SWE223: Digital Electronics
Fall 2015

Lecture 3
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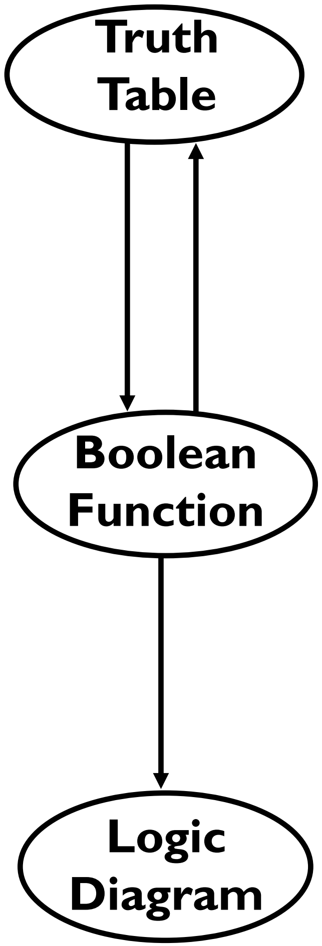
Textbooks

- Moris Mano and Kime, "Logic & Computer Design Fundamentals", Prentice Hall.

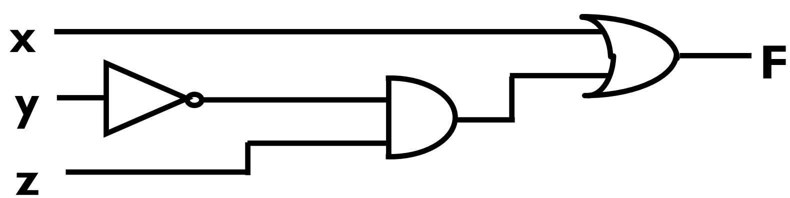
Theorems and Postulates

LOGIC CIRCUIT DESIGN



x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$F = x + y'z$



Boolean Algebra

Boolean identities

Name	AND version	OR version
Identity	$x \cdot 1 = x$	$x + 0 = x$
Complement	$x \cdot \overline{x} = 0$	$x + \overline{x} = 1$
Commutative	$x \cdot y = y \cdot x$	$x + y = y + x$
Distribution	$x \cdot (y+z) = xy+xz$	$x + (y \cdot z) = (x+y)(x+z)$
Idempotent	$x \cdot x = x$	$x + x = x$
Null	$x \cdot 0 = 0$	$x + 1 = 1$

Boolean Algebra (cont.)

Name	AND version	OR version
Involution	$\overline{\overline{x}} = x$	---
Absorption	$x \cdot (x+y) = x$	$x + (x \cdot y) = x$
Associative	$x \cdot (y \cdot z) = (x \cdot y) \cdot z$	$x + (y + z) = (x + y) + z$
de Morgan	$\overline{x \cdot y} = \overline{x} + \overline{y}$	$\overline{x + y} = \overline{x} \cdot \overline{y}$

Identifying postulates and theorems

- ***Basic Postulates***
- ***Postulate 1 (Definition)***: A Boolean algebra is a closed algebraic system containing a set K of two or more elements and the two operators $\bullet$ and $+$.
- ***Postulate 2 (Existence of 1 and 0 element)***:

  (a) $a + 0 = a$ (identity for $+$), (b) $a \bullet 1 = a$ (identity for $\bullet$)
- ***Postulate 3 (Commutativity)***:

  (a) $a + b = b + a$, (b) $a \bullet b = b \bullet a$
- ***Postulate 4 (Associativity)***:

  (a) $a + (b + c) = (a + b) + c$ (b) $a \bullet (b \bullet c) = (a \bullet b) \bullet c$
- ***Postulate 5 (Distributivity)***:

  (a) $a + (b \bullet c) = (a + b) \bullet (a + c)$ (b) $a \bullet (b + c) = a \bullet b + a \bullet c$
- ***Postulate 6 (Existence of complement)***:

  (a) $a + \overline{a} = 1$ (b) $a \bullet \overline{a} = 0$
- Normally $\bullet$ is omitted.

