**NAME : RAHUL RAI**

**ROLL NO : 205119073**

**RESOURCE MANAGEMENT TECHNIQUES**

UNIT 3

**Ques: What is Non-Linear Programming? Importance of Non-Linear Programming. Types of Non-Linear Programming. Applications of Non-Linear Programming.**

**Answer:**

Nonlinear Programming (NLP) is the process of solving an optimization problem where some of the constraints or the objective function are nonlinear. An optimization problem is one of calculation of the extrema (maxima, minima or stationary points) of an objective function over a set of unknown real variables and conditional to the satisfaction of a system of equalities and inequalities, collectively termed constraints. It is the sub-field of mathematical optimization that deals with problems that are not linear.

Let n, m, and p be positive integers. Let X be a subset of Rn, let f, gi, and hj be real-valued functions on X for each i in {1, …, m} and each j in {1, …, p}, with at least one of f, gi, and hj being nonlinear.

A nonlinear minimization problem is an optimization problem of the form

Minimize f(x)

Subject to gi(x) <=0 for each i = 1,2,…m

hj(x) =0 for each j ={1,2,….p

A nonlinear maximization problem is defined in a similar way.

**Importance of Non-Linear Programming:-**

For many general nonlinear programming problems, the objective function has many locally optimal solutions; finding the best of all such minima, the global solution, is often difficult. An important special case of nonlinear programming is convex programming in which all local solutions are global solutions.

The importance of Nonlinear Programming Applications is growing due to rapidly increasing sophistication of managers and operation researchers in implementing decision oriented mathematical models, as well as to the growing availability of computer routines capable of solving large-scale nonlinear problems.

**2. Types of Non-Linear Programming:-**

**1. CONSTRAINED OPTIMIZATION :**

Some constrained optimizations are:

**a**. Equality constrainted:

We are concerned with finding the smallest value of the function f(x) of the n real variables x in the case where x is required to satisfy a set of m equality constraints c(x) = 0. The first-order optimality or, as they are often known,

Karush-Kuhn-Tucker (KKT) conditions for this problem are that

∇xℓ(x, y) = 0, and c(x) = 0,

where the Lagrangian function ℓ(x, y)def = f(x) − c(x)T y and where the components of the m-vector y are Lagrange multipliers.

b. **Inequality constrainted:**

Nonlinear programming problems rarely exclusively involve equality constraints,but typically involve a mixture of equations and inequalities. We thus turn to the inequality constrained problem. The Karush-Kuhn-Tucker (first-order optimality) conditions for the inequality problem minimize

x∈ℜn f(x) subject to c(x) ≥ 0

are that

∇xℓ(x, λ) = 0, c(x) ≥ 0, λ ≥ 0, and c(x)T λ = 0,

where the Lagrangian function ℓ(x, λ) = f(x) − c(x)T λ.

**2. UNCONSTRAINED OPTIMIZATION :**

Here we begin by considering a significantly simplified (but nonetheless important) nonlinear programming problem, i.e., the domain and range of the function to be minimized are one-dimensional and there are no constraints. A necessary condition for a minimum of a function was developed in calculus and is simply f’(x) = 0.

Note that higher derivative tests can determine whether the function is a max or a min, or the value f(x + δ) may be compared to f(x).

Note that if we let g(x) = f’(x) then we may convert the problem of finding a minimum or maximum of a function to the problem of finding a zero.

* Newton’s Method:

Note that in the bisection method the actual value of the function g(x) was only being used to determine the correct bracket for the root. Root finding is accelerated considerably by using this function information more effectively.

For example, imagine we were seeking the root of a function that was a straight line, i.e., g(x) = ax + b and our initial guess for the root was x0. If we extend this straight line from the point x0 it is easy to determine where it crosses the axis, i.e., ax1 + b = 0 so x1 = −b/a.

Of course, if the function were truly linear then no first guess would be required. So now consider the case that g(x) is nonlinear but may be approximated locally about the point x0 by a line. Then the point of intersection of this line with the x-axis is an estimate, or second guess, for the root x ∗ .

The linear approximation comes from Taylor’s theorem,

i.e., g(x) = g(x0) + g’ (x0)(x – x0) + 1/ 2 g ‘’(x0)(x – x0) 2 + . . .

