ASSIGNMENT3 QUESTION4

4. You are given a set of n jobs where each job i has a deadline $d_i \geq 1$ and profit $p_i > 0$. Only one job can be scheduled at a time. Each job takes 1 unit of time to complete. We earn the profit if and only if the job is completed by its deadline. The task is to find the subset of jobs that maximises profit. Your algorithm should run in time $O(n^2)$.

Answer:

To get the maximum profit, we must do the most valuable job in every free time slot before the job deadline. Since each job takes 1 unit of time to complete, we will have the biggest d_i deadline as our biggest time slot.

First, we are going to sort all jobs profit in decreasing order. Then follow the order, we put them in time slot that is latest possible free slot that meets its deadline. If no such slot is there, ignore that job. Since we have n jobs and we will go through time slot every time , the time complexity for this will be $O(n^2)$.When we finish that, we can find the subset of jobs that have maximum profit.

If we put job in time slot that after the deadline, we will not earn profit. So, it is not be valid. And if we put job in time slot that is not latest possible free slot, it may decrease the total profit since there are some more valuable job deadline is small and only few choices to put. It will take more valuable job has no place to put. That is not optimal. If we put job in occupied slot, it will decrease the total profit since we put job in the decreasing order of profit. That is also not optimal. If we do not put job in time slot in decreasing order of profit, it will decrease the total profit since we put less value job in the time slot. That is not optimal.

So, our answer is the optimal answer.

For example, we have a list of job like this.

JOB ID	DEADLINE	Profit
1	2	100
2	1	19
3	2	27
4	1	25
5	3	15

And the biggest deadline is 3 we will have the 3 free time slot.

0- 1	1 - 2	2-3
free	free	free

First, we will sort the profit in decreasing order. We get

JOB ID	DEADLINE	Profit
1	2	100
3	2	27
4	1	25
2	1	19
5	3	15

We are going to put job 1 in the time slot 2 since it is the latest possible free slot that meets its deadline then go to next job 3.

0-1	1-2	2-3
free	JOB 1	free

Then we are going to put job 3 in the time slot 1 since it is the latest possible free slot that meets its deadline then go to next job 4.

0-1	1-2	2-3
JOB3	JOB 1	free

Since time slot is already be taken by job3 and no more possible time slot, we will ignore the job4, and same for job2.Then we put job 5 in the time slot 3.

0-1	1-2	2-3
JOB3	JOB 1	Job5

Then we get the job subset which have the maximum profit. It is 27 + 100 +15 = 142