ASSIGNMENT5 QUESTION2

2. You are given a usual $n \times n$ chess board with k white bishops on the board at the given cells (a_i, b_i), $(1 \leq a_i, b_i \leq n,\ 1 \leq i \leq k)$. You have to determine the largest number of black rooks you can place on the board so that no two rooks are in the same row or in the same column and are not under the attack of any of the k bishops (recall that bishops go diagonally).(20 pts)

Answer:

Make a bipartite graph with all the columns as vertices on the left hand side and all the rows as vertices on the right hand side. Each square s_{ij} can now be represented by an edge from column j to row i. Add a super-source on the left and connect it to all left vertices by edges of 1. And add a super-sink on the right and connect it to all right vertices by edges of 1.Since we would not want the two rook are in same row or same column. Connect each left vertices to all right vertices by edges of 1 capacities.

Then we know there will k white bishops on board at the given cells $(a_i, b_i), 1 \leq a_i, b_i \leq n, 1 \leq i \leq k$ and bishops go diagonally. We are going to delete the edge that is invalid. And we should delete all edge that represent square $s_{a_i b_i}$ and all edge represent diagonal square of (a_i, b_i). Which represent the square $s_{(a_i-x)(b_i-x)}, s_{(a_i-x)(b_i+x)}, s_{(a_i+x)(b_i-x)}, s_{(a_i+x)(b_i+x)}$ where $1 \leq a_i - x, a_i + x, b_i + x, b_i - x \leq n, 1 \leq i \leq k.$ $1 \leq x \leq n$

Then we turn this problem to a max flow problem and we can Edmonds-Karp Max Flow algorithm to find the maximum flow and we can find the largest number of black rooks can place on the board.