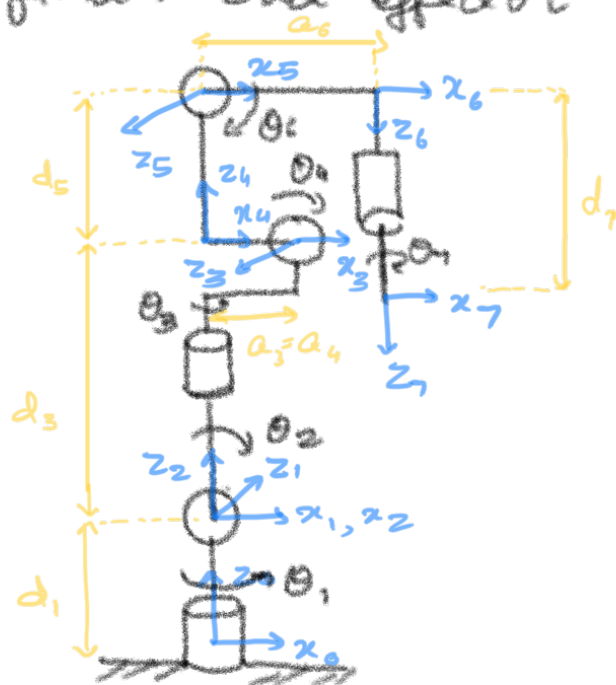


Prelab 3

1. Given: $\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4, \dot{q}_5, \dot{q}_6, \dot{q}_7$

To find: End effector $v_x, v_y, v_z, \omega_x, \omega_y, \omega_z$



$$\begin{aligned}
 d_1 &= 0.333 \text{ m} \\
 d_2 &= 0.316 \text{ m} \\
 a_3 &= a_4 = 0.0825 \text{ m} \\
 d_5 &= 0.384 \text{ m} \\
 d_6 &= 0.088 \text{ m} \\
 d_7 &= 0.21 \text{ m}
 \end{aligned}$$

DH Table ($\theta_5^* = 0$)

Link	a_i	d_i	d_i	θ_i^*
1	0	-90	d_1	θ_1^*
2	0	90	0	θ_2^*
3	a_3	90	d_3	θ_3^*
4	$-a_4$	-90	0	θ_4^*
5	0	90	d_5	0
6	a_6	90	0	θ_6^*
7	0	0	d_7	$\theta_7^* - 45^\circ$

Approach:

- ① Finding $T_1^0, T_2^0, T_3^0, T_4^0, T_5^0, T_6^0, T_7^0$
- ② Obtaining $\vec{O}_1^0, \vec{O}_2^0, \vec{O}_3^0, \vec{O}_4^0, \vec{O}_5^0, \vec{O}_6^0, \vec{O}_7^0$
- ③ Obtaining $R_1^0, R_2^0, R_3^0, R_4^0, R_5^0, R_6^0, R_7^0$
- ④ using $J_{v_i} = \hat{z}_{i-1} \times (\vec{O}_i^0 - \vec{O}_{i-1}^0)$ as all joints are revolute (using geometric approach)
- ⑤ using $J_{\omega_i} = p_i (R_{i-1}^0, \hat{z})$ when $p_i \neq 1$ for all cases, as all joints are revolute (geometric approach)
- ⑥ $T = T_1 T_2 T_3 T_4 T_5 T_6 T_7$

$$J_{\omega} = [J_{\omega_1} \ J_{\omega_2} \ J_{\omega_3} \ J_{\omega_4} \ J_{\omega_5} \ J_{\omega_6} \ J_{\omega_7}]$$

$$\textcircled{1} \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} J_v \\ J_{\omega} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \\ \dot{q}_7 \end{bmatrix}$$

Steps ①, ②, ③ are in the attached ipynb file.

