



DeepLearning.AI

# Math for Machine Learning

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## Linear algebra - Week 1

Systems of linear equations

Singular and non-singular matrices

Determinants

Rank of a matrix

Row reduction

Null space



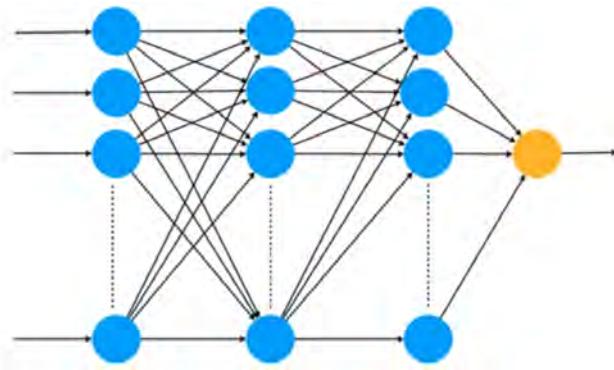
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# System of Linear Equations

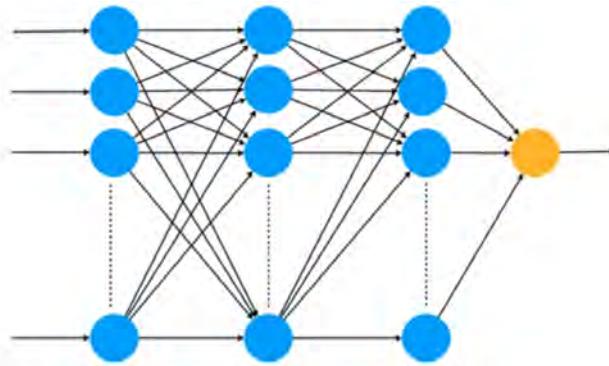
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## Machine learning motivation

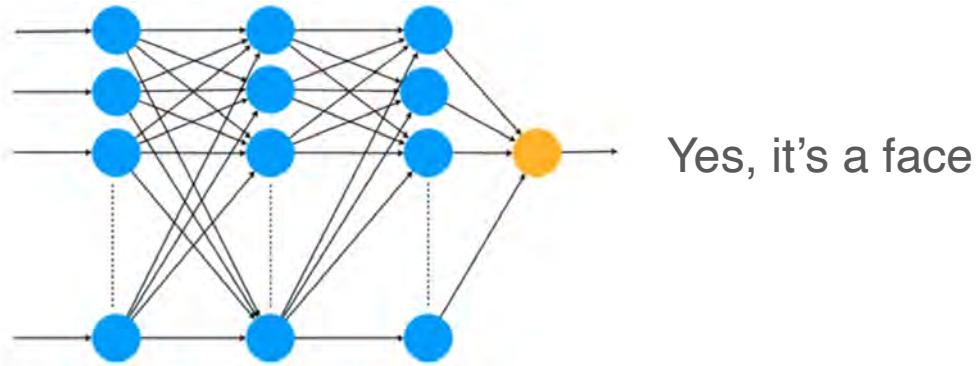
# Neural networks - Matrix operations



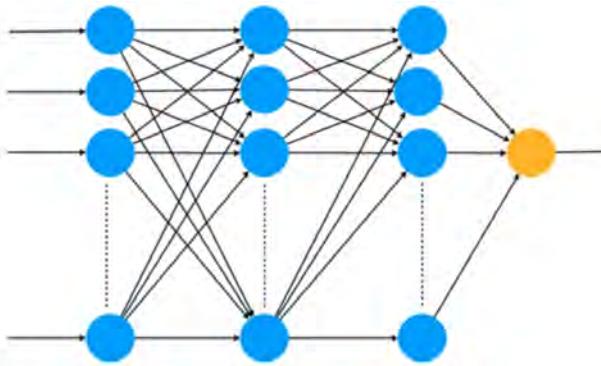
# Neural networks - Matrix operations



# Neural networks - Matrix operations

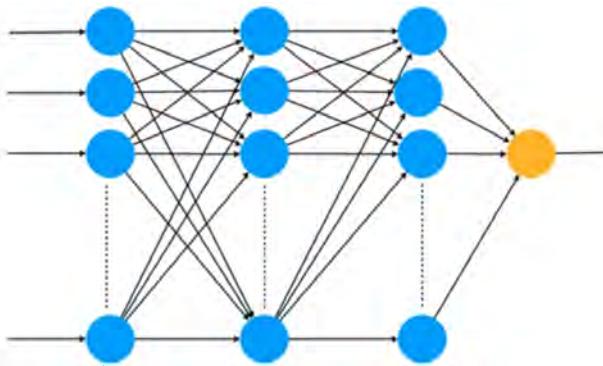
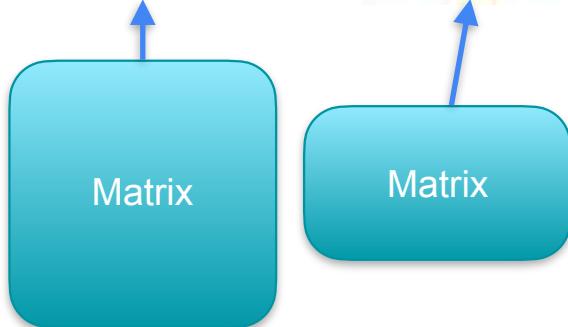


# Neural networks - Matrix operations



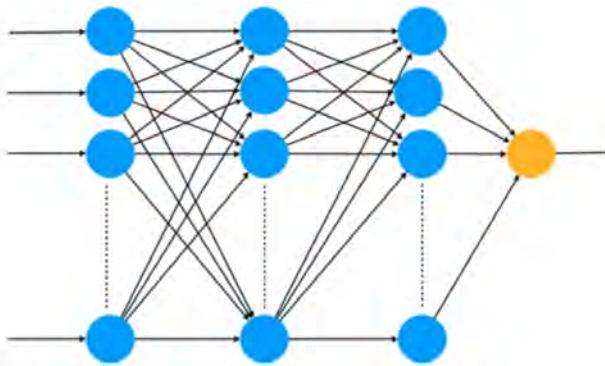
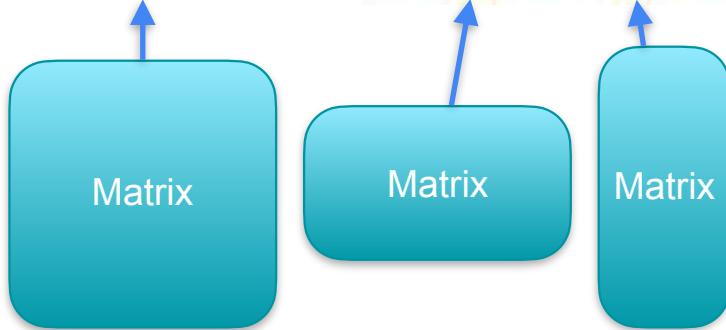
Yes, it's a face

# Neural networks - Matrix operations



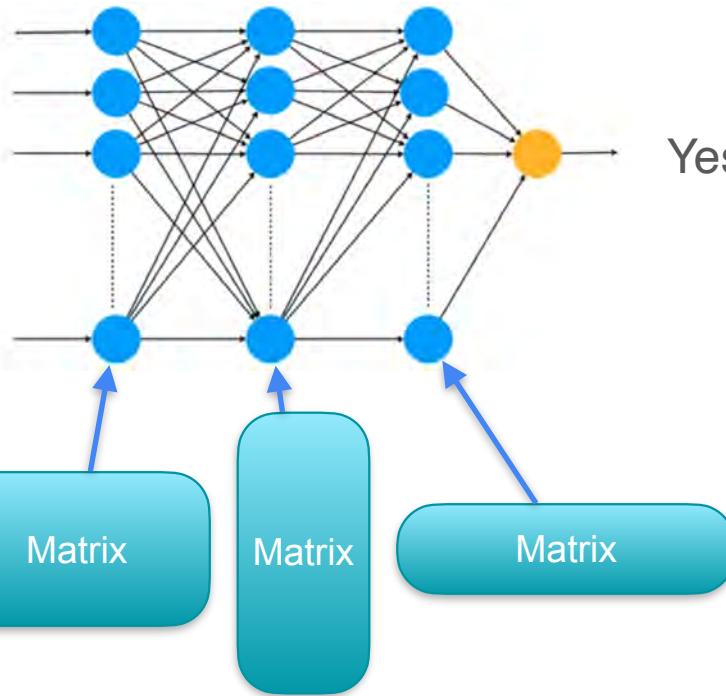
Yes, it's a face

# Neural networks - Matrix operations



Yes, it's a face

# Neural networks - Matrix operations



# Neural networks - image recognition

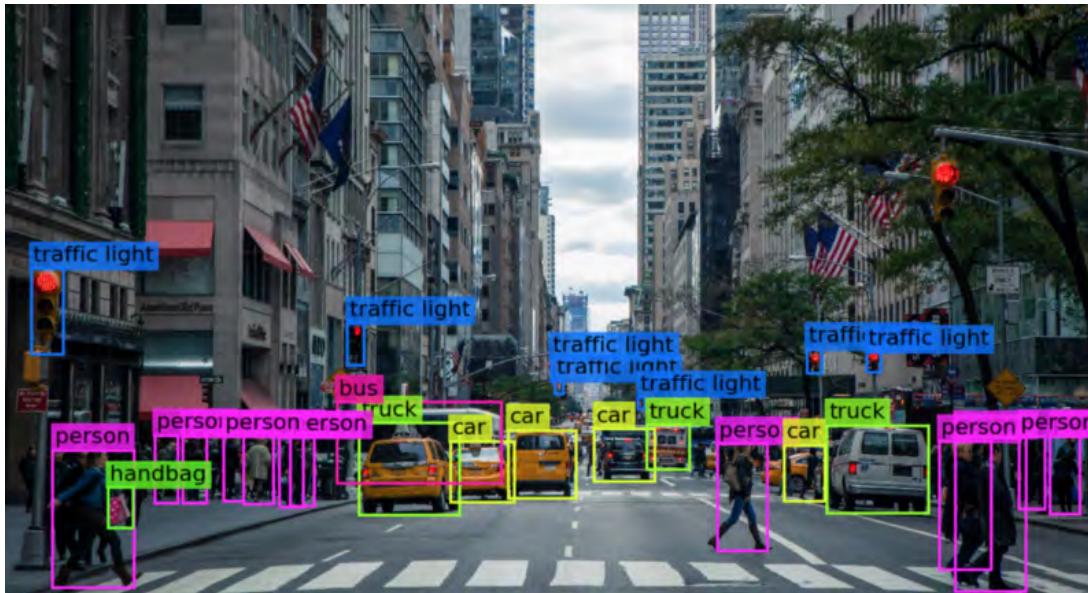


Image recognition in a busy street in New York.

- Image recognition: Getting the computer to see images and recognize what is on them.



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# System of Linear Equations

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## System of sentences

# Systems of sentences

# Systems of sentences

## System 1



The dog is **black**



The cat is **orange**

# Systems of sentences

System 1

 The dog is **black**  
 The cat is **orange**

System 2

 The dog is **black**  
 The dog is **black**

# Systems of sentences

## System 1

 The dog is **black**  
 The cat is **orange**

## System 2

 The dog is **black**  
 The dog is **black**

## System 3

 The dog is **black**  
 The dog is **white**

# Systems of sentences

## System 1

 The dog is **black**  
 The cat is **orange**

Complete

## System 2

 The dog is **black**  
 The dog is **black**

## System 3

 The dog is **black**  
 The dog is **white**

# Systems of sentences

## System 1

 The dog is **black**  
 The cat is **orange**

Complete

## System 2

 The dog is **black**  
 The dog is **black**

Redundant

## System 3

 The dog is **black**  
 The dog is **white**

# Systems of sentences

## System 1

 The dog is **black**  
 The cat is **orange**

Complete

## System 2

 The dog is **black**  
 The dog is **black**

Redundant

## System 3

 The dog is **black**  
 The dog is **white**

Contradictory

# Systems of sentences

## System 1

 The dog is **black**  
 The cat is **orange**

Complete

## System 2

 The dog is **black**  
 The dog is **black**

Redundant

**Singular**

## System 3

 The dog is **black**  
 The dog is **white**

Contradictory

**Singular**

# Systems of sentences

## System 1

 The dog is **black**  
 The cat is **orange**

Complete

**Non-singular**

## System 2

 The dog is **black**  
 The dog is **black**

Redundant

**Singular**

## System 3

 The dog is **black**  
 The dog is **white**

Contradictory

**Singular**

# Systems of sentences

System 1

 The dog is **black**  
 The cat is **orange**  
 The bird is **red**

System 2

 The dog is **black**  
 The dog is **black**  
 The bird is **red**

System 3

 The dog is **black**  
 The dog is **black**  
 The dog is **black**

System 4

 The dog is **black**  
 The dog is **white**  
 The bird is **red**

# Systems of sentences

## System 1

 The dog is **black**  
 The cat is **orange**  
 The bird is **red**

Complete

**Non-singular**

## System 2

 The dog is **black**  
 The dog is **black**  
 The bird is **red**

## System 3

 The dog is **black**  
 The dog is **black**  
 The dog is **black**

## System 4

 The dog is **black**  
 The dog is **white**  
 The bird is **red**

# Systems of sentences

## System 1

 The dog is **black**  
 The cat is **orange**  
 The bird is **red**

Complete

Non-singular

## System 2

 The dog is **black**  
 The dog is **black**  
 The bird is **red**

Redundant

Singular

## System 3

 The dog is **black**  
 The dog is **black**  
 The dog is **black**

## System 4

 The dog is **black**  
 The dog is **white**  
 The bird is **red**

# Systems of sentences

## System 1

 The dog is **black**  
 The cat is **orange**  
 The bird is **red**

Complete

**Non-singular**

## System 2

 The dog is **black**  
 The dog is **black**  
 The bird is **red**

Redundant

**Singular**

## System 3

 The dog is **black**  
 The dog is **black**  
 The dog is **black**

Redundant

**Singular**

## System 4

 The dog is **black**  
 The dog is **white**  
 The bird is **red**

# Systems of sentences

## System 1

 The dog is **black**  
 The cat is **orange**  
 The bird is **red**

Complete

**Non-singular**

## System 2

 The dog is **black**  
 The dog is **black**  
 The bird is **red**

Redundant

**Singular**

## System 3

 The dog is **black**  
 The dog is **black**  
 The dog is **black**

Redundant

**Singular**

## System 4

 The dog is **black**  
 The dog is **white**  
 The bird is **red**

Contradictory

**Singular**

# Quiz: Systems of sentences

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.

**Problem 1:**

What color is the bird?

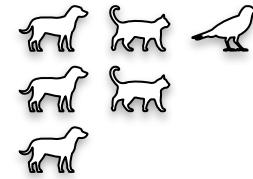
**Problem 2:**

Is this system singular or non-singular?

# Solution: Systems of information

Given this system:

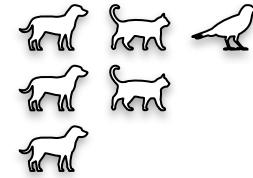
- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.

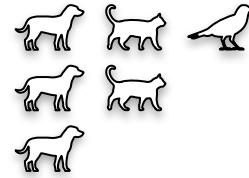


**Solution 1:**

# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



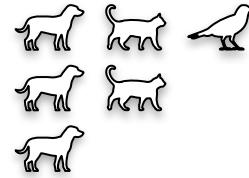
**Solution 1:**

The bird is red. 

# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



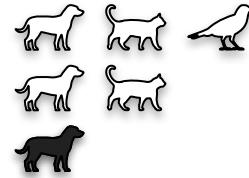
**Solution 1:**

The bird is red. 

# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



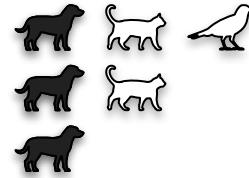
**Solution 1:**

The bird is red.

# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



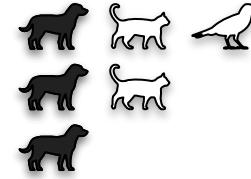
**Solution 1:**

The bird is red.

# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



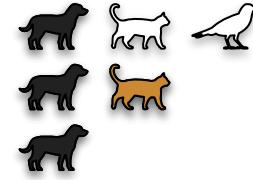
**Solution 1:**

The bird is red. 

# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



**Solution 1:**

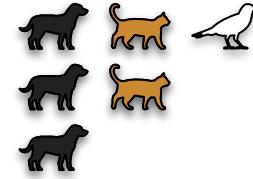
The bird is red.



# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



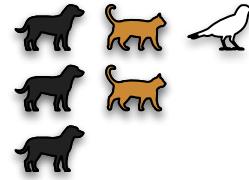
**Solution 1:**

The bird is red. 

# Solution: Systems of information

Given this system:

- 
- Between the dog, the cat, and the bird, one is red.
  - Between the dog and the cat, one is orange.
  - The dog is black.



**Solution 1:**

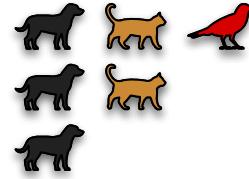
The bird is red. 

# Solution: Systems of information



Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



**Solution 1:**

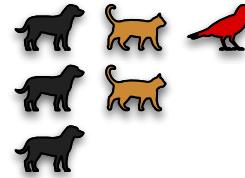
The bird is red.

# Solution: Systems of information



Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



**Solution 1:**

The bird is red.

**Solution 2:**

It is non-singular.



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# System of Linear Equations

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## System of equations

# Sentences → Equations

## Sentences

Between the dog and the cat, one is black.



# Sentences → Equations

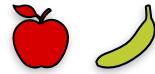
## Sentences

Between the dog and the cat, one is black.



## Sentences with numbers

The price of an apple and a banana is \$10.



# Sentences → Equations

## Sentences

Between the dog and the cat, one is black.



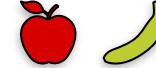
## Sentences with numbers

The price of an apple and a banana is \$10.



## Equations

$$a + b = 10$$



# Quiz: Systems of equations 1

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought an apple and two bananas and they cost \$12.

**Question:** How much does each fruit cost?

# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought an apple and two bananas and they cost \$12.
- **Solution:** An apple costs \$8, a banana costs \$2.

# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.
- **Solution:** An apple costs \$8, a banana costs \$2.

# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{🍎} + \text{🍌} + \text{🍌} = \$12$$

- **Solution:** An apple costs \$8, a banana costs \$2.

# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{🍎} + \text{🍌} + \boxed{\text{🍌}} = \$12$$

- **Solution:** An apple costs \$8, a banana costs \$2.

# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{🍎} + \text{🍌} + \boxed{\text{🍌}} = \$12$$

- **Solution:** An apple costs \$8, a banana costs \$2.

# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

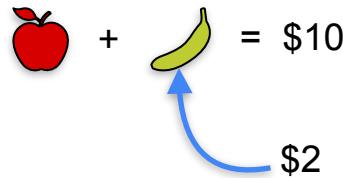
$$\text{🍎} + \text{🍌} + \boxed{\text{🍌}} = \$12$$

\$2

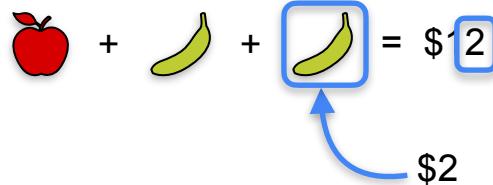
- **Solution:** An apple costs \$8, a banana costs \$2.

# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$


- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{apple} + \text{banana} + \text{banana} = \$12$$


- **Solution:** An apple costs \$8, a banana costs \$2.

# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

$\$8$    $+$    $= \$10$

$\$2$    $+$    $= \$10$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{apple} + \text{banana} + \text{banana} = \$12$$

  $+$    $+$    $= \$12$

$\$2$    $+$    $= \$12$

- **Solution:** An apple costs \$8, a banana costs \$2.

# Quiz: Systems of equations 2

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$20.

**Question:** How much does each fruit cost?

# Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$20.

# Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

# Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$\text{🍎🍎} + \text{🍌🍌} = \$20$$

# Solution: Systems of equations 2

- Day 1: You bought an apple and a banana and they cost \$10.

 $= \$10$ 

- Day 2: You bought two apples and two bananas and they cost \$20.

 $= \$20$ 

Same thing!!!



# Solution: Systems of equations 2

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- Day 2: You bought two apples and two bananas and they cost \$20.

$$2 \text{apple} + 2 \text{banana} = \$20$$

$$\begin{matrix} \text{apple} & \text{banana} \\ 8 & 2 \end{matrix}$$

Same thing!!!

# Solution: Systems of equations 2

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- Day 2: You bought two apples and two bananas and they cost \$20.

$$\text{🍎} + \text{🍌} = \$10$$
  
$$2\text{🍎} + 2\text{🍌} = \$20$$

Same thing!!!



8 2

5 5

# Solution: Systems of equations 2

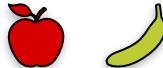
- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- Day 2: You bought two apples and two bananas and they cost \$20.

$$2\text{apple} + 2\text{banana} = \$20$$

Same thing!!!



8 2

5 5

8.3 1.7

# Solution: Systems of equations 2

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- Day 2: You bought two apples and two bananas and they cost \$20.

$$2\text{apple} + 2\text{banana} = \$20$$

Same thing!!!



8 2

5 5

8.3 1.7

0 10

# Solution: Systems of equations 2

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- Day 2: You bought two apples and two bananas and they cost \$20.

$$2\text{apple} + 2\text{banana} = \$20$$

Same thing!!!



8 2

5 5

8.3 1.7

0 10

Infinitely many solutions!

# Quiz: Systems of equations 3

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$24.

**Question:** How much does each fruit cost?

# Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$24.

# Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$24.

# Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10 \quad \rightarrow \quad \text{🍎🍎} + \text{🍌🍌} = \$20$$

- **Day 2:** You bought two apples and two bananas and they cost \$24.

# Solution: Systems of equations 3

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10 \quad \rightarrow \quad \text{apple} + \text{apple} + \text{banana} + \text{banana} = \$20$$

- Day 2: You bought two apples and two bananas and they cost \$24.

$$\text{apple} + \text{apple} + \text{banana} + \text{banana} = \$24$$

# Solution: Systems of equations 3

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10 \quad \rightarrow \quad \text{apple} + \text{apple} + \text{banana} + \text{banana} = \$20$$

- Day 2: You bought two apples and two bananas and they cost \$24.

$$\text{apple} + \text{apple} + \text{banana} + \text{banana} = \$24$$

Contradiction!

# Solution: Systems of equations 3

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10 \quad \rightarrow \quad \text{apple} + \text{apple} + \text{banana} + \text{banana} = \$20$$

- Day 2: You bought two apples and two bananas and they cost \$24.

$$\text{apple} + \text{apple} + \text{banana} + \text{banana} = \$24$$

Contradiction!

No solutions!

# Systems of equations

# Systems of equations

## System 1

-    $a + b = 10$
-    $a + 2b = 12$

# Systems of equations

## System 1

-    $a + b = 10$
-    $a + 2b = 12$

## System 2

-    $a + b = 10$
-    $2a + 2b = 20$

# Systems of equations

## System 1

-    $a + b = 10$
-    $a + 2b = 12$

## System 2

-    $a + b = 10$
-    $2a + 2b = 20$

## System 3

-    $a + b = 10$
-    $2a + 2b = 24$

# Systems of equations

## System 1

-    $a + b = 10$
-    $a + 2b = 12$

## System 2

-    $a + b = 10$
-    $2a + 2b = 20$

## System 3

-    $a + b = 10$
-    $2a + 2b = 24$

**Unique solution:**

# Systems of equations

## System 1

-   $a + \text{banana icon} = 10$
-   $a + 2\text{banana icon} = 12$

## System 2

-   $a + \text{banana icon} = 10$
-   $2a + 2\text{banana icon} = 20$

## System 3

-   $a + \text{banana icon} = 10$
-   $2a + 2\text{banana icon} = 24$

**Unique solution:**

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

# Systems of equations

## System 1

-   $a + \text{banana icon}$  = 10
-   $a + 2\text{banana icon}$  = 12

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

## System 2

-   $a + \text{banana icon}$  = 10
-   $2a + 2\text{banana icon}$  = 20

## System 3

-   $a + \text{banana icon}$  = 10
-   $2a + 2\text{banana icon}$  = 24

Complete

# Systems of equations

## System 1

-   $a + \text{banana icon}$  = 10
-   $a + 2\text{banana icon}$  = 12

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

## System 2

-   $a + \text{banana icon}$  = 10
-   $2a + 2\text{banana icon}$  = 20

## System 3

-   $a + \text{banana icon}$  = 10
-   $2a + 2\text{banana icon}$  = 24

Complete

Non-singular

# Systems of equations

## System 1

-   $a + \text{banana icon} = 10$
-   $a + 2\text{banana icon} = 12$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

Complete

Non-singular

## System 2

-   $a + \text{banana icon} = 10$
-   $2a + 2\text{banana icon} = 20$

Infinite solutions

## System 3

-   $a + \text{banana icon} = 10$
-   $2a + 2\text{banana icon} = 24$

# Systems of equations

## System 1

-   $a + \text{banana icon} = 10$
-   $a + 2\text{banana icon} = 12$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

Complete

Non-singular

## System 2

-   $a + \text{banana icon} = 10$
-   $2a + 2\text{banana icon} = 20$

Infinite solutions

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

## System 3

-   $a + \text{banana icon} = 10$
-   $2a + 2\text{banana icon} = 24$

# Systems of equations

## System 1

-   $a + \text{banana icon} = 10$
-   $a + 2\text{banana icon} = 12$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

## System 2

-   $a + \text{banana icon} = 10$
-   $2a + 2\text{banana icon} = 20$

Infinite solutions

$$\text{apple icon } a = 8, 7$$

$$\text{banana icon } b = 2, 3$$

## System 3

-   $a + \text{banana icon} = 10$
-   $2a + 2\text{banana icon} = 24$

Complete

Non-singular

# Systems of equations

## System 1

-   $a + \text{banana icon}$  = 10
-   $a + 2\text{banana icon}$  = 12

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

## System 2

-   $a + \text{banana icon}$  = 10
-   $2a + 2\text{banana icon}$  = 20

Infinite solutions

$$\text{apple icon } a = 8, 7, 6$$

$$\text{banana icon } b = 2, 3, 4$$

## System 3

-   $a + \text{banana icon}$  = 10
-   $2a + 2\text{banana icon}$  = 24

Complete

Non-singular

# Systems of equations

## System 1

-   $a + \text{banana icon} = 10$
-   $a + 2\text{banana icon} = 12$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

Complete

Non-singular

## System 2

-   $a + \text{banana icon} = 10$
-   $2a + 2\text{banana icon} = 20$

Infinite solutions

$$\begin{aligned}\text{apple icon } a &= 8, 7, 6, \dots \\ \text{banana icon } b &= 2, 3, 4\end{aligned}$$

## System 3

-   $a + \text{banana icon} = 10$
-   $2a + 2\text{banana icon} = 24$

# Systems of equations

## System 1

-    $a + b = 10$
-    $a + 2b = 12$

Unique solution:

$$\text{apple} \ a = 8$$

$$\text{banana} \ b = 2$$

Complete

Non-singular

## System 2

-    $a + b = 10$
-    $2a + 2b = 20$

Infinite solutions

$$\text{apple} \ a = 8, 7, 6, \dots$$

$$\text{banana} \ b = 2, 3, 4$$

## System 3

-    $a + b = 10$
-    $2a + 2b = 24$

# Systems of equations

## System 1

-    $a + b = 10$
-    $a + 2b = 12$

Unique solution:

$$\text{apple} \ a = 8$$

$$\text{banana} \ b = 2$$

Complete

Non-singular

## System 2

-    $a + b = 10$
-    $2a + 2b = 20$

Infinite solutions

$$\text{apple} \ a = 8, 7, 6, \dots$$

$$\text{banana} \ b = 2, 3, 4$$

Redundant

Singular

## System 3

-    $a + b = 10$
-    $2a + 2b = 24$

# Systems of equations

## System 1

-   $a + \text{banana icon} = 10$
-   $a + 2\text{banana icon} = 12$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

Complete

Non-singular

## System 2

-   $a + \text{banana icon} = 10$
-   $2a + 2\text{banana icon} = 20$

Infinite solutions

$$\begin{aligned}\text{apple icon } a &= 8, 7, 6, \dots \\ \text{banana icon } b &= 2, 3, 4\end{aligned}$$

Redundant

Singular

## System 3

-   $a + \text{banana icon} = 10$
-   $2a + 2\text{banana icon} = 24$

No solution

# Systems of equations

## System 1

-    $a + b = 10$
-    $a + 2b = 12$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

Complete

Non-singular

## System 2

-    $a + b = 10$
-    $2a + 2b = 20$

Infinite solutions

$$\text{apple icon } a = 8, 7, 6, \dots$$
$$\text{banana icon } b = 2, 3, 4$$

Redundant

Singular

## System 3

-    $a + b = 10$
-    $2a + 2b = 24$

No solution

# Systems of equations

## System 1

-    $a + b = 10$
-    $a + 2b = 12$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

Complete

Non-singular

## System 2

-    $a + b = 10$
-    $2a + 2b = 20$

Infinite solutions

$$\text{apple icon } a = 8, 7, 6, \dots$$

$$\text{banana icon } b = 2, 3, 4$$

Redundant

Singular

## System 3

-    $a + b = 10$
-    $2a + 2b = 24$

No solution

Contradictory

Singular

# What is a linear equation?

**Linear**

**Non-linear**

# What is a linear equation?

**Linear**

$$a + b = 10$$

**Non-linear**

# What is a linear equation?

## Linear

$$a + b = 10$$

$$2a + 3b = 15$$

## Non-linear

# What is a linear equation?

## Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

## Non-linear

# What is a linear equation?

**Linear**

$$a + b = 10$$

$$2a + 3b = 15$$

**Non-linear**

$$3.4a - 48.99b + 2c = 122.5$$

Numbers

# What is a linear equation?

## Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers

## Non-linear

$$a^2 + b^2 = 10$$

# What is a linear equation?

## Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers

## Non-linear

$$a^2 + b^2 = 10$$

$$\sin(a) + b^5 = 15$$

# What is a linear equation?

## Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers

## Non-linear

$$a^2 + b^2 = 10$$

$$\sin(a) + b^5 = 15$$

$$2^a - 3^b = 0$$

# What is a linear equation?

## Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers

## Non-linear

$$a^2 + b^2 = 10$$

$$\sin(a) + b^5 = 15$$

$$2^a - 3^b = 0$$

$$ab^2 + \frac{b}{a} - \frac{3}{b} - \log(c) = 4^a$$



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# System of Linear Equations

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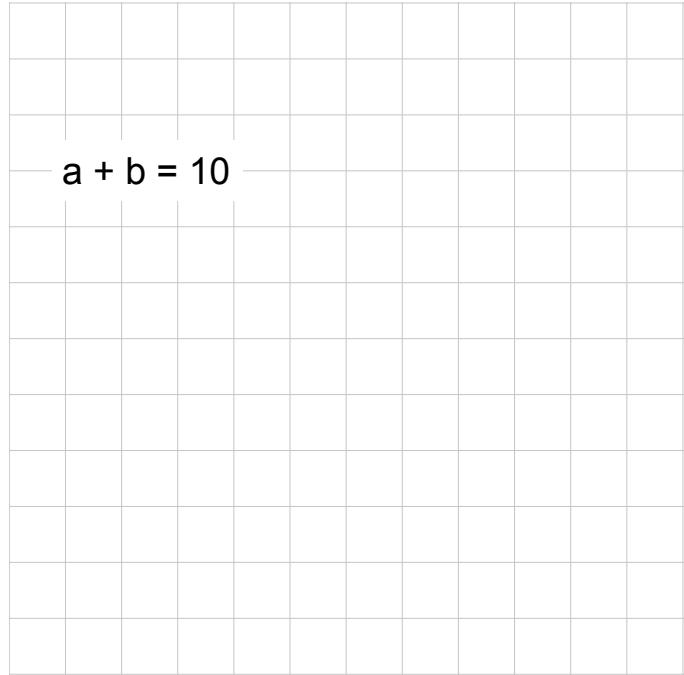
**System of equations as lines**

# Linear equation → line

# Linear equation → line

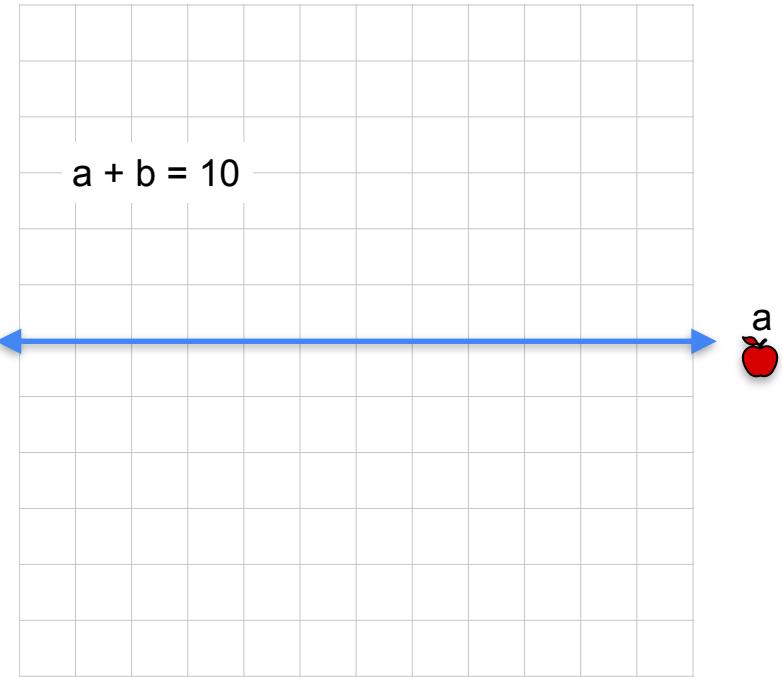
$$a + b = 10$$

# Linear equation → line

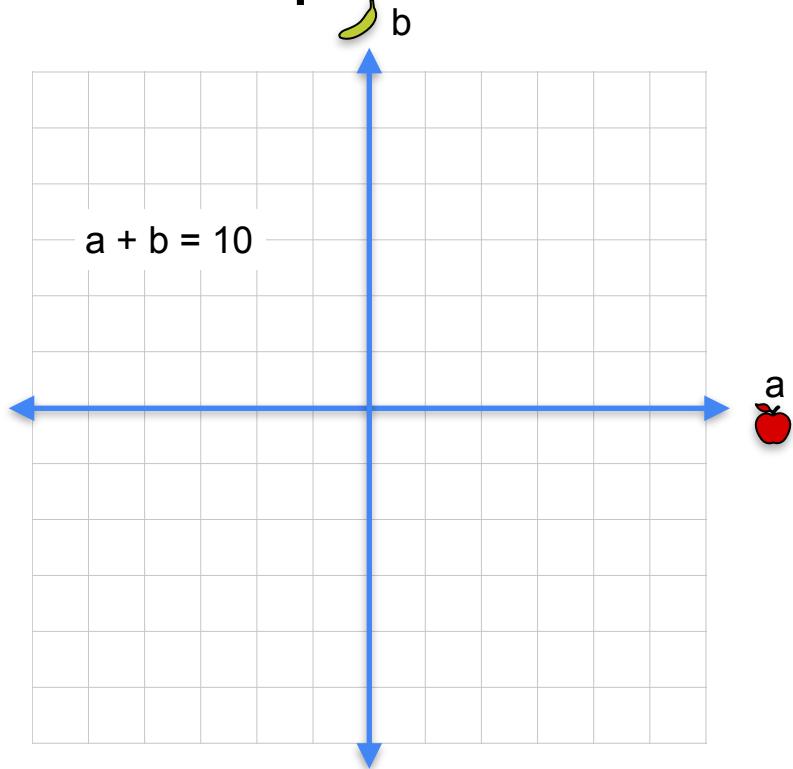


$$a + b = 10$$

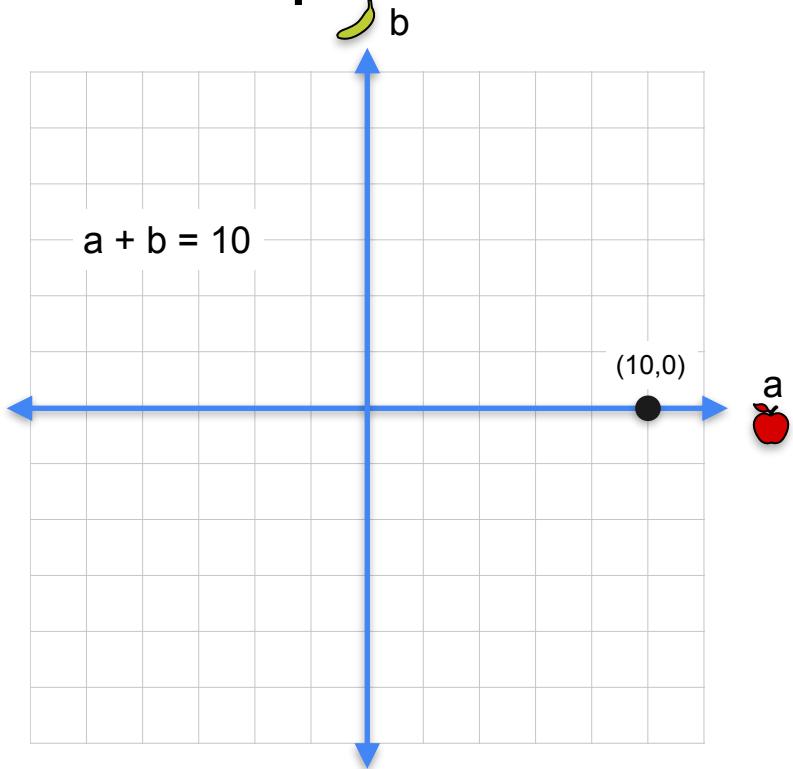
# Linear equation $\rightarrow$ line



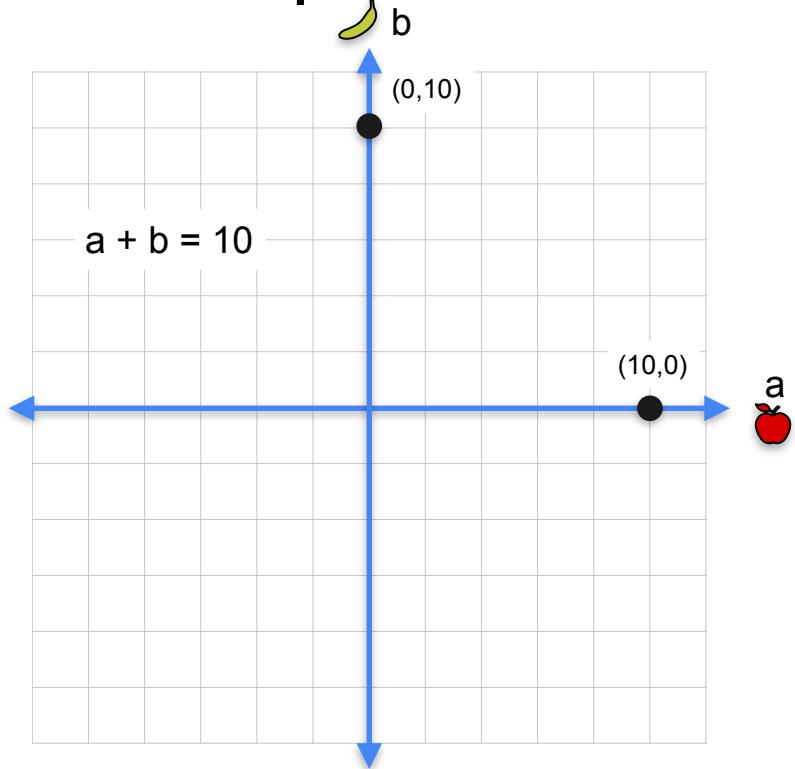
# Linear equation $\rightarrow$ line



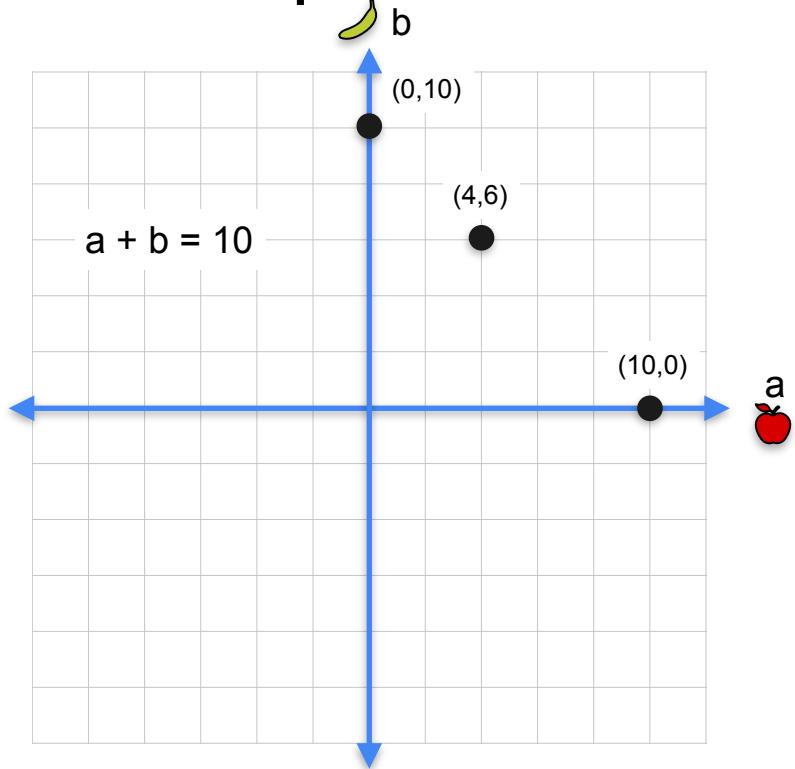
# Linear equation $\rightarrow$ line



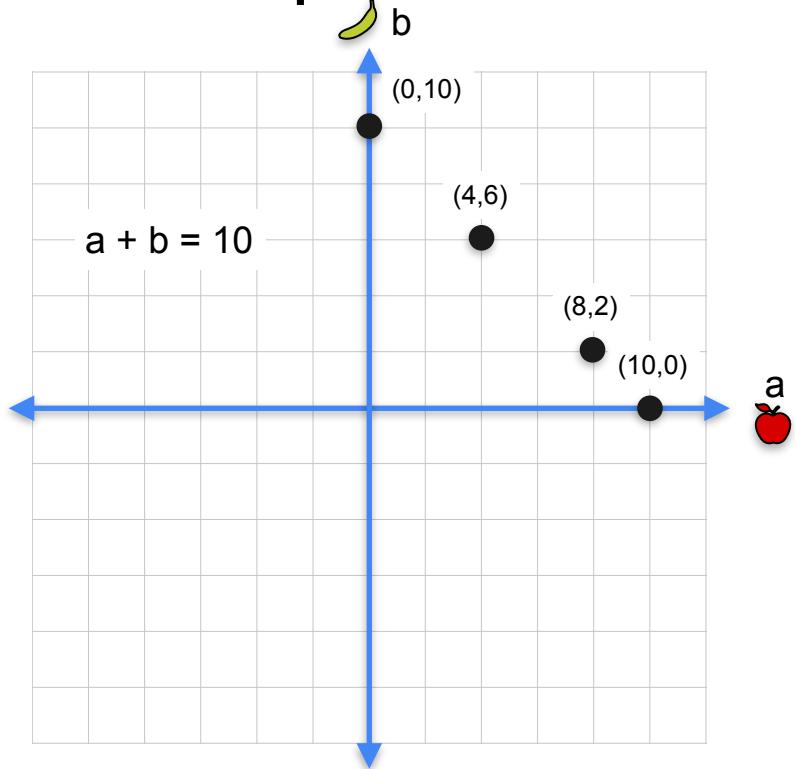
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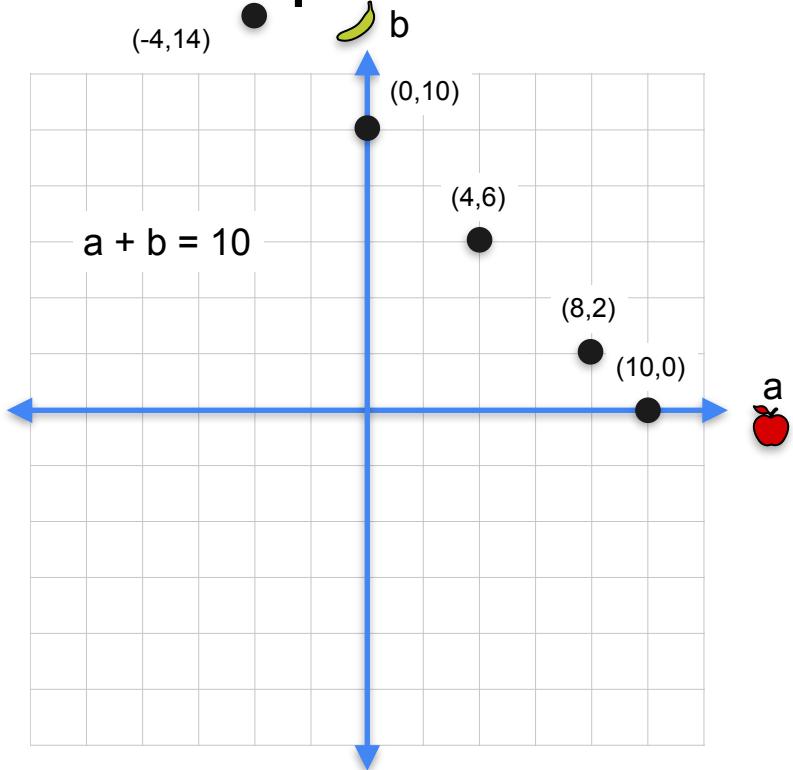
# Linear equation $\rightarrow$ line