Identifying postulates and theorems (cont')

- ***Fundamental Theorems of Boolean Algebra***
- ***Theorem 1 (Idempotency)***:

  (a) $a + a = a$ (b) $aa = a$
- ***Theorem 2 (Null element)***:

  (a) $a + 1 = 1$ (b) $a0 = 0$
- ***Theorem 3 (Involution)***

  $\overline{\overline{a}} = a$
- ***Theorem 4 (Absorption)***

  (a) $a + ab = a$ (b) $a(a + b) = a$
- ***Examples***:
  - $(X + Y) + (X + Y)Z = X + Y$ [T4(a)]
  - $AB'(AB' + B'C) = AB'$ [T4(b)]
- ***Theorem 5***
- (a) $a + a'b = a + b$ (b) $a(a' + b) = ab$

Identifying postulates and theorems (cont')

- ***Fundamental Theorems of Boolean Algebra***
- ***Examples***:
  - $B + AB'C'D = B + AC'D$ [T5(a)]
  - $(X + Y)((X + Y)' + Z) = (X + Y)Z$ [T5(b)]
- ***Theorem 6***

(a) $ab + ab' = a$ (b) $(a + b)(a + b') = a$

- ***Examples***:
  - $ABC + AB'C = AC$ [T6(a)]
  - $(W' + X' + Y' + Z')(W' + X' + Y' + Z)(W' + X' + Y + Z')(W' + X' + Y + Z)$

    $= (W' + X' + Y')(W' + X' + Y + Z')(W' + X' + Y + Z)$ [T6(b)]

    $= (W' + X' + Y')(W' + X' + Y)$ [T6(b)]

    $= (W' + X')$ [T6(b)]

Fundamentals of Boolean Algebra (5)

- ***Theorem 7***

  (a) $ab + ab'c = ab + ac$ (b) $(a + b)(a + b' + c) = (a + b)(a + c)$

- ***Examples***:
  - $wy' + wx'y + wxyz + wxz' = wy' + wx'y + wxy + wxz'$ [T7(a)]
    $= wy' + wy + wxz'$ [T7(a)]
    $= w + wxz'$ [T7(a)]
    $= w$ [T7(a)]
  - $(x'y' + z)(w + x'y' + z') = (x'y' + z)(w + x'y')$ [T7(b)]

Fundamentals of Boolean Algebra (6)

- ***Theorem 8 (DeMorgan's Theorem)***
  (a) $(a + b)' = a'b'$ (b) $(ab)' = a' + b'$

- Generalized DeMorgan's Theorem
  (a) $(a + b + \ldots z)' = a'b' \ldots z'$ (b) $(ab \ldots z)' = a' + b' + \ldots z'$

- ***Examples***:
  - $(a + bc)' = (a + (bc))'$
    $= a'(bc)'$ [T8(a)]
    $= a'(b' + c')$ [T8(b)]
    $= a'b' + a'c'$ [P5(b)]
  - Note: $(a + bc)' \neq a'b' + c'$

Fundamentals of Boolean Algebra (7)

- ***More Examples for DeMorgan's Theorem***
  - $(a(b + z(x + a')))' = a' + (b + z(x + a'))'$ [T8(b)]
    $= a' + b'(z(x + a'))'$ [T8(a)]
    $= a' + b'(z' + (x + a')')$ [T8(b)]
    $= a' + b'(z' + x'(a')')$ [T8(a)]
    $= a' + b'(z' + x'a)$ [T3]
    $= a' + b'(z' + x')$ [T5(a)]
  - $(a(b + c) + a'b)' = (ab + ac + a'b)'$ [P5(b)]
    $= (b + ac)'$ [T6(a)]
    $= b'(ac)'$ [T8(a)]
    $= b'(a' + c')$ [T8(b)]

DeMorgan's: Example

Solution

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

$$F_2 = \overline{(\overline{X} + Z)} + (\overline{\overline{XY}}) \quad \text{;Theorem \#14A}$$

$$F_2 = \overline{(\overline{X} + Z)} + (XY) \quad \text{;Theorem \#9}$$

$$F_2 = (\overline{\overline{X}}\,\overline{Z}) + (XY) \quad \text{;Theorem \#14B}$$

$$F_2 = (X\,\overline{Z}) + (XY) \quad \text{;Theorem \#9}$$

$$F_2 = X\,\overline{Z} + X\,Y \quad \text{; Rewritten without AND symbols}$$

Fundamentals of Boolean Algebra (8)

- ***Theorem 9 (Consensus)***

  (a) $ab + a'c + bc = ab + a'c$ (b) $(a + b)(a' + c)(b + c) = (a + b)(a' + c)$

- ***Examples***:
  - $AB + A'CD + BCD = AB + A'CD$ [T9(a)]
  - $(a + b')(a' + c)(b' + c) = (a + b')(a' + c)$ [T9(b)]
  - $ABC + A'D + B'D + CD$
    $= ABC + (A' + B')D + CD$ [P5(b)]
    $= ABC + (AB)'D + CD$ [T8(b)]
    $= ABC + (AB)'D$ [T9(a)]
    $= ABC + (A' + B')D$ [T8(b)]
    $= ABC + A'D + B'D$ [P5(b)]

Boolean Algebra (cont.)

