

# Multivariate statistics: Assignment 1

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## Abstract

### 1 Introduction and data exploration

This report assesses hearing thresholds for a sample of 546 healthy male volunteers. The subjects were 52 years old on average at the start of the study and are followed for an average of 7.57 years. The hearing threshold is measured, on average, every 1.59 years. Table 1 describes the demographics in more detail. It can be seen that the data is highly unbalanced; there is a lot of variation in the time a volunteer is followed and in the number of times a volunteer's hearing threshold is measured. Normally, each ear is measured at each visit but this only happened in 93.22% of all visits. The left (right) ear was tested in 96.72% (96.5%) of all visits.

Previous research on the hearing data of males in the Baltimore Longitudinal Study of Aging (BLSA) showed a change in hearing threshold for all age groups but especially the older population (Brant and Fozard 1990). In contrast to our dataset, Pearson et al. (1995) and Morrell et al. (1997) consider females in their study. They similarly found a decrease in hearing sensitivity for all ages at 500Hz and included a quadratic function of age to predict the hearing threshold, as in Verbeke, Spiessens, and Lesaffre (2001). Additionally, a statistically significant learning effect from the first visit to subsequent visits was found (Morrell et al. 1997; Verbeke, Spiessens, and Lesaffre 2001). Morrell et al. (1997) observed that hearing levels are slightly poorer on average on the left compared to the right ear and that the variance in the hearing threshold is higher for people with a higher age.

Figures 1 and 2 show the trends in the hearing threshold for all volunteers over time. These figures shows that many volunteers' hearing threshold over time have an erratic pattern meaning there is likely high variability **within subjects**. Additionally, variability **between subjects** is also high, especially for older volunteers.

#### 1.1 Mean, variance and correlation structure

Figure 3 shows the mean and 95% confidence interval for the hearing threshold (dB) for different age groups. The thresholds for the age groups are chosen such that each group is approximately the same size (between 141 and 149 measurements in each group). The graph shows that the variance increases with age and there doesn't seem to be a significant difference, on average, between left and right ears.

Figure 4 shows Loess smoothing curves by age category and side of the ear. The trend in the

