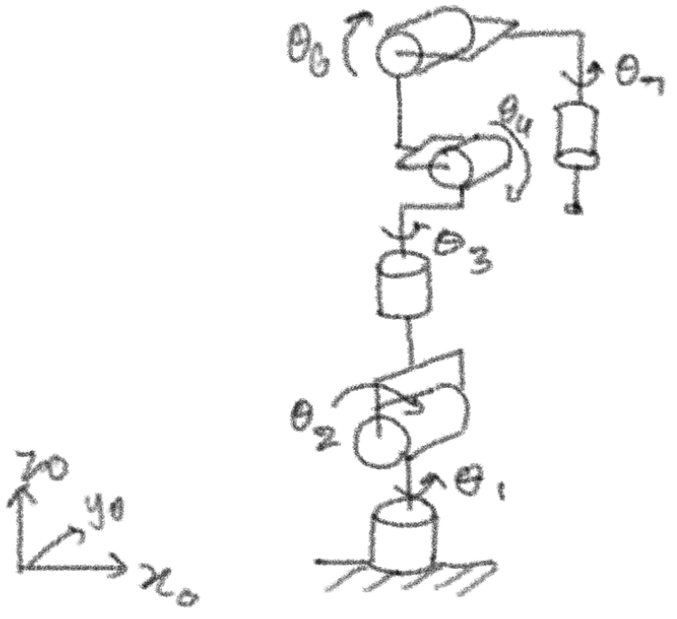
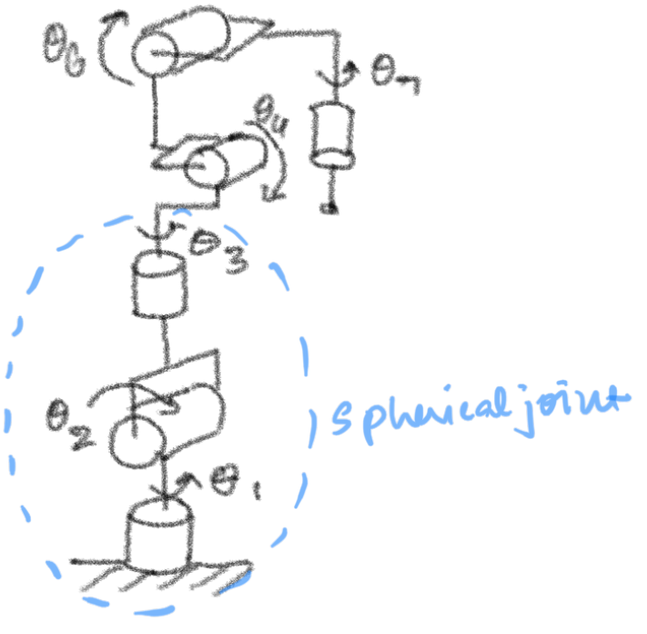
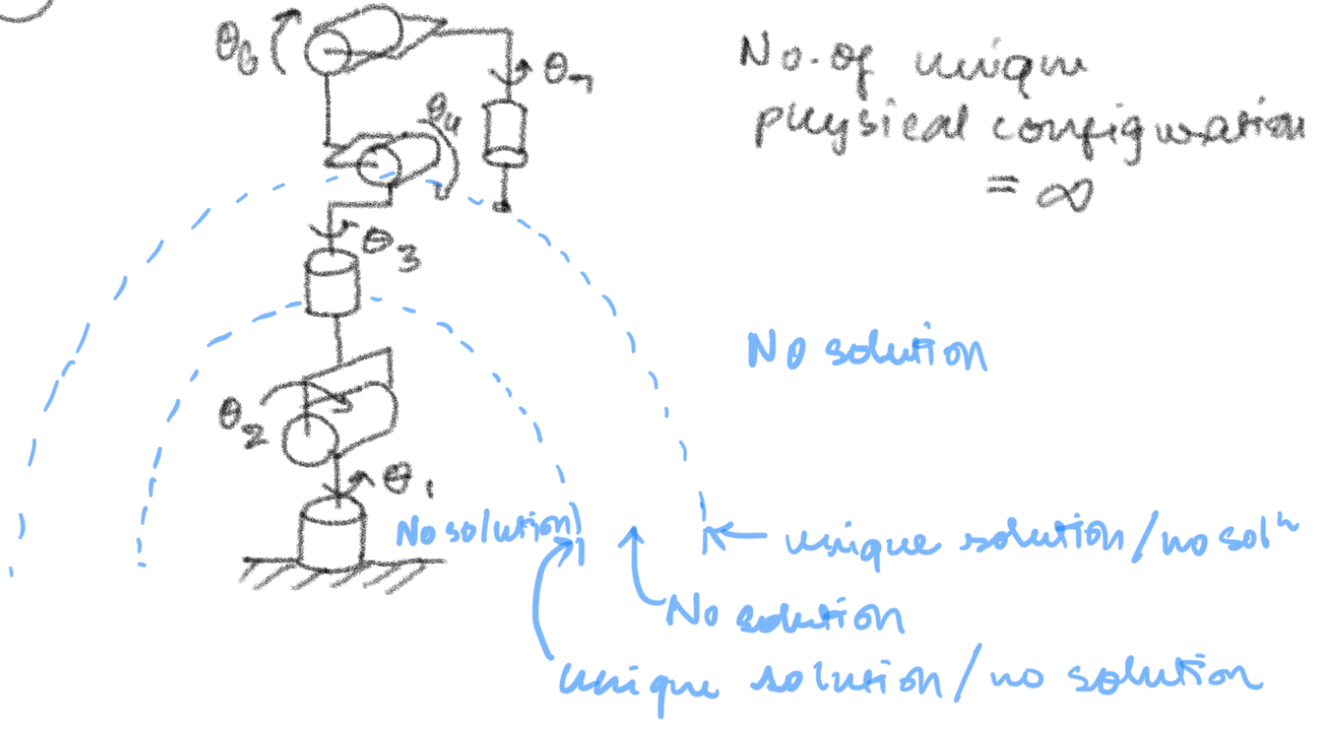
Prelab 2 Tool used: Python (SymPy)
(Constraint $\theta_5 = 0$)