- **Usefulness of these Tables**

  - Simplification of the Boolean function

  - Derivation of equivalent Boolean functions to obtain logic diagrams utilizing different

K-MAP

Algebraic Forms

- ***Literal***: A variable, complemented or uncomplemented.
- ***Product term***: A literal or literals ANDed together.
- ***Sum term***: A literal or literals ORed together.

- ***SOP (Sum of Products)***:
- ORing product terms
- $f(A, B, C) = ABC + A'C + B'C$

- ***POS (Product of Sums)***
- ANDing sum terms
- $f(A, B, C) = (A' + B' + C')(A + C')(B + C')$

Algebraic Forms cont'

- A ***minterm*** is a product term in which all the variables appear exactly once either complemented or uncomplemented.
- ***Canonical Sum of Products (canonical SOP)***:
  - Represented as a sum of minterms only.
  - ***Example***: $f_1(A,B,C) = A'BC' + ABC' + A'BC + ABC$
- Minterms of three variables:

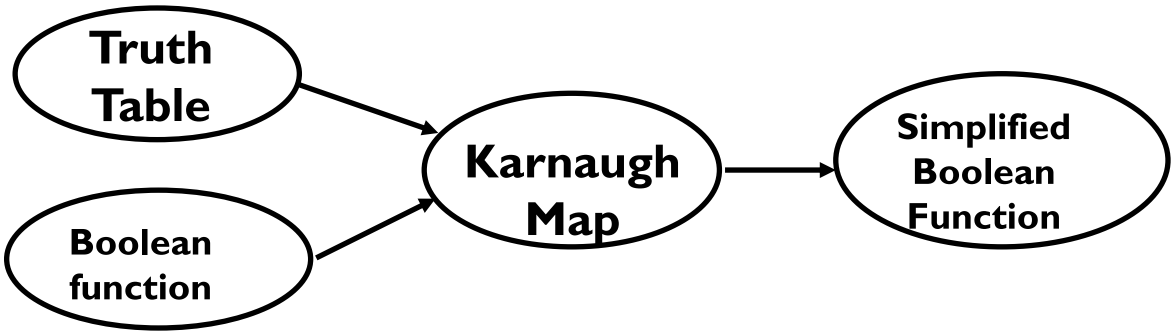
Minterm	Minterm Code	Minterm Number
$A'B'C'$	000	m_0
$A'B'C$	001	m_1
$A'BC'$	010	m_2
$A'BC$	011	m_3
$AB'C'$	100	m_4
$AB'C$	101	m_5
ABC'	110	m_6
ABC	111	m_7

Algebraic Forms cont'

- A ***maxterm*** is a sum term in which all the variables appear exactly once either complemented or uncomplemented.
- ***Canonical Product of Sums (canonical POS)***:
  - Represented as a product of maxterms only.
  - ***Example***: $f_2(A,B,C) = (A+B+C)(A+B+C')(A'+B+C)(A'+B+C')$ (2.7)
- Maxterms of three variables:

Maxterm	Maxterm Code	Maxterm Number
$A+B+C$	000	M_0
$A+B+C'$	001	M_1
$A+B'+C$	010	M_2
$A+B'+C'$	011	M_3
$A'+B+C$	100	M_4
$A'+B+C'$	101	M_5
$A'+B'+\mathrm{C}$	110	M_6
$A'+B'+C'$	111	M_7

KARNAUGH MAP

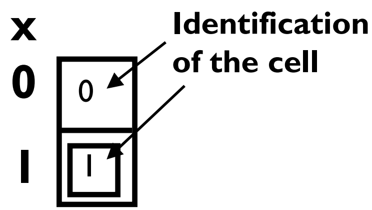
- **Karnaugh Map (K-map) is a simple procedure for simplifying Boolean expressions.**
- 

- **Karnaugh Map for an n-input digital logic circuit (n-variable sum-of-products**
- **form of Boolean Function, or Truth Table) is**
- **- Rectangle divided into 2^n cells**
- **- Each cell is associated with a *Minterm***
- **- An output(function) value for each input value associated with a**
- **minterm is written in the cell representing the minterm**
- **→ 1-cell, 0-cell**

