Statistics Assignment

Question-1:
The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not.
Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.) Propose the type of probability distribution that would accurately portray the above scenario, and list out the three conditions that this distribution follows.

b.) Calculate the required probability.

Answer:
The probability that drug is able to produce satisfactory result = 4 * (Probability that drug is not able to produce a satisfactory result)
Thus,

P(drug is able to produce satisfactory result) = ⅘ = 0.8
P(drug is not able to produce satisfactory result) = ⅕ = 0.2

The type of probability distribution that would accurately portray the above scenario is **Binomial Distribution**. The three conditions that this distribution follows are:

1. Total number of trials is fixed at n
2. Each trial is binary, i.e., has only two possible outcomes - success or failure, heads or tail etc.
3. The probability of success is same (not varying) in all trials, denoted by p

The formula for calculating binomial probability is given by:

$$P(X = r) = nCr(p)^r(1-p)^{n-r}$$

Where **n** is the number of trials, **r** is the number of successful events, **p** is the probability of success.

As per the question, we need to calculate the probabilities for X<=3, i.e. X=0, X=1, X=2, and X=3.
Thus, probability that the drug was able to do a satisfactory job

$= P(X=0) + P(X=1) + P(X=2) + P(X=3)$
$= (10C0(0.2)^0(0.8)^{10}) + (10C1(0.2)^1(0.8)^9) + (10C2(0.2)^2(0.8)^8) + (10C3(0.2)^3(0.8)^7)$
= (0.1073) + (0.2684) + (0.3019) + (0.2013)
= 0.8789

Thus, there is a probability of **87.89%** or **0.8789** that at most, 3 drugs are not able to do a satisfactory job.

Question-2:
For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

a.)Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.

b.)Find the required range.

Answer:
The above problem could be solved using **Centra Limit Theorem** which says that, for any kind of data, provided a high number of samples has been taken (**n>30**), the following properties hold true:

1. Sampling distribution's mean equals to Population mean

   $$\mu_{\widehat{X}} = \mu$$

2. Sampling distribution's standard deviation (Standard error)

   $$= \frac{\sigma}{\sqrt{n}}$$

   Where **σ** is the standard deviation of the original distribution and **n** is sample size
3. For **n > 30**, the sampling distribution becomes a normal distribution

According to the question,
The sample of size, n=100
The sample mean, $\overline{X}$=207
Standard deviation, S=65
The confidence level is 95%

We can find the corresponding Z^* value using Table of z-values for Confidence Intervals.
Thus, corresponding Z^* value is ± 1.96

The range of population mean (μ) is given by $\left(\overline{X} - \frac{Z^* S}{\sqrt{n}} \text{ , } \overline{X} + \frac{Z^* S}{\sqrt{n}}\right)$

Thus, the range for our case is $\left(207 - \frac{1.96*65}{\sqrt{100}} \text{ , } 207 + \frac{1.96*65}{\sqrt{100}}\right)$

Which is equal to (194.26, 219.74)

Thus, the range in which the population mean time (of the effect of the drug) will lie with 95% confidence level is from **194.26 seconds to 219.74 seconds**.

Question-3:
a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

b) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by α and β respectively. For the current hypothesis test conditions (sample size, mean, and standard deviation), the value of α and β come out to 0.05 and 0.45 respectively.

Now, a different sampling procedure is proposed so that when the same hypothesis test is conducted, the values of α and β are controlled at 0.15 each. Explain under what conditions would either method be more preferred than the other.

Answer:
a) The Null hypothesis will be:
The painkiller drug have a time of effect of at most 200 seconds
Or $H_0 : \mu \leq 200$
The Alternate hypothesis will be:
The painkiller drug have a time of effect of more than 200 seconds
Or $H_1 : \mu > 200$
Here,
The population mean, μ=200, the sample mean, $\mu_{\hat{X}} = 207$, the sample of size, n=100, the sample mean, X=207, standard deviation, S=65, significance level = 5%

Utilizing **critical value method** and **p-value method** to make the decision.

i) Using the critical value method,
Standard Error = $\sigma_{\bar{x}}$ = $\frac{\sigma}{\sqrt{n}}$ = $\frac{65}{\sqrt{100}}$ = 6.5
Cumulative probability of UCV from the value of
α = 0.95 (since it is one tail test)

