Deep Hedging : Enhancing Options Hedging Strategies with Deep Learning Models

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DEFINITIONS

- ***Derivative:*** A financial derivative is a security whose value is tied to the value of another security, called the underlying.
- ***Derivative Securities***: A financial contract is a derivative security, or a contingent claim if its value at expiration is exactly determined by the market price of one or more cash instruments called the underlying.
- ***Forward and Futures Contracts****:* A forward or futures contract is a security that obligates the holder to buy a certain underlying asset at a certain price at a certain time in the future. See
- ***Options****:* An option is a security that gives its owner, the holder, the right (not the obligation) to buy or sell an underlying asset at a certain price on a certain date or dates.
- ***European Options****:* A European-style option on a security is the right (not obligation) to buy in a call option or to sell in a put option the security at strike price K at expiry T.

CALL AND PUT

- Consider a case in which a trader is long has bought a call with strike K = 100, then F(T) =max[S(T) – K, 0].
- Consider a case in which a trader is long or has bought a put with strike K = 100, then F(T) =max[K – S(T), 0].
- Consider a case in which a trader is short--has sold or written--a call with strike K = 100, then F(T) =– max[S(T) – K, 0].
- Consider a case in which a trader is long--or has sold or written--a put with strike K = 100, then F(T)= – max[K – S(T), 0].

- Notations:T = expiration date, the expiry, of the derivative, t = time index (typically at some time t < T),S(t) = the price of the underlying at time t, F(S(t), t) = the derivative price at time t given S(t), F(t) = F(S(t), t), when the context is clear

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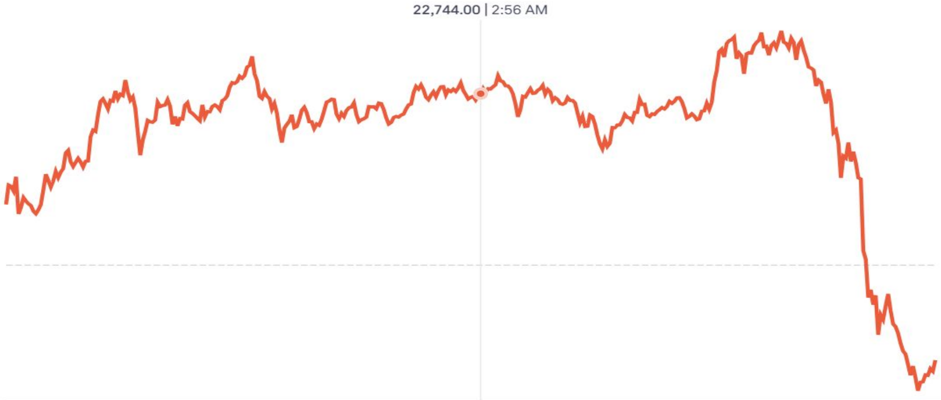
NIFTY 50

Option Chain

22604.85 -38.55 (0.17%) 1D

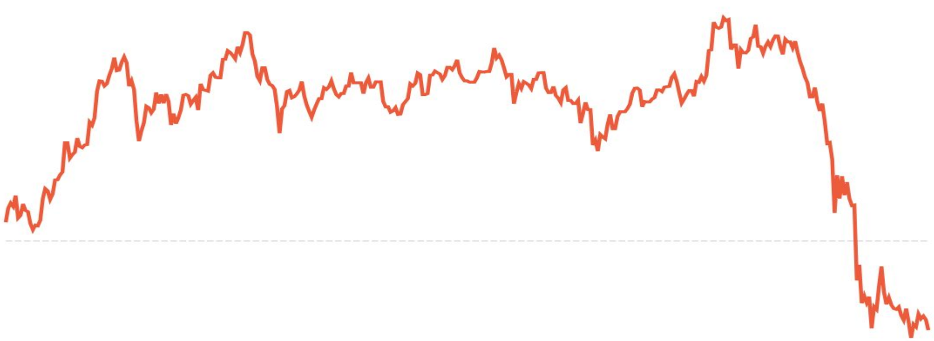


1D 1W 1M 1Y 3Y 5Y All Terminal

NIFTY 02 May 22350 Call

Create Alert Watchlist

₹314.85 -31.20 (9.02%) 1D



1D 1W 1M 3M Terminal

NIFTY 50 Option Chain Expiry 02 May

Quick Basket Terminal

OI (lots)	CALL PRICE	STRIKE PRICE	PUT PRICE	OI (lots)
22,882 -6.71%	₹314.85 -31.20 (9.02%)	22,350.00	₹12.20 -12.40 (50.41%)	1,16,673 +49.12%
44,455 -14.05%	₹273.85 -27.20 (9.04%)	22,400.00	₹18.15 -12.25 (40.30%)	2,27,725 +16.54%
17,047 -32.39%	₹229.15 -29.80 (11.51%)	22,450.00	₹25.20 -12.20 (32.62%)	1,14,795 +26.74%
1,27,021 -7.43%	₹188.30 -29.55 (13.56%)	22,500.00	₹34.55 -11.40 (24.81%)	2,80,835 -10.73%
54,383 -17.96%	₹149.25 -29.60 (16.55%)	22,550.00	₹46.05 -11.30 (19.70%)	1,08,801 -13.06%

BLACK-SCHOLES

C = Call option price

S = Current stock price

K = Strike price of the option

r = risk-free interest rate (a number between 0 and

σ = volatility of the stock return (between 0 and 1)

t = time to option maturity (in years)

N = normal cumulative distribution function

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-rt}$$

$$d_1 = \frac{1}{\sigma\sqrt{t}}\left(\ln\left(\frac{S}{K}\right) + t\left(r + \frac{\sigma^2}{2}\right)\right)$$

$$d_2 = \frac{1}{\sigma\sqrt{t}}\left(\ln\left(\frac{S}{K}\right) + t\left(r - \frac{\sigma^2}{2}\right)\right)$$

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}z^2}\,dz$$

CHALLENGES IN OPTIONS TRADING

- Volatility Risk: Options prices are affected by changes in implied volatility. Traders may be exposed to increased volatility risk.
- Directional risk : They are vulnerable to losses if the price of the underlying asset moves unfavorably.
- Unintended Losses: Losses can accumulate rapidly, particularly if the market experiences significant price swings.
- Profit Potential: Traders may miss out on opportunities to capture profits or mitigate losses as market conditions change.

