1) Determine the equation of the curve $2x^2+3y^2-8x+6y-7=0$ when the ~~origor~~ origin is transfered to the point $(2,-1)$

When the origin is shifted to a new point (h,k), we replace

x with $(X+h)$

y with $(Y+k)$

where (X,Y) are the new coordinates with respect to shifted origin.

In this case, the origin is shifted to $(2,-1)$. So, $h=2$, $k=1$

Therefore, $x = X+2$

$y = Y-1$

Given equation : $2x^2+3y^2-8x+6y-7=0$

~~Substitute~~ Substituting and Simplifying

1) Substitute : $2(X+2)^2+3(Y^2-1)^2-8(X+2)+6(Y-1)-7=0$

$2(X^2+4X+4)+3(Y^2-2Y+1)-8X-16+6Y-6-7=0$

$2X^2+8X+8+3Y^2-6Y+3-8X-16+6Y-6-7=0$

$2X^2+3Y^2+(8X-8X)+(-6Y+6Y)+(8+3-16-6-7)=0$

$2X^2+3Y^2-18=0$

Replacing X and Y with x' and y', $2x'^2+3y'-18=0$

2) Transform to parallel axes through the new origin (1, -2) of the equation $2x^2+y^2-4x+4y=0$

Determine the Transformation

New origin : $(h,k) = (1,-2)$

Substitute $x = X+h = X+1$

$y = Y+k = Y-2$

Given equation $2x^2+y^2-4x+4y=0$

Substitute it with x and y. ~~2(x~~

$$2(X+1)^2+(Y-2)^2-4(X+1)+4(Y-2)=0$$

$$= 2(X^2+2X+1)+(Y^2-4Y+4)-4X-4+4Y-8=0$$

$$2X^2+4X+2+Y^2-4Y+4-4X-4+4Y-8=0$$

$$2X^2+Y^2+(4X-4X)+(-4Y+4Y)+(2+4-4-8)=0$$

$$2X^2+Y^2-6=0$$

The transformed equation is $2X^2+Y^2-6=0$

Replacing X and Y with x' and y', $2x'^2+y'^2-6=0$

3) Transform to parallel axes through the new origin (3,1) of the equation $x^2-6x+2y^2+7=0$

Define the Transformation

New origin : $(h,k)=(3,1)$

Substitute $x = X+h = X+3$

$y = Y+k = Y+1$

Given equation $x^2-6x+2y^2+7=0$

Substitute it with x and y.

$(X+3)^2-6(X+3)+2(Y+1)^2+7=0$

$= X^2+6X+9-6X-18+2(Y^2+2Y+1)+7=0$

$X^2+6x+9-6X-18+2Y^2+4Y+2+7=0$

$X^2+2Y^2+\cancel{(6x-6x)}+(6X-6X)+4Y+(9-18+2+7)=0$

$X^2+2Y^2+4Y=0$

The transformed equation is $X^2+2Y^2+4Y=0$

Replacing ~~it with~~ X and Y with x' and y'. $x'^2+2y'^2+4y'=0$

4) Determine the equation of parabola $x^2-2xy+y^2+2x-4y+3=0$ after rotating the axes through the angle $45°$

When ~~we~~ rotating axes counterclockwise by angle θ, we use the following transformations:

$$x = X\cos\theta - Y\sin\theta$$
$$y = X\sin\theta + Y\sin\theta$$

Applying the rotation, $\theta = 45°$. ~~The~~ Therefore $\cos 45 = \frac{1}{\sqrt{2}}$, $\sin 45° = \frac{1}{\sqrt{2}}$

$$x = (X-Y)/\sqrt{2}, \quad y = (X+Y)/\sqrt{2}$$

Substituting these with equation ; $x^2-2xy+y^2+2x-4y+3=0$

$$= \left(\frac{X-Y}{\sqrt{2}}\right)^2 - 2\left(\frac{X-Y}{\sqrt{2}}\right)\left(\frac{X+Y}{\sqrt{2}}\right) + \left(\frac{X+Y}{\sqrt{2}}\right)^2 + 2x\left(\frac{X-Y}{\sqrt{2}}\right) - 4x\left(\frac{X+Y}{\sqrt{2}}\right) + 3 = 0$$

$$= \frac{X^2-2XY+Y^2}{2} - 2\,\frac{X^2-Y^2}{2} + \frac{X^2+2XY+Y^2}{2} + 2\frac{X-Y}{\sqrt{2}} - 4x\frac{X+Y}{\sqrt{2}} + 3 = 0$$

$$= \frac{X^2-2XY+Y^2}{2} - X^2+Y^2 + \frac{X^2+2XY+Y^2}{2} + \frac{X-Y}{\sqrt{2}} - 4x\frac{X+Y}{\sqrt{2}} + 3 = 0$$

$$= \frac{X^2-2XY+Y^2}{2} + \frac{X^2+2XY+Y^2}{2} - X^2+Y^2 + \frac{X-Y}{\sqrt{2}} - 4x\frac{X+Y}{\sqrt{2}} + 3 = 0$$

$$= \frac{2X^2+2Y^2}{2} - X^2+Y^2 + \frac{X-Y}{\sqrt{2}} - 4x\frac{X+Y}{\sqrt{2}} + 3 = 0$$

