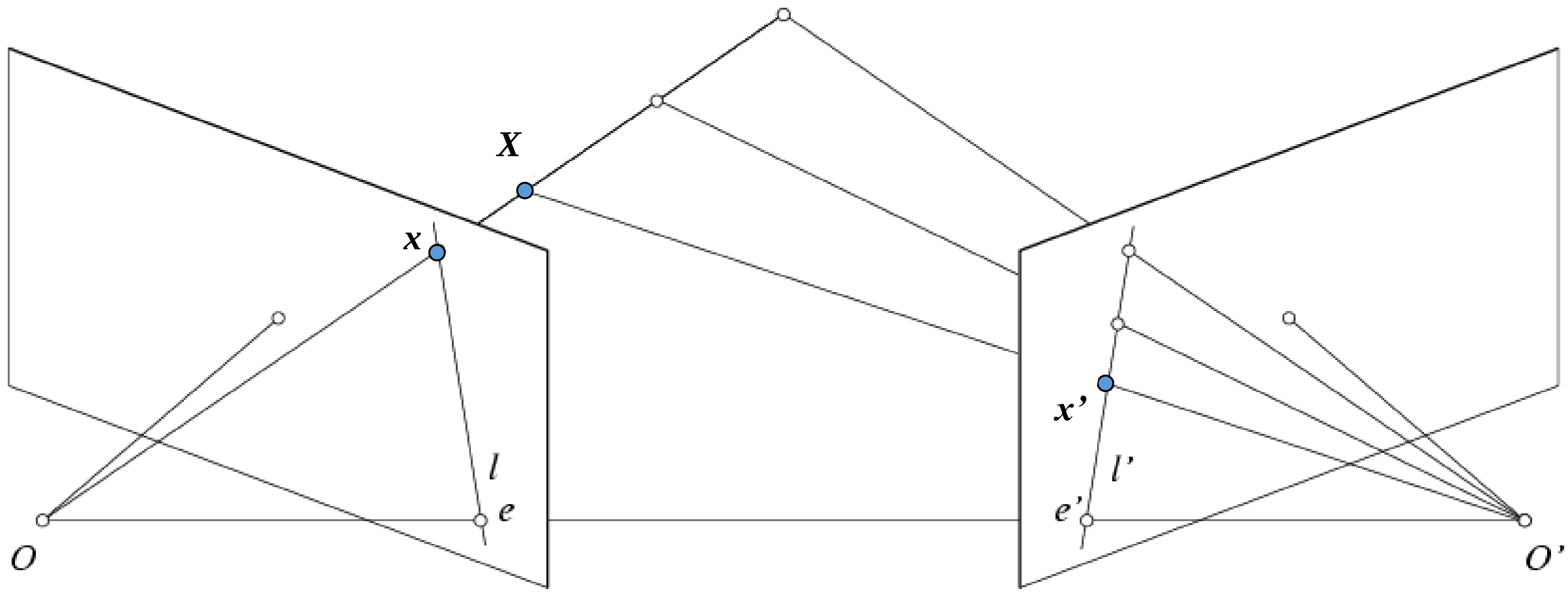


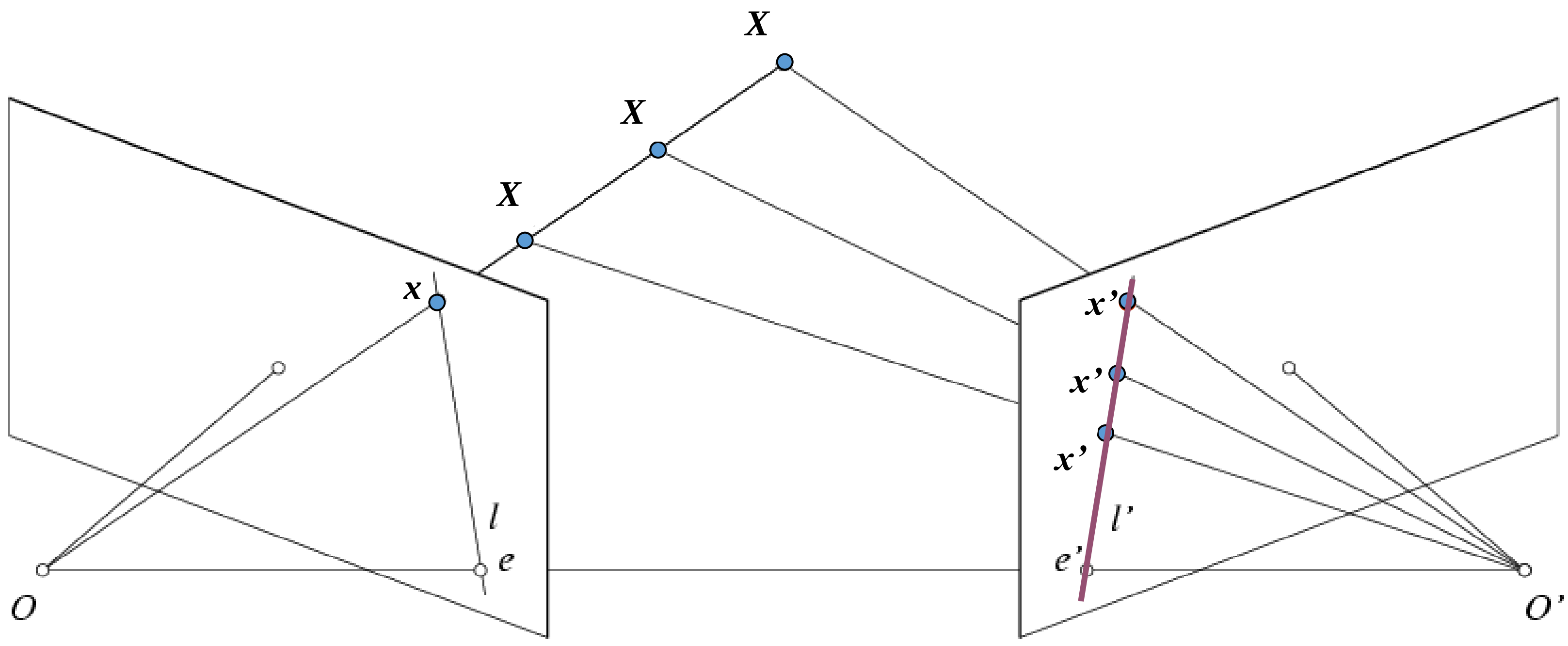
Key idea: Epipolar constraint



Key idea: Epipolar constraint

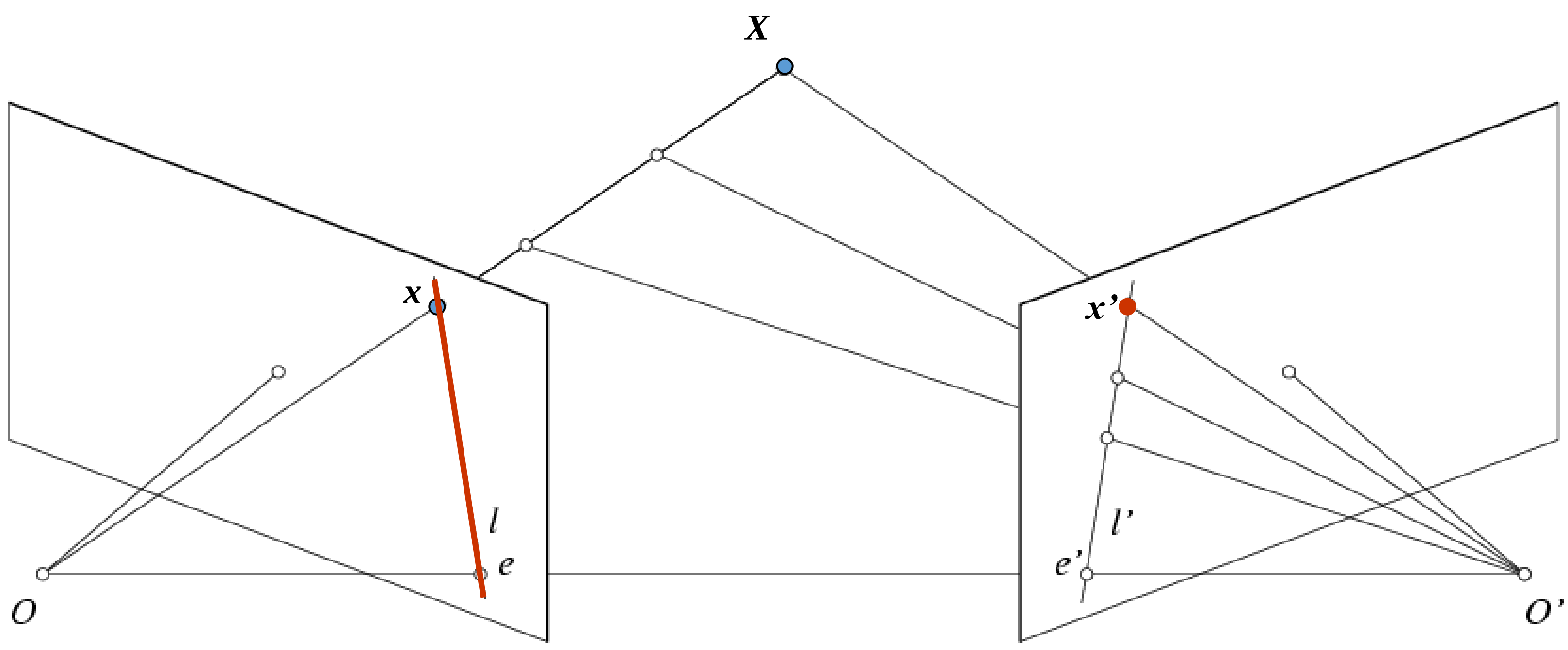


Key idea: Epipolar constraint



Potential matches for *x* have to lie on the corresponding line *l'*.

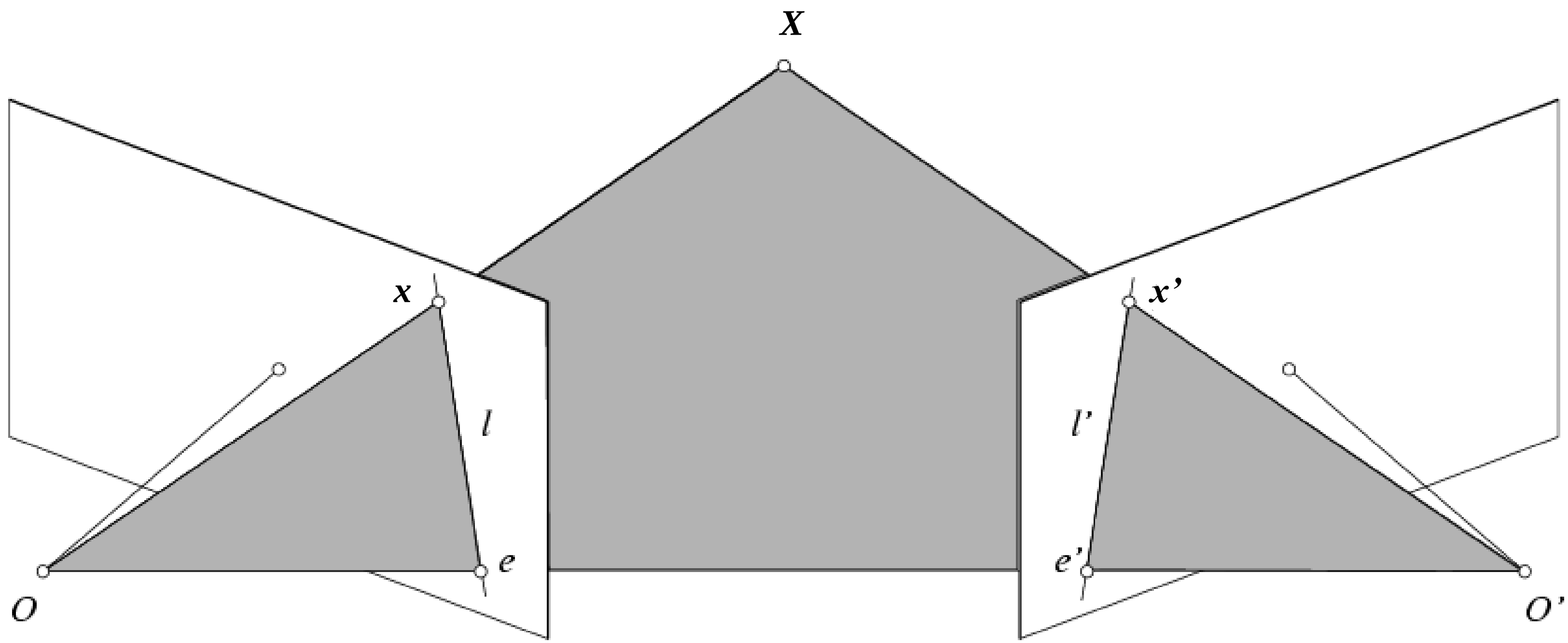
Key idea: Epipolar constraint



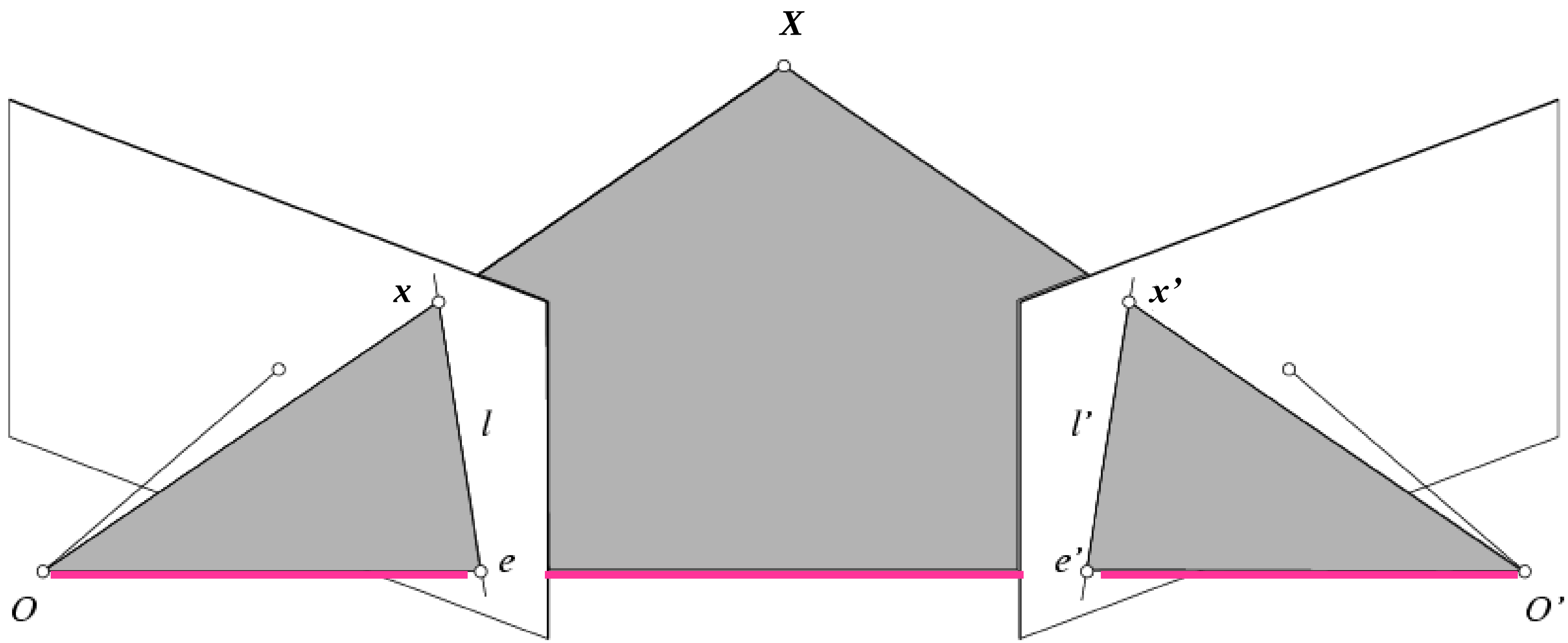
Potential matches for *x* have to lie on the corresponding line *l'*.

Potential matches for *x'* have to lie on the corresponding line *l*.

Epipolar geometry: notation

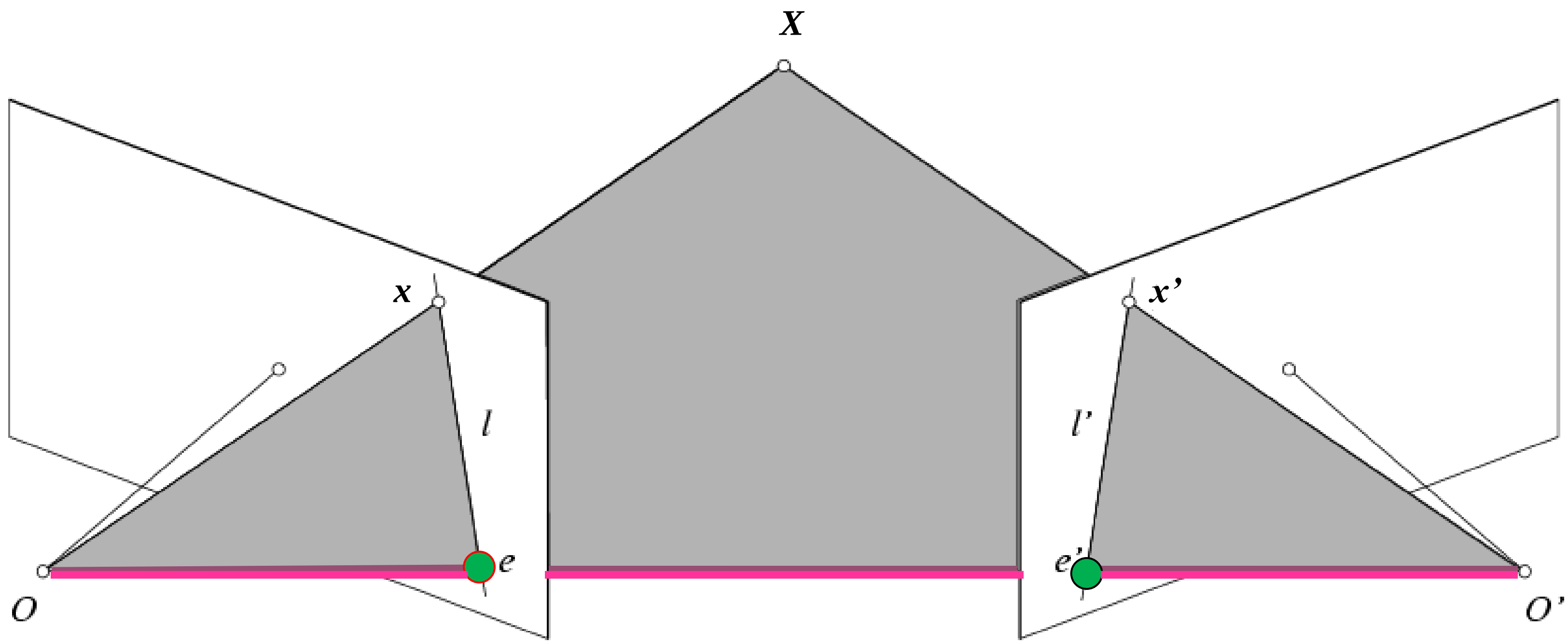


Epipolar geometry: notation



- **Baseline** – line connecting the two camera centers

Epipolar geometry: notation

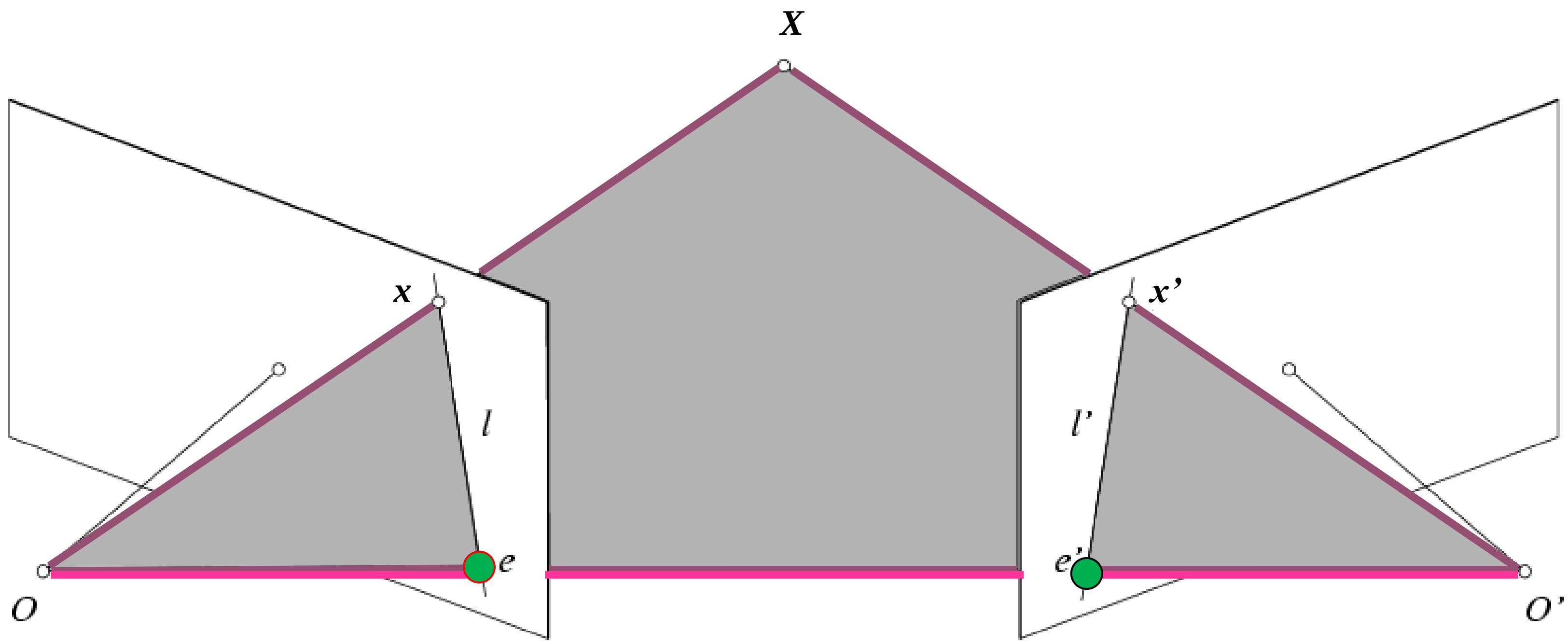


- **Baseline** – line connecting the two camera centers
- **Epipoles**

= intersections of baseline with image planes

= projections of the other camera center

Epipolar geometry: notation



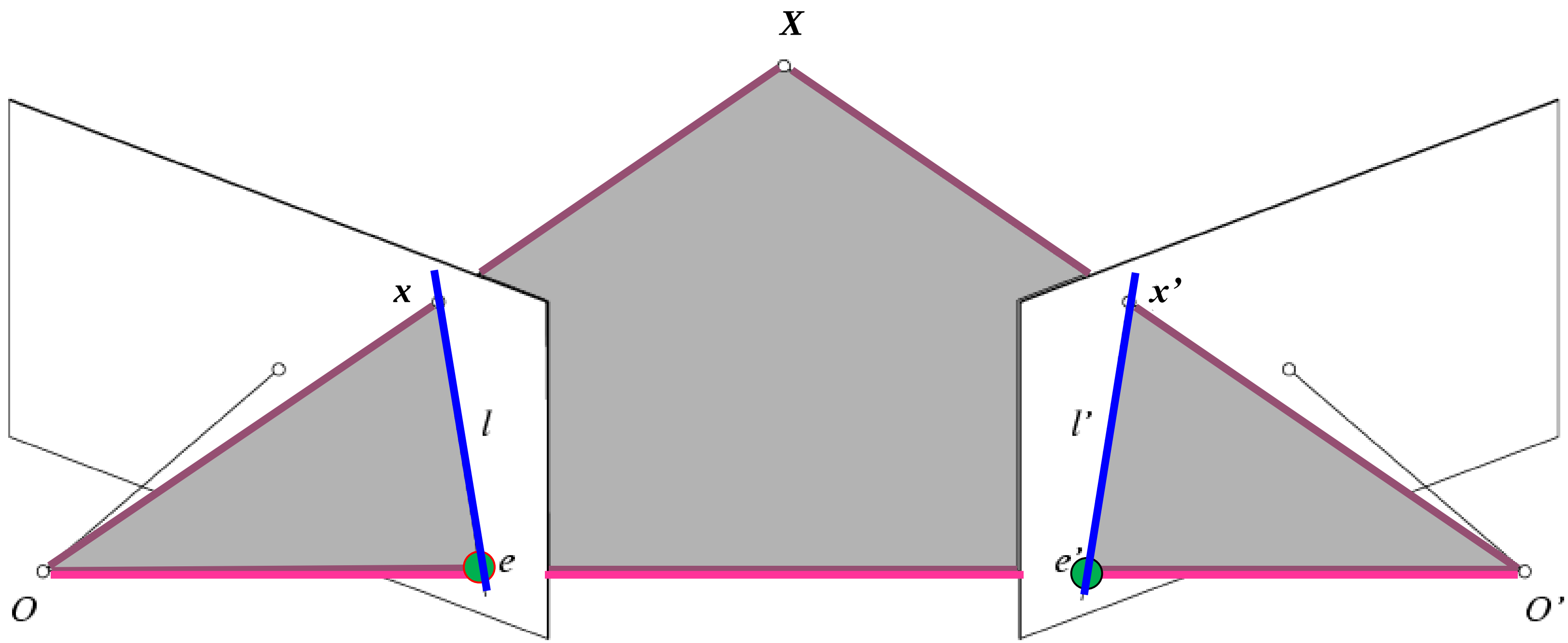
- **Baseline** – line connecting the two camera centers
- **Epipoles**