$$\textcircled{4} \quad J_{v_1} = \hat{z}_{i-1} \times (\vec{O}_n - \vec{O}_{i-1})$$

$$\begin{aligned} \therefore J_{v_1} &= \hat{z}_0 \times (\vec{O}_7 - \vec{O}_0) \\ &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} \vec{O}_{7x} \\ \vec{O}_{7y} \\ \vec{O}_{7z} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} -\vec{O}_{7y} \\ \vec{O}_{7x} \\ 0 \end{bmatrix} \end{aligned}$$

$$J_{v_2} = \hat{z}_1 \times (\vec{O}_7 - \vec{O}_1)$$

$$= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \left(\begin{bmatrix} \vec{O}_{7x} \\ \vec{O}_{7y} \\ \vec{O}_{7z} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0.333 \end{bmatrix} \right) = \begin{bmatrix} \vec{O}_{7z} - 0.333 \\ 0 \\ -\vec{O}_{7x} \end{bmatrix}$$

$$J_{v_3} = \hat{z}_2 \times (\vec{O}_7 - \vec{O}_2)$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} \vec{O}_{7x} \\ \vec{O}_{7y} \\ \vec{O}_{7z} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0.333 \end{bmatrix} \right) = \begin{bmatrix} -\vec{O}_{7y} \\ \vec{O}_{7x} \\ 0 \end{bmatrix}$$

$$J_{v_4} = \hat{z}_3 \times (\vec{O}_7 - \vec{O}_3)$$

$$= \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \times \left(\begin{bmatrix} \vec{O}_{7x} \\ \vec{O}_{7y} \\ \vec{O}_{7z} \end{bmatrix} - \begin{bmatrix} \vec{O}_{3x} \\ \vec{O}_{3y} \\ \vec{O}_{3z} \end{bmatrix} \right)$$

$$= \begin{bmatrix} -(\vec{O}_{7z} - \vec{O}_{3z}) \\ 0 \\ (\vec{O}_{7x} - \vec{O}_{3x}) \end{bmatrix}$$

$$J_{v_5} = \hat{z}_4 \times (\vec{O}_7 - \vec{O}_4)$$

$$J_{v5} = \hat{z}_4 \times (\vec{O}_7 - \vec{O}_4) \\ = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} \vec{O}_{7x} - \vec{O}_{4x} \\ \vec{O}_{7y} - \vec{O}_{4y} \\ \vec{O}_{7z} - \vec{O}_{4z} \end{bmatrix} = \begin{bmatrix} -(\vec{O}_{7z} - \vec{O}_{4z}) \\ (\vec{O}_{7x} - \vec{O}_{4x}) \\ 0 \end{bmatrix}$$

$$J_{v6} = \hat{z}_5 \times (\vec{O}_7 - \vec{O}_5) \\ = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \times \begin{bmatrix} \vec{O}_{7x} - \vec{O}_{5x} \\ \vec{O}_{7y} - \vec{O}_{5y} \\ \vec{O}_{7z} - \vec{O}_{5z} \end{bmatrix} = \begin{bmatrix} -(\vec{O}_{7z} - \vec{O}_{5z}) \\ 0 \\ (\vec{O}_{7x} - \vec{O}_{5x}) \end{bmatrix}$$

$$J_{v7} = \hat{z}_6 \times (\vec{O}_7 - \vec{O}_6) \\ = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \times \begin{bmatrix} \vec{O}_{7x} - \vec{O}_{6x} \\ \vec{O}_{7y} - \vec{O}_{6y} \\ \vec{O}_{7z} - \vec{O}_{6z} \end{bmatrix} = \begin{bmatrix} \vec{O}_{7y} - \vec{O}_{6y} \\ -(\vec{O}_{7x} - \vec{O}_{6x}) \\ 0 \end{bmatrix}$$

$$J_v = [J_{v1} \quad J_{v2} \quad J_{v3} \quad J_{v4} \quad J_{v5} \quad J_{v6} \quad J_{v7}]$$