Dextrous WS: points that manipulator can reach with arbitrary orientation of End effector.



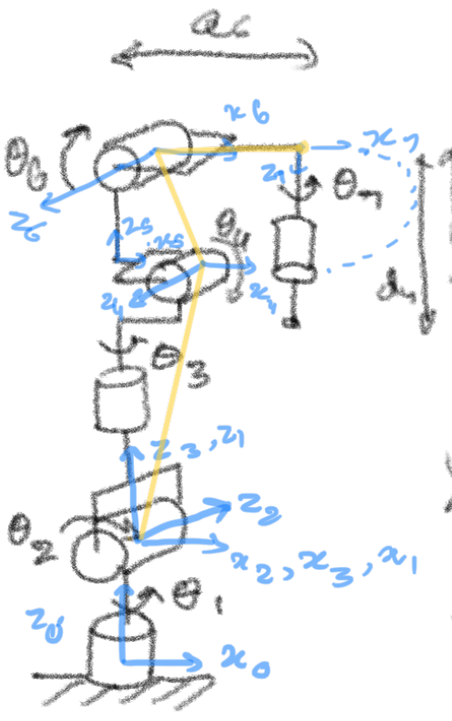
1.



No. of unique physical configuration $= \infty$

2. a) $T_7^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \end{bmatrix} = \begin{bmatrix} R_7^0 & d_7^0 \end{bmatrix}$

$$\begin{bmatrix} r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 \end{bmatrix}$$