= intersections of baseline with image planes

= projections of the other camera center

- **Epipolar Plane** – plane containing baseline and 3d point

Epipolar geometry: notation



- **Baseline** – line connecting the two camera centers

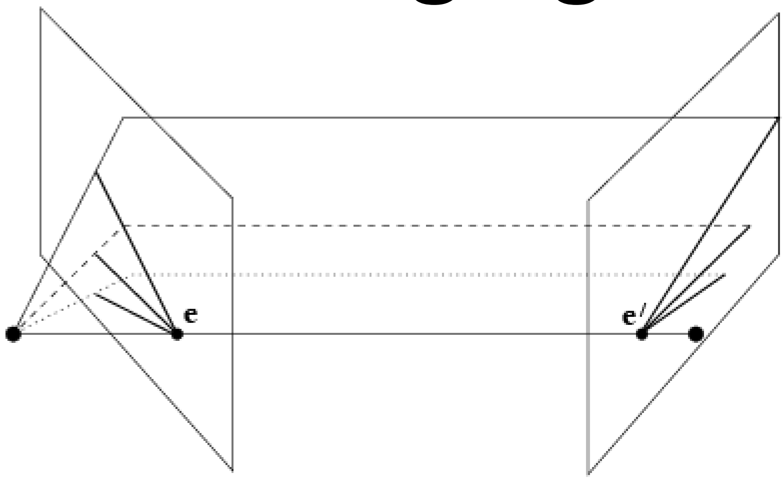
- **Epipoles**

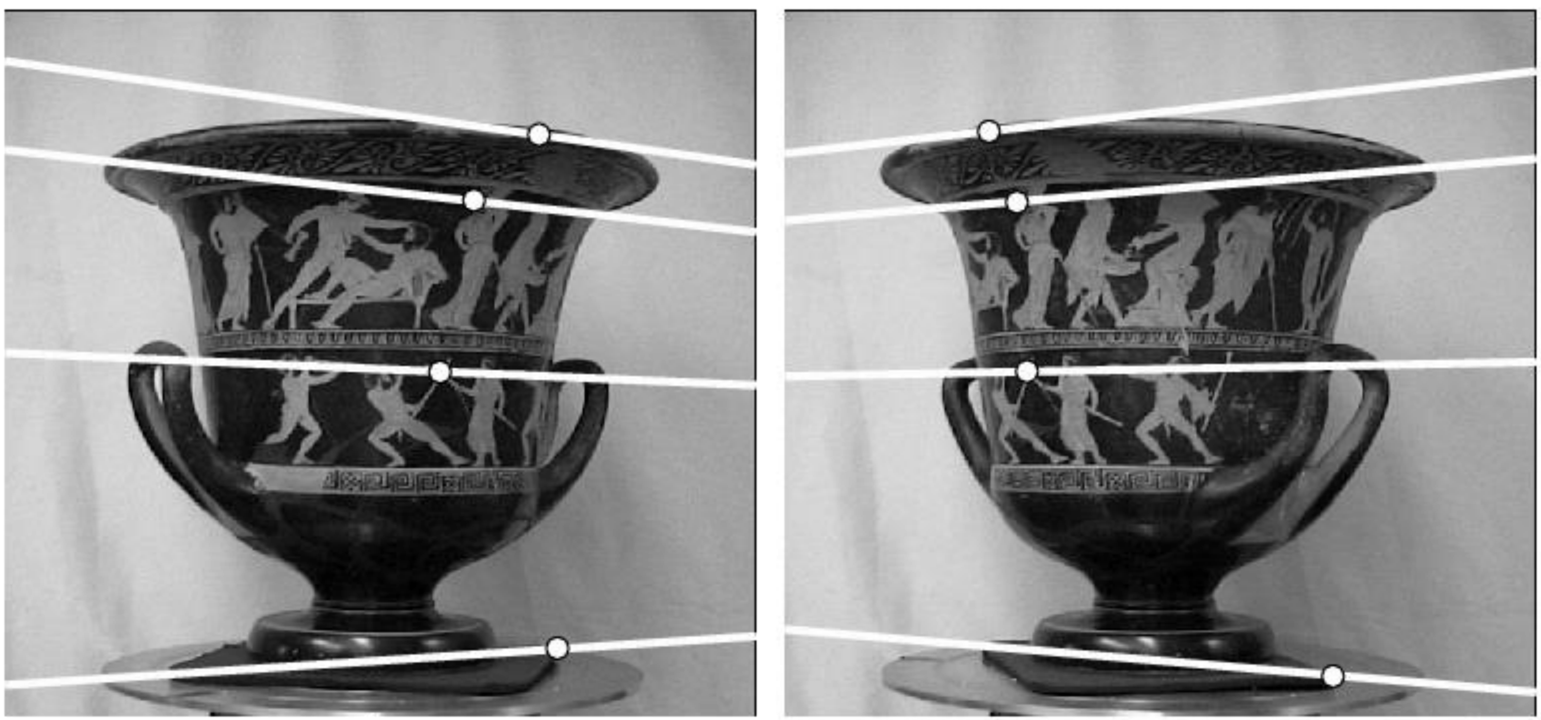
= intersections of baseline with image planes

= projections of the other camera center

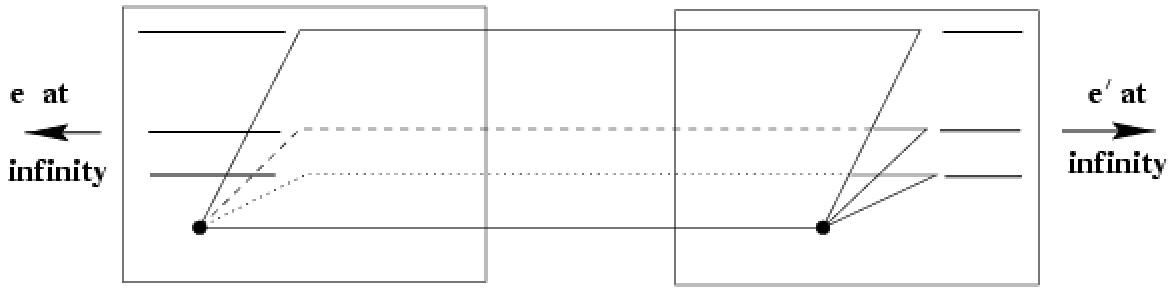
- **Epipolar Plane** – plane containing baseline and 3d point
- **Epipolar Lines** - intersections of epipolar plane with image planes (always come in corresponding pairs)

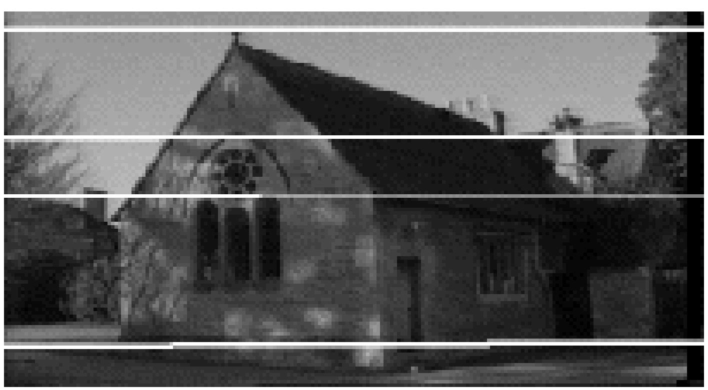
Example: Converging cameras





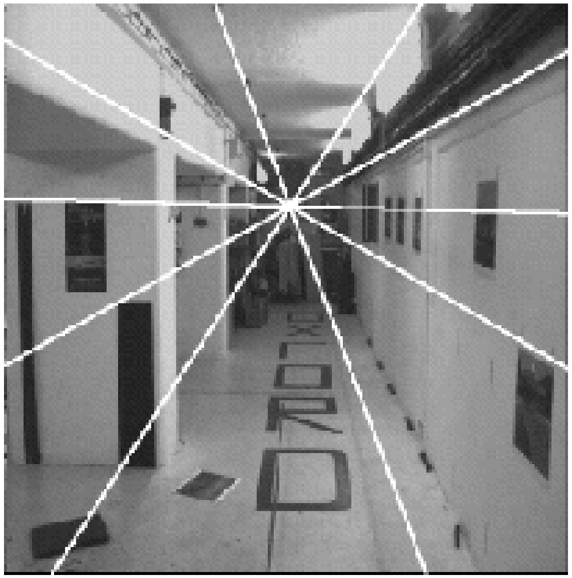
Example: Parallel cameras

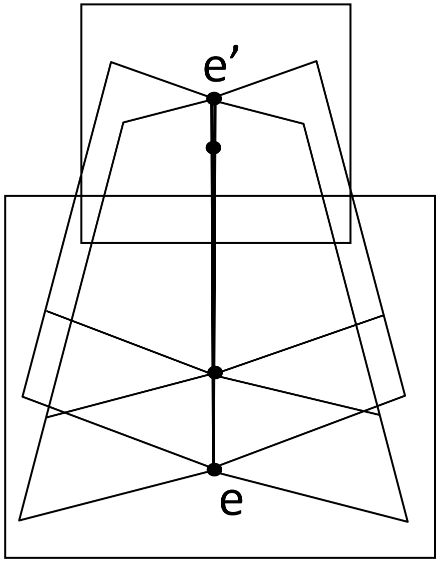




Example: Forward motion

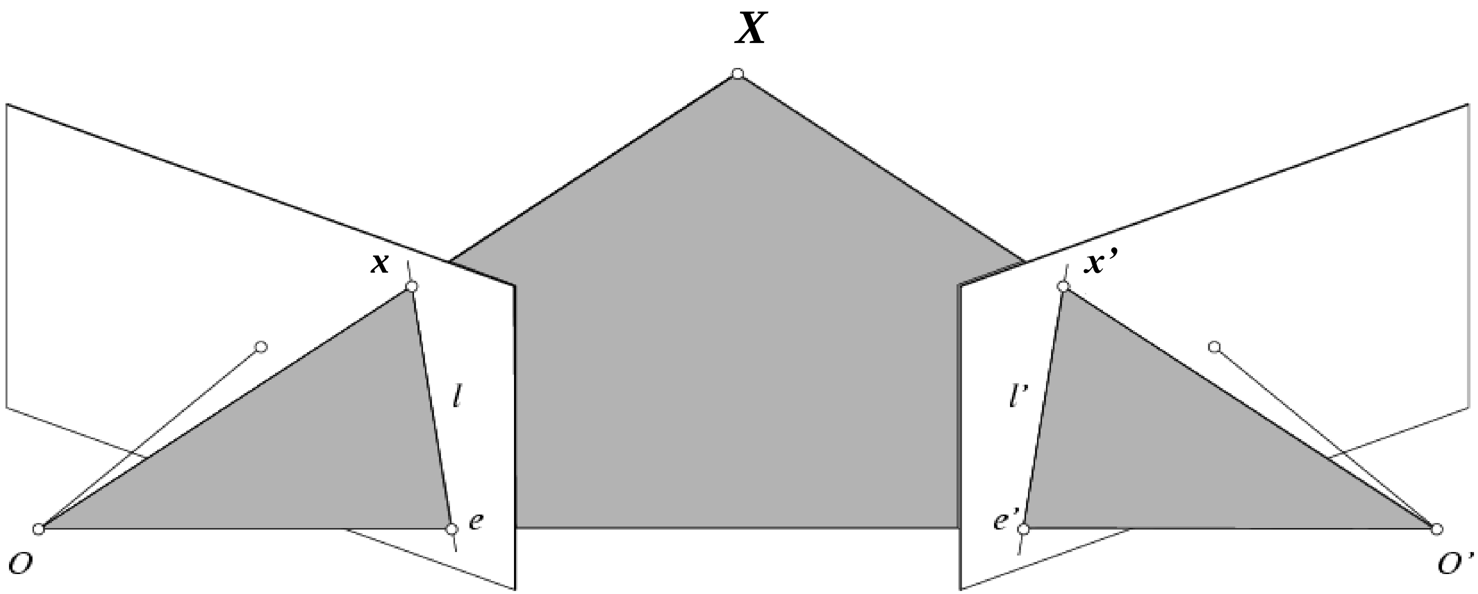
The camera moves directly forward





Epipole has same coordinates in both images.
Points move along lines radiating from e:
"Focus of expansion"

Epipolar constraint: Calibrated case



Given the intrinsic parameters of the cameras:

1. Convert to normalized coordinates by pre-multiplying all points with the inverse of the calibration matrix; set first camera's coordinate system to world coordinates

$$\hat{x} = K^{-1}x = X$$

Homogeneous 2d point (3D ray towards X)

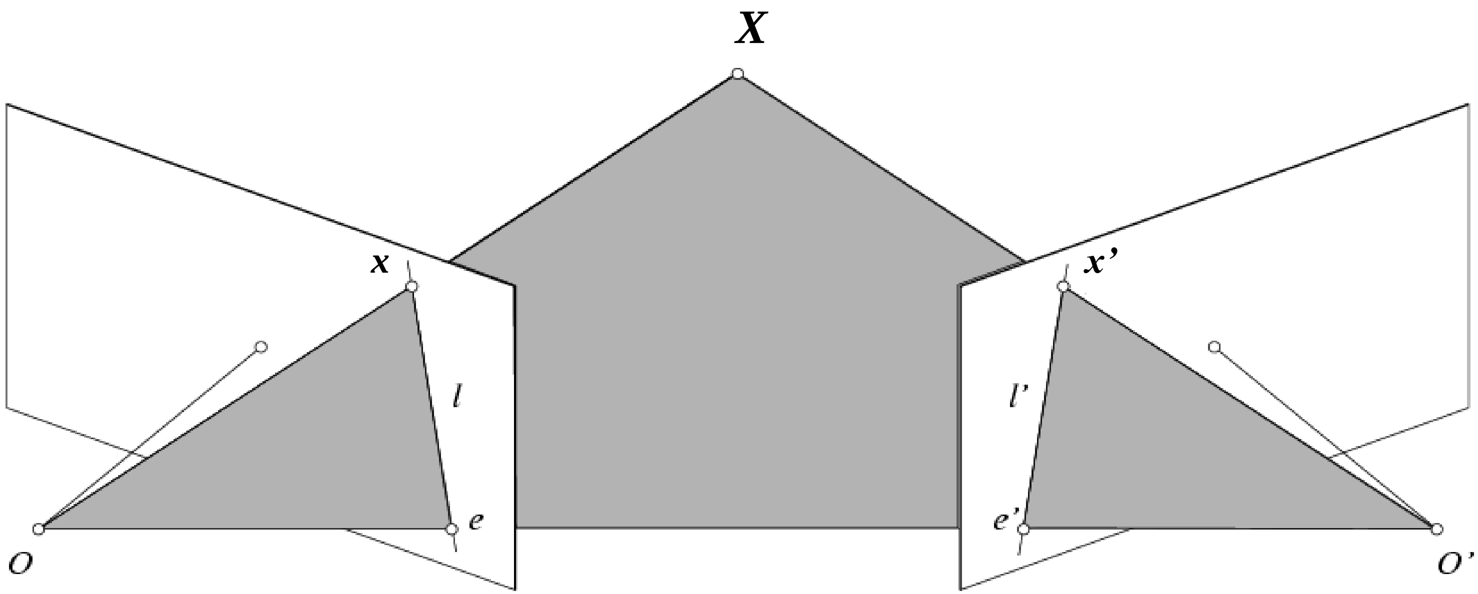
2D pixel coordinate (homogeneous)

3D scene point

$$\hat{x}' = K'^{-1}x' = X'$$

3D scene point in 2nd camera's 3D coordinates

Epipolar constraint: Calibrated case



Given the intrinsic parameters of the cameras:

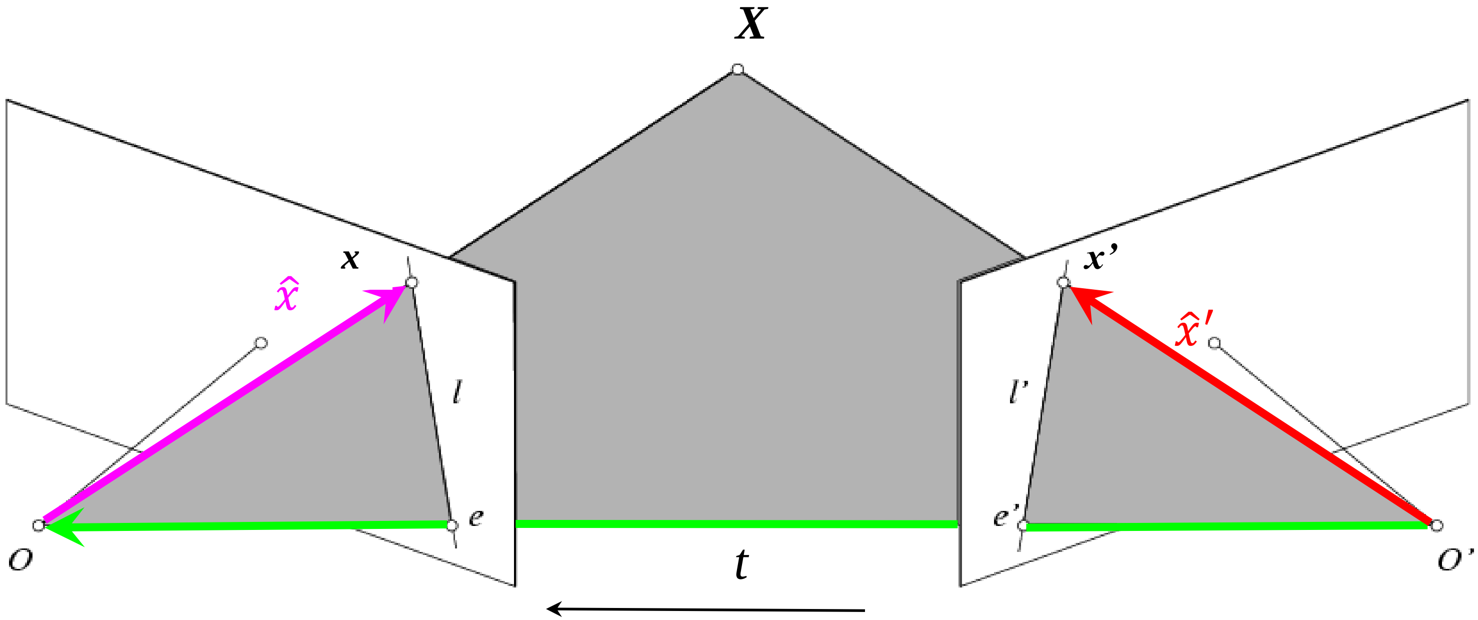
1. Convert to normalized coordinates by pre-multiplying all points with the inverse of the calibration matrix; set first camera's coordinate system to world coordinates
2. Define some *R* and *t* that relate X to X' as below

for some scale factor

$$\hat{x} = K^{-1} x = X \qquad \hat{x}' = K'^{-1} x' = X'$$

$$\boxed{\hat{x} = R\hat{x}' + t}$$

Epipolar constraint: Calibrated case



$$\hat{x} = K^{-1}x = X$$

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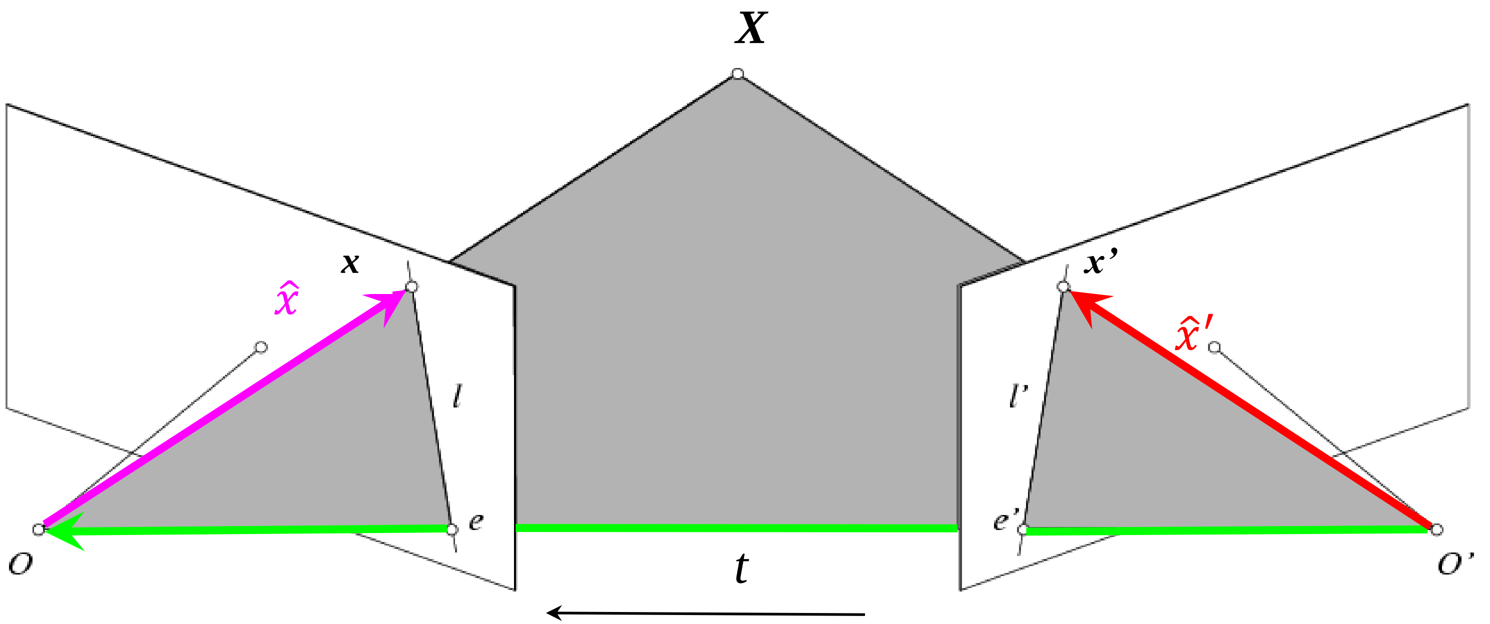
$$\hat{x} = R\hat{x}' + t$$

Epipolar constraint: Calibrated case



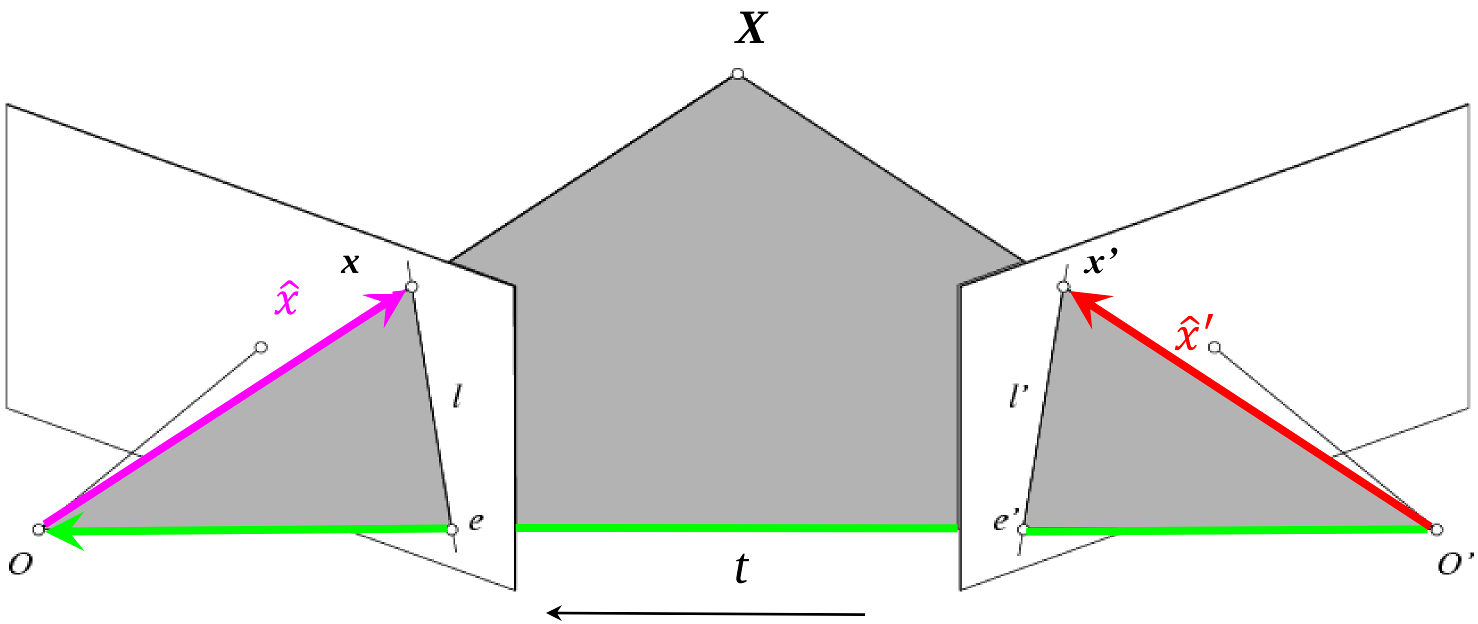
$$\hat{x} = K^{-1}x = X$$

$$\hat{x}' = K'^{-1}x' = X'$$

$$\hat{x} = R\hat{x}' + t$$

$$\hat{x}.[t \times \hat{x}] = 0$$

Epipolar constraint: Calibrated case



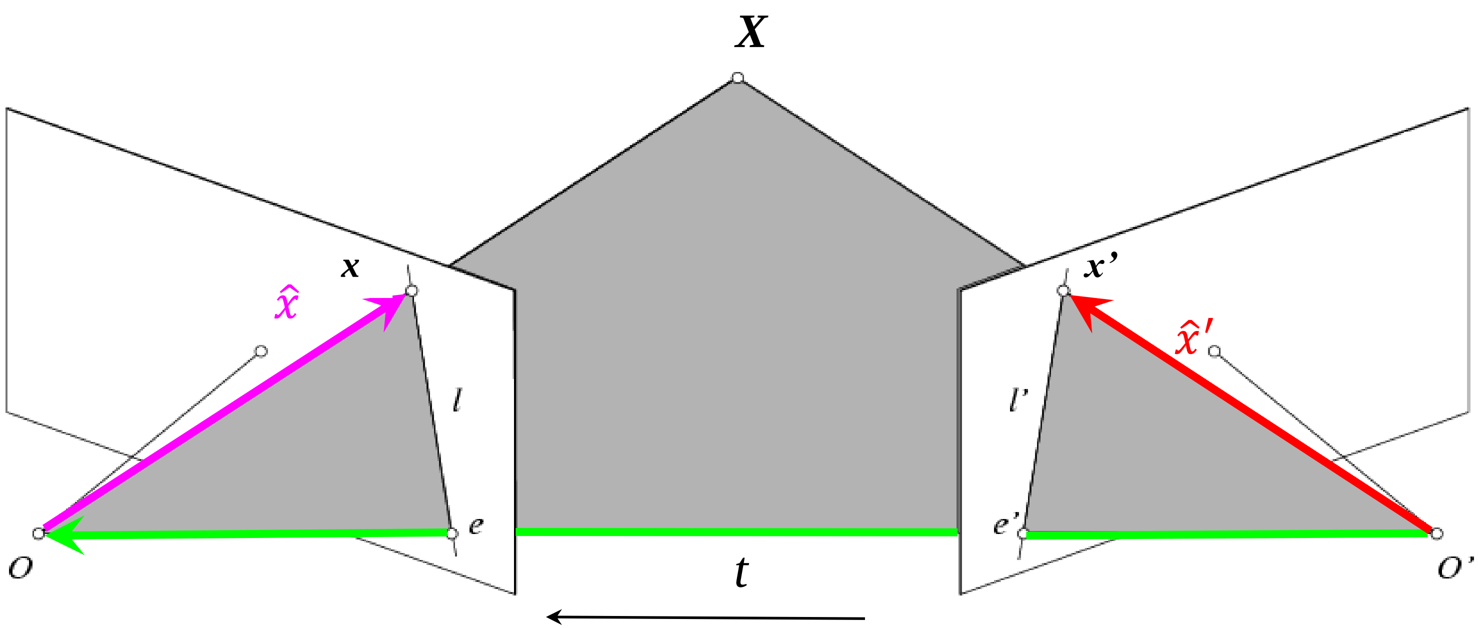
$$\hat{x} = K^{-1} x = X$$

$$\hat{x}' = K'^{-1} x' = X'$$

$$\hat{x} = R\hat{x}' + t$$

$\hat{x}.[t \times \hat{x}] = 0 \Rightarrow \hat{x}.[t \times (R\,\hat{x}' + t)] = 0$

