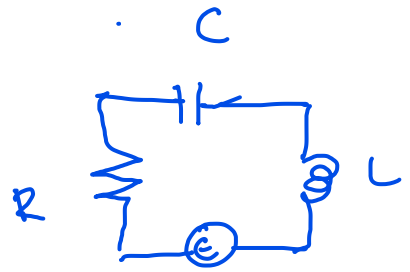
03.03.2022. Electric circuits.



Let a source of emf be connected in series with a capacitor, a coil and a resistor as shown in figure.

I – current

By Ohm's law, the voltage drop across the resistance R is IR.

By the Law of Faraday, the voltage drop across the inductance L is $L\frac{dI}{dt}$.

If V is the voltage across the capacitor and $E(t)$ is the voltage across the source, then the voltage equn is

$$L\frac{dI}{dt} + RI + V = E(t) \quad \text{—(i)}.$$

If C is the capacitance, it is known that the charge Q on a capacitor plate satisfies the relation $Q = CV$ (or) $\frac{Q}{C} = V$.

$$\text{(i)} \longrightarrow L\frac{dI}{dt} + RI + \frac{Q}{C} = E(t) \quad \text{—(ii)}$$

But by definition $I = \frac{dQ}{dt}$

$$\therefore L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E(t)$$ ✓

If $E(t)=0$, electrical vibrations of the circuit are free.

Auxiliary eqn. is $Lm^2+Rm+\frac{1}{C}=0$

$$m=\frac{-R\pm\sqrt{R^2-4L/C}}{2L}$$

(i) Over damped if $R^2-\frac{4L}{C}>0$

(ii) Critically damped if $R^2-\frac{4L}{C}=0$

(iii) under damped if $R^2-\frac{4L}{C}<0$.

1. Find the charge $q(t)$ on the capacitor in an LCR circuit when $L=0.25$ henry (h), $R=10$ ohms (Ω), $C=0.001$ farad, $E(t)=0$, $q(0)=q_0$ coulombs (C), $i(0)=0$.

$C=0.001 \Rightarrow \frac{1}{C}=1000$

Eqn.: $\frac{1}{4}q''+10q'+1000q=0 \Rightarrow q''+40q'+4000q=0$.

$m^2+40m+4000=0 \qquad m=-20\pm 60i$

$$q=e^{-20t}(c_1\cos 60t+c_2\sin 60t) \quad \rightarrow (1).$$

When $t=0$, $q=q_0$ Sub

$$q_0=e^0(c_1\cos 0+c_2\sin 0) \Rightarrow c_1=q_0.$$

$i=0$ when $t=0$ (i.e) $\frac{dq}{dt}=0$ when $t=0$.

Diff (1).

$$\frac{dq}{dt} = e^{-20t}(-60C_1\sin 60t + 60C_2\cos 60t) + (C_1\cos 60t + C_2\sin 60t)(-20e^{-20t}).$$

$i(0)=0$ | $i(t)=0/t=0$ | $\left(\frac{dq}{dt}\right)_{t=0} = 0.$

$$0 = 1(60)C_2 + C_1(-20)(1).$$

$$\Rightarrow +20C_1 = 60C_2 \Rightarrow 20q_0 = 60C_2 \Rightarrow C_2 = \tfrac{1}{3}q_0$$

$$\therefore q(t) = e^{-20t}\left(q_0\cos 60t + \tfrac{1}{3}q_0\sin 60t\right)$$

$$= q_0 e^{-20t}\left(\cos 60t + \tfrac{1}{3}\sin 60t\right).$$