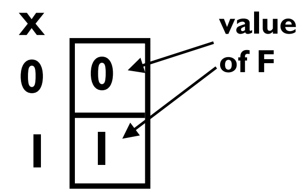
- **Each Minterm is identified by a decimal number whose binary representation**
- **is identical to the binary interpretation of the input values of the minterm.**

KARNAUGH MAP cont’

x	F
0	1
1	0



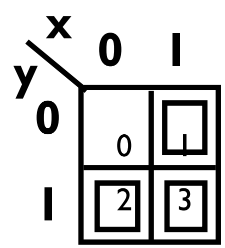
Karnaugh Map

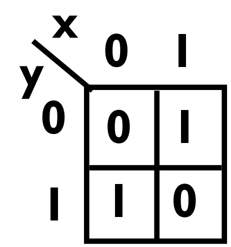


$$F(x) = \Sigma\,(1)$$

1-cell

x	y	F
0	0	0
0	1	1
1	0	1
1	1	1





$$F(x,y) = \Sigma\,(1,2)$$

KARNAUGH MAP cont'

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

y, z

x \ yz	00	01	11	10
0	0	$\boxed{1}$	3	$\boxed{2}$
1	$\boxed{4}$	5	7	6

x \ yz	00	01	11	10
0	0	1	0	1
1	1	0	0	0

$F(x,y,z) = \sum (1,2,4)$

u	v	w	x	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

w, v, u, x

uv \ wx	00	01	11	10
00	0	$\boxed{1}$	$\boxed{3}$	2
01	4	5	7	$\boxed{6}$
11	12	13	15	$\boxed{14}$
10	$\boxed{8}$	$\boxed{9}$	$\boxed{11}$	10

uv \ wx	00	01	11	10
00	0	1	1	0
01	0	0	0	1
11	0	0	0	1
10	1	1	1	0

$F(u,v,w,x) = \sum (1,3,6,8,9,11,14)$

MAP SIMPLIFICATION - 2 ADJACENT CELLS

Adjacent cells $\boxed{\textbf{Rule: } xy' + xy = x(y+y') = x}$

- binary identifications are different in one bit
→ minterms associated with the adjacent cells have one variable complemented each other

Cells (1,0) and (1,1) are adjacent
Minterms for (1,0) and (1,1) are
x • y' --> x=1, y=0
x • y --> x=1, y=1

F = xy'+ xy can be reduced to F = x
From the map

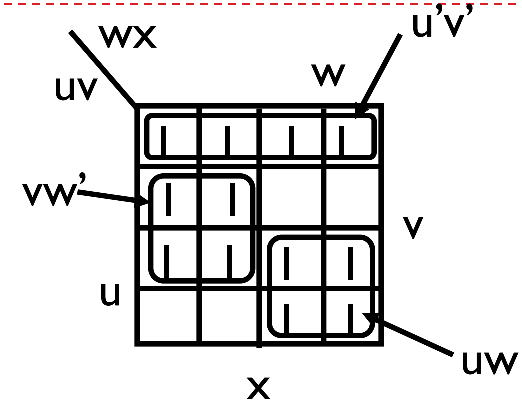
x \ y	0	1
0	0	0
1	1	1

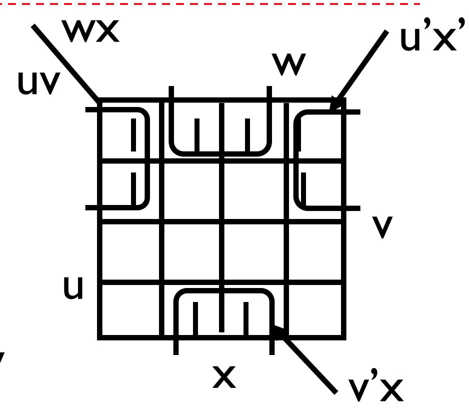
2 adjacent cells xy' and xy
→ merge them to a larger cell x

$$F(x,y) = \sum(2,3)$$
$$= xy' + xy$$
$$= x$$

MAP SIMPLIFICATION - MORE THAN 2 CELLS

u'v'w'x' + u'v'w'x + u'v'wx + u'v'wx'
= u'v'w'(x'+x) + u'v'w(x+x')
= u'v'w' + u'v'w
= u'v'(w'+w)
= u'v'





u'v'w'x'+u'v'w'x+u'vw'x'+u'vw'x+uvw'x'+uvw'x+uv'w'x'+uv'w'x
= u'v'w'(x'+x) + u'vw'(x'+x) + uvw'(x'+x) + uv'w'(x'+x)
= u'(v'+v)w' + u(v'+v)w'
= (u'+u)w' = w'