23.02.2022.

1. Solve $y'' - 6y' + 9y = 5e^{3x}$ (1)

A.E $m^2 - 6m + 9 = 0 \Rightarrow (m-3)^2 = 0 \Rightarrow m = 3$ is a double root.

$y_{PI} = ce^{3x}$ $y'' = 9ce^{3x}$ $y' = 3ce^{3x}$

$(9c - 18c + 9c)e^{3x} = 0 \neq 5e^{3x}$ ✓.

$y_{PI_2} = cxe^{3x}$ on substitution LHS $= 0 \neq$ RHS

C.F or hom solution $y_h = (c_1 + c_2x)e^{3x}$.

To find the P.I

let the proposed solution be $y_{PI} = cx^2e^{3x}$

$$y' = c\left[x^2 \cdot 3e^{3x} + e^{3x} \cdot 2x\right]$$

$$= c(3x^2 + 2x)e^{3x}$$

$$y'' = c\left[(3x^2+2x) \cdot 3e^{3x} + e^{3x}(6x+2)\right]$$

LHS of (1) $= c\left[(3x^2+2x)3e^{3x} + e^{3x}(6x+2)\right] - 6\left[c(3x^2+2x)e^{3x}\right] + 9\left[cx^2e^{3x}\right]$

$$= ce^{3x}\left[9x^2 + 6x + 6x + 2 - 18x^2 - 12x + 9x^2\right]$$

$$= 2ce^{3x}.$$

RHS (1) $= 5e^{3x}$. Equating $2ce^{3x} = 5e^{3x}$

$c = 5/2$

$\therefore y_c = y_h + y_{PI}$

$$= (c_1 + c_2x)e^{3x} + \frac{5}{2}x^2e^{3x}.$$

$$y_h = e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x)$$ If roots are imaginary

2. Solve $y'' + 9y = -4x\sin 3x$.

A.E: $m^2 + 9 = 0 \Rightarrow m^2 = -9 \Rightarrow m = \pm 3i = \alpha \pm i\beta$ (say)

$\alpha = 0, \beta = 3$

$$y_h = e^{0x}(c_1\cos 3x + c_2\sin 3x)$$

$$= c_1\cos 3x + c_2\sin 3x.$$

To find PI: Let the proposed solution be

$$y_p = (ax+b)\cos 3x + (cx+d)\sin 3x$$

$$y_p' = (ax+b)(-3\sin 3x) + \cos 3x(a) + (cx+d)3\cos 3x + \sin 3x(c).$$

$$= \sin 3x(-3ax - 3b + c) + \cos 3x(a + 3cx + 3d).$$

$$y_p'' = (-3ax - 3b + c)(3\cos 3x) + \sin 3x(-3a) + (a + 3cx + 3d)(-3\sin 3x) + \cos 3x(3c).$$

$$= \cos 3x(-9ax - 9b + 3c + 3c) + \sin 3x(-3a - 3a - 9cx - 9d)$$

$$= \cos 3x(-9ax - 9b + 6c) + \sin 3x(-6a - 9cx - 9d).$$

LHS of (1) $= \cos 3x(-9ax - 9b + 6c) + \sin 3x(-6a - 9cx - 9d) + 9\cos 3x(ax+b) + 9\sin 3x(cx+d).$

$$= \cos 3x(6c) + \sin 3x(-6a).$$

RHS of (1) $= -4x\sin 3x$

Equating

$a = 0 \quad c = 0 \quad \therefore y_p = ?$ ✗.

✓ $y_p = (ax+b)x\cos 3x + (cx+d)x\sin 3x = (ax^2+bx)\cos 3x$

$+(cx^2+dx)\sin 3x$

$y_p' = (ax^2+bx)(-\sin 3x(3)) + \cos 3x(2ax+b).$ ✓

$+ (cx^2+dx)(3\cos 3x) + \sin 3x\ (2cx+d).$ ✓

$= \sin 3x(-3ax^2 - 3bx + 2cx + d) + \cos 3x(2ax + b + 3cx^2 + 3dx).$

$y_p'' =$

$\cos 3x(2a + 6c + 3d - 9ax^2 - 9bx + 6x + 3d)$

$+ \sin 3x(-6ax - 3b + 2c - 6ax - 3b - 9cx^2 - 9dx)$

On substitution

$\cos 3x(2a + 6c\cancel{x} + 6d + 6cx) + \sin 3x(-12ax - 6b + 2c) = -4x\sin 3x$

$x\cos 3x$ terms: $12cx\cos 3x = 0.$ $\Rightarrow c = 0.$

$x\sin 3x$ terms $-12a\cancel{x\sin 3x} = -4\cancel{x\sin 3x}$

$a = 1/3$

$\cos 3x$ terms: $2a + 6d = 0$

$a = 1/3$

So $2/3 + 6d = 0$

$6d = -2/3$

$d = \frac{-2}{18} = -1/9.$

$\sin 3x$ terms $-6b + 2c = 0$

$b = 0$ as $c = 0$

$y_p = \frac{1}{3}x^2\cos 3x - 1/9\, x\sin 3x.$

$y_c = y_h + y_p.$

3. Solve : $y'' + 4y = x + 2e^{-2x}$

RHS	Proposed Solution
1. $P(x)$	$Q(x)$
2. ce^{ax}	de^{ax} if a is not a root of the auxiliary eqn.
3. ce^{ax}	dxe^{ax} if a is one of the roots of the auxiliary eqn.
4. ce^{ax}	dx^2e^{ax} if a is a double root of the auxiliary eqn.
To be checked 5. $a\cos bx$ (or) $b\sin bx$	$c\cos bx + d\sin bx$ if b is not a root of the a. eqn.
6. $P(x)\cos bx / P(x)\sin bx$	$Q(x)\cos bx + R(x)\sin bx$
7. $P(x)e^{ax}\cos bx / P(x)e^{ax}\sin bx$	$Q(x)e^{ax}\cos bx + R(x)e^{ax}\sin bx$

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3(ii). $2y'' - 5y = x^2 + 5e^{-4x}$

A.E $2m^2 - 5 = 0 \Rightarrow m^2 = 5/2 \Rightarrow m = \pm\sqrt{5/2}$.

$\therefore$ CF or $y_h = c_1 e^{\sqrt{5/2}\,x} + c_2 e^{-\sqrt{5/2}\,x}$.

To find PI$_1$:

Let the proposed soln. be $ax^2 + bx + c = y_{p_1}$.

$y_{p_1}' = 2ax + b \quad y_{p_1}'' = 2a$.

LHS $= 2(2a) - 5(ax^2 + bx + c) = x^2 = $ RHS.

$= -5ax^2 - 5bx + (4a - 5c) = x^2 \Rightarrow -5a = 1 \Rightarrow a = -1/5$

Coeff of x: $b = 0$

Const/: $4a - 5c = 0 \Rightarrow 5c = 4a$

$\Rightarrow c = 4/5\, a = 4/5(-1/5)$

$= -4/25$

$\therefore y_{p_1} = -1/5\, x^2 - 4/25$

To find PI$_2$: Let the proposed soln. be $ce^{-4x} = y_{p_2}$

$y_{p_2}' = -4ce^{-4x}$, $y_{p_2}'' = 16ce^{-4x}$.

Sub. $2(16ce^{-4x}) - 5ce^{-4x} = 5e^{-4x}$

$27ce^{-4x} = 5e^{-4x} \Rightarrow c = \frac{5}{27}$

$\therefore y_{p_2} = \frac{5}{27} e^{-4x}$ $\qquad y = y_h + y_{p_1} + y_{p_2}$.