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HEDGING

- Hedging : Hedging iis an investment position intended to offset potential losses or gains that may be incurred by a companion investment.

- Greek Hedging : A legacy approach once justified by lack of data and computational power.

- Statistical Hedging : Brings data-driven risk management but still relies on classic models for pricing

GREEKS

- Greeks : First partial differential of Black-Scholes formula.
- Delta (Δ): Delta measures the sensitivity of an option's price to changes in the price of the underlying asset.
- Gamma (Γ): Gamma measures the rate of change of delta with respect to changes in the price of the underlying asset.
- Theta (Θ): Theta measures the rate of change of an option's price with respect to the passage of time.
- Vega (ν): Vega measures the sensitivity of an option's price to changes in implied volatility.
- Rho (ρ): Rho measures the sensitivity of an option's price to changes in the risk-free interest rate.

DELTA HEDGING

- A delta-neutral position is one in which the overall delta is zero, thus reducing the movement of the option price relative to the underlying asset.

- The Delta values range from -1 to +1.

- $\Delta t = (t, St) = N(d1)$

- Any option on the underlying asset has a delta of 0.2 if the price of the underlying asset increases by 1 USD per share, the value of the option on that asset will increase by 0.2 USD per share.

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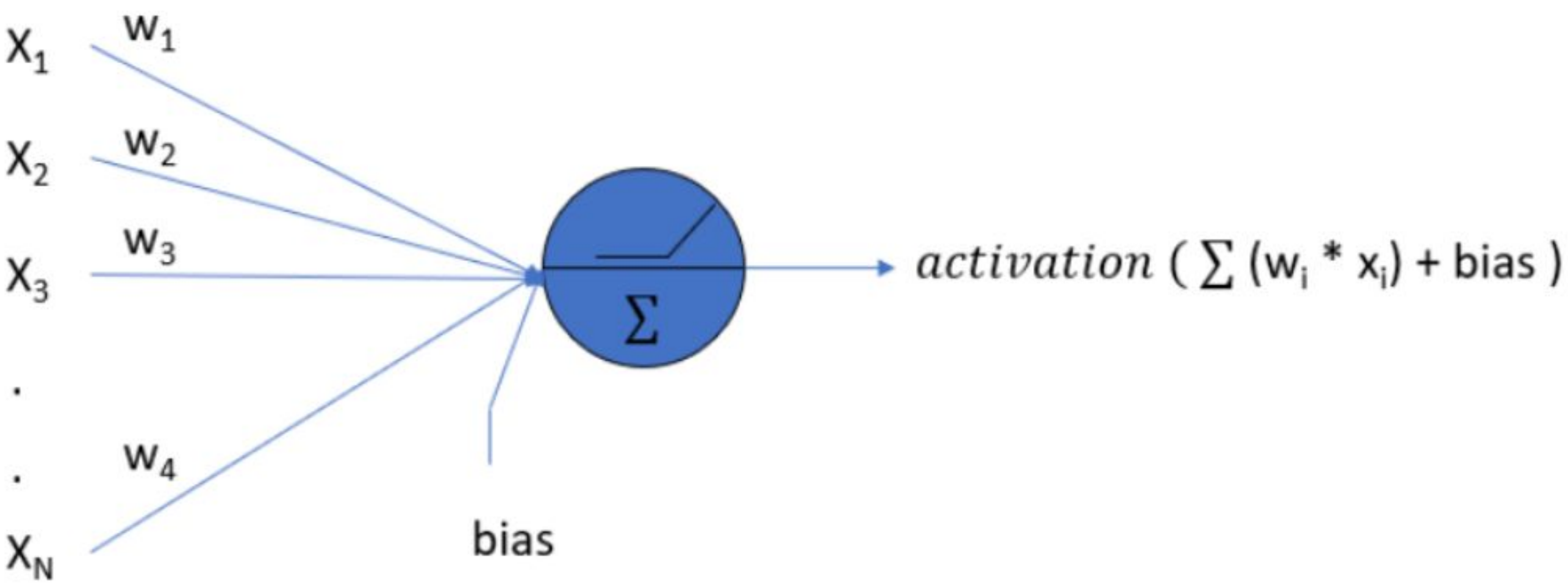
DEEP HEDGING

- Utilizing data-driven analysis, instead of assumptions and approximations.
- Deep hedging uses neural network and expects market information and the current delta position as input.
- Predictions for the next delta using the input data.
- Deep hedging strategy
    - (Market data, Current delta) $\longrightarrow$ Next Delta

Algorithm

- ➢ Define number of paths
- ➢ Initialize initial stock price(S), current delta, and stochastic integral.
- ➢ For each t, t $\in$ [0, T], iterate this
  - ○ Generate noise (G) with respect to stock price.
  - ○ Compute dt (t/T) and using volatility calculate the delta of the stock price.
  - ○ Pass the stock price and initial delta in the neural network and get the output.
  - ○ Using the output, initial stochastic integral, delta of stock price and S update stochastic integral.
  - ○ Update stock price (S + delta of stock price)
- ➢ Calculate the payoff using the final stock price and target price
- ➢ Compute hedging error (theoretical Black-Scholes price + stochastic integral-Payoff)
- ➢ Calculate the Loss function of hedging error

FEED FORWARD NEURAL NETWORK (FNN)



A single neuron shown with X_i inputs with their respective weights W_i and a bias term and applied activation function

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FEED FORWARD NEURAL NETWORK (FNN)

- Input Layer : Sum of weights and input data along with bais
- Activation Function : Used to capture non-linearity in data. e.g : ReLu, tanh, Sigmoid,linear etc.
- Input layer data passes through hidden layer consists of defined number of nodes
- Number of Hidden Layer is tuning parameter
- Output layer calculate total loss using loss function. e.g : MSE, Hinge etc.