Thus the value of Z_c from Z-Table comes out to be
Z_c = 1.645 (for given value of α = 0.95)

Now we calculate the Critical Value
UCV = $\mu + (Z_c * \sigma_{\bar{x}})$ = 200 + (1.645*6.5) = 210.6925

Since our sample mean $\mu_{\hat{X}} = 207$ which is less than our Upper Critical Value of 210.6925
Thus, we **fail to reject the Null Hypothesis**.

ii) Using the p-value method,
Z-score = $(\bar{x} - \mu)/(\sigma/\sqrt{n})$ = (207-200)/(65/10) = (7)/(6.5) = 1.0769 $\approx$ 1.077

The sample mean is on the right side of the distribution mean (the z-score is positive)

Cumulative probability of sample point (using Z-table) = 0.8577 (since it is a one tail test)

Therefore, cumulative probability = 0.8577
For one-tailed test → p = 1 - 0.8577 = 0.1423

As p-value (0.1423) is greater than the value of α (0.05).
Thus, we **fail to reject the Null Hypothesis**.

b) Type I error occurs when the null hypothesis is true but we reject it,
i.e. reject H_0 when it is true
Type II error occurs when the null hypothesis is false but we fail to reject it,
i.e. fail to reject H_0 when it is false

Earlier, α = 0.05 and β = 0.45
This means that in 5% of cases, the painkiller drug have a time of effect of at most 200 seconds but we rejected that. And that in 45% of cases, the painkiller drug have a time of effect of at more than 200 seconds but we accepted that.
This means roughly 45% of our drugs did not produce a satisfactory result and 5% of drugs that could produce satisfactory results were rejected.

Inference: The result of the painkiller drug will be unsatisfactory in roughly 45% cases, hence, its a concern for the company. Only 5% of good drugs are being rejected, hence, it will reduce the wastage.
It will reduce the wastage of good drugs but the accepted drugs will be very less effective.
If the company wants to focus only on profits and reduce operational cost and wastage, this sampling method is better.

Later, α = 0.15 and β = 0.15
This means that in 15% of cases, the painkiller drug have a time of effect of at most 200 seconds but we rejected that. And that in 15% of cases, the painkiller drug have a time of effect of at more than 200 seconds but we accepted that.
This means roughly 15% of our drugs did not produce a satisfactory result and 15% of drugs that could produce satisfactory results were rejected.

Inference: The result of the painkiller drug will be unsatisfactory in roughly 15% cases, hence. Also, 15% of drugs are being rejected.
It will have more wastage of good drugs but the accepted drugs will be much more effective.
If the company wants to focus on the effectiveness of painkiller drug and is comfortable with higher operational cost and wastage, this sampling method is better.

Question-4:
Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign for its existing subscribers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.

Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.

Answer:
A/B testing is a direct industry application of the two-sample proportion test sample. It can help us statistically figure out the better version of the two taglines. We can implement A/B testing by the following steps:

<u>Step 1: Identify the test variable</u>
Here, in our case, there are two different taglines on the online ad campaign platform. Thus our test variable is the tagline.

<u>Step 2: Create a 'control' and a 'challenger'</u>
One of the taglines will be considered control while the other will be considered challenger, throughout the A/B test.

<u>Step 3: Form a hypothesis</u>
Let's say our hypothesis is that the control tagline is equally good or better than challenger tagline.

<u>Step 4: Split your sample groups equally and randomly</u>
Almost half of the random visitors must see the control tagline and remaining visitors should see the challenger tagline.

<u>Step 5: Test both variations simultaneously</u>
Test both the variations simultaneously and collect feedback from visitors on both the platform. If there is an action associated with the ad campaign, measure the action response on the two variations of the ad campaign.

<u>Step 6: Give the A/B test enough time to produce useful data</u>
A significant number of visitors must visit both the variations of the ad campaign before we can derive/conclude any result from the A/B test.

<u>Step 7: Take action based on the result of A/B testing</u>
We can now decide on which tagline is better than the other. There may be a case that the tagline does not have any significant effect on the ad campaign. We may use the result of A/B testing and then make changes to the ad campaign accordingly.