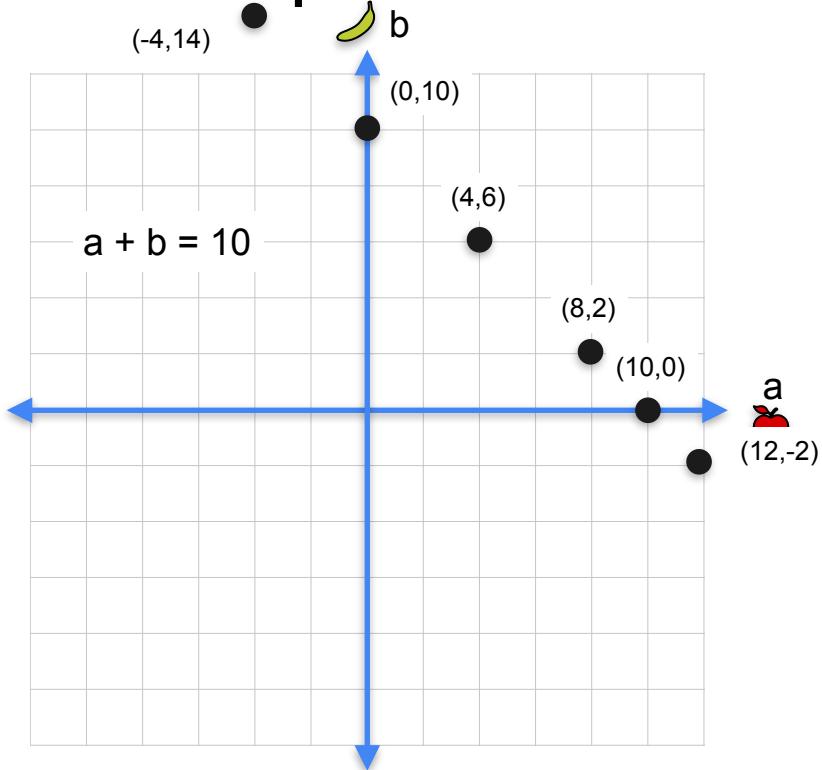
# Linear equation $\rightarrow$ line



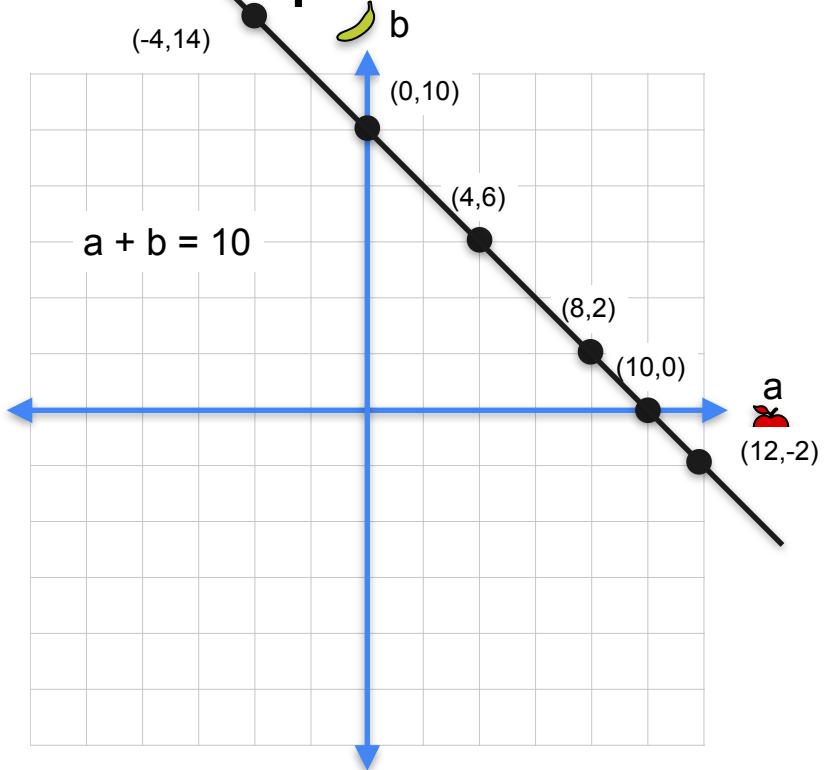
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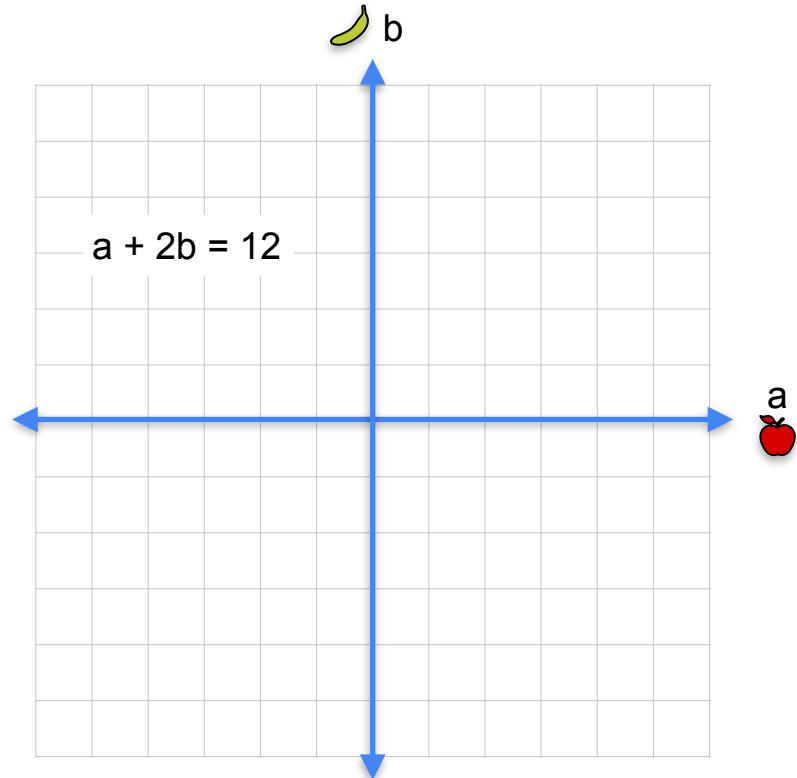
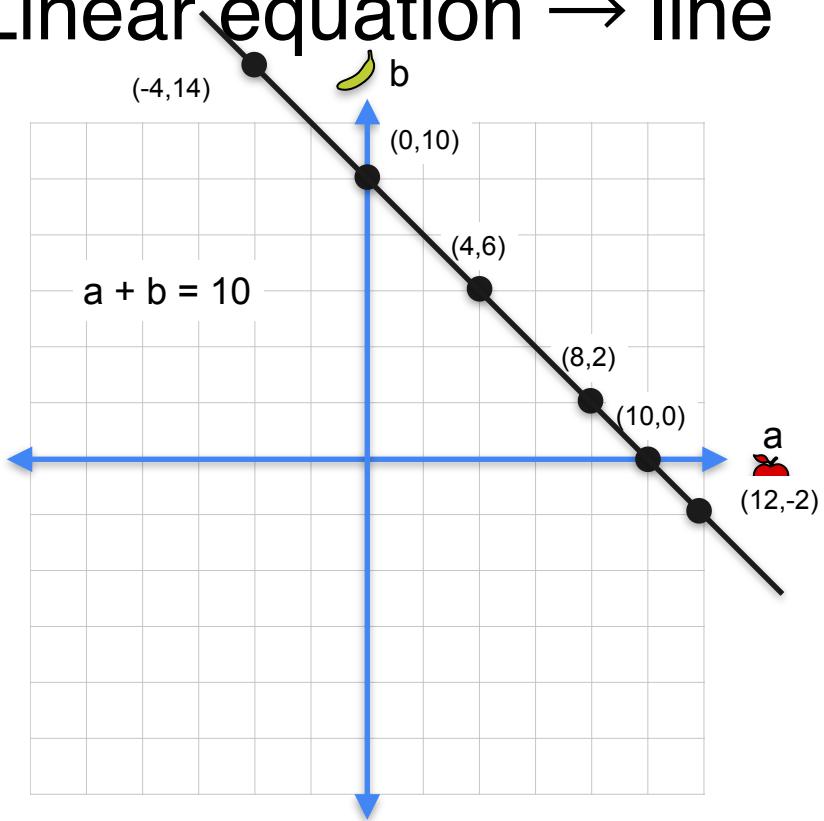
# Linear equation $\rightarrow$ line



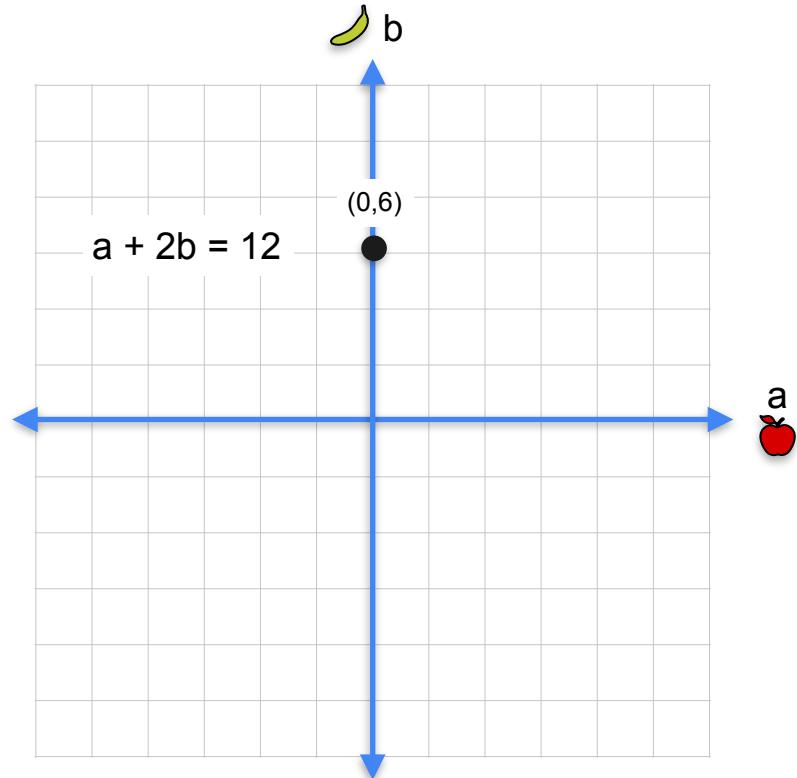
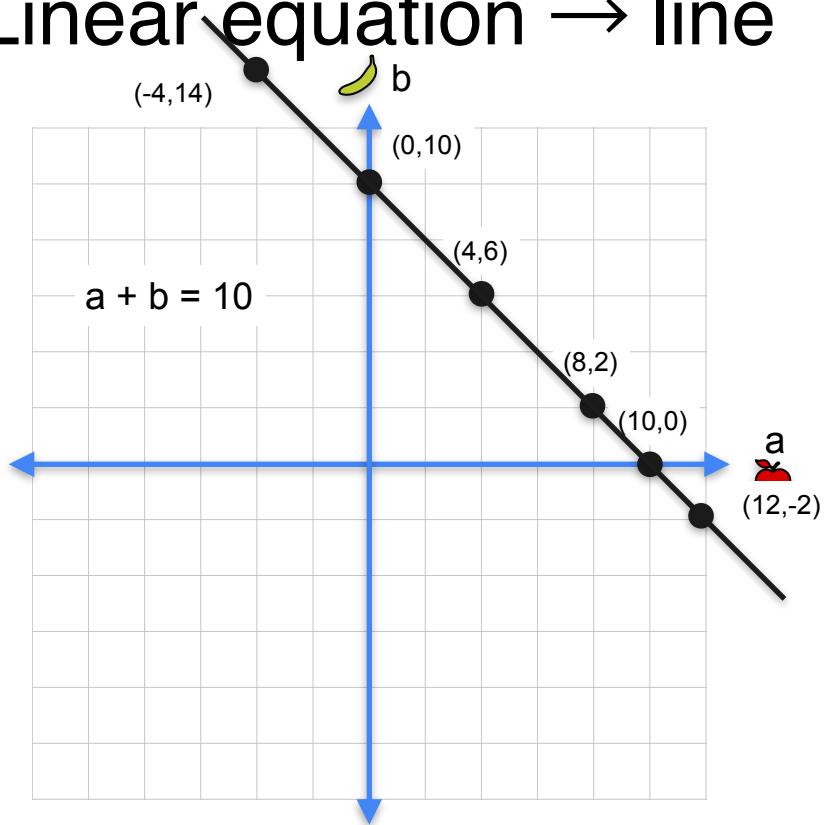
# Linear equation $\rightarrow$ line



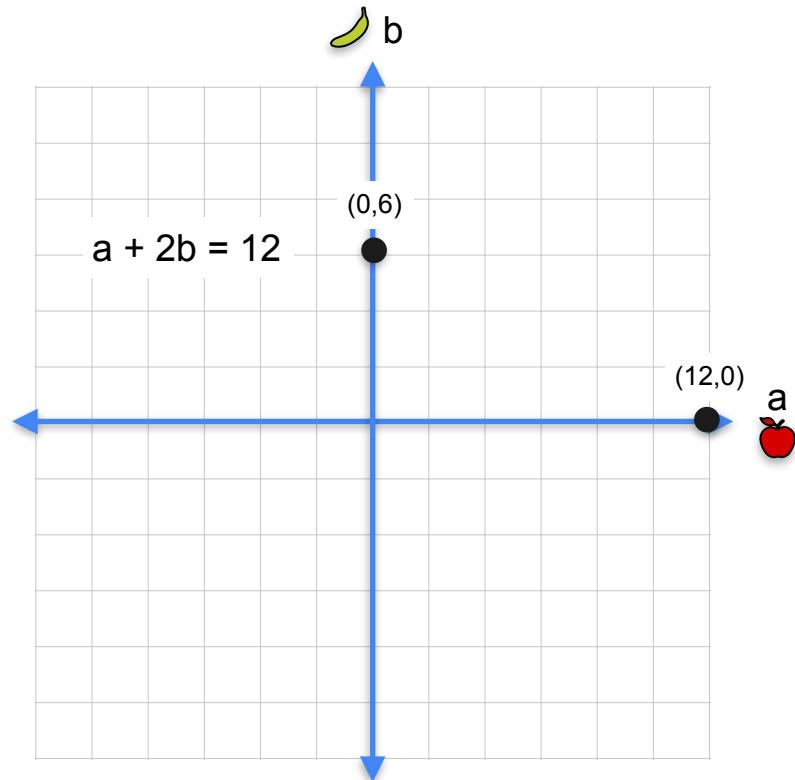
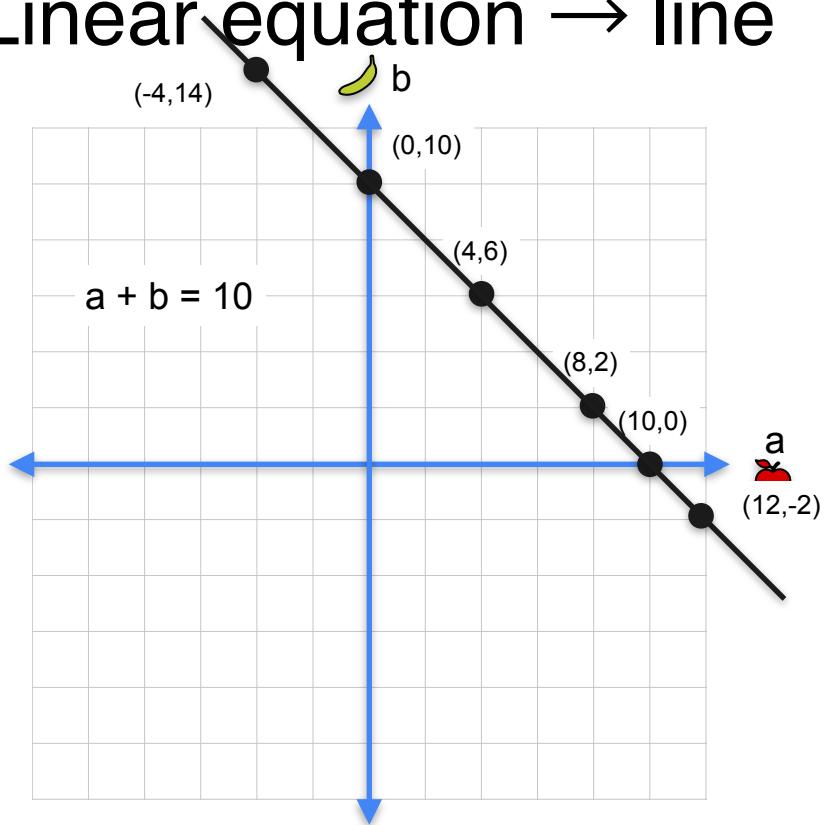
# Linear equation $\rightarrow$ line



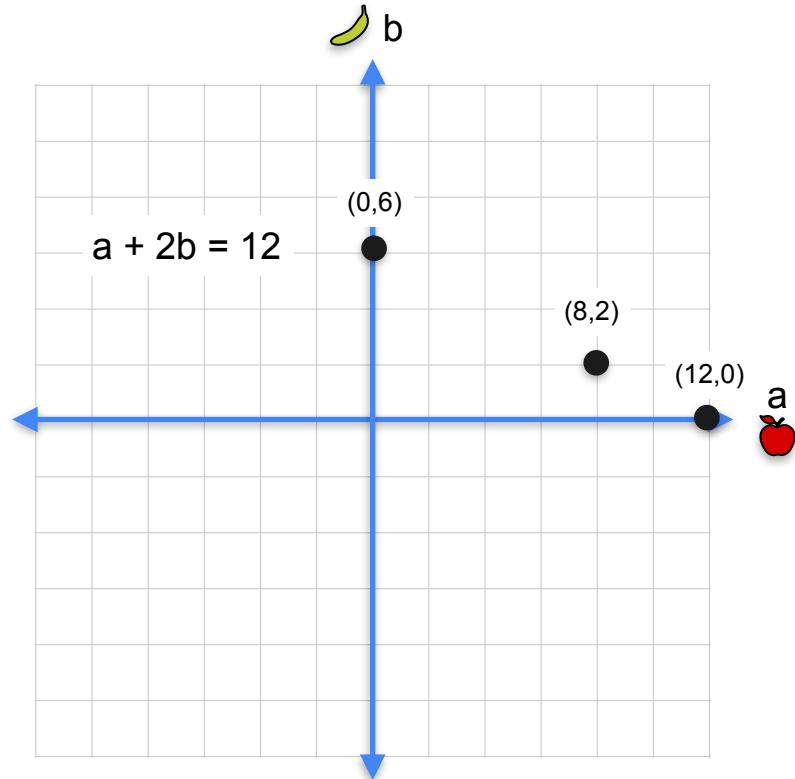
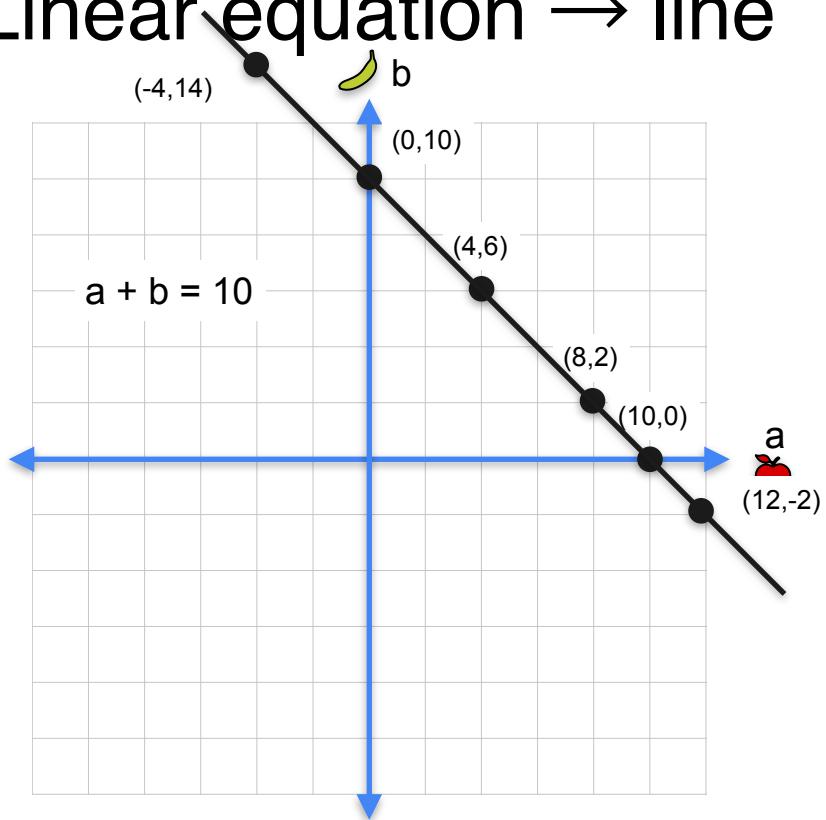
# Linear equation $\rightarrow$ line



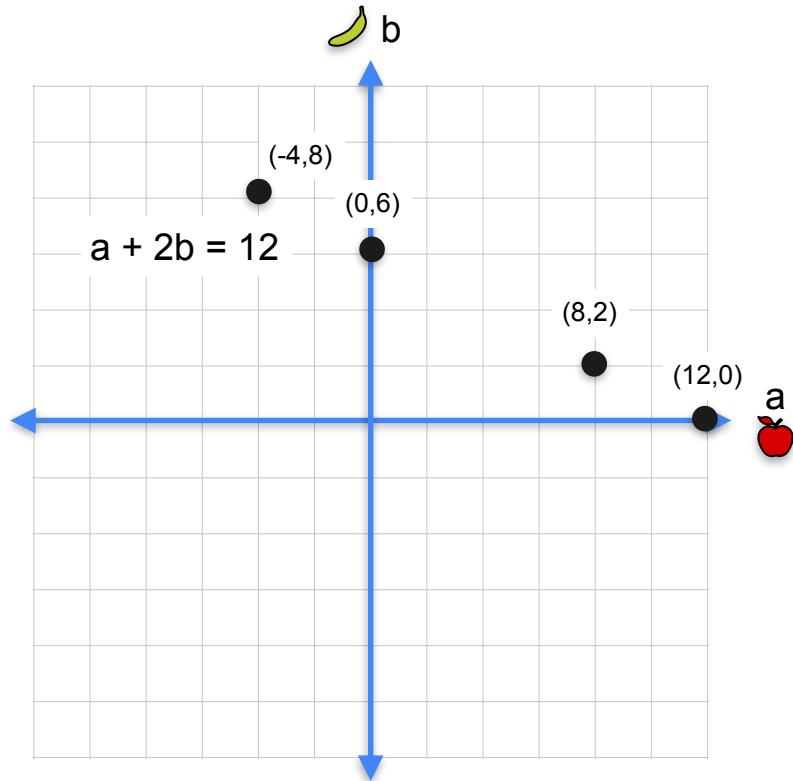
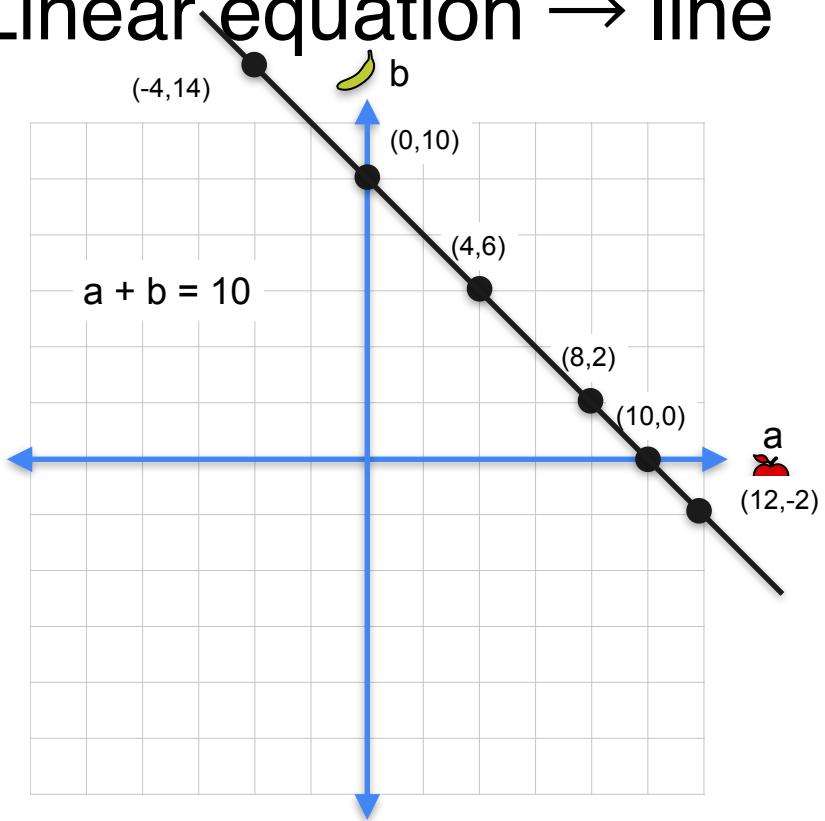
# Linear equation $\rightarrow$ line



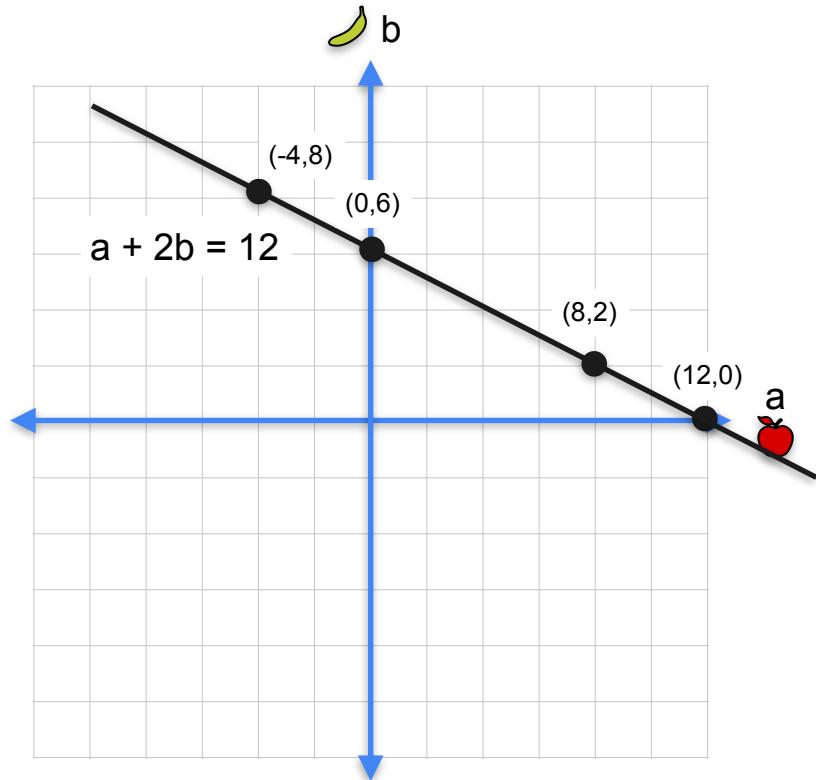
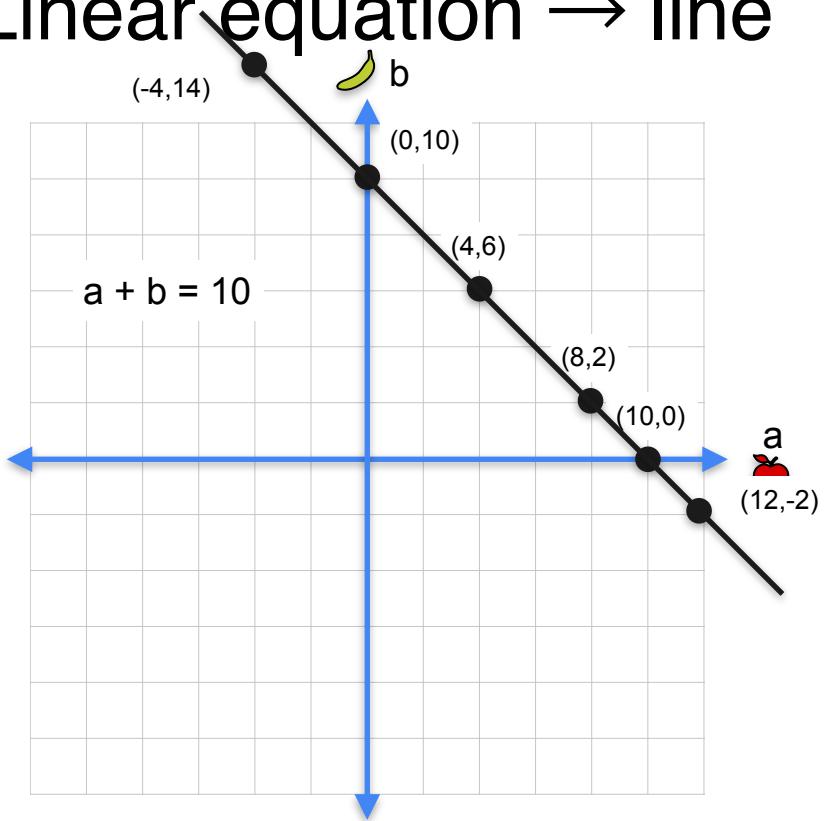
# Linear equation $\rightarrow$ line



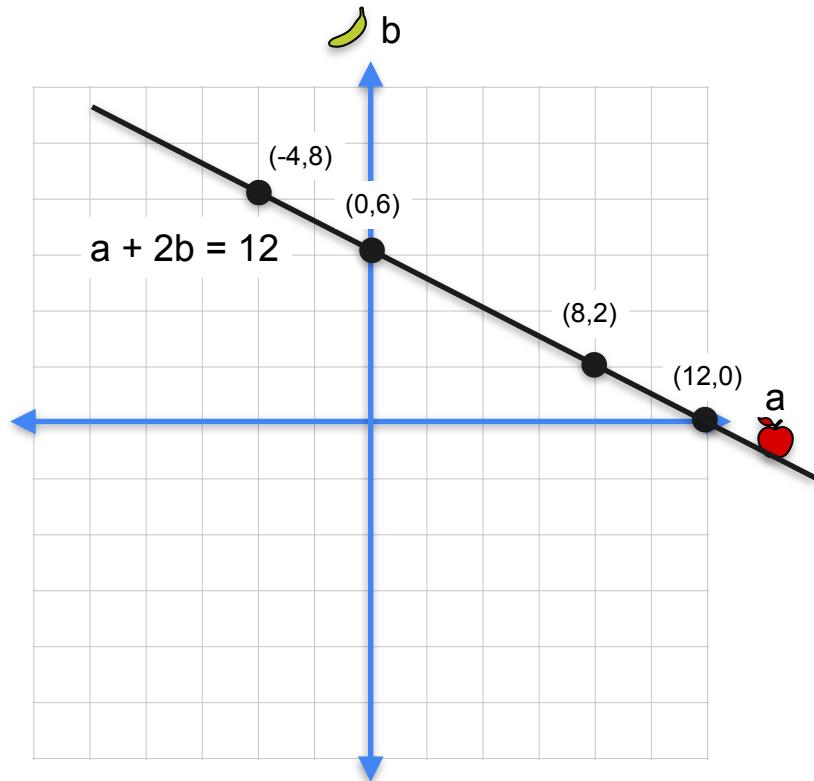
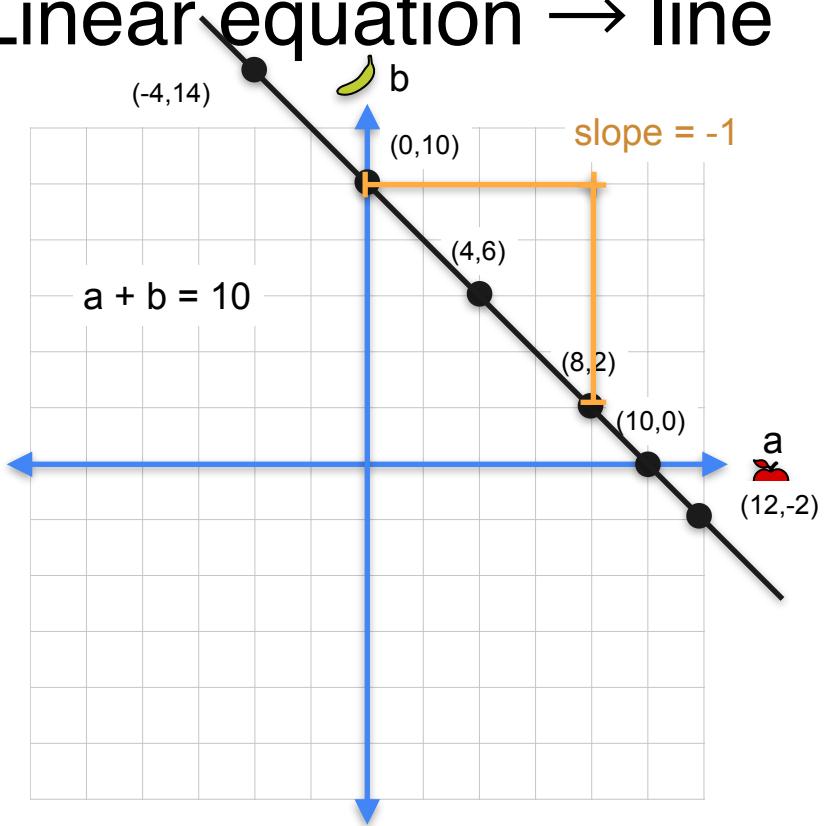
# Linear equation $\rightarrow$ line