D H Table

Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1^*
2	0	-90	0	θ_2^*
3	0	90	0	θ_3^*
4	a_3	90	d_3	θ_4^*
5	$-a_4$	-90	0	0
6	0	90	d_5	θ_6^*
7	a_6	90	0	$\theta_7^* - \frac{\pi}{4}$

$$T_7^0 = T_1^0 T_2^1 T_3^2 T_4^3 T_5^4 T_6^5 T_7^6$$

$$T_1^0 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_2^1 = \begin{bmatrix} c\theta_2 & 0 & -s\theta_2 & 0 \\ s\theta_2 & 0 & c\theta_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} c\theta_3 & 0 & s\theta_3 & 0 \\ s\theta_3 & 0 & -c\theta_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_4^3 = \begin{bmatrix} c\theta_4 & 0 & s\theta_4 & a_3 c\theta_4 \\ s\theta_4 & 0 & -c\theta_4 & a_3 s\theta_4 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5^4 = \begin{bmatrix} 1 & 0 & 0 & -a_4 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_6^5 = \begin{bmatrix} c\theta_6 & 0 & s\theta_6 & 0 \\ s\theta_6 & 0 & -c\theta_6 & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_7^6 = \begin{bmatrix} c(\theta_7 - \frac{\pi}{4}) & 0 & s(\theta_7 - \frac{\pi}{4}) & a_6 c(\theta_7 - \frac{\pi}{4}) \\ s(\theta_7 - \frac{\pi}{4}) & 0 & -c(\theta_7 - \frac{\pi}{4}) & a_6 s(\theta_7 - \frac{\pi}{4}) \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End Eff. has offset by d_7, considering that,

$$T_7^6 = \begin{bmatrix} c(\theta_7 - \frac{\pi}{4}) & 0 & s(\theta_7 - \frac{\pi}{4}) & a_6 c(\theta_7 - \frac{\pi}{4}) \\ s(\theta_7 - \frac{\pi}{4}) & 0 & -c(\theta_7 - \frac{\pi}{4}) & a_6 s(\theta_7 - \frac{\pi}{4}) \\ 0 & 1 & 0 & d_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(Please refer attached document for values of elements of T_7^0)

$$T_0^7 = \begin{bmatrix} (R_7^0)^{-1} & -(R_7^0)^{-1} d_7^0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (R_7^0)^T & -(R_7^0)^T d_7^0 \\ 0 & 1 \end{bmatrix}$$

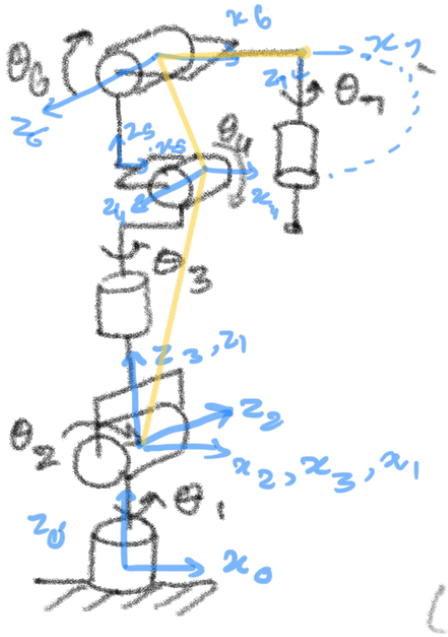
$$= \begin{bmatrix} r_{11} & r_{21} & r_{31} & -(r_{11} t_1 + r_{21} t_2 + r_{31} t_3) \\ r_{12} & r_{22} & r_{32} & -(r_{12} t_1 + r_{22} t_2 + r_{32} t_3) \\ r_{13} & r_{23} & r_{33} & -(r_{13} t_1 + r_{23} t_2 + r_{33} t_3) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) $T_2^7 = T_0^7 \, T_2^0$

$$O_2^7 = \begin{bmatrix} T_2^7[0][3] \\ T_2^7[1][3] \\ T_2^7[2][3] \end{bmatrix}$$

(Please refer the attached document for expression of O_2^7)

c)



This is true because if joint 7 is seen as the base & the robot is turned upside down, the joints 1, 2 & 3 form a spherical wrist. Now if $\theta_7 = 0$ (joint 7 is fixed), The upside down robot reduces to a planar R-R robot. If θ_7 is any variable then the robot is effectively a 3-D R-R robot. So the position of the spherical wrist is only affected by Joints 4, 6, 7 (as joint 5 is fixed). Therefore it is true that only joints 4, 6, 7 affect position vector O_2^7.

d) The following solution assumes the robot to be upside down. So for a constant orientation of the wrist ($\theta_1 = \theta_2 = \theta_3 = 0$ & $\theta_5 = 0$), we calculate θ_4, θ_6 & θ_7.

