Conformal GAN Model Pseudocode

Algorithm 1 Conformal GAN Training

Input: Training data $\{\mathbf{x}_i, y_i\}_{i=1}^{n}$, number of epochs E, batch size B, learning rates η_D, η_G, regularization parameters λ, μ
Initialize generator G and discriminator D with appropriate architectures
for epoch = 1 to E **do**
  for batch = 1 to $\frac{n}{B}$ **do**
    Train Discriminator:
    Sample real batch $\{\mathbf{x}_r, y_r\}$ from training data
    Generate fake batch $\{\mathbf{x}_f, y_f\}$ using G
    Compute discriminator loss:

$$L_D = L_{\text{real}} + L_{\text{fake}} - \lambda \cdot L_{\text{reg}}$$

    Update discriminator parameters using gradient descent with learning rate η_D
    Train Generator:
    Compute generator loss:

$$L_G = L_{\text{gen}} + \mu \cdot L_{\text{conform}}$$

    Update generator parameters using gradient descent with learning rate η_G
  end for
end for

Algorithm 2 Discriminator and Generator Loss Computations

1: **Discriminator Losses:**

2: Real loss:

$$L_{\text{real}} = -E_{\mathbf{x}_r, y_r}[\log D(\mathbf{x}_r, y_r)]$$

3: Fake loss:

$$L_{\text{fake}} = -E_{\mathbf{x}_f, y_f}[\log(1 - D(\mathbf{x}_f, y_f))]$$

4: Conformity Regularization Loss:

$$L_{\text{reg}} = \frac{1}{B} \sum_{i=1}^{B} |\text{conf}(\mathbf{x}_{r,i}, y_{r,i}) - \text{conf}(\mathbf{x}_{f,i}, y_{f,i})|$$

5: **Generator Loss:**

6: Generator adversarial loss:

$$L_{\text{gen}} = -E_{\mathbf{x}_f, y_f}[\log D(\mathbf{x}_f, y_f)]$$

7: Conformity loss:

$$L_{\text{conform}} = \frac{1}{B} \sum_{i=1}^{B} |\text{conf}(\mathbf{x}_{f,i}, y_{f,i}) - \text{target\_conf}|^2$$

Algorithm 3 Conformity Score Functions

Inductive Conformal Prediction (ICP):

Compute conformity scores for a batch $\{\mathbf{x}_b, y_b\}$:

$$\text{conf}(\mathbf{x}_b, y_b) = \|\mathbf{x}_b - \bar{\mathbf{x}}_b\|_2$$

where $\bar{\mathbf{x}}_b$ is the mean of $\mathbf{x}_b$.

Mondrian Conformal Prediction:

Compute conformity scores for each class:

$$\text{conf}_c(\mathbf{x}_b) = \|\mathbf{x}_b - \bar{\mathbf{x}}_{b,c}\|_2$$

where $\bar{\mathbf{x}}_{b,c}$ is the mean of $\mathbf{x}_b$ for class c.

Cross-Conformal Prediction:

Compute conformity scores using cross-validation:

For each fold i in k-fold cross-validation:

  Define training and calibration sets based on fold i

  Compute mean of training set: $\bar{\mathbf{x}}_i$

  Compute conformity scores for calibration set:

$$\text{conf}_i(\mathbf{x}_b) = \|\mathbf{x}_b - \bar{\mathbf{x}}_i\|_2$$

Venn-Abers Predictors:

Fit an Isotonic Regression model to the data:

$$\hat{y} = \text{IsotonicRegression}(\mathbf{x}_b, y_b)$$

Compute conformity scores:

$$\text{conf}(\mathbf{x}_b, y_b) = \hat{y}$$

Algorithm 4 Conformal Prediction Interval Computation

Input: Synthetic data $\mathbf{X}_s$, significance level α

Compute non-conformity scores for synthetic data:

$$\text{scores} = \text{compute\_conformity\_scores}(\mathbf{X}_s)$$

Compute the $(1-\alpha)$-quantile of the scores:

$$q = \text{quantile}(\text{scores}, 1-\alpha)$$

Define prediction intervals for each synthetic sample:

$$\text{interval}_i = [\mathbf{x}_{s,i} - q, \mathbf{x}_{s,i} + q]$$

Output: List of prediction intervals