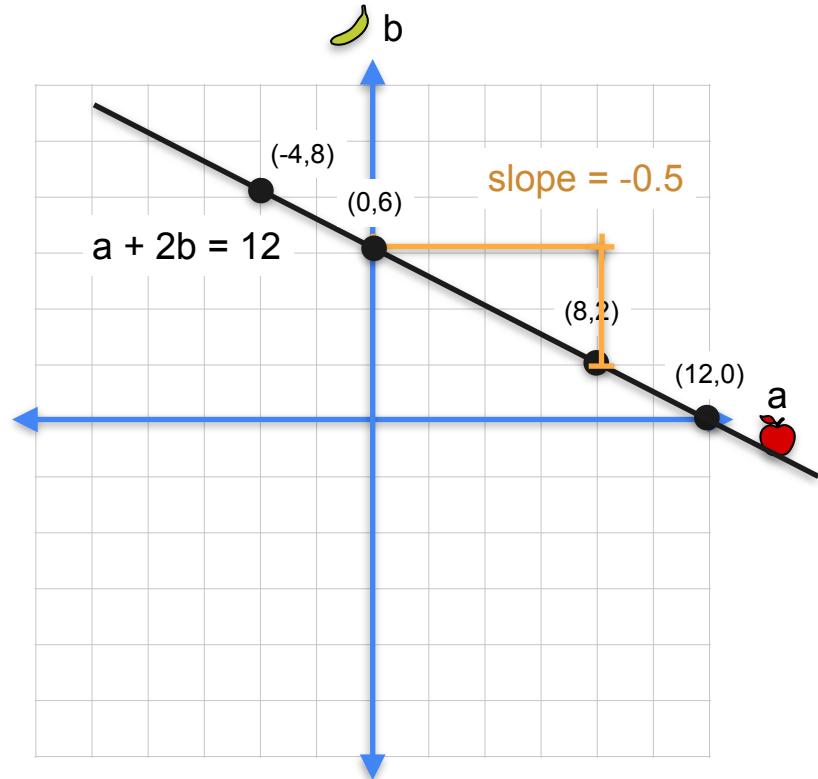
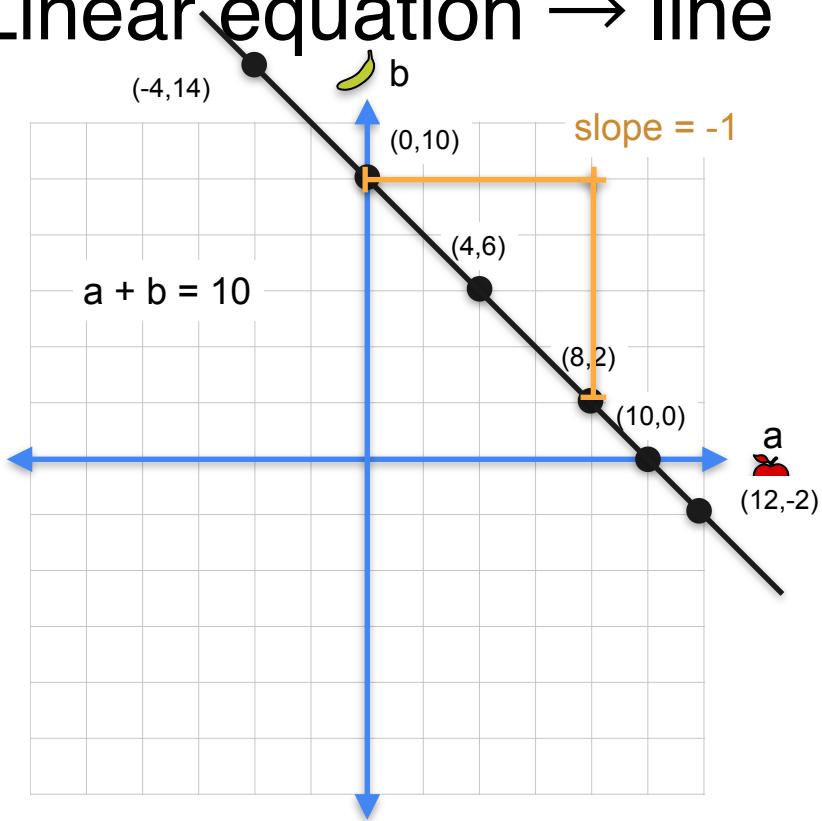
# Linear equation $\rightarrow$ line



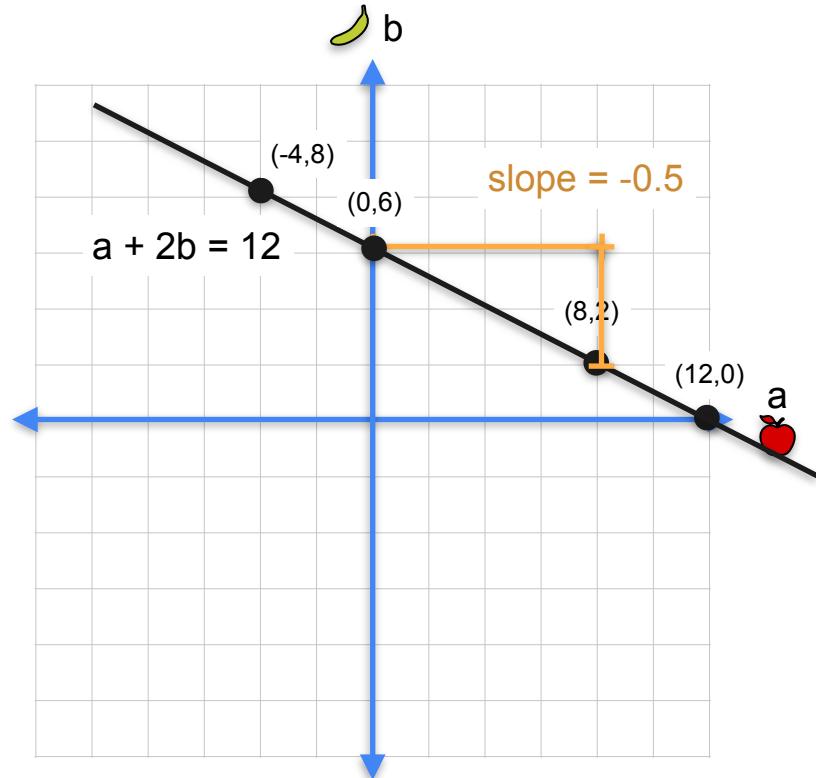
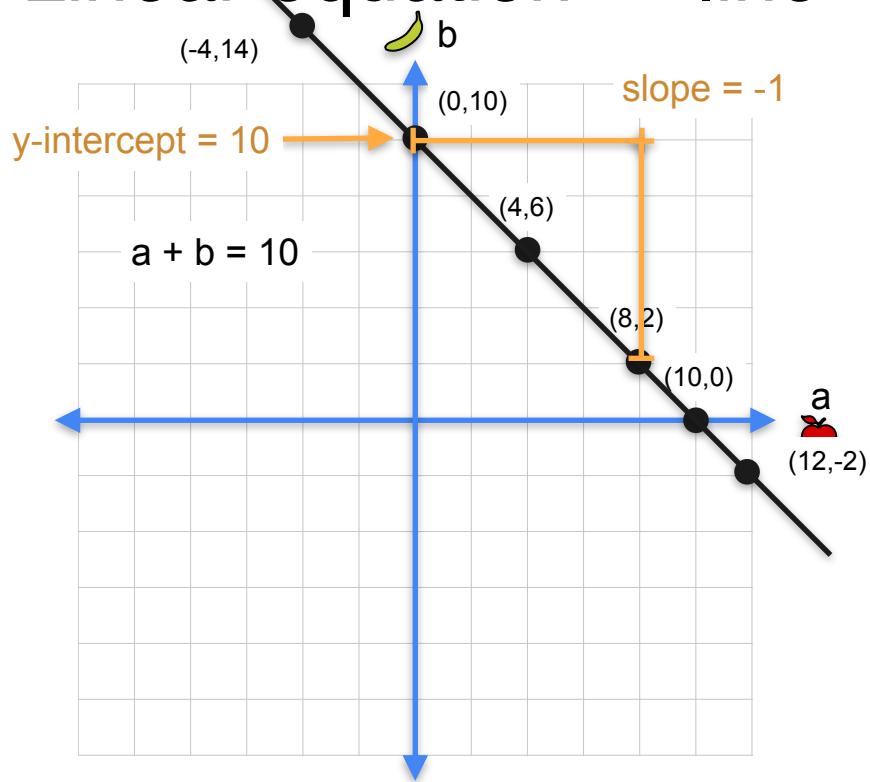
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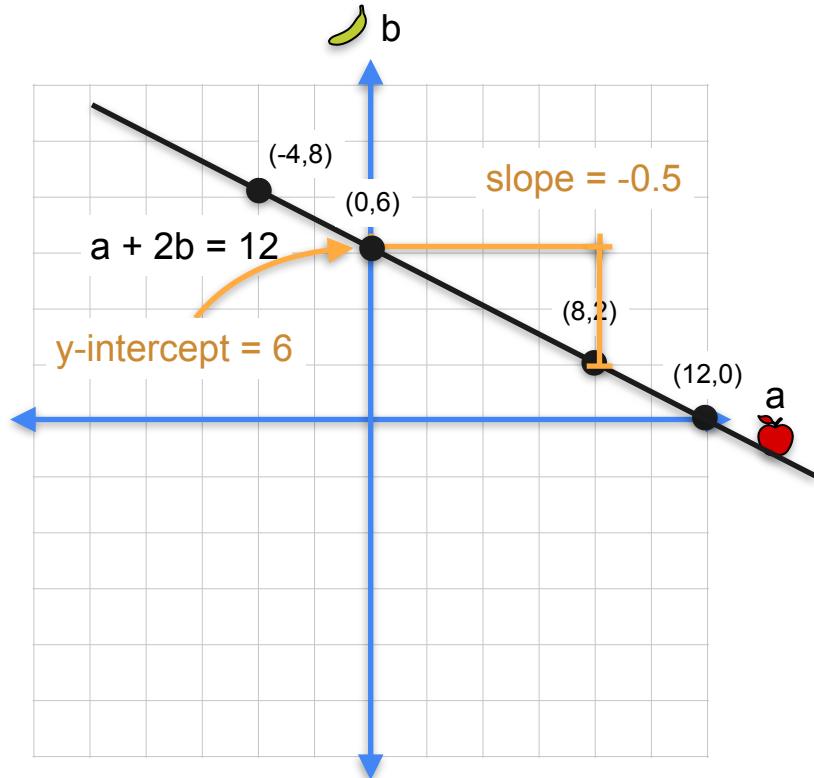
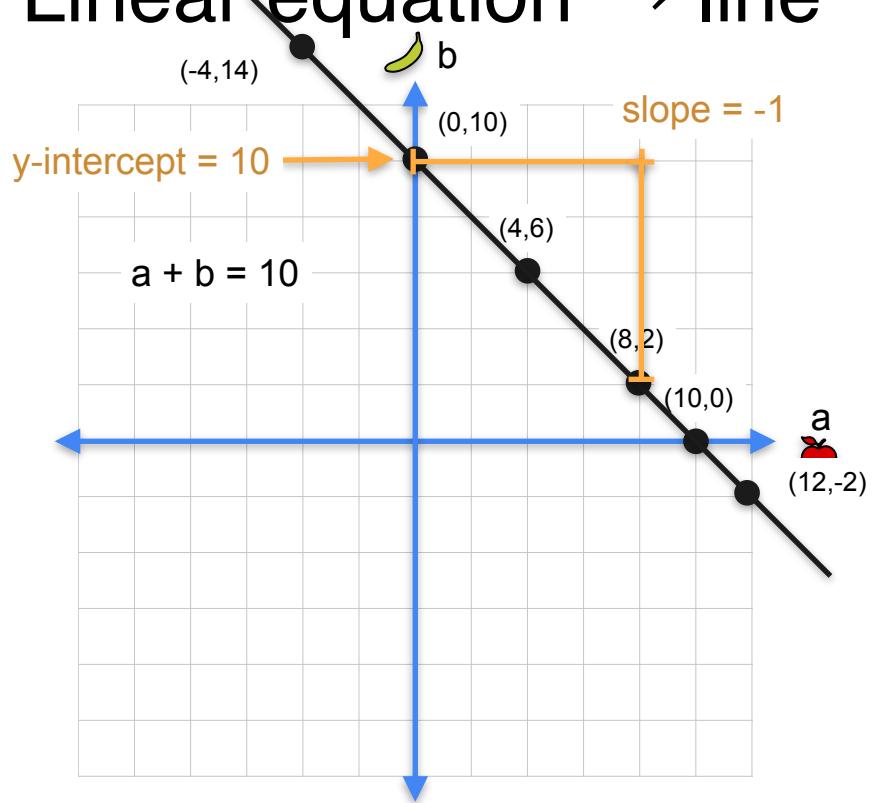
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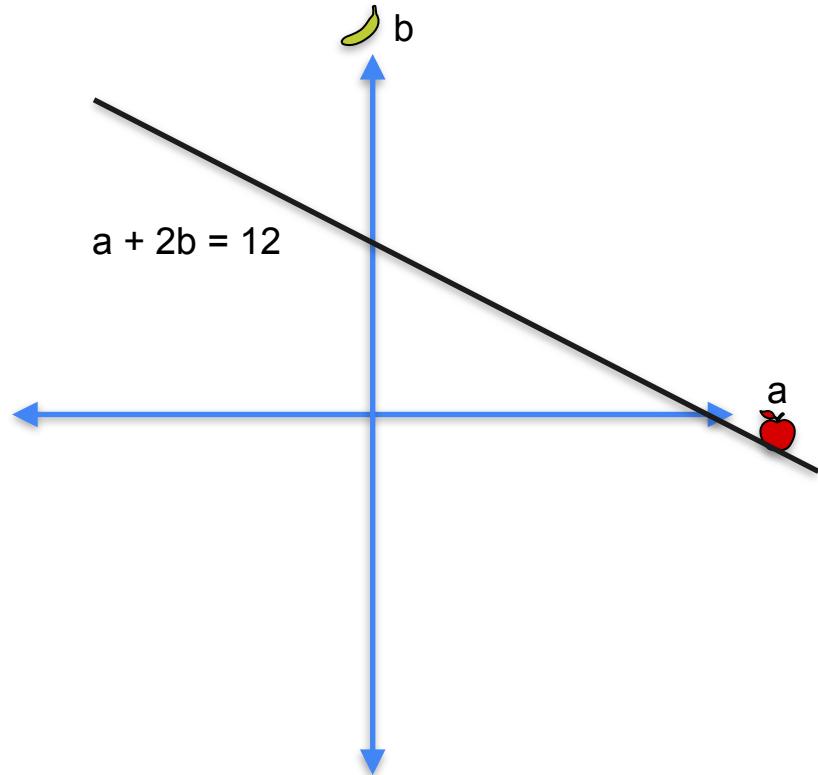
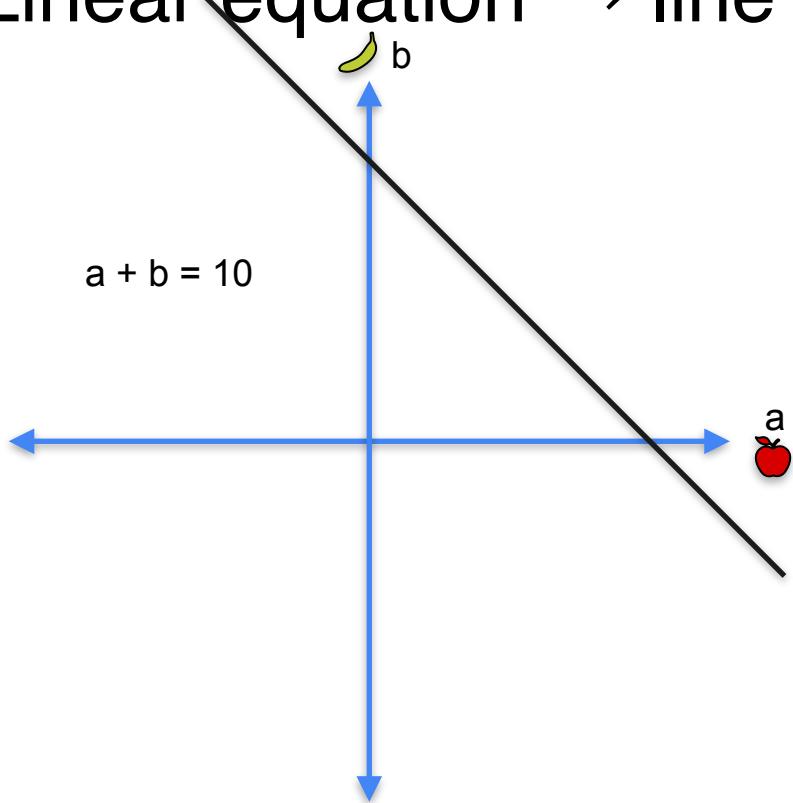
# Linear equation $\rightarrow$ line



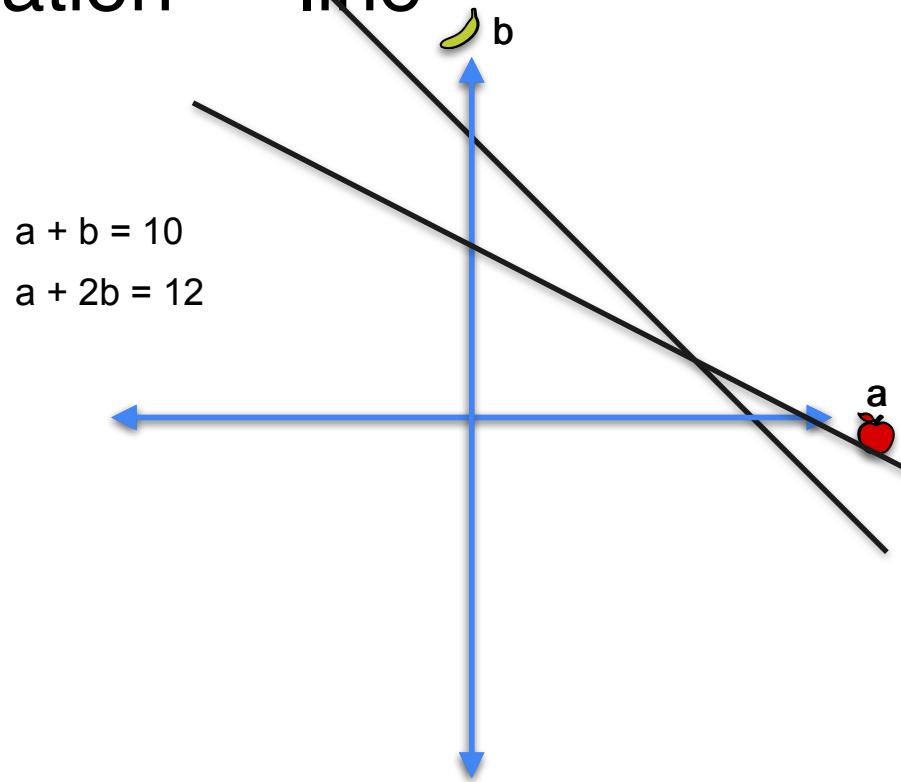
# Linear equation $\rightarrow$ line



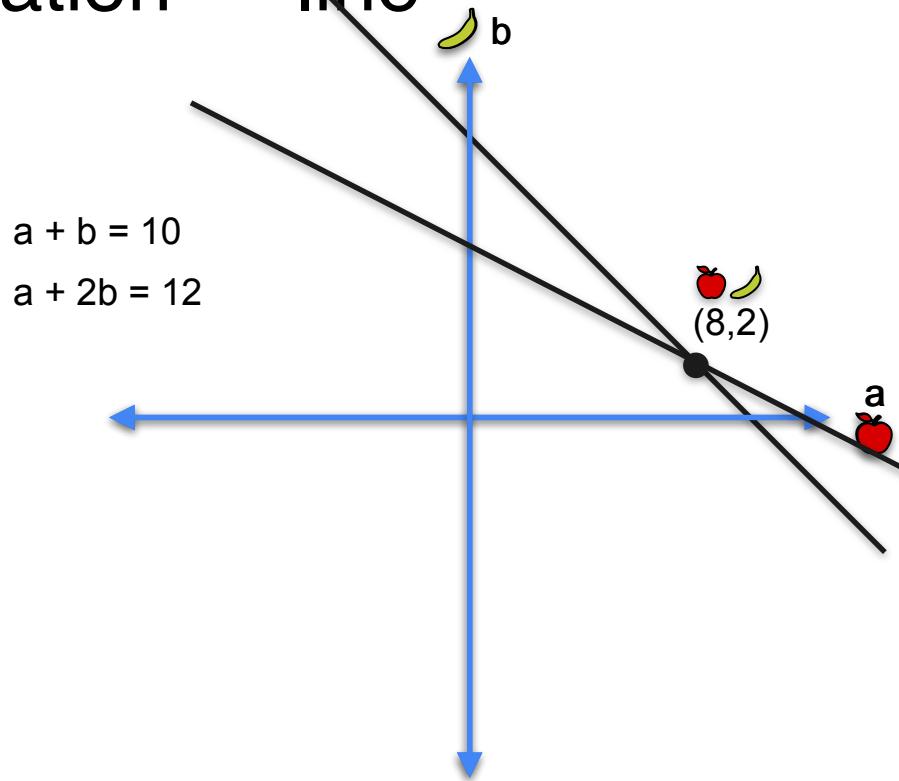
# Linear equation → line



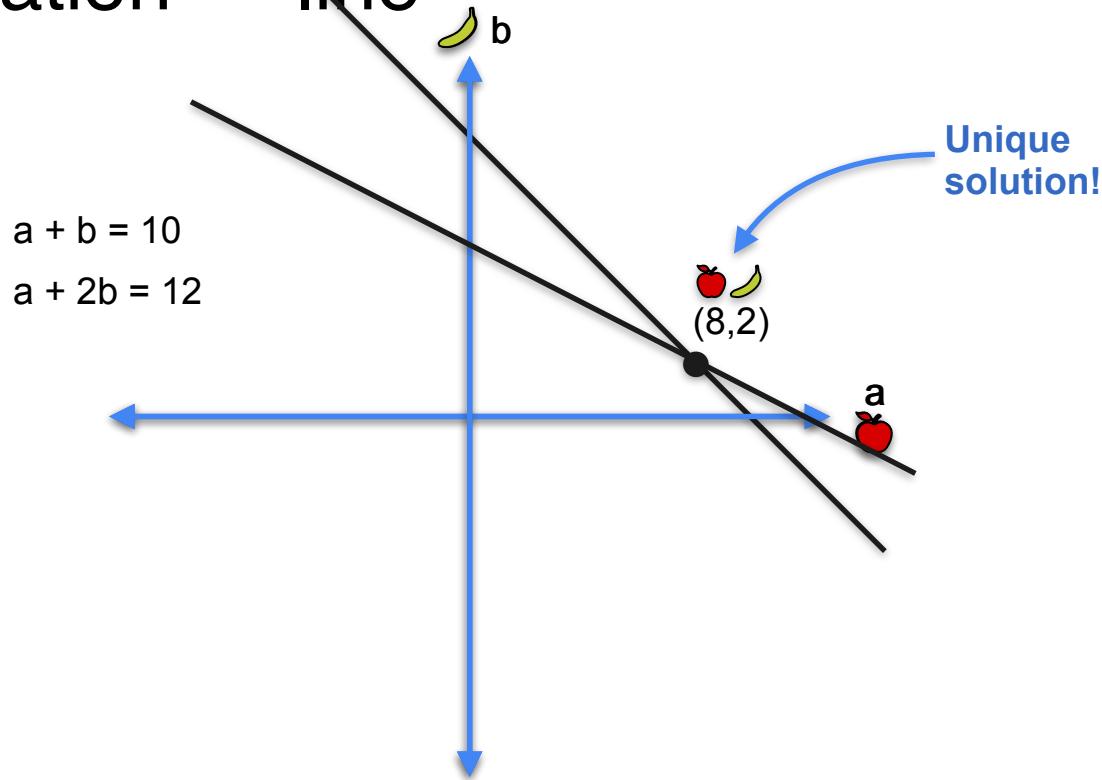
# Linear equation → line



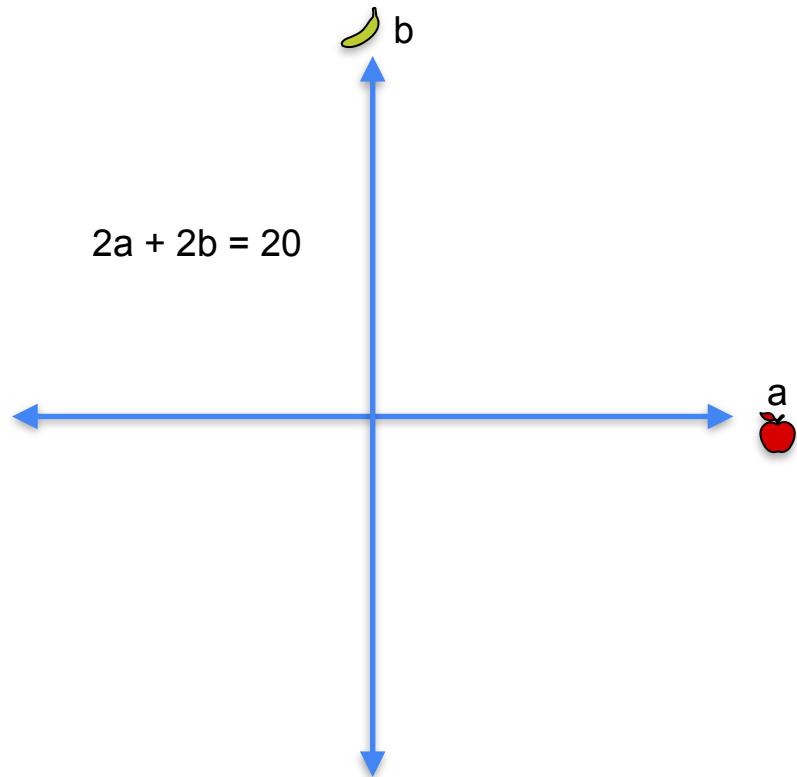
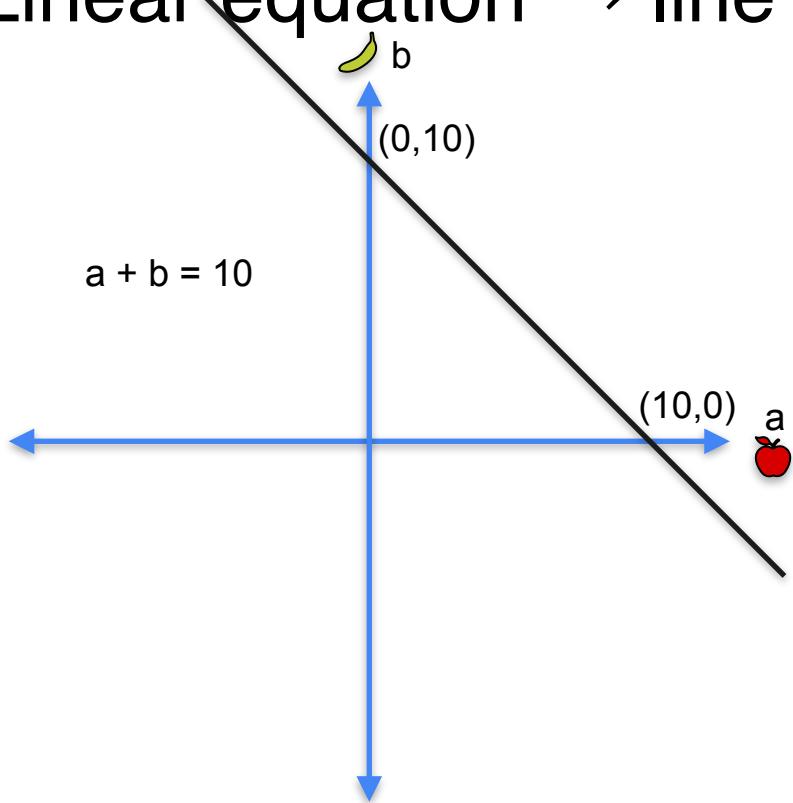
# Linear equation → line



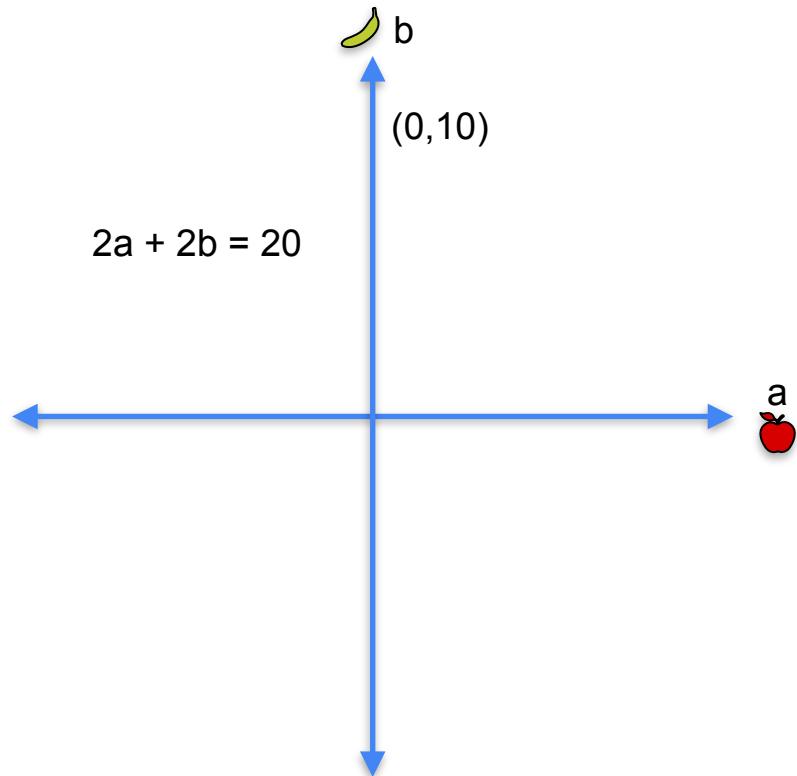
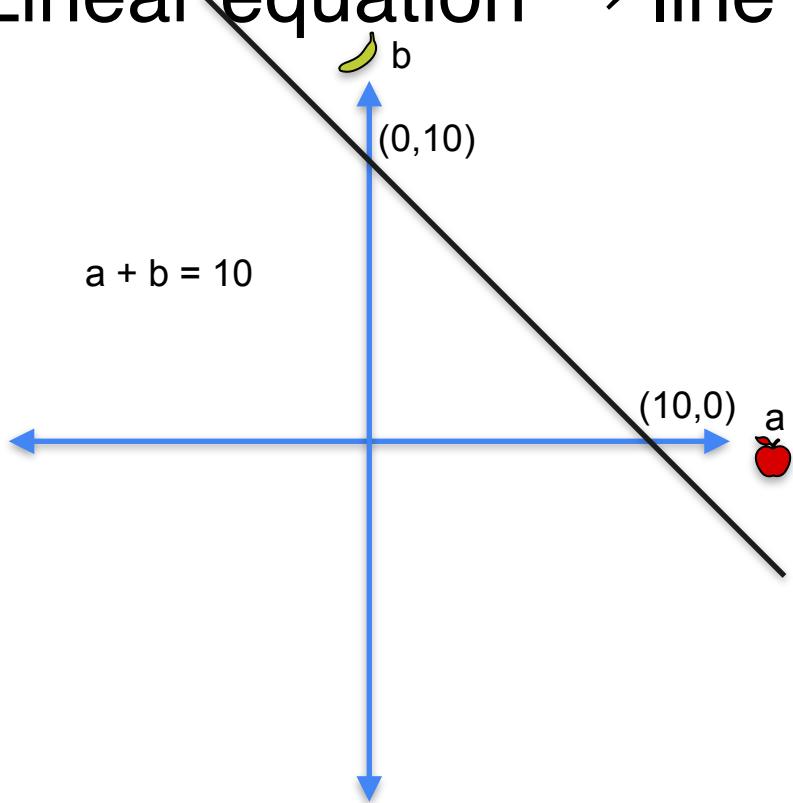
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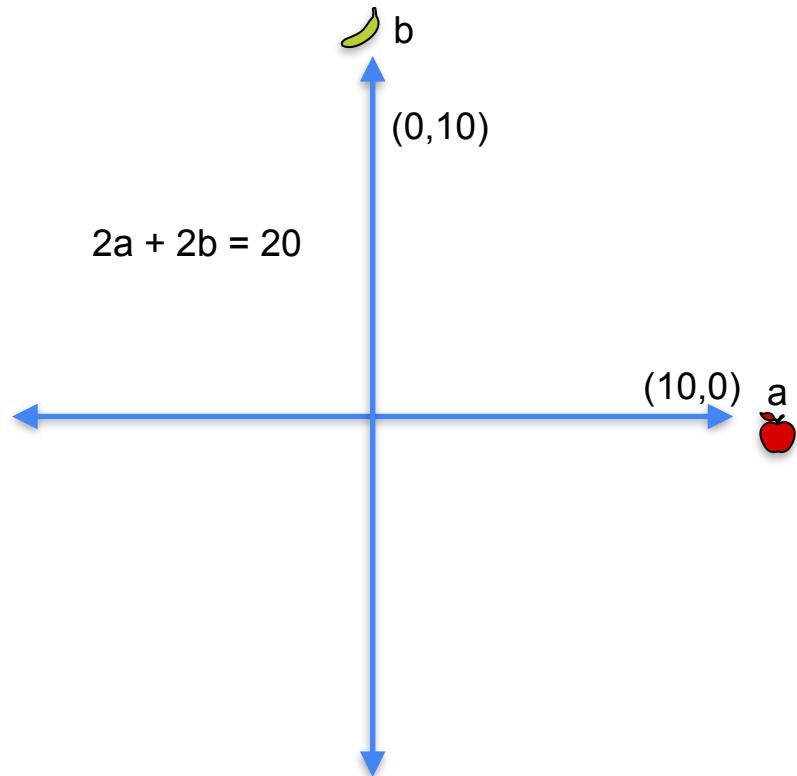
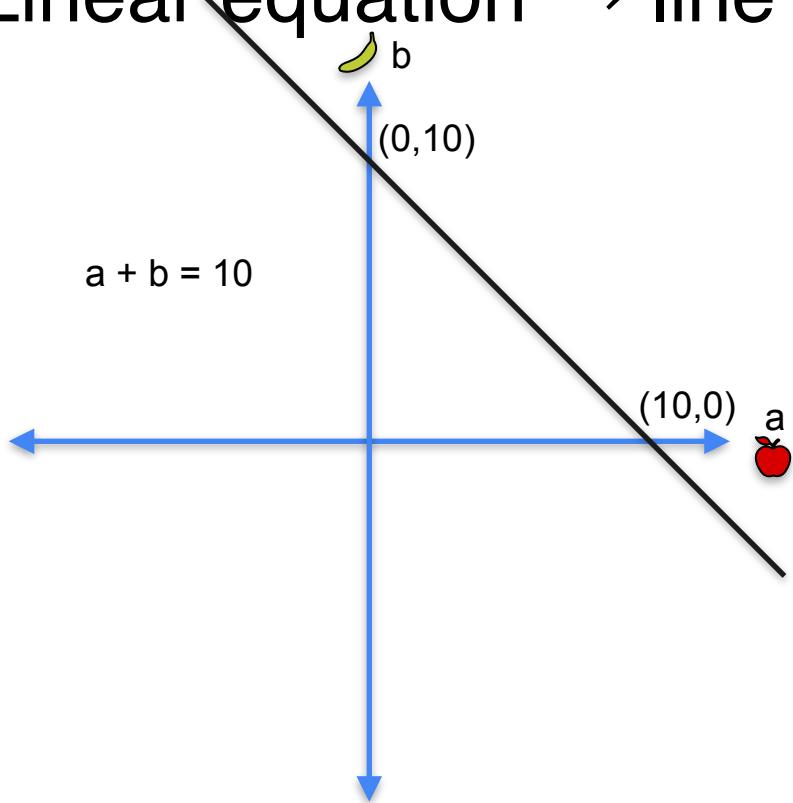
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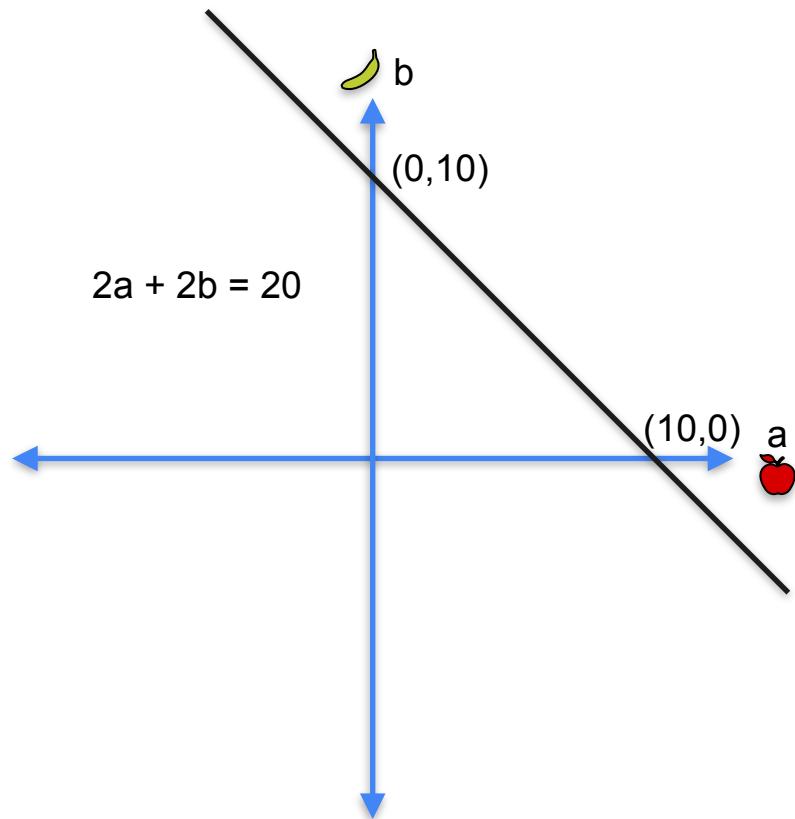
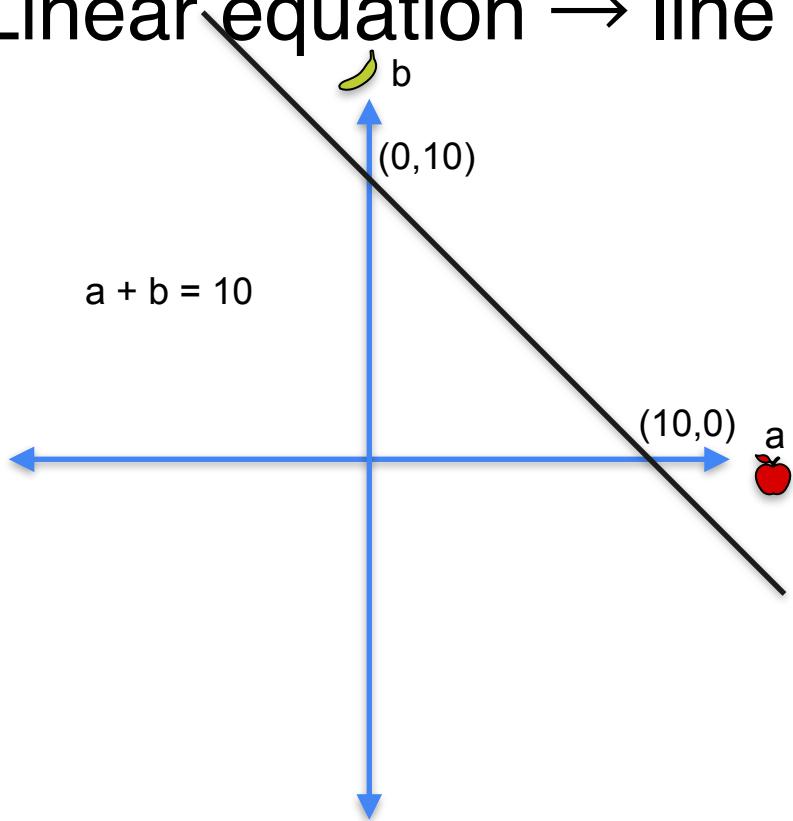
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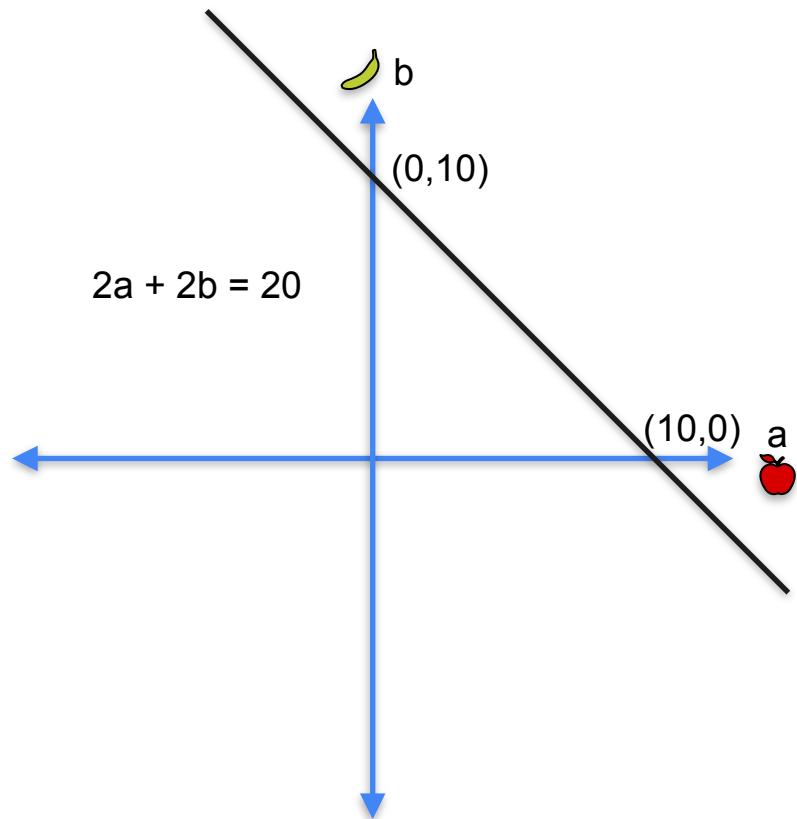
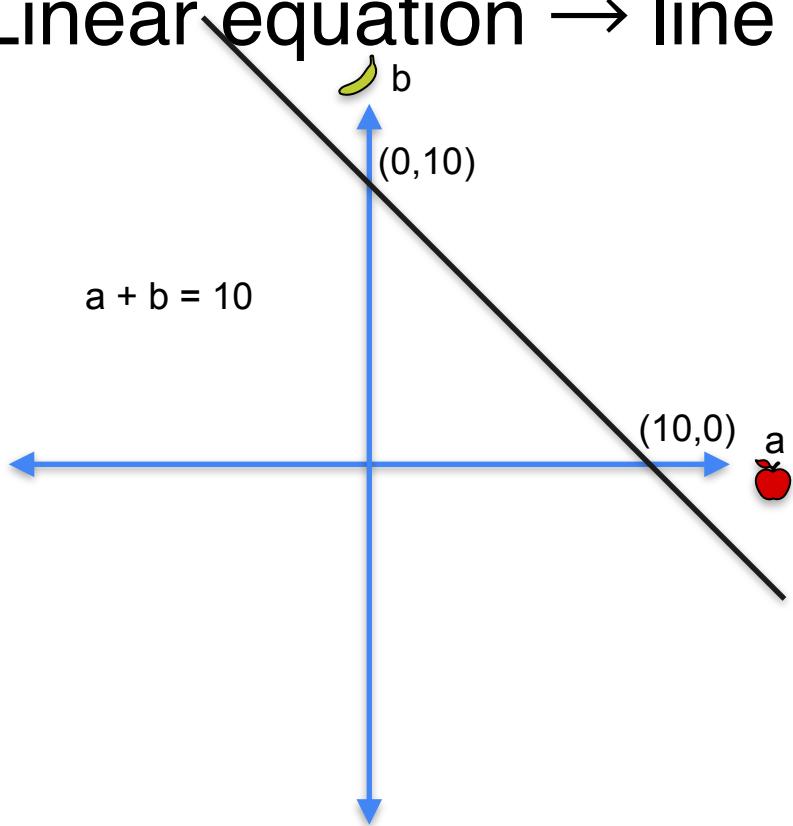
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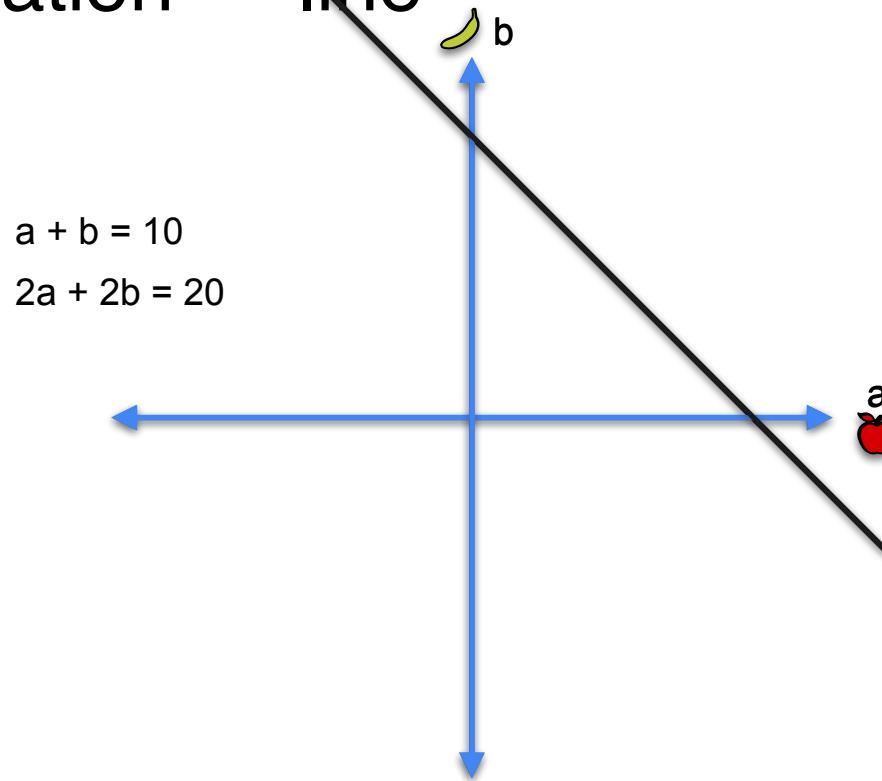
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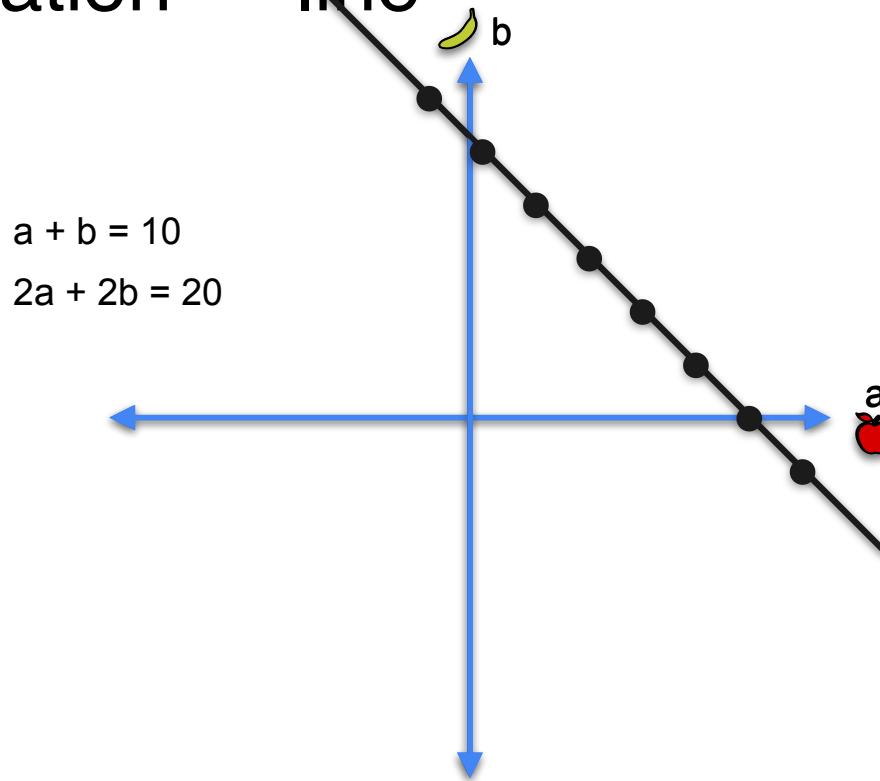
# Linear equation $\rightarrow$ line



