Assignment 3

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- Exercise 0.1
  Suppose L is a regular language with alphabet Σ. Give an algorithm to tell whether L contains at least 100 strings.

  If L is a regular language, there is exists some DFA $M = (Q, \Sigma, \delta, q_0, F)$ which describes it. The algorithm is as follows.

  1. Begin with a counter = 0.
  2. For every state s in Q, run Djisktra's algorithm on M starting from s to visit every state reachable from s. If no final state is reachable, color s red.
  3. Run a depth-first search on M. This search will visit every state in Q. If there is a non-red state in Q that is accessible from itself via a path of non-red state (in its own path), stop searching since there are an infinite number of valid strings in $L(M)$. If there are no such cycles in M, then a depth-first search on M is a straight-forward tree traversal. We may simple search from $q0$ and count all the non-red paths that terminate in final states, incrementing our counter for each one. If when our search terminates, our counter ≥ 100, we have found more than 100 valid strings $L(M)$.

- Exercise 0.2
  If $w = a_1a_2 \ldots a_n$ and $x = b_1b_2 \ldots b_n$ are strings of the same length, define $alt(w, x)$ to be the string in which the symbols of w and x alternate, starting with w, that is, $a_1b_1a_2b_2 \ldots a_nb_n$. If L and M are languages, define $alt(L, M)$ to be the set of strings of the form $alt(w, x)$, where w is any string in L and x is any string in M of the same length. Prove that if L and M are regular, so is $alt(L, M)$.

  Let a_i and b_i be defined as the ith elements of Σ_L and Σ_M respectively, and f and g be homomorphisms where

  $f(a_i) = a_i$
  $f(b_i) = \epsilon$
  $g(b_i) = b_i$
  $g(a_i) = \epsilon$

  If we take $L' = f^{-1}(L)$, we will have the set of all strings created by taking each element in L with every combination of $b \in \Sigma_M$ inserted between each letter. Similarly, $M' = g^{-1}(M)$ will give us the set of all strings created by taking each element in M with every combination of $a \in \Sigma_L$ inserted between each letter.

Since L' by construction will only have strings with valid strings of L embedded in them (interspered with some b's) and M' by construction will only have strings with valid strings of M embedded, to get the set of all strings alt(L,M), we take the intersection of these sets intersected with the regular expression $(ab)^*$ (to ensure that we get only strings that are strictly alternating). So,

$alt(L, M) = f^{-1}(L) \cap g^{-1}(M) \cap (ab)^*$

Since homomorphisms and regular expressions are closed under regularity, alt(L,M) is regular.

- Exercise 0.3

  If L is a language, and a is a symbol, then $a \setminus L$ is the set of strings w such that aw is in L. For example, if $L = \{a, aab, baa\}$ then $a \setminus L = \{\epsilon, ab\}$. Prove that if L is regular, so is $a \setminus L$.

  Let us define homomorphisms f and g such that
  $f(a) = a$
  $f(\hat{a}) = a$
  $g(a) = a$
  $g(\hat{a}) = \epsilon$

  Let $L' = f^{-1}(L)$. This will give us a language L' that contains all the strings of L and also all possible variants on those strings such that some number of a in each string is substituted out with $\hat{a}$. For example, if $L = \{a, aab, baa\}$ then $L' = \{a, \hat{a}, aab, \hat{a}ab, a\hat{a}b, \hat{a}\hat{a}b, baa, b\hat{a}a, ba\hat{a}, b\hat{a}\hat{a}\}$. Because regularity is closed under homomorphisms, L' is regular. Now we can apply the regular expression $\hat{a}(a + b)^*$ to L', which will return a set of strings L'' equal to L except that for every string in L aw, we now have instead $\hat{a}w$. Finally, we can apply the inverse of g to L''. Taking $g^{-1}(L'')$ will replace all $\hat{a}$ with the empty string. Since by our construction in L'' $\hat{a}$ occurs only at the beginning of strings, we have created $a \setminus L$.

  $a \setminus L = g^{-1}(f(L) \cap (\hat{a}(a + b)^*))$

  Because L is regular and all the operations we have used, homomorphisms and regular expression, are closed under regularity, $a \setminus L$ is also regular.

- Exercise 0.4

B	X							
C	X	X						
D	0	X	X					
E	X	0	X	X				
F	X	X	0	X	X			
G	0	X	X	0	X	X		
H	X	0	X	X	0	X	X	
I	X	X	0	X	X	0	X	X
	A	B	C	D	E	F	G	H

So this system can actually be reduced to 3 states: $\{C, I, F\}\{B, E, H\}\{A, D, G\}$.