- Based on total losses optimizers such ADAM used for updating weights, this process is Back-propagation.
- Depending on hyper parameters, forward propagation and backpropagation continue until defined hyper parameters found

FEED FORWARD NEURAL NETWORK (FNN)

- Total 64 nodes with 6 layers
- Activation function : LeakyReLu and Tanh
- Total iterations 10000 with 256 paths
- Loss function : MSE
- Optimizers : ADAM
- Training method : Custom training

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SUPPORT VECTOR REGRESSION (SVR)

Example: Applications in Statistical Estimation

- Y = regressant (target variable)
- $\boldsymbol{X} = (X_1, \dots, X_d)$ = regressors (factors)
- $Z_f = Y - f(\boldsymbol{X}) - C$ = residual
- $\bar{Z}_f = Y - f(\boldsymbol{X})$ = residual w/o intercept

Generalized Regression

$$\min_{f \in \mathcal{F}, C} \mathcal{E}(Z_f) = \min_{f \in \mathcal{F}} \underbrace{\min_{C} \mathcal{E}(\bar{Z}_f - C)}_{\text{error projection}}$$

$$= \min_{f \in \mathcal{F}} \mathcal{D}(\bar{Z}_f) \text{ and } C \in \mathcal{S}(\bar{Z}_f) = \operatorname*{argmin}_{C} \mathcal{E}(\bar{Z}_f - C)$$

SUPPORT VECTOR REGRESSION (SVR)

ε-SVR: Discrete Case

Given: training data $X^\ell = (x_i, y_i)_{i=1}^{\ell}, x_i \in \mathbb{R}^n, y_i \in \mathbb{R}, y = (y_1, \dots, y_\ell)$
Find: hyperplane $f_{w,b}(x) = w^\top x + b,\ (w, b) \in \mathbb{R}^{n+1}$ that optimally fits the data

Let $z = z(\boldsymbol{w}, b)$ be a random variable taking with equal probabilities $p = 1/\ell$ the components $\left(y_i - f_{\boldsymbol{w},b}(\boldsymbol{x}_i)\right)_{i=1}^{\ell}$, $C > 0, \nu \in (0,1]$

Denote $\mathcal{L}(\xi) = \max\{0, |\xi| - \varepsilon\} = [|\xi| - \varepsilon]_+ =$" ε-insensitive loss function"

$$\min_{\boldsymbol{w},b} \mathbb{E}[\mathcal{L}(z(\boldsymbol{w}, b))] + \frac{1}{2C} \| \boldsymbol{w} \|_2^2$$

Let $\nu = 1 - \alpha$. Consider

$$\min_{\varepsilon} (1 - \alpha) \left(\varepsilon + \frac{1}{1 - \alpha} \mathbb{E}[|z(\boldsymbol{w}, b)| - \varepsilon]_+ \right) = (1 - \alpha) \bar{q}_\alpha(|z(\boldsymbol{w}, b)|)$$

$= \langle\langle z(\boldsymbol{w}, b) \rangle\rangle_\alpha =$ CVaR norm [Bertsimas et.al., 2014; Mafusalov & Uryasev, 2016]

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SUPPORT VECTOR REGRESSION (SVR)

ν-SVR: Discrete Case

The ν-SVR [Schölkopf et al., 2000]

$$\min_{\boldsymbol{w},b,\varepsilon} \nu\left(\varepsilon + \frac{1}{\nu}\mathbb{E}[\mathcal{L}(z(\boldsymbol{w},b))]\right) + \frac{1}{2C}\parallel \boldsymbol{w} \parallel_2^2$$

The ν-SVR [Malandii & Uryasev, 2022]

$$\min_{\boldsymbol{w},b} \langle\langle z(\boldsymbol{w},b)\rangle\rangle_\alpha + \frac{1}{2C}\parallel \boldsymbol{w} \parallel_2^2$$

The ν-SVR [Takeda et al., 2010]

$$\min_{\boldsymbol{w},b} \mathrm{CVaR}_\alpha\left(\left|\boldsymbol{y} - C\frac{f(\boldsymbol{w},b)}{\parallel \boldsymbol{w} \parallel}\right|\right)$$

