Comparative Analysis of Risk Measure Rankings: Implications for Portfolio Optimization Under Varying Risk Aversion

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Abstract

In this project, we extend the work of Fred Viole by adding a few other risk measures beyond the original scope. In addition to EDoR, CDaR, CVaR, and LPM1–LPM4, we analyze Max Drawdown, Value at Risk (VaR), Volatility, and Downside Beta to provide a more comprehensive assessment of downside risk. We also enhance the correlation analysis by introducing Kendall's correlation alongside the previously used Spearman correlation. Our study encompasses a detailed analysis of these risk measures, their behavior over time, portfolio construction using CVaR minimization, and the stability of risk rankings across different time windows.

1 Introduction

In the original work by Fred Viole, an analysis was conducted on SP 500 stock data from 2018 to 2025, where stock returns were used to compute various risk measures, including EDoR, CDaR, CVaR, LPM1, LPM2, LPM3, and LPM4. Stocks were then ranked based on these measures, and Spearman correlation was applied to assess the relationships among them. Building on this foundation, we have extended the study by incorporating additional risk metrics such as Max Drawdown, VaR, Volatility, and Downside Beta, as well as alternative correlation methods like Kendall's Tau. Our work further explores key questions related to Risk Measure Analysis, Portfolio Construction, and the Time Stability and Predictive Power of these risk metrics.

2 Risk Measures

Risk measures are used to evaluate the potential for losses in an investment or portfolio and help in comparing the downside risk across assets. In this work, we have implemented and analyzed several such measures:

2.1 Value-at-Risk (VaR)

VaR at confidence level α measures the maximum loss not exceeded with probability α:

$$q(\alpha) = \inf\{x \in R \mid P(X \leq x) \geq \alpha\}, \quad sothat \quad q(\alpha) \leq x$$

2.2 Conditional Value-at-Risk (CVaR)

CVaR represents the expected loss given that a loss exceeds the Value-at-Risk (VaR) threshold at confidence level α:

$$\bar{q}(\alpha) = E\{X \mid X > q(\alpha)\}$$

- $\bar{q}(\alpha)$: Conditional Value at Risk (CVaR) at confidence level α.
- X: A random variable representing the loss (or negative return).
- $q(\alpha)$: Value at Risk (VaR) at confidence level α; the α-quantile of the loss distribution.
- $X > q(\alpha)$: Condition specifying losses that exceed the VaR threshold.

2.3 Drawdowns

Let $DD(t, \omega)$ denote the drawdown at time t on scenario ω,
t is time and T is last inivestment interval,
ω is scenario (sample-path) from set ,
$\alpha\ isconfidencelevel,$
$and(1 - \alpha)$T = an integer number.

- **Maximum Drawdown of Return(Max Drawdown):**

$$MaxDD(\omega) = \max_{t \in [0,T]} DD(t, \omega)$$

- **Expectd Drawdown of Return(EDoR):**

$$AvDD(\omega) = \frac{1}{T} \int_0^T DD(t, \omega)\, dt$$

- **Conditional Drawdown-at-Risk (CDaR):**

$$CDaR(\alpha) = E\left[DD(t, \omega) \mid DD(t, \omega) \geq q(\alpha)\right]$$

2.4 Lower Partial Moments (LPM)

$$LPM1 = E\left[\max(0 - R_t, 0)\right] = \frac{1}{T}\sum_{t=1}^{T}\max(0 - R_t, 0)$$

$$LPM2 = E\left[\max(0 - R_t, 0)^2\right] = \frac{1}{T}\sum_{t=1}^{T}\max(0 - R_t, 0)^2$$

$$LPM3 = E\left[\max(0 - R_t, 0)^3\right] = \frac{1}{T}\sum_{t=1}^{T}\max(0 - R_t, 0)^3$$

$$LPM4 = E\left[\max(0 - R_t, 0)^4\right] = \frac{1}{T}\sum_{t=1}^{T}\max(0 - R_t, 0)^4$$

where, $\mathrm{R}_t isreturns.$

2.5 Volatility

Volatility is the standard deviation of returns and represents total risk:

$$Volatility = \sqrt{252} \times \sigma_{returns}$$

2.6 Downside Beta

Downside Beta measures an asset's sensitivity to market returns when the market is down. It is calculated as:

$$\beta^- = \frac{Cov(R_i, R_m \mid R_m < 0)}{Var(R_m \mid R_m < 0)}$$

Where, R_i is the return of asset i,
R_m is the market return.