$$J_v = \left[\begin{array}{c|c|c|c|c|c|c} -\vec{O}_{7y} & \vec{O}_{7z} - 0.333 & -\vec{O}_{7y} & -(\vec{O}_{7z} - \vec{O}_{3z}) & -(\vec{O}_{7z} - \vec{O}_{4z}) & -(\vec{O}_{7z} - \vec{O}_{5z}) & \\ \vec{O}_{7x} & 0 & \vec{O}_{7x} & 0 & \vec{O}_{7x} - \vec{O}_{4x} & 0 & \\ 0 & -\vec{O}_{7z} & 0 & (\vec{O}_{7x} - \vec{O}_{3x}) & 0 & \vec{O}_{7x} - \vec{O}_{5x} & \end{array} \right]$$

$$\hookrightarrow \begin{bmatrix} \vec{O}_{7y} - \vec{O}_{6y} \\ -(\vec{O}_{7x} - \vec{O}_{6x}) \\ 0 \end{bmatrix}$$

$$⑤ \quad J_{w_i} = \rho_i (R_i^0 \hat{z})$$

$$J_{w_1} = 1 (R_0^0 \hat{z}) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_{w_2} = 1 (R_1^0 \hat{z})$$

$$J_{w_3} = R_2^0 \hat{z}$$

$$J_{w_4} = R_3^0 \hat{z}$$

$$J_{w_5} = R_4^0 \hat{z}$$

$$J_{w_6} = R_5^0 \hat{z}$$

$$J_{w_7} = R_6^0 \hat{z}$$

$$J_w = [J_{w1} \ J_{w2} \ J_{w3} \ J_{w4} \ J_{w5} \ J_{w6} \ J_{w7}]$$

For complete expression see ipynb file

2. Linear velocity end effector when $\dot{q}_1 = 50 \text{ rad/s}$

$$\dot{q}_2, \dot{q}_3, \dot{q}_4, \dot{q}_5, \dot{q}_6, \dot{q}_7 = 0$$

$$V_x = 0$$

$$V_y = 4.4 \text{ m/s}$$

$$V_z = 0$$

Yes FK reflects this velocity.

3. Condition for no solution:

Under singular configuration., Eg: $\theta_3 = 0$,
in general, when $\det(J_v) = 0$

Condition for unique solution:

Overdetermined system, (kinematically deficient robot). i.e. when $\text{rank}(J) \leq \min(n, 6)$

Condition for infinitely many solutions:

Underdetermined system (redundant robot like franka arm) i.e. when $\text{rank}(J) \leq \min(m, 6)$

4. Lab testing plan:

After writing code for velocity FK & IK, I shall use an arbitrary set of joint speed values & obtain some E.E. velocity values by passing into velocity FK. Using these E.E. velocity values in velocity IK, will see if I obtain the same joint velocities that I started with.

