BERGISCHE UNIVERSITÄT WUPPERTAL

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Tools

Summer Term 2022

Exercise Sheet 3

Exercise 9 (Git, Mean Values, 4 p.)

Perform the following steps using Git:

- Create a bare repository (e. g. in a directory `repo.git`).
- Create a new empty directory (e. g. `wcopy`) that will be used as a working copy (*not* inside the repository from above!).
- Clone the empty bare repository into the directory of the working copy (ignore the warning).
- Write a C++ program that asks the user to enter two numbers, reads in the values, and prints the entered values.
- Add your source code to the staging area and commit to the local repository.
- Add a function that computes the arithmetic mean of two values. Call that function for the entered values and print the result. Commit your changes.
- Add a function that computes the geometric mean of two values. Also call that function for the entered values and print the result. Commit your changes again.
- Add a loop to the main program such that the input of the values and the computations are repeated until at least one negative value was entered. Commit these final changes.
- Push your changes from the local repository into the bare repository.
- Pack your bare repository into an archive by a command like
  `tar cfvz repo.tgz repo.git` and send this file by email as your assignment for this exercise.

Exercise 10 (Fortran, Prime Factors, 4 p.)

Write a Fortran program that prints the prime factorization of an integer number.

Idea for an algorithm:

- Check all possible factors $2, 3, \ldots$ if they are a divisor of the number.
- Divide the number by the factor you found and continue with that new number.
- A factor can occur multiple times.
- All found factors are automatically prime.

Example:

```
> primefactors
n = 1440
2 * 2 * 2 * 2 * 2 * 3 * 3 * 5
> primefactors
n = 12345
3 * 5 * 823
> primefactors
n = 123457
123457
```

Hint: The Fortran function for m mod n is `mod(m, n)`.

Exercise 11 (Fortran, Polynomials, Newton's Method, 6 p.)

Write a Fortran program that can find zeros of polynomials.

In the directory `/home/tools/Exercises/11` (or in Moodle) you will find the file `polynomial.f90` that contains a main program and a subroutine for the output of polynomials.

Polynomials in that program are stored in arrays of size 5, i. e. the degree is at most 4. The array $(c_0, c_1, c_2, c_3, c_4)$ represents the polynomial $p(x) = c_4 x^4 + c_3 x^3 + c_2 x^2 + c_1 x + c_0$.

Add code for the following three functions and uncomment the corresponding lines in the main program:

a) (2 p.) Add a function `polyEval(p, x)` that evaluates and returns `p(x)`. (Use Horner's scheme if you know it.)

b) (2 p.) Add a function `polyDeriv(p)` that returns the derivative $\mathtt{p}'(x)$.

c) (2 p.) Add a function `newton(p, x0)` that applies Newton's method to find a root near `x0`.

algorithm Newton's method
input function f, start value x_0

for $i = 0, 1, \ldots$
$$x_{i+1} := x_i - \frac{f(x_i)}{f'(x_i)}$$

Stop the iteration when $|x_i - x_{i-1}| < 10^{-14}$. If $i > 100$, print an error message ("no convergence") and return from the function.

Exercise 12 (Fortran, Power Method, 5 p.)

Write a Fortran function that computes the eigenvalue with largest absolute value $\lambda_{\max}$ of a matrix $A \in \mathbb{R}^{n\times n}$ using the power method. Start with the vector $x^0 = (1, \ldots, 1)^T \in \mathbb{R}^n$. Perform the iteration $x^{k+1} = \frac{Ax^k}{\|Ax^k\|_2}$ until $\|x^{k+1} - x^k\|_2 < 10^{-12}$. Return $\|Ax^{k+1}\|_2$. Use matrix and vector operations where possible. Since Fortran 2008 there is a function `norm2(v)` for the Euclidian norm of a vector `v`. (The method fails if A has multiple eigenvalues with maximum absolute value or if x^0 has no component in the direction of the eigenvector associated with $\lambda_{\max}$. We assume that these situtations will not occur.)

Add a main program that reads in a matrix size n and creates the following Toeplitz matrix of size $n \times n$:

$$A = \begin{pmatrix} 4 & -1 & -2 & 0 & 0 & 0 & \cdots \\ -1 & 4 & -1 & -2 & 0 & 0 & \cdots \\ -2 & -1 & 4 & -1 & -2 & 0 & \cdots \\ 0 & -2 & -1 & 4 & -1 & -2 & \\ \vdots & & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix}$$

Determine the maximum eigenvalue of A.

Use an allocatable matrix; the above function should be contained in the main program.

Hand in by: Mon., 13.06.2022 until 14:00 by email to `tools@studs.math.uni-wuppertal.de`.