3 Correlation Measures

Correlation measures help in understanding how risk metrics relate to one another across assets. They provide insight into whether different risk measures produce consistent rankings or evaluations. In this work, we used the following methods:

3.1 Spearman's Rank Correlation

Spearman's rank correlation coefficient, denoted by ρ_s, measures the strength and direction of the monotonic relationship between two ranked variables. It is defined as:

$$\rho_s = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)} \tag{1}$$

where d_i is the difference between the ranks of corresponding values of the two variables, and n is the number of observations.

This non-parametric measure is especially useful when the relationship between variables is not linear but monotonic.

3.2 Kendall's Tau Correlation

Kendall's tau coefficient, denoted by τ, assesses the ordinal association between two measured quantities. It is based on the number of concordant and discordant pairs:

$$\tau = \frac{(C - D)}{\frac{1}{2}n(n-1)} \tag{2}$$

where:

- C is the number of concordant pairs,
- D is the number of discordant pairs,
- n is the total number of observations.

Kendall's tau provides a more robust measure of correlation when dealing with ordinal data and is less sensitive to outliers than Spearman's ρ.

4 Case Study

We conducted a case study using SP 500 stock data from 2018 to 2024, with each step of the analysis thoroughly explained.

4.1 Data Prepartion

We began by retrieving the list of SP 500 stocks from Wikipedia. For these stocks, we collected daily closing prices spanning from January 2018 to December 2024. Due to excessive missing values, 21 stocks were excluded from the dataset. Additionally, we fetched the SP 500 index closing prices for the same period to serve as a market benchmark. After merging the index and stock data, we computed the log returns to prepare the data for further risk analysis. Log returns are calculated as:

$$r_t = \ln \left(\frac{P_t}{P_{t-1}} \right)$$

where:

- r_t is the log return at time t,
- P_t is the price of the asset at time t,
- P_{t-1} is the price of the asset at the previous time step $t-1$,

4.2 Risk Measure Computation

First, we set the threshold Q to 0.2 and the Q level to 0.1. Then, for each stock, we calculate the various risk measures mentioned earlier. This involves using the defined threshold and quantile levels to focus on specific parts of the return distribution and compute the associated risk metrics for each stock accordingly.

Stock	EDoR	CDaR	CVaR	LPM1	LPM2	LPM3	LPM4
A	0.042306	0.418839	0.002980	0.006256	0.000169	0.000008	5.02e-07
AAPL	0.010724	0.302217	0.002667	0.006264	0.000180	0.000009	6.17e-07
ABBV	0.041015	0.451449	0.002534	0.005420	0.000164	0.000012	1.35e-06
ABT	0.020925	0.326556	0.002552	0.005265	0.000122	0.000005	2.87e-07
ACGL	0.036305	0.500120	0.002854	0.005992	0.000194	0.000013	1.48e-06

Table 1: Risk Measures for Selected Stocks and risk metrics

4.3 Rank and correlations

After computing the risk measures, we ranked each stock according to its risk measure in ascending order, and then computed Spearman and Kendall rank correlations to assess the relationships among the measures.

Stock	EDoR	CDaR	CVaR	LPM1	LPM2	LPM3	LPM4
A	241	207	166	216	165	99	85
AAPL	93	97	97	218	199	136	105
ABBV	236	240	67	95	147	224	253
ABT	145	125	74	78	38	32	19
ACGL	217	277	139	178	225	254	272

Table 2: Ranking of selected Selected Stocks and risk metrics

4.4 Risk measure analysis

We compared Spearman and Kendall correlations of risk measures. We found that the correlation of downside beta with other risk measures is lower, and Kendall's correlation shows less correlation among risk measures as compared to Spearman's.

