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Second order linear non-homogeneous differential equⁿ with const. coeffts.

1. $y'' - 4y = 8x^2 - 2x \quad —(1)$

Auxiliary equⁿ: $m^2 - 4 = 0$

$m^2 = 4 \quad , m = \pm 2.$

$m_1 = 2$
$m_2 = -2$

The roots are real and distinct.

$8x^2 - 2x$

$16x - 2$

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complementary fn: $c_1 e^{2x} + c_2 e^{-2x}$

mtd of Undetermined coeffts:

$\therefore$ Let the initial Solution be $y_p = ax^2 + bx + c$

$a_0 \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = f(x)$

$y_p' = 2ax + b$

$y_p'' = 2a$

$2a - 4(ax^2 + bx + c) = 8x^2 - 2x$

$-4ax^2 - 4bx + (2a - 4c) = 8x^2 - 2x$

coeffts of x^2: $-4a = 8 \Rightarrow a = -2$

Coeffts of x: $-4b = -2 \Rightarrow b = 1/2$

const. terms: $2a - 4c = 0$

$2a = 4c \quad$ (or) $-4 = 4c \Rightarrow c = -1.$

$y_p = -2x^2 + \frac{1}{2}x - 1$

$X \begin{cases} y_p' = -4x + 1/2 \\ y_p'' = -4 \end{cases}$

$y_p'' - 4y_p = -4 - 4(-2x^2 + 1/2 x - 1)$

$= 8x^2 - 2x$

Complete solution = Comp fn + PI

$$y = \underbrace{c_1 e^{2x} + c_2 e^{-2x}} \underbrace{- 2x^2 + 1/2 - 1}.$$

2. $y = y_h + y_{PI}$.

Solve $y'' + 2y' - 3y = 4e^{2x}$ — (1).

Auxiliary eqn, $m^2 + 2m - 3 = 0$. ← To find y_h.

$(m+3)(m-1) = 0$

$m = -3, 1$. The roots are real & distinct.

$\therefore \quad y_h = c_1 e^{-3x} + c_2 e^{x}$.

To find y_p: Let the initial soln. be $y_p = ce^{2x}$

$y_p' = 2ce^{2x} \qquad y_p'' = 4ce^{2x}$.

Sub in (1). $4ce^{2x} + 2(2ce^{2x}) - 3ce^{2x} = 4e^{2x}$.

$5ce^{2x} = 4e^{2x}$

$\Rightarrow c = 4/5$

Thus $y_p = \frac{4}{5} e^{2x}$

GS = CF + PI

$y = y_h + y_p = c_1 e^{-3x} + c_2 e^{x} + \frac{4}{5} e^{2x}$.

$m^2+2m-3=0$ $\quad (m+3)(m-1)=0$ $\quad m=-3, \boxed{+1}$ $\quad c_1e^{-3x}+c_2e^{x}$

3. $y''+2y'-3y=8e^{x}$

$y_p = ce^{x}$

$y_p' = ce^{x}$

$y_p'' = ce^{x}$

$ce^{x}+2ce^{x}-3ce^{x} = 0$

Let the initial soln. be $cxe^{x} = y_p$

$$y_p' = c\left(xe^{x}+e^{x}.1\right)$$

$$= ce^{x}(x+1)$$

$$y_p'' = c\left[e^{x}(1)+(x+1)e^{x}\right]$$

$$= ce^{x}+cxe^{x}+ce^{x}$$

$$= 2ce^{x}+cxe^{x} = ce^{x}(x+2).$$

$$(x+2)ce^{x}+2ce^{x}(x+1)-3cxe^{x} = 8e^{x}$$

$$ce^{x}\left[x+2+2x+2-3x\right] = 8e^{x}$$

$$\Rightarrow ce^{x}[4] = 8e^{x} \Rightarrow 4c=8 \Rightarrow c=2.$$

$$\therefore y_p = 2xe^{x}$$

$$y = y_h+y_p = c_1e^{-3x}+c_2e^{+x}+2xe^{x}.$$

4.