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M1IF

Exercice 2:

1) The (ξ_i)'s are iid with a finite variance

$$\overline{X_n} = \mu + \sqrt{n}\left(\frac{1}{n}\sum_{i=1}^{n} \xi_i - \underbrace{0}_{E(\xi_1)}\right) \times \frac{1}{\sqrt{n}}$$

Thanks to the central-limit theorem, for n big enough, $E(\overline{X_n}) \simeq \mu$ and $V(\overline{X_n}) \simeq 0$ so the empirical mean will tend to μ.

2) $\overline{X_n}$ is a real RV (almost surely) non negative and $E(\overline{X_n}) = \frac{1}{n} n E(X_1) = \mu$.

Bienaymé-Chebychev inequality:

$$P(|\overline{X_n} - \mu| \geq 0,1) \leq \frac{V(\overline{X_n})}{0,1^2}$$

$$V(\overline{X_n}) = \frac{1}{n^2} V(X_1 + \dots + X_n) \underset{(X_i)\text{'s are independent}}{=} \frac{1}{n} V(X_1)$$

$$V(X_1) = V(\underbrace{\mu}_{\text{constant}} + \xi_1) = V(\xi_1) = 1$$

$$P(|\overline{X_n} - \mu| \geq 10^{-2}) \leq \frac{1}{10^{-2} n}$$

$$\frac{1}{10^{-2} n} \leq 0,01 \iff n \geq \frac{1}{10000}.$$