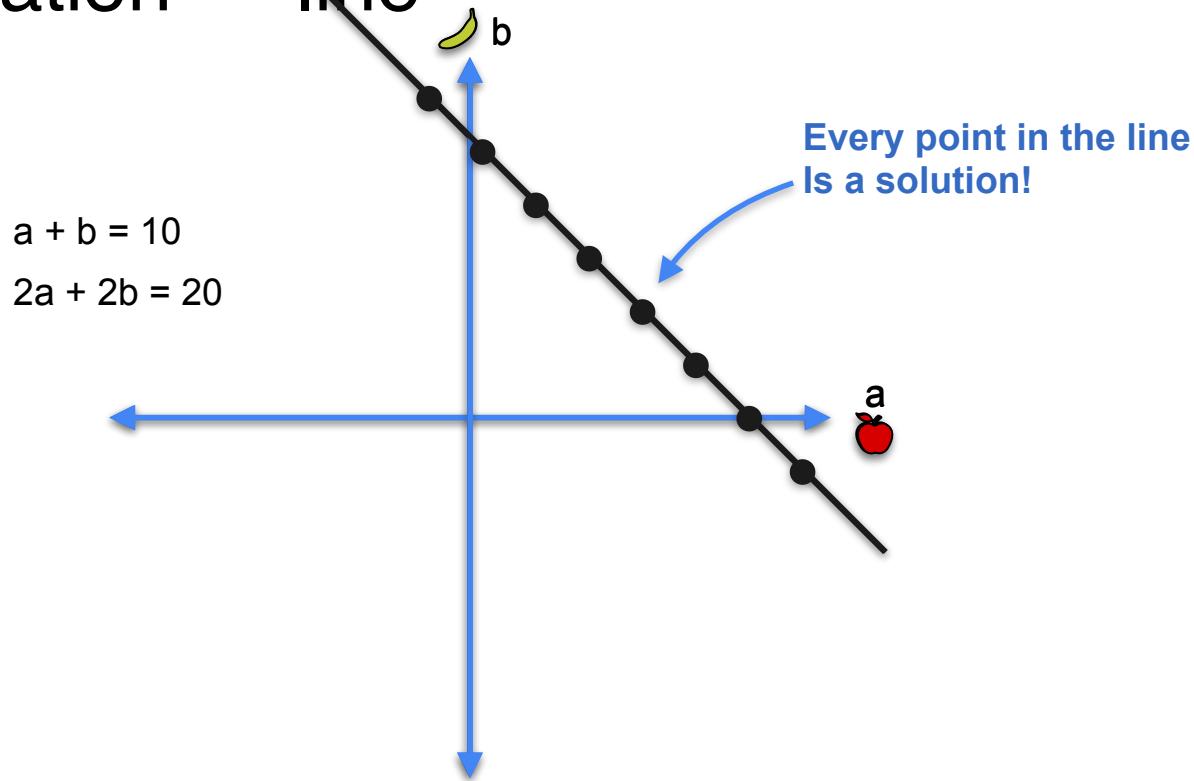
# Linear equation → line



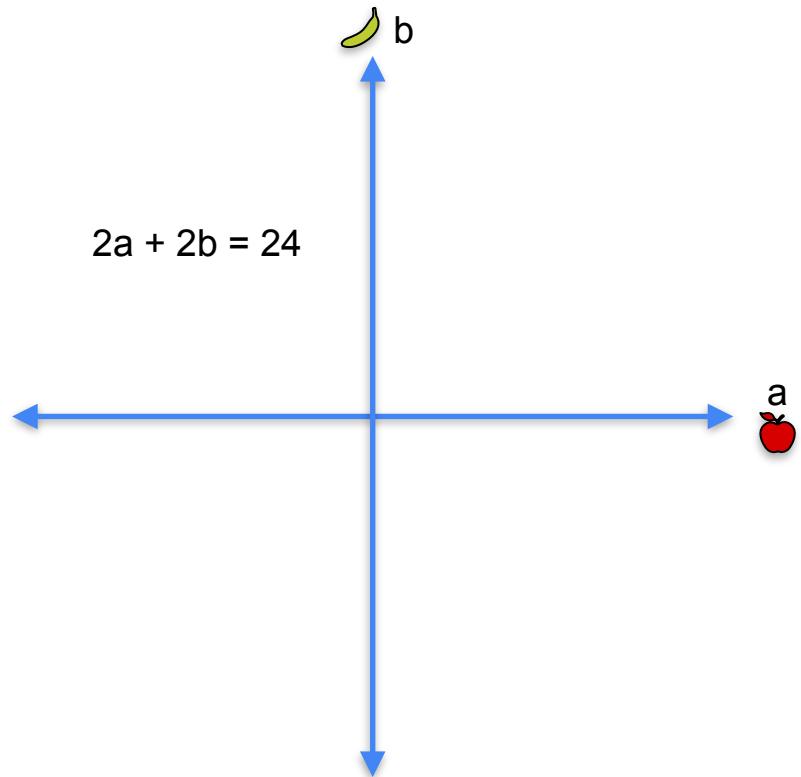
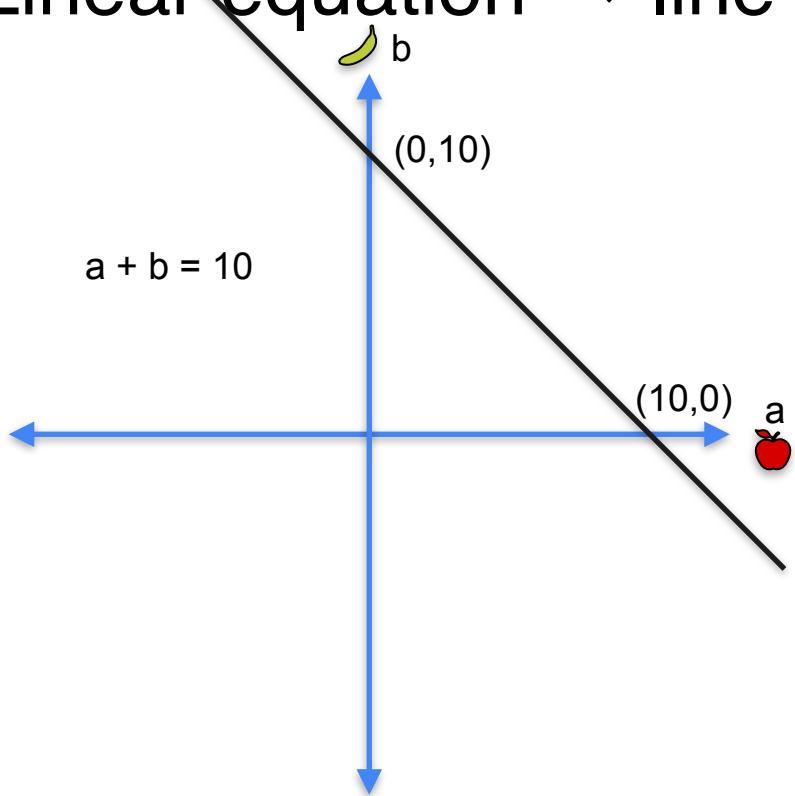
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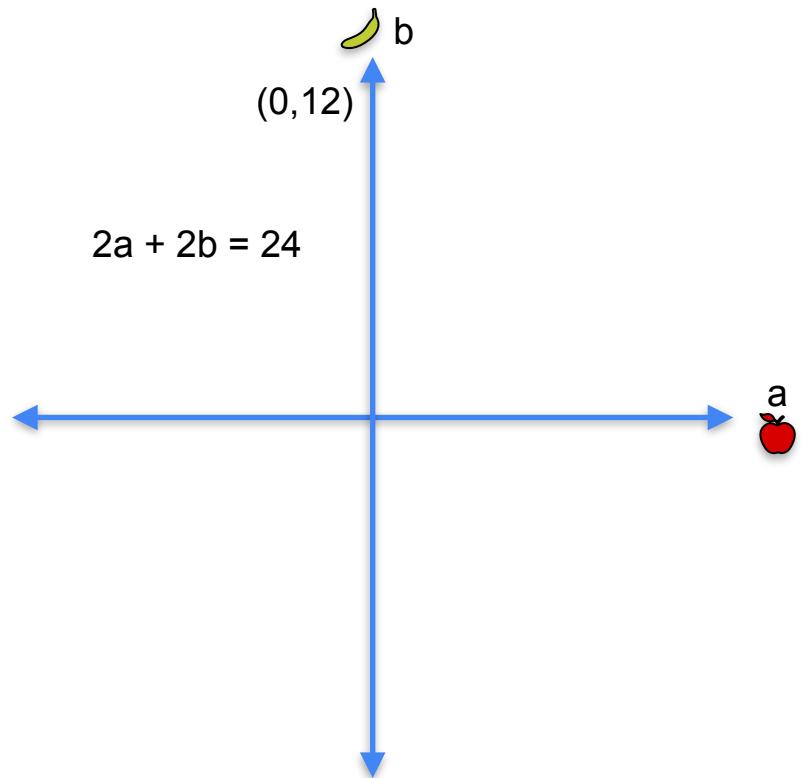
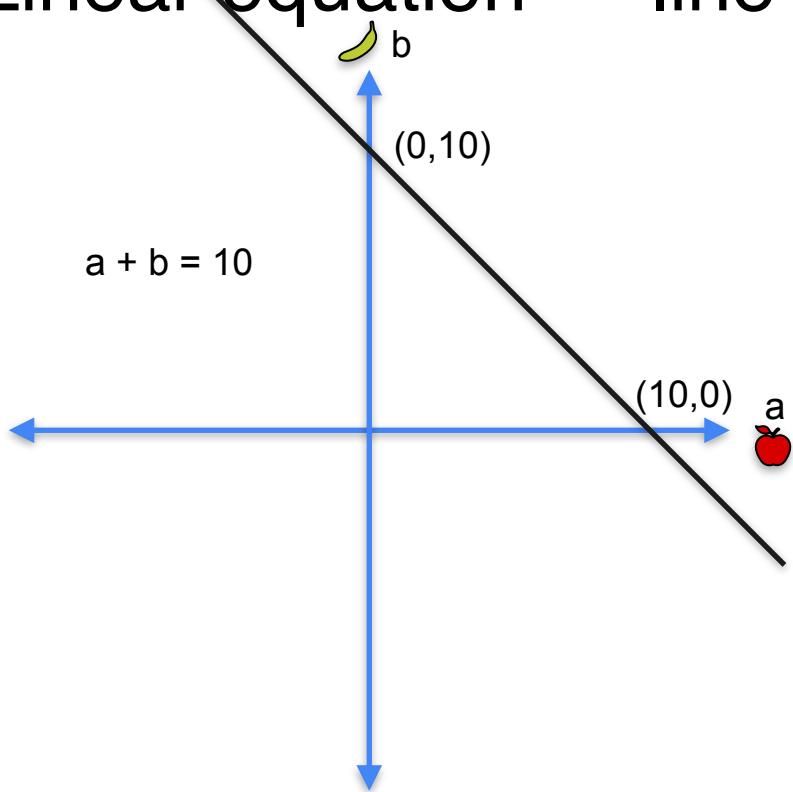
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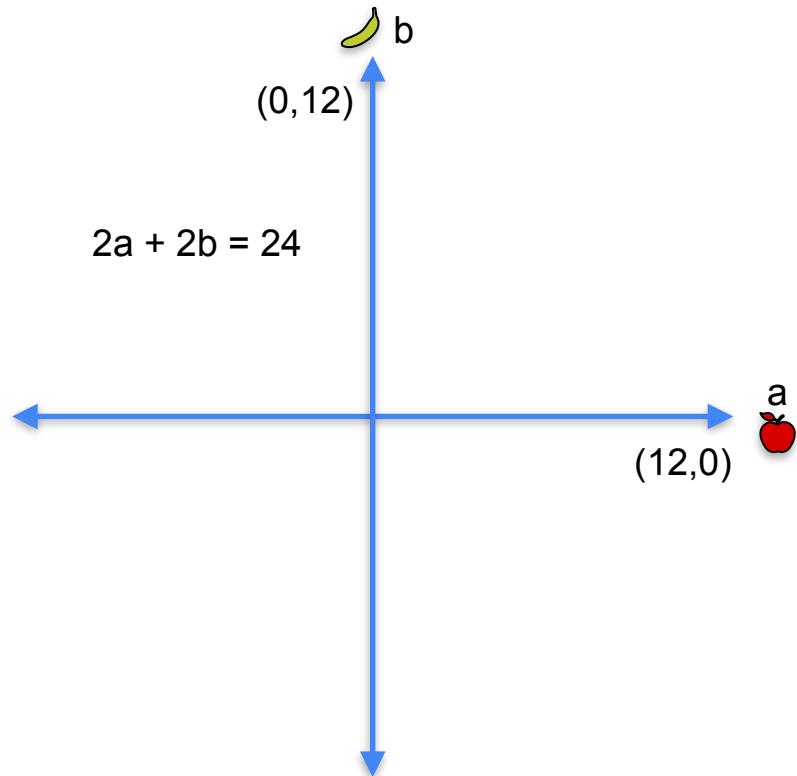
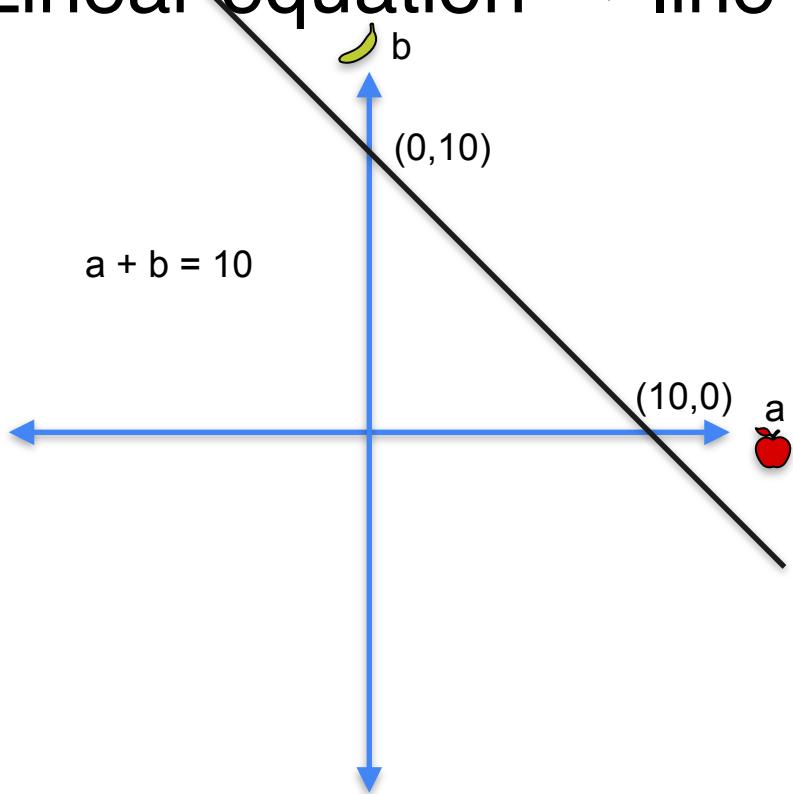
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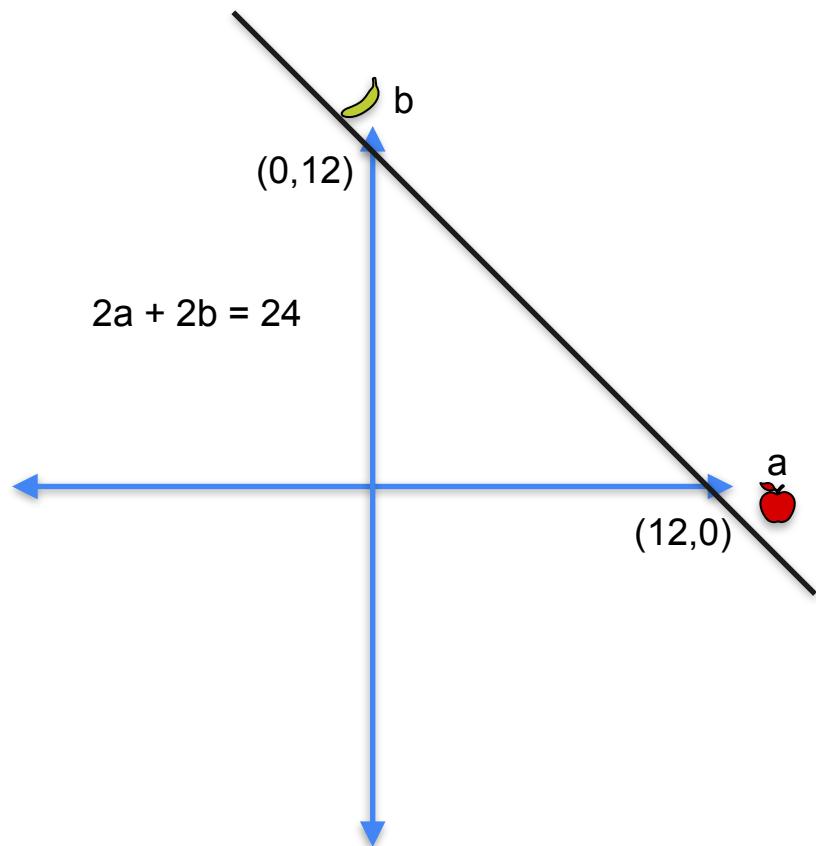
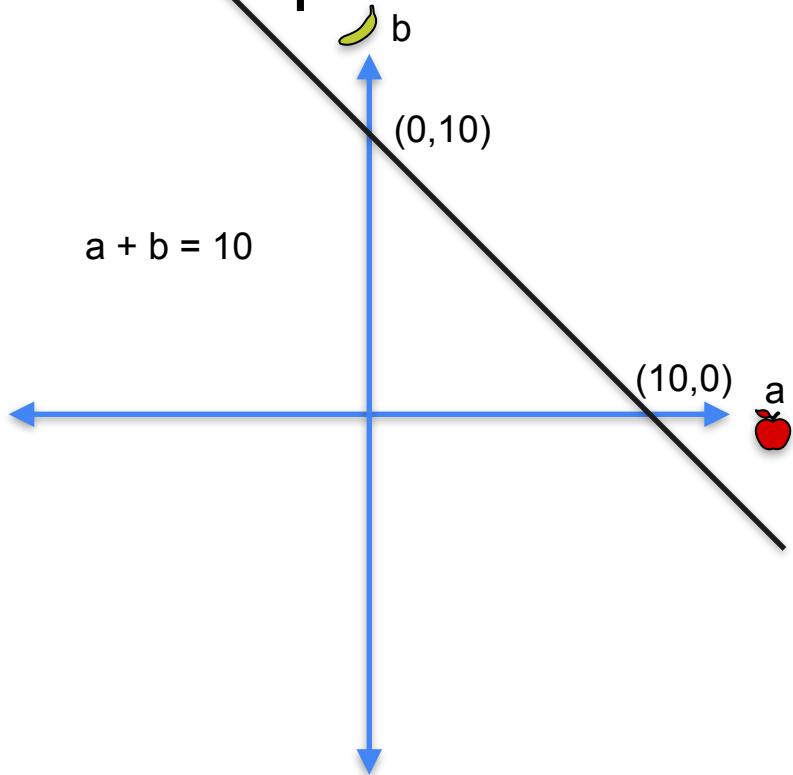
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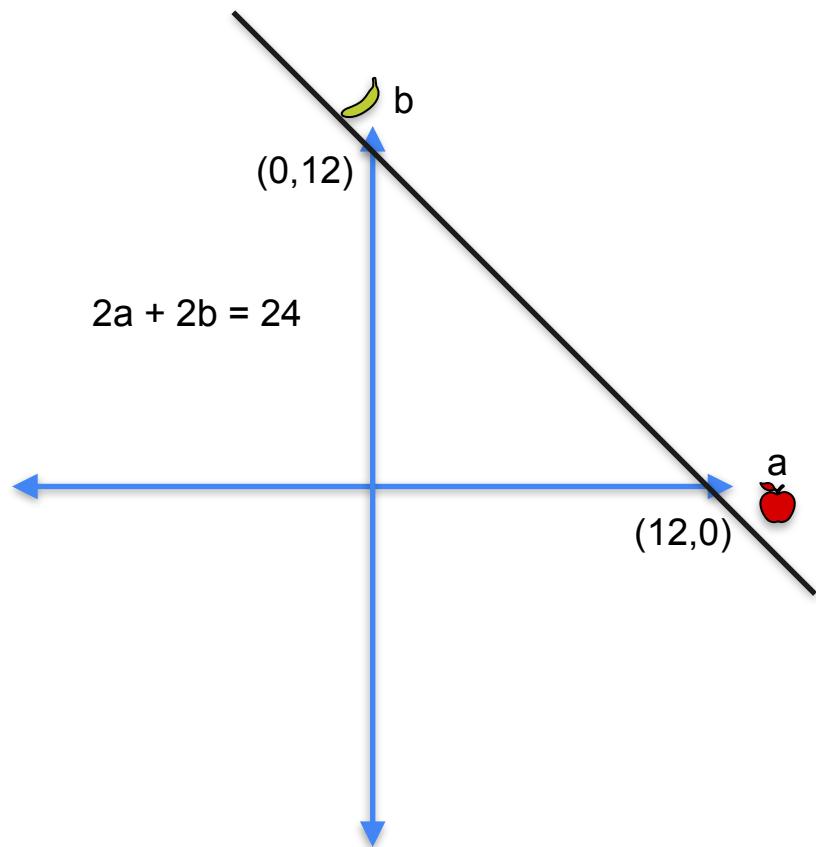
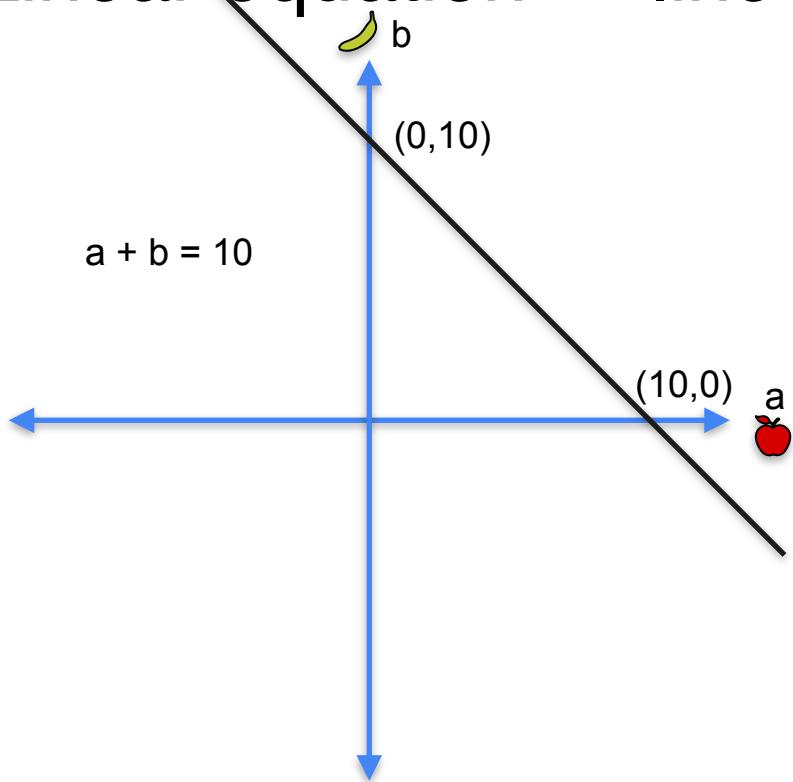
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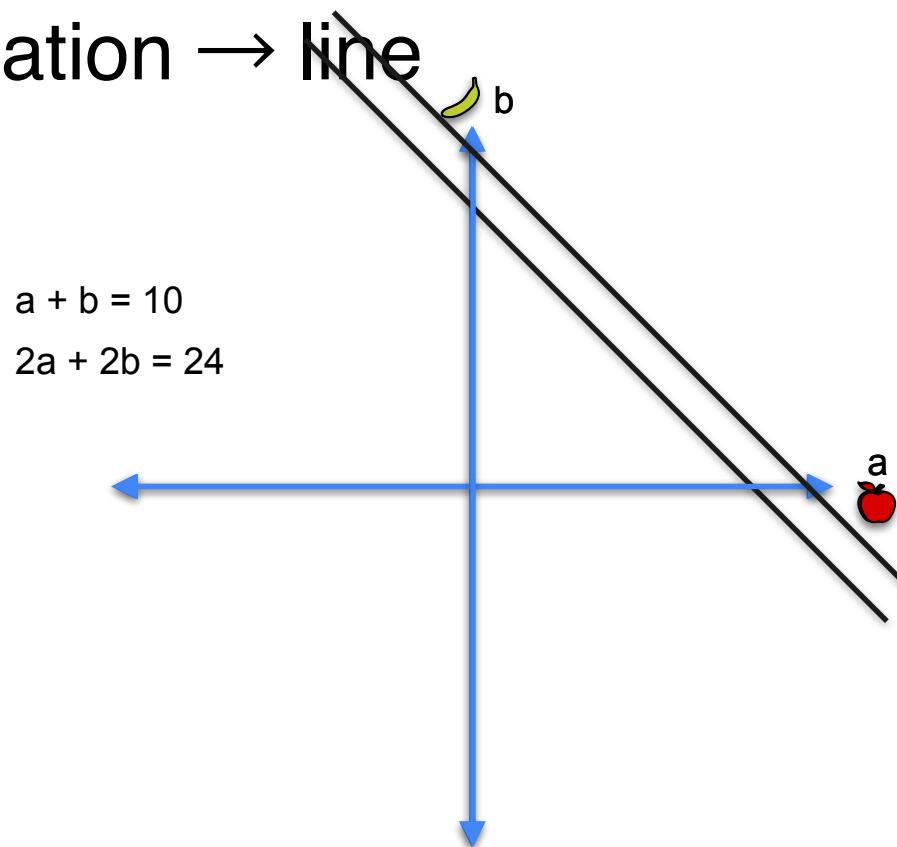
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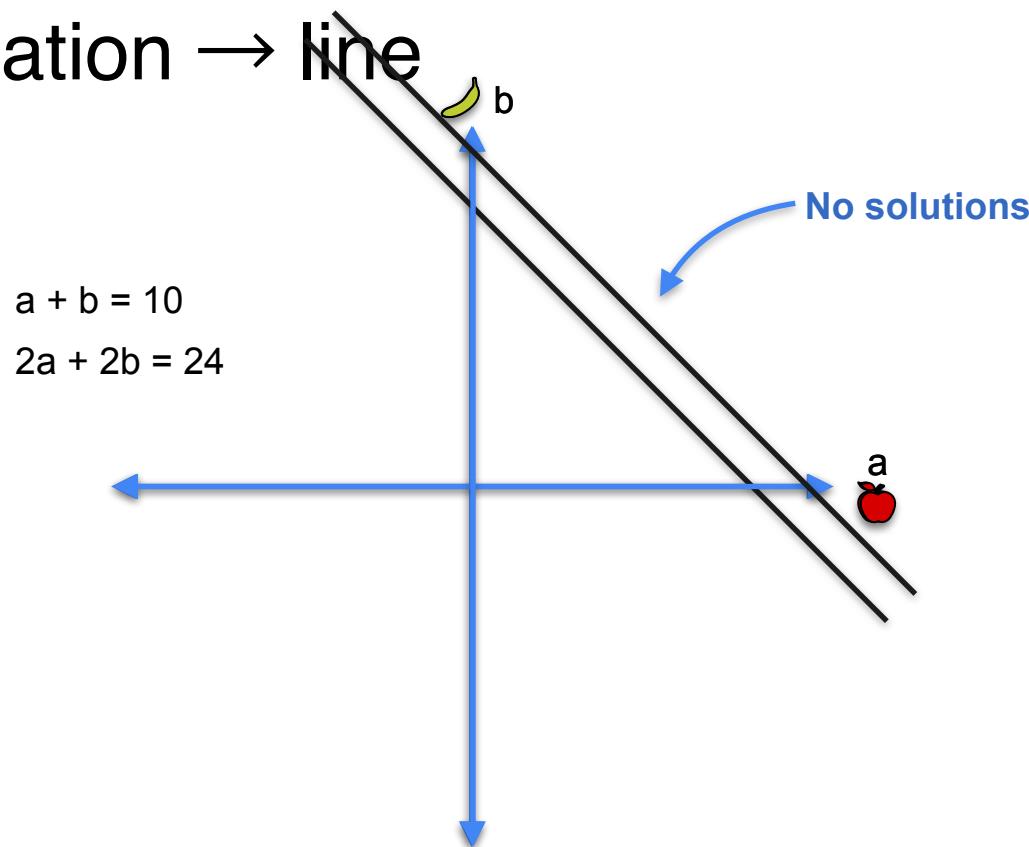
# Linear equation $\rightarrow$ line



# Linear equation → line



# Linear equation → line



# Systems of equations as lines

# Systems of equations as lines

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

# Systems of equations as lines

## System 1

- $a + b = 10$   

- $a + 2b = 12$   


## System 2

- $a + b = 10$   

- $2a + 2b = 20$   


# Systems of equations as lines

## System 1

- $a + b = 10$   

- $a + 2b = 12$   


## System 2

- $a + b = 10$   

- $2a + 2b = 20$   


## System 3

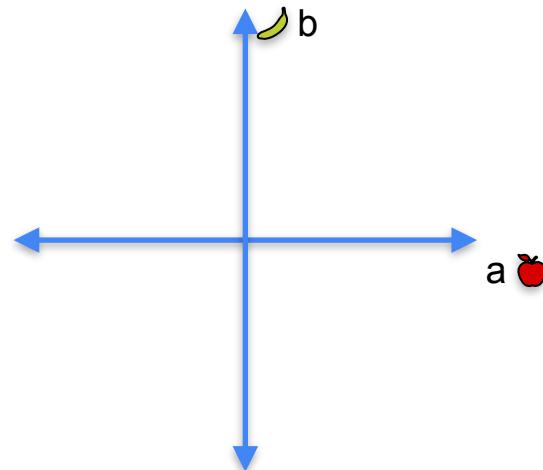
- $a + b = 10$   

- $2a + 2b = 24$   


# Systems of equations as lines

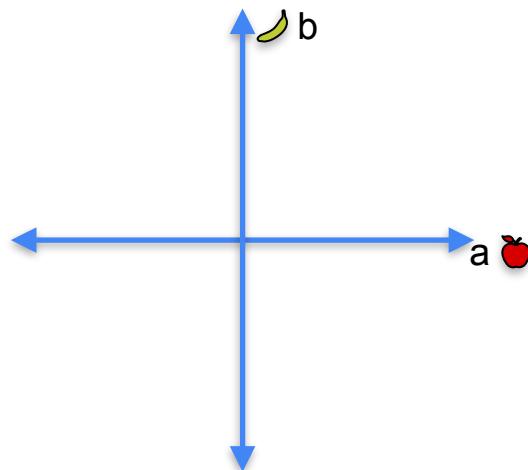
System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 



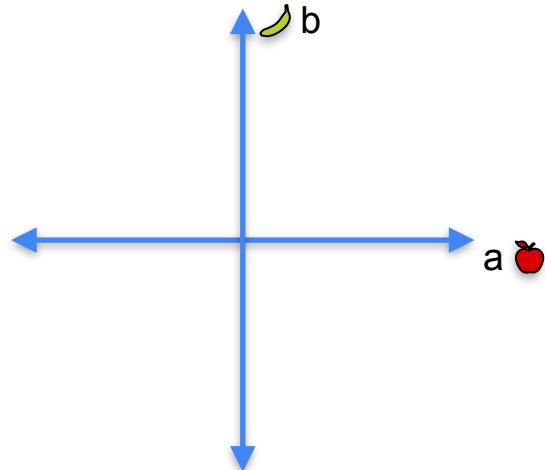
System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
 



System 3

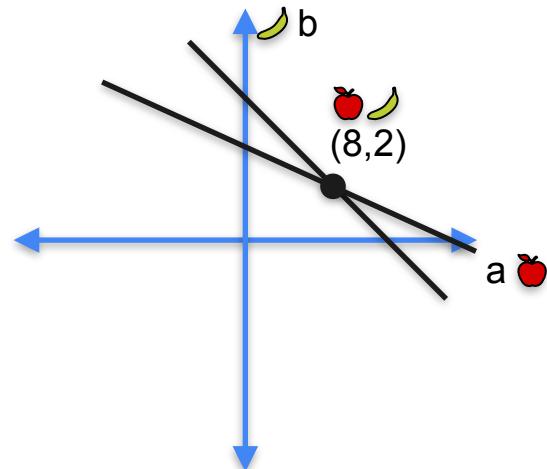
- $a + b = 10$   
 
- $2a + 2b = 24$   
 



# Systems of equations as lines

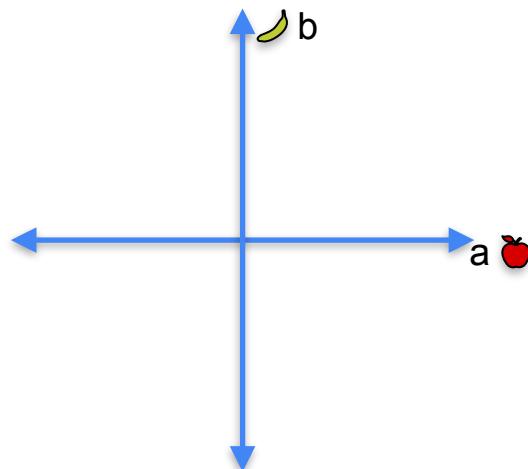
System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 



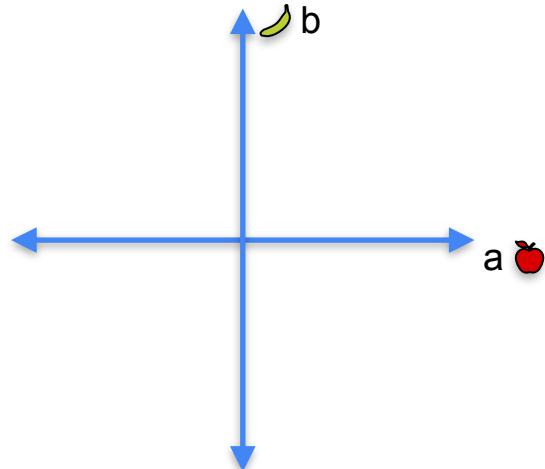
System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
 