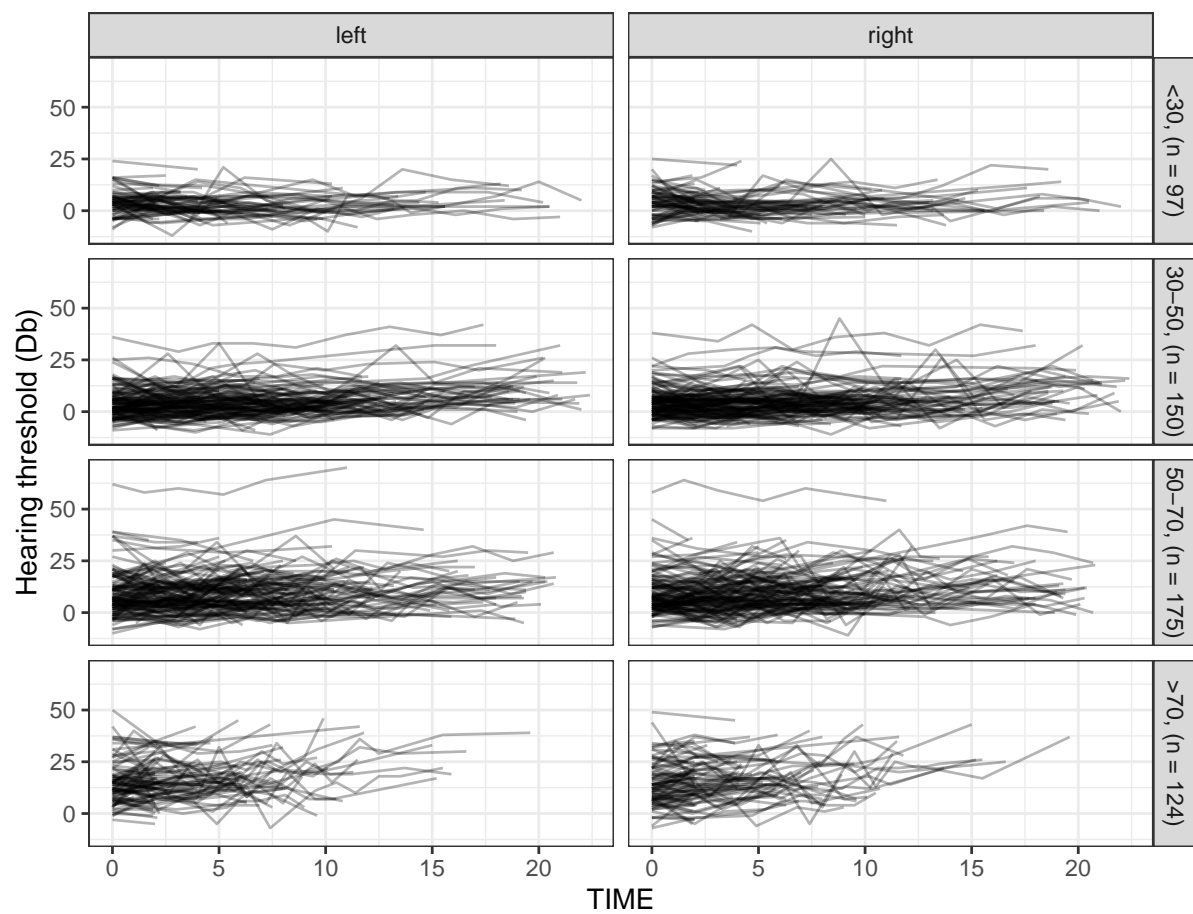


Figure 1: Hearing threshold over time, divided by left and right ear and by age group at the start of the study