Epipolar constraint: Calibrated case



$$\hat{x} = K^{-1} x = X \qquad\qquad \hat{x}' = K'^{-1} x' = X'$$

$$\hat{x} = R\hat{x}' + t$$

$$\hat{x}.[t \times \hat{x}] = 0 \Rightarrow \hat{x}.[t \times (R\,\hat{x}' + t)] = 0$$

$$\Rightarrow \hat{x}.[t \times (R\,\hat{x}')] = 0$$

(because $\hat{x}$, $R\hat{x}'$, and t are co-planar)

Matrix representation of cross product

$$\mathbf{a} = (a_1 \ a_2 \ a_3)^{\mathrm{T}}$$

$$\mathbf{b} = (b_1 \ b_2 \ b_3)^{\mathrm{T}}$$

$$[\mathbf{a}]_\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

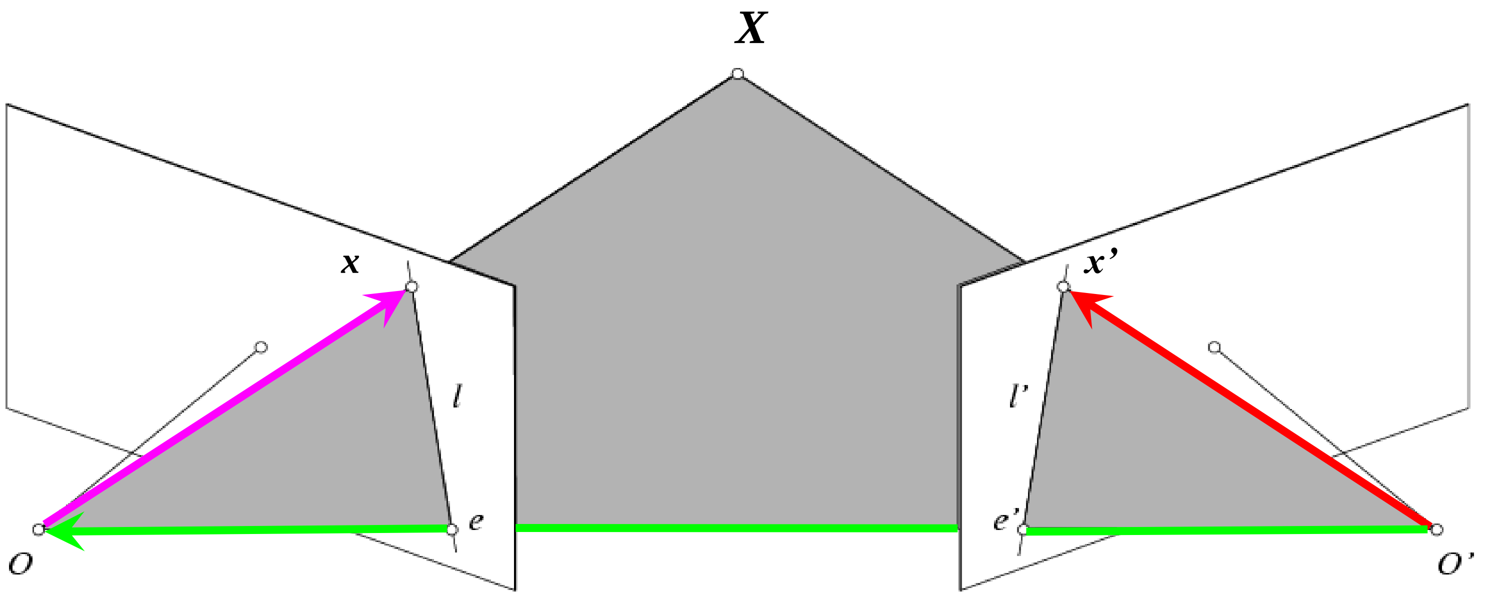
Skew-symmetric matrix

(Rank 2)

$$\mathbf{a} \times \mathbf{b} = [\mathbf{a}]_\times \mathbf{b}$$

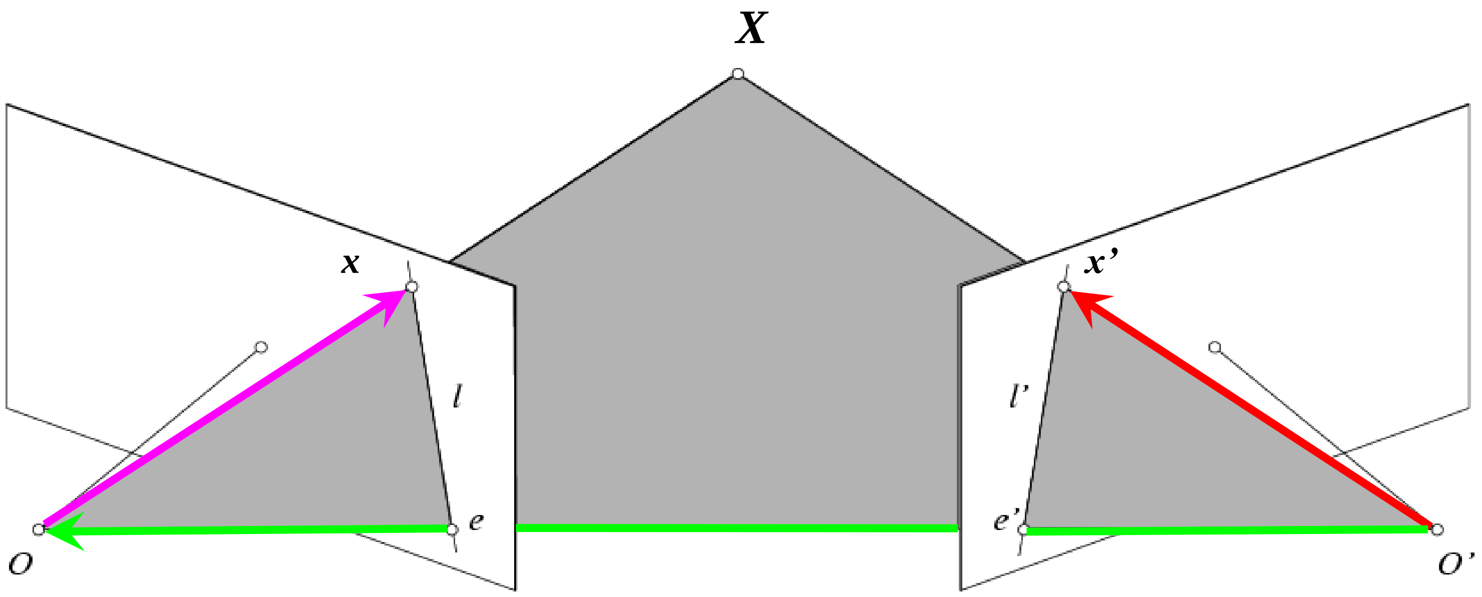
[Matrix representation of the cross product]

Essential matrix



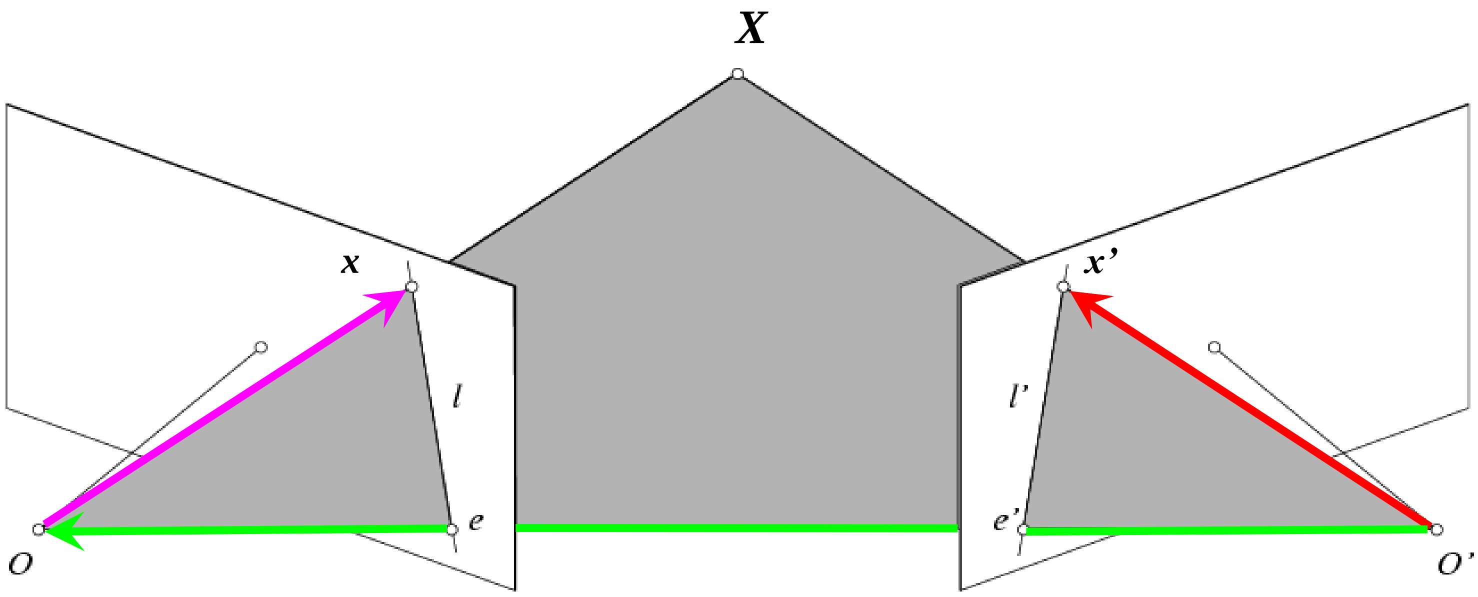
$$\hat{x}.[t \times (R\hat{x}')] = \hat{x}^T [t]_\times R\hat{x}'$$

Essential matrix



$$\hat{x}.[t \times (R\hat{x}')] = \hat{x}^T [t]_\times R\hat{x}' = \hat{x}^T E \hat{x}' = 0 \quad \text{with} \quad E = [t]_\times R$$

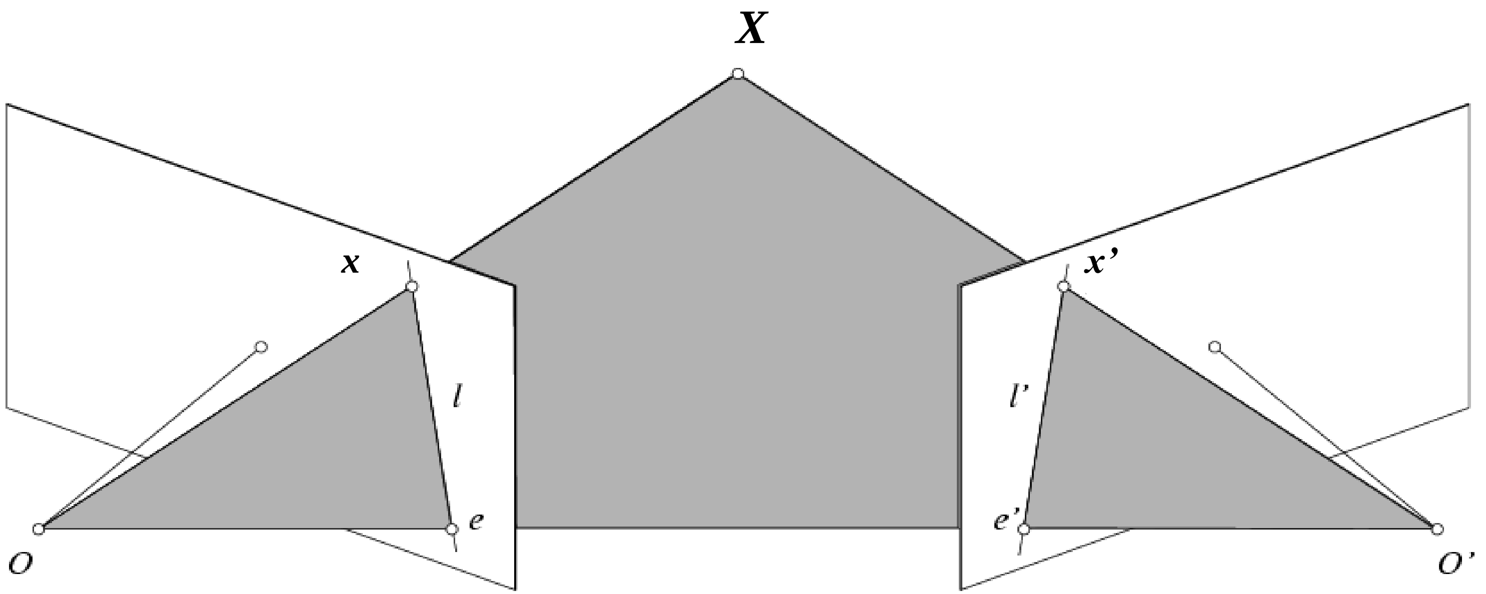
Essential matrix



$$\hat{x}.[t\times(R\hat{x}')] = \hat{x}^T[t]_\times R\hat{x}' = \hat{x}^T E\hat{x}' = 0 \quad \text{with} \quad E = [t]_\times R$$

Essential Matrix
(Longuet-Higgins, 1981)

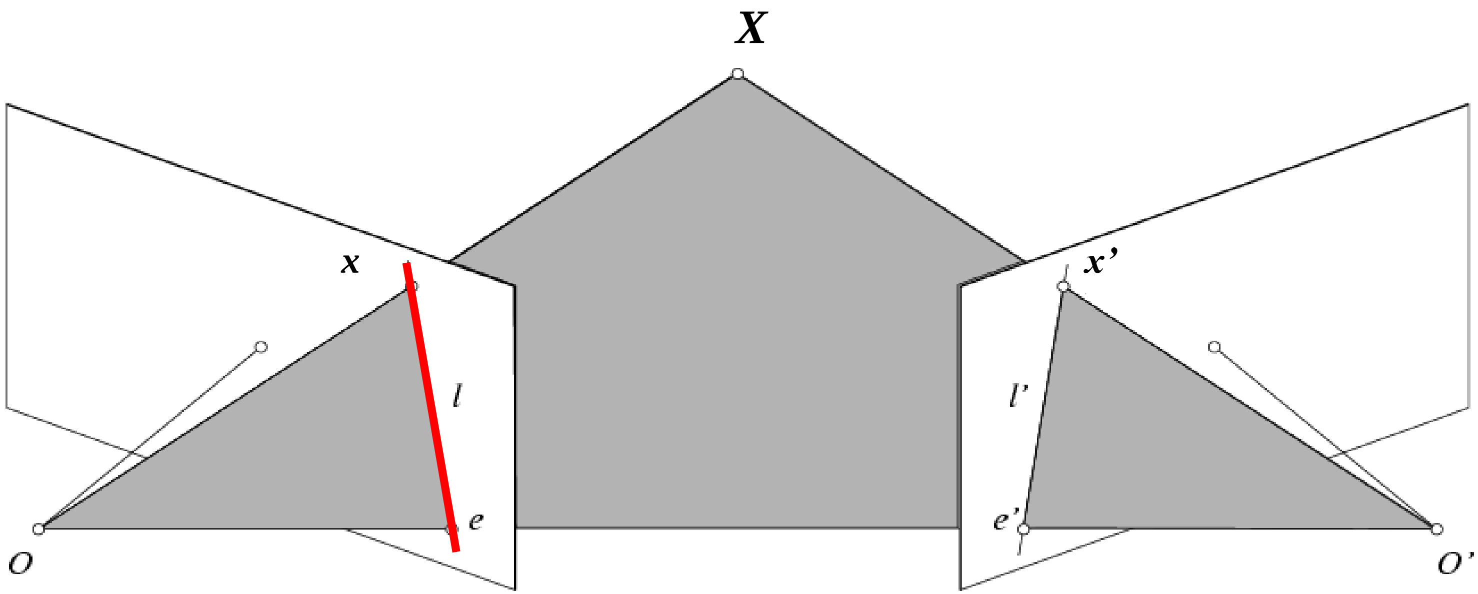
Properties of the Essential matrix



$$\hat{x} \cdot [t \times (R\hat{x}')] = 0 \quad \Rightarrow \quad \hat{x}^T E \hat{x}' = 0 \quad \text{with} \quad E = [t]_\times R$$

Skew-symmetric matrix

Properties of the Essential matrix

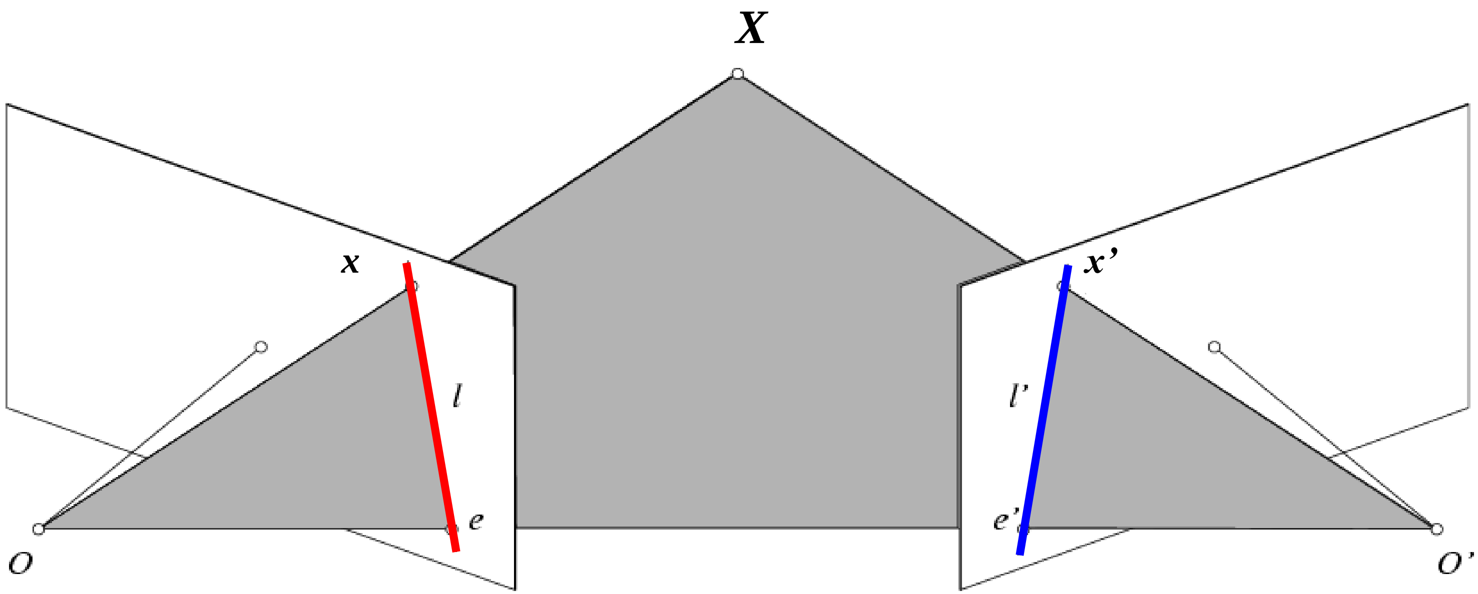


$$\hat{x} \cdot [t \times (R\hat{x}')] = 0 \quad \Rightarrow \quad \hat{x}^T E \hat{x}' = 0 \quad \text{with} \quad E = [t]_{\times} R$$

- *E x'* is the epipolar line associated with *x'* (*l* = *E x'*)

Skew-symmetric matrix

Properties of the Essential matrix

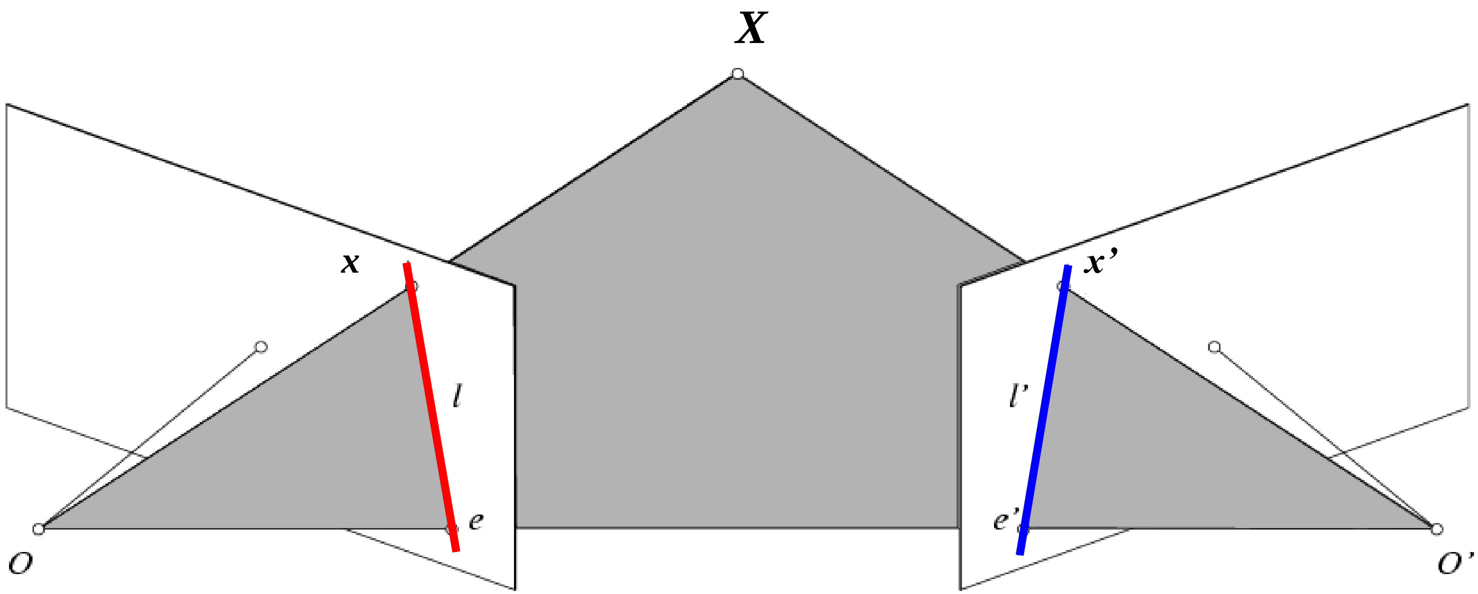


$$\hat{x} \cdot [t \times (R\hat{x}')] = 0 \quad \Rightarrow \quad \hat{x}^T E \hat{x}' = 0 \quad \text{with} \quad E = [t]_\times R$$

Skew-symmetric matrix

- *E x'* is the epipolar line associated with *x'* (*l = E x'*)
- $E^T x$ is the epipolar line associated with *x* ($l' = E^T x$)

Properties of the Essential matrix

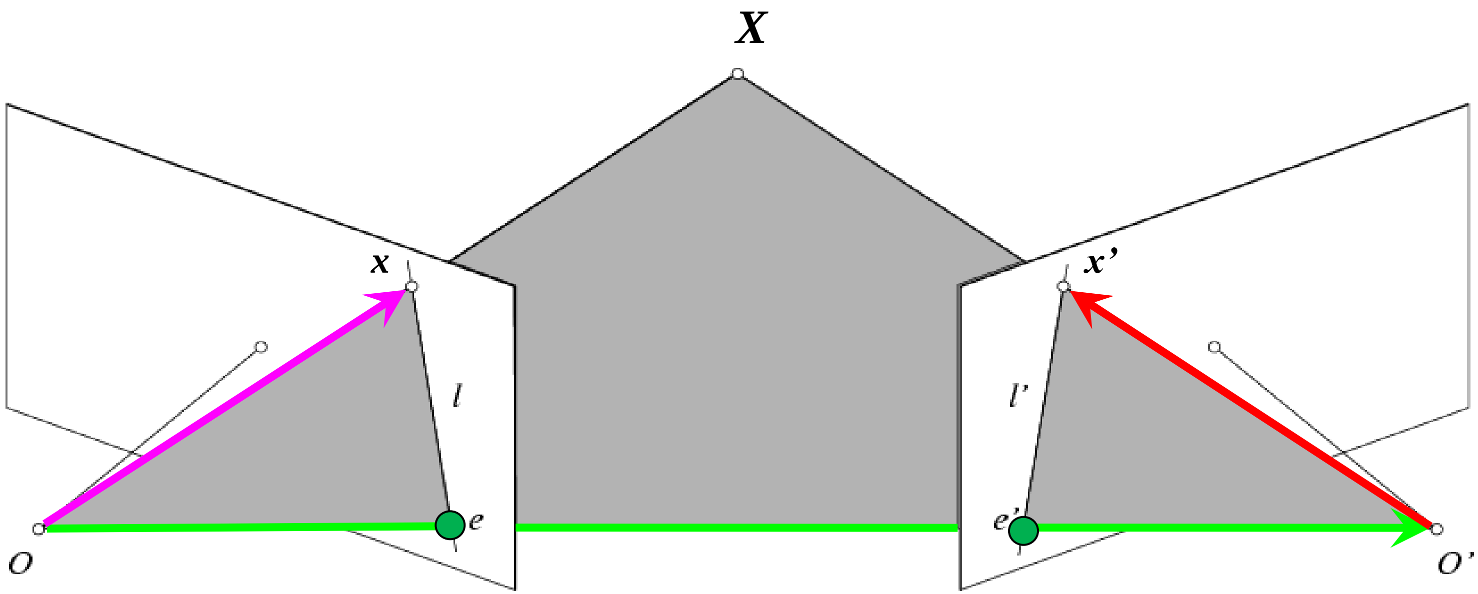


$$\hat{x} \cdot [t \times (R\hat{x}')] = 0 \quad \Rightarrow \quad \hat{x}^T E \hat{x}' = 0 \quad \text{with} \quad E = [t]_\times R$$

Skew-symmetric matrix

- *E x'* is the epipolar line associated with *x'* (*l* = *E x'*)
- $E^T x$ is the epipolar line associated with *x* ($l' = E^T x$)
- *E* is singular (rank two): det(E)=0