System 3

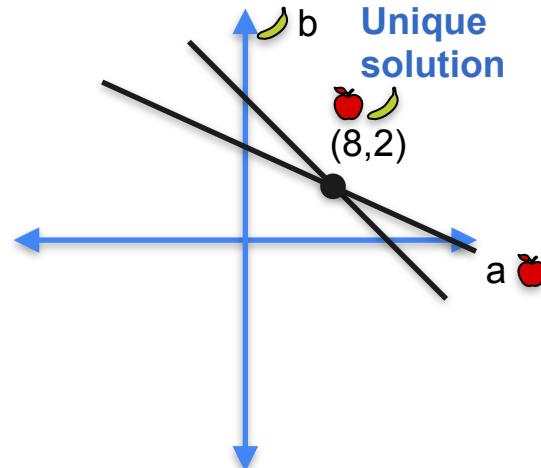
- $a + b = 10$   
 
- $2a + 2b = 24$   
 



# Systems of equations as lines

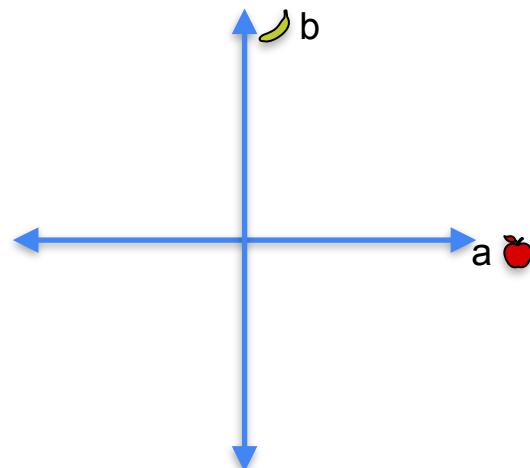
System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 



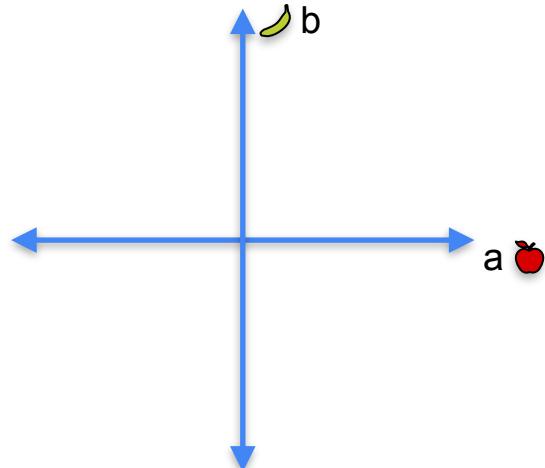
System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
 



System 3

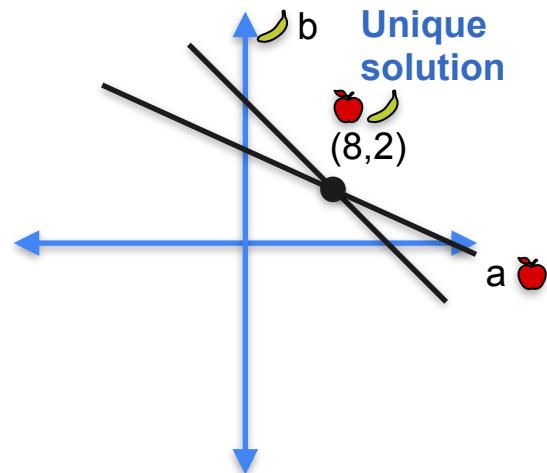
- $a + b = 10$   
 
- $2a + 2b = 24$   
 



# Systems of equations as lines

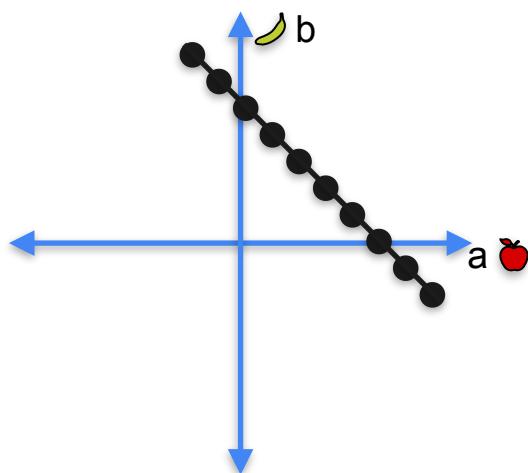
System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 



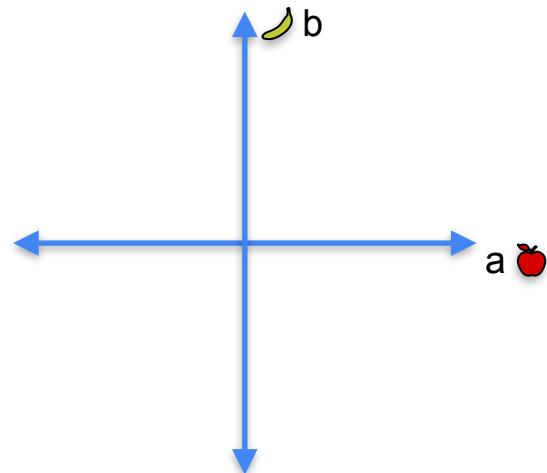
System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
 



System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
 



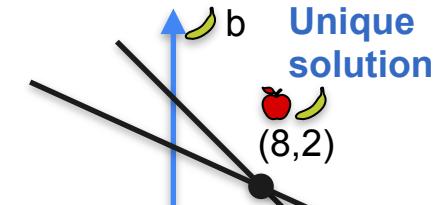
# Systems of equations as lines

System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

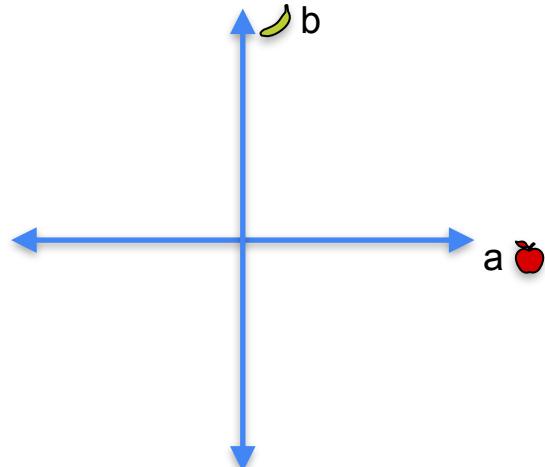
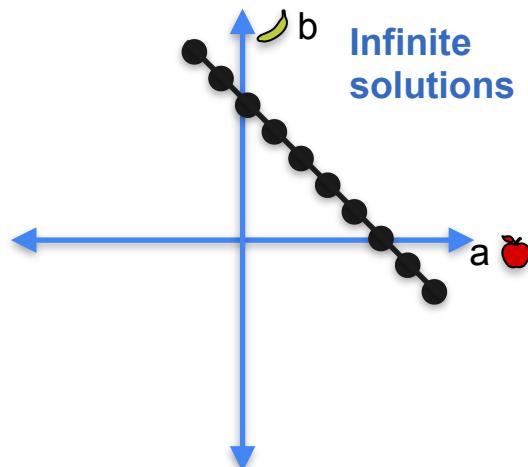
System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
 



System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
 



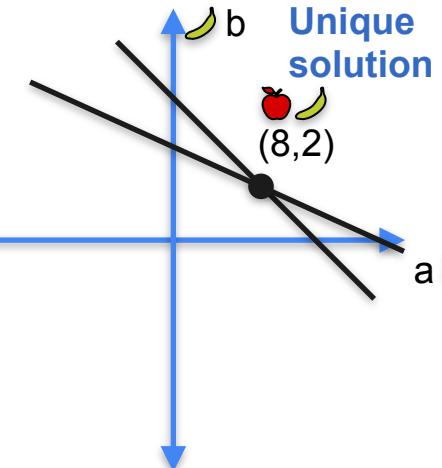
# Systems of equations as lines

System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

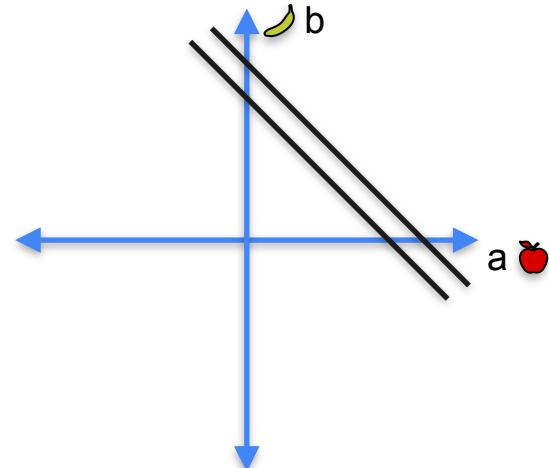
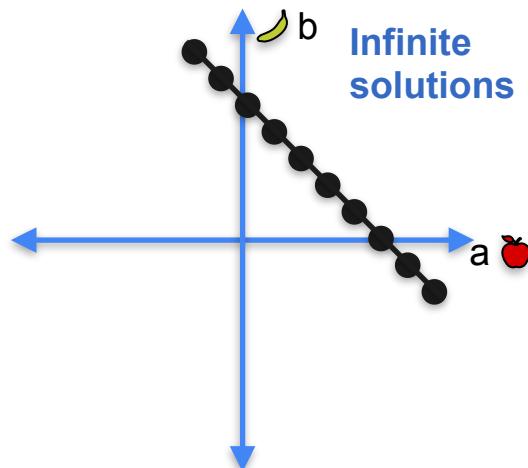
System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
 



System 3

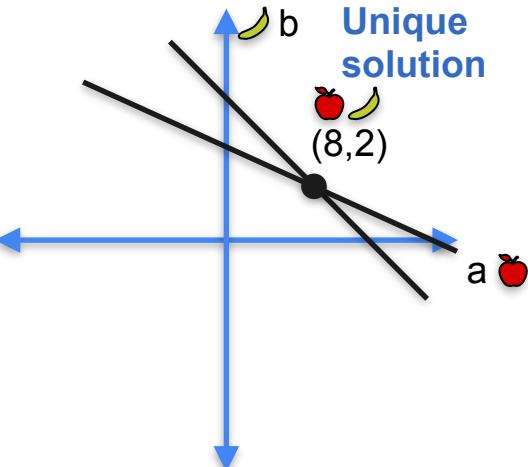
- $a + b = 10$   
 
- $2a + 2b = 24$   
 



# Systems of equations as lines

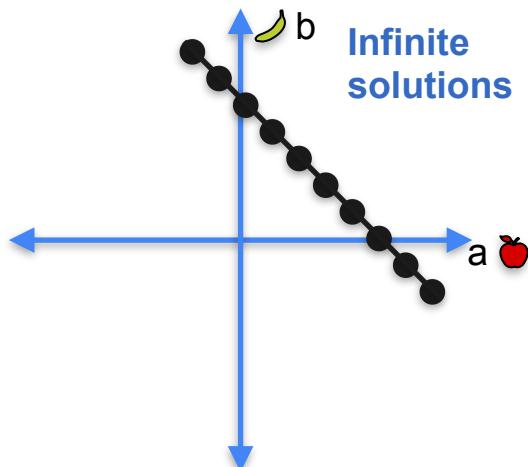
## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 



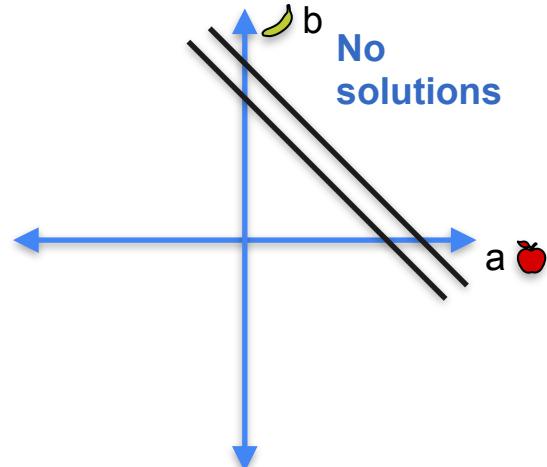
## System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
 



## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
 



# Systems of equations as lines

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

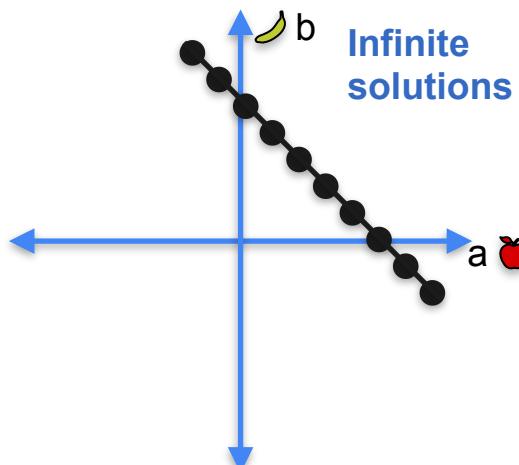
 **Unique solution**  
  $(8, 2)$

**Complete  
Non-singular**

## System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
 

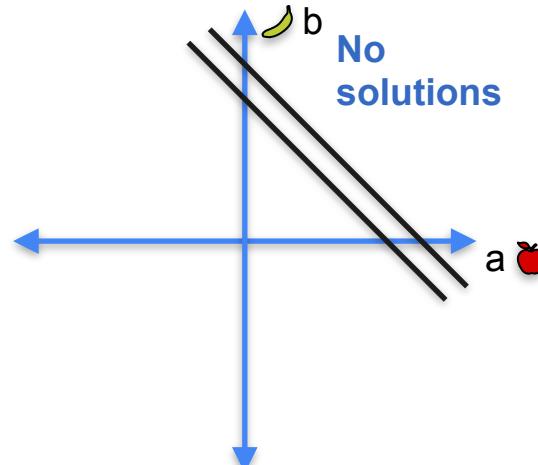
 **Infinite solutions**



## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
 

 **No solutions**



# Systems of equations as lines

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

 **b**  
 **a**

**Unique solution**

**(8,2)**

**Complete  
Non-singular**

## System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
 

 **b**  
 **a**

**Redundant**

**Singular**

## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
 

 **b**  
 **a**

**No solutions**

# Systems of equations as lines

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

 **b**  
 **a**

**Unique solution**

**(8,2)**

**Complete  
Non-singular**

## System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
 

 **b**  
 **a**

**Infinite solutions**

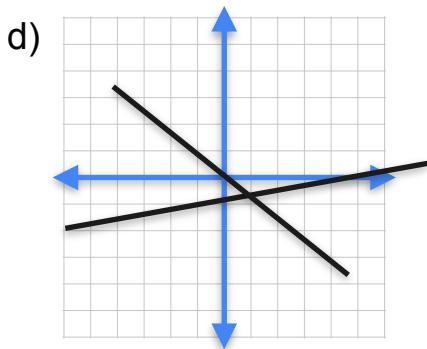
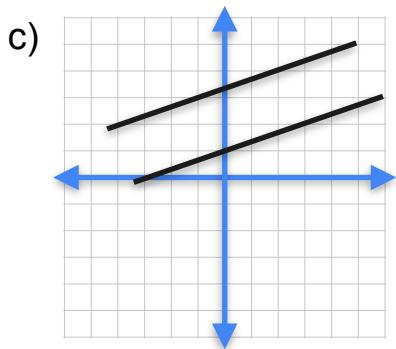
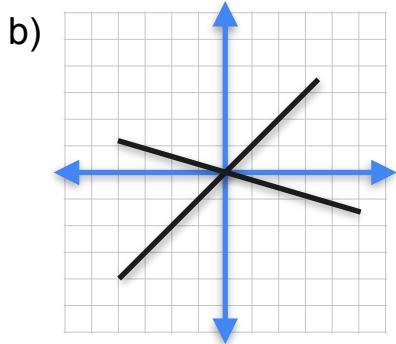
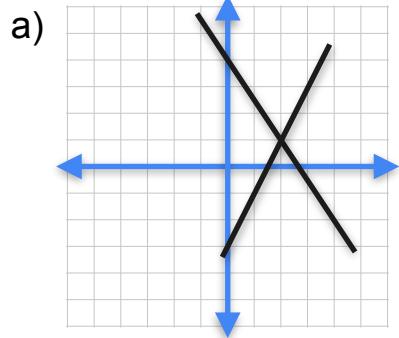
 

# Quiz

## Problem 1

Which of the following plots corresponds to the system of equations:

- $3a + 2b = 8$
- $2a - b = 3$



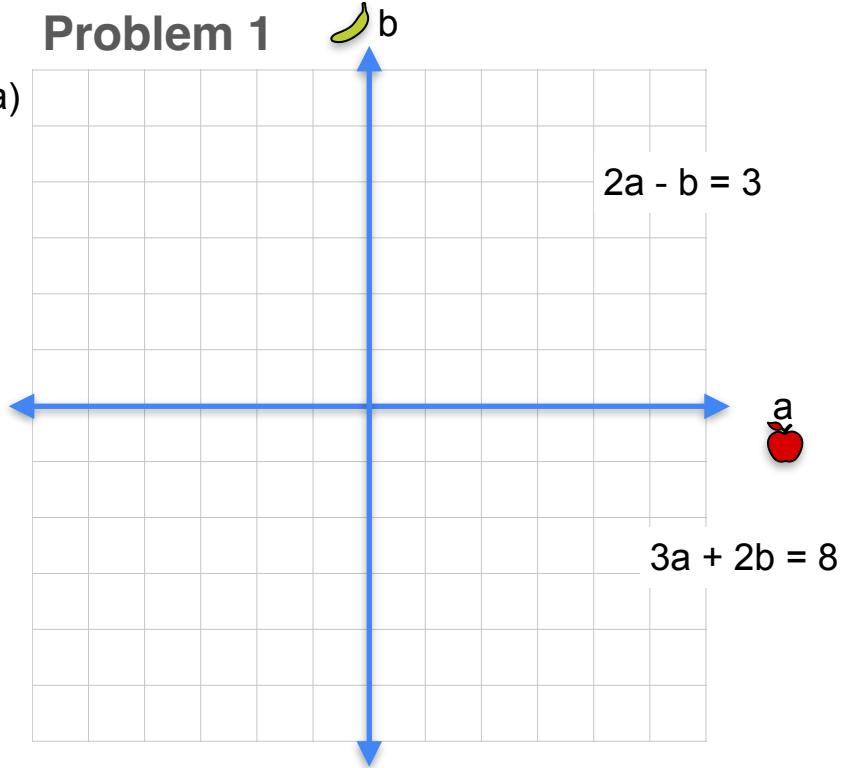
## Problem 2

Is this system singular or non-singular?

# Solution

## Problem 1

a)



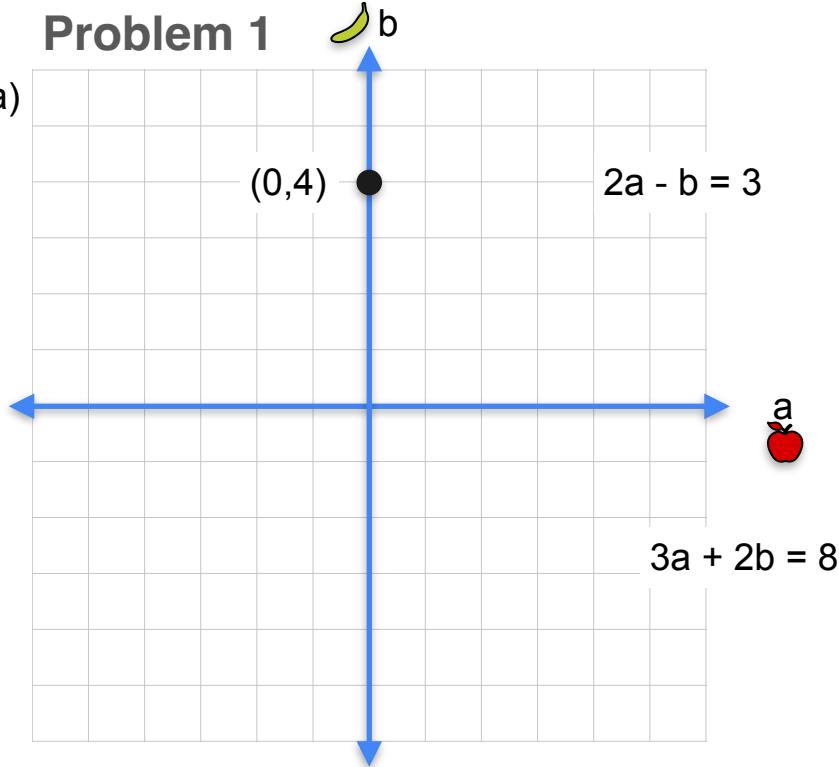
## Problem 2

Since the lines cross at a unique point, the system is non-singular.

# Solution

## Problem 1

a)



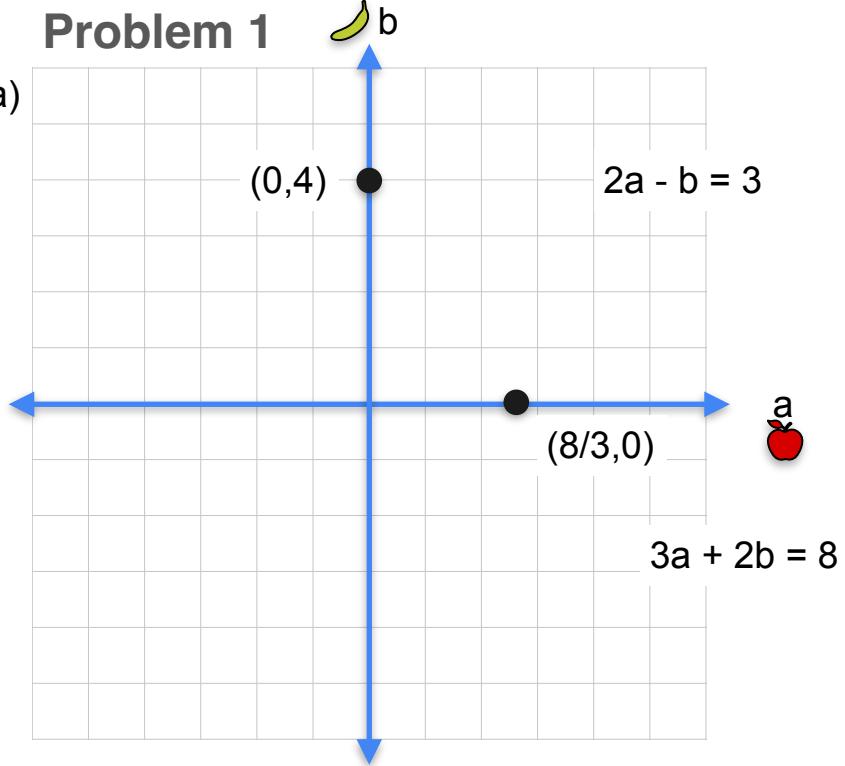
## Problem 2

Since the lines cross at a unique point, the system is non-singular.

# Solution

## Problem 1

a)



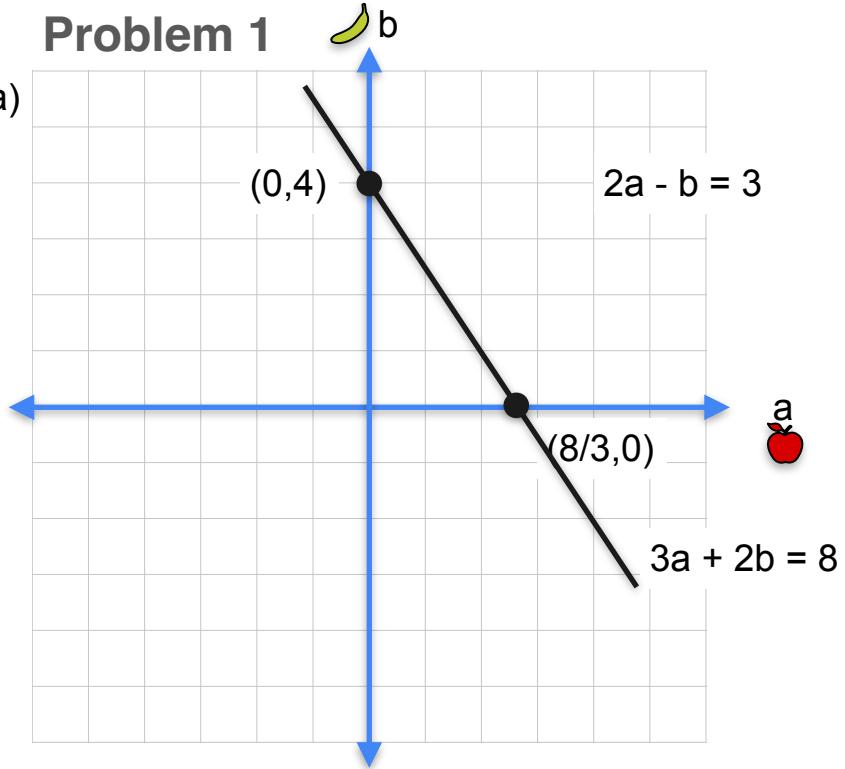
## Problem 2

Since the lines cross at a unique point, the system is non-singular.

# Solution

## Problem 1

a)



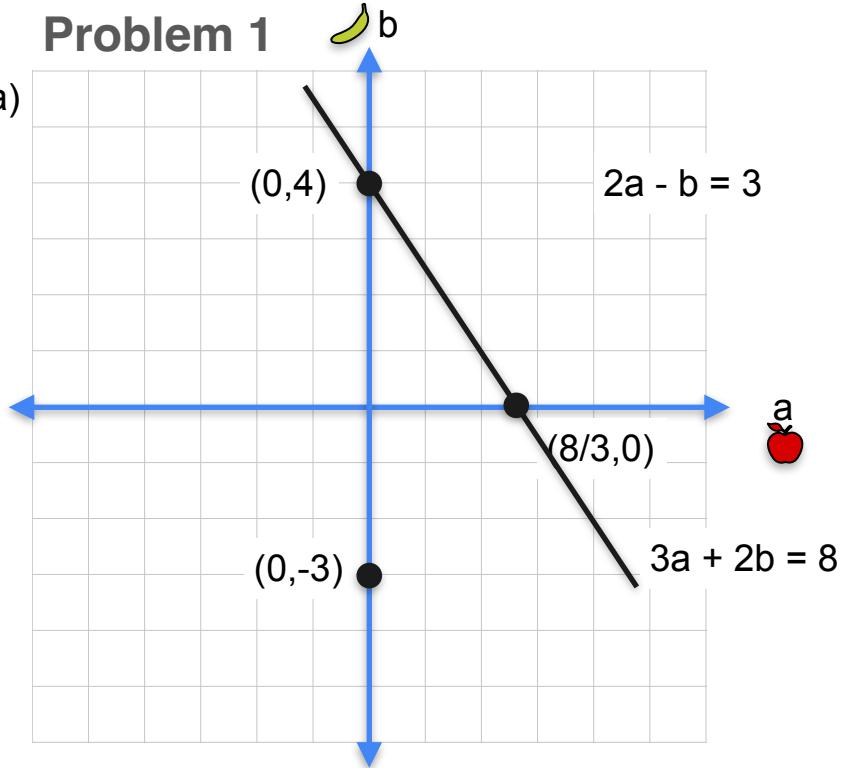
## Problem 2

Since the lines cross at a unique point, the system is non-singular.

# Solution

## Problem 1

a)



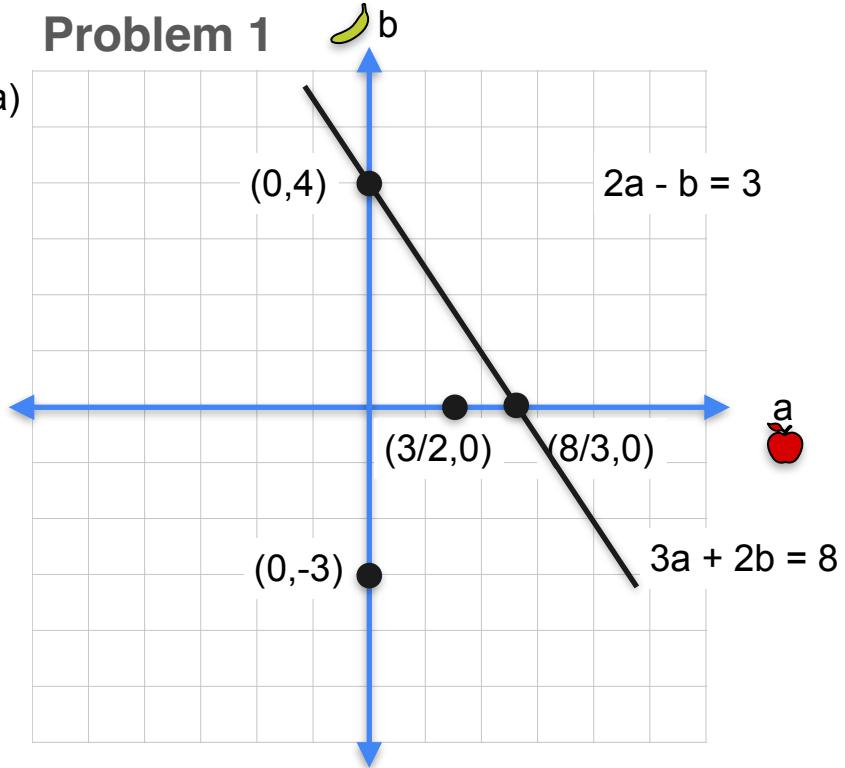
## Problem 2

Since the lines cross at a unique point, the system is non-singular.

# Solution

## Problem 1

a)



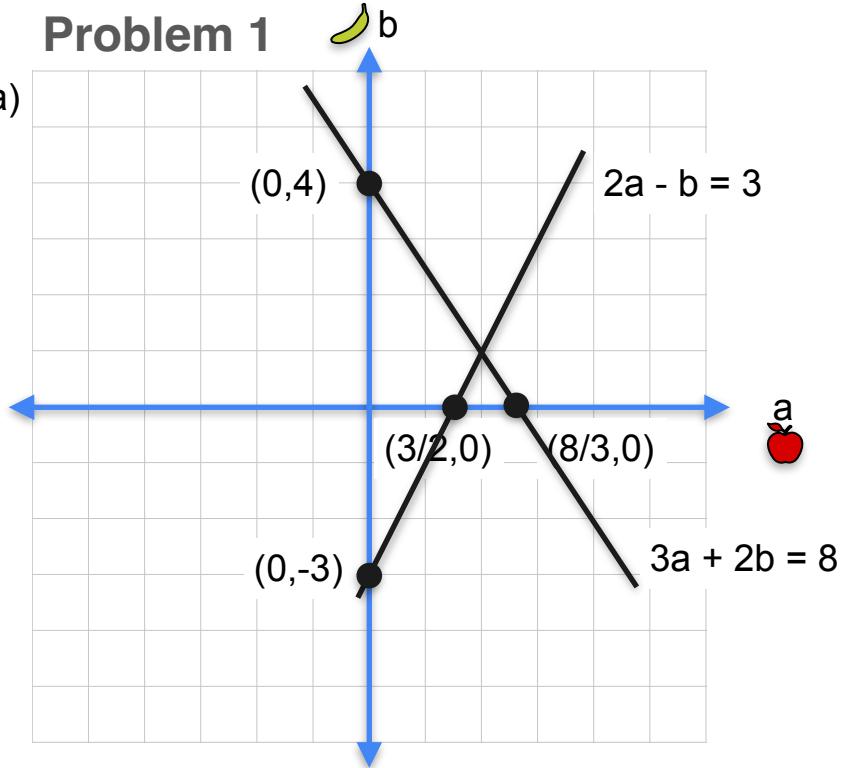
## Problem 2

Since the lines cross at a unique point, the system is non-singular.

# Solution

## Problem 1

a)



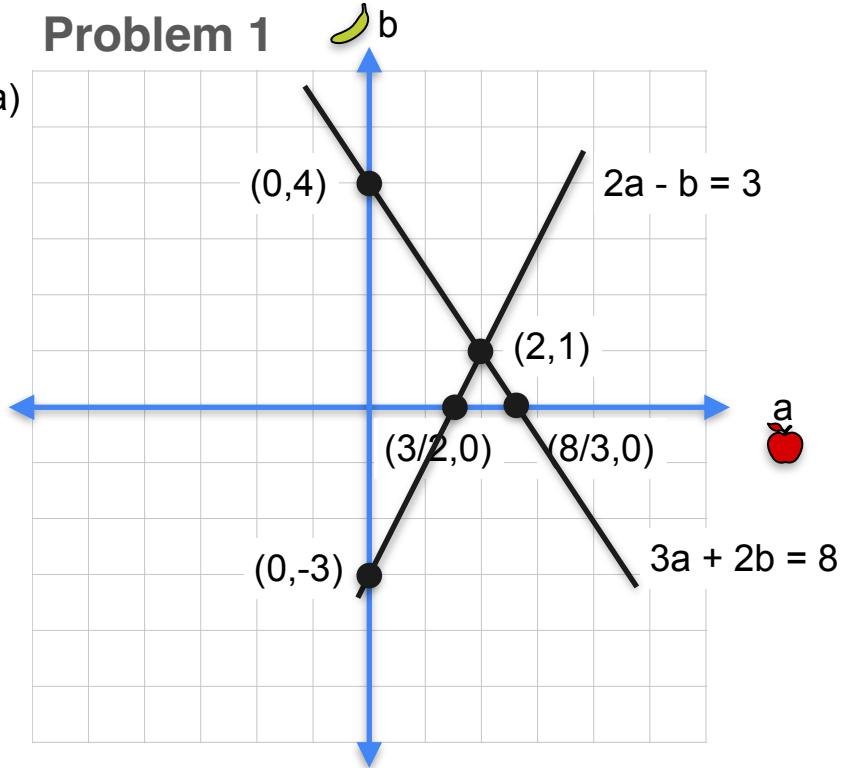
## Problem 2

Since the lines cross at a unique point, the system is non-singular.

# Solution

## Problem 1

a)



## Problem 2

Since the lines cross at a unique point, the system is non-singular.



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# System of Linear Equations

---

**A geometric notion of  
singularity**

# Systems of equations as lines

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

Unique  
solution

(8,2)

Complete  
Non-singular

## System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
 

Infinite  
solutions

Redundant  
Singular

## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
 

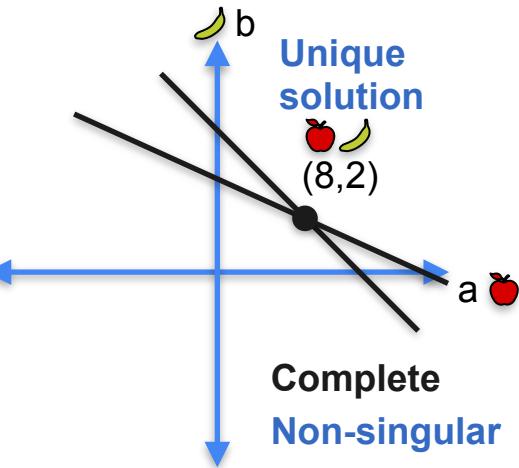
No  
solutions

Contradictory  
Singular

# Systems of equations as lines

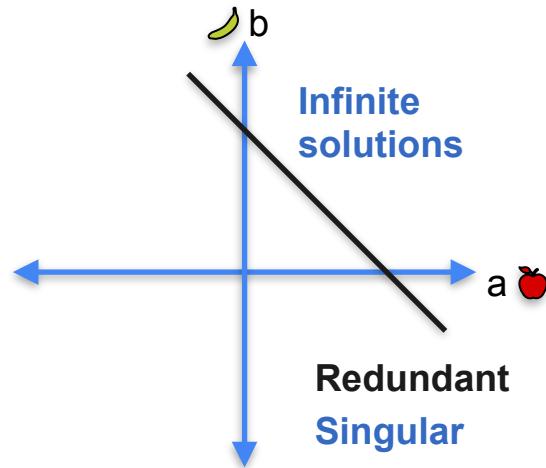
System 1

- $a + b = 10$
- $a + 2b = 12$



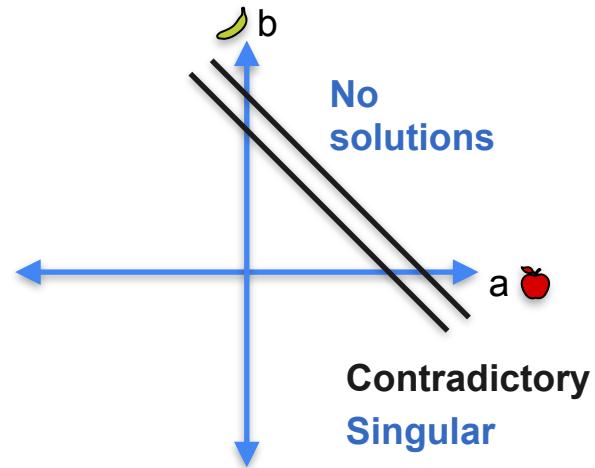
System 2

- $a + b = 10$
- $2a + 2b = 20$



System 3

- $a + b = 10$
- $2a + 2b = 24$



# Systems of equations as lines

System 1

-    $= 0$
-    $= 0$

Unique  
solution

Complete  
Non-singular

System 2

-    $= 10$
-    $= 20$

Infinite  
solutions

Redundant  
Singular

System 3

-    $= 10$
-    $= 24$

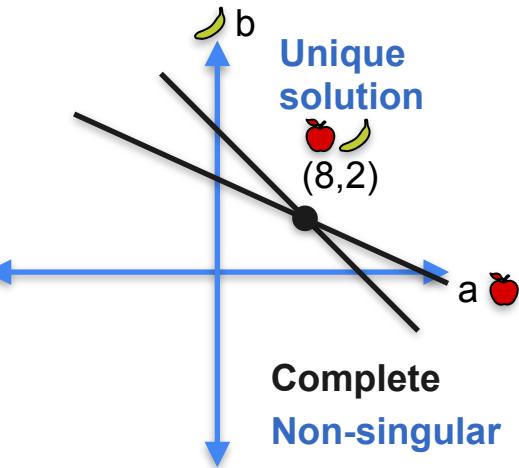
No  
solutions

Contradictory  
Singular

# Systems of equations as lines

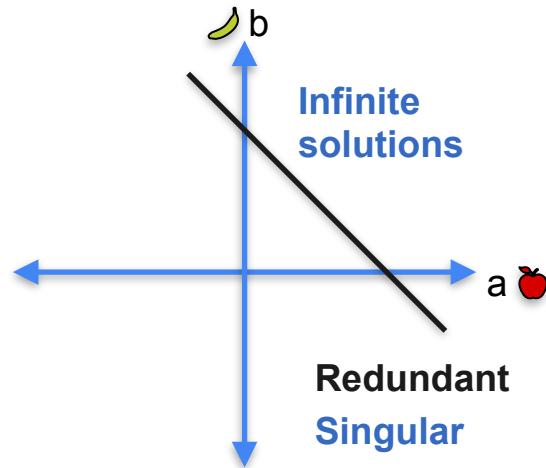
System 1

- $a + b = 0$
- $a + 2b = 0$



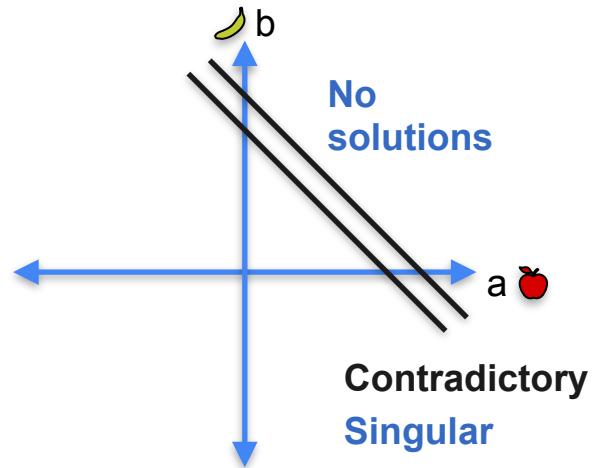
System 2

- $a + b = 0$
- $2a + 2b = 0$



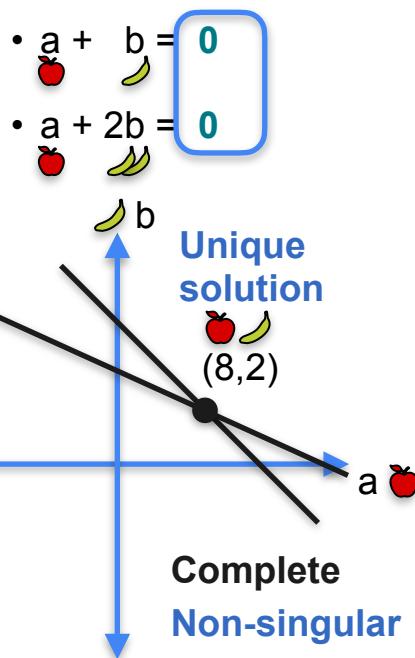
System 3

- $a + b = 10$
- $2a + 2b = 24$

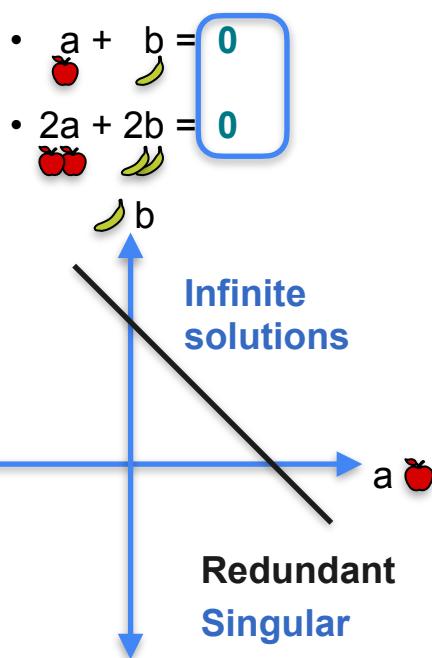


# Systems of equations as lines

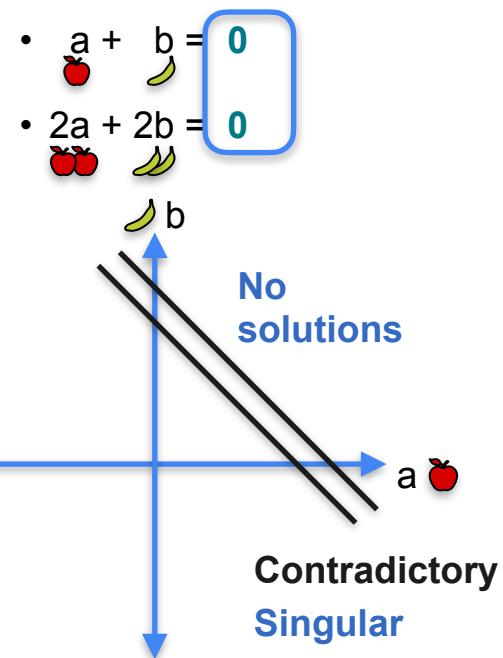
System 1



System 2

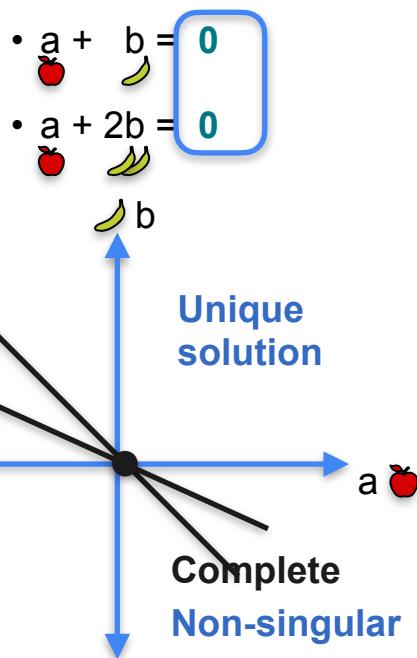


System 3

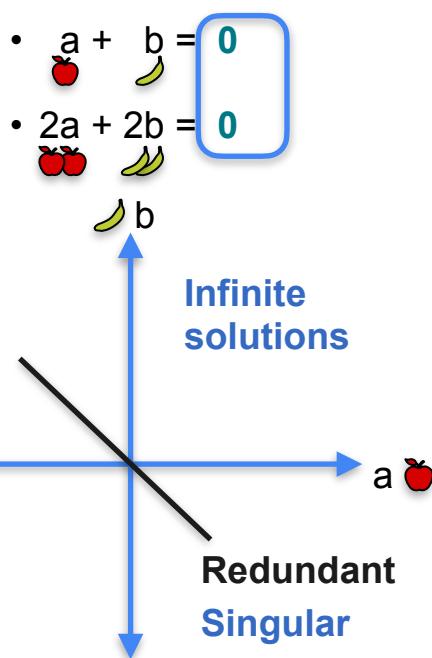


# Systems of equations as lines

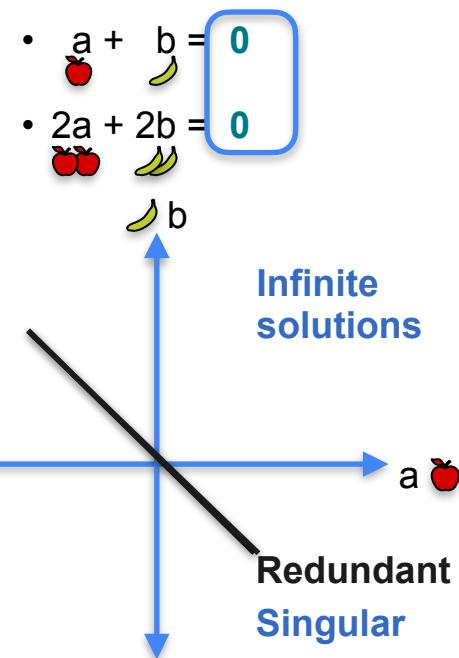
System 1



System 2



System 3





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# System of Linear Equations