$\therefore$ When $\theta_1 = \theta_2 = \theta_3 = 0$

$$T_0^7 = \begin{bmatrix} r_{11}' & r_{12}' & r_{13}' & t_1' \\ r_{21}' & r_{22}' & r_{23}' & t_2' \\ r_{31}' & r_{32}' & r_{33}' & t_3' \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_{11}' = \cos(\theta_4 + \theta_6)\cos(\theta_7 - \tfrac{\pi}{4})$$

$$n'_{12} = \sin(\theta_4+\theta_6)\cos(\theta_7-\frac{\pi}{4})$$

$$n'_{13} = \sin(\theta_7-\frac{\pi}{4})$$

$$n'_{21} = \sin(\theta_4+\theta_6)$$

$$n'_{22} = -\cos(\theta_4+\theta_6)$$

$$n'_{23} = 0$$

$$n'_{31} = \sin(\theta_7-\frac{\pi}{4})\cos(\theta_4+\theta_6)$$

$$n'_{32} = \sin(\theta_4+\theta_6)\sin(\theta_7-\frac{\pi}{4})$$

$$n'_{33} = a_3\sin\theta_4 - a_4\sin\theta_4 + a_6\sin(\theta_4+\theta_6)\cos(\theta_7-\frac{\pi}{4}) - d_7\cos(\theta_4+\theta_6)$$

$$t'_1 = -a_3\cos\theta_6\cos(\theta_7-\frac{\pi}{4}) + a_4\cos\theta_6\cos(\theta_7-\frac{\pi}{4}) - a_6 - d_1\sin(\theta_7-\frac{\pi}{4}) - d_3\sin(\theta_7-\frac{\pi}{4}) - d_5\sin(\theta_7-\frac{\pi}{4})$$

$$t'_2 = -a_3\sin\theta_6 + a_4\sin\theta_6 - d_7$$

$$t'_3 = -a_3\sin(\theta_7-\frac{\pi}{4})\cos\theta_6 + a_4\sin(\theta_7-\frac{\pi}{4})\cos\theta_6 + d_1\cos(\theta_7-\frac{\pi}{4}) + d_3\cos(\theta_7-\frac{\pi}{4}) + d_5\cos(\theta_7-\frac{\pi}{4})$$

from n'_{13},

$$n_{13} = \sin(\theta_7 - \pi/4)$$

$$\boxed{\theta_7 = \frac{\pi}{4} + \sin^{-1}(n'_{13})} \quad - (1)$$

from t'_2,

$$t'_2 + d_7 = (a_4 - a_3)\sin\theta_6$$

$$\sin\theta_6 = \frac{t'_2 + d_7}{a_4 - a_3}$$

$$\boxed{\theta_6 = \sin^{-1}\left(\frac{t'_2 + d_7}{a_4 - a_3}\right)} \quad - (2)$$

from n'_{21} & n'_{22},

$$-\frac{n'_{21}}{n'_{22}} = \frac{\sin(\theta_4 + \theta_6)}{\cos(\theta_4 + \theta_6)}$$

$$\tan^{-1}\left(-\frac{n'_{21}}{n'_{22}}\right) = \theta_4 + \theta_6$$

using eqn (2),

$$\boxed{\theta_4 = \tan^{-1}\left(-\frac{n'_{21}}{n'_{22}}\right) - \sin^{-1}\left(\frac{t'_2 + d_7}{a_4 - a_3}\right)} \quad (3)$$

e) Now, considering $\theta_1, \theta_2, \theta_3$ as non zero values, (θ_5 still 0)

$$n'_{22} = -\sin\theta_6 \sin(\theta_4 + \theta_6)$$

known from eq^n 2 & 3

$$\therefore \quad \frac{-r'_{23}}{\sin(\theta_4+\theta_6)} = \sin\theta_3$$

$$\boxed{\theta_3 = \sin^{-1}\left(\frac{-r'_{23}}{\sin(\theta_4+\theta_6)}\right)}$$

$$\text{I} - r'_{12} = A\sin(\theta_1+\theta_2) + B\cos(\theta_1+\theta_2) + C$$

$$\text{II} - r'_{22} = P\sin(\theta_1+\theta_2) - T\cos(\theta_1+\theta_2)$$

- where A, B, C, P, T are all values that were previously evaluated.
- Adding I & II gives us $\theta_1+\theta_2$

3

a) Please refer attached document for test cases.

b) For test case 1, I have chosen zero pose. On physical evaluation, there should be no displacement in the y direction

For test case 2, I have chosen a pose where the final pose is the end effector on the same level as the base frame

as the base frame.

