ACADÉMIE DE ______________________

A remplir par les candidats avec le plus grand soin
(ni signature, ni marque)

EXAMEN : APPD 2

SESSION : ______________________

COMPOSITION D ______________________

Note sur 20	Coefficient	Note à reporter au P.V.

Établissement d'origine :
Centre de :
Nom :
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Appréciations du correcteur

3/ Task graph scheduling

3.1/ Task graph scheduling without communications

1/ Recall that a valid schedule σ satisfies
$\sigma(u) + w(u) \leq \sigma(v) \quad \forall (u,v) \in E$

where $w : V \longrightarrow \mathbb{N}^*$ ($w(u)$ = time to complete task u)

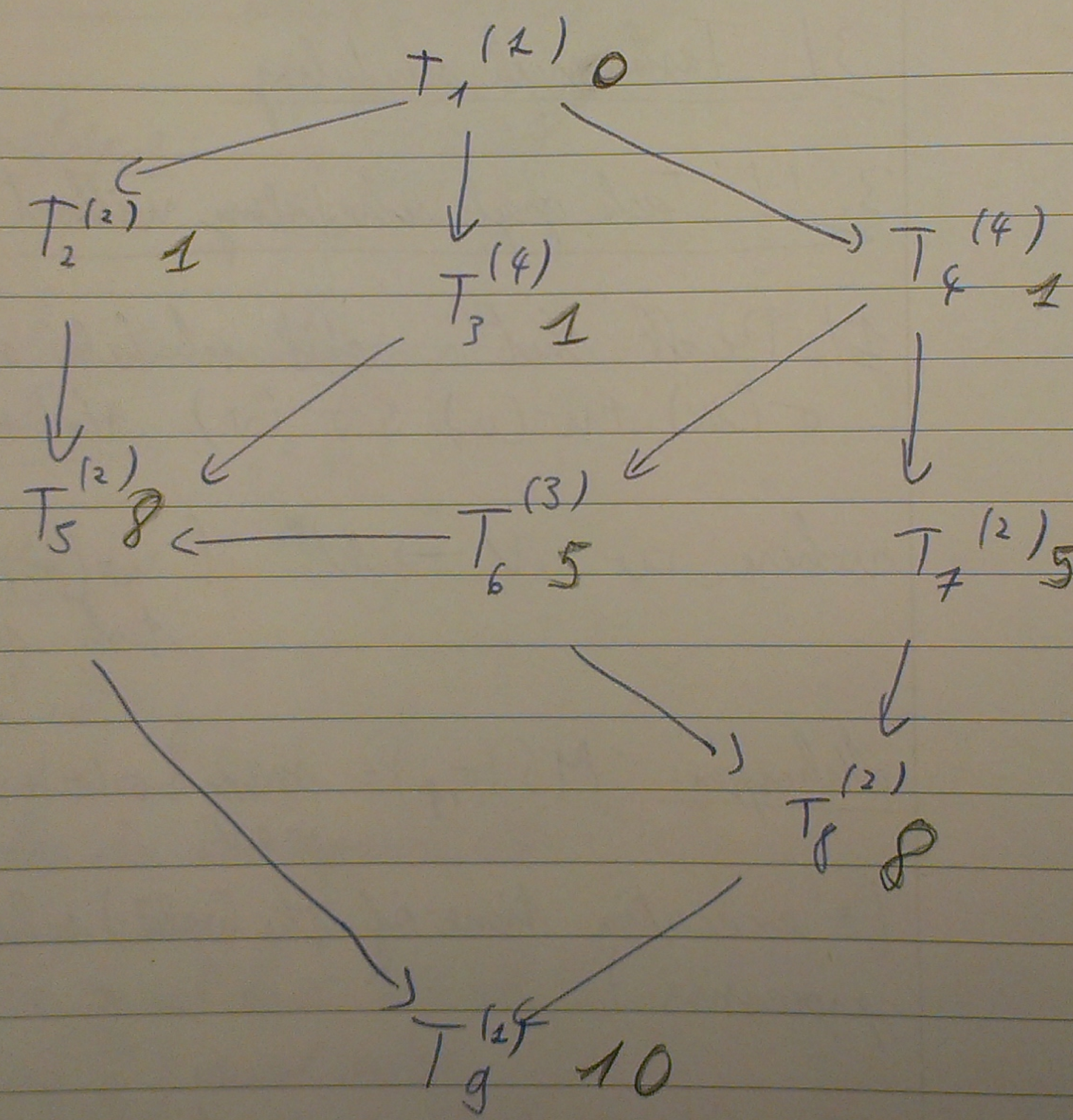
Makespan: $MS(\sigma, p) = \max_{v \in V} \{\sigma(v) + w(v)\} - \min_{v \in V} \{\sigma(v)\}$

(= execution time of the (valid) schedule σ on p processors)

We always have $MS_{opt}(1) \geq MS_{opt}(2) \geq \dots \geq MS_{opt}(\infty)$.