Epipolar constraint: Uncalibrated case



- If we don't know *K* and *K'*, then we can write the epipolar constraint in terms of *unknown* normalized coordinates:

$$\hat{x}^T E \hat{x}' = 0 \qquad\qquad x = K\hat{x}, \quad x' = K'\hat{x}'$$

The Fundamental Matrix

Without knowing K and K', we can define a similar relation using *unknown* normalized coordinates

$$\hat{x}^T E \hat{x}' = 0$$

$$\hat{x} = K^{-1} x$$

$$\hat{x}' = K'^{-1} x'$$

The Fundamental Matrix

Without knowing K and K', we can define a similar relation using *unknown* normalized coordinates

$$\hat{x}^T E \hat{x}' = 0$$

$$\hat{x} = K^{-1} x$$

$$\hat{x}' = K'^{-1} x'$$

$$\Rightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

The Fundamental Matrix

Without knowing K and K', we can define a similar relation using *unknown* normalized coordinates

$$\hat{x}^T E \hat{x}' = 0$$

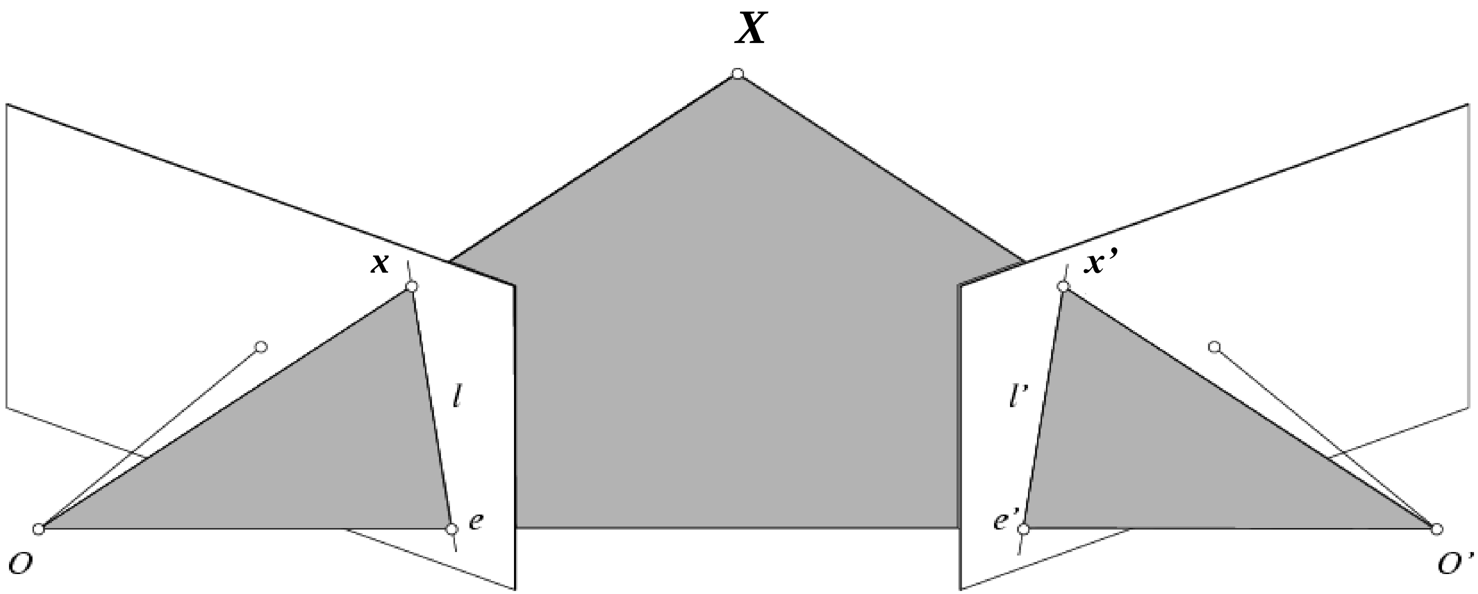
$$\hat{x} = K^{-1} x$$

$$\hat{x}' = K'^{-1} x'$$

$$\Rightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

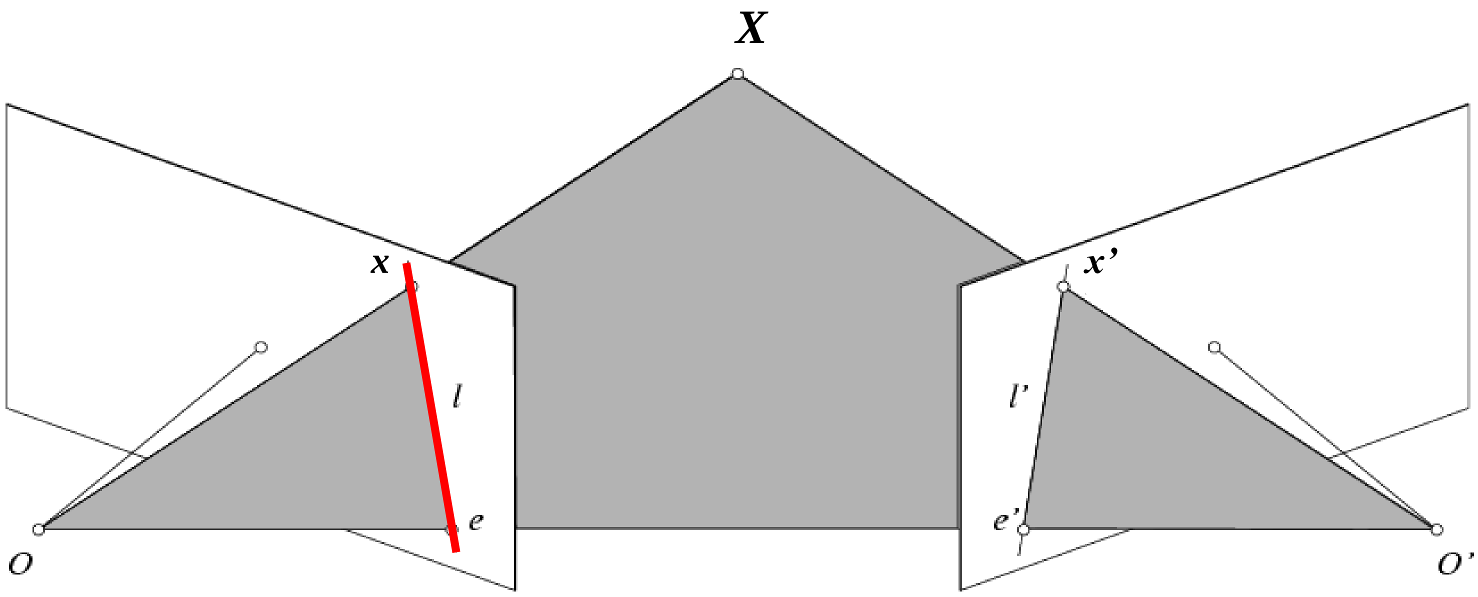
Fundamental Matrix
(Faugeras and Luong, 1992)

Properties of the Fundamental matrix



$$x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

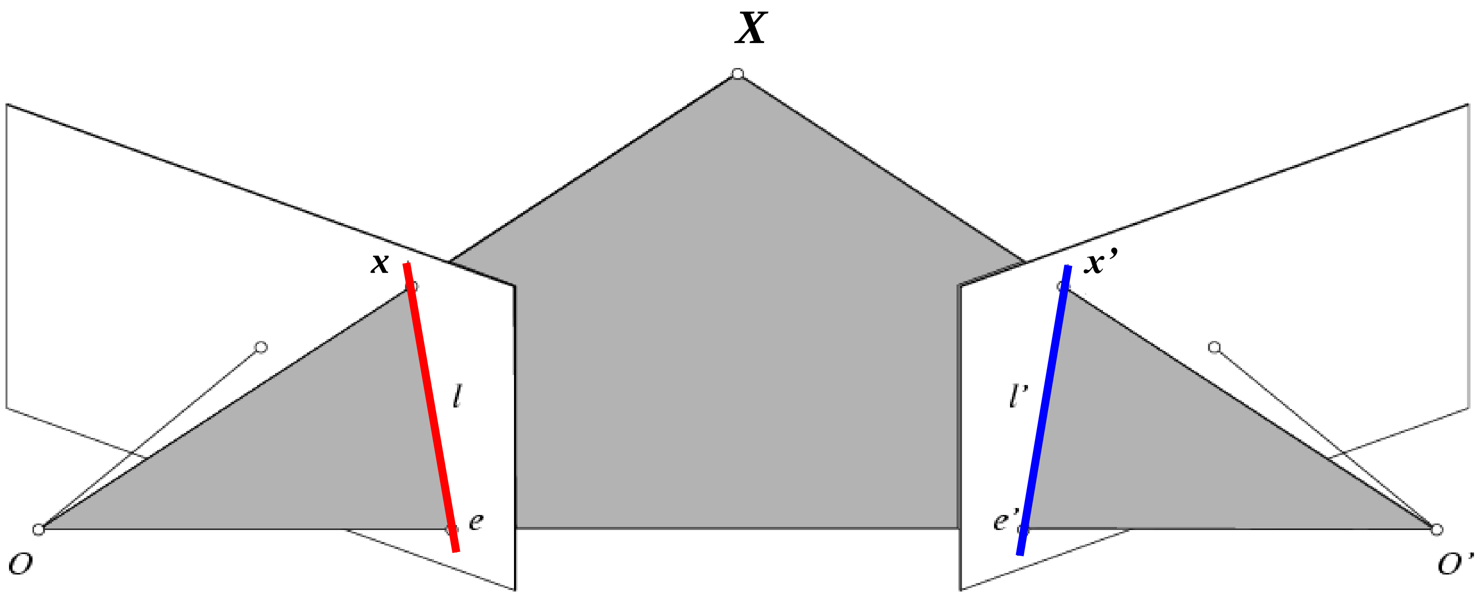
Properties of the Fundamental matrix



$$x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

- *F x'* is the epipolar line associated with *x'* (*l* = *F x'*)

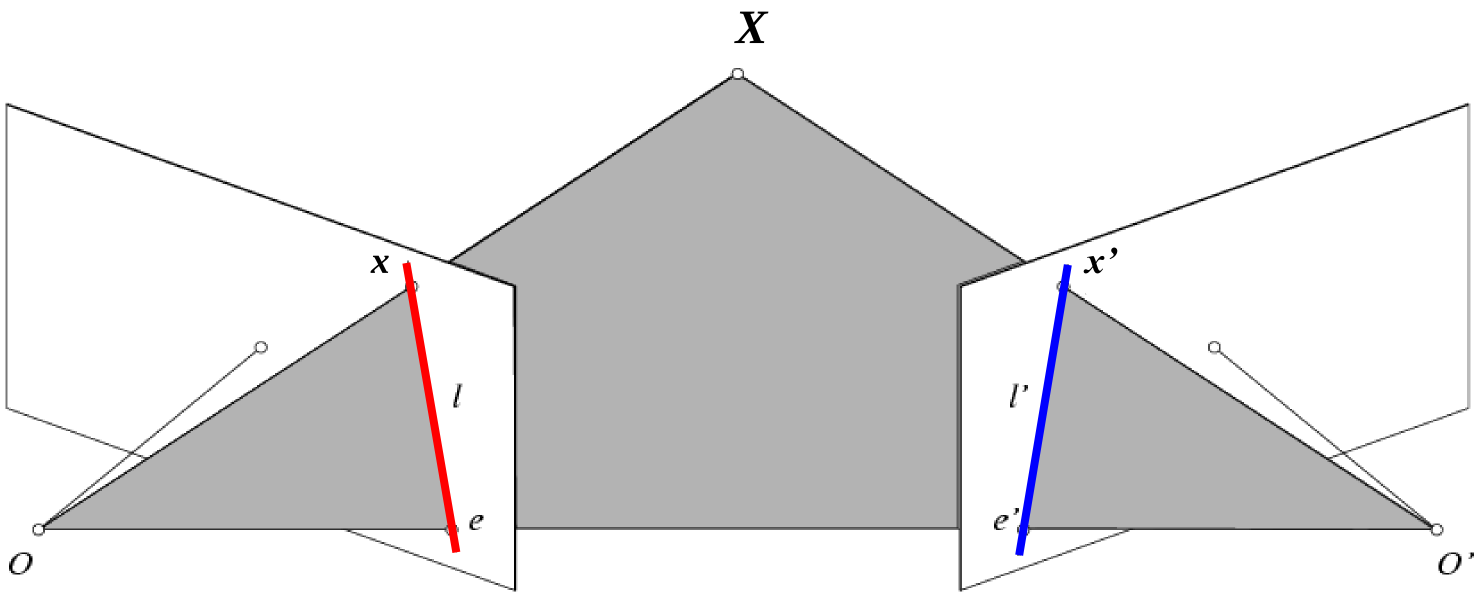
Properties of the Fundamental matrix



$$x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

- $F x'$ is the epipolar line associated with x' ($l = F x'$)
- $F^T x$ is the epipolar line associated with x ($l' = F^T x$)

Properties of the Fundamental matrix



$$x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

- $F x'$ is the epipolar line associated with x' ($l = F x'$)
- $F^T x$ is the epipolar line associated with x ($l' = F^T x$)
- F is singular (rank two): det(F)=0

Estimating the Fundamental Matrix

- 8-point algorithm
  - Least squares solution using SVD on equations from 8 pairs of correspondences
  - Enforce det(F)=0 constraint using SVD on F

Note: estimation of F (or E) is degenerate for a planar scene.

8-point algorithm

1. Solve a system of homogeneous linear equations
    a. Write down the system of equations

$$\mathbf{x}^T F \mathbf{x}' = 0$$

$$uu'f_{11} + uv'f_{12} + uf_{13} + vu'f_{21} + vv'f_{22} + vf_{23} + u'f_{31} + v'f_{32} + f_{33} = 0$$

8-point algorithm

1. Solve a system of homogeneous linear equations
    a. Write down the system of equations

$$\mathbf{x}^T F \mathbf{x}' = 0$$

$$uu'f_{11} + uv'f_{12} + uf_{13} + vu'f_{21} + vv'f_{22} + vf_{23} + u'f_{31} + v'f_{32} + f_{33} = 0$$

$$\mathrm{A}\boldsymbol{f} = \begin{bmatrix} u_1 u_1{}' & u_1 v_1{}' & u_1 & v_1 u_1{}' & v_1 v_1{}' & v_1 & u_1{}' & v_1{}' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_n u_v' & u_n v_n{}' & u_n & v_n u_n{}' & v_n v_n{}' & v_n & u_n{}' & v_n{}' & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ \vdots \\ f_{33} \end{bmatrix} = \mathbf{0}$$

8-point algorithm

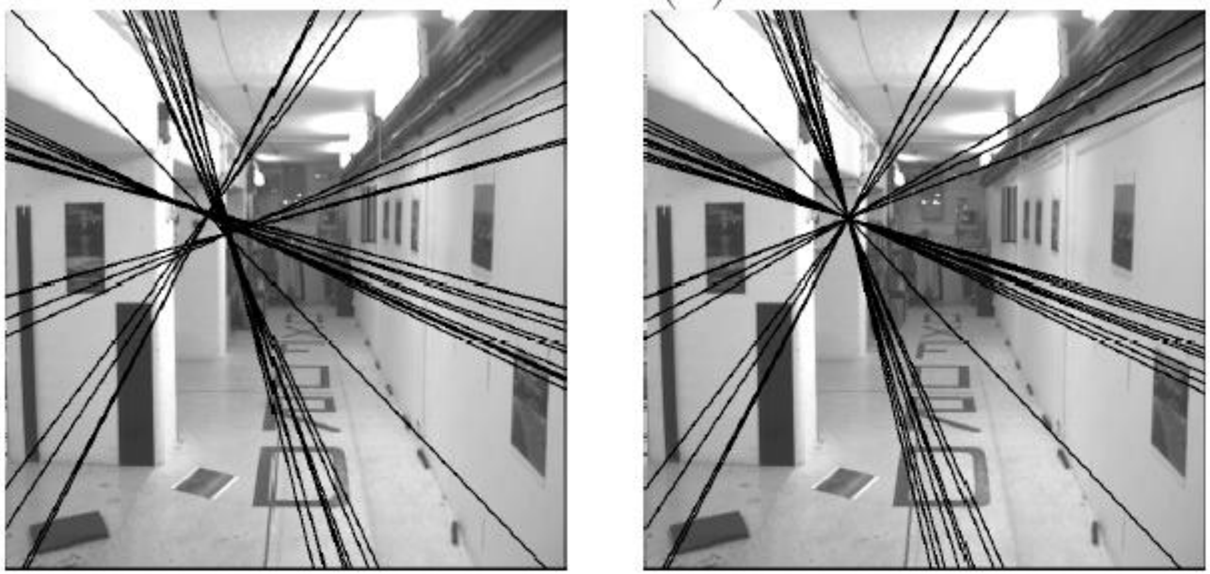
1. Solve a system of homogeneous linear equations
   a. Write down the system of equations
   b. Solve **f** from A**f**=**0** using SVD

Matlab:

```
[U, S, V] = svd(A);
f = V(:, end);
F = reshape(f, [3 3])';
```

Need to enforce singularity constraint

Fundamental matrix has rank 2 : $\det(\mathtt{F}) = 0$.



Left : Uncorrected $\mathtt{F}$ – epipolar lines are not coincident.

Right : Epipolar lines from corrected $\mathtt{F}$.

8-point algorithm

1. Solve a system of homogeneous linear equations
   a. Write down the system of equations
   b. Solve **f** from A**f**=**0** using SVD

   Matlab:
```
[U, S, V] = svd(A);
f = V(:, end);
F = reshape(f, [3 3])';
```

2. Resolve det(F) = 0 constraint using SVD

   Matlab:
```
[U, S, V] = svd(F);
S(3,3) = 0;
F = U*S*V';
```

8-point algorithm

Notes:

- Use RANSAC to deal with outliers (sample 8 points)
  - How to test for outliers? $|\boldsymbol{x^T F x'}| < \boldsymbol{threshold}$
- Solve in normalized coordinates
  - mean=0
  - standard deviation ~= (1,1,1)

Moving on to stereo…

Fuse a calibrated binocular stereo pair to produce a depth image

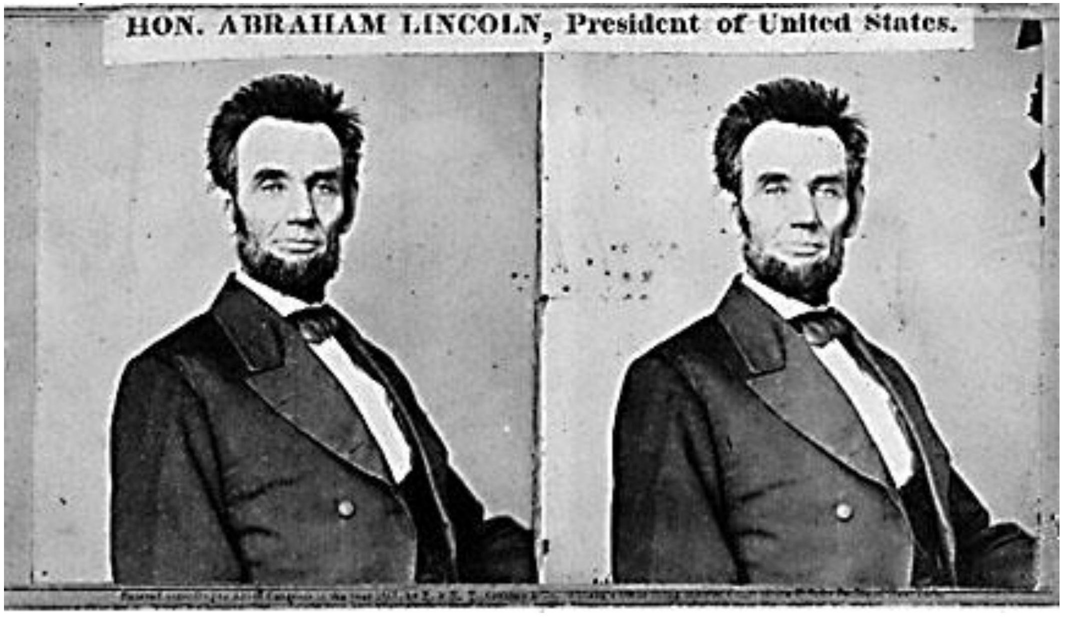
image 1

image 2

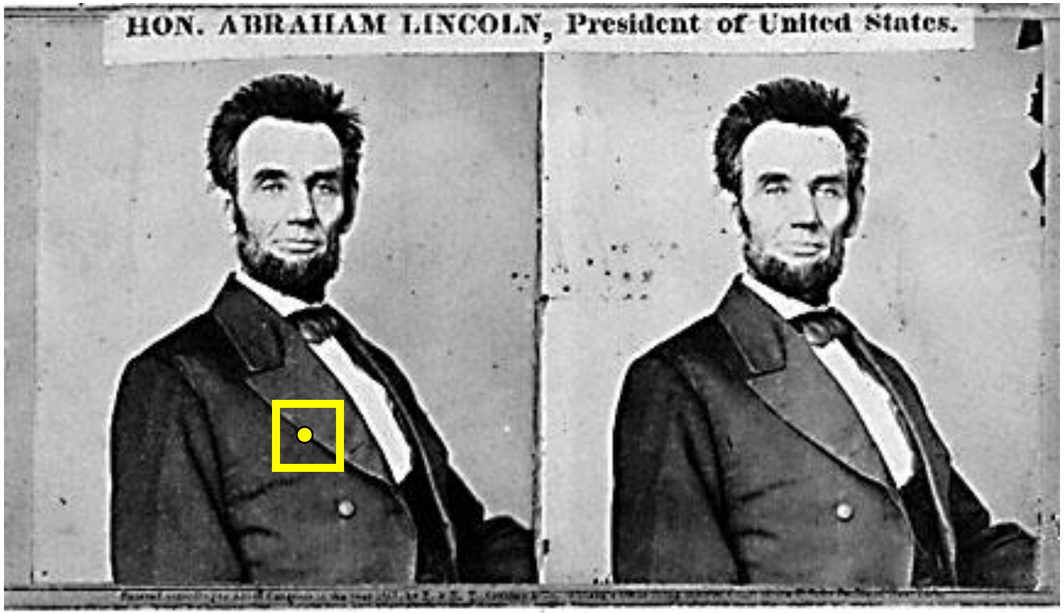
Depth map

Many of these slides adapted from Steve Seitz and Lana Lazebnik

Basic stereo matching algorithm

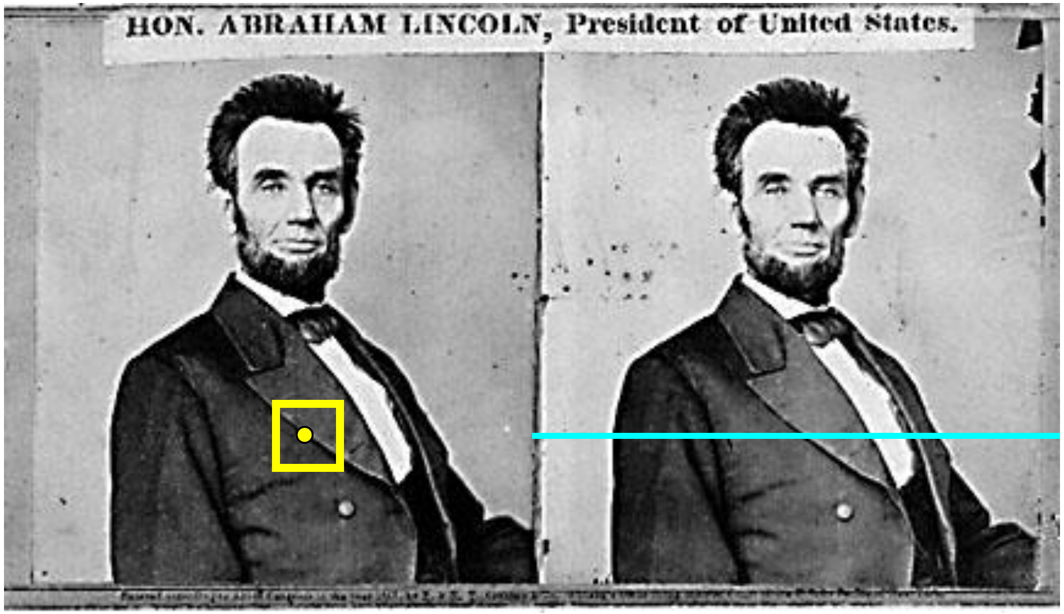


Basic stereo matching algorithm



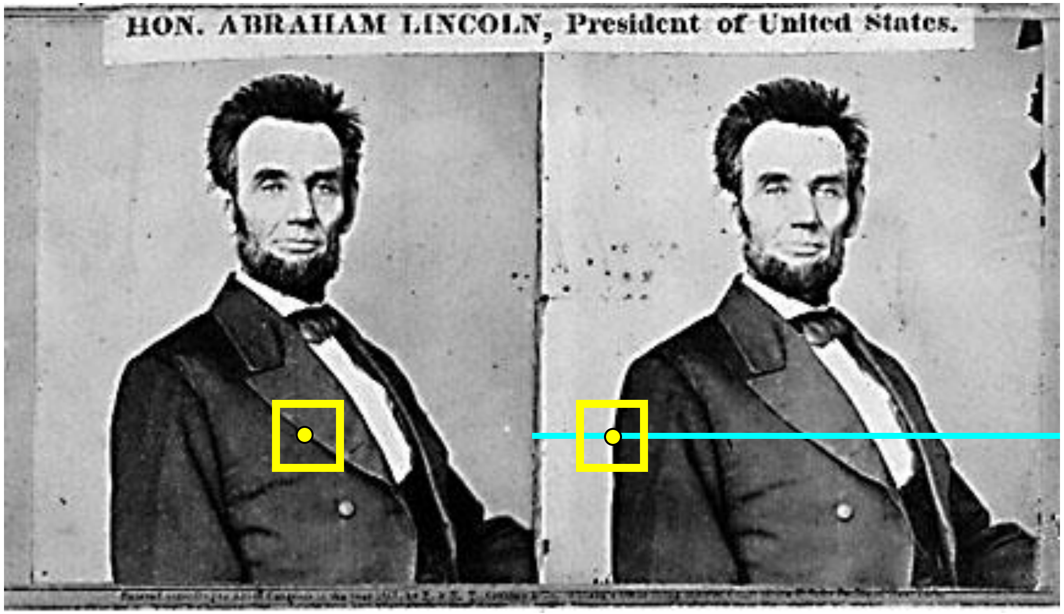
- For each pixel in the first image

Basic stereo matching algorithm



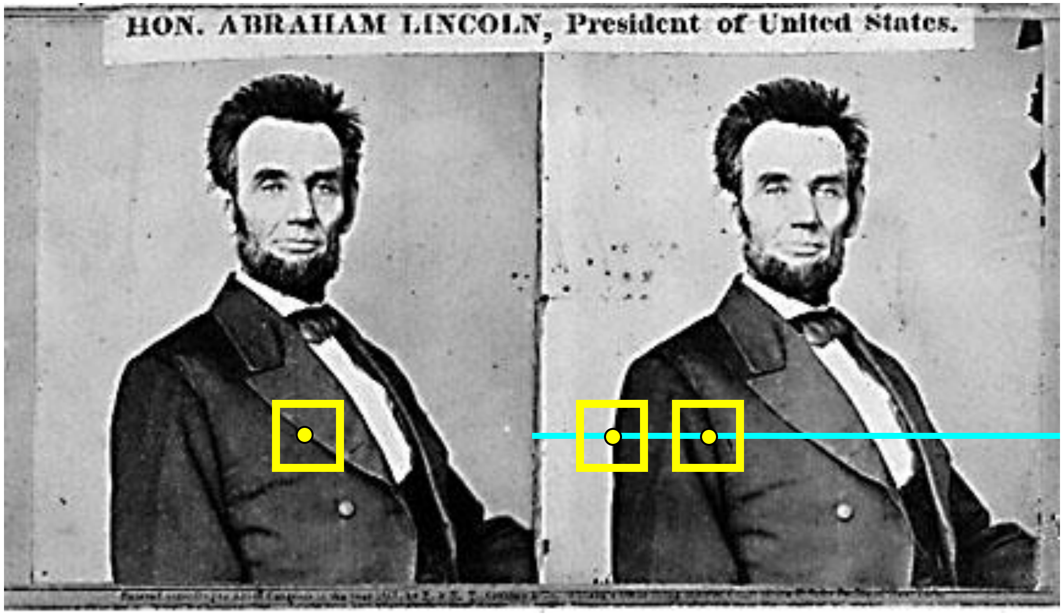
- For each pixel in the first image
  - Find corresponding epipolar line in the right image

