ASSIGNMENT5 QUESTION1

1. Today was just a regular day for everyone in Krypton until a news flashed that a meteor is going to destroy Krypton in X days. Krypton has N cities, some of which are connected by bidirectional roads. You are given a road map of Krypton; for every two cities C_i and C_j which are connected by a (direct) road from C_i straight to C_j you are given the value $t(i, j)$ which is the number of days to travel from city C_i to city C_j. (You can of course also go from a city C_m to city C_k without a direct road from C_m to C_k by going through a sequence of intermediate cities connected by direct roads.)

   In each city C_i the Krypton Government built q_i pods to carry inhabitants in case of any calamity, which will transport them to Earth. City C_i has population p_i. As soon as the people hear this news they try to save themselves by acquiring these pods either at their own city or in other city before the meteor destroys everything. Note that a pod can carry only one person. Find the largest number of invaders the Earth will have to deal with. (20 pts)

Answer:

This is a max flow problem. To find the largest number of invaders the earth will deal, we should determine for each city C_i the set of cities that can reach within X days. We can use Dijkstra algorithm to determine every city that the shortest path (time) to other city. And filter city that can reach within X days. Then we get the set of cities that can reach within X for each city. To make the representation compact, make a bipartite graph with vertices corresponding to all cities both on the left and on the right side but with different interpretation: on the left vertices represent populations of the corresponding cities; on the right the vertices represent the set of ponds in the corresponding cities. Add a super-source on the left and connect it to all left vertices by edges of P_i. Add a super-sink on the right and connect it to all right vertices by edges of q_i.Then connect the left side city with the set of right city that can be reached in X days include itself by edges of infinite capacities. Then we turn this to a flow network and we need to find the maximum flow. We can Edmonds-Karp Max Flow algorithm to find the maximum flow and we can find the largest number of invaders the Earth will have to deal with.