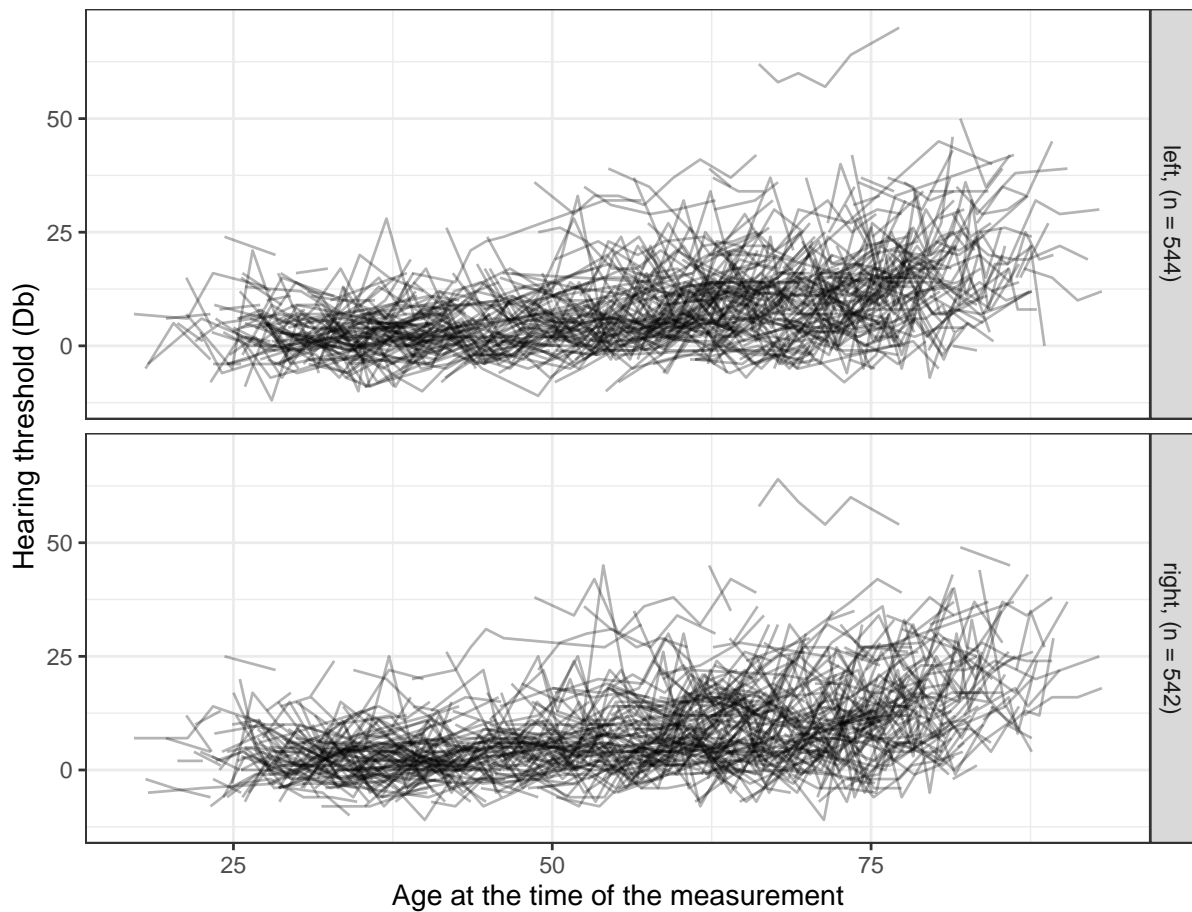


Figure 2: Hearing threshold over time, divided by left and right ear.

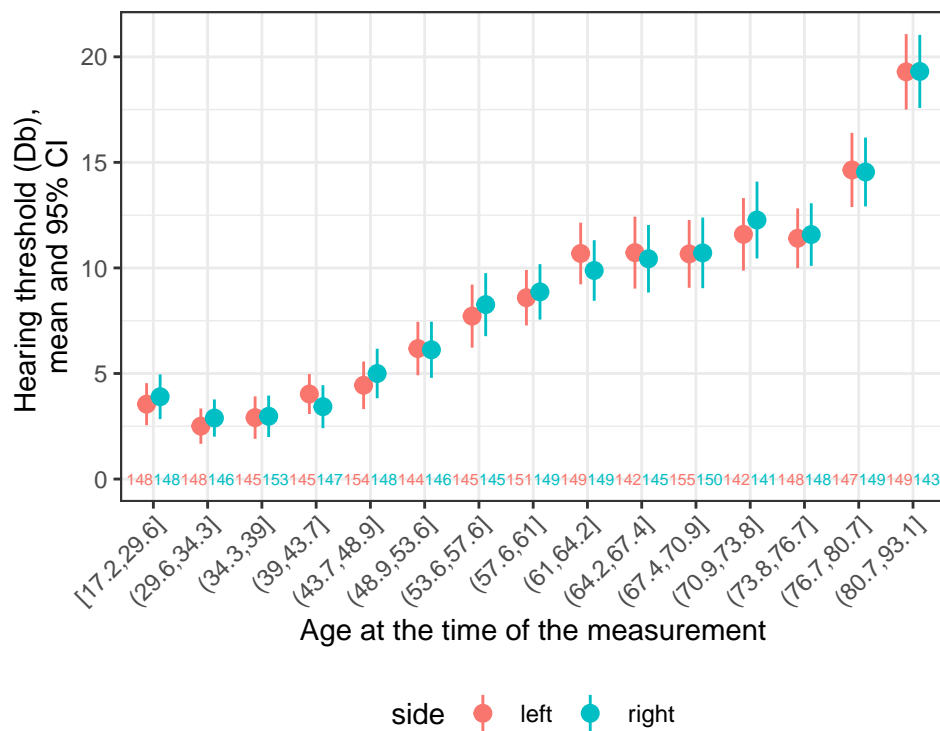


Figure 3: Hearing threshold over time, divided by left and right ear. The numbers in the bottom show the number of measurements that were taken.

average is slightly increasing for most groups. For the oldest group, there is a more pronounced increase in the hearing threshold. Additionally, the estimated standard errors around the mean are larger as time increases, likely due to dropout. Based on the estimated averages from 4, squared residuals can be calculated. Figure 5 shows the Loess smoothed variance function for each group.

The data has a hierarchical (grouped) structure since each individual (*id*) is measured several times over the years. Some basic descriptive statistics were derived using the *statsBy* function from the *psych* package. The intraclass correlation (percentage of variance due to groups) is 0.69 for the hearing threshold (*y*) and, as expected, very high (0.93) for age at the time of measurement.