Basic stereo matching algorithm



- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Search along epipolar line and pick the best match

Basic stereo matching algorithm



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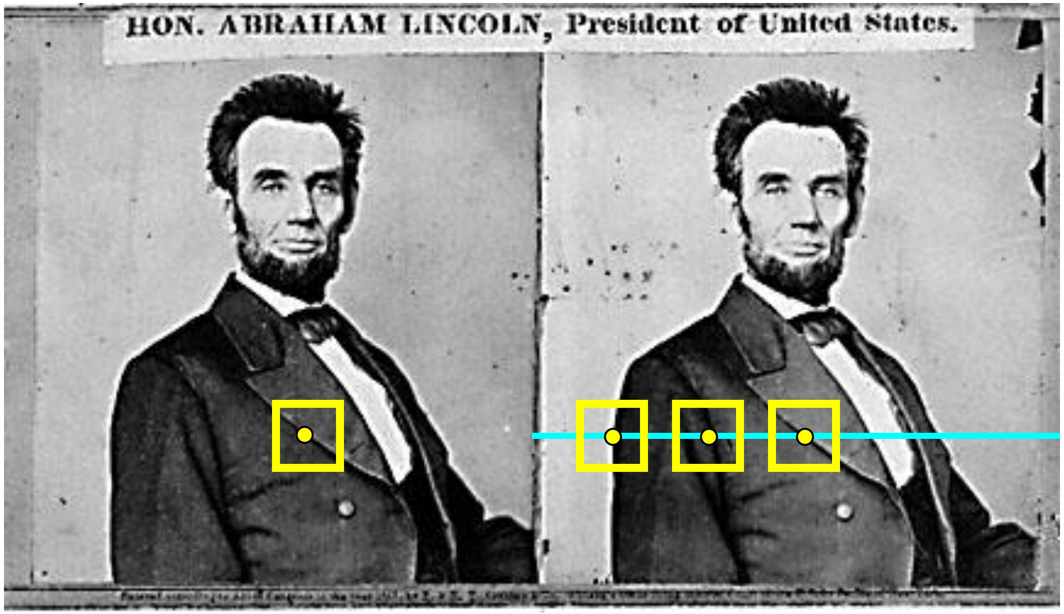

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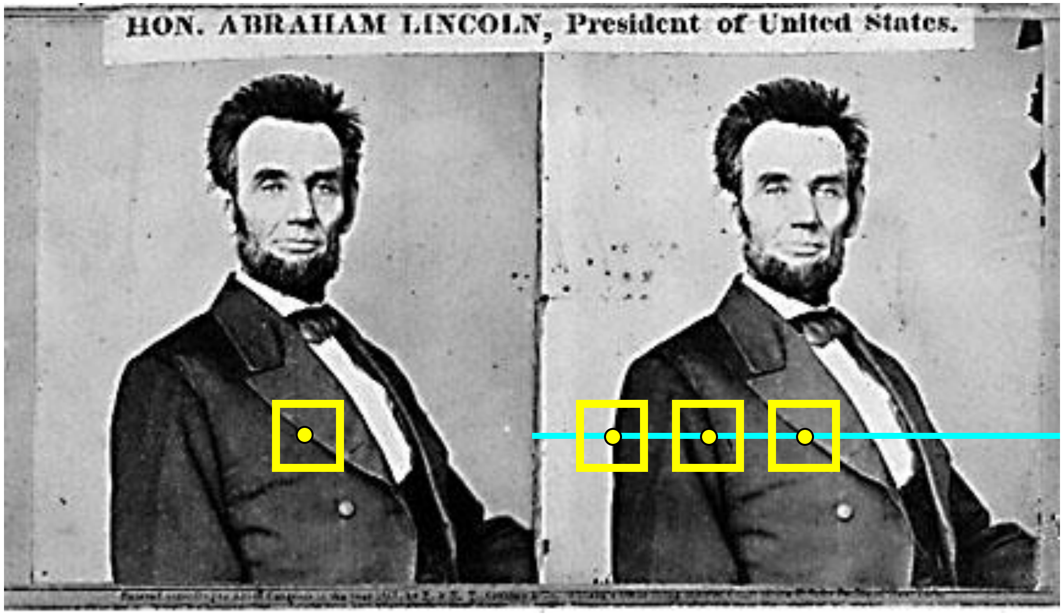
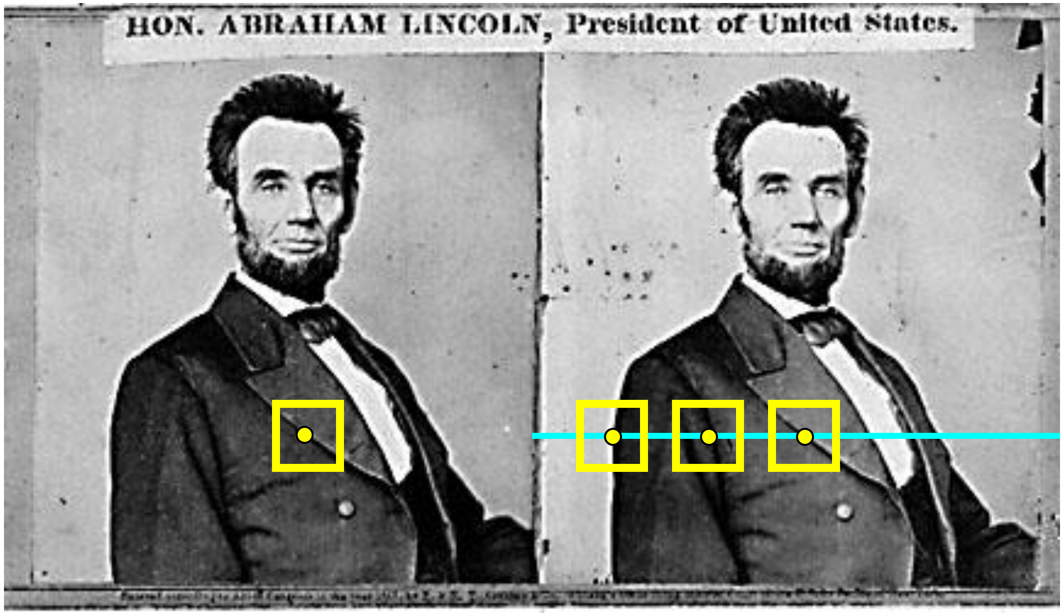


- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Search along epipolar line and pick the best match
  - Triangulate the matches to get depth information

Basic stereo matching algorithm



- For each pixel in the first image
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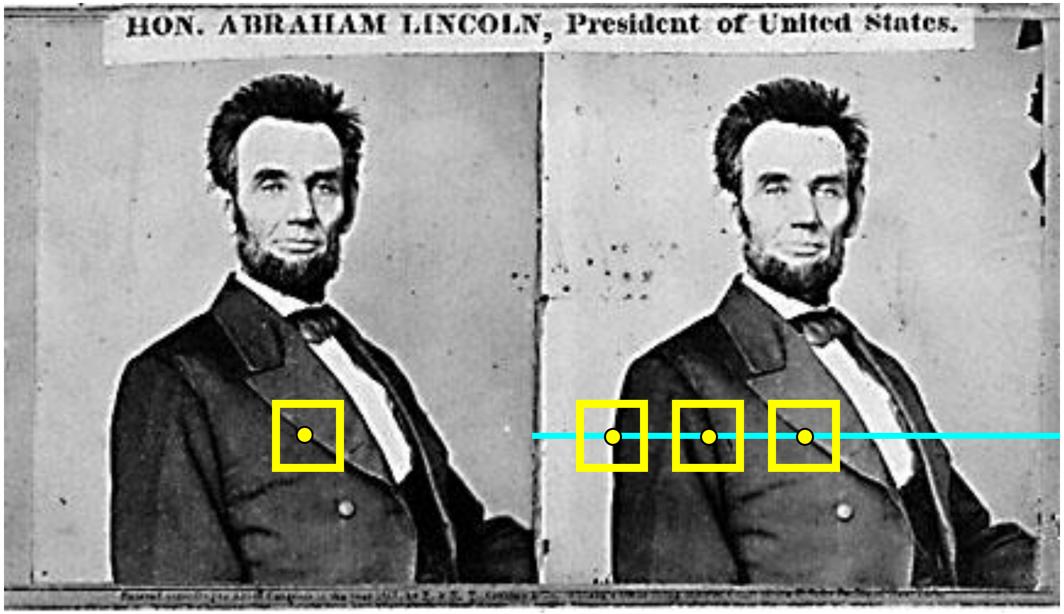
  - Triangulate the matches to get depth information
- Simplest case: epipolar lines are scanlines

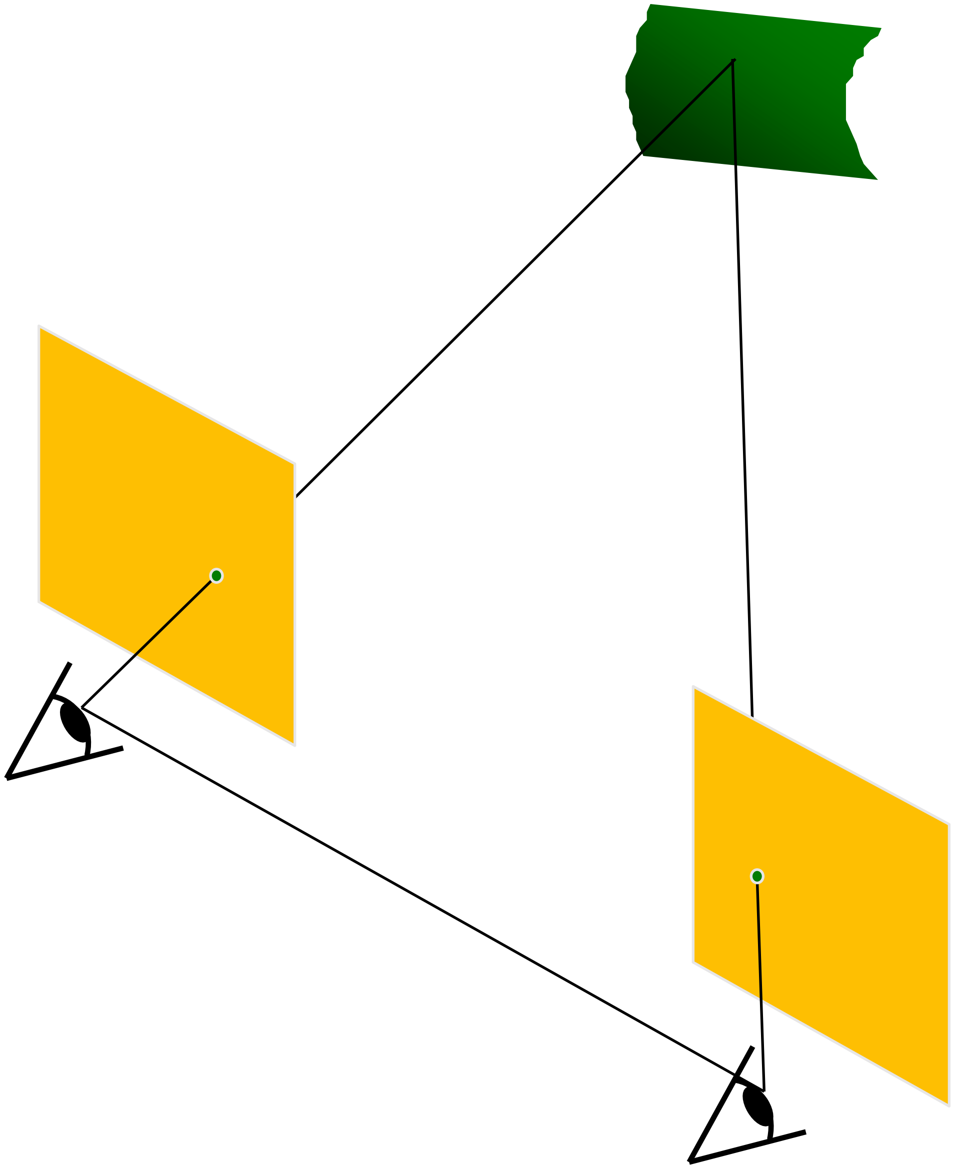
Basic stereo matching algorithm



- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Search along epipolar line and pick the best match
  - Triangulate the matches to get depth information

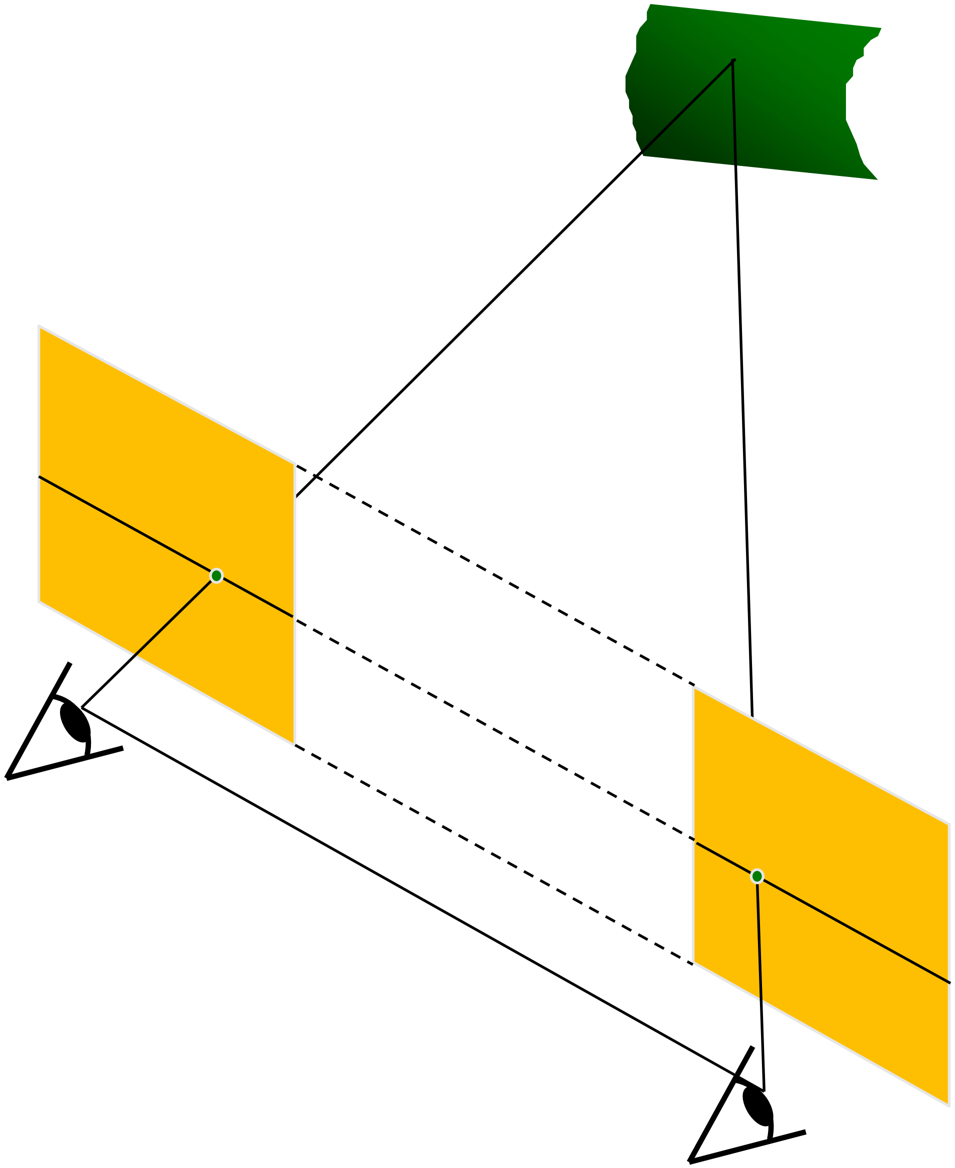
- Simplest case: epipolar lines are scanlines
  - When does this happen?

Simplest Case: Parallel images



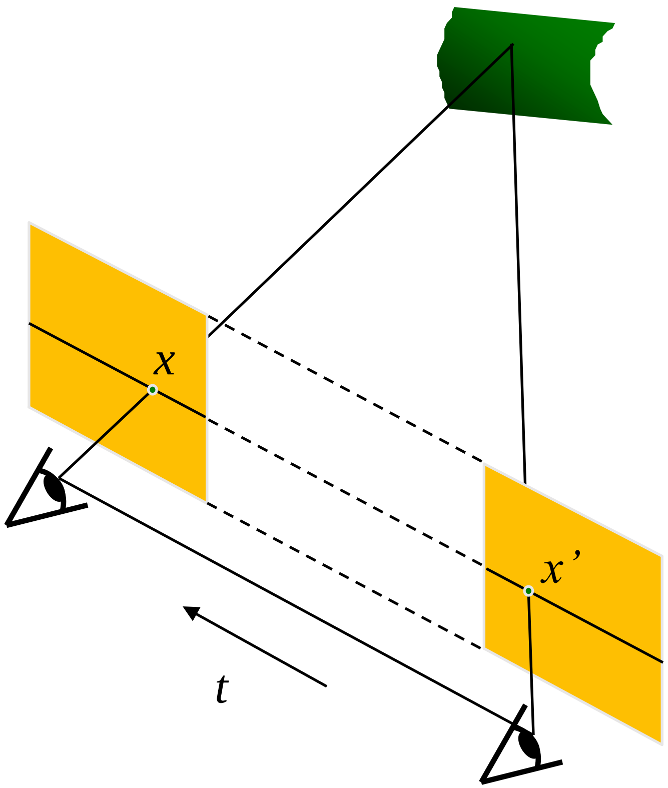
- Image planes of cameras are parallel to each other and to the baseline
- Camera centers are at same height
- Focal lengths are the same

Simplest Case: Parallel images



- Image planes of cameras are parallel to each other and to the baseline
- Camera centers are at same height
- Focal lengths are the same
- Then, epipolar lines fall along the horizontal scan lines of the images

Simplest Case: Parallel images



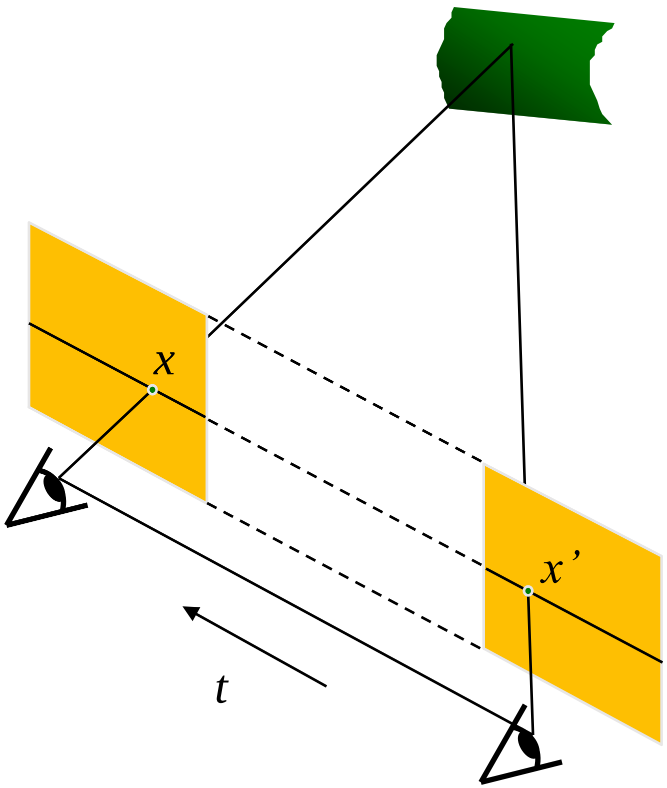
Epipolar constraint:

$$x^T E x' = 0, \quad E = t \times R$$

$R = I \qquad t = (T, 0, 0)$

$$E = t \times R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

Simplest Case: Parallel images



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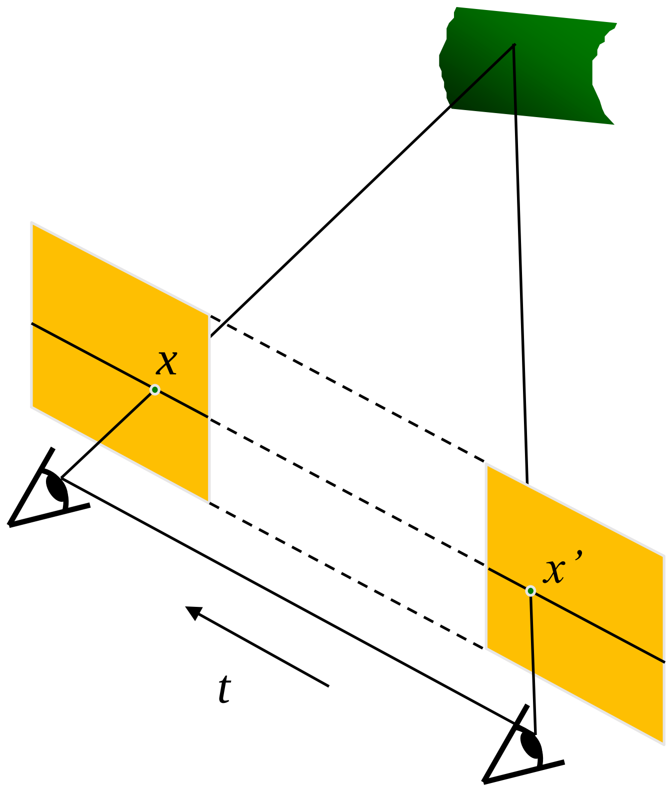
$$\begin{pmatrix} u & v & 1 \end{pmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \qquad \begin{pmatrix} u & v & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -T \\ Tv' \end{pmatrix} = 0 \qquad Tv = Tv'$$

Simplest Case: Parallel images



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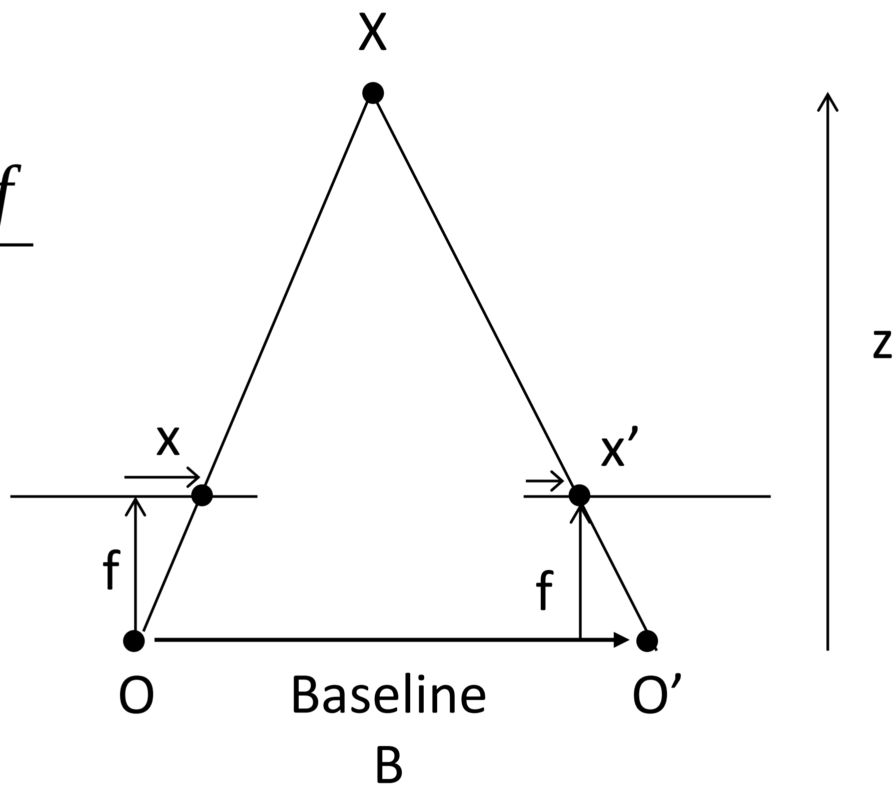
The y-coordinates of corresponding points are the same

Depth from disparity

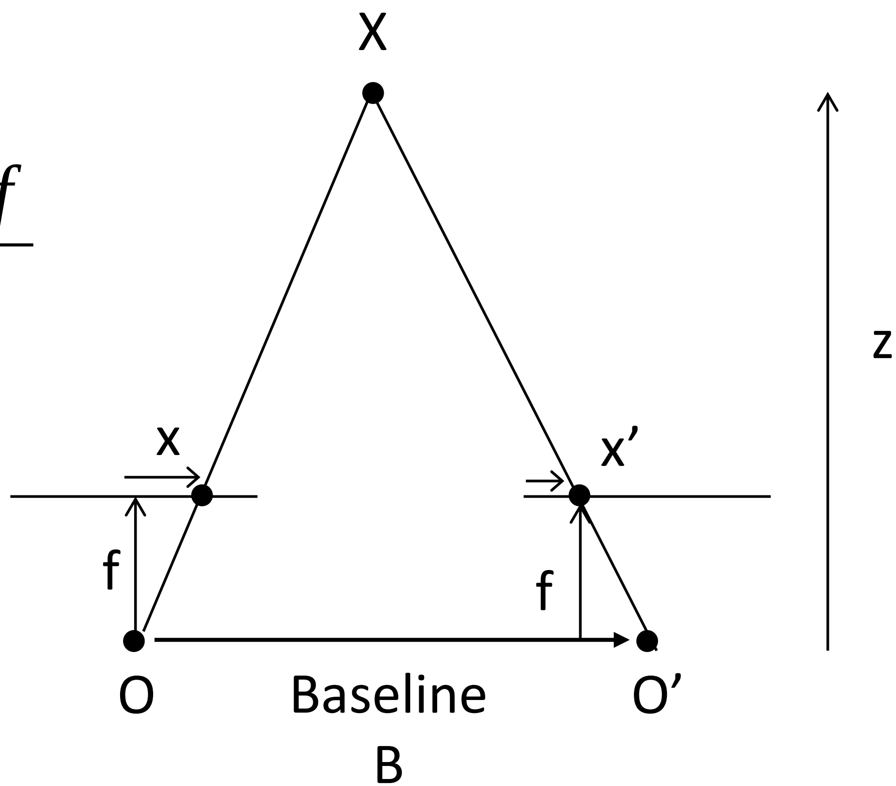
$$\frac{B - x + x'}{B} = \frac{z - f}{z}$$