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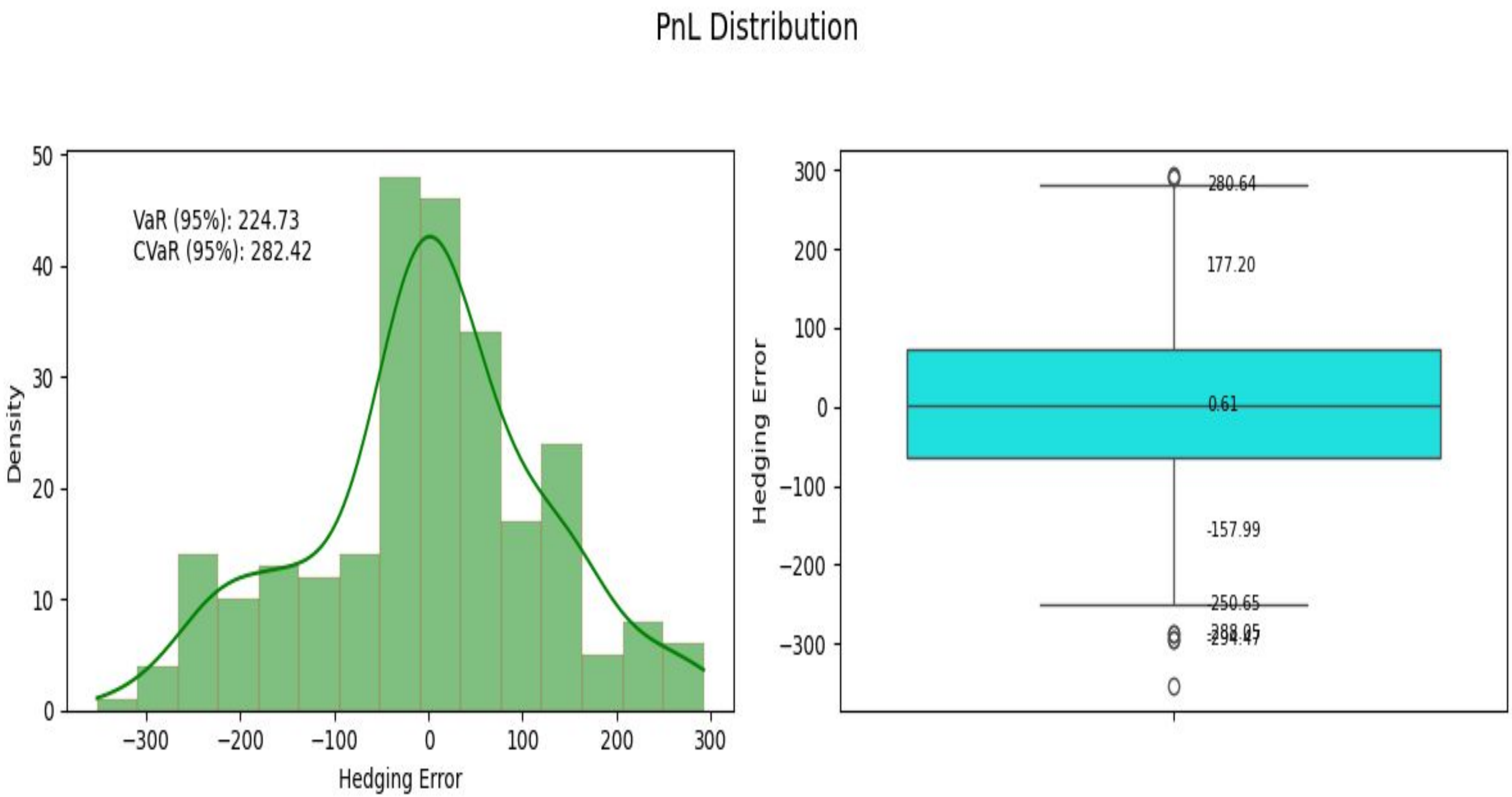
RESULTS 1: FNN