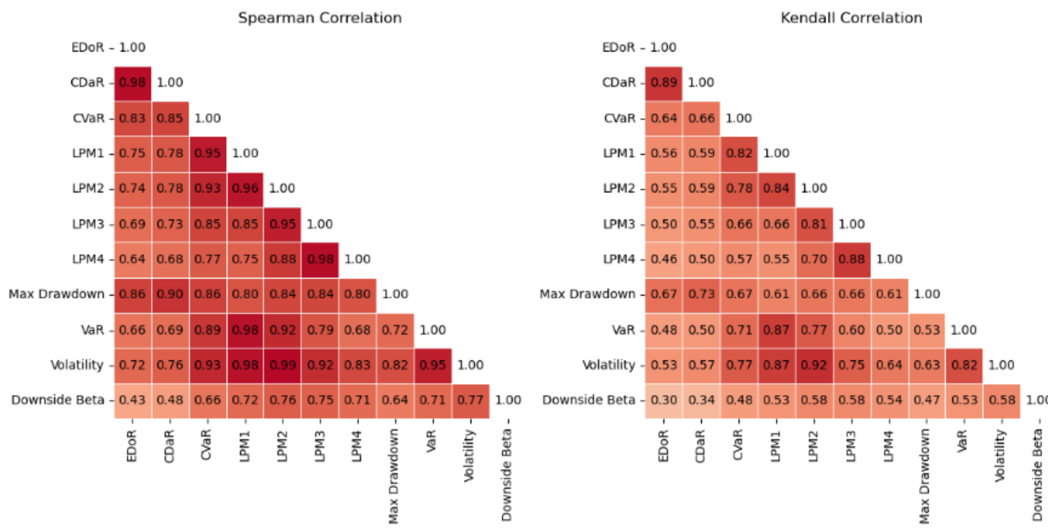

Figure 1: Spearman and Kendall Correlation Heatmap

4.4.1 Measure Sensitivity

We repeated the analysis using two different thresholds and Q levels to evaluate the sensitivity of the risk measures. As shown in the middle plot, the CDaR values for Q=0.1 and Q=0.05 lie close to the diagonal, indicating a very high correlation of 1. This suggests that varying the Q level in CDaR does not significantly affect the results. In contrast, CVaR exhibits a slightly lower correlation of around 0.97, with values appearing more scattered. Similarly, when analyzing EDoR with thresholds Q=0.2 and Q=0.3, the correlation remained nearly perfect.

We performed the same analysis using both Spearman and Kendall correlation methods. As observed earlier, the Kendall correlation generally indicated weaker relationships among the risk measures. However, for threshold Q=0.3 and level Q=0.05, the Kendall correlations slightly increased, suggesting a modest improvement in agreement between the measures under these parameter settings.