For test case 3, I have chosen a pose that can be achieved by only changing Joint 2 & 4.

c) To ensure correctness of solution in simulation, we verify that the joint angles obtained analytically, when used for forward kinematics, yield the same pose. We can do this on the simulation by 1st obtaining the joint angles & then in the FK simulation, checking if the origin of each joint frame remains in place throughout the simulation.

d) To ensure correctness of solution in hardware, we can measure the position of the end effector from the base using a measuring tape. (This pose is reached from the θ that we have calculated.)

- If the EE origin (located on the robot) is close to the last column of T_7^0, calculated using newly obtained θ, then the joint angles are correct.

r11 = -(sin(θ1 + θ2)*sin(θ4 + θ6) - cos(θ3)*cos(θ1 + θ2)*cos(θ4 + θ6))*cos(θ7 - pi/4) + sin(θ3)*sin(θ7 - pi/4)*cos(θ1 + θ2)

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r12 = sin(θ1 + θ2)*cos(θ4 + θ6) + sin(θ4 + θ6)*cos(θ3)*cos(θ1 + θ2)

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r13 = -(sin(θ1 + θ2)*sin(θ4 + θ6) - cos(θ3)*cos(θ1 + θ2)*cos(θ4 + θ6))*sin(θ7 - pi/4) - sin(θ3)*cos(θ1 + θ2)*cos(θ7 - pi/4)

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r21 = (sin(θ1 + θ2)*cos(θ3)*cos(θ4 + θ6) + sin(θ4 + θ6)*cos(θ1 + θ2))*cos(θ7 - pi/4) + sin(θ3)*sin(θ1 + θ2)*sin(θ7 - pi/4)

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r22 = sin(θ1 + θ2)*sin(θ4 + θ6)*cos(θ3) - cos(θ1 + θ2)*cos(θ4 + θ6)

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r23 = (sin(θ1 + θ2)*cos(θ3)*cos(θ4 + θ6) + sin(θ4 + θ6)*cos(θ1 + θ2))*sin(θ7 - pi/4) - sin(θ3)*sin(θ1 + θ2)*cos(θ7 - pi/4)

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r31 = -sin(θ3)*cos(θ4 + θ6)*cos(θ7 - pi/4) + sin(θ7 - pi/4)*cos(θ3)

---

r32 = -sin(θ3)*sin(θ4 + θ6)

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r33 = -sin(θ3)*sin(θ7 - pi/4)*cos(θ4 + θ6) - cos(θ3)*cos(θ7 - pi/4)

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t1 = -a3*sin(θ4)*sin(θ1 + θ2) + a3*cos(θ3)*cos(θ4)*cos(θ1 + θ2) + a4*(sin(θ4)*sin(θ1 + θ2) - cos(θ3)*cos(θ4)*cos(θ1 + θ2)) - a6*(sin(θ1 + θ2)*sin(θ4 + θ6) - cos(θ3)*cos(θ1 + θ2)*cos(θ4 + θ6))*cos(θ7 - pi/4) + a6*sin(θ3)*sin(θ7 - pi/4)*cos(θ1 + θ2) + d3*sin(θ3)*cos(θ1 + θ2) + d5*sin(θ3)*cos(θ1 + θ2) + d7*(sin(θ1 + θ2)*cos(θ4 + θ6) + sin(θ4 + θ6)*cos(θ3)*cos(θ1 + θ2))