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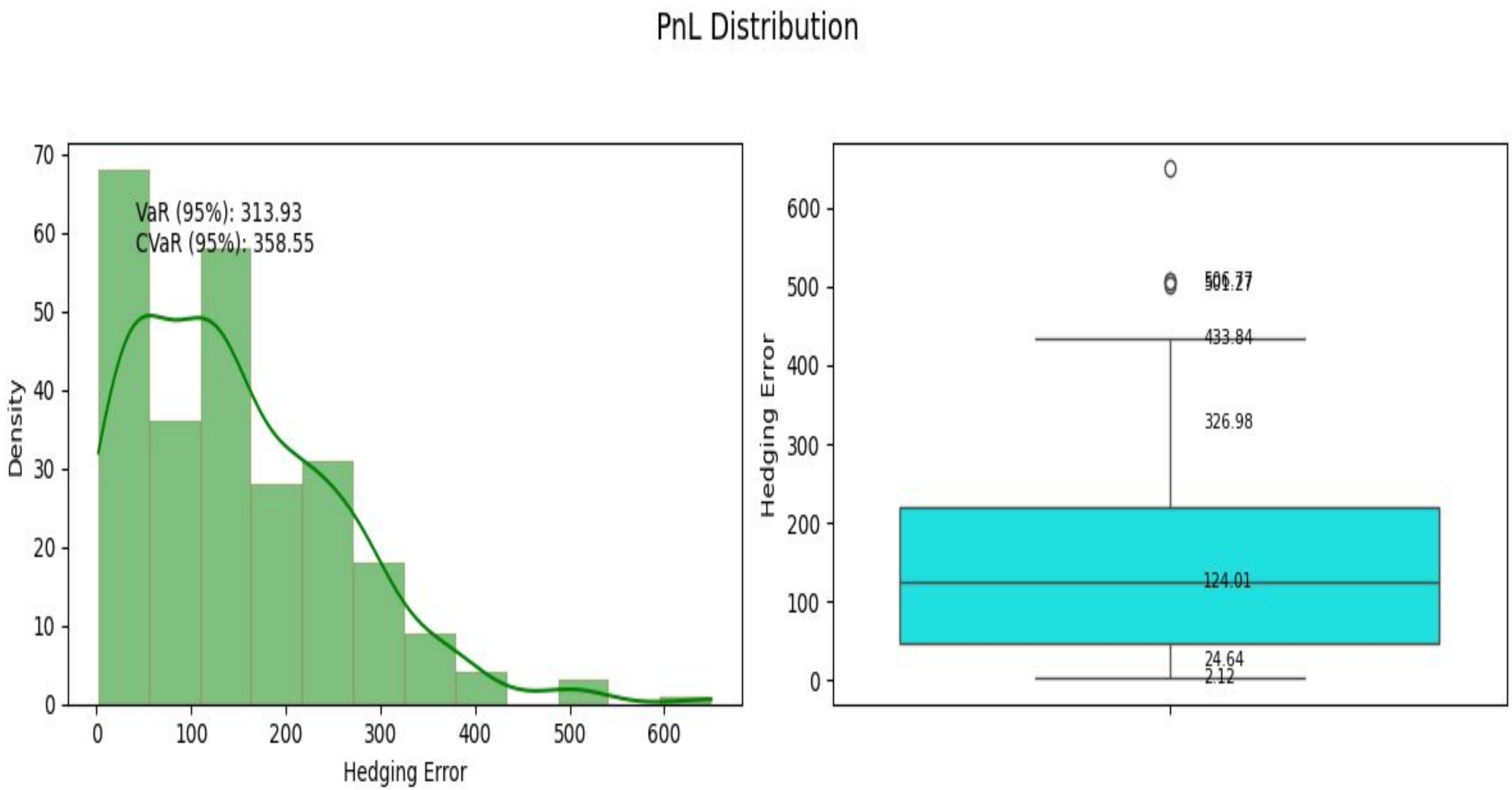
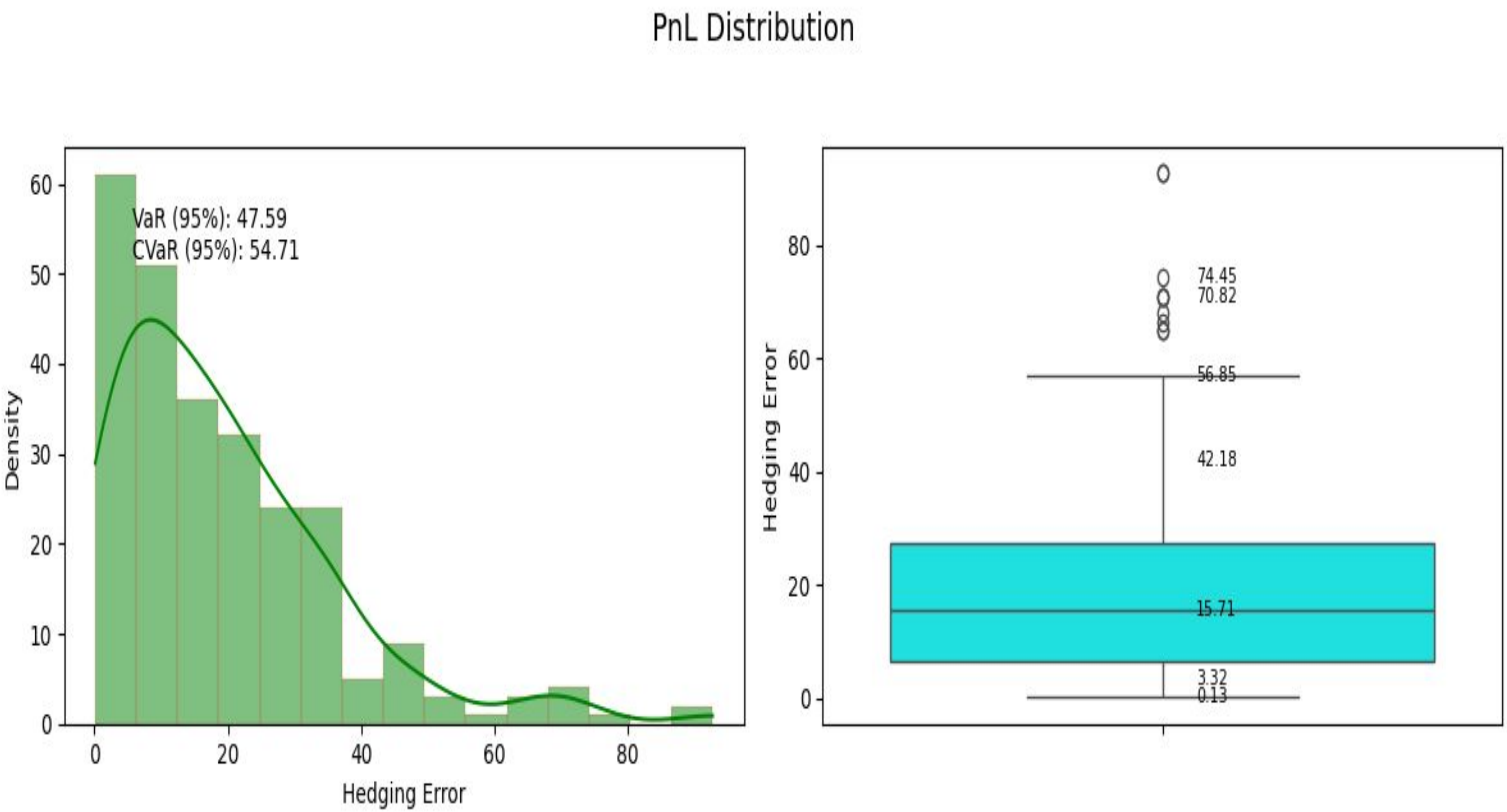
RESULTS 1: ε-SVR