So the linear approximation to g(x) about the point x0 can be written l(x) = g(x0) + g’ (x0)(x − x0)

If we take x1 to be the root of the linear approximation we have

l(x1) = 0 = g(x0) + g’(x0)(x1 – x0)

Solving for x1 gives x1 = x0 − g(x0)/g’(x0) or at the nth iteration xn+1 = xn − g(xn)/g’(xn)

The iteration above is for determining a zero of a function g(x). To determine a maximum or minimum value of a function f we employ condition that f 0 (x) = 0. Now the iteration is modified as as xn+1 = xn – f’ (xn)/ f’’(xn).

* Bisection Algorithm:

Let x ∗ be a root, or zero, of g(x), i.e., g(x\*) = 0. If an initial bracket [a, b] is known such that x ∗ ∈ [a, b], then a simple and robust approach to determining the root is to bisect this interval into two intervals [a, c] and [c, b] where c is the midpoint, i.e., c = (a + b)/2

If g(a)g(c) < 0

then we conclude x ∗ ∈ [a, c]

while if g(b)g(c) < 0

then we conclude x ∗ ∈ [b, c]

This process may now be iterated such that the size of the bracket (as well as the actual error of the estimate) is being divided by 2 every iteration.

**3. Applications of Non-Linear Programming:-**

**1. Applications of Non-Linear Programming:-**

i). NLP technique is successfully applied to the overall cost minimization of transformer active and mechanical part,

ii). Transformer design variables such as the conductors cross-section and windings are added to the optimization algorithm for an enlarged and transverse optimum transformer designs.

The proposed methods find acceptable optimum transformer design by minimizing either the overall transformer material cost (i.e. the transformer active part cost plus mechanical part cost) or the overall transformer materials and operating cost taking into consideration proper loss evaluation factors, while simultaneously satisfying all the constraints imposed by international standards and transformer user needs, instead of focusing on the optimization of only one parameter of transformer performance (e. g no-load losses or short circuit impedance). Using the proposed technique, a graphic user interface (GUI) software package is developed that combine‟s transformer design with analysis, optimization and visualization tools, useful for both design optimization and educational use. The technique is applied to the design of power transformers of several ratings and loss. Categories and the results are compared with transformer design optimization method (which is already used by transformer industry), resulting to significant cost savings.

**2. Application of Nonlinear Programming for Optimization of Nutrient Requirements for Maximum Weight Gain in Buffaloes:**

Nonlinear effects of nutrient ingredients are introduced as an approach closer to the true effects of nutrient ingredients. A nonlinear model is developed to take consideration of nutrient ingredients more effectively. Proposed model with nonlinear programming measures its performance and gives a comparative result with linear programming models.. Leading to the same guideline a ration can be formulated using all its nutrient ingredients simultaneously at the optimum level. In this paper, it is envisaged to develop a mathematical model using non-linear programming to take simultaneous effects of all nutrient ingredients and the diet is optimized by using Kuhn- Tucker conditions. This result is also compared to than that of linear programming formulation of the model.

*1. Weightage of Variables*

First of all, linear relationship for dependent and independent variables is formulated to decide the weightage of the variables. Assuming a linear relationship between weight gain of buffaloes and intake of DM, CP and TDN, the weightage of these variables was decided.

Using least square method, the relationship is depicted in the following equation which describes the weightage of the variables x1, x2 and x3.

|  |  |
| --- | --- |
|  | (1) |

*2. Relationship between Variables*

By using least square method, the relations between y and x1, y and x2, y and x3 of different degrees were established and then by using F-test the relation of best fit was decided. Applying the F-test, the following most appropriate relationship between the variables were derived:

|  |  |
| --- | --- |
|  | (2) |

*3. Formulation of Objective Function*

The objective function was established by using the appropriate relations of the variables x1, x2, x3 according to their weightage on weight gain of the buffalo calves. The weightage with respect to total effect of this weightage was considered:

|  |  |
| --- | --- |
|  | (3) |

*4. Constraints*

The constraints according to feeding standards on the above-mentioned variables according to feeding standards of NRC (1981) were applied

|  |  |
| --- | --- |
|  | (4) |

*5. Problem Defined*

The main problem is formulated to maximise weight gain of the animal:



subject to:

|  |  |
| --- | --- |
|  | (5) |

*6. Solution of the Problem*

Introducing Kuhn-Tucker conditions, the weight gain of the buffalo calves could be maximized as:



Using Kuhn-Tucker conditions, the following set of equations were obtained for optimal solutions:













x1 ≤ 396.311

x2 ≤ 37.708

x3 ≤ 368.1687036



Solving these equations the optimum values of the three nutrients is found out to maximize the body weight gain. Accordingly we have:

x1= 381.3028, x2= 7.708, x3= 368.1687036 g/kg W0.75It also gives, λ1= 0.393301399, λ2= 0.027548012 which satisfied all the conditions.

The problem is also formulized and solved by simplex method and it gives,

x1= 396.311, x2= 37.708, x3= 368.1687036 g/kg W0.75

Comparison shows that by linear programming result is obtained at corner points of feasible area and optimization is at comparatively at higher values of nutrient ingredients. This comparison represents that non-linear programming is better way to take simultaneous effect of all nutrient ingredients together and maximize the weight gain in animal with optimized value of nutrient ingredients.

UNIT 4

**Ques. What is Integer Programming? Importance of Interger Programming. Types of Interger Programming. Applications of Inter Programming.**

**Answer:**

An integer programming problem is a mathematical optimization or feasibility program in which some or all of the variables are restricted to be integers. In many settings the term refers to integer linear programming (ILP), in which the objective function and the constraints (other than the integer constraints) are linear.Integer programming is NP-complete. In particular, the special case of 0-1 integer linear programming, in which unknowns are binary, and only the restrictions must be satisfied, is one of Karp's 21 NP-complete problems.If some decision variables are not discrete the problem is known as a mixed-integer programming problem.

An integer linear program in canonical form is expressed as:

Maximize cT x

Subject to Ax<=b,

x>=0,

and x € Z

and an ILP in standard form is expressed as

maximize cT x

subject to Ax +s =b,

x>=0, s>=0,

and x € Z

where c, b are vectors and A is a matrix, where all entries are integers. As with linear programs, ILPs not in standard form can be converted to standard form  by eliminating inequalities, introducing slack variables (s) and replacing variables that are not sign-constrained with the difference of two sign-constrained variables.

**Importance of Integer Programming:-**

There are two main reasons for using integer programming when modeling problems as a linear program: The integer variables represent quantities that can only be integer. ... The integer variables represent decisions (e.g. whether to include an edge in a graph) and so should only take on the value 0 or 1.

There are two main reasons for using integer variables when modeling problems as a linear program:

1. The integer variables represent quantities that can only be integer. For example, it is not possible to build 3.7 cars.

2. The integer variables represent decisions (e.g. whether to include an edge in a graph) and so should only take on the value 0 or 1.

**2. Types of Integer Programming:-**

There are three types of IP models:

1. Mixed integer programming, only some of the variables are restricted to integer

values.

2. In pure integer programming, all the variables are integers.

3. In binary integer programming or 0-1 integer programming, all the variables are binary (restricted to the values 0 or 1).

Mixed Integer Programming:

The problems most commonly solved by the Gurobi Parallel Mixed Integer Programming solver are of the form:

|  |  |
| --- | --- |
| Objective: | minimize cT x |
| Constraints: | A x = b (linear constraints) |
|  | l ≤ x ≤ u (bound constraints) |
|  | some or all xj must take integer values (integrality constraints) |

The integrality constraints allow MIP models to capture the discrete nature of some decisions.  For example, a variable whose values are restricted to 0 or 1, called a binary variable, can be used to decide whether or not some action is taken, such as building a

Binary Integer Programming:

Binary Integer Programming (BIP) is an approach to solve a system of linear inequalities in binary unknowns (0 or 1 in what follows). Integer programming has been studied in mathematics, computer science, and operations research for more than 40 years. It has been successfully applied to solve a huge number of large-scale combinatorial problems.