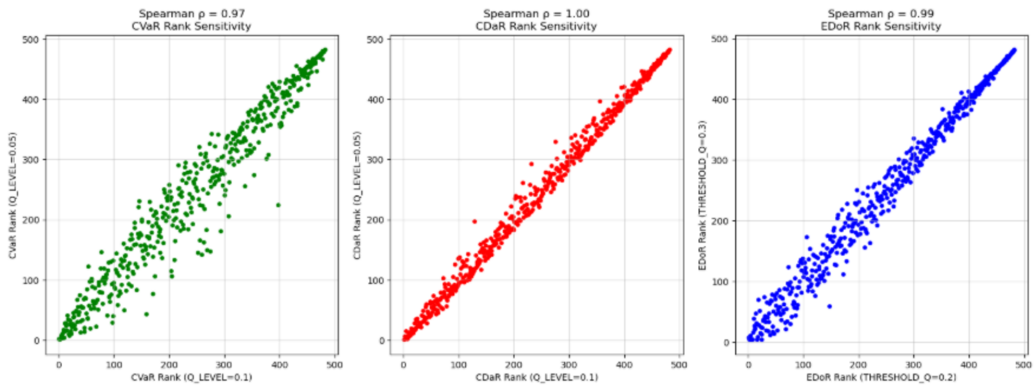

Figure 2: Sensitivity for different threshold Q and level Q

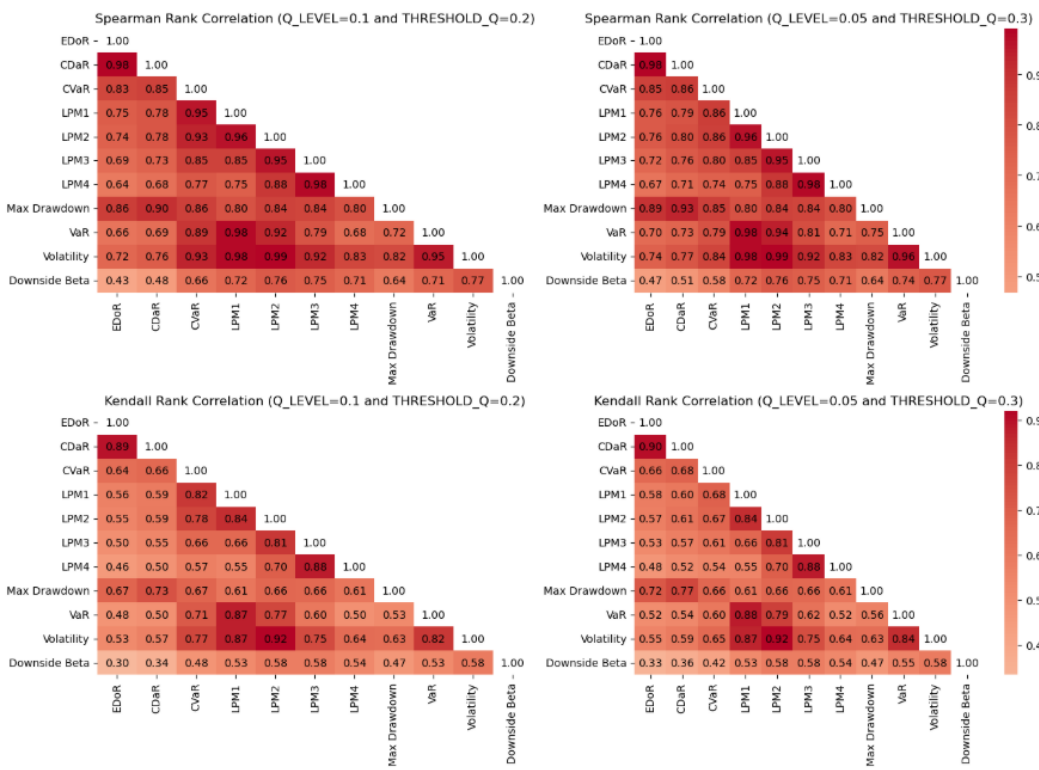

Figure 3: Spearman and Kendall Correlation Heatmap

4.4.2 Sector Effects

We extended our analysis beyond individual stocks to the sector level, evaluating all eleven GICS sectors. Notably, the Real Estate sector exhibited consistently low correlations across various risk measures, indicating a distinct risk profile. This sector-level approach provided deeper insights into how risk behaves differently across industries.