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t2 = a3*sin(θ4)*cos(θ1 + θ2) + a3*sin(θ1 + θ2)*cos(θ3)*cos(θ4) - a4*(sin(θ4)*cos(θ1 + θ2) + sin(θ1 + θ2)*cos(θ3)*cos(θ4)) + a6*(sin(θ1 + θ2)*cos(θ3)*cos(θ4 + θ6) + sin(θ4 + θ6)*cos(θ1 + θ2))*cos(θ7 - pi/4) + a6*sin(θ3)*sin(θ1 + θ2)*sin(θ7 - pi/4) + d3*sin(θ3)*sin(θ1 + θ2) + d5*sin(θ3)*sin(θ1 + θ2) - d7*(-sin(θ1 + θ2)*sin(θ4 + θ6)*cos(θ3) + cos(θ1 + θ2)*cos(θ4 + θ6))

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t3 = -a3*sin(θ3)*cos(θ4) + a4*sin(θ3)*cos(θ4) - a6*sin(θ3)*cos(θ4 + θ6)*cos(θ7 - pi/4) + a6*sin(θ7 - pi/4)*cos(θ3) + d1 + d3*cos(θ3) + d5*cos(θ3) - d7*sin(θ3)*sin(θ4 + θ6)

Q2.

a) T_0^7 =

[[-(sin(θ1 + θ2)*sin(θ4 + θ6) - cos(θ3)*cos(θ1 + θ2)*cos(θ4 + θ6))*cos(θ7 - pi/4) + sin(θ3)*sin(θ7 - pi/4)*cos(θ1 + θ2), (sin(θ1 + θ2)*cos(θ3)*cos(θ4 + θ6) + sin(θ4 + θ6)*cos(θ1 + θ2))*cos(θ7 - pi/4) + sin(θ3)*sin(θ1 + θ2)*sin(θ7 - pi/4), -sin(θ3)*cos(θ4 + θ6)*cos(θ7 - pi/4) + sin(θ7 - pi/4)*cos(θ3), -a3*cos(θ6)*cos(θ7 - pi/4) + a4*cos(θ6)*cos(θ7 - pi/4) - a6 + d1*sin(θ3)*cos(θ4 + θ6)*cos(θ7 - pi/4) - d1*sin(θ7 - pi/4)*cos(θ3) - d3*sin(θ7 - pi/4) - d5*sin(θ7 - pi/4)], [sin(θ1 + θ2)*cos(θ4 + θ6) + sin(θ4 + θ6)*cos(θ3)*cos(θ1 + θ2), sin(θ1 + θ2)*sin(θ4 + θ6)*cos(θ3) - cos(θ1 + θ2)*cos(θ4 + θ6), -sin(θ3)*sin(θ4 + θ6), -a3*sin(θ6) + a4*sin(θ6) + d1*sin(θ3)*sin(θ4 + θ6) - d7], [-(sin(θ1 + θ2)*sin(θ4 + θ6) - cos(θ3)*cos(θ1 + θ2)*cos(θ4 + θ6))*sin(θ7 - pi/4) - sin(θ3)*cos(θ1 + θ2)*cos(θ7 - pi/4), (sin(θ1 + θ2)*cos(θ3)*cos(θ4 + θ6) + sin(θ4 + θ6)*cos(θ1 + θ2))*sin(θ7 - pi/4) - sin(θ3)*sin(θ1 + θ2)*cos(θ7 - pi/4), a3*sin(θ4)*cos(θ1 + θ2) + a3*sin(θ1 + θ2)*cos(θ3)*cos(θ4) - a4*(sin(θ4)*cos(θ1 + θ2) + sin(θ1 + θ2)*cos(θ3)*cos(θ4)) + a6*(sin(θ1 + θ2)*cos(θ3)*cos(θ4 + θ6) + sin(θ4 + θ6)*cos(θ1 + θ2))*cos(θ7 - pi/4) + a6*sin(θ3)*sin(θ1 + θ2)*sin(θ7 - pi/4) + d3*sin(θ3)*sin(θ1 + θ2) + d5*sin(θ3)*sin(θ1 + θ2) - d7*(-sin(θ1 + θ2)*sin(θ4 + θ6)*cos(θ3) + cos(θ1 + θ2)*cos(θ4 + θ6)), -a3*sin(θ7 - pi/4)*cos(θ6) + a4*sin(θ7 - pi/4)*cos(θ6) + d1*sin(θ3)*sin(θ7 - pi/4)*cos(θ4 + θ6) + d1*cos(θ3)*cos(θ7 - pi/4) + d3*cos(θ7 - pi/4) + d5*cos(θ7 - pi/4)], [0, 0, 0, 1]]