The general form of an integer linear programming problem is max { c T x | Ax ≤ b, x ∈ Zn } (1.1) with a real matrix A of a dimension m by n, and vectors c ∈ Rn , b ∈ Rm, c T x being the scalar product of the vectors c and x. If the system Ax ≤ b includes the constraints 0 ≤ x ≤ 1, we get a binary integer linear programming problem (BIP). A vector x\* in Zn with Ax\* ≤ b is called a feasible solution. If moreover, c T x\* = max { c T x | Ax ≤ b, x ∈ Zn }, then x\* is called an optimal solution and c T x\* the optimal value.

We use specialized branch and bound method for solving BIP known as Balas Additive Algorithm.

The keys how Balas Additive Algorithm works lies in its special structure:

* The objective function is minimization and all of the coefficients are nonnegative, so we would prefer to set all the variables to zero to give the smallest value of Z.
* If we cannot set all the variables to zero without violating one or more constraints, then we prefer to set all the variables that has smallest index 1. This is because variables are ordered so that those earlier in the list increase by Z by smallest amount.

**3. Applications of Integer Programming:-**

**1. Production planning:-**

Mixed-integer programming has many applications in industrial productions, including job-shop modelling. One important example happens in agricultural production planning involves determining production yield for several crops that can share resources (e.g. Land, labor, capital, seeds, fertilizer, etc.). A possible objective is to maximize the total production, without exceeding the available resources. In some cases, this can be expressed in terms of a linear program, but the variables must be constrained to be integer.

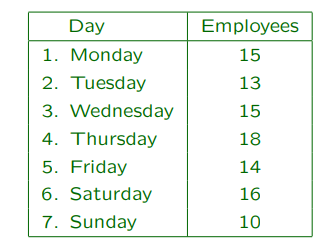
**2. Cellular networks:-**

The task of frequency planning in GSM mobile networks involves distributing available frequencies across the antennas so that users can be served and interference is minimized between the antennas. This problem can be formulated as an integer linear program in which binary variables indicate whether a frequency is assigned to an antenna.

**3. Cash flow matching:-**

Cash flow matching is a process of hedging in which a company or other entity matches its cash outflows (i.e. financial obligations) with its cash inflows over a given time horizon. It is a subset of immunization strategies in finance. Cash flow matching is of particular importance to defined benefit pension plans.

**Example :** The number of employees needed –



Employees work five consecutive days and have the next two days off.

The objective is to employ the minimum number of workers.

Decision variables:

xj : number of employees whose working shift starts on day j, j = 1, . . . , 7.

min z = x1 + x2 + x3 + x4 + x5 + x6 + x7

subject to

x1 + x4 + x5 + x6 + x7 ≥ 15

x1 + x2 + x5 + x6 + x7 ≥ 13

x1 + x2 + x3 + x6 + x7 ≥ 15

x1 + x2 + x3 + x4 + x7 ≥ 18

x1 + x2 + x3 + x4 + x5 ≥ 14

x2 + x3 + x4 + x5 + x6 ≥ 16

x3 + x4 + x5 + x6 + x7 ≥ 10

x1, x2, x3, x4, x5, x6, x7 ≥ 0 and integer.

UNIT 5

**Ques. What is Queuing Theory? Importance of Queuing Theory.**

**Models of Queuing Theory and Applications of QT.**

**Answer:**

**Queuing theory** is the mathematical study of the congestion and delays of waiting in line. Queuing theory examines every component of waiting in line to be served, including the arrival process, service process, number of servers, number of system places, and the number of customers—which might be people, data packets, cars, etc.

**Importance of Queuing Theory:-**

Waiting in line is a part of everyday life because as a process it has several important functions. Queues are a fair and essential way of dealing with the flow of customers when there are limited resources. Negative outcomes arise if a queue process isn’t established to deal with overcapacity.

For example, when too many visitors navigate to a website, the website will slow and crash if it doesn’t have a way to change the speed at which it processes requests or a way to queue visitors.

Queuing are the most frequently encountered problems in everyday life. For example, queue at a cafeteria, library, bank, etc. Common to all of these cases are the arrivals of objects requiring service and the attendant delays when the service mechanism is busy.

**Models of Queuing Theory:-**

**Kendall’s Notation for Queues:-**

A Inter-arrival time distribution

B Service time distribution

C Number of servers

D Maximum number of jobs that can be there in the system (waiting and in service)

Default ¥ for infinite number of waiting positions

E Queueing Discipline (FCFS, LCFS, SIRO etc.)