→ Let's compute $MS_{opt}(\infty)$ by doing a BFS on the graph. This means we construct an optimal schedule using as many processors as we need.

On est prié de ne rien écrire au-dessus de cette ligne

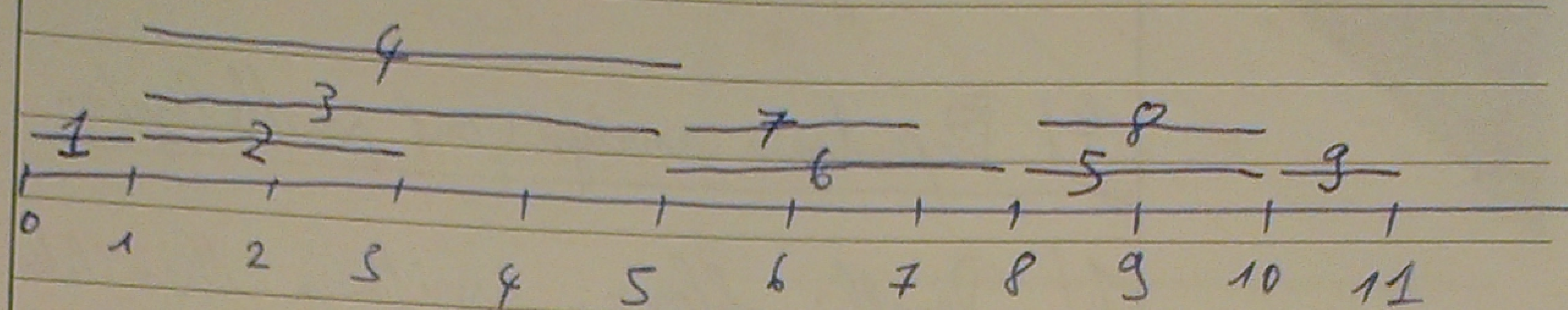


$\sigma(T_i) = t_i$ = time when we can start task T_i.

$$MS_{opt}(\infty) = \underbrace{10}_{\sigma(T_9)} + \underbrace{1}_{\text{execution time of } T_9.}$$

We can also notice that no more than 3 processors are required for this schedule.

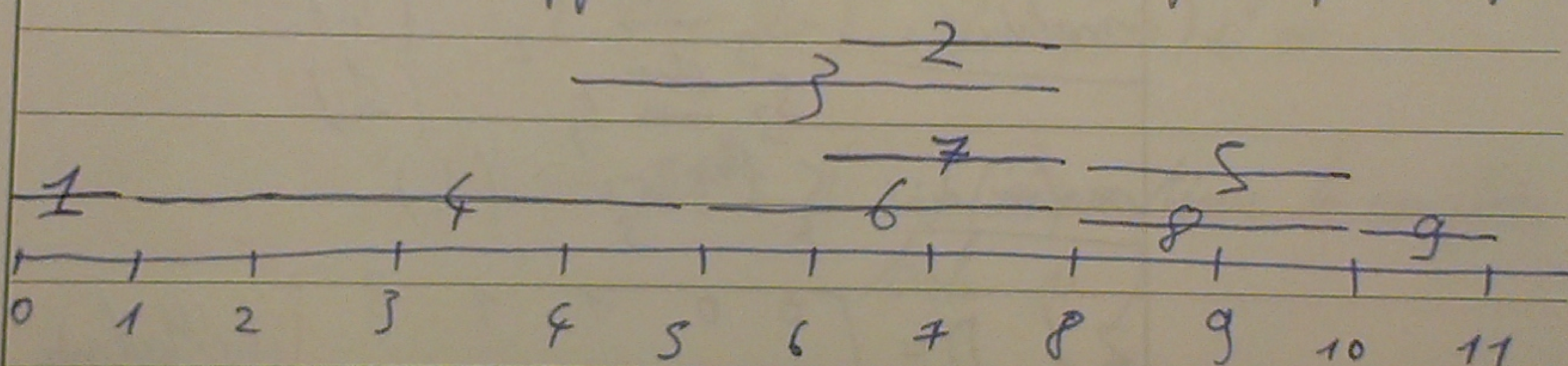
2/ In fact this is exactly what we just did...
Let's put it in an other way. Here is the ASAP ("As soon as possible") schedule:



Once again, only 3 processors are sufficient.