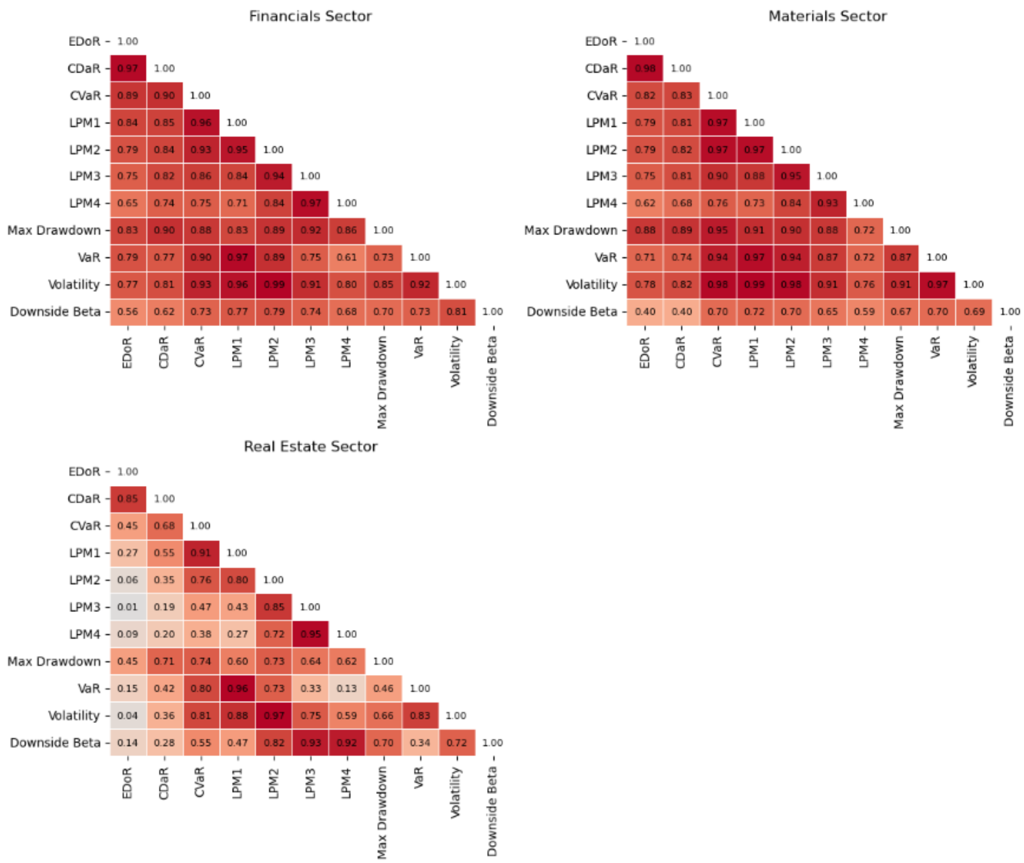

Figure 4: Spearman and Kendall Correlation Heatmap

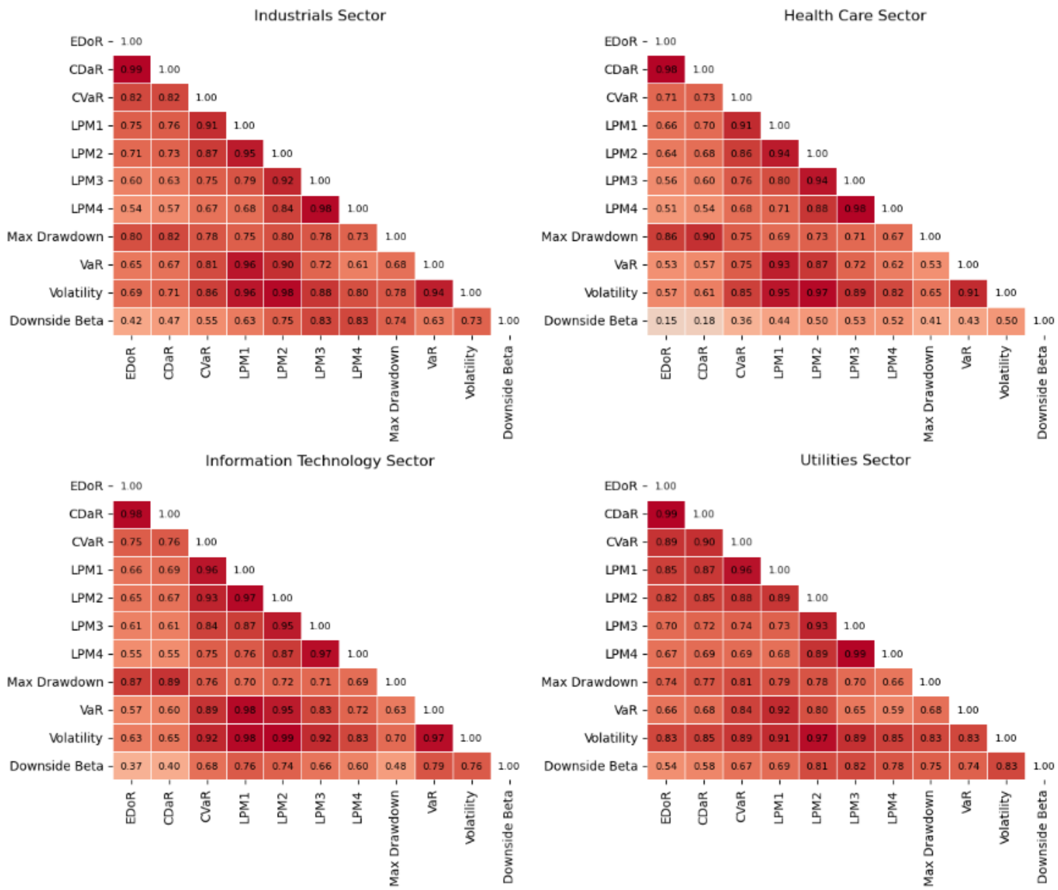

Figure 5: Spearman and Kendall Correlation Heatmap

4.5 Portfolio Construction

For portfolio construction, we utilized two years of historical data from 2018 and 2019. After calculating the daily returns for each stock, we constructed minimum-risk portfolios by optimizing Conditional Value at Risk (CVaR). The stock weights were obtained by solving the following optimization problem focused on minimizing portfolio risk.

minimize:
cvar risk(0.90, matrix scenarios)
Constraint: == 1
linear(matrix$_{b}udget$)
$Box :>= 0$

Later, we divided the stocks into the highest and lowest weights. The tables below show 10 stocks with the highest and lowest weights.

Table 3: Top 10 Stocks by Weight

Ticker	Weight
FAST	0.014720
UNH	0.013843
AXP	0.013415
SYF	0.011988
MNST	0.011688
ELV	0.011457
APTV	0.011195
MPWR	0.010893
MPC	0.010872
KLAC	0.010728

Table 4: Bottom 10 Stocks by Weight

Ticker	Weight
MRK	0.000195
CMG	0.000167
NDAQ	0.000150
LH	0.000146
WY	0.000131
VRSK	0.000130
SBUX	0.000123
NTAP	0.000055
FCX	0.000040
ROL	0.000007

Based on the obtained weights, we focused on the top and bottom ten stocks from the portfolio. We then tested these stocks by evaluating their Max Drawdown and Volatility. The analysis revealed that the top-ranked stocks exhibited higher Max Drawdown and Volatility compared to the bottom-ranked stocks, indicating a greater level of risk associated with the higher-ranked stocks in terms of downside risk and overall risk fluctuations.