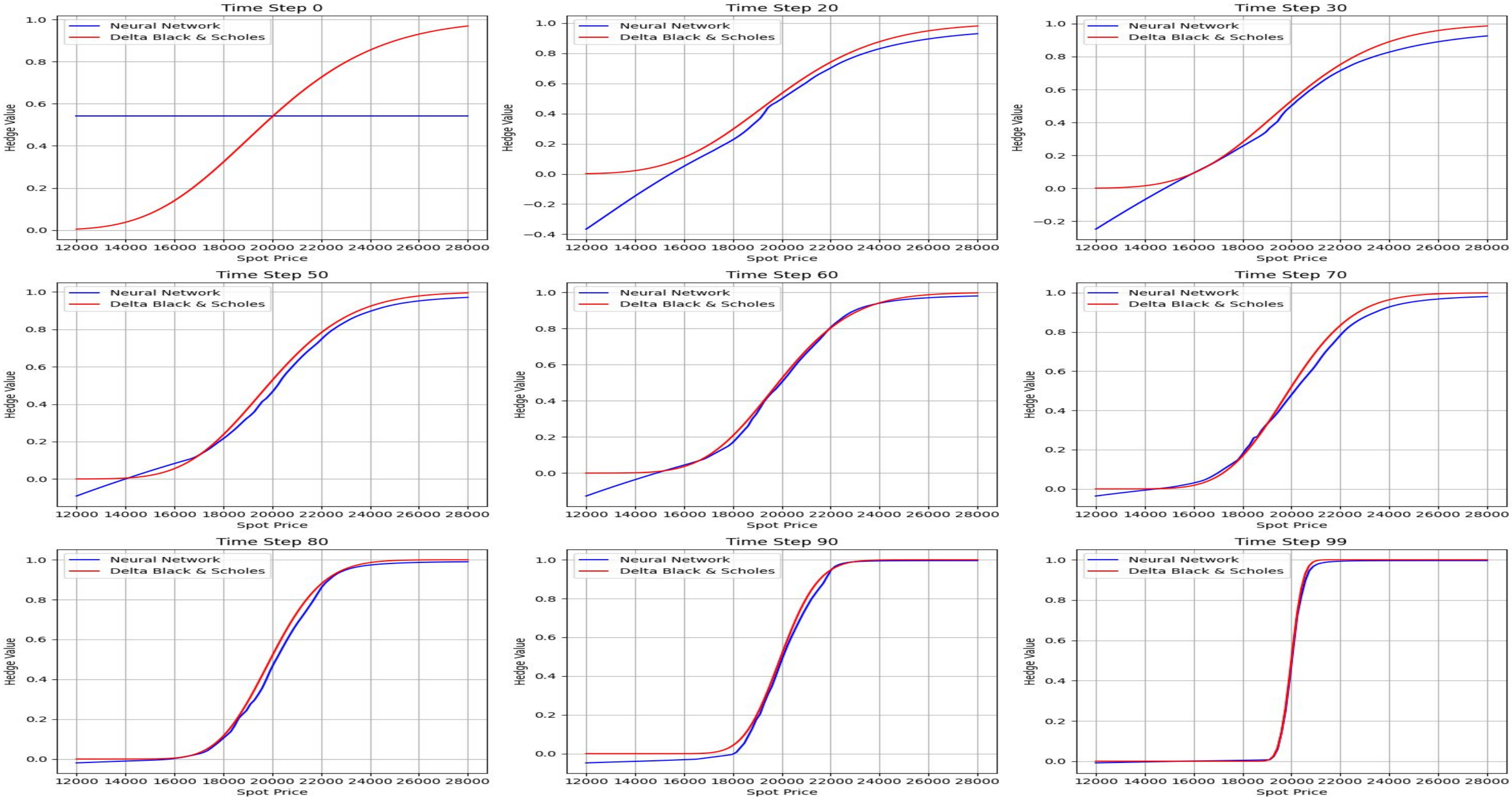
RESULTS 1: ν-SVR



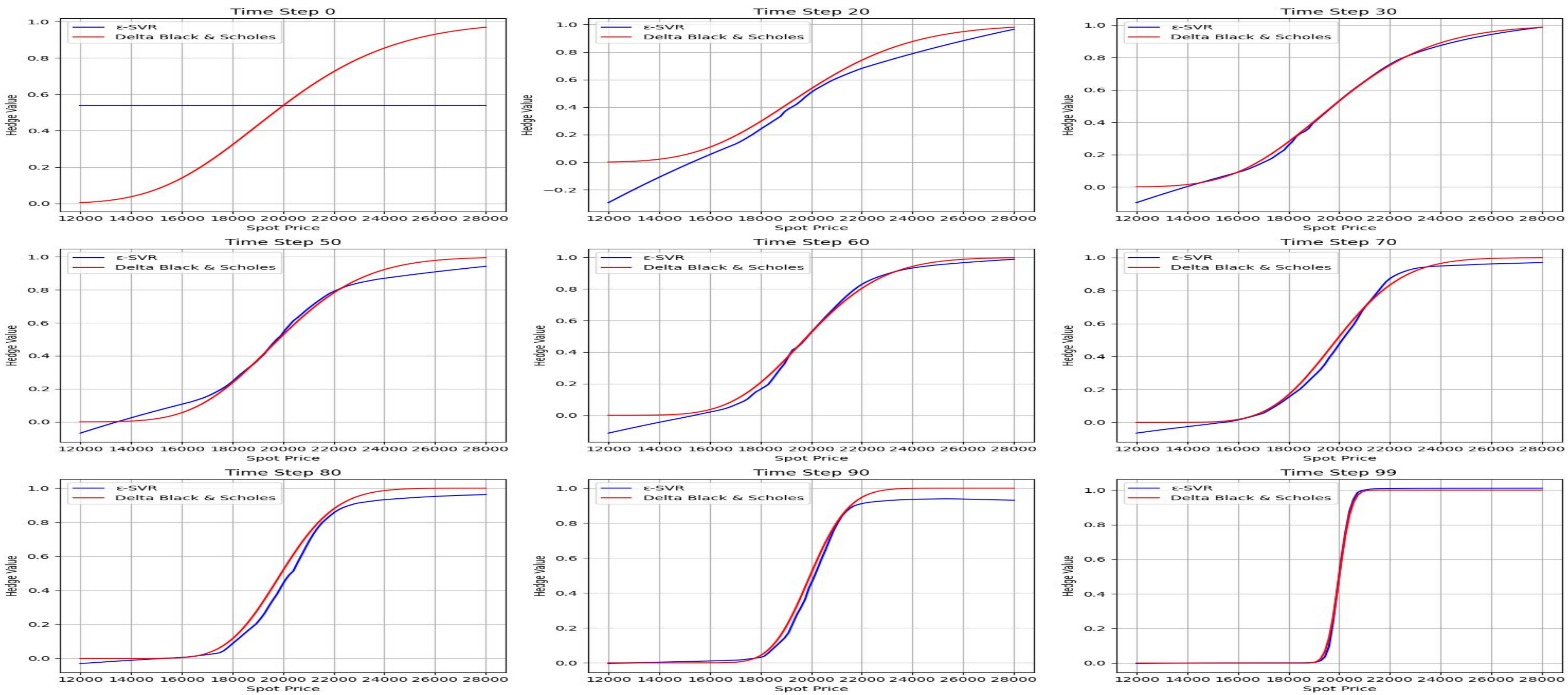
RESULTS 2 : METRICS

Metrics	Neural Network(FNN)	ε-SVR	ν-SVR
Mean	-2.339	138.315	19.539
Standard Deviation	138.051	106.767	17.051
Skewness	0.049	1.182	1.426
Kurtosis	0.099	1.952	2.255
Percentile(5%)	-228.037	10.619	1.189