b) o_2^7 =

[[-a3*cos(θ6)*cos(θ7 - pi/4) + a4*cos(θ6)*cos(θ7 - pi/4) - a6 - d3*sin(θ7 - pi/4) - d5*sin(θ7 - pi/4)],

[-a3*sin(θ6) + a4*sin(θ6) - d7],

[-a3*sin(θ7 - pi/4)*cos(θ6) + a4*sin(θ7 - pi/4)*cos(θ6) + d1*(a3*sin(θ4)*cos(θ1 + θ2) + a3*sin(θ1 + θ2)*cos(θ3)*cos(θ4) - a4*(sin(θ4)*cos(θ1 + θ2) + sin(θ1 + θ2)*cos(θ3)*cos(θ4)) + a6*(sin(θ1 + θ2)*cos(θ3)*cos(θ4 + θ6) + sin(θ4 + θ6)*cos(θ1 + θ2))*cos(θ7 - pi/4) + a6*sin(θ3)*sin(θ1 + θ2)*sin(θ7 - pi/4)

+ d3*sin(θ3)*sin(θ1 + θ2) + d5*sin(θ3)*sin(θ1 + θ2) + d7*(sin(θ1 + θ2)*sin(θ4 + θ6)*cos(θ3) - cos(θ1 + θ2)*cos(θ4 + θ6))) + d1*sin(θ3)*sin(θ7 - pi/4)*cos(θ4 + θ6) + d1*cos(θ3)*cos(θ7 - pi/4) + d3*cos(θ7 - pi/4) + d5*cos(θ7 - pi/4)]]

d) T_0^7=
[[cos(θ4 + θ6)*cos(θ7 - pi/4), sin(θ4 + θ6)*cos(θ7 - pi/4),
sin(θ7 - pi/4), -a3*cos(θ6)*cos(θ7 - pi/4) + a4*cos(θ6)*cos(θ7 - pi/4) - a6 - d1*sin(θ7 - pi/4) - d3*sin(θ7 - pi/4) - d5*sin(θ7 - pi/4)],
[sin(θ4 + θ6), -cos(θ4 + θ6), 0, -a3*sin(θ6) + a4*sin(θ6) - d7],
[sin(θ7 - pi/4)*cos(θ4 + θ6), sin(θ4 + θ6)*sin(θ7 - pi/4), a3*sin(θ4) - a4*sin(θ4) + a6*sin(θ4 + θ6)*cos(θ7 - pi/4) - d7*cos(θ4 + θ6), -a3*sin(θ7 - pi/4)*cos(θ6) + a4*sin(θ7 - pi/4)*cos(θ6) + d1*cos(θ7 - pi/4) + d3*cos(θ7 - pi/4) + d5*cos(θ7 - pi/4)],
[0, 0, 0, 1]]

Q3.
a) Following values of T_7^0 shall be tested
Test case 1 :
[[1/√2, 0, -1/√2, a3 - a4 + 1/√2*a6], [0, -1, 0, 0], [-1/√2, 0, -1/√2, -1/√2*a6 + d1 + d3 + d5], [0, 0, 0, 1]]

Test case 2 :
[[-1/√2, 0, 1/√2, -a3 + a4 - 1/√2*a6], [0, 1, 0, 0], [-1/√2, 0, -1/√2, -1/√2*a6 + d1 + d3 + d5], [0, 0, 0, 1]]

Test case 3 :
[[-0.0.3535, -0.866, 0.3535, -a3/2 + a4/2 – 0.3535*a6- 0.866*d7], [-0.61, 1/2, 0.61, -0.86*a3+0.86*a4 – 0.612*a6 + d7/2], [-0.707, 0, -0.707, -0.707*a6 + d1 + d3 + d5], [0, 0, 0, 1]])