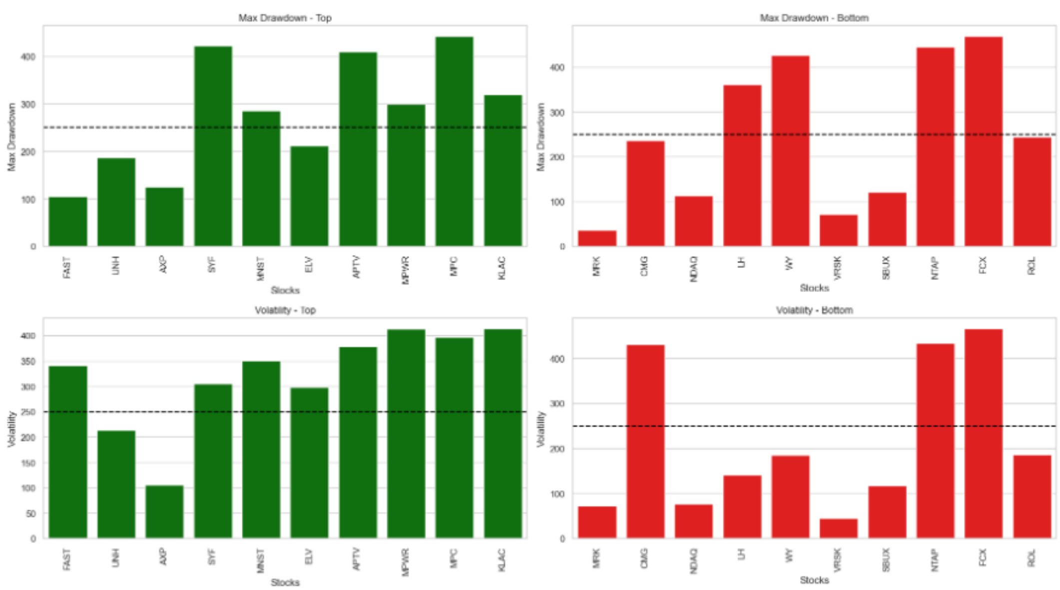

Figure 6: Portfolio Analysis

4.5.1 Combining measures

Considering a conservative risk appetite, we assigned the following weights to each risk measure: CVaR (0.20), Max Drawdown (0.20), Volatility (0.15), LPM4 (0.15), LPM2 and LPM3 (0.10 each), LPM1 and Downside Beta (0.05 each), and EDoR (0.00). Using these weights, we computed a composite risk score for each stock to reflect overall downside risk in line with conservative investment preferences.

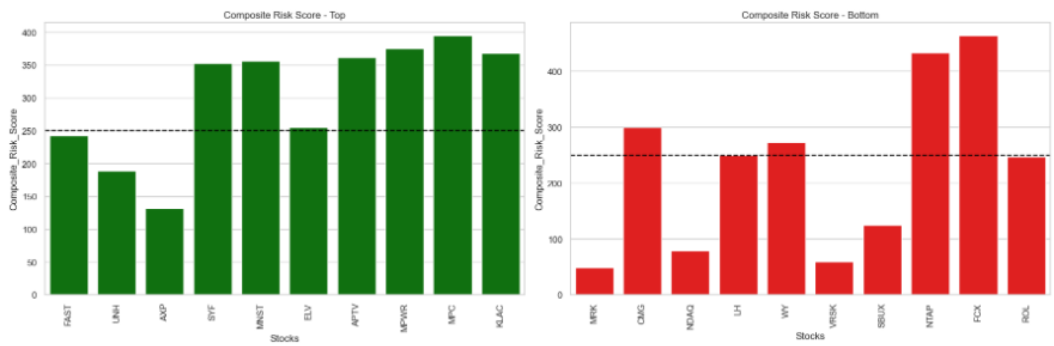

Figure 7: Portfolio Analysis

4.6 Time Stability and Predictive Power

After preparing the daily returns data, we created additional datasets for six separate periods spanning from 2018 to 2024, each corresponding to a two-year period with a one-year rolling window. For each dataset, we calculated the various risk measures and ranked the stocks in ascending order based on their risk scores. Our analysis revealed that Value at Risk (VaR) and Conditional Value at Risk (CVaR) performed relatively well in capturing downside risk across different time periods, whereas the performance of Lower Partial Moment 3 (LPM3) was less effective. Below is the plot showcasing the results for two distinct rolling window periods.

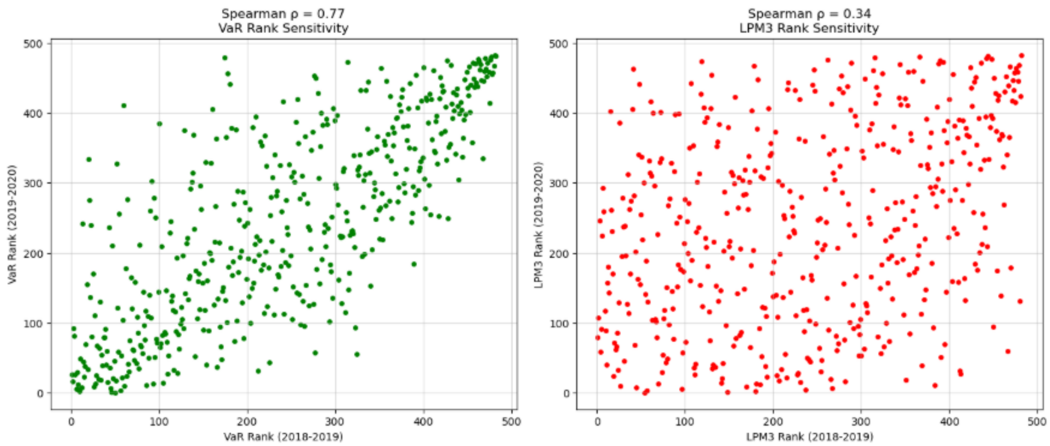

Figure 8: Time Stability

In the plot below, we observe that when comparing the VaR between the 2018-2019 and 2023-2024 time periods, there is a strong positive correlation. Similarly, LPM3 shows a close relationship across these periods. However, this consistency is not observed when comparing the 2018-2019 and 2019-2020 time periods, where the correlation is less pronounced and the risk measures behave differently.