The total correlation between *y* and the age at the measurement, ignoring the hierarchical structure, is 0.45. Following Marzban et al. (2013) and Montgomery (2017), the correlation matrix for each subject that has at least 2 measurements for both ears is calculated, one can obtain within-group correlations and variances. Figure 6 shows the histograms over all individuals for the correlation between the age at the measurement and *y*, and the variances for both age and *y* (though the variance in age is not very informative). The mean or median of all within-group correlations is often used as a measure of within-group correlation. The mean (0.15) and median (0.23) of the within-group correlations are indicated in red and blue respectively on the graph. It can be seen that within-group correlation spans the entire range from -1 to 1, meaning that age has a strong positive relationship to the hearing threshold for some and a strong negative relationship for others. Age will be a good predictor for hearing threshold for some but not for others. Between-group correlation is obtained by averaging the age at measurement and the

Table 1: Demographics for all respondents

<b>Age at the beginning of the study</b>	
min	17.20
max	87.00
median	54.10
mean (sd)	51.99 $\pm$ 18.70
<b>Years of follow-up</b>	
min	0.00
max	22.40
median	6.30
mean (sd)	7.57 $\pm$ 6.30
<b>Number of visits</b>	
min	1
max	15
median	3.00
mean (sd)	4.19 $\pm$ 2.88

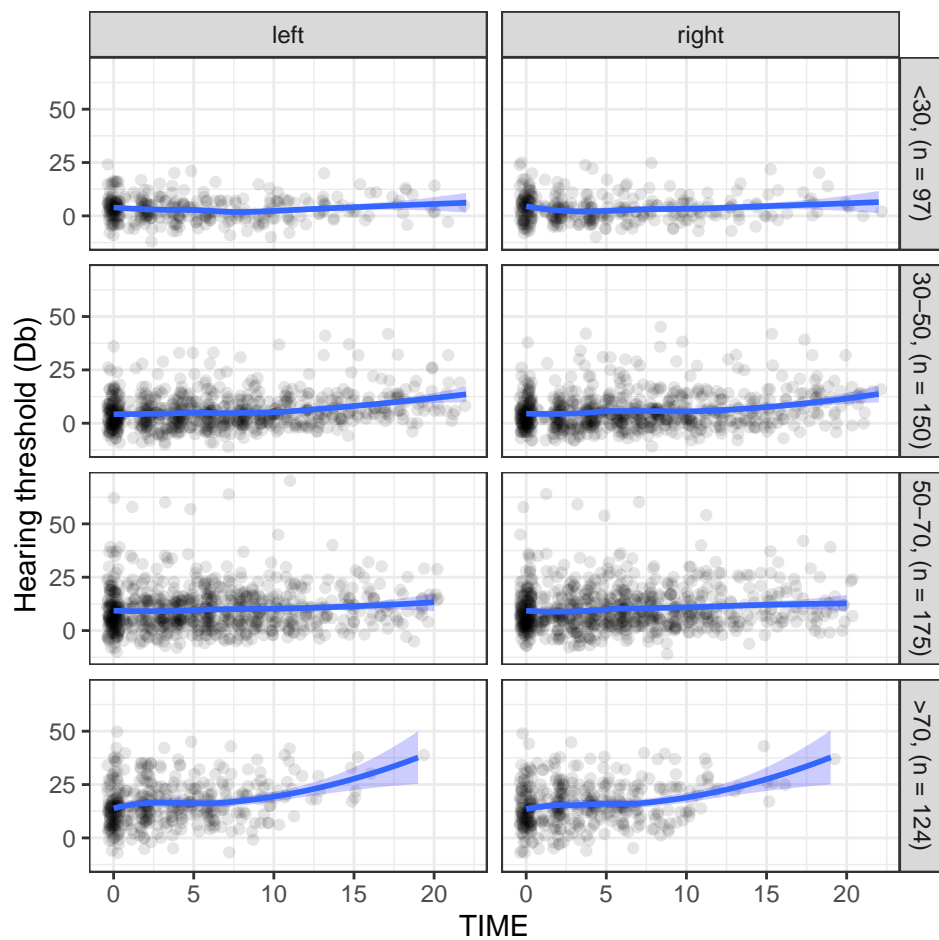


Figure 4: Loess smoothing on the hearing threshold since start of the study, divided by left and right ear and age group at the start of the study.