For the hardware testing, I will use a tachometer at each joint motor to get the values of joint velocities (when E.E values are set for testing)

```
import sympy as sym
import math

a = [0, 0, 0.0825, -0.0825, 0, 0.088, 0] #sym.Symbol('a3')
alpha = [-math.pi/2, math.pi/2, math.pi/2, -math.pi/2, math.pi/2, math.pi/2, 0]
d = [0.333, 0, 0.316, 0, 0.384, 0, 0.210] #sym.Symbol('d5')
theta = [sym.Symbol('theta_1'), sym.Symbol('theta_2'), sym.Symbol('theta_3'), sym.Symbol('theta_4'), 0, sym.Symbol('theta_6'), sym.Symbol('theta_7')]
zero_config = [0,0,0,0,0,0,0]

def T_calc(a, alpha, d, theta):
    T = []
    for i in range(7):
        c, s = sym.cos(theta[i]), sym.sin(theta[i])
        ca, sa = math.cos(alpha[i]), math.sin(alpha[i])
        t = sym.Matrix([[c, -s*ca, s*sa, a[i]*c], [s, c*ca, -c*sa, a[i]*s], [0, sa, ca, d[i]], [0, 0, 0, 1]])
        T.append(t)

    return T

def T_wrt_0(T):
    T_1_0 = sym.simplify(sym.nsimplify(T[0],tolerance=1e-10,rational=True))
    T_2_0 = sym.simplify(sym.nsimplify(T[0] * T[1],tolerance=1e-10,rational=True))
    T_3_0 = sym.simplify(sym.nsimplify(T[0] * T[1] * T[2],tolerance=1e-10,rational=True))
    T_4_0 = sym.simplify(sym.nsimplify(T[0] * T[1] * T[2] * T[3],tolerance=1e-10,rational=True))
    T_5_0 = sym.simplify(sym.nsimplify(T[0] * T[1] * T[2] * T[3] * T[4],tolerance=1e-10,rational=True))
    T_6_0 = sym.simplify(sym.nsimplify(T[0] * T[1] * T[2] * T[3] * T[4] * T[5],tolerance=1e-10,rational=True))
    T_7_0 = sym.simplify(sym.nsimplify(T[0] * T[1] * T[2] * T[3] * T[4] * T[5] * T[6],tolerance=1e-10,rational=True))

    return T_1_0, T_2_0, T_3_0, T_4_0, T_5_0, T_6_0, T_7_0

def o_wrt_0(T_1_0, T_2_0, T_3_0, T_4_0, T_5_0, T_6_0, T_7_0):
    o_1_0 = sym.Matrix([T_1_0[3], T_1_0[7], T_1_0[11]])
    o_2_0 = sym.Matrix([T_2_0[3], T_2_0[7], T_2_0[11]])
    o_3_0 = sym.Matrix([T_3_0[3], T_3_0[7], T_3_0[11]])
    o_4_0 = sym.Matrix([T_4_0[3], T_4_0[7], T_4_0[11]])
    o_5_0 = sym.Matrix([T_5_0[3], T_5_0[7], T_5_0[11]])
    o_6_0 = sym.Matrix([T_6_0[3], T_6_0[7], T_6_0[11]])
    o_7_0 = sym.Matrix([T_7_0[3], T_7_0[7], T_7_0[11]])

    return o_1_0, o_2_0, o_3_0, o_4_0, o_5_0, o_6_0, o_7_0

def R_wrt_0(T_1_0, T_2_0, T_3_0, T_4_0, T_5_0, T_6_0, T_7_0):
    R_1_0 = sym.Matrix([T_1_0[0], T_1_0[1], T_1_0[2]], [T_1_0[4], T_1_0[5], T_1_0[6]], [T_1_0[8], T_1_0[9], T_1_0[10]])
    R_2_0 = sym.Matrix([T_2_0[0], T_2_0[1], T_2_0[2]], [T_2_0[4], T_2_0[5], T_2_0[6]], [T_2_0[8], T_2_0[9], T_2_0[10]])
    R_3_0 = sym.Matrix([T_3_0[0], T_3_0[1], T_3_0[2]], [T_3_0[4], T_3_0[5], T_3_0[6]], [T_3_0[8], T_3_0[9], T_3_0[10]])
