Finding root

Age	A<25	25≤A≤35	A>35
Y	2	4	3
N	3	0	2

A<25:

Gini: $1-P(y)^2-P(N)^2=1-\frac{2}{5}^2-\frac{3}{5}^2=0.48$

25≤Age≤35:

Gini: $1-P(y)^2-P(N)^2=1-\frac{4}{4}^2-\frac{0}{4}^2=0$

A>35:

Gini: $1-\frac{3}{5}^2-\frac{2}{5}^2=0.48$

AVg Gini $=0.48\times\frac{5}{14}+0\times\frac{4}{14}+0.48\times\frac{5}{14}=$ **0.34**

Income	high	medium	low
Y	2	4	3
N	2	2	1

high: Gini $=1-\frac{2}{4}^2-\frac{2}{4}^2=0.5$

medium: Gini $1-\frac{4}{6}^2-\frac{2}{6}^2=0.44$

Low: Gini $=1-(\frac{3}{4})^2-(\frac{1}{4})^2=0.375$

AVg Gini $=0.5\times\frac{4}{14}+\frac{6}{14}\times0.44+0.375\times\frac{4}{14}=$ **0.438**

Two way split for age which is an ordinal categorical value so we only have two options.

$A \leq 25 + 25 \leq A \leq 35 = A \leq 35$

Age	$A \leq 35$	$A > 35$
Y	6	3
N	3	2

$A \leq 35$

Gini: $1 - \frac{6}{9}^2 - \frac{3}{9}^2 = 0.44$

$A > 35$:

Gini $= 1 - \frac{3}{5}^2 - \frac{2}{5}^2 = 0.48$

Avg Gini $= 0.44 \times \frac{9}{14} + 0.48 \times \frac{5}{14} =$ **0.45**

Age	$A < 25$	$A \geq 25$
Y	2	7
N	3	2

$A < 25$:

Gini $= 1 - \frac{2}{5}^2 - \frac{3}{5}^2 = 0.48$

$A \geq 25$:

Gini $= 1 - \frac{7}{9}^2 - \frac{2}{9}^2 = 0.34$

Avg Gini $= 0.48 \times \frac{5}{14} + 0.34 \times \frac{9}{14} =$ **0.39**

we found multi split Average Gini to equal = 0.34 which is less than both ways of the Two way splits

$0.34 < 0.45$ & $0.34 < 0.39$

We choose multi split

is student	yes	No
buy Y	6	3
buy N	1	4

yes:
$Gini = 1 - \frac{6}{7}^2 - \frac{1}{7}^2 = 0.244$

No:
$Gini = 1 - \frac{3}{7}^2 - \frac{4}{7}^2 = 0.489$

$Avg\ Gini = 0.244 \times \frac{7}{14} + 0.489 \times \frac{7}{14} = \boxed{0.3665}$

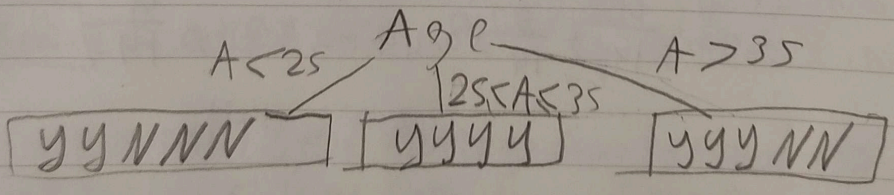
credit rating	Excellent	Fair
Y	3	6
N	3	2

Excellent: $Gini = 1 - \frac{3}{6}^2 - \frac{3}{6}^2 = 0.5$

Fair: $Gini = 1 - \left(\frac{6}{8}\right)^2 - \left(\frac{2}{8}\right)^2 = 0.375$

$Avg\ Gini = 0.5 \times \frac{6}{14} + 0.375 \times \frac{8}{14} = \boxed{0.428}$

After finding and comparing the Avg Gini of all feature we find that Age has the lowest Avg Gini between them, which is 0.34 so we pick the Age to be the root



we find that the Age $25 \leq A \leq 35$ results in a pure leaf which means it doesn't require any further splitting, so we only repeat process for the rest