Default is FCFS

( where M exponential,D deterministic,Ek Erlangian (order k),G general,¥ infinity)

M/M/1 or M/M/1/¥ Single server queue with Poisson arrivals,

exponentially distributed service times and infinite number of

waiting positions

There are four types of queuing model:

* **M/M/1 Queuing Model:-**

In queueing theory, a discipline within the mathematical theory of probability, an M/M/1 queue represents the queue length in a system having a single server, where arrivals are determined by a Poisson process and job service times have an exponential distribution. The model name is written in Kendall's notation. The model is the most elementary of queueing models[1] and an attractive object of study as closed-form expressions can be obtained for many metrics of interest in this model. An extension of this model with more than one server is the M/M/c queue.

* **M/M/C Queuing Model:-**

In queueing theory, a discipline within the mathematical theory of probability, the M/M/c queue is a multi-server queueing model. In Kendall's notation it describes a system where arrivals form a single queue and are governed by a Poisson process, there are c servers and job service times are exponentially distributed. It is a generalisation of the M/M/1 queue which considers only a single server. The model with infinitely many servers is the M/M/∞ queue.

* **M/M/1/ Queuing Model**
* **M/M/C/N Queuing Model**

Queuing Model:

It is a suitable model used to represent a service oriented problem, where customers arrive randomly to receive some service, the service time being also a random variable.

Arrival:

The statistical pattern of the arrival can be indicated through the probability distribution of the number of the arrivals in an interval.

Service Time:

The time taken by a server to complete service is known as service time.

Server:

It is a mechanism through which service is offered.

Queue Discipline:

It is the order in which the members of the queue are offered service. i.e, It is the rule accordingly to which customers are selected for service when queue has been formed.

The most common disciplines are

1. First come First services(FCFS)

2. First in First Out(FIFO)

3. Last in First out(LIFO)

4. Selection for service in Random order(SIRO)

Poisson Process:

It is a probabilistic phenomenon where the number of arrivals in an interval of length t follows a Poisson distribution with parameter t, where is the rate of arrival.

Queue (Waiting lines):

A group of items waiting to receive service, including those receiving the service, is known as queue.

Waiting time in queue (Wq):

Time spent by a customer in the queue before being served.

Waiting time in the system (WS):

It is the total time spent by a customer in the system. It can be calculated as follows:

Waiting time in the system = Waiting time in queue + Service time

Queue length (Lq):

Number of persons in the system at any time Average length of line. The number of customers in the queue per unit of time.

Average idle time (p0):

The average time for which the system remains idle Bulk Arrivals If more than one customer enters the system at an arrival event, it is known as bulk arrivals. Note that bulk arrivals are not embodied in the models of the subsequent sections.

Queuing System:

A queuing system can be completely described by The input (or arrival Patten) The service mechanism (or service pattern) The queuing discipline Customer’s behaviour.

**3. Applications of Queuing theory**

Queuing theory is powerful because the ubiquity of queue situations means there are countless and diverse applications of queuing theory.

Queuing theory has been applied, just to name a few, to:

a.. telecommunications

b.. transportation

c.. logistics

d.. finance

e.. emergency services

f.. computing

g.. industrial engineering

h.. project management

i.. Tobacco Supply

j.. Customers and Synchronous Vacations

1**. Applications of Queuing Theory in Library management:-**

A library is an organized collection of books, and some special materials like audio or visual materials, CDs, cassettes, video tape, DVDs, e-books, audio books and many other types of electronic resources. Scope of Queuing application in Libraries are circulation of books,counter service and allied services like reprography. The basic tasks in library are stacks maintenance, membership management, selection of library materials, and planning the acquisition of materials**.**

**2. Applications of Queuing Theory in Traffic system:-**

The vehicular traffic flow and explore could be minimized using queuing theory in order to reduce the delay on the roads. The role of transportation in human life cannot be overemphasized. A basic model of vehicular traffic based on queuing theory. It will determine the best times of the red, amber, and green lights to be either on or off in order to reduce traffic congestion on the roads. Queuing also helps to reduce fuel consumption thereby saving money for the Government to tackle problem of other sectors of the economy.