    R_4_0 = sym.Matrix([T_4_0[0], T_4_0[1], T_4_0[2]], [T_4_0[4], T_4_0[5], T_4_0[6]], [T_4_0[8], T_4_0[9], T_4_0[10]])
    R_5_0 = sym.Matrix([T_5_0[0], T_5_0[1], T_5_0[2]], [T_5_0[4], T_5_0[5], T_5_0[6]], [T_5_0[8], T_5_0[9], T_5_0[10]])
    R_6_0 = sym.Matrix([T_6_0[0], T_6_0[1], T_6_0[2]], [T_6_0[4], T_6_0[5], T_6_0[6]], [T_6_0[8], T_6_0[9], T_6_0[10]])
    R_7_0 = sym.Matrix([T_7_0[0], T_7_0[1], T_7_0[2]], [T_7_0[4], T_7_0[5], T_7_0[6]], [T_7_0[8], T_7_0[9], T_7_0[10]])

    return R_1_0, R_2_0, R_3_0, R_4_0, R_5_0, R_6_0, R_7_0

def cross(b, c):
    t1 = b[1]*c[2] - c[1]*b[2]
    t2 = -(b[0]*c[2] - b[2]*c[0])
    t3 = b[0]*c[1] - b[1]*c[0]
    return sym.Matrix([t1, t2, t3])

def Jv_calc(o_1_0, o_2_0, o_3_0, o_4_0, o_5_0, o_6_0, o_7_0):
    z0 = sym.Matrix([0,0,1])
    z1 = sym.Matrix([0,1,0])
    z2 = sym.Matrix([0,0,1])
    z3 = sym.Matrix([0,-1,0])
    z4 = sym.Matrix([0,0,1])
    z5 = sym.Matrix([0,-1,0])
    z6 = sym.Matrix([0,0,-1])
    Jv1 = sym.simplify(cross(z0, o_7_0))
    Jv2 = sym.simplify(cross(z1, o_7_0 - o_1_0))
    Jv3 = sym.simplify(cross(z2, o_7_0 - o_2_0))
    Jv4 = sym.simplify(cross(z3, o_7_0 - o_3_0))
    Jv5 = sym.simplify(cross(z4, o_7_0 - o_4_0))
    Jv6 = sym.simplify(cross(z5, o_7_0 - o_5_0))
    Jv7 = sym.simplify(cross(z6, o_7_0 - o_6_0))
    Jv = sym.Matrix([Jv1.T, Jv2.T, Jv3.T, Jv4.T, Jv5.T, Jv6.T, Jv7.T]) #yields a row major ordered matrix (7 rows 3 columns)
    Jv = Jv.T #converts to 3 rows and 7 columns
    print('Jv shape =', Jv.shape)
    return Jv

def Jw_calc(R_1_0, R_2_0, R_3_0, R_4_0, R_5_0, R_6_0, R_7_0):
    z_hat = sym.Matrix([0,0,1])

    Jw1 = sym.simplify(z_hat)
    Jw2 = sym.simplify(R_1_0*z_hat)
    Jw3 = sym.simplify(R_2_0*z_hat)
    Jw4 = sym.simplify(R_3_0*z_hat)
    Jw5 = sym.simplify(R_4_0*z_hat)
    Jw6 = sym.simplify(R_5_0*z_hat)
    Jw7 = sym.simplify(R_6_0*z_hat)
    Jw = sym.Matrix([Jw1.T, Jw2.T, Jw3.T, Jw4.T, Jw5.T, Jw6.T, Jw7.T]) #yields a row major ordered matrix (7 rows 3 columns)
    Jw = Jw.T #converts to 3 rows and 7 columns
    print('Jw shape =', Jw.shape)
    return Jw

def J(Jv, Jw):
    J = sym.Matrix([Jv, Jw])
    print("Jacobian shape is =", J.shape)
    return J

#function calls (symbolic)
T = T_calc(a, alpha, d, theta)
T_1_0, T_2_0, T_3_0, T_4_0, T_5_0, T_6_0, T_7_0 = T_wrt_0(T)
o_1_0, o_2_0, o_3_0, o_4_0, o_5_0, o_6_0, o_7_0 = o_wrt_0(T_1_0, T_2_0, T_3_0, T_4_0, T_5_0, T_6_0, T_7_0)
R_1_0, R_2_0, R_3_0, R_4_0, R_5_0, R_6_0, R_7_0 = R_wrt_0(T_1_0, T_2_0, T_3_0, T_4_0, T_5_0, T_6_0, T_7_0)

print("T_1_0 =", T_1_0)
print("T_2_0 =", T_2_0)
```