Percentile(95%)	227.848	330.813	54.555
VaR(95%)	224.734		
CVaR(95%)	282.420		

RESULTS 3 : FNN



RESULTS 3: ε-SVR



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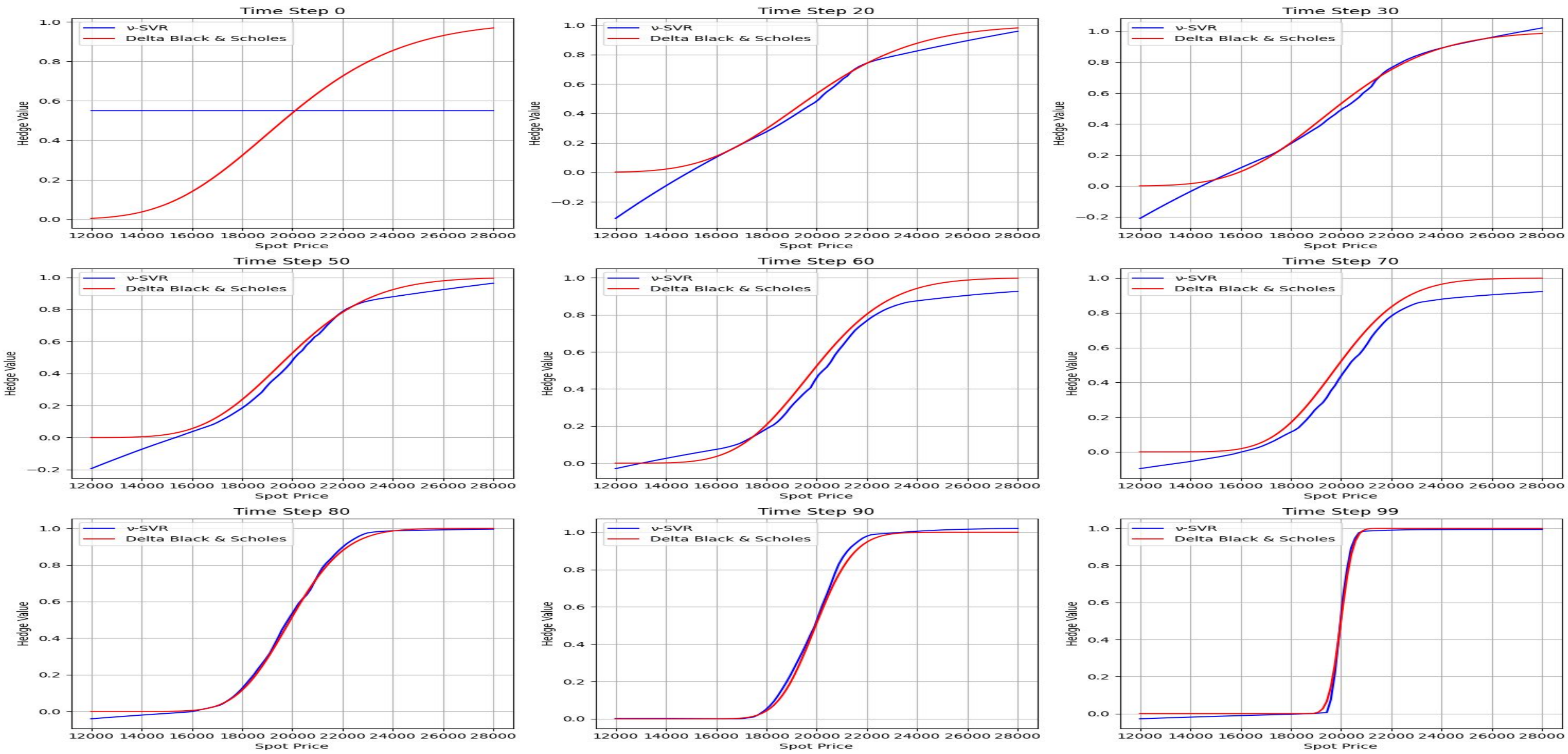
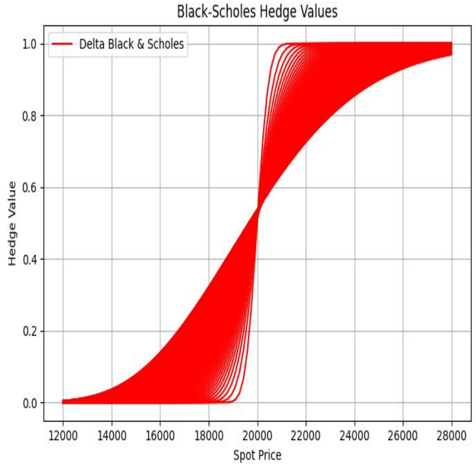
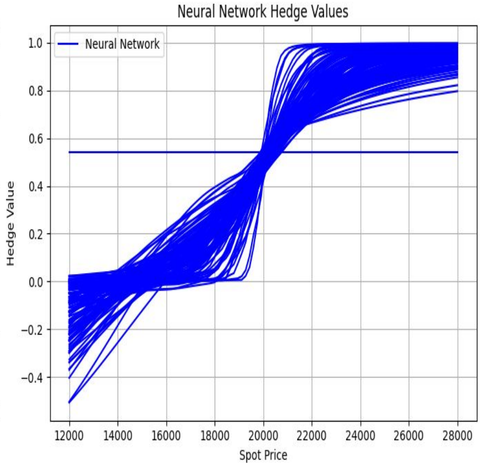
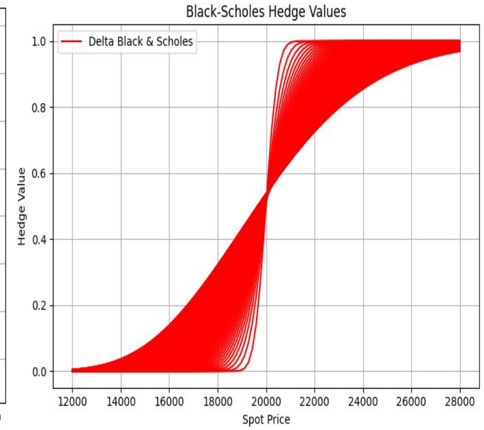
RESULTS 3: ν-SVR