Depth from disparity

$$\frac{B - x + x'}{B} = \frac{z - f}{z}$$

$$\Rightarrow \quad \frac{x - x'}{B} = \frac{f}{z}$$



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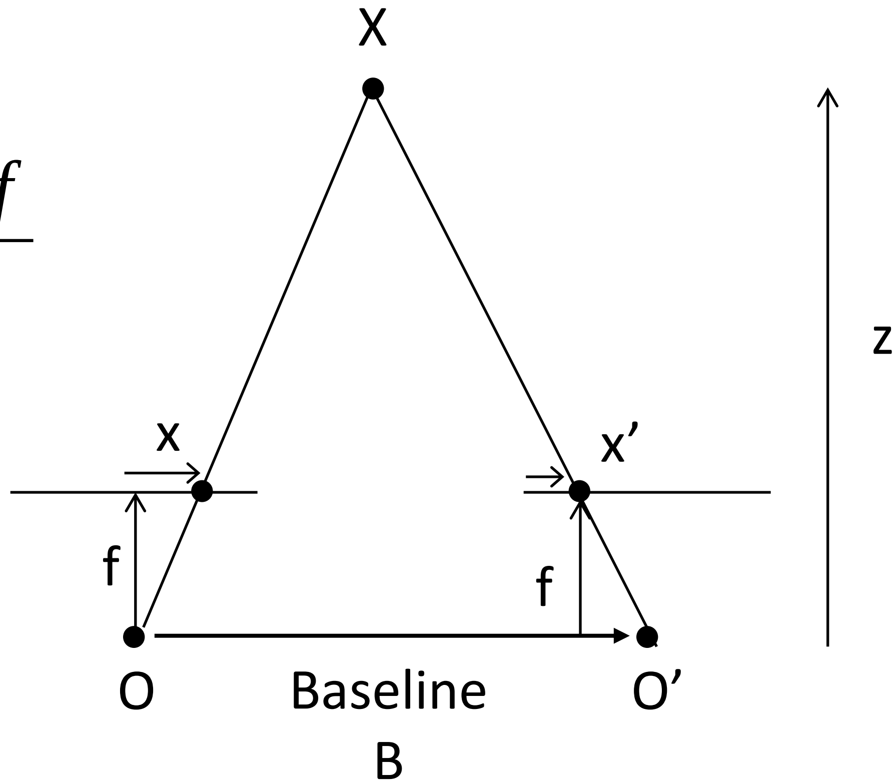
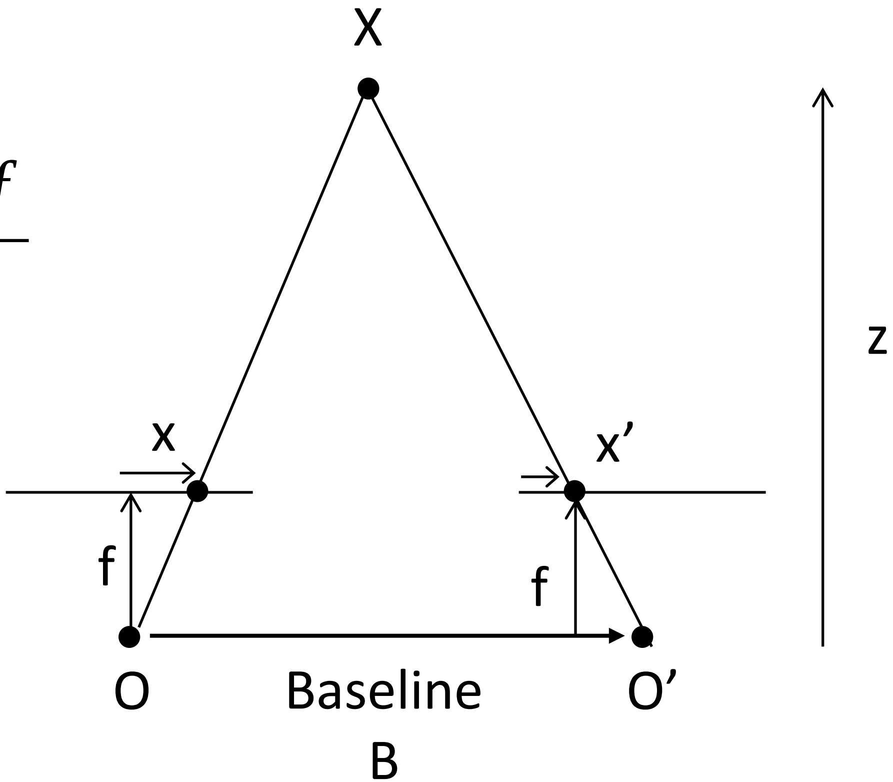


$$disparity = x - x' = \frac{B \cdot f}{z}$$

Depth from disparity

$$\frac{B - x + x'}{B} = \frac{z - f}{z}$$

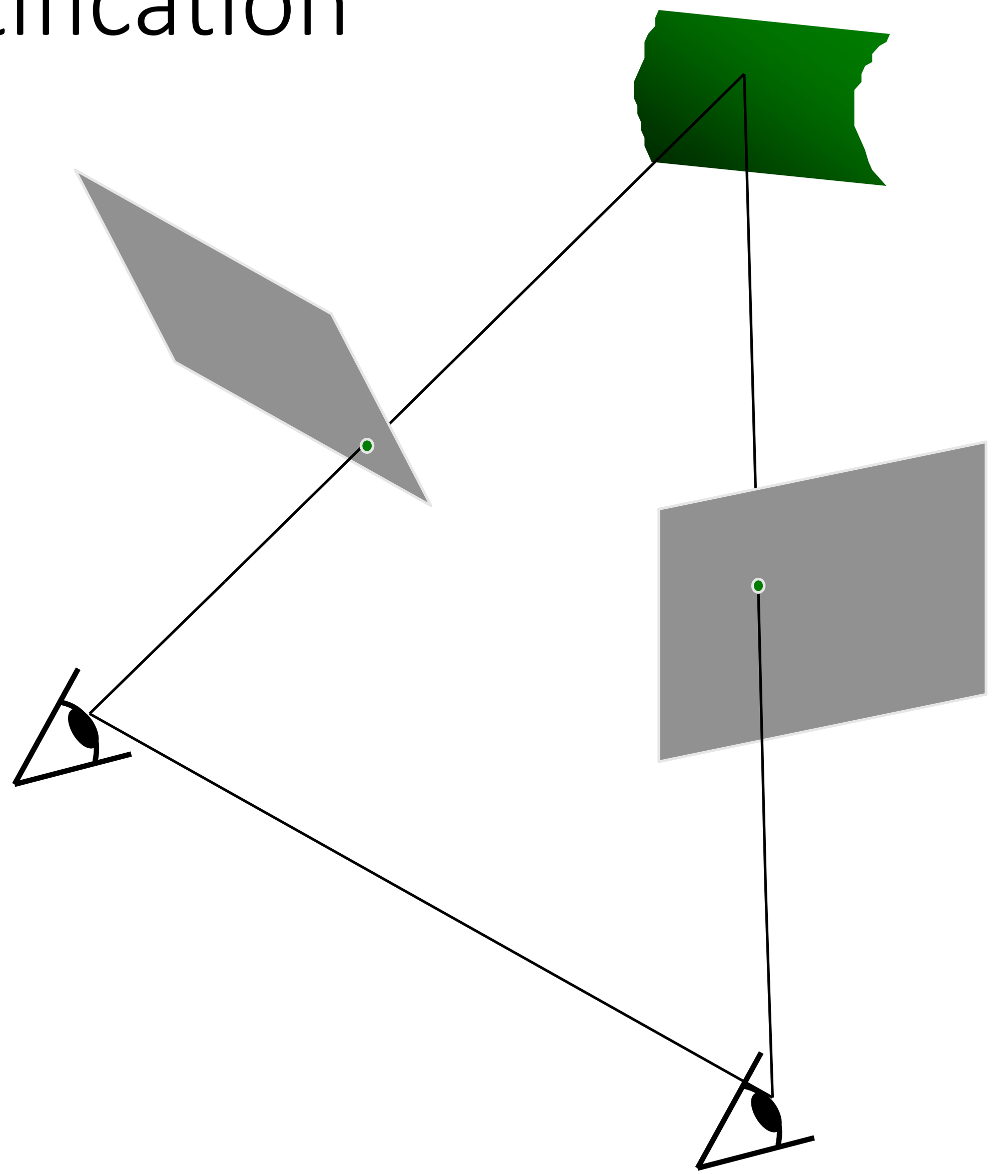
$$\Rightarrow \frac{x - x'}{B} = \frac{f}{z}$$



$$disparity = x - x' = \frac{B \cdot f}{z}$$

Disparity is inversely proportional to depth.

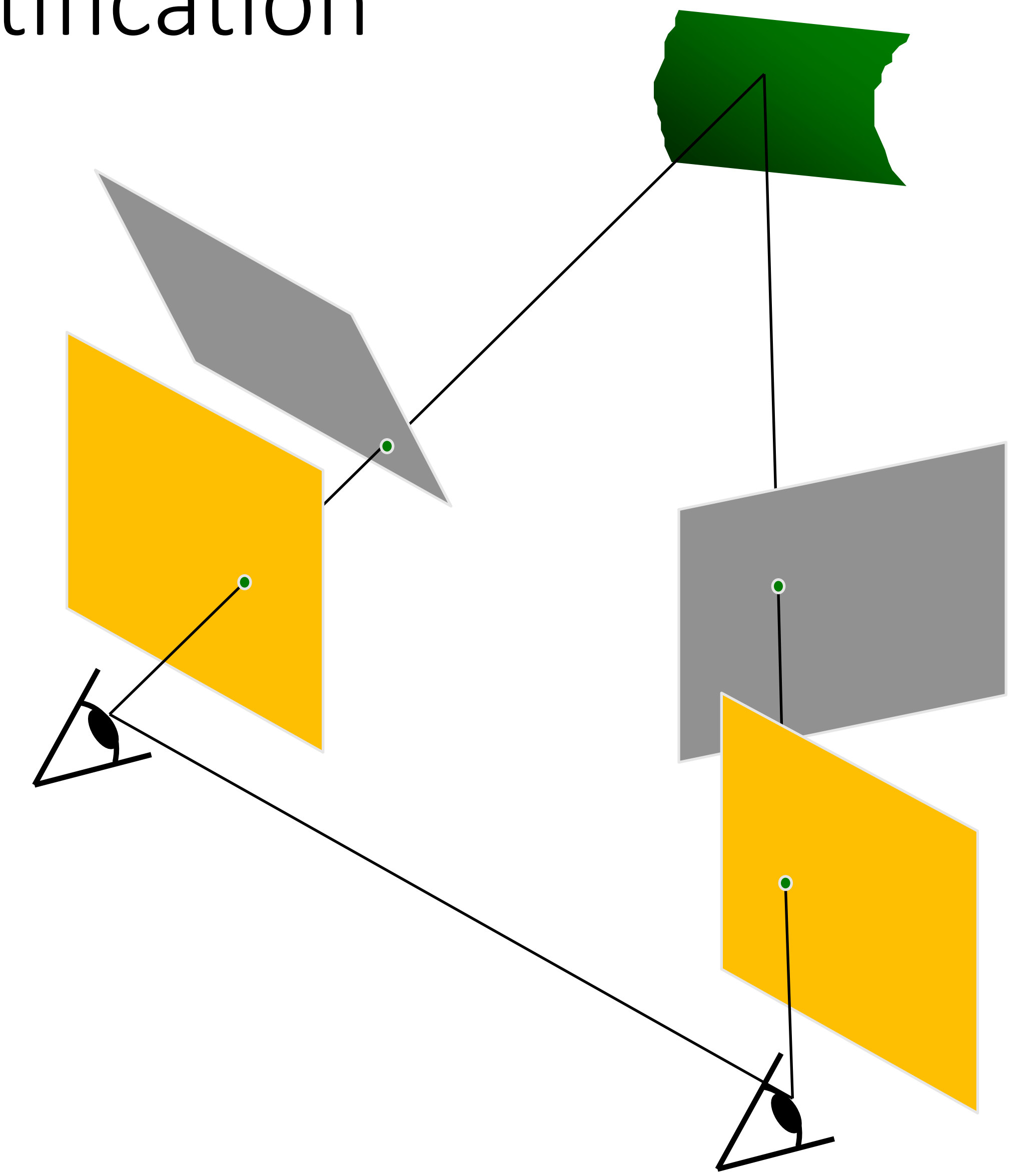
Stereo image rectification



C. Loop and Z. Zhang. [Computing Rectifying Homographies for Stereo Vision](). IEEE Conf. Computer Vision and Pattern Recognition, 1999.

Stereo image rectification

- Reproject image planes onto a common plane parallel to the line between camera centers



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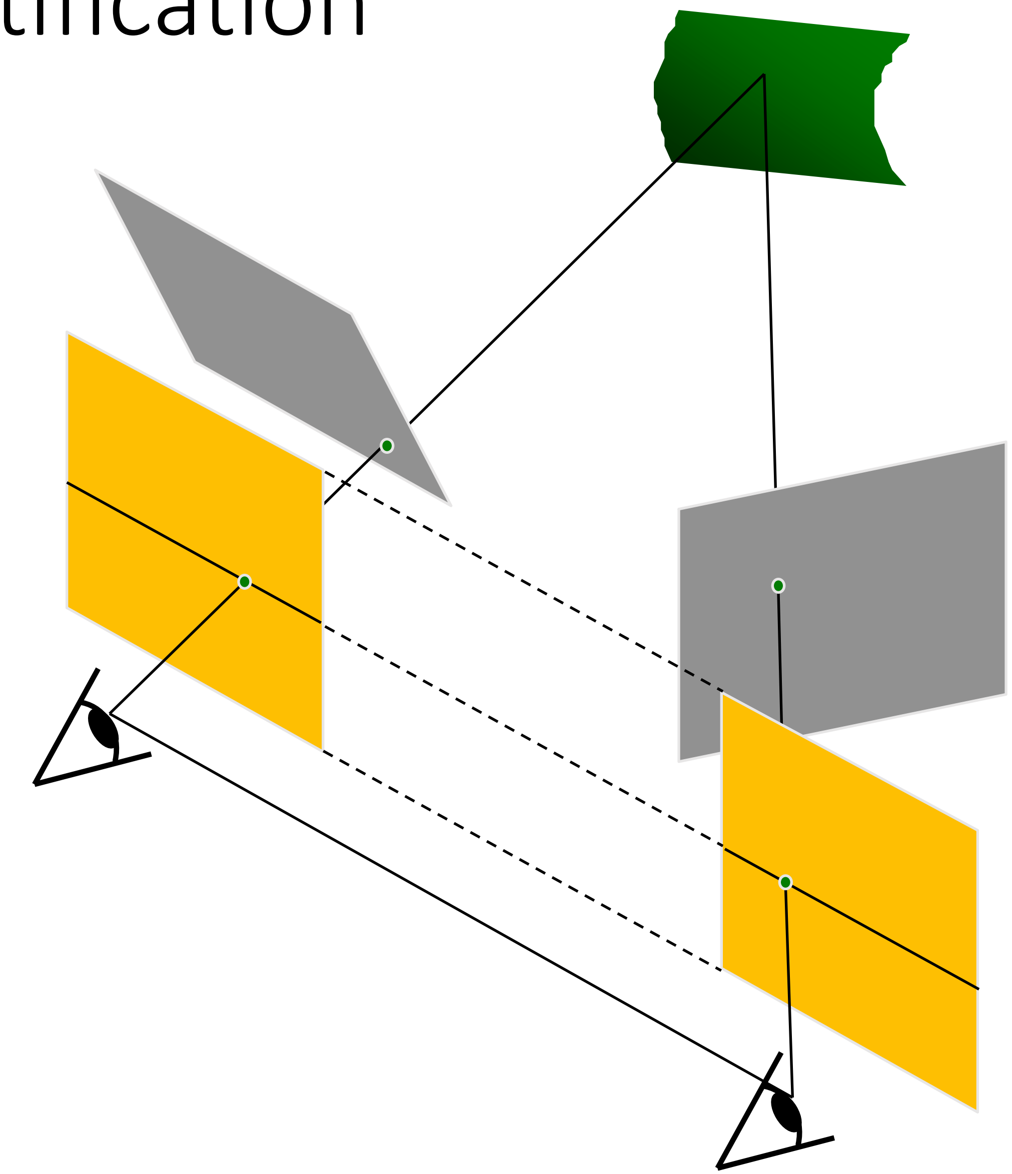
Stereo image rectification

- Reproject image planes onto a common plane parallel to the line between camera centers

- Pixel motion is horizontal after this transformation



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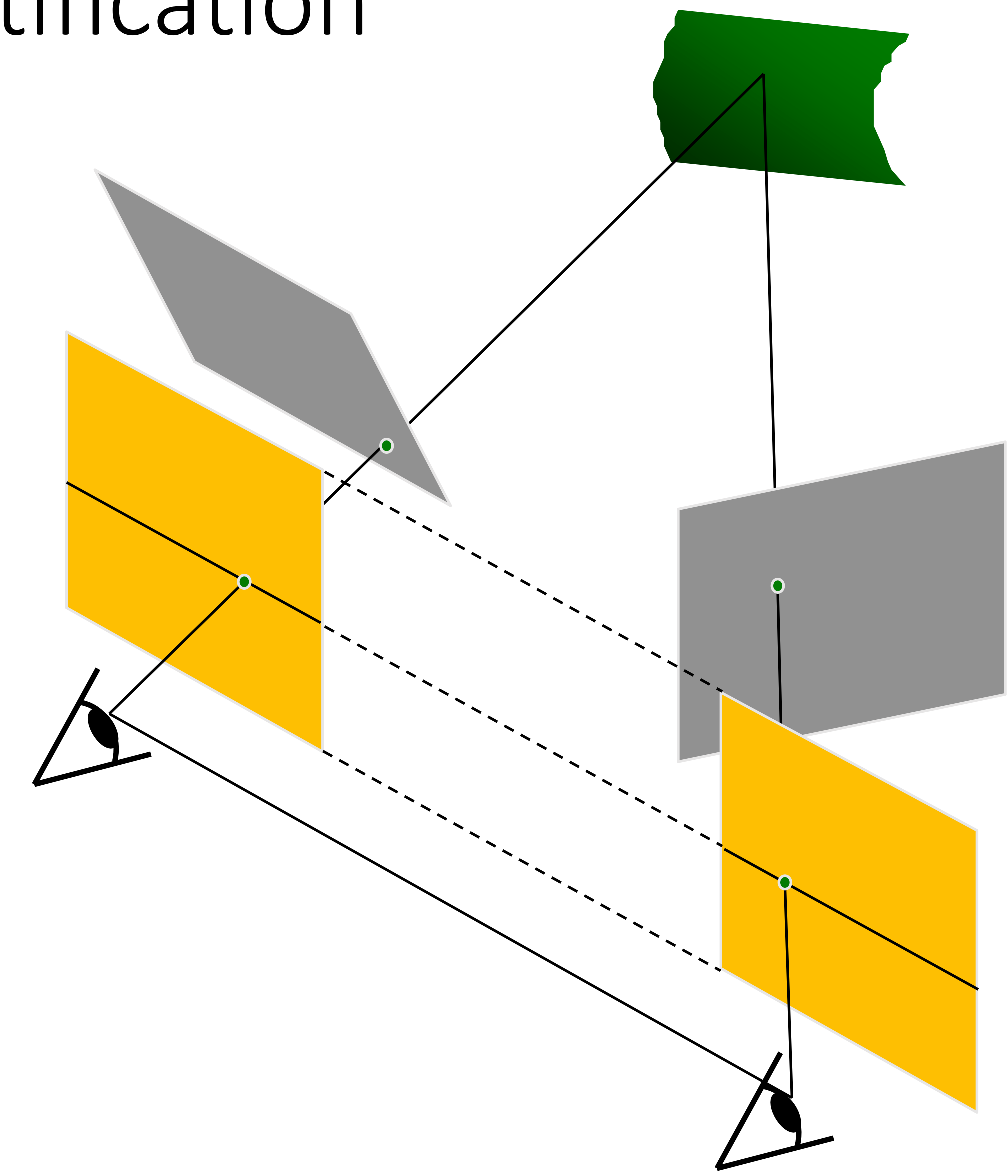
Stereo image rectification

- Reproject image planes onto a common plane parallel to the line between camera centers
- Pixel motion is horizontal after this transformation
- Two homographies (3x3 transform), one for each input image reprojection



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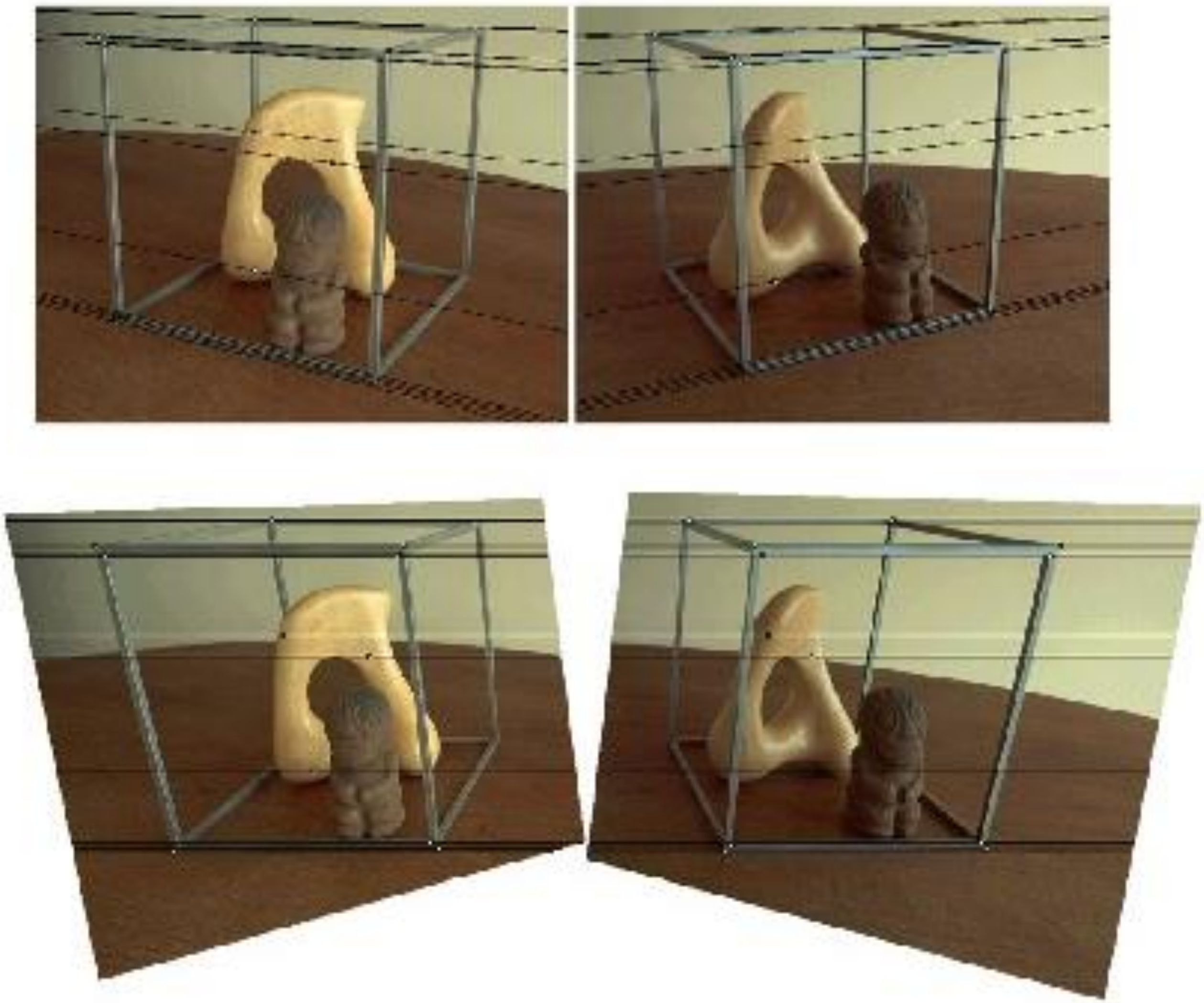
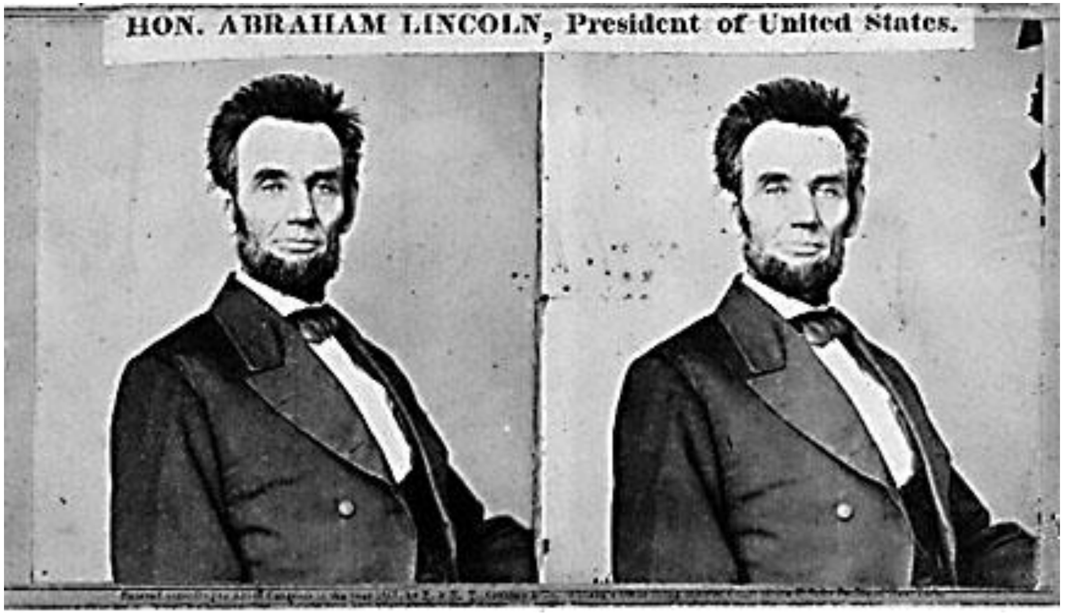
Rectification example



