ASSIGNMENT4 QUESTION4

4. Given a weighted **directed** graph $G(V, E)$, find a path in G (possibly self-intersecting) of length exactly K that has the maximum total weight. The path can visit a vertex multiple times and can traverse an edge also multiple times. It can also start and end at arbitrary vertices or even start and end at the same vertex. (30 pts)

Answer:

For every node i and every $1 \leq k \leq K$ we need to find the maximum weight path of length exactly k.

Let $P(i,k)$ be the maximum weight path of length exactly k which end at i.

Let $Weight(x,y)$ is the weight of edge from node x to node y.

Solve the subproblems in the order $P(i,1), \ldots, P(i,K)$ for every node i.

For every $1 \leq k \leq K$ we have:

$P(i,k) = \max\left(P(m,k-1) + Weight(m,i)\right)$ For all node m can go to i.

To be noticed, if $P(m,k-1)$ = 0, then $Weight(m,i)$ should also be 0, since there is no path that have length exactly (k-1) to m node. Try other m. If there is no more m make $P(m,k-1) \neq$ 0, It means there is no path that have exactly length (k) to the node I, so $P(i,k) = 0$

And we have some base case:

$P(i,1) = \max\left(Weight(x,i)\right)$ $For\ all\ node\ x\ can\ go\ to\ node\ i$. If there is no such node x, $P(i,1) = 0$

The final solution will be given by $\max\left(P(i,K)\right)$ $for\ all\ node\ i$.

The time complexity for this solution will be $O(V^2E)$. Since we will check every node and every edge it connected the node.