RESULTS 4


Black-Scholes Hedge Values
Delta Black & Scholes
Hedge Value
Spot Price


Neural Network Hedge Values
Neural Network
Hedge Value
Spot Price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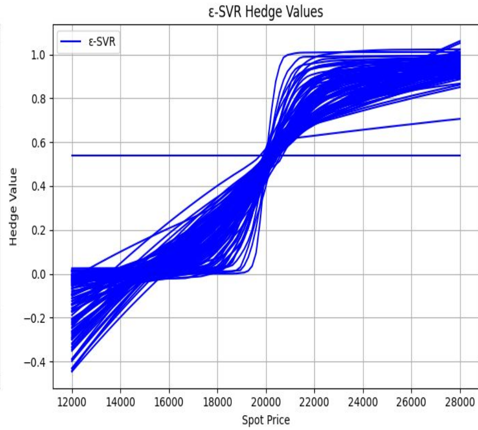
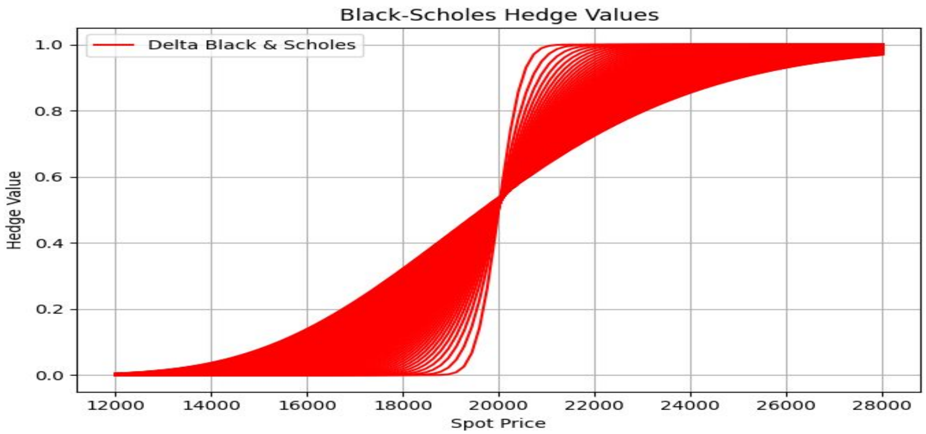

ε-SVR Hedge Values
ε-SVR
Hedge Value
Spot Price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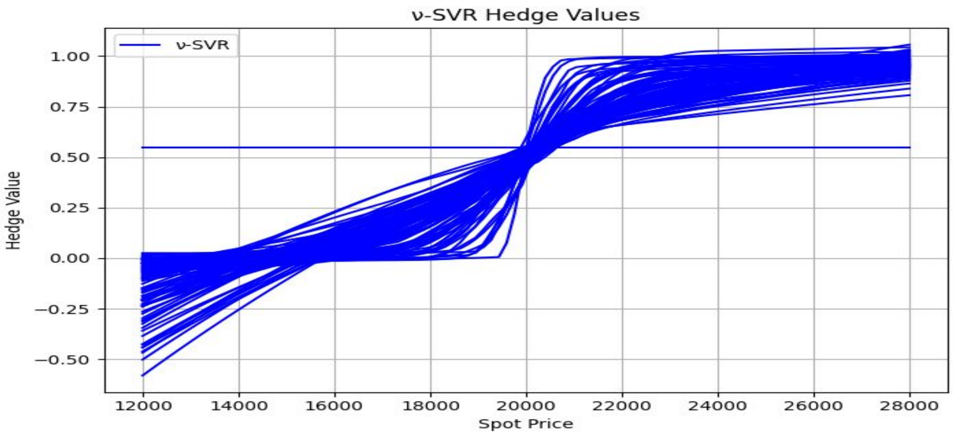
ν-SVR Hedge Values
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THANK YOU !