---

**Singular vs nonsingular  
matrices**

# Systems of equations as matrices

## System 1

- $a + b = 0$   
 
- $a + 2b = 0$   
 

## System 2

- $a + b = 0$   
 
- $2a + 2b = 0$   
 

# Systems of equations as matrices

System 1

-    $= 0$
-    $= 0$

	
1	1
1	2

System 2

-    $= 0$
-    $= 0$

# Systems of equations as matrices

System 1

-    $= 0$
-    $= 0$

	
1	1
1	2

System 2

-    $= 0$
-    $= 0$

	
1	1
2	2

# Systems of equations as matrices

System 1

-    $= 0$
-    $= 0$

	
1	1
1	2

Non-singular  
system

(Unique solution)

System 2

-    $= 0$
-    $= 0$

	
1	1
2	2

# Systems of equations as matrices

System 1

-    $= 0$
-    $= 0$

	
1	1
1	2

Non-singular  
system

Non-singular  
matrix

(Unique solution)

System 2

-    $= 0$
-    $= 0$

	
1	1
2	2

# Systems of equations as matrices

System 1

-    $= 0$
-    $= 0$

	
1	1
1	2

Non-singular  
system

Non-singular  
matrix

(Unique solution)

System 2

-    $= 0$
-    $= 0$

	
1	1
2	2

Singular  
system

(Infinitely many solutions)

# Systems of equations as matrices

System 1

-    $= 0$
-    $= 0$

	
1	1
1	2

Non-singular  
system

Non-singular  
matrix

(Unique solution)

System 2

-    $= 0$
-    $= 0$

	
1	1
2	2

Singular  
system

Singular  
matrix

(Infinitely many solutions)



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# System of Linear Equations

---

**Linear dependence and  
independence**

# Linear dependence between rows

## Non-singular

- $a + b = 0$   
 
- $a + 2b = 0$   
 

	
1	1
1	2

## Singular system

- $a + b = 0$   
 
- $2a + 2b = 0$   
 

	
1	1
2	2

# Linear dependence between rows

## Non-singular

-    $= 0$
-    $= 0$

	
1	1
1	2

## Singular system

-    $= 0$
-    $= 0$

	
1	1
2	2

Second equation is  
a multiple of the  
first one

# Linear dependence between rows

## Non-singular

-    $a + b = 0$
-    $a + 2b = 0$

	
1	1
1	2

## Singular system

-    $a + b = 0$
-    $2a + 2b = 0$

	
1	1
2	2

Second equation is a multiple of the first one

Second row is a multiple of the first row

# Linear dependence between rows

## Non-singular

-    $a + b = 0$
-    $a + 2b = 0$

	
1	1
1	2

## Singular system

-    $a + b = 0$
-    $2a + 2b = 0$

	
1	1
2	2

Second equation is a multiple of the first one

Second row is a multiple of the first row

Rows are *linearly dependent*

# Linear dependence between rows

## Non-singular

- $a + b = 0$   
 
- $a + 2b = 0$   
 

	
1	1
1	2

No equation is a multiple of the other one

## Singular system

- $a + b = 0$   
 
- $2a + 2b = 0$   
 

	
1	1
2	2

Second equation is a multiple of the first one

Second row is a multiple of the first row

Rows are *linearly dependent*

# Linear dependence between rows

## Non-singular

- $a + b = 0$   
 
- $a + 2b = 0$   
 

	
1	1
1	2

No equation is a multiple of the other one

No row is a multiple of the other one

## Singular system

- $a + b = 0$   
 
- $2a + 2b = 0$   
 

	
1	1
2	2

Second equation is a multiple of the first one

Second row is a multiple of the first row

Rows are *linearly dependent*

# Linear dependence between rows

## Non-singular

- $a + b = 0$   
 
- $a + 2b = 0$   
 

	
1	1
1	2

No equation is a multiple of the other one

No row is a multiple of the other one

Rows are *linearly independent*

## Singular system

- $a + b = 0$   
 
- $2a + 2b = 0$   
 

	
1	1
2	2

Second equation is a multiple of the first one

Second row is a multiple of the first row

Rows are *linearly dependent*



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# System of Linear Equations

---

## The determinant

# Linear dependence between rows

Non-singular matrix



1	1
1	2

Singular matrix



1	1
2	2

# Linear dependence between rows

Non-singular matrix



1	1
1	2

Singular matrix



1	1
2	2



# Linear dependence between rows

Non-singular matrix



1	1
1	2

Singular matrix



1	1
2	2



1	1
---	---

# Linear dependence between rows

Non-singular matrix



1	1
1	2

Singular matrix



1	1
2	2



$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline \end{array} \times 2 =$$

# Linear dependence between rows

Non-singular matrix



1	1
1	2

Singular matrix



1	1
2	2



$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

# Linear dependence between rows

Non-singular matrix



1	1
1	2

Singular matrix



1	1
2	2

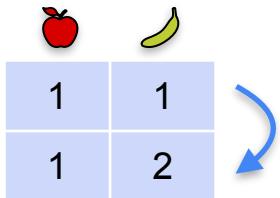


$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

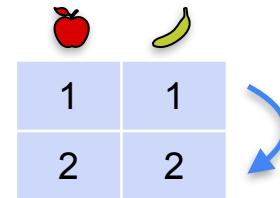
# Linear dependence between rows

Non-singular matrix



apple	banana
1	1
1	2

Singular matrix



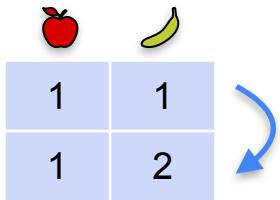
apple	banana
1	1
2	2

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

# Linear dependence between rows

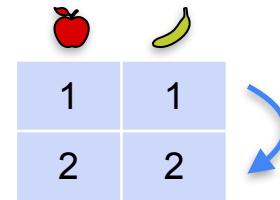
Non-singular matrix



1	1
1	2

$$\begin{bmatrix} 1 & 1 \end{bmatrix}$$

Singular matrix



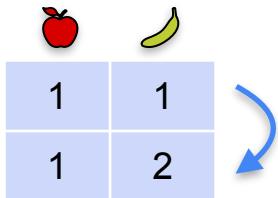
1	1
2	2

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

# Linear dependence between rows

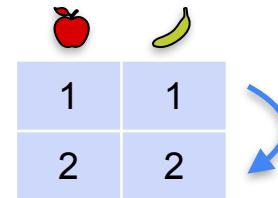
Non-singular matrix



1	2
---	---

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times ? =$$

Singular matrix



2	2
---	---

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

# Linear dependence between rows

Non-singular matrix



1	1
1	2

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times ? = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Singular matrix



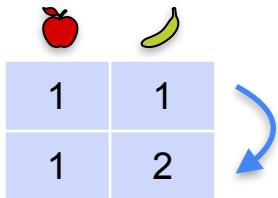
1	1
2	2

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

# Linear dependence between rows

Non-singular matrix

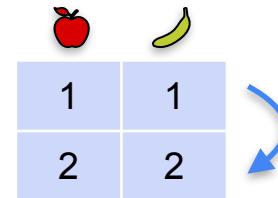


	
1	1
1	2

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times ? = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Rows linearly independent

Singular matrix

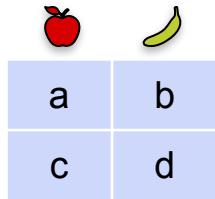


	
1	1
2	2

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

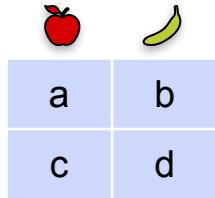
# Determinant



**Matrix is singular if**

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

# Determinant

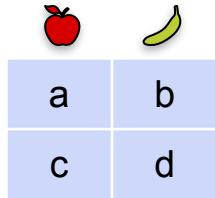


$$ak = c$$

**Matrix is singular if**

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

# Determinant



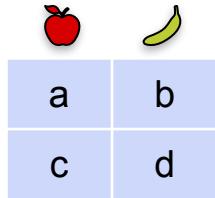
$$ak = c$$

$$bk = d$$

**Matrix is singular if**

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

# Determinant



$$ak = c$$

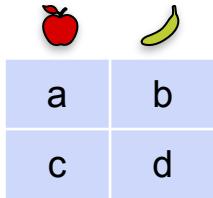
$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

**Matrix is singular if**

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

# Determinant



$$ak = c$$

$$bk = d$$

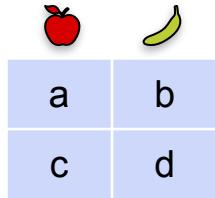
$$\frac{c}{a} = \frac{d}{b} = k$$

**Matrix is singular if**

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

$$ad = bc$$

# Determinant



$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

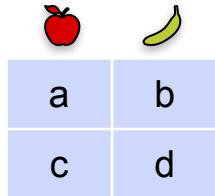
**Matrix is singular if**

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

$$ad = bc$$

$$ad - bc = 0$$

# Determinant



$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

Determinant

$$ad = bc$$

$$ad - bc = 0$$

# Determinant

	
a	b
c	d

$$\text{Determinant} = ad - bc$$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

Determinant

$$ad = bc$$

$$ad - bc = 0$$

# Determinant

	
a	b
c	d

$$\text{Determinant} = ad - bc$$

$$\begin{matrix} a \\ & d \end{matrix}$$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{matrix} a & b \end{matrix} * k = \begin{matrix} c & d \end{matrix}$$

Determinant

$$ad = bc$$

$$ad - bc = 0$$

# Determinant

	
a	b
c	d

$$\text{Determinant} = ad - bc$$

$$\begin{matrix} a & \\ & d \end{matrix} -$$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{matrix} a & b \end{matrix} * k = \begin{matrix} c & d \end{matrix}$$

Determinant

$$ad = bc$$

$$ad - bc = 0$$

# Determinant

	
a	b
c	d

$$\text{Determinant} = ad - bc$$

$$\begin{matrix} a & & b \\ & d & - \\ & c & \end{matrix}$$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{matrix} a & b \\ c & d \end{matrix} * k = \begin{matrix} a & b \\ c & d \end{matrix}$$

Determinant

$$ad = bc$$

$$ad - bc = 0$$

# Determinant

Non-singular matrix

	
1	1
1	2

Singular matrix

	
1	1
2	2

# Determinant

Non-singular matrix

	
1	1
1	2

Singular matrix

	
1	1
2	2

Determinant

$$1 \begin{matrix} 1 \\ 2 \end{matrix} - \begin{matrix} 1 \\ 1 \end{matrix} 1$$

# Determinant

Non-singular matrix

	
1	1
1	2

Singular matrix

	
1	1
2	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & - & \\ 2 & & 1 \end{matrix}$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

# Determinant

Non-singular matrix

	
1	1
1	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & - & \\ 2 & & 1 \end{matrix}$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

	
1	1
2	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & - & \\ 2 & & 2 \end{matrix}$$

# Determinant

Non-singular matrix

	
1	1
1	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & - & \\ 2 & & 1 \end{matrix}$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

	
1	1
2	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & - & \\ 2 & & 2 \end{matrix}$$

$$1 \cdot 2 - 2 \cdot 1 = 0$$

# Determinant

Non-singular matrix

	
1	1
1	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & 2 & \\ & & 1 \end{matrix} -$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

	
1	1
2	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & 2 & \\ & & 2 \end{matrix} -$$

$$1 \cdot 2 - 2 \cdot 1 = 0$$

# Determinant

Non-singular matrix

	
1	1
1	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & 2 & \\ & & 1 \end{matrix} -$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

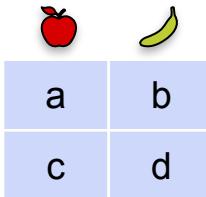
	
1	1
2	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & 2 & \\ & & 1 \end{matrix} -$$

$$1 \cdot 2 - 2 \cdot 1 = 0$$

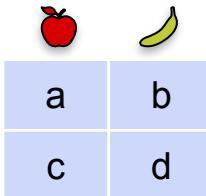
# Determinant and singularity



a	b
c	d

$$ad - bc$$

# Determinant and singularity



a	b
c	d

$$ad - bc$$

Matrix is singular



Determinant is zero

# Quiz: Determinant

**Problem 1:** Find the determinant of the following matrices

**Matrix 1**

5	1
-1	3

**Matrix 2**

2	-1
-6	3

**Problem 2:** Are these matrices singular or non-singular?

# Solutions: Determinant

**Matrix 1:**  $\det = 5 \cdot 3 - 1 \cdot (-1) = 15 + 1 = 16$

5	1
-1	3

**Non-singular**

**Matrix 2:**  $\det = 2 \cdot 3 - (-1) \cdot (-6) = 6 - 6 = 0$

2	-1
-6	3

**Singular**



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# System of Linear Equations

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## System of equations (3x3)

# Quiz: Systems of equations

**Problem 1:** You're trying to figure out the price of apples, bananas, and cherries at the store. You go three days in a row, and bring this information.

- **Day 1:** You bought an apple, a banana, and a cherry, and paid \$10.
- **Day 2:** You bought an apple, two bananas, and a cherry, and paid \$15.
- **Day 3:** You bought an apple, a banana, and two cherries, and paid \$12.

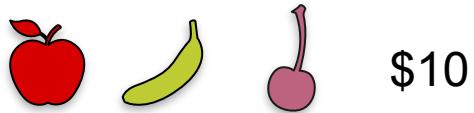
How much does each fruit cost?

# Solution: Systems of equations

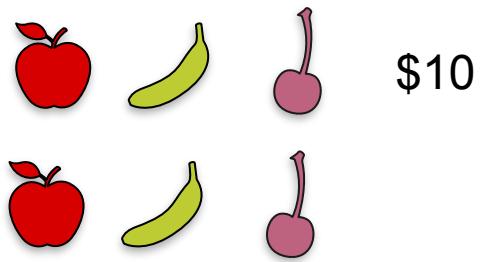
# Solution: Systems of equations



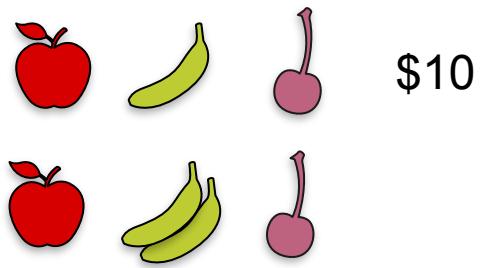
# Solution: Systems of equations



# Solution: Systems of equations

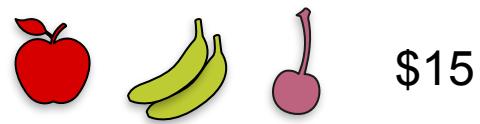


# Solution: Systems of equations



# Solution: Systems of equations


$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \\ \$10 \end{array}$$


$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \\ \$15 \end{array}$$

# Solution: Systems of equations



# Solution: Systems of equations

$$\begin{array}{c} \text{apple} \quad \text{banana} \quad \text{cherry} \\ \text{apple} \quad \text{banana} \quad \text{cherry} \end{array} \quad \$10$$

$$\begin{array}{c} \text{apple} \quad \text{banana} \quad \text{cherry} \\ \text{apple} \quad \text{banana} \quad \text{cherry} \end{array} \quad \$15 \quad \rightarrow \quad \text{banana} \quad \$5$$

$$\begin{array}{c} \text{apple} \quad \text{banana} \quad \text{cherry} \\ \text{apple} \quad \text{banana} \quad \text{cherry} \end{array}$$

# Solution: Systems of equations



# Solution: Systems of equations

$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \\ \text{1} & \text{1} & \text{1} \end{array} \quad \$10$$

$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \\ \text{1} & \text{2} & \text{1} \end{array} \quad \$15 \quad \rightarrow \quad \text{banana} \quad \$5$$

$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \\ \text{1} & \text{1} & \text{2} \end{array} \quad \$12$$

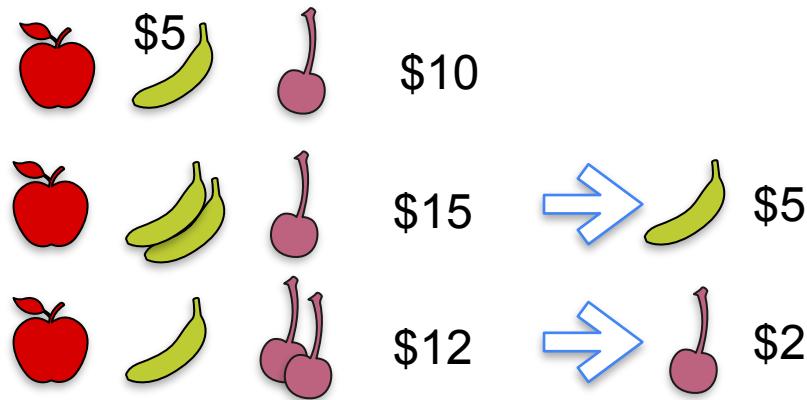
# Solution: Systems of equations

$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \\ \text{1} & \text{1} & \text{1} \end{array} \quad \$10$$

$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \\ \text{1} & \text{2} & \text{1} \end{array} \quad \$15 \quad \xrightarrow{\text{banana}} \quad \text{banana} \quad \$5$$

$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \\ \text{1} & \text{1} & \text{2} \end{array} \quad \$12 \quad \xrightarrow{\text{cherry}} \quad \text{cherry} \quad \$2$$

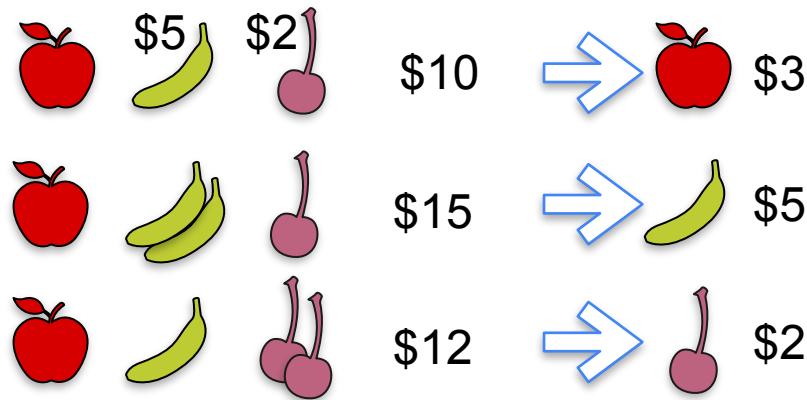
# Solution: Systems of equations



# Solution: Systems of equations

$$\begin{array}{ccc} \text{apple} & \$5 & \text{banana} & \$2 \\ \text{cherry} & \$2 & & \\ \hline & \$10 & & \end{array} \rightarrow \begin{array}{ccc} \text{apple} & \$3 & \\ & & \end{array}$$
$$\begin{array}{ccc} \text{apple} & & \text{banana} \\ \text{banana} & & \\ \text{cherry} & & \\ \hline & \$15 & \end{array} \rightarrow \begin{array}{ccc} & \text{banana} & \$5 \\ & & \end{array}$$
$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \\ & \$2 & \\ \hline & \$12 & \end{array} \rightarrow \begin{array}{ccc} & \text{cherry} & \$2 \\ & & \end{array}$$

# Solution: Systems of equations



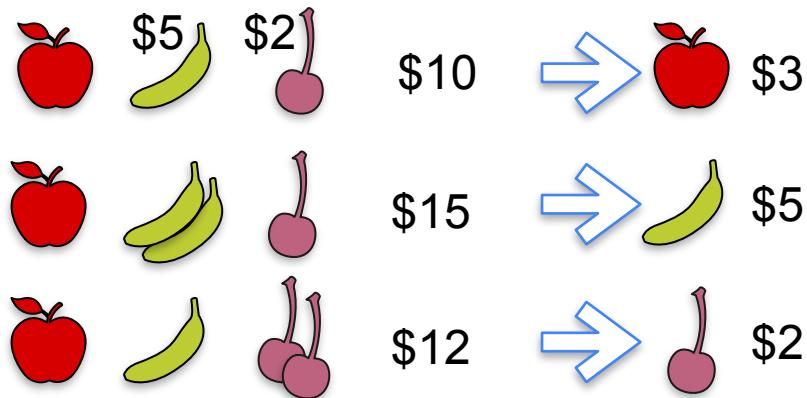
## System of equations 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

# Solution: Systems of equations



## System of equations 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## Solution

$$a = 3$$

$$b = 5$$

$$c = 2$$

# Quiz: More systems of equations

## System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

## System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

# Solutions: More systems of equations

## System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

## System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

# Solutions: More systems of equations

## System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

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$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

## System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

**Infinitely many sols.**

# Solutions: More systems of equations

## System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

## System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

**Infinitely many sols.**

$$c = 5$$

# Solutions: More systems of equations

## System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

## System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

**Infinitely many sols.**

$$c = 5$$

$$a + b = 5$$

# Solutions: More systems of equations

## System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

## System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

**Infinitely many sols.**

$$c = 5$$

$$a + b = 5$$

$$(0,5,5), (1,4,5), (2,3,5), \dots$$

# Solutions: More systems of equations

## System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

**Infinitely many sols.**

$$c = 5$$

$$a + b = 5$$

$$(0,5,5), (1,4,5), (2,3,5), \dots$$

## System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

**No solutions**

## System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

# Solutions: More systems of equations

## System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

**Infinitely many sols.**

$$\begin{aligned}c &= 5 \\a + b &= 5 \\(0,5,5), (1,4,5), (2,3,5), \dots\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

**No solutions**

$$\begin{aligned}\text{From 1st and 2nd:} \\c &= 5 \\\text{From 2nd and 3rd:} \\c &= 3\end{aligned}$$

## System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

# Solutions: More systems of equations

## System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

**Infinitely many sols.**

$$\begin{aligned}c &= 5 \\a + b &= 5 \\(0,5,5), (1,4,5), (2,3,5), \dots\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

**No solutions**

$$\begin{aligned}\text{From 1st and 2nd:} \\c &= 5 \\\text{From 2nd and 3rd:} \\c &= 3\end{aligned}$$

## System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

**Infinitely many solutions**

# Solutions: More systems of equations

## System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

**Infinitely many sols.**

$$\begin{aligned}c &= 5 \\a + b &= 5 \\(0,5,5), (1,4,5), (2,3,5), \dots\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

**No solutions**

$$\begin{aligned}\text{From 1st and 2nd:} \\c &= 5 \\\text{From 2nd and 3rd:} \\c &= 3\end{aligned}$$

## System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

**Infinitely many solutions**

$$\begin{aligned}\text{Any 3 numbers that add} \\ \text{to 10 work.} \\ (0,0,10), (2,7,1), \dots\end{aligned}$$



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# System of Linear Equations

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**Singular vs non-singular  
matrices**

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## System 2

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## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Unique solution

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Unique solution

Infinite solutions

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Unique solution

Infinite solutions

No solutions

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

Redundant

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Unique solution

Complete

Infinite solutions

Redundant

No solutions

Contradictory

Infinite solutions

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Unique solution

Complete

Infinite solutions

Redundant

No solutions

Contradictory

Infinite solutions

Redundant

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Unique solution

Complete

Non-singular

Infinite solutions

Redundant

No solutions

Contradictory

Infinite solutions

Redundant

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Unique solution

Complete

Non-singular

Infinite solutions

Redundant

Singular

No solutions

Contradictory

Infinite solutions

Redundant

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Unique solution

Complete

Non-singular

Infinite solutions

Redundant

Singular

No solutions

Contradictory

Singular

Infinite solutions

Redundant

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Unique solution

Infinite solutions

No solutions

Infinite solutions

Complete

Redundant

Contradictory

Redundant

Non-singular

Singular

Singular

Singular

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$



$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$



$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$



$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$



$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

**Unique solution:**

$$a = 0$$

$$b = 0$$

$$c = 0$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

**Unique solution:**

$$a = 0$$

$$b = 0$$

$$c = 0$$

**Complete**

**Non-singular**

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

Infinite solutions:

$$c = 0$$

$$a + b = 0$$

(i.e.,  $a = -b$ )

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

**Unique solution:**

$$a = 0$$

$$b = 0$$

$$c = 0$$

**Complete**

**Non-singular**

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

**Infinite solutions:**

$$c = 0$$

$$a + b = 0$$

(i.e.,  $a = -b$ )

**Redundant**

**Singular**

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

**Unique solution:**

$$a = 0$$

$$b = 0$$

$$c = 0$$

**Complete**

**Non-singular**

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

**Infinite solutions:**

$$c = 0$$

$$a + b = 0$$

(i.e.,  $a = -b$ )

**Redundant**

**Singular**

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

**Infinite solutions:**

$$a + b + c = 0$$

(i.e.,  $c = -a - b$ )

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

Infinite solutions:

$$c = 0$$

$$a + b = 0$$

(i.e.,  $a = -b$ )

Redundant

Singular

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

Infinite solutions:

$$a + b + c = 0$$

(i.e.,  $c = -a - b$ )

Redundant

Singular

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

1	1	1
1	2	1
1	1	2

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

1	1	1
1	2	1
1	1	2

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

1	1	1
1	1	2
1	1	3

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

1	1	1
1	2	1
1	1	2

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

1	1	1
1	1	2
1	1	3

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

1	1	1
2	2	2
3	3	3

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

1	1	1
1	2	1
1	1	2

Non-singular

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

1	1	1
1	1	2
1	1	3

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

1	1	1
2	2	2
3	3	3

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

1	1	1
1	2	1
1	1	2

Non-singular

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

1	1	1
1	1	2
1	1	3

Singular

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

1	1	1
2	2	2
3	3	3

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

1	1	1
1	2	1
1	1	2

Non-singular

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

1	1	1
1	1	2
1	1	3

Singular

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

1	1	1
2	2	2
3	3	3

Singular



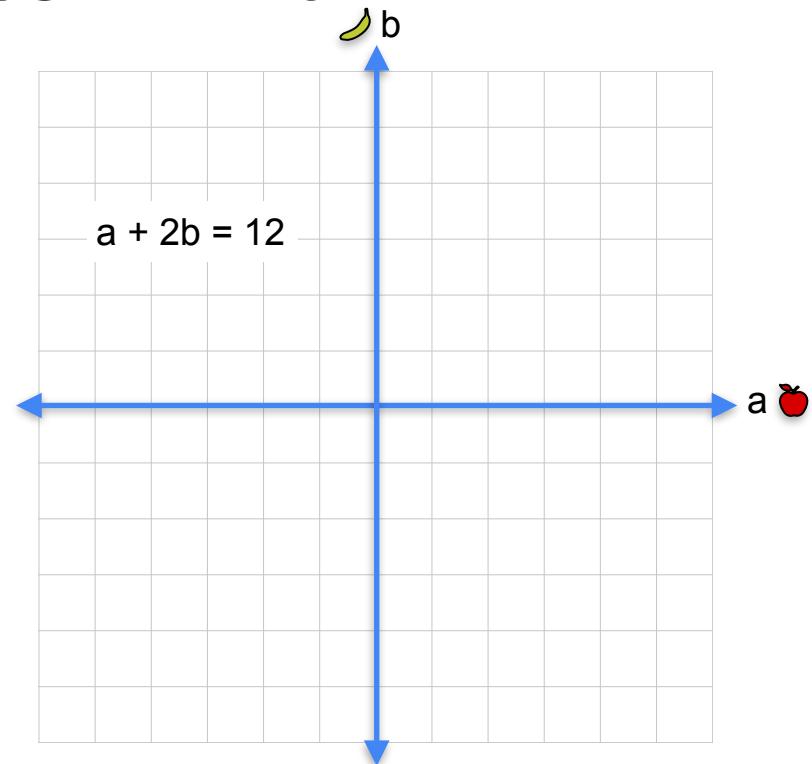
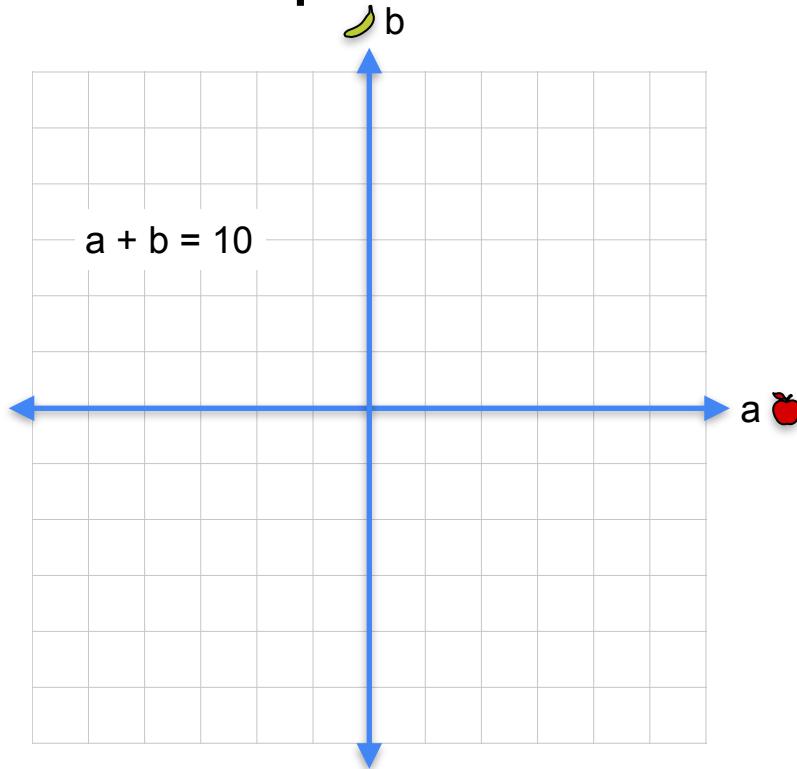
DeepLearning.AI

# System of Linear Equations

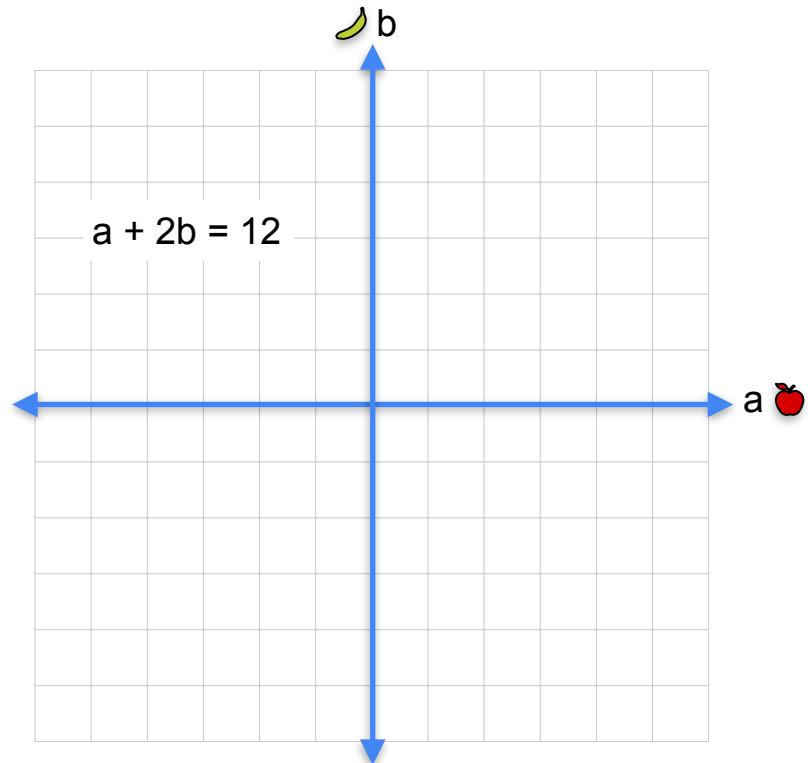
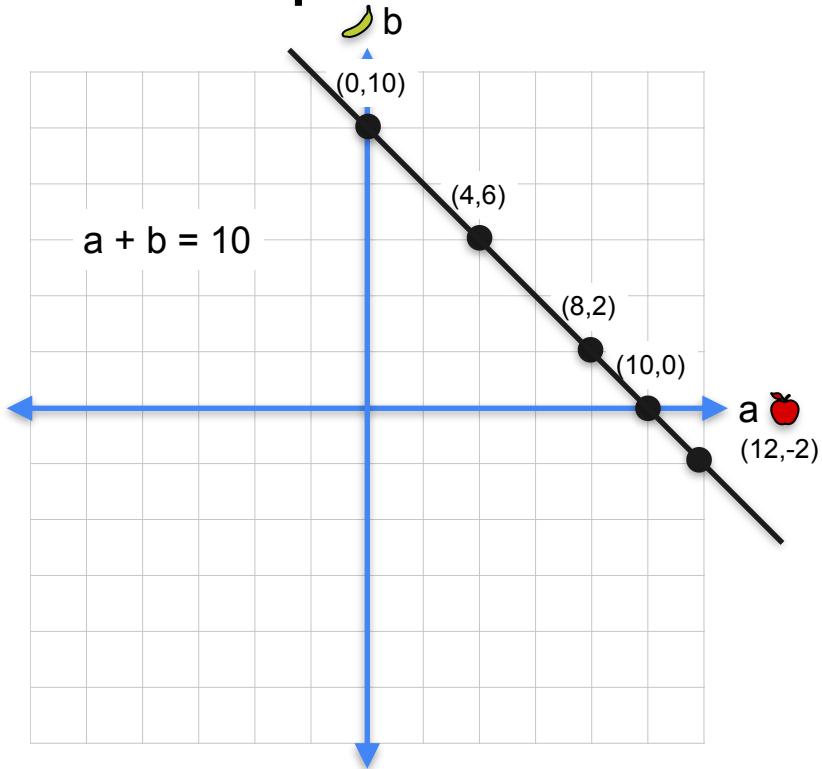
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**System of equations as  
planes (3x3)**