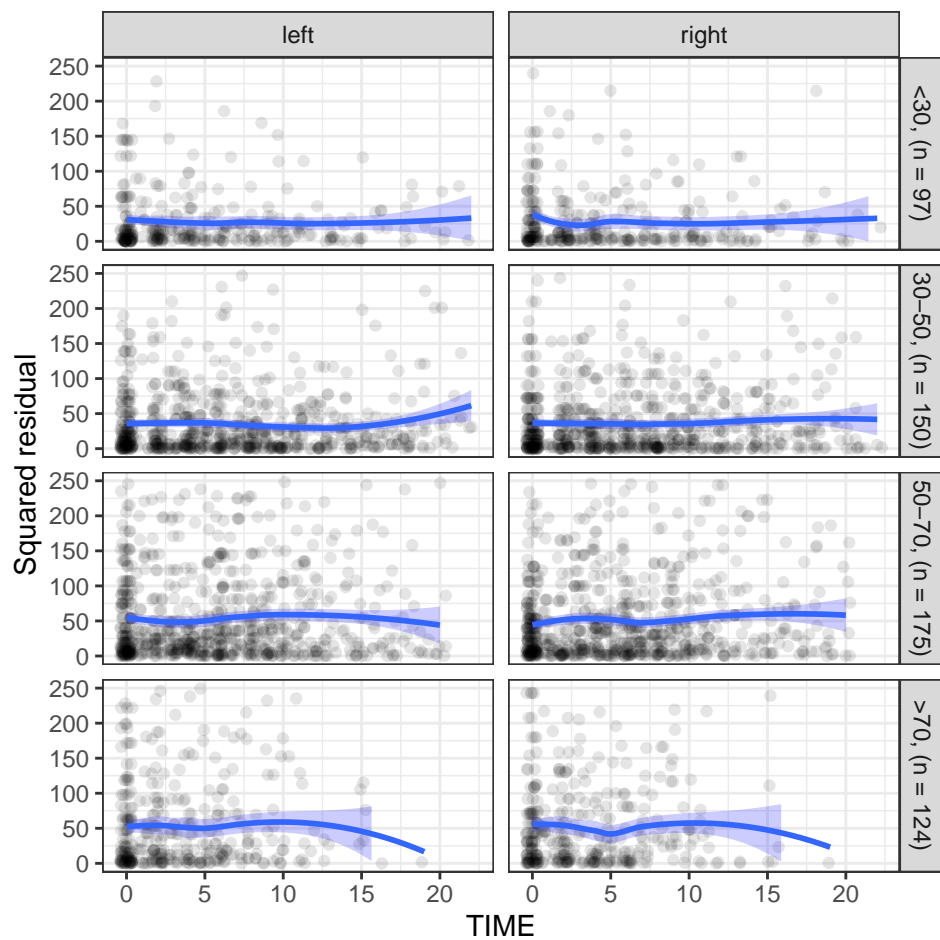


Figure 5: Loess smoothed variance in the hearing threshold since start of the study, divided by left and right ear and age group at the start of the study.

hearing threshold for each group (see scatterplot in Figure 6) and then computing the correlation across the groups, obtaining 0.54.

Panel B in Figure 6 shows that the correlation between the side of the ear that is measured and the hearing threshold ( $y$ ) is high for some but small for most individuals. Panel C in Figure 6 shows that the variance in the hearing threshold is usually less than 50 but can be very large for some individuals.