Basic stereo matching algorithm



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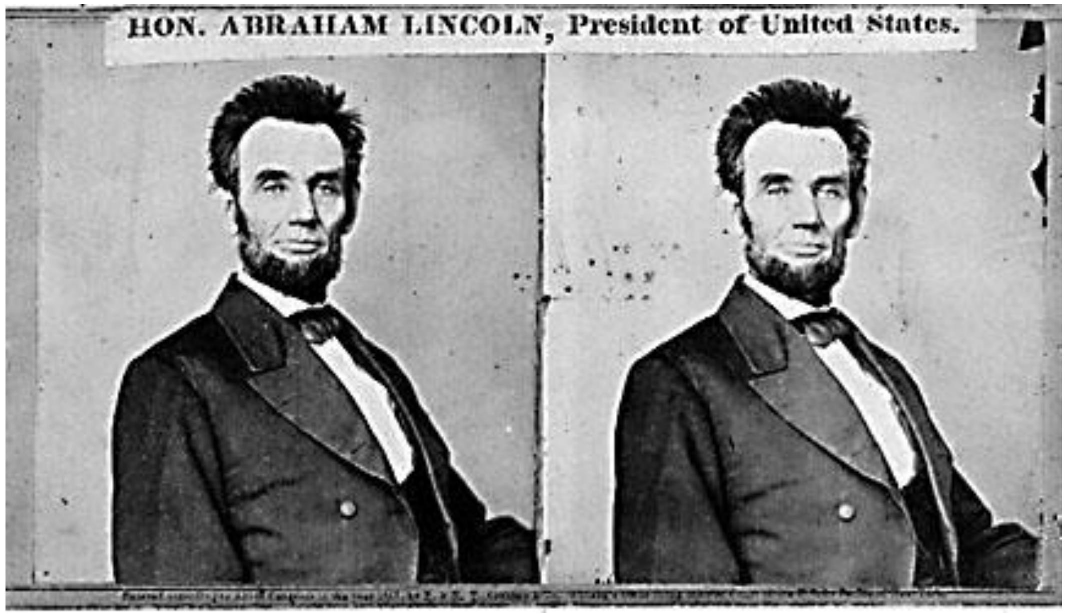

- If necessary, rectify the two stereo images to transform epipolar lines into scanlines

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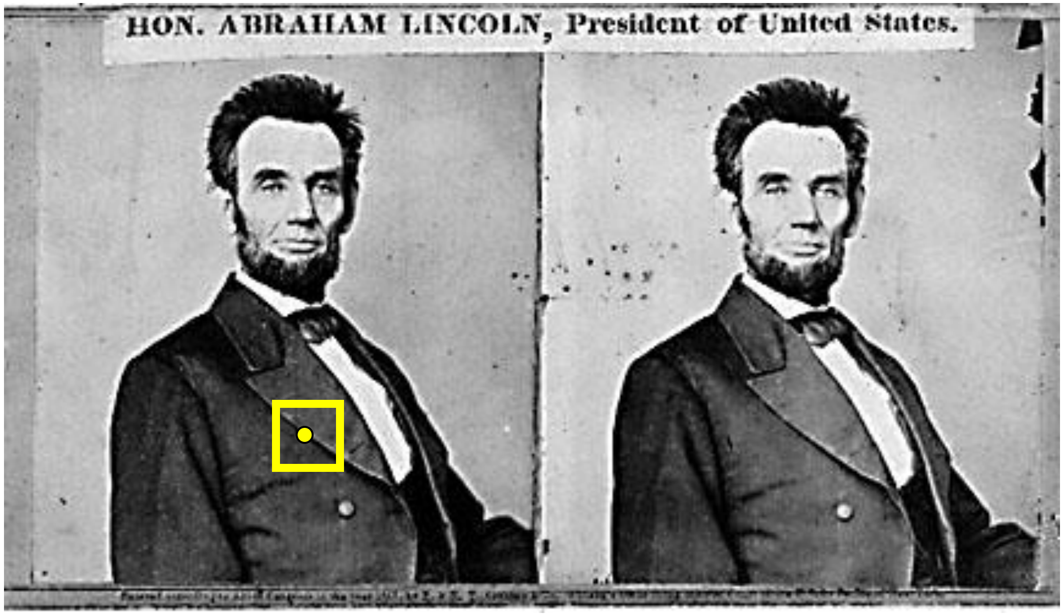
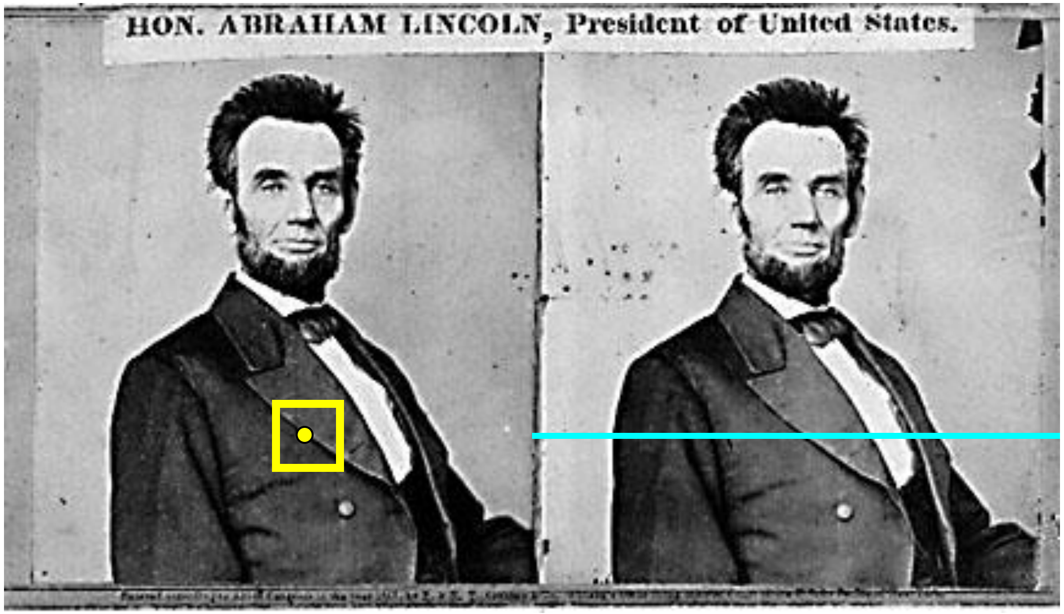


- If necessary, rectify the two stereo images to transform epipolar lines into scanlines
- For each pixel x in the first image

Basic stereo matching algorithm



- If necessary, rectify the two stereo images to transform epipolar lines into scanlines
- For each pixel x in the first image
  - Find corresponding epipolar scanline in the right image

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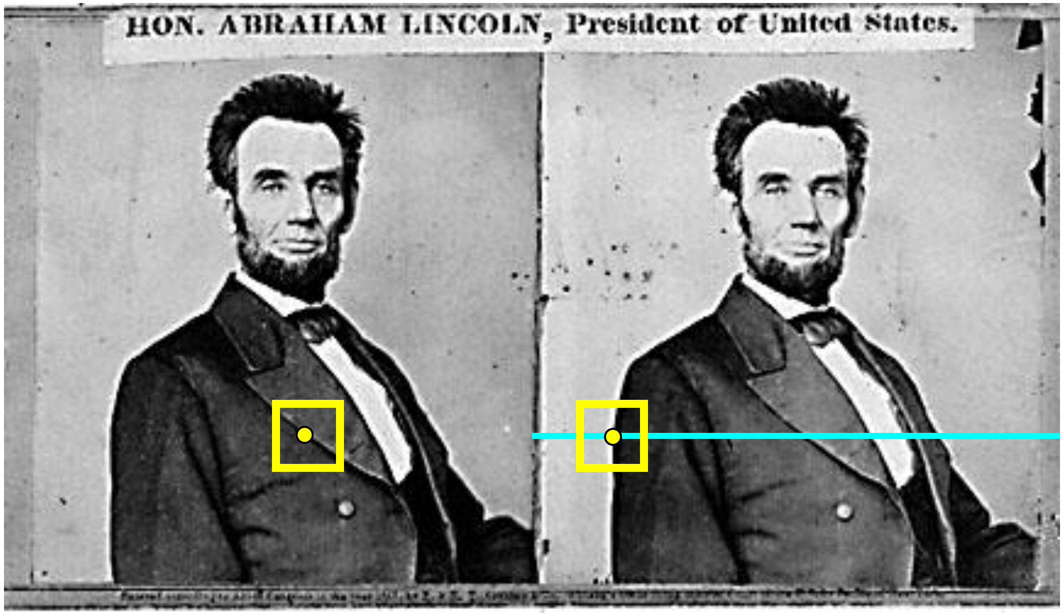

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- For each pixel x in the first image
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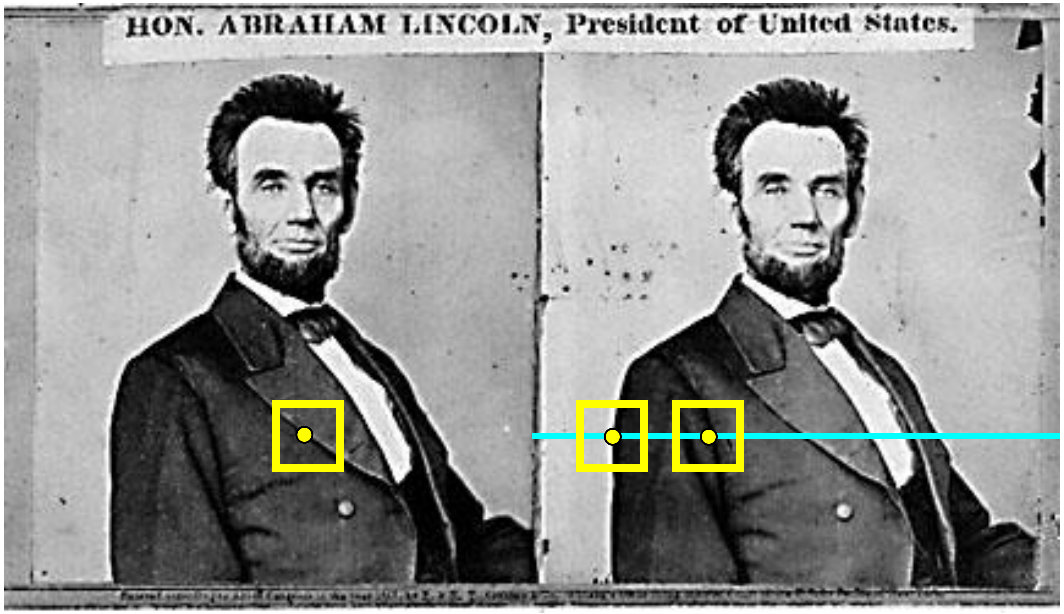
  - Search the scanline and pick the best match x’

Basic stereo matching algorithm



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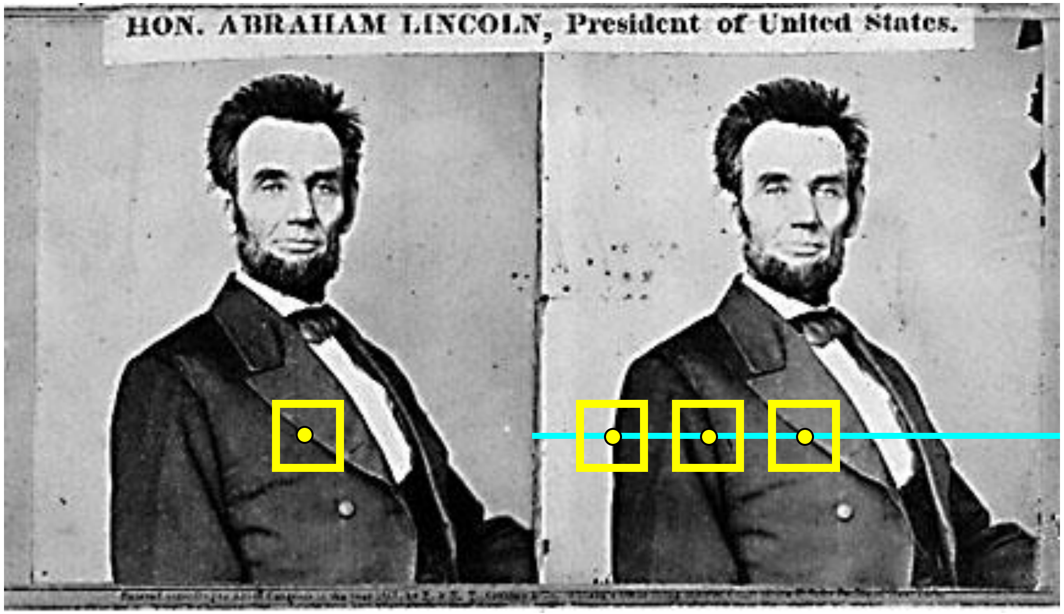

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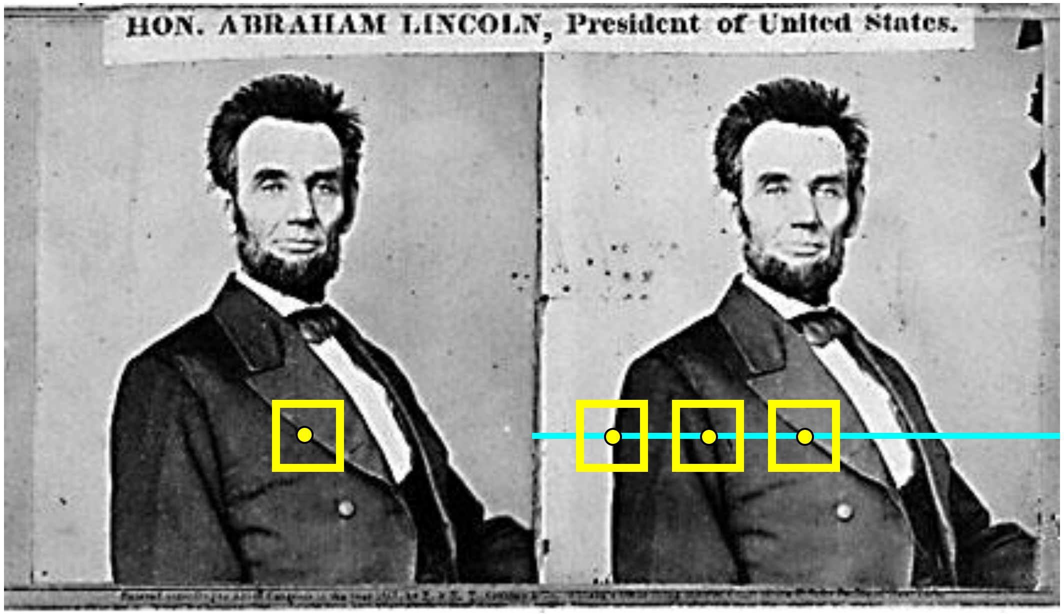
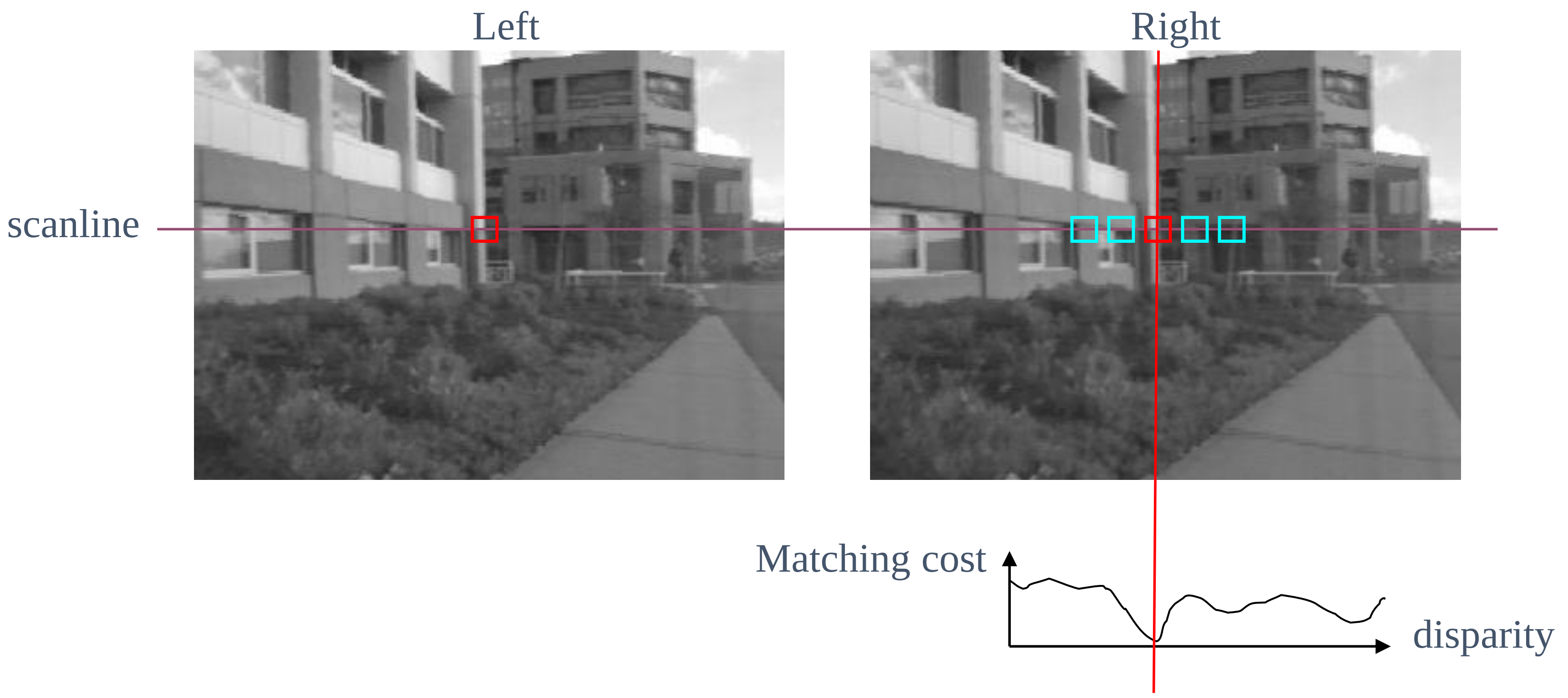


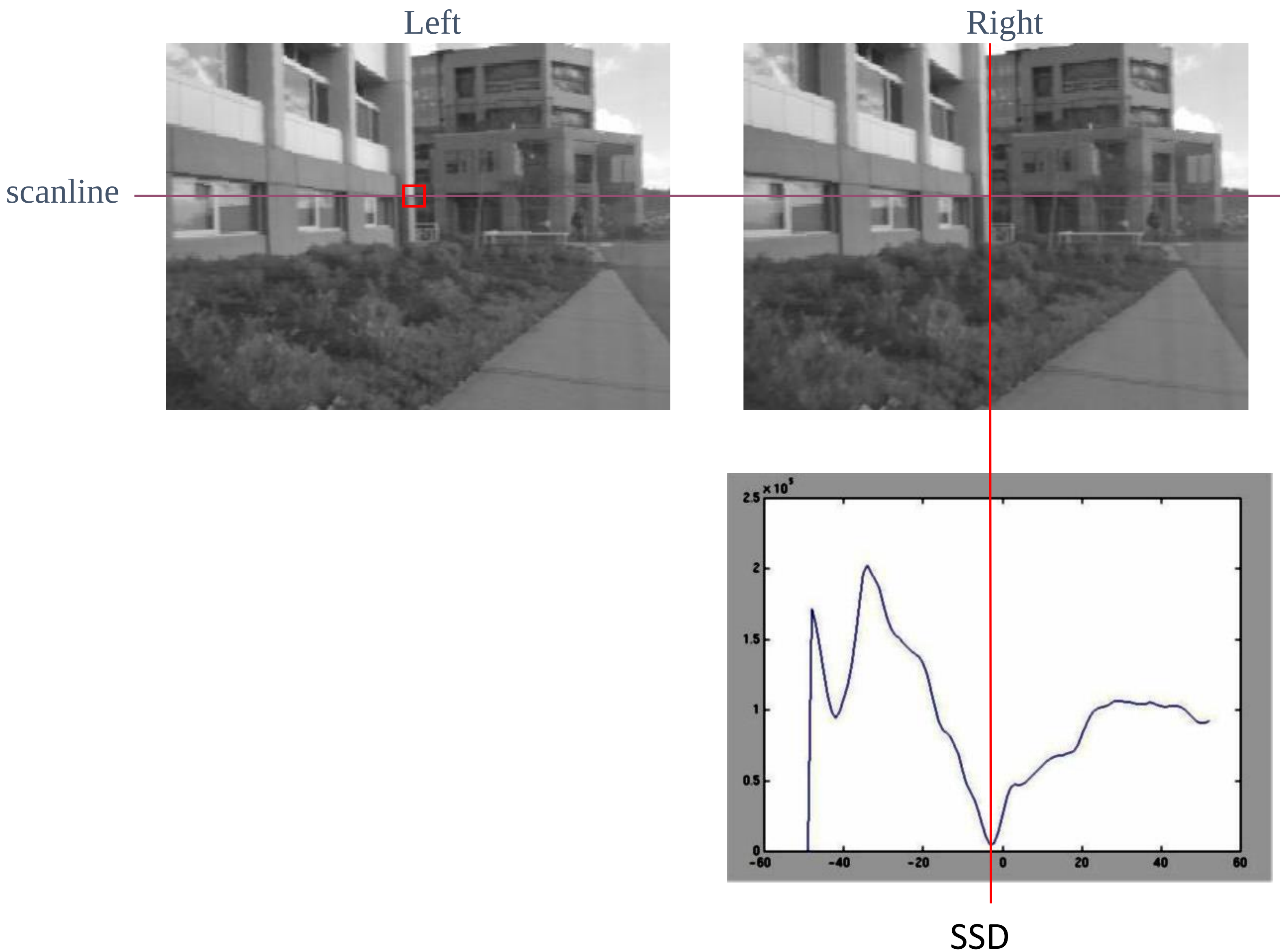
- If necessary, rectify the two stereo images to transform epipolar lines into scanlines
- For each pixel x in the first image
  - Find corresponding epipolar scanline in the right image
  - Search the scanline and pick the best match x'
  - Compute disparity x-x' and set depth(x) = fB/(x-x')

Correspondence search

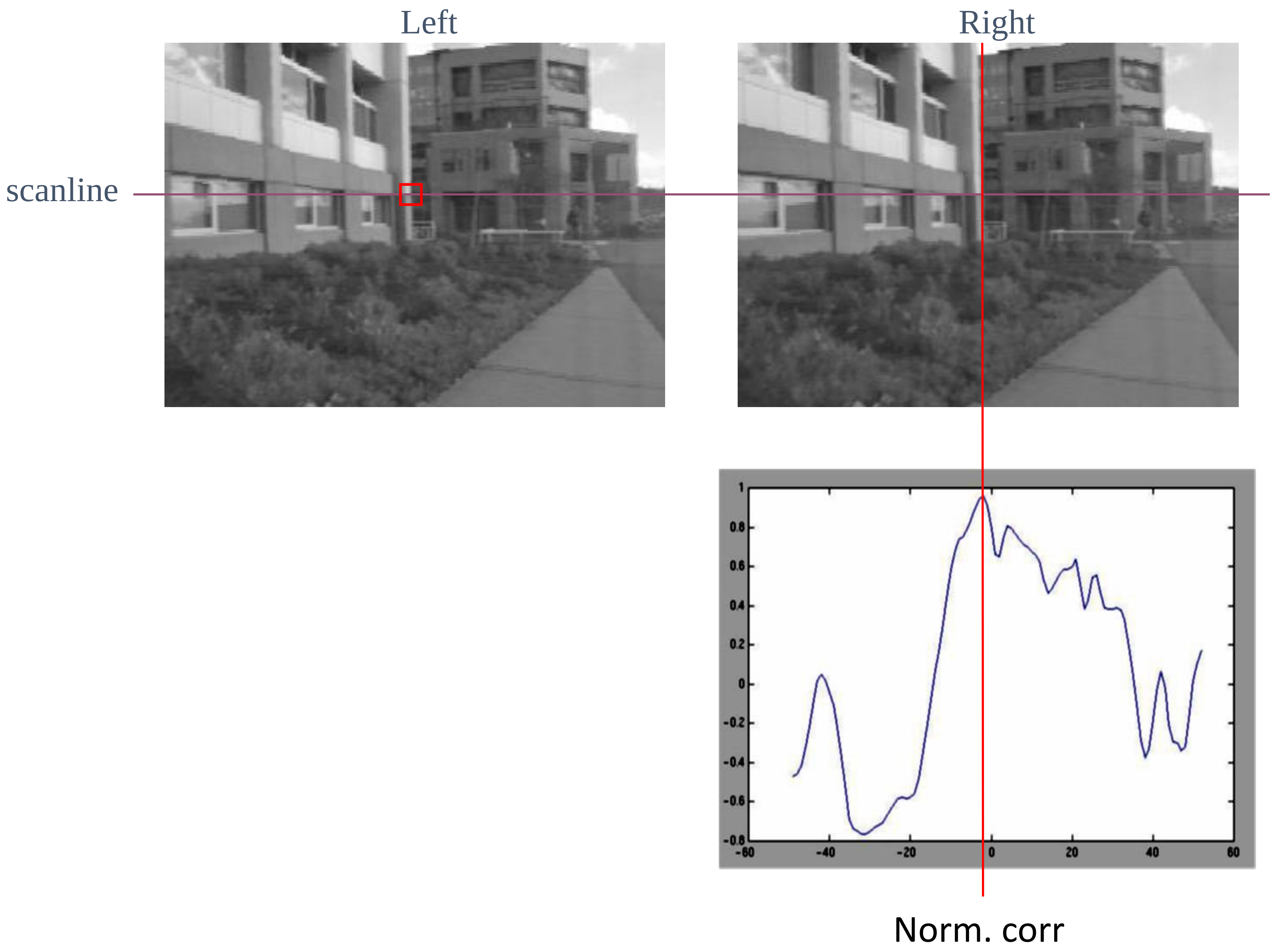


- Slide a window along the right scanline and compare contents of that window with the reference window in the left image
- Matching cost: SSD or normalized correlation

Correspondence search

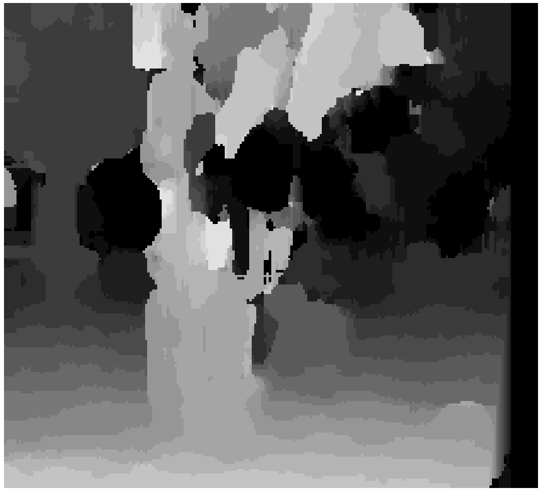


Correspondence search



Effect of window size

W = 3

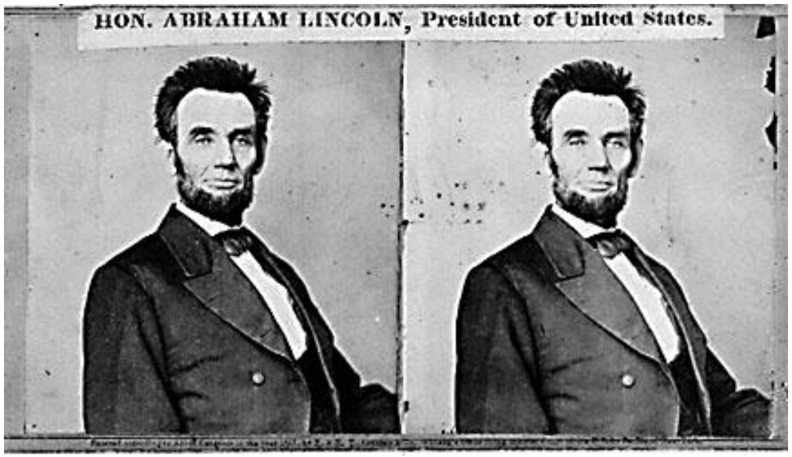


W = 20

- Smaller window
  - + More detail
  - – More noise
- Larger window
  - + Smoother disparity maps
  - – Less detail
  - – Fails near boundaries