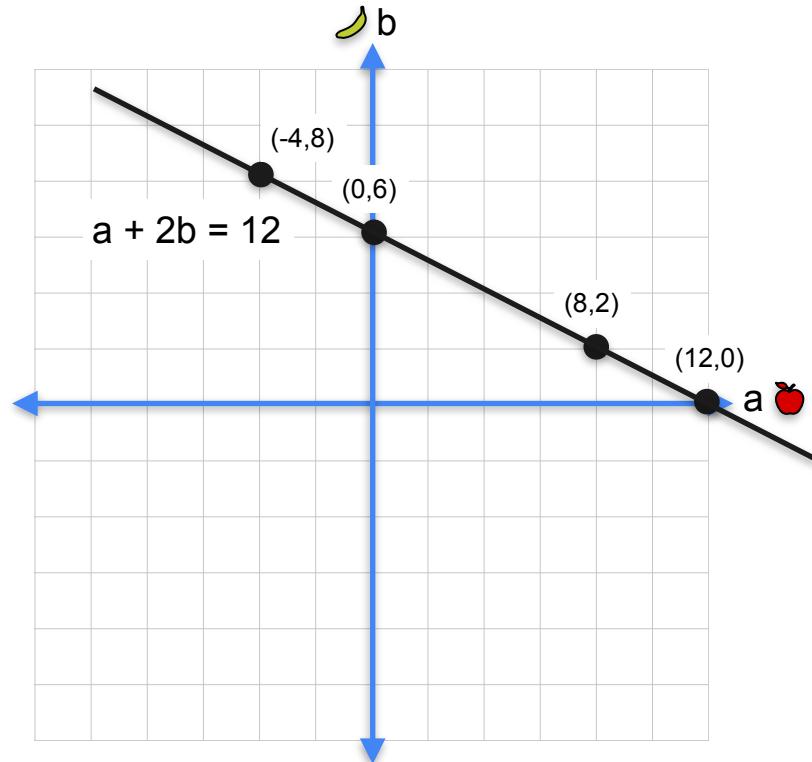
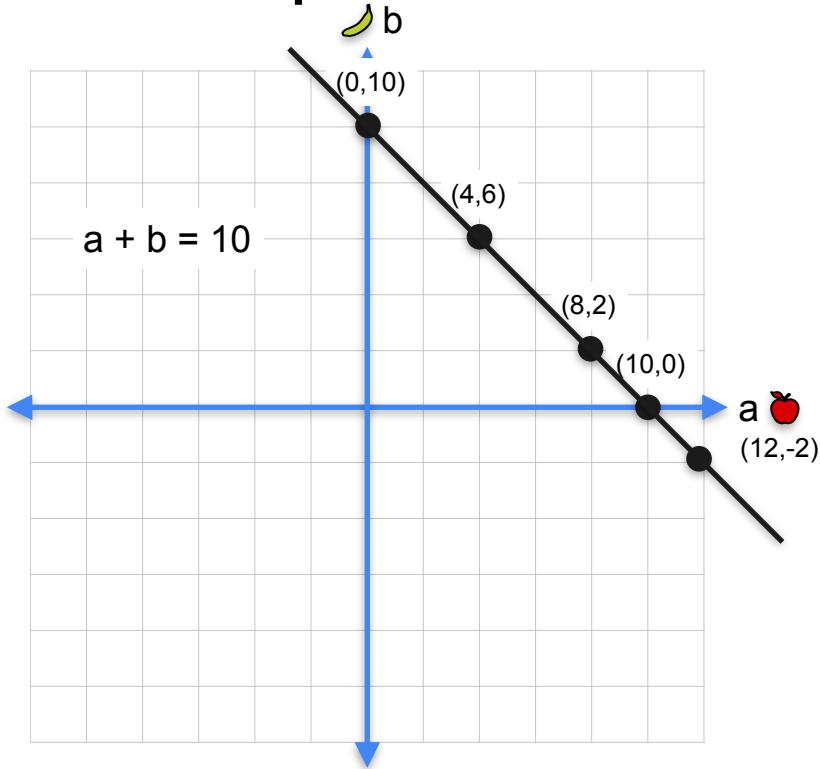
# Linear equation in 2 variables -> Line



# Linear equation in 2 variables -> Line

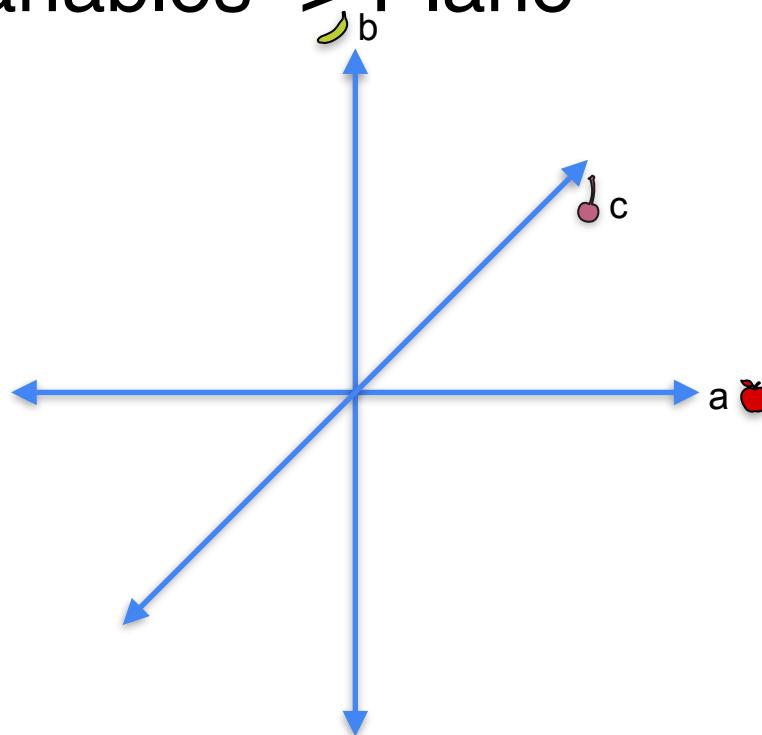


# Linear equation in 2 variables -> Line



# Linear equation in 3 variables -> Plane

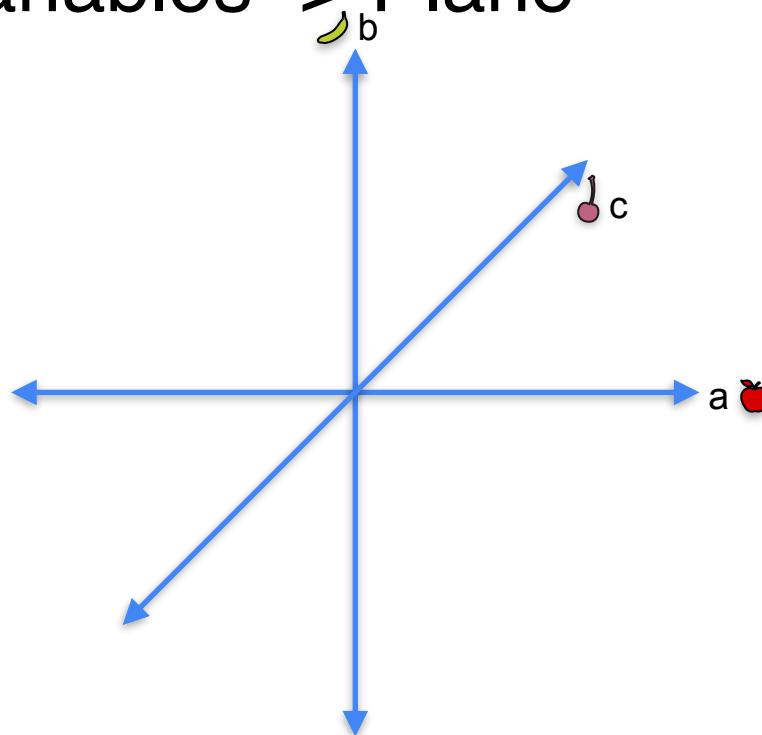
$$a + b + c = 1$$



# Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

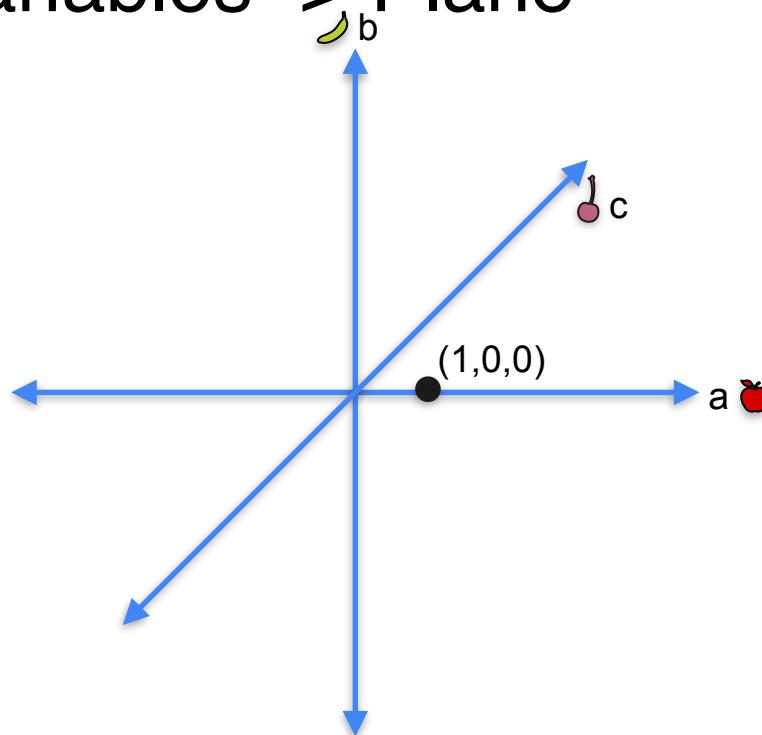
$$1 + 0 + 0 = 1$$



# Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

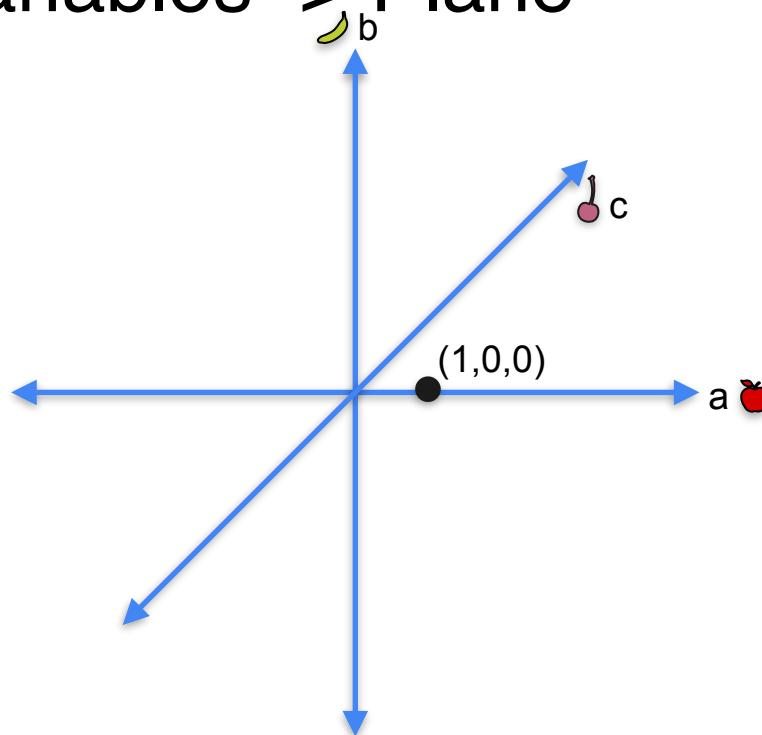


# Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

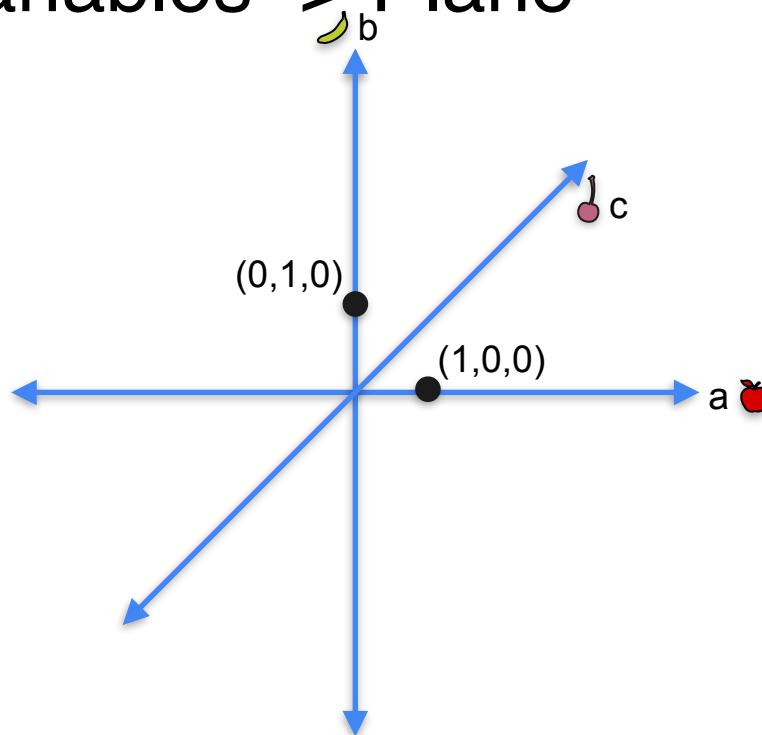


# Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$



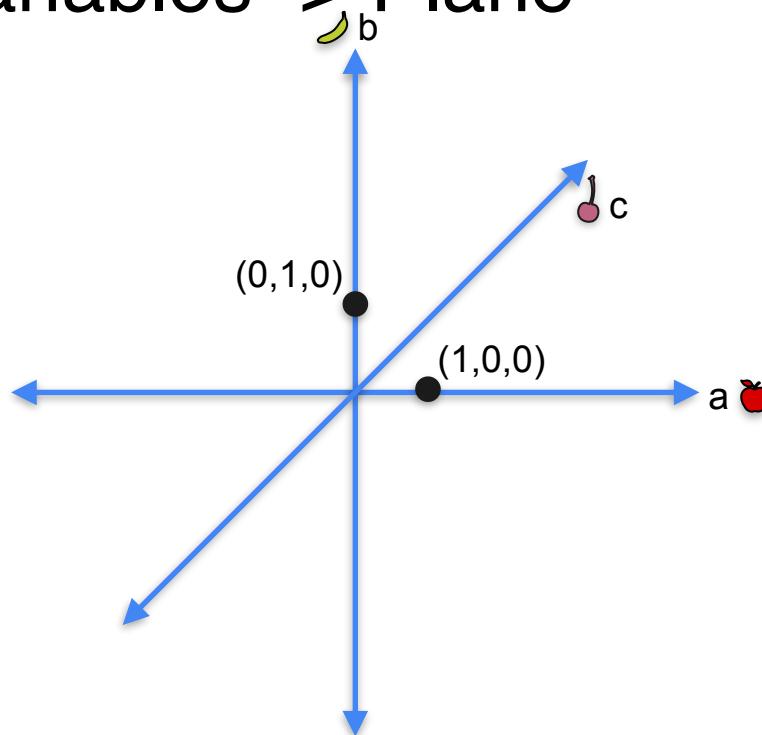
# Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



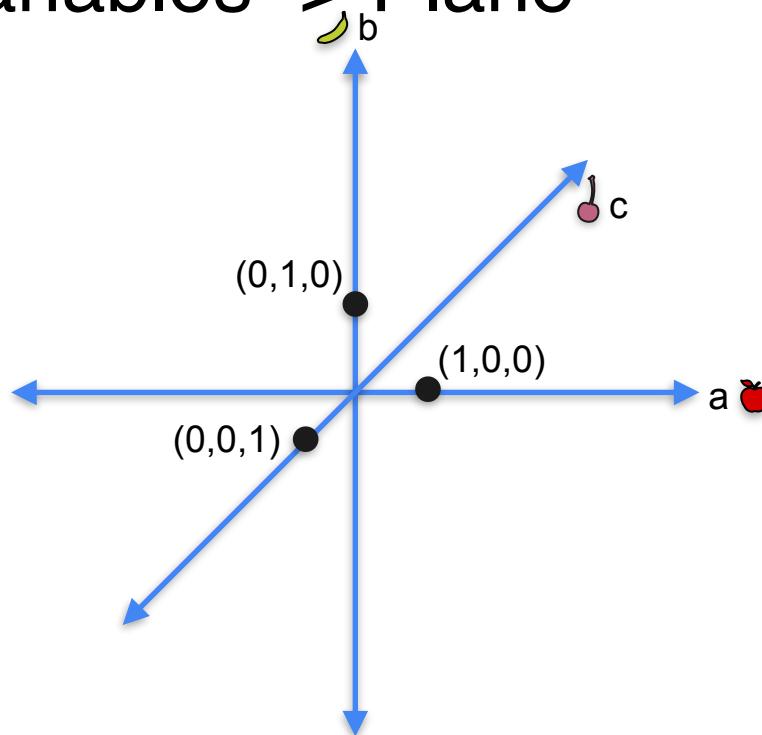
# Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



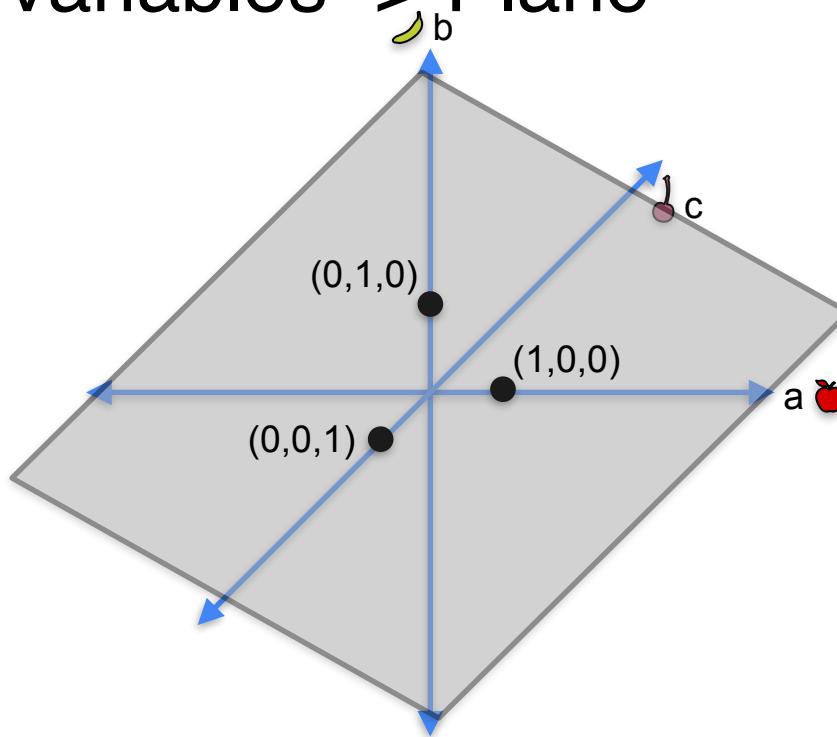
# Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

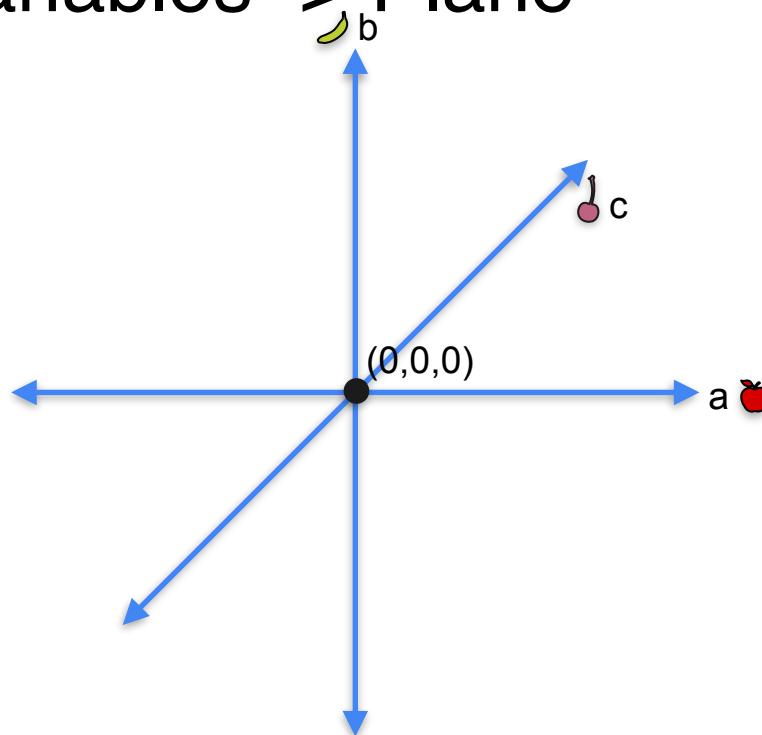
$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



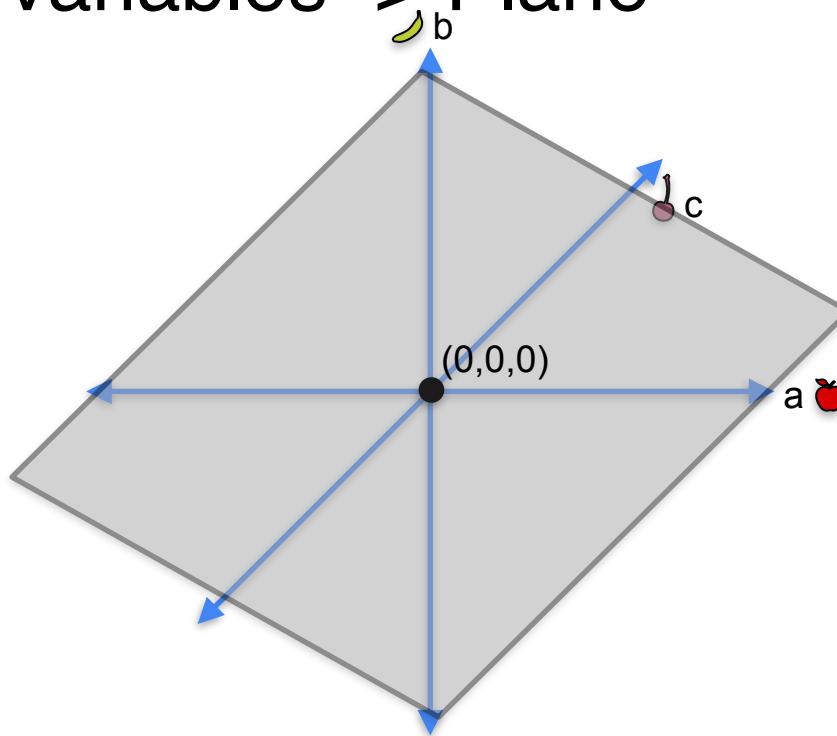
# Linear equation in 3 variables -> Plane

$$3a - 5b + 2c = 0$$



# Linear equation in 3 variables -> Plane

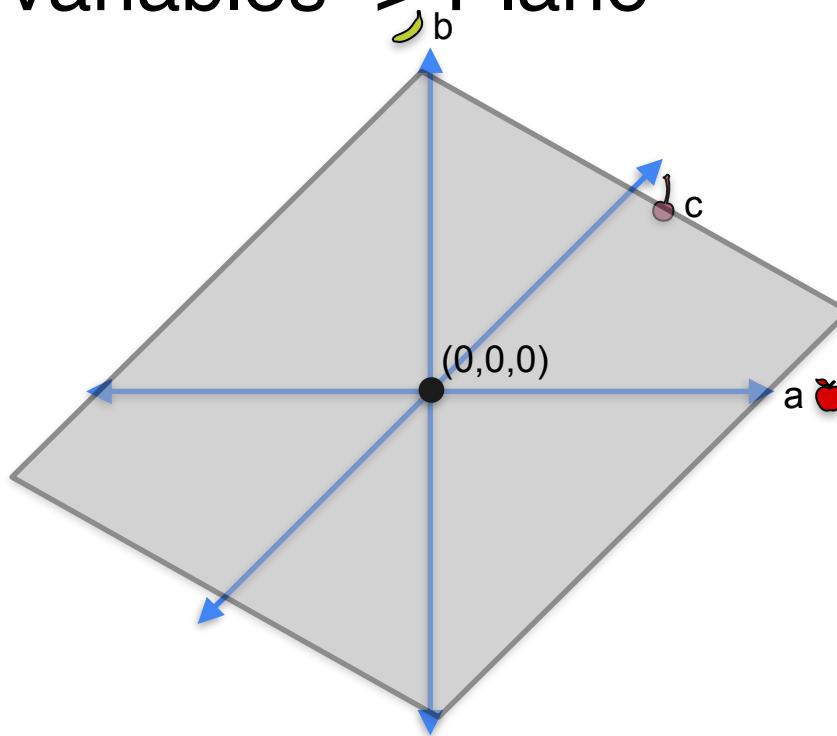
$$3a - 5b + 2c = 0$$



# Linear equation in 3 variables -> Plane

$$3a - 5b + 2c = 0$$

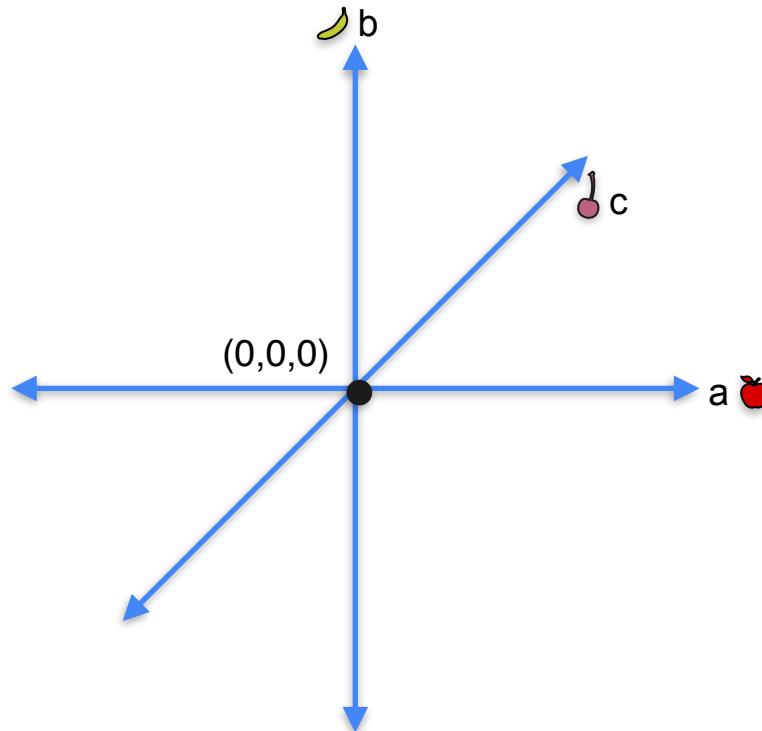
$$3(0) + 5(0) + 2(0) = 0$$



# System 1

## System 1

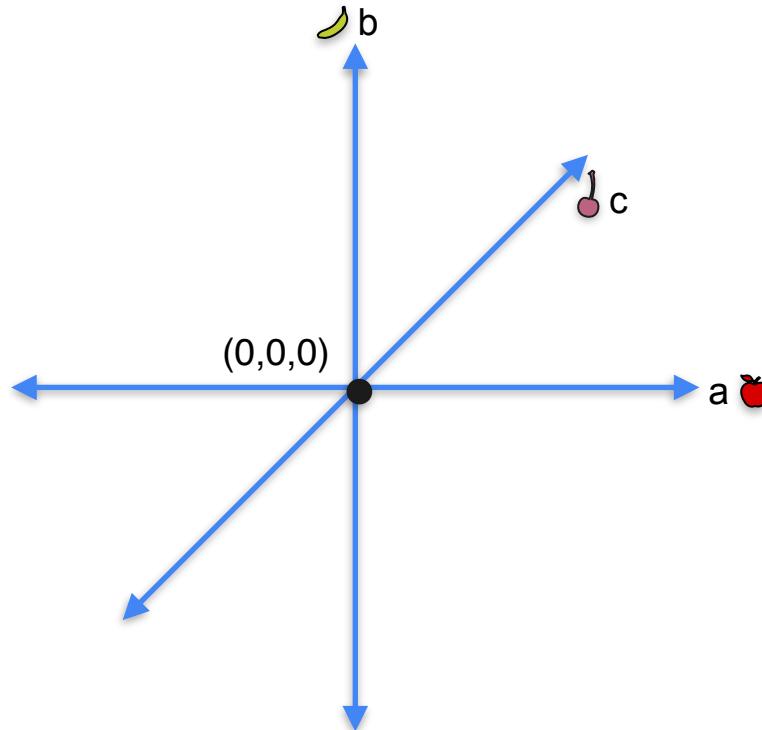
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



# System 1

## System 1

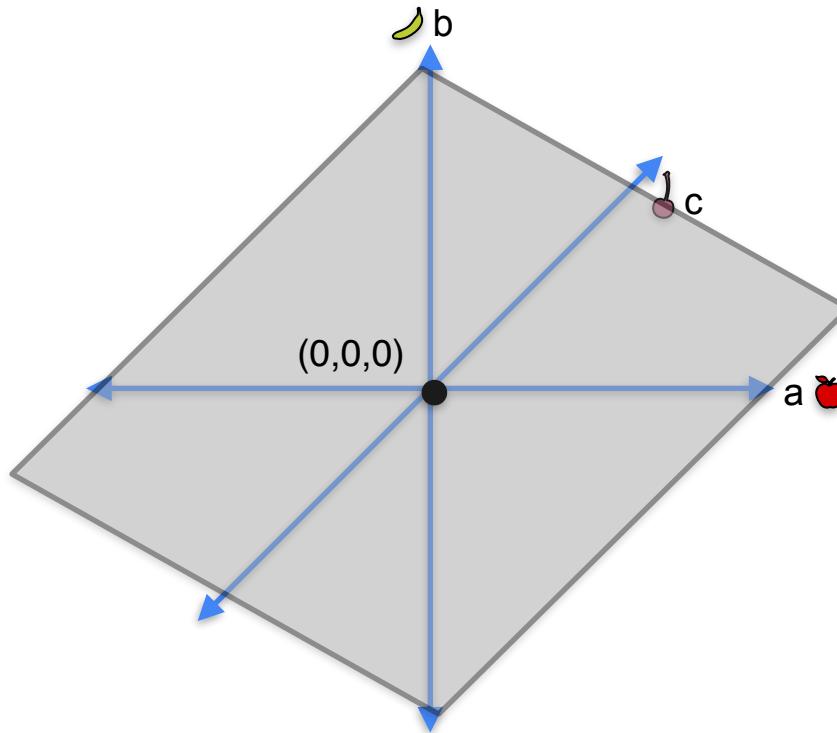
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



# System 1

## System 1

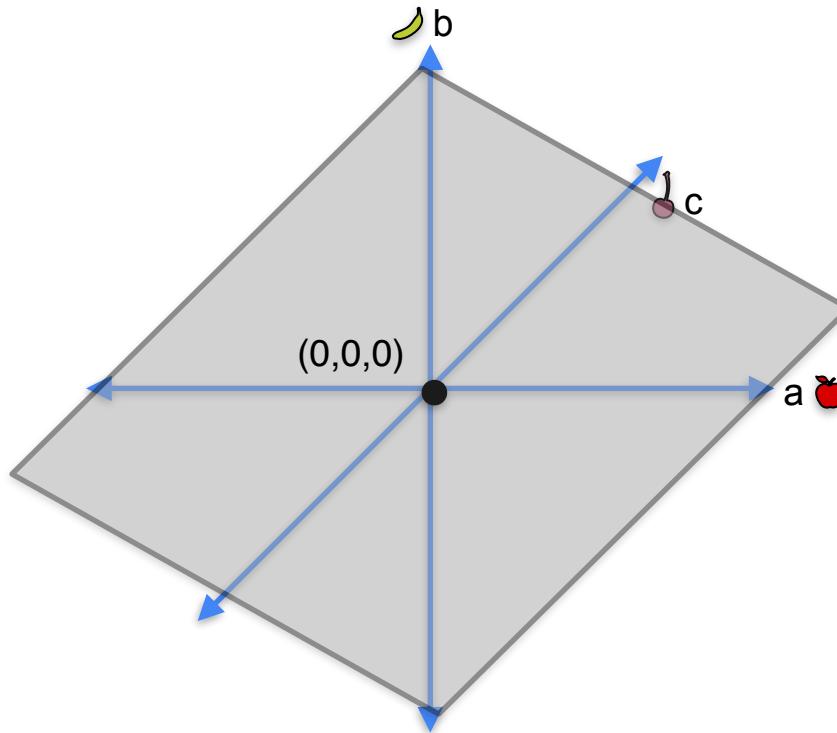
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



# System 1

## System 1

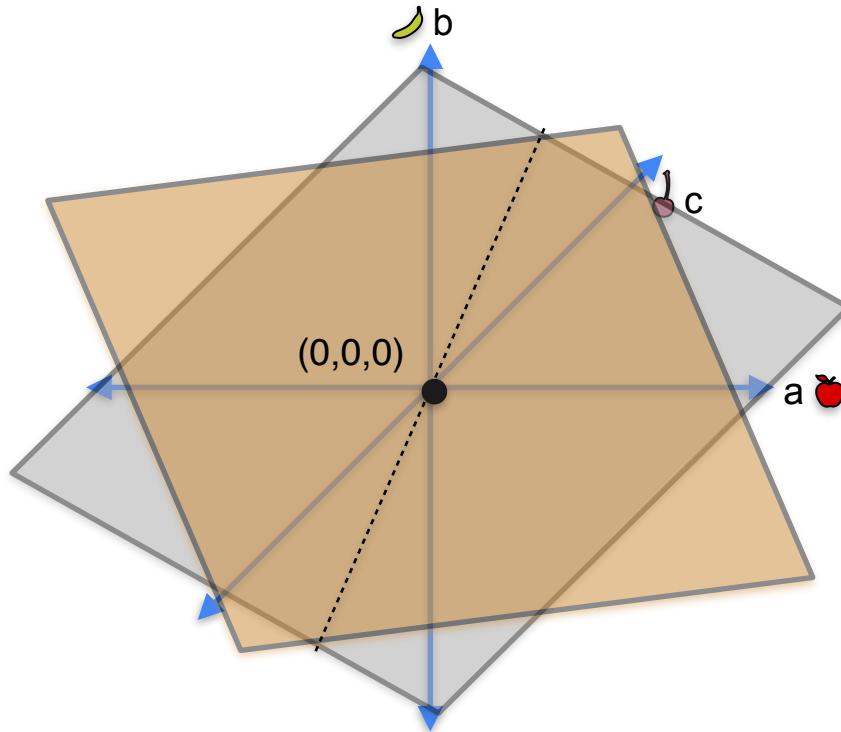
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



# System 1

## System 1

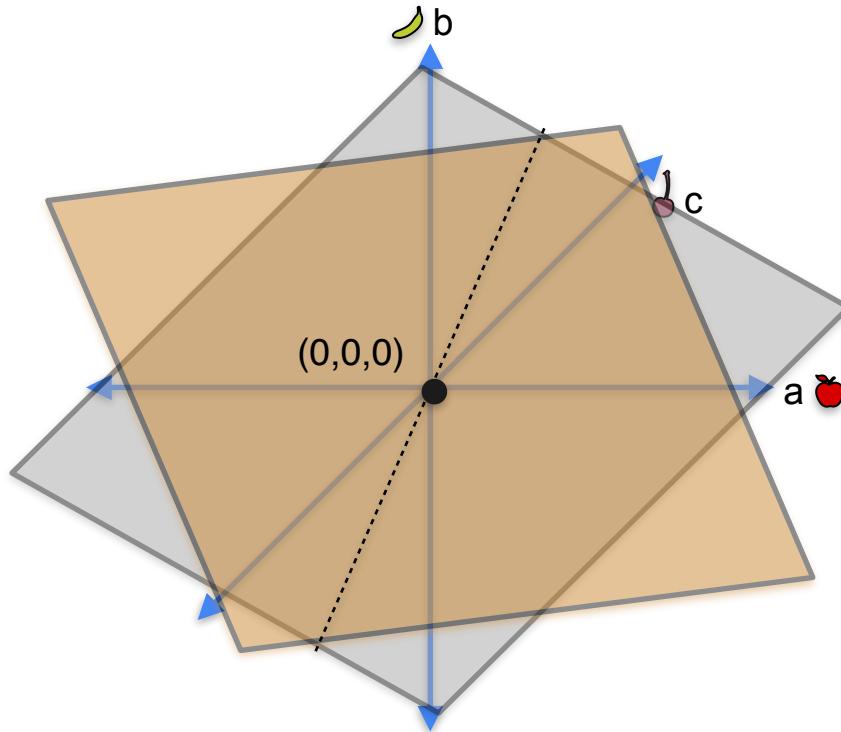
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



# System 1

## System 1

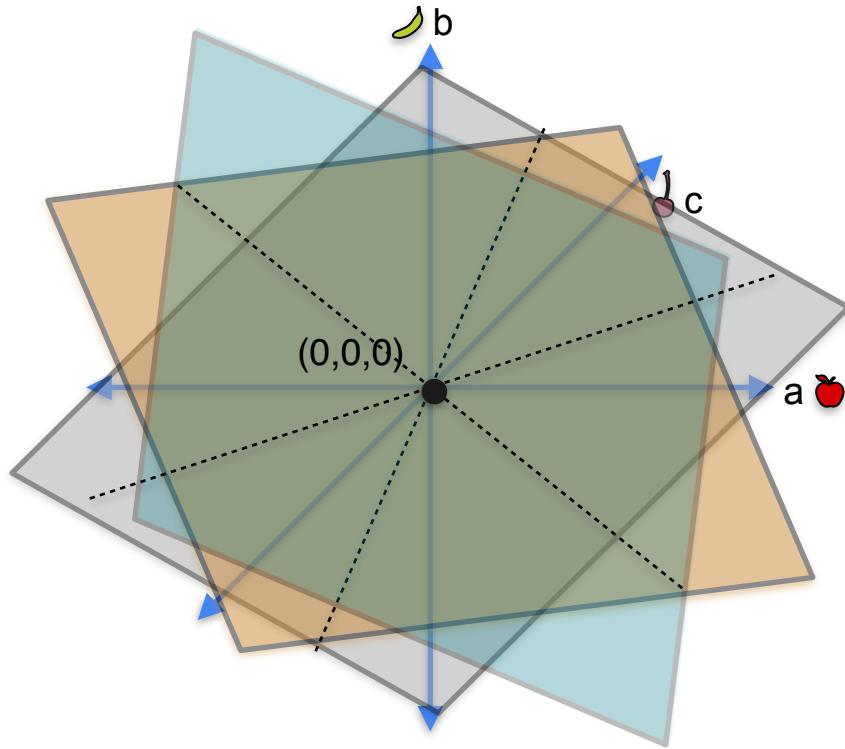
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



# System 1

## System 1

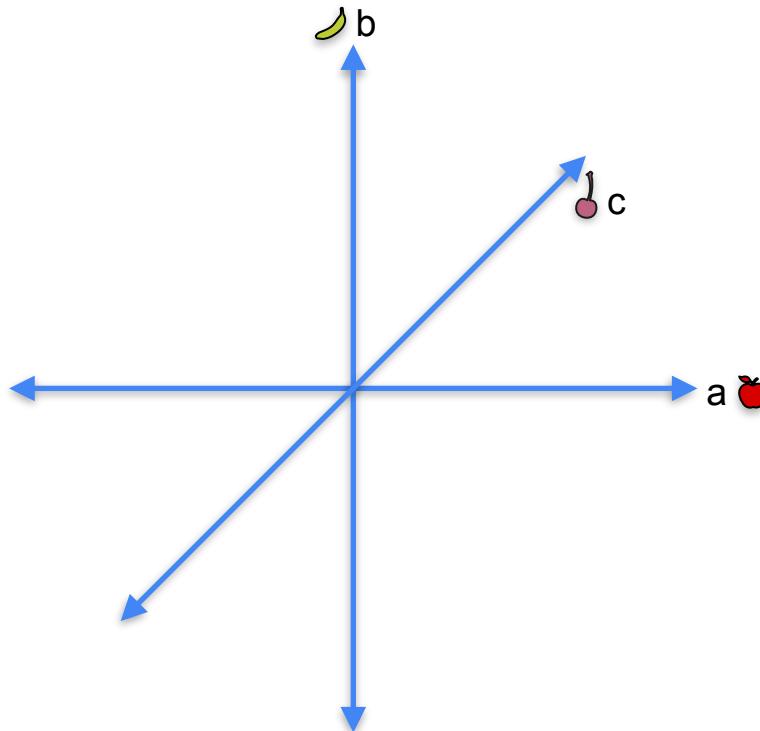
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



# System 2

## System 2

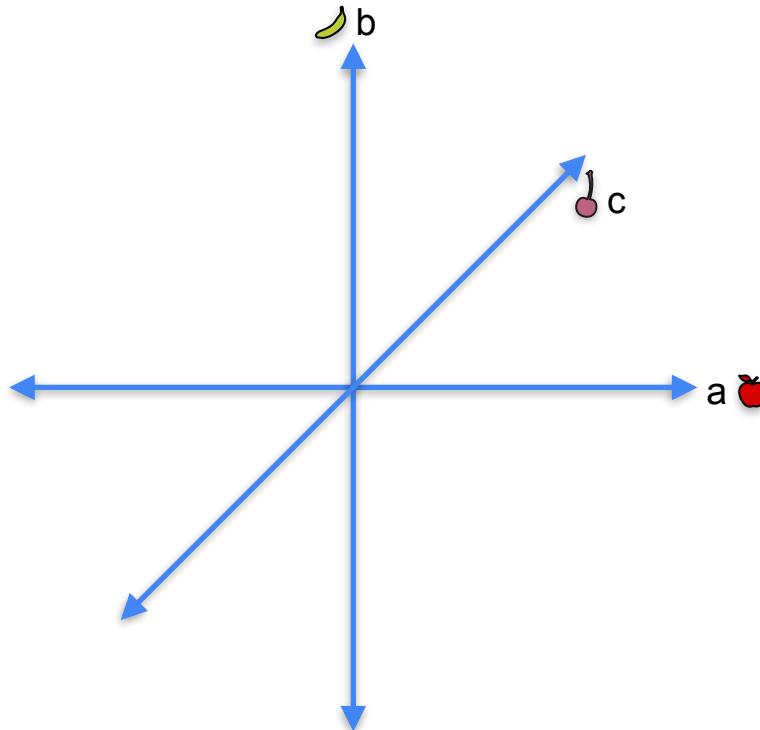
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



