The Plane

① Find eqn of plane passing through the intersection of $x+2y+3z+4=0$ & $4x+3y+2z+1=0$ —① —② and point $(1,2,3)$.

is given by

eqn ③ passes point $(1,2,3)$.

$$x+2y+3z+4 + k(4x+3y+2z+1)=0 \quad —③$$
$$1+2\times2+3\times3+4+k(4\times1+3\times2+2\times3+1)=0, \quad k=-\frac{18}{17}$$

From ③, putting $k=-\frac{18}{17}$, $x+2y+3z+4-\frac{18}{17}(4x+3y+2z+1)=0$, $55x+20y-15z-50=0$, which is required of plane.

② Find the eqn of plane through points $(4,-2,1)$ & parallel to plane whose direction numbers are $7,2,-3$.

The eqn of a plane through $(4,-2,1)$ is $a(x-4)+b(y+2)+c(z-1)=0$ —①

The condition that the co-efficients a,b,c are proportional to direction numbers $7,2,-3$.

Therefore, the eqn of the required plane is ~~×××××××~~

$$\frac{a}{7}=\frac{b}{2}=\frac{-c}{3}=k(say), \quad a=7k, b=2k, c=-3k$$

substituting k in ①, $7k(x-4)+2k(y+2)-3k(z-1)=0 \Rightarrow 7(x-4)+2(y+2)-3(z-1)=0$

$$7x+2y-3z-21=0$$

③ Find eqn of plane through $(2,-3,1)$ and is normal to the line ~~joing~~ joining the points $(3,4,-1)$ & $(2,-1,5)$

Let the eqn of plane through the point $(2,-3,1)$ is $a(x-2)+b(y+3)+c(z-1)=0$ —①
direction ratios of the line joining the points $(3,4,-1)$ & $(2,-1,5)$ is $(3-2,4+1,-1-5)$ or $(1,5,-6)$
The direction ratio's of ① and the direction ratio's of line joining the given points are proportional.

$$\frac{a}{1}=\frac{b}{5}=\frac{c}{-6}=k(say), \quad a=k, b=5k, c=-6k.$$ From ① putting these values $k(x-2)+5k(y+3)-6k(z-1)=0$

$$x+5y-6z+19=0$$

④ Find eqn of plane through points $(8,-2,2)$, $(2,1,-9)$ & $(2,4,-6)$ and find also perpendicular distance from the origin to the plane

Let the eqn pass through the point $(8,-2,2)$ is $a(x-8)+b(y+2)+c(z-2)=0$ —①

eqn pass through $(2,1,-9)$ & $(2,4,-6)$, So we get $a(2-8)+b(1+2)+c(-9-2)=0$
$$\Rightarrow 2a-b+2c=0 \quad —②$$
$$a(2-8)+b(4+2)+c(-6-2)=0$$

Solving ② & ③, $a=2, b=-2, c=-3$.

$$\Rightarrow 3a-3b+k=0 \quad —③$$

from ①, the required eqn is $2(x-8)-2(y+2)-3(z-2)=0 \Rightarrow$

$$2x-2y-3z-14=0 \quad —④$$

Now perpendicular distance from origin to the plane ④ is $=\left|\frac{-14}{\sqrt{2^2+(-2)^2+(-3)^2}}\right| = \frac{14}{\sqrt{17}}$

⑤ Find eqn of plane through $(2,2,1)$ & $(9,3,6)$ and perpendicular to plane $2x+6y+6z=9$.

Let eqn of plane through $(2,2,1)$ is $a(x-2)+b(y-2)+c(z-1)=0$ —① also pass $(9,3,6)$
$$a(9-2)+b(3-2)+c(6-1)=0 \text{ or } 7a+b+5c=0 \quad —②$$
also perpendicular to plane $2x+6y+6z=9$ —③ $\Rightarrow 2a+6b+6c=0$ —④. Solving ③ & ④

$$\frac{a}{6-30}=\frac{b}{10-42}=\frac{c}{42-2}=k \quad \therefore a=3k, b=4k, c=-5k$$

From ①, $3k(x-2)+4k(y-2)-5k(z-1)=0, \Rightarrow 3x+4y-5z-9=0$

⑥ Find eqn of plane which passes points $(1,0,-1)$ & $(2,1,3)$ and perpendicular to plane $2x+y+z=1$

find vector in the plane, $\vec{AB}=(2-1,1-0,3-(-1))=(1,1,4)$, let's label the given point as $A(1,0,-1)$ & $B(2,1,3)$
Normal y $\vec{n}=(2,1,1)$. Normal vector in plane $(\vec{n})$ is perpendicular to both $\vec{AB}$ & $\vec{n}$,
$\vec{n}=\vec{AB}\times\vec{n}=\begin{vmatrix}i & j & k \\ 1 & 1 & 4 \\ 2 & 1 & 1\end{vmatrix}=(-3,7,-1)$. Using point $A(1,0,-1)$ & normal vector $(-3,7,-1):-3(x-1)+7(y-0)-1(z-(-1))=0$
$$-3x+7y-z+2=0$$