$$= X^2+Y^2 - X^2+Y^2 + \frac{X-Y}{\sqrt{2}} - 4x\frac{X+Y}{\sqrt{2}} + 3 = 0$$

$$= 2Y^2 + \frac{X-Y}{\sqrt{2}} - 4\frac{X+Y}{\sqrt{2}} + 3 = 0$$

$$= (4Y^2)/2 + (-2X-6Y)/\sqrt{2} + 3 = 0$$

$$= 2Y^2 - \frac{2X}{\sqrt{2}} - \frac{6Y}{\sqrt{2}} + 3 = 0 \quad = 2Y^2 - \sqrt{2}X - 3\sqrt{2}Y + 3 = 0 \; \cancel{= 2Y^2 - \sqrt{2}X - 3\sqrt{2}Y + 3 = 0}$$

$$= 2Y^2 - \sqrt{2}x + 3\sqrt{2}Y + 3 = 0$$

The equation of the parabola after rotating the axes through $45°$ is $2Y^2 - \sqrt{2}X - 3\sqrt{2}Y + 3$

5) Removing the first degree terms in the equation $3x^2+4y^2-12x+4y+13=0$ prove that $3x^2+4y^2=0$

Given equation $3x^2+4y^2-12x+4y+13=0$

$$\Rightarrow 3(x^2-4x)+4(y^2+y)+13=0$$

$$\Rightarrow 3(x^2-2x.2+2^2-2^2)+4\left(y^2+2y\times\frac{1}{2}+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2\right)+13=0$$

$$\Rightarrow 3(x-2)^2-12+4(y+\frac{1}{2})^2-1+13=0$$

$$\Rightarrow 3(x-2)^2+4(y+\frac{1}{2})^2=0$$

Let $X=x-2$, $Y=y+2$, $\therefore 3X^2+4Y^2=0$, this is possible only $X=0$ & $Y=0$

$x-2=0$ | $y+\frac{1}{2}$

$x=2$ | $y=-\frac{1}{2}$

6) Remove the first degree terms in the equation $3x^2-4y^2-6x-8y-10$

Given equation $3x^2-4y^2-6x-8y-10=0$

$$3x^2-6x-4y^2-8y-10=0$$

$$3(x^2-2x)-4(y^2-2y)-10=0$$

$$3(x^2-2x\times1+1^2-1^2)-4(y^2-2y+1-1)-10=0$$

$$3\{(x-1)^2-1\}-4\{(y-1)^2-1\}-10=0$$

$$\Rightarrow 3(x-1)^2-4(y-1)^2-9=0$$

$$3X^2-4Y^2-9=0$$

$X=x-1$, $Y=y-1$

It possible if $X=0$ & $Y=0$

$x-1=0$ $y-1=0$

$\therefore x=1$ $y=1$

(Q6) Transform the equation $11x^2+24xy+4y^2-20x-40y-5=0$ to rectangular axes through point $(2,-1)$ and inclined at angle $\tan^{-1}(-4/3)$

Given equation $11x^2+24xy+4y^2-20x-40y-5=0$ ——— (1)

After shifting the origin to the point $(2,-1)$ we consider $x=x'+2$, $y=y'-1$

~~Subsit~~ substituting x & y in equation (1) $11(x'+2)^2+24(x'+2)(y'-1)+4(y'-1)^2-20(x'+2)-40(y'-1)-5=0$

$\Rightarrow 11x'^2+44x'+44+24(x'+2)\times(y'-1)+4(y'^2-2y'+1)-20x'-40-40y'+40-5=0$

$\Rightarrow 11x'^2+44x'+44+24(x'y'+2y'-x'-2)+4y'^2-8y'+4-20x'-40y'-5=0$

$\Rightarrow 11x'^2+44x'+44+24(x'y'-x'+2y'-2)+4y'^2-8y'+4-20x'\cancel{-40y'+40-5}-40-40y'+40-5=0$

$\Rightarrow 11x'^2+24x'y'+4y'^2=0$ ——— (2)

Now, we need to rotate the axes by the angle $\theta=\tan^{-1}(-4/3)$. The rotation formulas are

$x'=x\cos\theta-y\sin\theta$ $[\cos\theta=3/5,\ \sin\theta=-4/5]$ $=\frac{3}{5}x+\frac{4}{5}y$

$y'=x\sin\theta+y\cos\theta$ $[\cos\theta=3/5,\ \sin\theta=-4/5]$ $=-\frac{4}{5}x+\frac{3}{5}y$

Put those values in equation (2), $11\left(\frac{3}{5}x+\frac{4}{5}y\right)^2+24\left(\frac{3}{5}x+\frac{4}{5}y\right)\left(-\frac{4}{5}x+\frac{3}{5}y\right)+4\left(-\frac{4}{5}x+\frac{3}{5}y\right)^2=0$

$\Rightarrow 11\left(\frac{9}{25}x^2+\frac{24}{25}xy+\frac{16}{25}y^2\right)+24\left(-\frac{12}{25}x^2+\frac{9}{25}xy+\frac{-16}{25}xy+\frac{12}{25}y^2\right)+4\left(\frac{16}{25}x^2+\frac{-24}{25}xy+\frac{9}{25}y^2\right)=0$

$\Rightarrow \frac{99}{25}x^2+\frac{264}{25}xy+\frac{176}{25}y^2+\frac{-288}{25}x^2+\frac{-168}{25}xy+\frac{288}{25}y^2+\frac{64}{25}x^2+\frac{-96}{25}xy+\frac{36}{25}y^2=0$

$\Rightarrow -\frac{125}{25}x^2+\frac{500}{25}y^2=0$

$\Rightarrow -5x^2+20y^2=0$

$\Rightarrow x^2=4y^2$