Age > 35

Credit rating	Excellent	Fair
Y	0	3
N	2	0

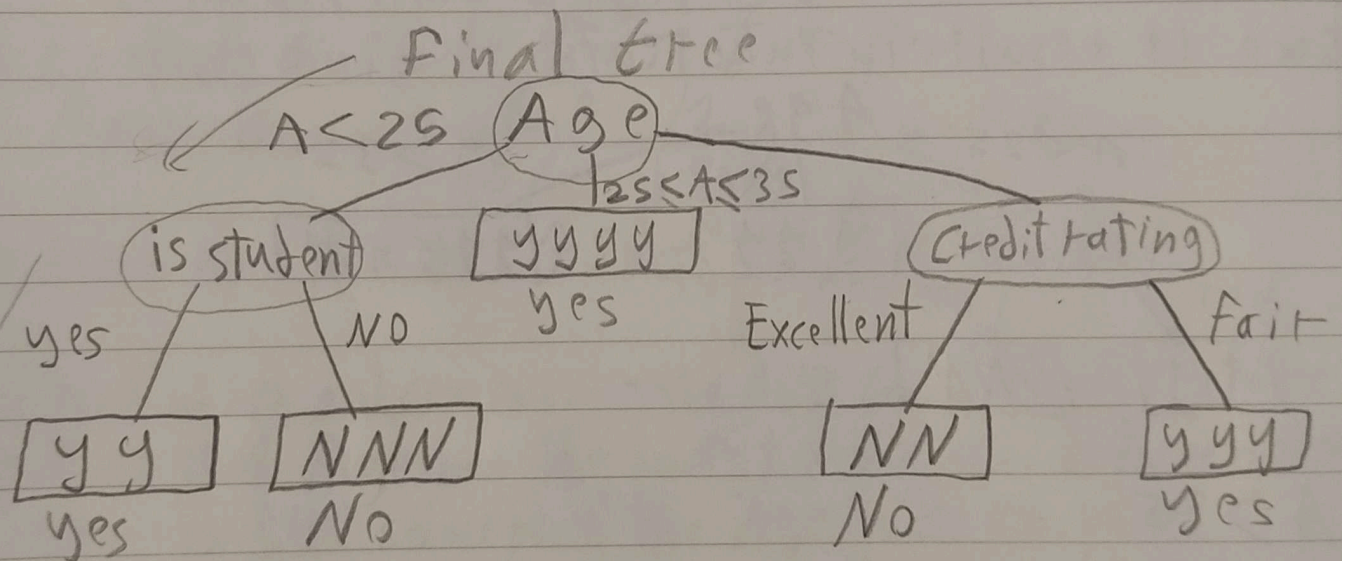
Excellent Gini $= 1 - \frac{2}{2}^2 - \frac{0}{2}^2 = 0$

Fair Gini $= 1 - \frac{3}{3}^2 - \frac{0}{3}^2 = 0$

Avg Gini $= 0 \times \frac{2}{5} + 0 \times \frac{3}{5} = 0$

Credit rating Avg Gini is less than is student Credit rating Avg Gini is zero which means it results in pure leafs so we can choose it and there's no need to test the features

Final tree



b) if Age ≤ 20 and income = medium and is_student = yes and credit fair the class will be (yes) after traversing our final decision tree

Age > 35

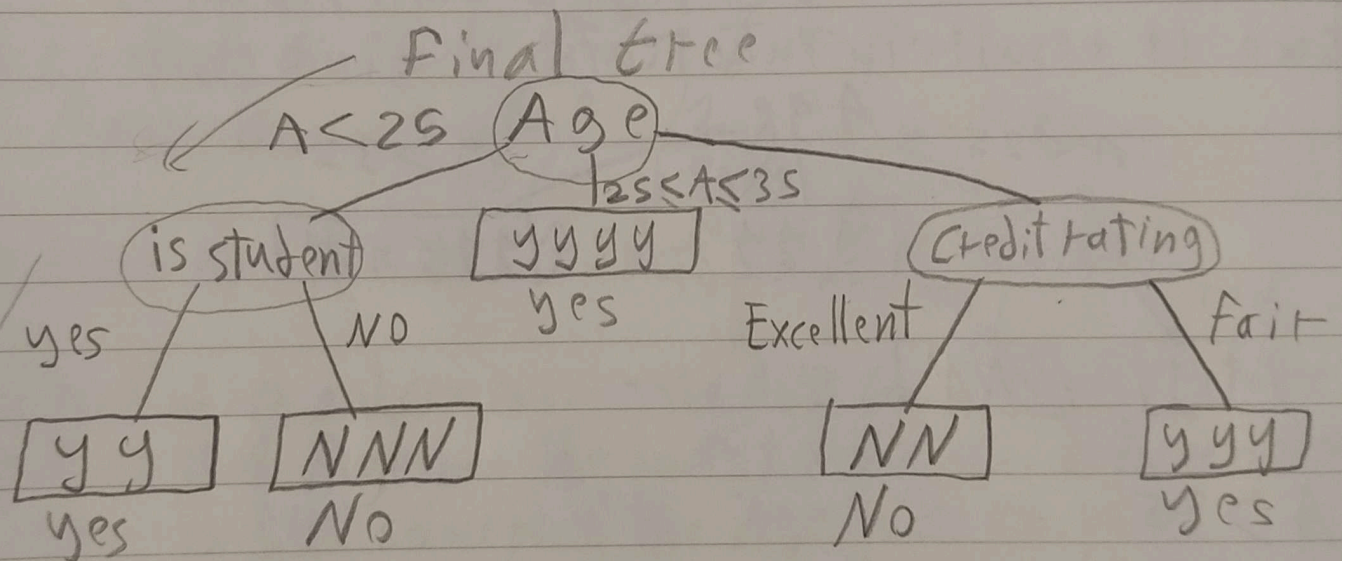
Credit rating	Excellent	Fair
Y	0	3
N	2	0

Excellent Gini $= 1 - \frac{2}{2}^2 - \frac{0}{2}^2 = 0$

Fair Gini $= 1 - \frac{3}{3}^2 - \frac{0}{3}^2 = 0$

Avg Gini $= 0 \times \frac{2}{5} + 0 \times \frac{3}{5} = 0$

Credit rating Avg Gini is less than is student Credit rating Avg Gini is zero which means it results in pure leafs so we can choose it and there's no need to test the features



b) if Age ≤ 20 and income = medium and is_student = yes and credit fair the class will be (yes) after traversing our final decision tree