Conversion terms

Mean Available Potential Energy to Mean Kinetic Energy

$$C(P_m, K_m) = -\int_V g\overline{\rho}\,\overline{w}dV$$

Eddy Available Potential Energy to Eddy Kinetic Energy

$$C(P_e, K_e) = -\int_V g\overline{\rho' w'}dV$$

Eddy Kinetic Energy to Mean Kinetic Energy

$$C(K_e, K_m) = \int_V (\rho_o \overline{u' \boldsymbol{u}'} \nabla \overline{u} + \rho_o \overline{v' \boldsymbol{u}'}. \nabla \overline{v})dV$$

Eddy Available Potential Energy to Mean Available Potential Energy

$$C(P_e, P_m) = -\int_V \frac{g}{n_o} \overline{\rho' \boldsymbol{u}'}. \nabla_h \overline{\rho} dV$$

Generation Terms

Generation of Mean Available Potential Energy

$$G(P_m) = -\int_S g \frac{\alpha_{o,1}}{n_o} \overline{J}_s \overline{\rho *} dS - \int_S g \frac{\beta_{o,1}}{n_o} \overline{G}_s \overline{\rho *} dS$$

here J is surface temperature flux

G is surface salinity flux

$\alpha_{o,1}$ and $\beta_{o,1}$ are coefficient of expansions at the top layer

$J_s = \frac{H}{\rho_s c}$ and H is surface heat flux

$G_s = S_1(E - P)$

E is evaporation and P is precipitation

S_1 is the surface salinity </br>

$\rho* = \rho - \rho_{ref}$
$\rho_{ref} = \langle\overline{\rho}\rangle$ where $\langle\ldots\rangle$ refer to area averaging

Generation of mean Kinetic Energy

$$G(K_m) = \int_S (\overline{\tau_x}\,\overline{u} + \overline{\tau_y}\,\overline{v})\,dS$$

Generation of Eddy Kinetic Energy

$$G(K_e) = \int_S (\overline{\tau_x' u'} + \overline{\tau_y' v'})\,dS$$

Generation of Eddy Available Potential Energy

$$G(P_e) = -\int_S g\frac{\alpha_{o,1}}{n_o}\overline{J_s'\rho'}\,dS - \int_S g\frac{\beta_{o,1}}{n_o}\overline{G_s'\rho'}\,dS$$

Energy reservoirs

Mean Potential Energy

$$P_m = -\int_V \frac{g}{2n_0}\overline{\rho*}^2 dV$$

Mean Kinetic Energy

$$K_m = \int_V \frac{1}{2}\rho_0\left(\overline{u}^2 + \overline{v}^2\right)$$

Eddy Potential Energy

$$P_e = -\int_V \frac{g}{2n_0}\overline{\rho*'^2} dV$$

Eddy Kinetic Energy

$$K_e = \int_V \frac{1}{2}\rho_0\left(\overline{u'^2} + \overline{v'^2}\right)$$

Fields to be writtten from POP2 Output

Primes to be calculated

$$\overline{\rho' \boldsymbol{u}'}$$

$$\overline{u' \boldsymbol{u}'} = \overline{u' u'} + \overline{u' v'} + \overline{u' w'}$$

$$\overline{v' \boldsymbol{u}'} = \overline{v' u'} + \overline{v' v'} + \overline{v' w'}$$

$$\overline{\rho' w'}$$

$$\overline{J_s' \rho'}$$

$$\overline{G_s' \rho'}$$

$$\overline{\tau_x' u'}$$

$$\overline{\tau_y' v'}$$

On the value of n_o and Assumptions made for $\varrho,\ \alpha,\ and\ \beta$

$$\frac{d\rho}{dt} = \left(\frac{\partial\rho}{\partial\theta}\right)_{p,S}\frac{d\theta}{dt} + \left(\frac{\partial\rho}{\partial S}\right)_{p,\theta}\frac{dS}{dt} + \left(\frac{\partial\rho}{\partial z}\right)_{\theta,S} w$$

$$\frac{d\theta}{dt} = \frac{\partial J}{\partial z}\ and\ \frac{dS}{dt} = \frac{\partial G}{\partial z}$$

This gives us

$$\frac{\partial\rho}{\partial t} + \boldsymbol{u_h}.\nabla_h\rho + w\left(\frac{\partial\rho}{\partial z} - \left(\frac{\partial\rho}{\partial z}\right)_{S,\theta}\right) = \left(\frac{\partial\rho}{\partial\theta}\right)_{S,p}\frac{\partial J}{\partial z} + \left(\frac{\partial\rho}{\partial S}\right)_{S,\theta}\frac{\partial G}{\partial z}$$

$$\frac{\partial\rho}{\partial t} + \boldsymbol{u_h}.\nabla_h\rho + w\left(\frac{\partial\varrho}{\partial z}\right) = \alpha_o(\lambda,\phi,z)\frac{\partial J}{\partial z} + \beta_o(\lambda,\phi,z)\frac{\partial G}{\partial z}$$

$$\frac{\partial\rho}{\partial z} - \left(\frac{\partial\rho}{\partial z}\right)_{S,\theta} = \frac{\partial\varrho}{\partial z}$$

is vertial local potential density gradient

$$\frac{\partial\varrho}{\partial z} \approx \frac{\partial\langle\overline{\varrho}\rangle}{\partial z} = n_o(z)$$

$$\left(\frac{\partial\rho}{\partial\theta}\right)_{S,p} \approx \left(\frac{\partial\overline{\rho}}{\partial\theta}\right)_{S,p} = \alpha_o(\lambda,\phi,z)$$

$$\left(\frac{\partial\rho}{\partial S}\right)_{\theta,p} \approx \left(\frac{\partial\overline{\rho}}{\partial S}\right)_{\theta,p} = \beta_o(\lambda,\phi,z)$$

The value of α and β used in POP2 in linear Equation of State

$$\alpha_o = \left(\frac{\partial\rho}{\partial T}\right)_{S,p} = 2.55\times 10^{-4} gr/cm^3/K$$

$$\beta_o = \left(\frac{\partial\rho}{\partial S}\right)_{T,p} = 7.64\times 10^{-}1 gr/cm^3/msu$$

$$1 gram/kg = 0.001 msu$$

Results

Red are for relative wind forcing

Blue are for absolute wind forcing

Black is from paper [An Estimate of the Lorenz Energy Cycle for the World Ocean Based on the 1/10 STORM/NCEP Simulation](https://journals.ametsoc.org/doi/full/10.1175/JPO-D-12-079.1?searchText=Complex%2BTerrain%2BMesoscale%2BConvective%2BSystems&af=R) (https://journals.ametsoc.org/doi/full/10.1175/JPO-D-12-079.1? searchText=Complex%2BTerrain%2BMesoscale%2BConvective%2BSystems&af=I