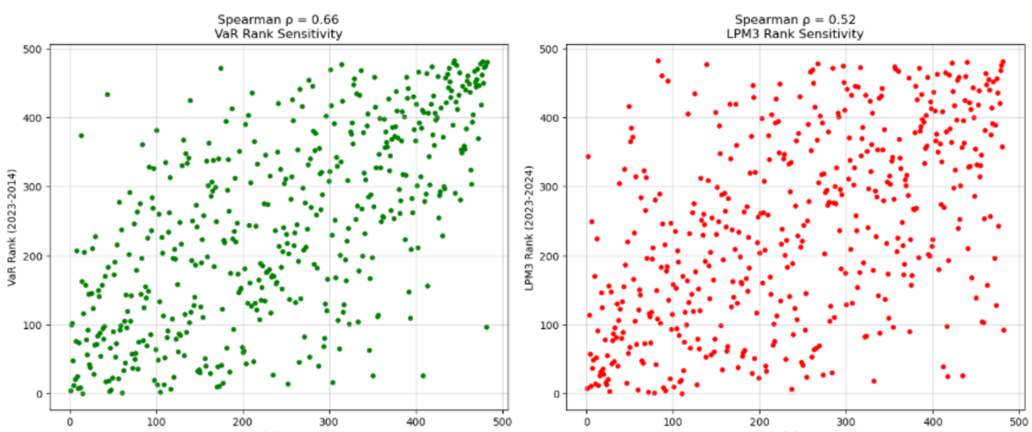

Figure 9: Time Stability

Additionally, we analyzed stocks with a rank higher than 400 and observed that their rankings remained largely unchanged across the different periods.

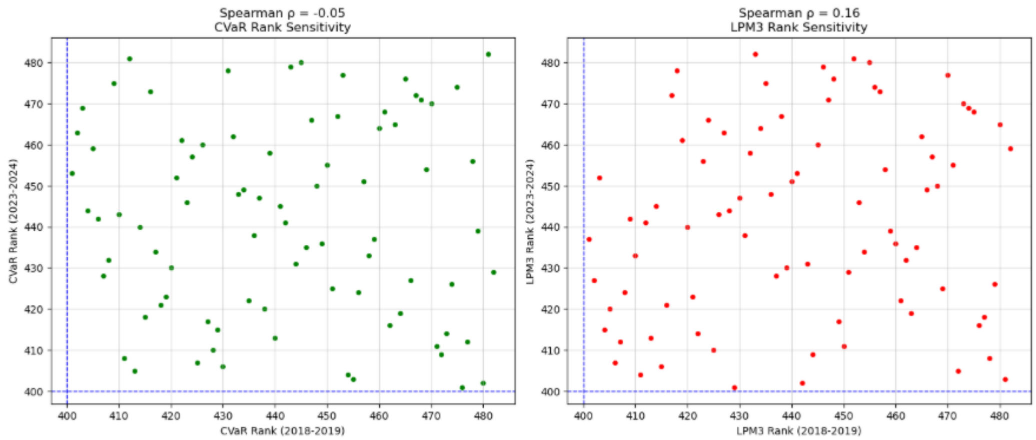

Figure 10: Time Stability for higher ranked stocks

5 Conclusion

We conducted a comprehensive risk measure analysis at both the stock and sector levels, examining their sensitivity across various risk metrics. Our portfolio construction focused on minimizing CVaR risk, and we also explored how the portfolio composition changes when assuming a conservative risk appetite profile. Additionally, we analyzed multiple rolling window time periods and observed that the rankings for higher-ranked stocks remained consistent across different periods.

References

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- https://en.wikipedia.org/wiki/Risk_measure
- https://en.wikipedia.org/wiki/Rank_correlation