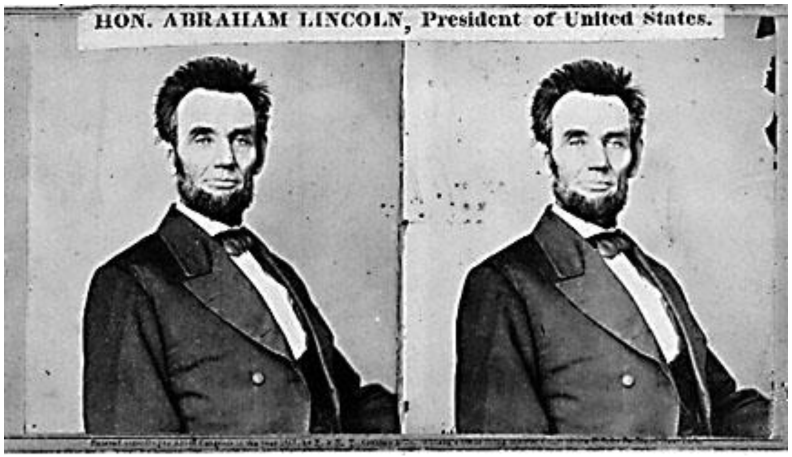
Failures of correspondence search

Failures of correspondence search



Textureless surfaces

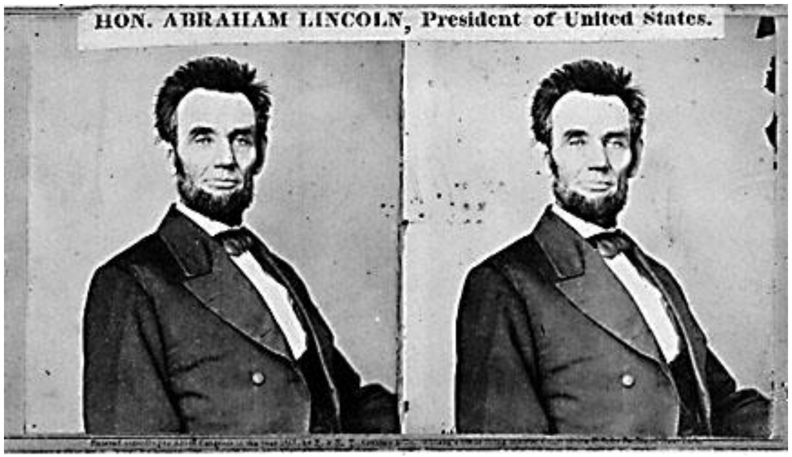
Failures of correspondence search



Textureless surfaces

Occlusions, repetition

Failures of correspondence search



Textureless surfaces

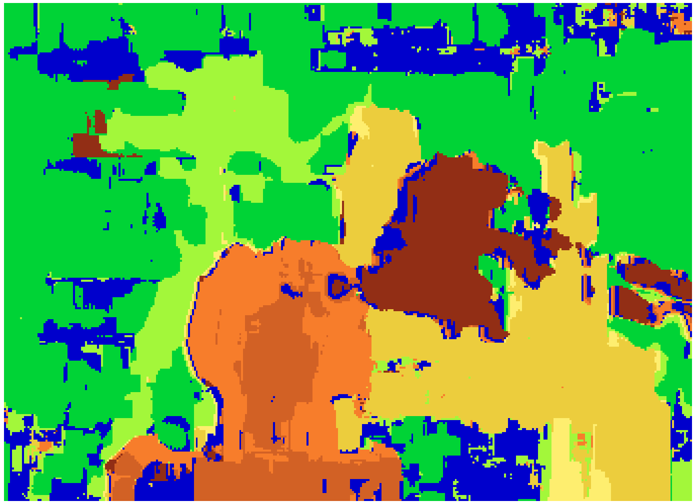
Occlusions, repetition

Non-Lambertian surfaces, specularities

Results with window search

Data

Window-based matching



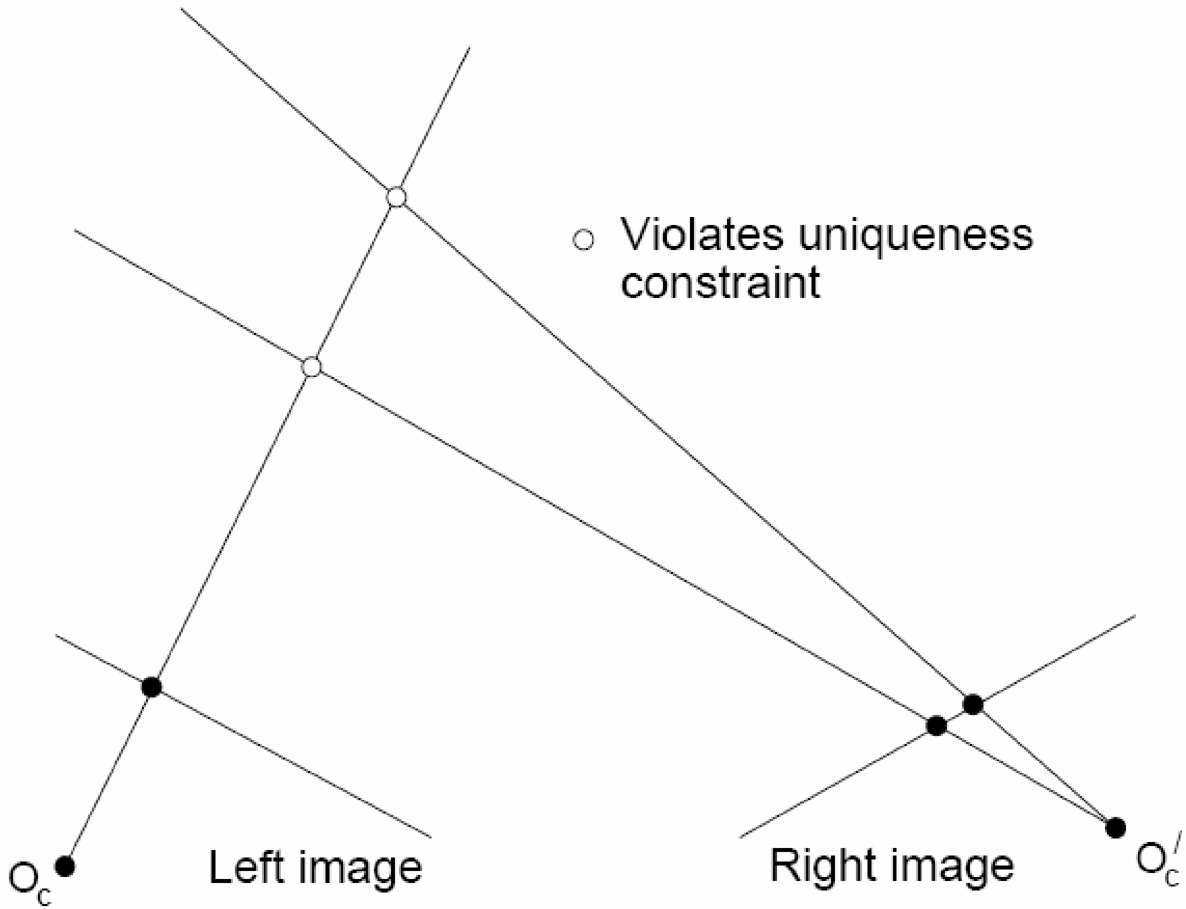
Ground truth

How can we improve window-based matching?

- So far, matches are independent for each point
- What constraints or priors can we add?

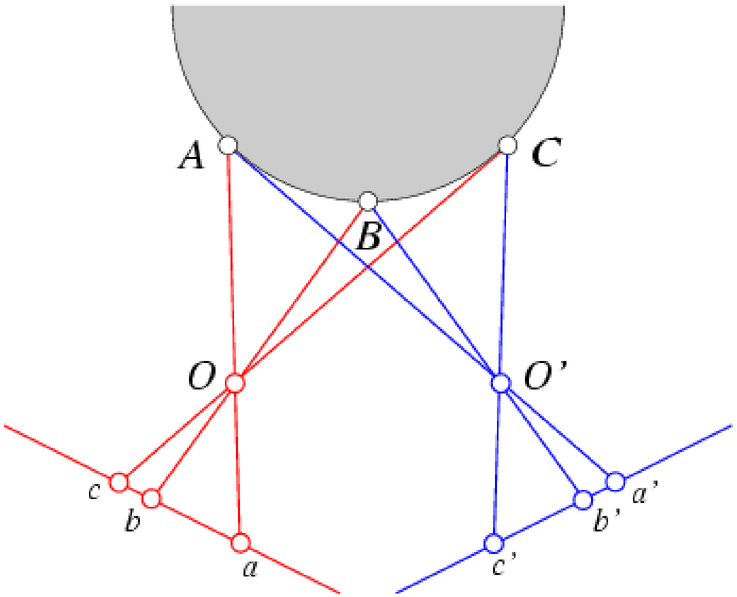
Stereo constraints/priors

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image



Stereo constraints/priors

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image
- Ordering
  - Corresponding points should be in the same order in both views



Stereo constraints/priors

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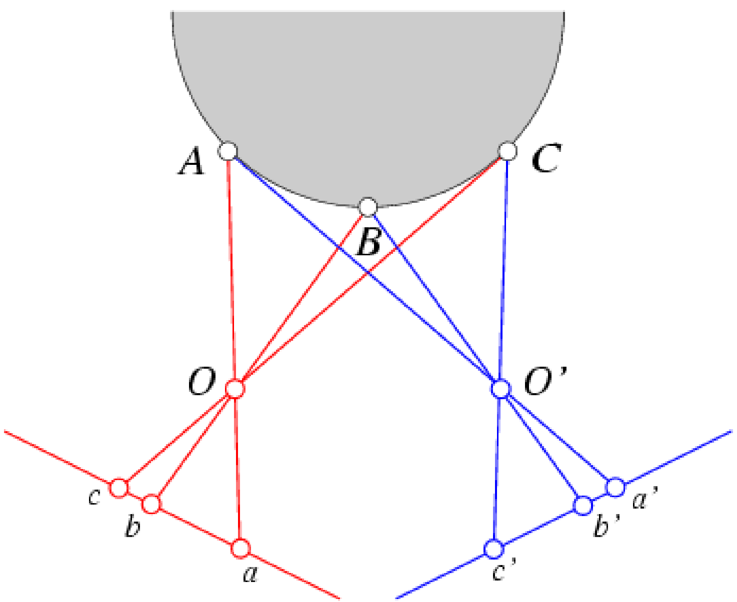
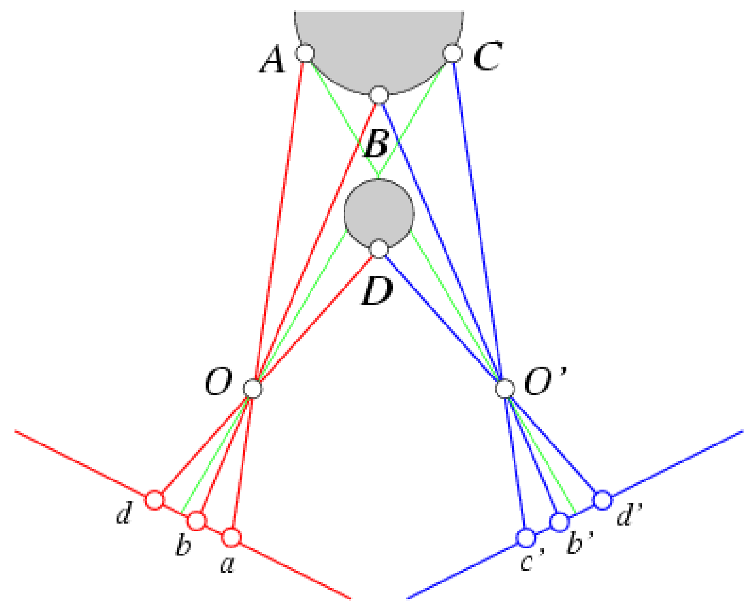


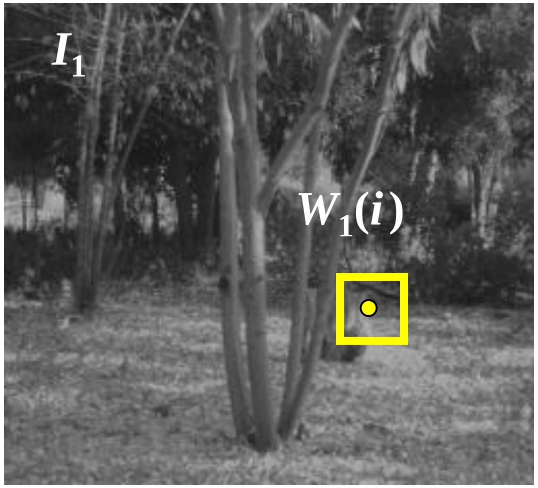


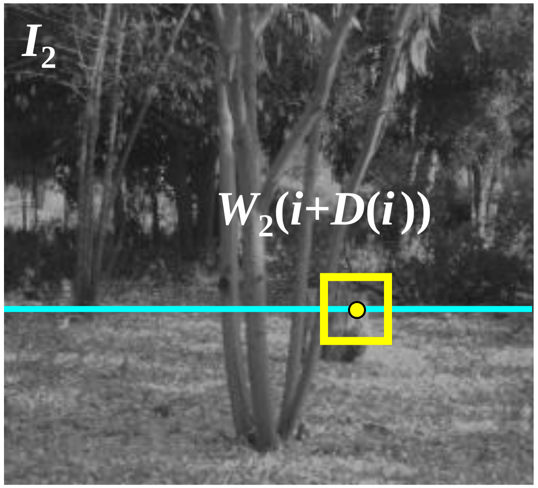
Ordering constraint doesn't hold

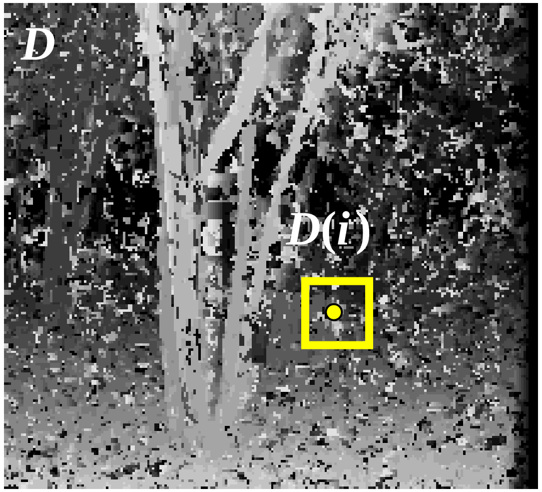
Stereo constraints/priors

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image
- Ordering
  - Corresponding points should be in the same order in both views
- Smoothness
  - We expect disparity values to change slowly (for the most part)

Stereo matching as energy minimization

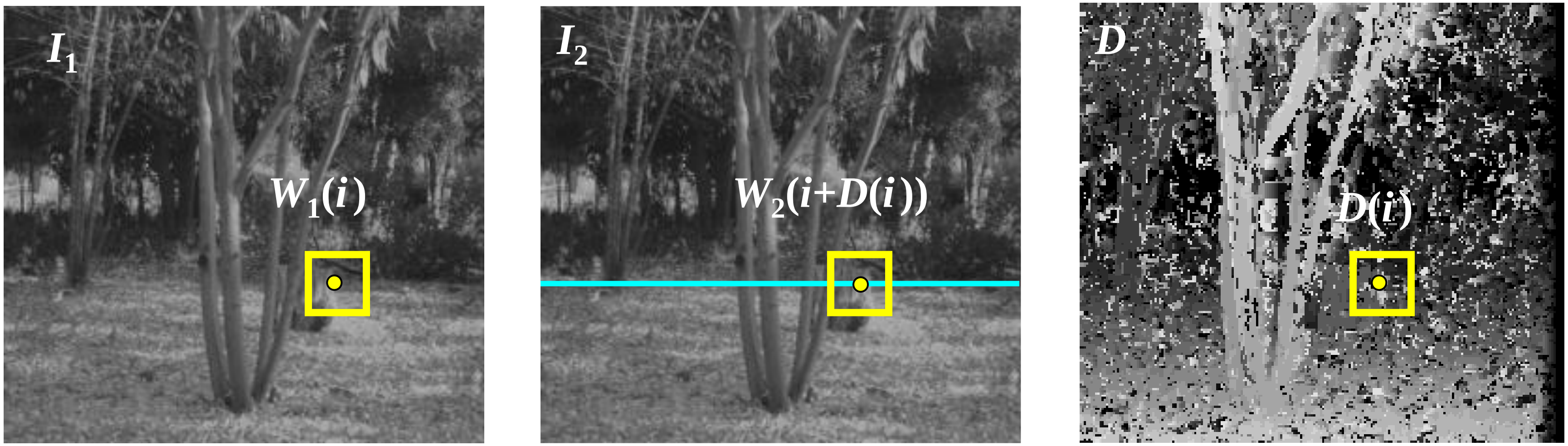






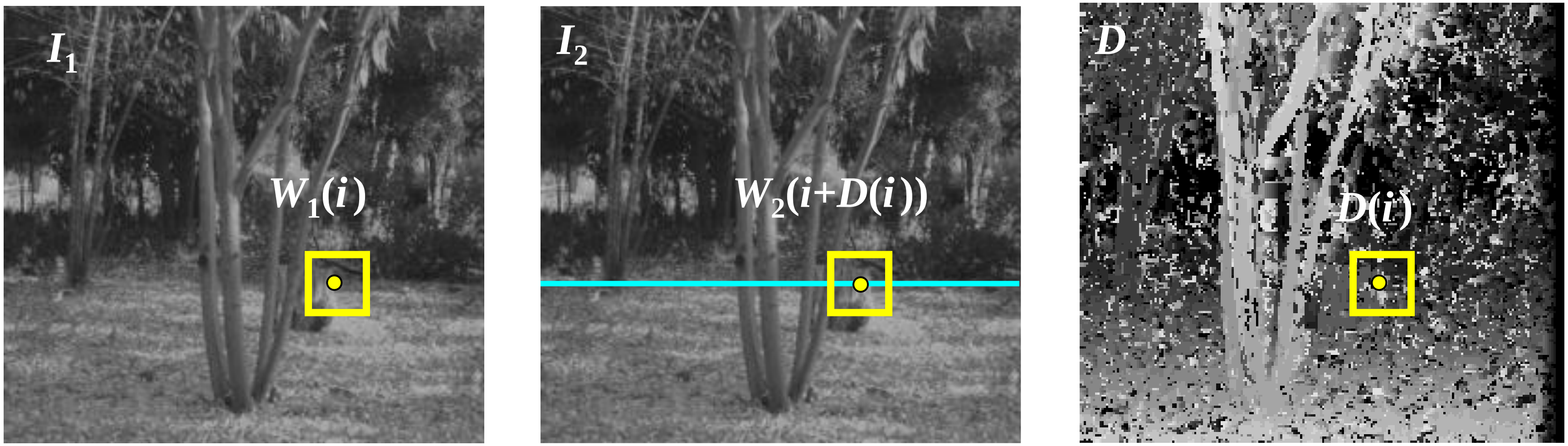
$$E_{\text{data}} = \sum_i \left(W_1(i) - W_2(i + D(i))\right)^2$$

Stereo matching as energy minimization



$$E_{\text{data}} = \sum_i \left(W_1(i) - W_2(i + D(i)) \right)^2 \qquad E_{\text{smooth}} = \sum_{\text{neighbors } i,j} \left\| D(i) - D(j) \right\|^2$$

Stereo matching as energy minimization



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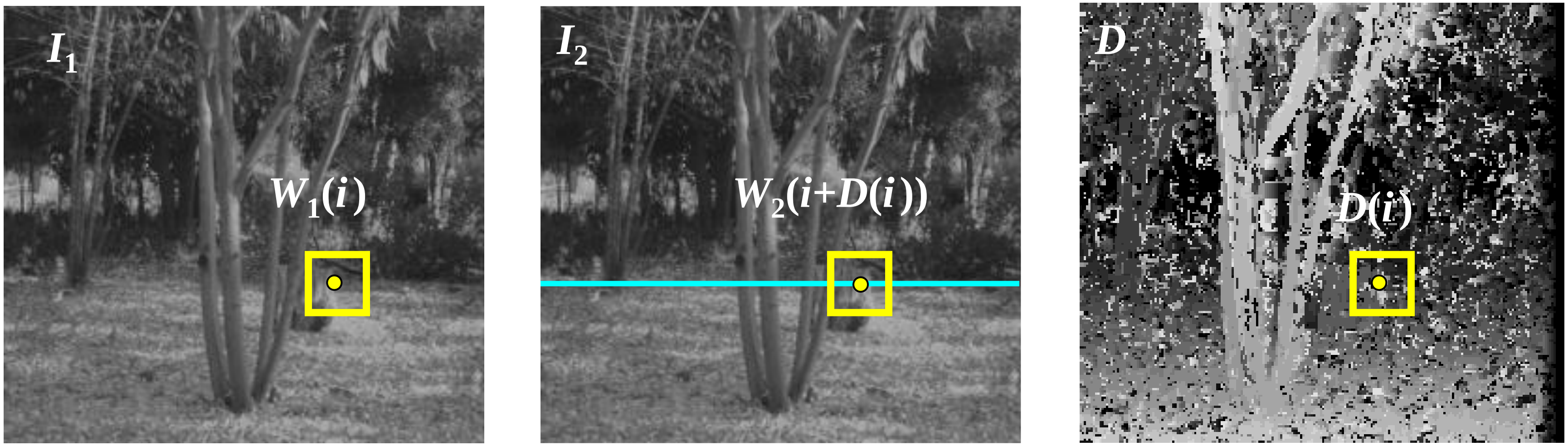
$$E = E_{\text{data}}(D; I_1, I_2) + \beta E_{\text{smooth}}(D)$$

Stereo matching as energy minimization



$$E_{\text{data}} = \sum_i \left(W_1(i) - W_2(i + D(i))\right)^2 \qquad E_{\text{smooth}} = \sum_{\text{neighbors } i,j} \left\| D(i) - D(j) \right\|^2$$

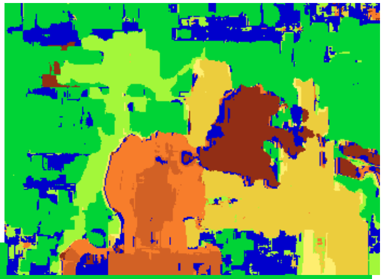
$$E = E_{\text{data}}(D; I_1, I_2) + \beta E_{\text{smooth}}(D)$$

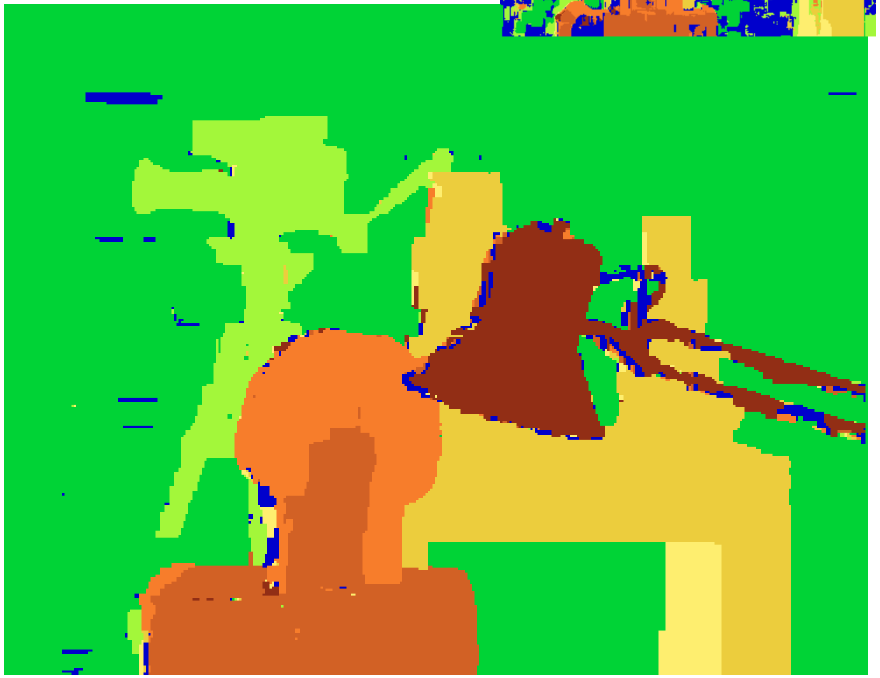
- Energy functions of this form can be minimized using *graph cuts*

Y. Boykov, O. Veksler, and R. Zabih, [Fast Approximate Energy Minimization via Graph Cuts](), PAMI 2001

Many of these constraints can be encoded in an energy function and solved using graph cuts

Before





Graph cuts

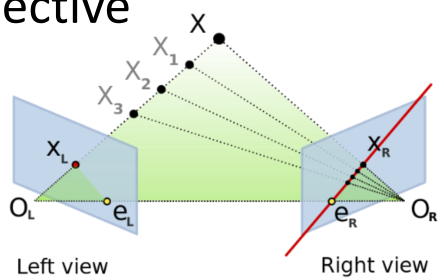
Ground truth

Y. Boykov, O. Veksler, and R. Zabih, [Fast Approximate Energy Minimization via Graph Cuts](), PAMI 2001

For the latest and greatest: http://www.middlebury.edu/stereo/

Things to remember

- **Epipolar geometry**
  - Epipoles are intersection of baseline with image planes
  - Matching point in second image is on a line passing through its epipole
  - Fundamental matrix maps from a point in one image to a line (its epipolar line) in the other
  - Can solve for F given corresponding points (e.g., interest points)
  - Can recover canonical camera matrices from F (with projective ambiguity)



- **Stereo depth estimation**
  - Estimate disparity by finding corresponding points along scanlines
  - Depth is inverse to disparity

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  - And many more ……….

Next class:

structure from motion