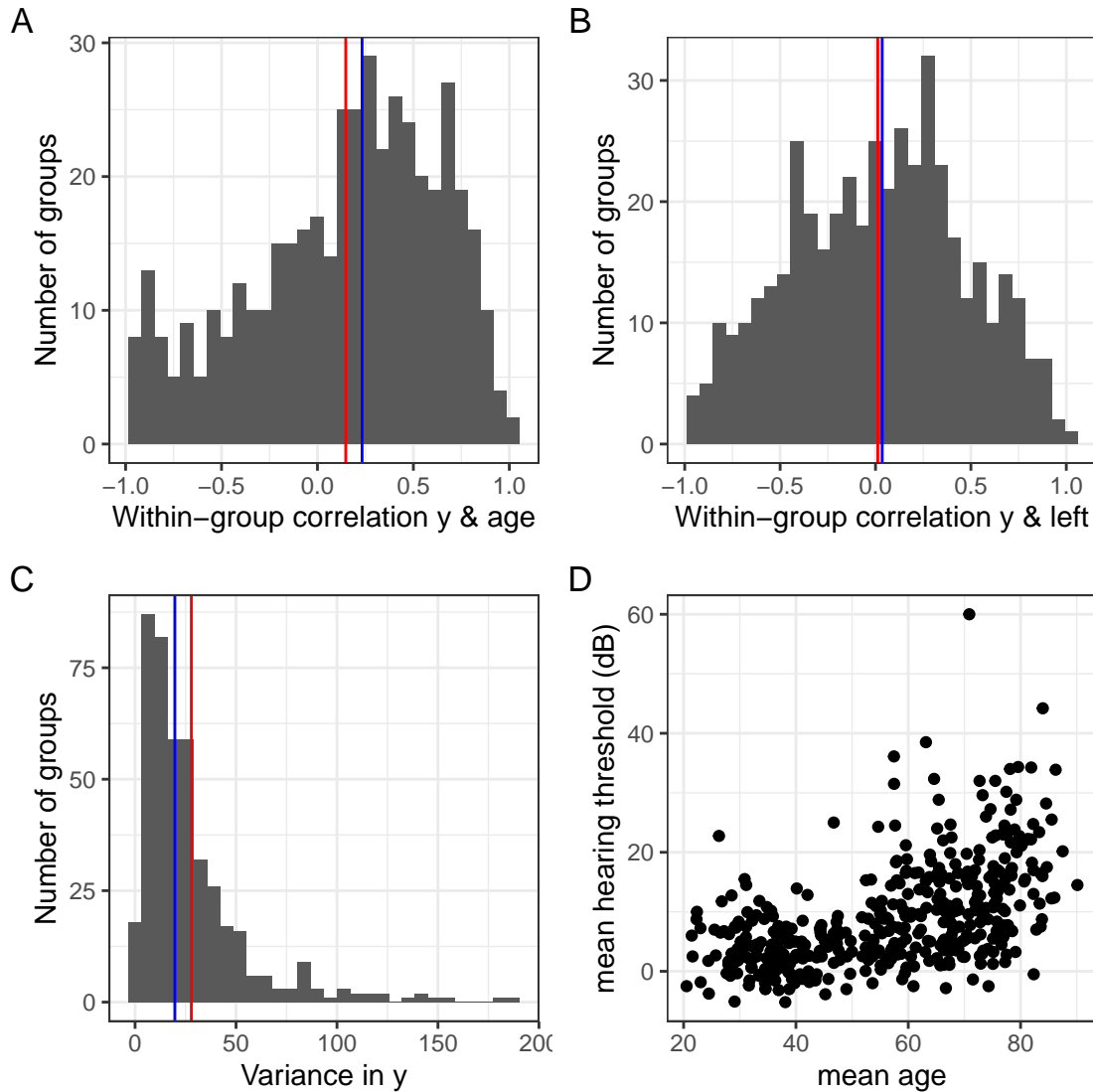


Figure 6: The mean is indicated in red, the median in blue

Describe the data, and use graphical techniques to explore the mean structure, the variance structure and the correlation structure. Summarize your conclusions. What are the implications with respect to statistical modeling ?

## 2 Methodology

In this section, we explore a couple of different methods to analyze the data. All analysis was carried out with the statistical software R. All scripts are freely available at this git repository.

### 2.1 Summary statistics

One possibility to deal with the hierarchical structure of the data is to summarize the data and reduce the number of measurements per subject to one.

This can be done, for instance, by calculating the average change in the hearing threshold for each subject  $i$ ,  $\Delta_i$  as in equation (1) where  $Y_{ijk}$  is the  $k^{th}$  measurement for ear  $j$  of subject  $i$  and  $n_{ij}$  is the total number of measurements of ear  $j$  of subject  $i$ .

$$\Delta_i = \frac{(Y_{il1} - Y_{iln_{ij}}) + (Y_{ir1} - Y_{irn_{ij}})}{2} \quad (1)$$

An immediate problem with this method is that we have learned the the within-group variance in the hearing threshold is quite large. Focusing only on the first and last measurement is therefore risky. Additionally, if time would influence hearing threshold, we need to correct for the time between the first and last measurement since this differs a lot between subjects. This method also requires that each subjects' both ears have been measured at least twice.

An even simpler summary statistic would be to use the average hearing threshold over all measurements and both ears. This would, however, not allow us to analyse changes in individuals over time; we can only infer on the population effect between age an hearing threshold.



## **2.2 Multivariate model**

## **2.3 Two-stage analysis**

## **2.4 Random-effects model**

# **3 Results**

## **3.1 Summary statistics**

## **3.2 Multivariate model**

## **3.3 Two-stage analysis**

## **3.4 Random-effects model**

# **4 Discussion and conclusion**

## **4.1 further research**

# **Bibliography**

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