Three Dimensional Geometry.

Solid Geometry

Rectangular Co-ordinates.

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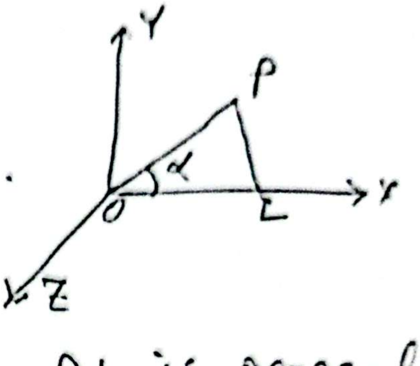
[3] If a line makes angles α, β, γ with the axes. Show that $l^2+m^2+n^2=1$

or $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

and also $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$

Solution



Let $O(0,0,0)$ be the origin and $P(x,y,z)$ be any point. and l, m, n are the direction cosines of OP and r is length of OP. PL is perpendicular on x-axis. So $OL=x$, $\angle POL=\alpha$.

$\therefore \frac{OL}{OP}=\cos\alpha$

$\Rightarrow OL = OP\cos\alpha$

$\Rightarrow x = lr \quad [l=\cos\alpha]$

Similarly, $y=mr$, $z=nr$.

squaring and adding x, y, z. we get

$x^2+y^2+z^2=r^2(l^2+m^2+n^2)$

$\Rightarrow r^2=r^2(l^2+m^2+n^2) \quad \left[\begin{array}{l} r^2=(x-0)^2+(y-0)^2+(z-0)^2 \\ =x^2+y^2+z^2 \end{array}\right]$

$\therefore l^2+m^2+n^2=1$

$\Rightarrow \cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\Rightarrow 1-\sin^2\alpha+1-\sin^2\beta+1-\sin^2\gamma=1$

$\therefore \sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$ proved

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III If a line makes equal angles (θ, θ, θ) with the axes show that $\sin\theta = \pm\sqrt{2/3}$

Solution: If α, β, γ be the angles made by a line with the axes, we have

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \quad \text{——— (1)}$$

A/Q $\quad \alpha = \beta = \gamma = \theta.$

$$\therefore \cos^2\theta + \cos^2\theta + \cos^2\theta = 1$$

$$\Rightarrow 3\cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = \frac{1}{3}$$

$$\Rightarrow \boxed{\cos\theta = \pm\sqrt{\frac{1}{3}}}$$

$$\Rightarrow 1 - \sin^2\theta = \frac{1}{3}$$

$$\Rightarrow \sin^2\theta = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\Rightarrow \sin\theta = \pm\sqrt{\frac{2}{3}}.$$

proved