Can we do better? (→ question 4!)

3/ Same strategy (we build the schedule from right to left!)



This time, 4 processors are required.

4/ We have noticed in 1/ that 3 processors are enough to achieve $MS_{opt}(\infty) = 11$.
Let's try with only 2 processors.

* T_1 and T_9 must be computed alone.
* The total time is 21

→ So we still have $21 - 1 - 1 = 19$ time units to share between our two processors.

However, to achieve $MS_{opt}(\infty) = 11$ we would need to give only 9 time units to each processor which only makes $9 \times 2 = 18 < 19$.

$\rightarrow MS_{opt}(\infty)$ is not reachable with only 2 processors.

$\boxed{p_{opt} = 3}$

4/ Dependence analysis and parallelization

1/ Recall: flow: written then (later) read
anti-dependence: read then (later) written
output: written then (later) overwritten

matrix a: $S_2 \xrightarrow{anti} S_1 \quad \begin{pmatrix} 1 \\ -4 \end{pmatrix} \quad \left(\geq_{lex} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right)$

$S_1 \xrightarrow{flow} S_4 \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

to check if we are not wrong!

matrix b: nothing!

matrix c: $S_3 \xrightarrow{flow} S_1 \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$S_3 \xrightarrow{flow} S_4 \quad \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

matrix d: $S_4 \xrightarrow{flow} S_3 \quad \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

2/ $D = \begin{bmatrix} 0 & 0 & 1 & 1 & 3 \\ \underbrace{1}_{d_1} & \underbrace{1}_{d_2} & \underbrace{-4}_{d_3} & \underbrace{-2}_{d_4} & \underbrace{-1}_{d_5} \end{bmatrix}$ (Matrix of dependence vectors in lexicographic order (just to be organised!))

// We must have $\forall k \in [\![1,5]\!], \lfloor \pi \cdot d_k \rfloor \geq 1$

$\pi = \begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix}$ must satisfy: $\begin{cases} \pi_2 \geq 1 \\ \pi_2 \geq 1 \\ \pi_1 - 4\pi_2 \geq 1 \\ \pi_1 - 2\pi_2 \geq 1 \\ 3\pi_1 - \pi_2 \geq 1 \end{cases}$

We can take $\pi = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$.

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3/ Recall that once we have found a vector π that satisfy the constraints, we can build a valid schedule:

$$\sigma_\pi\left(\begin{pmatrix} i \\ j \end{pmatrix}\right) = \left\lfloor \pi \cdot \begin{pmatrix} i \\ j \end{pmatrix} \right\rfloor = \pi_1 i + \pi_2 j$$

$$Dom = \left\{ \begin{pmatrix} i \\ j \end{pmatrix} \,/\, 1 \leq i, j \leq N \right\}$$

2 nested loops

$$T_{\sigma_\pi} = 1 + \max\{\sigma_\pi(p) \,/\, p \in Dom\} - \min\{\sigma_\pi(p) \,/\, p \in Dom\}$$

optimal execution time associated to the schedule built thanks to Lamport's vector

$$T_{\sigma_\pi} = 1 + \begin{bmatrix} 5 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} N \\ N \end{bmatrix} - \begin{bmatrix} 5 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= 1 + 6N - 6$$

$$T_{\sigma_\pi} = -5 + 6N$$

Is this optimal among all T_{σ_π} (where π is a Lamport's vector)?

On est prié de ne rien écrire au-dessus de cette ligne

$\pi_{opt} = \begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix}$ such that $\forall h \in [\![1, 5]\!]$, $\lfloor \pi \cdot d_h \rfloor \geq 1$ (*)

$$T_{\sigma_{\pi_{opt}}} = 1 + \max \left\{ \left\lfloor \pi_{opt} \cdot \begin{bmatrix} i \\ j \end{bmatrix} \right\rfloor / 1 \leq i, j \leq N \right\}$$

$$- \min \left\{ \left\lfloor \pi_{opt} \cdot \begin{bmatrix} i \\ j \end{bmatrix} \right\rfloor / 1 \leq i, j \leq N \right\}$$

$$T_{\sigma_{\pi_{opt}}} = 1 + \pi_1 N + \pi_2 N - \pi_1 - \pi_2$$

Because of (*) we must have $\begin{cases} \pi_1 \geq 1 + 4\pi_2 \\ \pi_2 \geq 1 \end{cases}$

This is optimal for $\pi_2 = 1$ and $\pi_1 = 5$ (which is exactly what we found!)