# System 2

## System 2

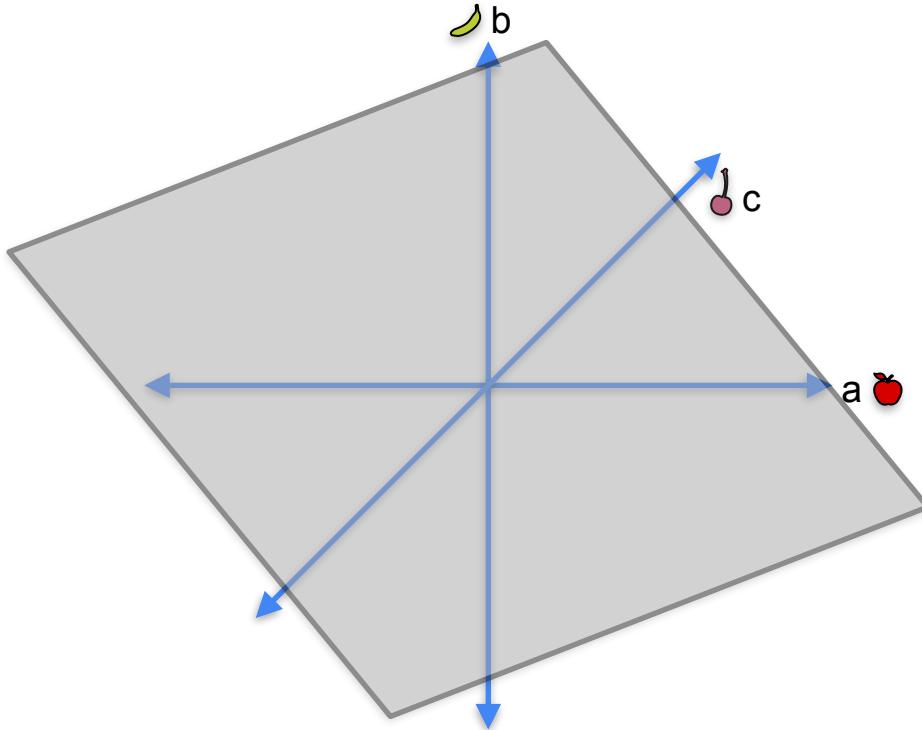
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



# System 2

## System 2

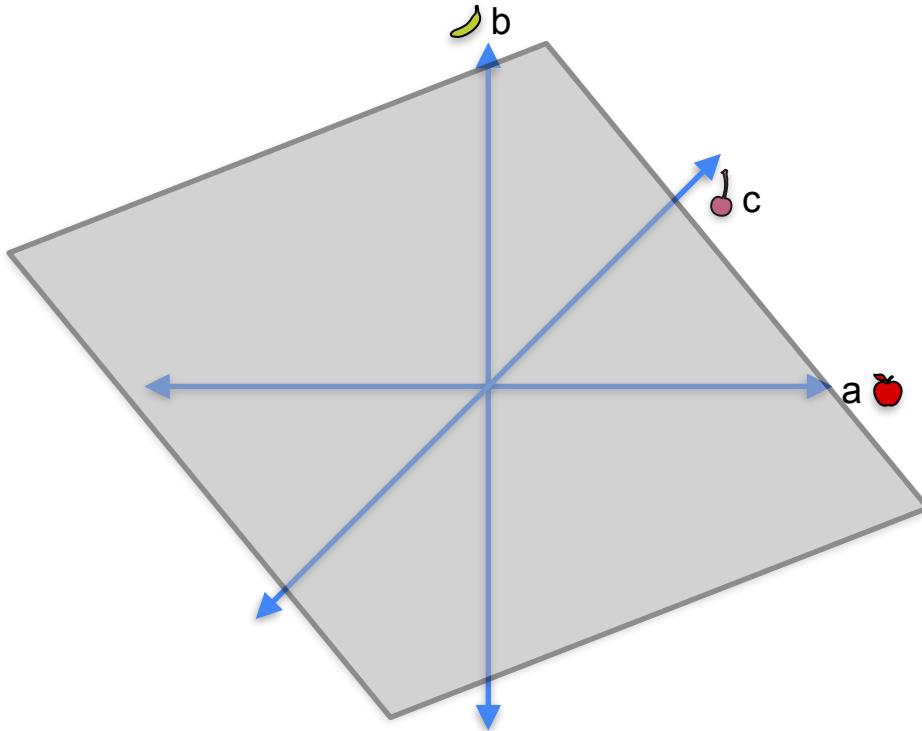
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



# System 2

## System 2

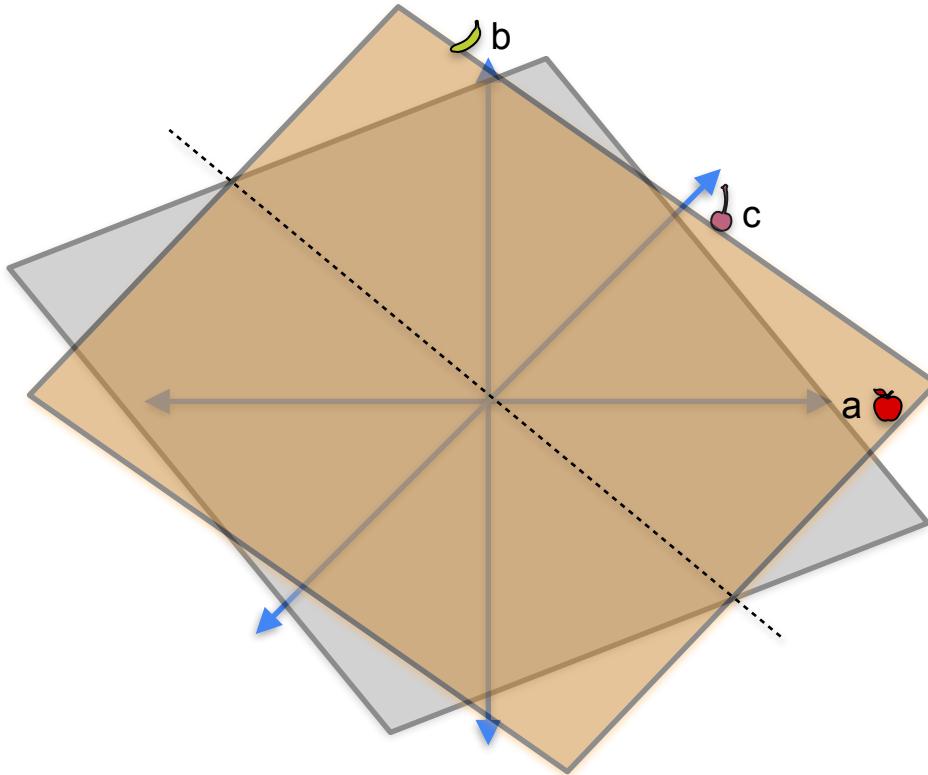
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



# System 2

## System 2

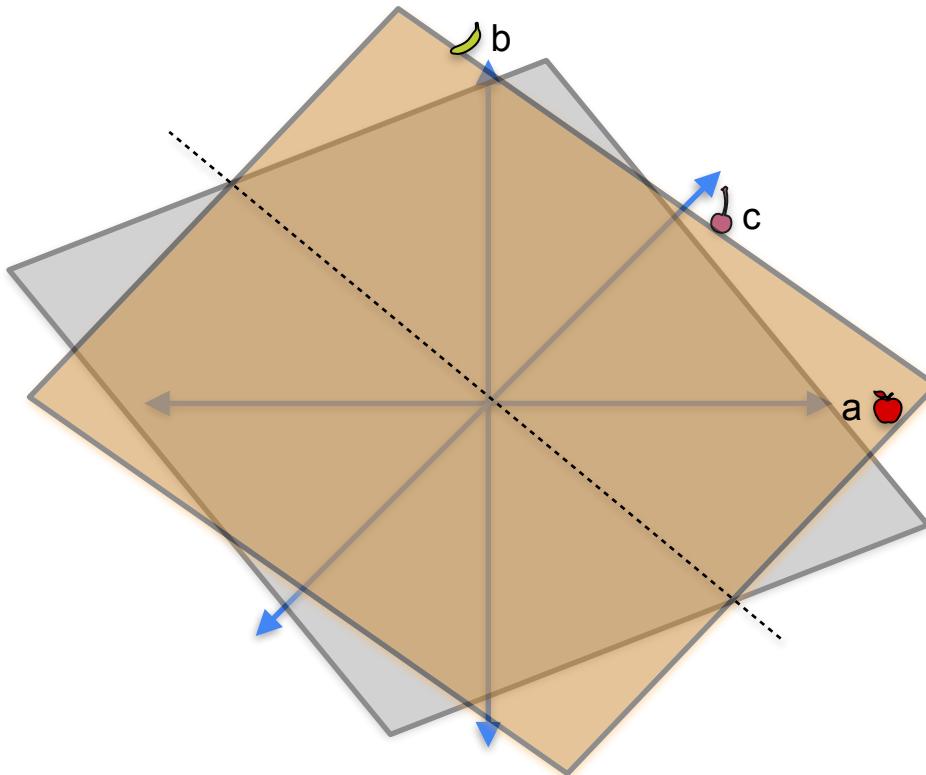
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



# System 2

## System 2

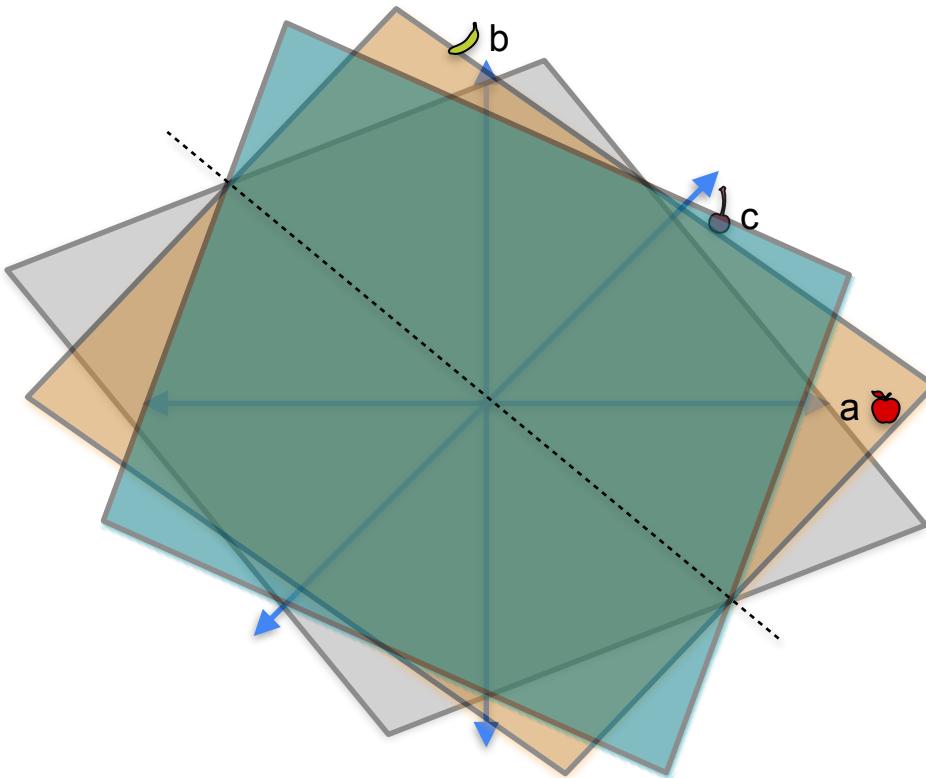
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



# System 2

## System 2

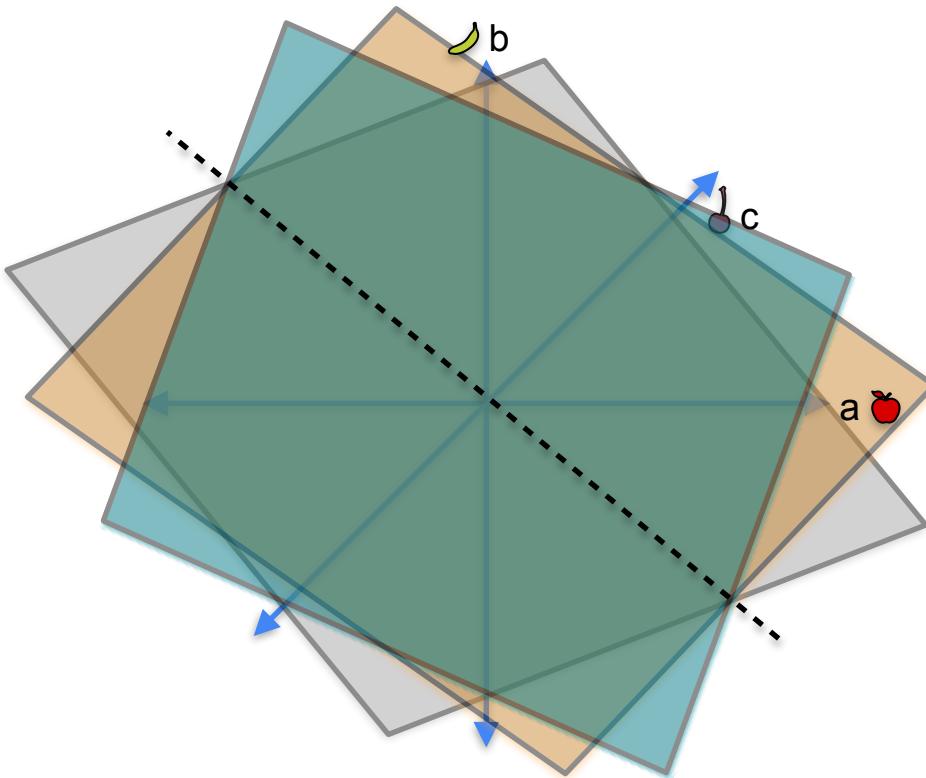
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



# System 2

## System 2

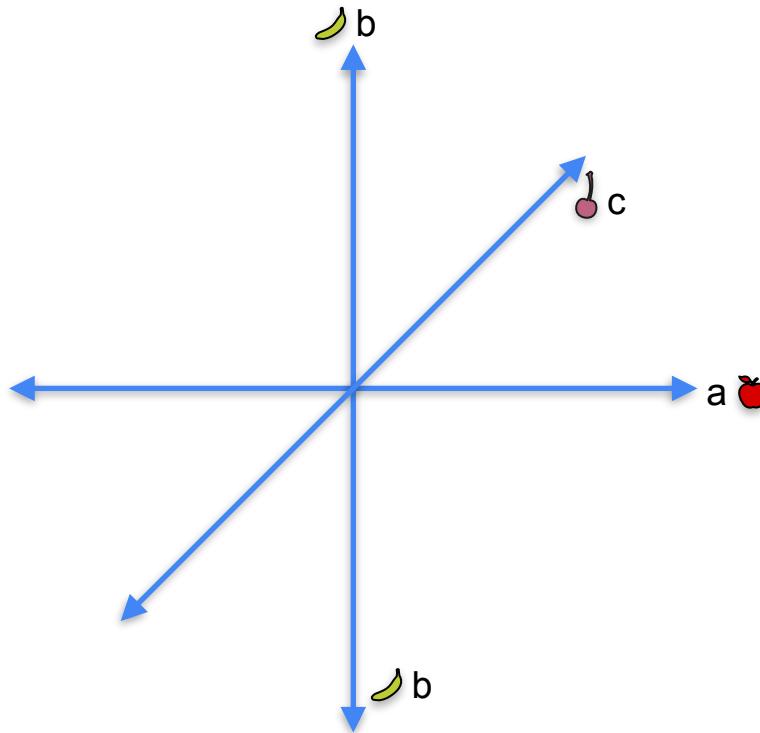
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



# System 3

## System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



# System 3

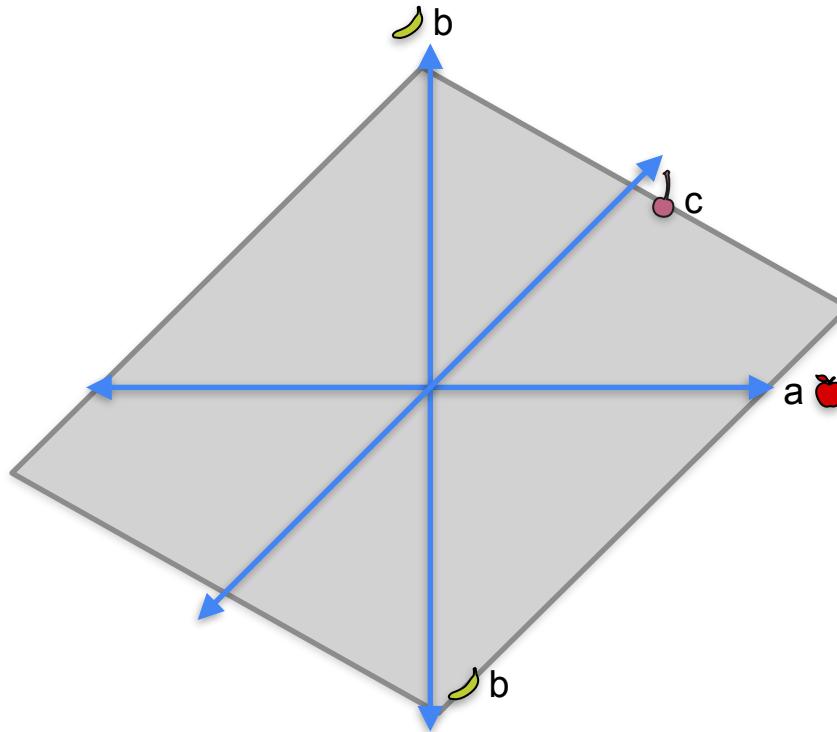
## System 3

- $a + b + c = 0$



- $2a + 2b + 2c = 0$

- $3a + 3b + 3c = 0$



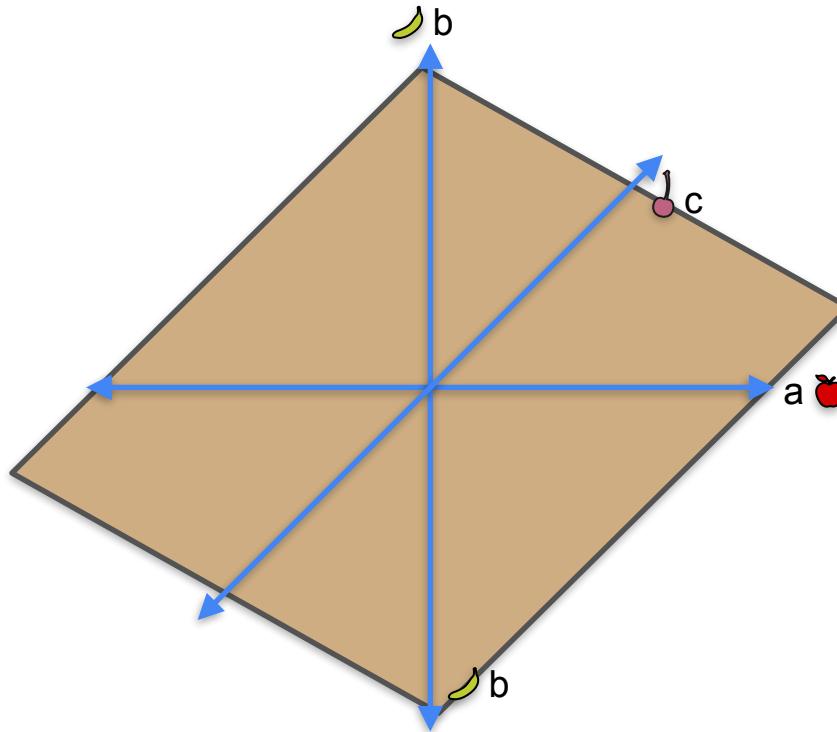
# System 3

## System 3

- $a + b + c = 0$

- $2a + 2b + 2c = 0$  

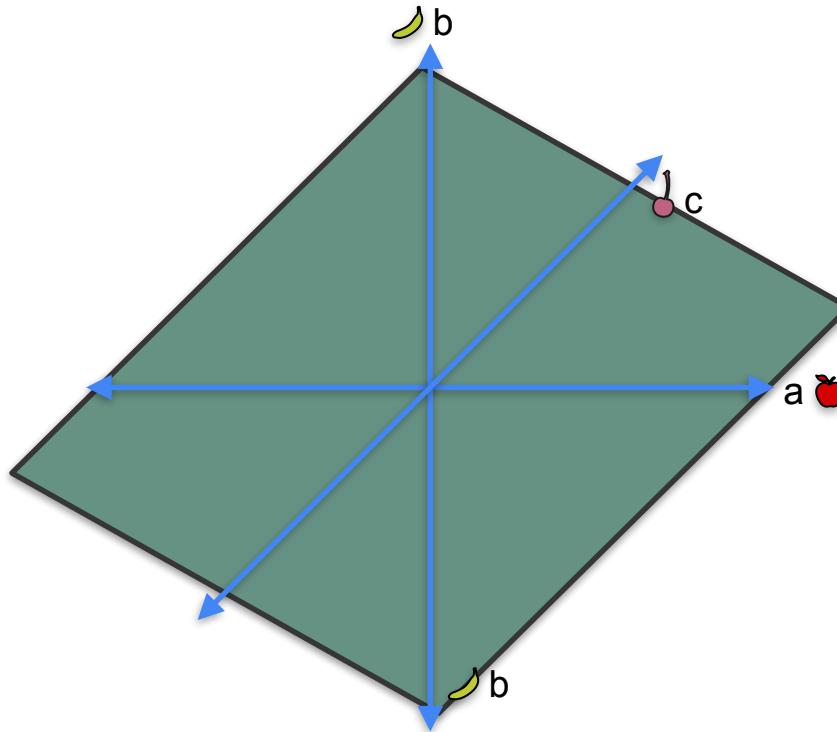
- $3a + 3b + 3c = 0$



# System 3

## System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$





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# System of Linear Equations

---

**Linear dependence and  
independence (3x3)**

# Linear dependence and independence

$$a = 1$$

$$b = 2$$

$$a + b = 3$$

# Linear dependence and independence

$$a = 1$$

$$b = 2$$

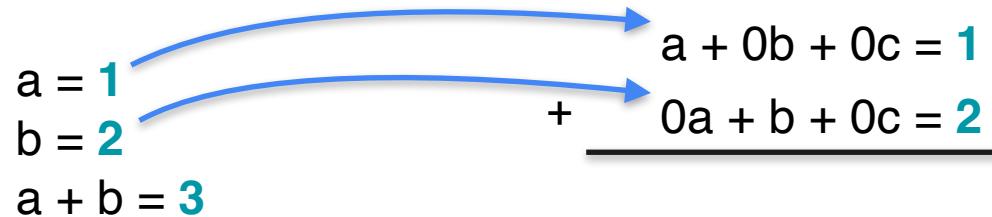
$$a + b = 3$$

$$a + 0b + 0c = 1$$

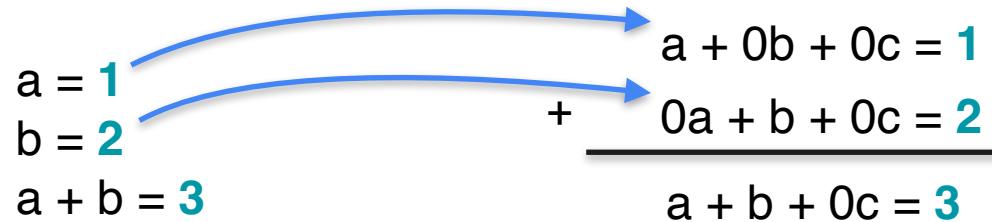
# Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{l} \xrightarrow{\text{blue curve}} a + 0b + 0c = 1 \\ \xrightarrow{\text{blue curve}} 0a + b + 0c = 2 \end{array}$$

# Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{r} + \\ \hline \end{array} \quad \begin{array}{l} a + 0b + 0c = 1 \\ 0a + b + 0c = 2 \end{array}$$


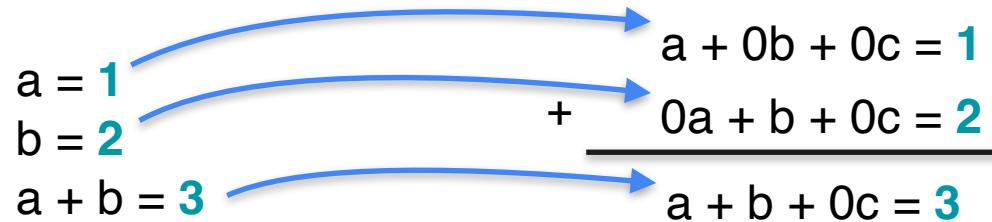
# Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{r} + \\ \hline \end{array} \quad \begin{array}{l} a + 0b + 0c = 1 \\ 0a + b + 0c = 2 \\ \hline a + b + 0c = 3 \end{array}$$


# Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{r} \xrightarrow{\hspace{1cm}} a + 0b + 0c = 1 \\ \xrightarrow{\hspace{1cm}} + \quad \quad \quad 0a + b + 0c = 2 \\ \hline \xrightarrow{\hspace{1cm}} \quad \quad \quad a + b + 0c = 3 \end{array}$$

# Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{r} a + 0b + 0c = 1 \\ + 0a + b + 0c = 2 \\ \hline a + b + 0c = 3 \end{array}$$


1	0	0
0	1	0
1	1	0

# Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{l} \xrightarrow{\hspace{10em}} a + 0b + 0c = 1 \\ \xrightarrow{\hspace{10em}} + \quad 0a + b + 0c = 2 \\ \hline \xrightarrow{\hspace{10em}} a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

# Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{l} \xrightarrow{\hspace{10em}} a + 0b + 0c = 1 \\ \xrightarrow{\hspace{10em}} + \quad 0a + b + 0c = 2 \\ \hline \xrightarrow{\hspace{10em}} a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

# Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{l} \xrightarrow{\hspace{10em}} a + 0b + 0c = 1 \\ \xrightarrow{\hspace{10em}} + \quad 0a + b + 0c = 2 \\ \hline \xrightarrow{\hspace{10em}} a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

# Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{l} \xrightarrow{\hspace{10em}} a + 0b + 0c = 1 \\ \xrightarrow{\hspace{10em}} + \quad 0a + b + 0c = 2 \\ \hline \xrightarrow{\hspace{10em}} a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

# Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{l} \xrightarrow{\hspace{10em}} a + 0b + 0c = 1 \\ \xrightarrow{\hspace{10em}} + \quad 0a + b + 0c = 2 \\ \hline \xrightarrow{\hspace{10em}} a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

# Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{l} \xrightarrow{\hspace{10em}} a + 0b + 0c = 1 \\ \xrightarrow{\hspace{10em}} 0a + b + 0c = 2 \\ + \quad \quad \quad \hline a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

# Linear dependence and independence

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

1	1	1
2	2	2
3	3	3

# Linear dependence and independence

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$


1	1	1
2	2	2
3	3	3

# Linear dependence and independence

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned} \quad \begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \end{aligned}$$


1	1	1
2	2	2
3	3	3

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} a + b + c = 0 \\ \xrightarrow{\quad} + \quad \underline{2a + 2b + 2c = 0} \end{array}$$

1	1	1
2	2	2
3	3	3

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} a + b + c = 0 \\ \xrightarrow{+} 2a + 2b + 2c = 0 \\ \hline 3a + 3b + 3c = 0 \end{array}$$

1	1	1
2	2	2
3	3	3

# Linear dependence and independence

$$\begin{array}{rcl}
 a + b + c = 0 & & a + b + c = 0 \\
 2a + 2b + 2c = 0 & + & 2a + 2b + 2c = 0 \\
 3a + 3b + 3c = 0 & & 3a + 3b + 3c = 0
 \end{array}$$

1	1	1
2	2	2
3	3	3

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} a + b + c = 0 \\ \xrightarrow{+} \underline{2a + 2b + 2c = 0} \\ \xrightarrow{\quad} 3a + 3b + 3c = 0 \end{array}$$

1	1	1
2	2	2
3	3	3

Row 1 + Row 2 = Row 3

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} a + b + c = 0 \\ \xrightarrow{+} \underline{2a + 2b + 2c = 0} \\ \xrightarrow{\quad} 3a + 3b + 3c = 0 \end{array}$$

1	1	1
2	2	2
3	3	3

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} a + b + c = 0 \\ \xrightarrow{+} \underline{2a + 2b + 2c = 0} \\ \xrightarrow{\quad} 3a + 3b + 3c = 0 \end{array}$$

1	1	1
2	2	2
3	3	3

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

# Linear dependence and independence

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

1	1	1
1	1	2
1	1	3

# Linear dependence and independence

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

$$a + b + c = 0$$



1	1	1
1	1	2
1	1	3

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{l} \xrightarrow{\hspace{10em}} a + b + c = 0 \\ \xrightarrow{\hspace{10em}} a + b + 3c = 0 \end{array}$$

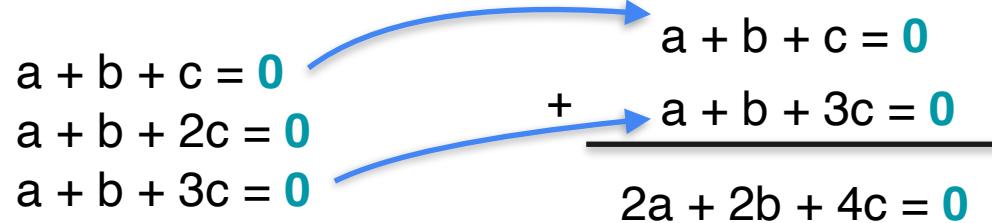
1	1	1
1	1	2
1	1	3

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} a + b + c = 0 \\ + \quad \xrightarrow{\quad} \underline{a + b + 3c = 0} \end{array}$$

1	1	1
1	1	2
1	1	3

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{l} a + b + c = 0 \\ + \\ \hline a + b + 3c = 0 \\ 2a + 2b + 4c = 0 \end{array}$$


1	1	1
1	1	2
1	1	3

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{l} a + b + c = 0 \\ + \quad a + b + 3c = 0 \\ \hline 2a + 2b + 4c = 0 \end{array}$$

↓  $\div 2$

1	1	1
1	1	2
1	1	3

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{l} a + b + c = 0 \\ + \quad a + b + 3c = 0 \\ \hline 2a + 2b + 4c = 0 \end{array}$$

$$\downarrow \div 2$$

$$a + b + 2c = 0$$

1	1	1
1	1	2
1	1	3

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{r} a + b + c = 0 \\ + a + b + 3c = 0 \\ \hline 2a + 2b + 4c = 0 \end{array}$$

÷ 2

$$a + b + 2c = 0$$

A diagram illustrating the elimination of the third equation from the first two. A blue curved arrow points from the first equation to the second. Another blue curved arrow points from the second equation to the third. A blue plus sign is placed between the second and third equations. A blue horizontal line is drawn under the second and third equations. A blue arrow points from the third equation down to the result  $a + b + 2c = 0$ . A blue circle is drawn around the first three equations.

1	1	1
1	1	2
1	1	3

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{l} a + b + c = 0 \\ + \\ \hline a + b + 3c = 0 \\ 2a + 2b + 4c = 0 \\ \div 2 \\ a + b + 2c = 0 \end{array}$$

Average of Row 1 and Row 3 is Row 2  
Row 2 **depends** on rows 1 and 3

1	1	1
1	1	2
1	1	3

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{l} a + b + c = 0 \\ + \\ \hline a + b + 3c = 0 \\ 2a + 2b + 4c = 0 \\ \div 2 \\ a + b + 2c = 0 \end{array}$$

A blue curved arrow points from the three equations on the left to the first two equations on the right. Another blue arrow points from the bottom row of the matrix to the equation  $a + b + 2c = 0$ .

1	1	1
1	1	2
1	1	3

Average of Row 1 and Row 3 is Row 2  
Row 2 **depends** on rows 1 and 3  
Rows are **linearly dependent**

# Linear dependence and independence

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

1	1	1
1	2	1
1	1	2

# Linear dependence and independence

$$a + b + c = 0$$

$a + 2b + c = 0$   No relations between equations

$$a + b + 2c = 0$$

1	1	1
1	2	1
1	1	2

# Linear dependence and independence

$$a + b + c = 0$$

$a + 2b + c = 0$   No relations between equations

$$a + b + 2c = 0$$

1	1	1
1	2	1
1	1	2

No relations between rows

# Linear dependence and independence

$$a + b + c = 0$$

$a + 2b + c = 0$   No relations between equations

$$a + b + 2c = 0$$

1	1	1
1	2	1
1	1	2

No relations between rows

Rows are **linearly independent**

# Quiz: Linear dependence and independence

**Problem:** Determine if the following matrices have linearly dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

# Solution: Linear dependence and independence

**Problem:** Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

# Solution: Linear dependence and independence

**Problem:** Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

**Dependent (singular)**

# Solution: Linear dependence and independence

**Problem:** Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

$$\text{Row1} - \text{Row2} = \text{Row3}$$

Dependent (singular)

Dependent (singular)

# Solution: Linear dependence and independence

**Problem:** Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

$$\text{Row1} - \text{Row2} = \text{Row3}$$

No relations

Dependent (singular)

Dependent (singular)

Independent  
(Non-singular)

# Solution: Linear dependence and independence

**Problem:** Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

$$\text{Row1} - \text{Row2} = \text{Row3}$$

No relations

$$2\text{Row1} = \text{Row3}$$

**Dependent (singular)**

**Dependent (singular)**

**Independent (Non-singular)**

**Dependent (singular)**



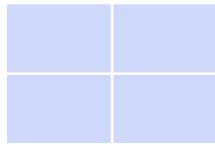
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# System of Linear Equations

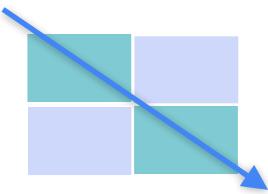
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**The determinant (3x3)**

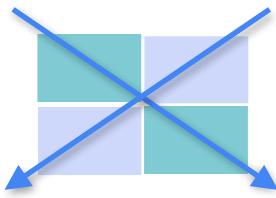
# Diagonals in a 3x3 matrix



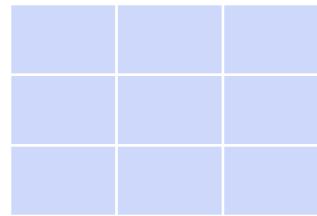
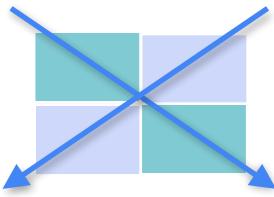
# Diagonals in a 3x3 matrix



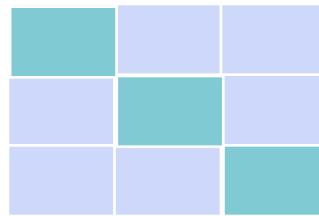
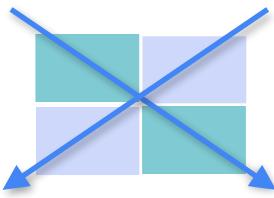
# Diagonals in a 3x3 matrix



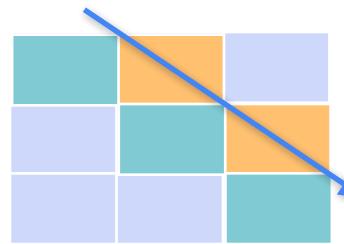
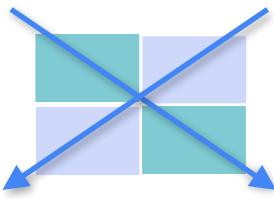
# Diagonals in a 3x3 matrix



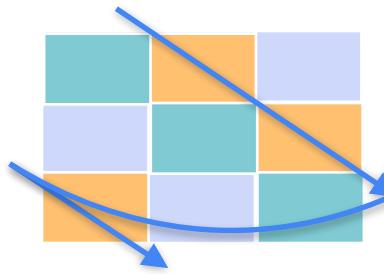
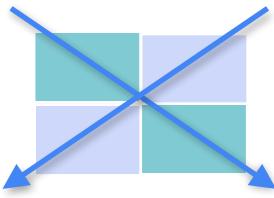
# Diagonals in a 3x3 matrix



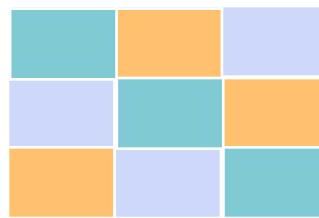
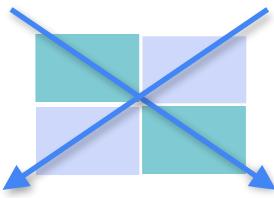
# Diagonals in a 3x3 matrix



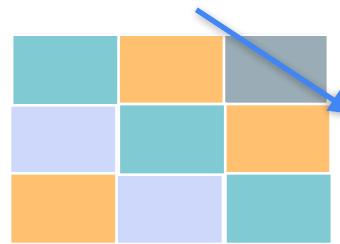
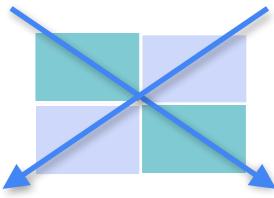
# Diagonals in a 3x3 matrix



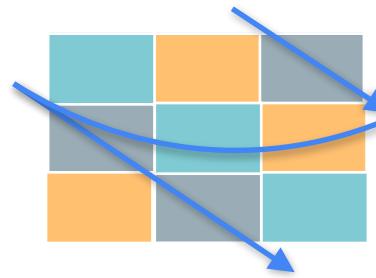
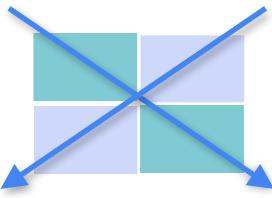
# Diagonals in a 3x3 matrix



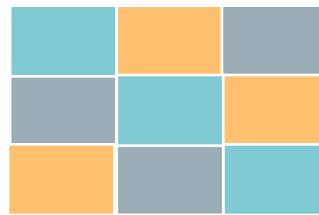
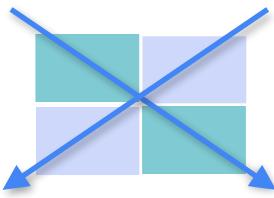
# Diagonals in a 3x3 matrix



# Diagonals in a 3x3 matrix



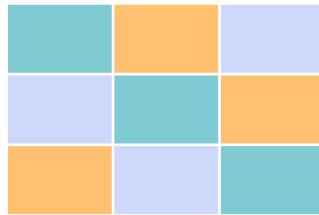
# Diagonals in a 3x3 matrix



# Determinant

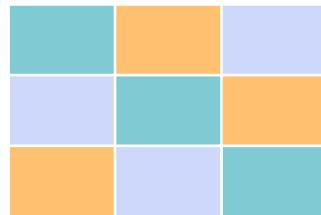
# Determinant

Add

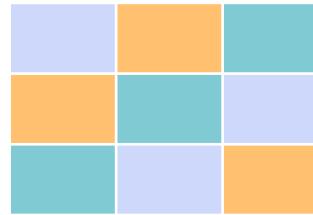


# Determinant

Add



Subtract



# The determinant

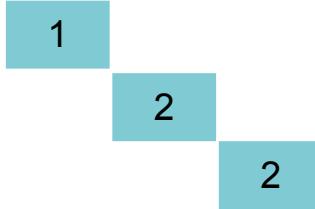
1	1	1
1	2	1
1	1	2

# The determinant

1	1	1
1	2	1
1	1	2

# The determinant

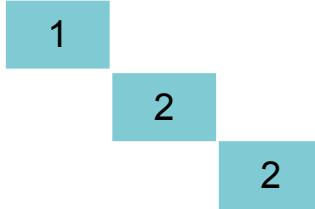
1	1	1
1	2	1
1	1	2



$$+ 1 \cdot 2 \cdot 2$$

# The determinant

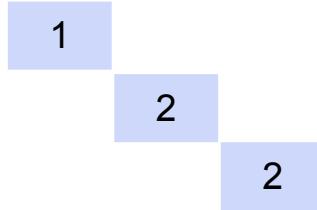
1	1	1
1	2	1
1	1	2



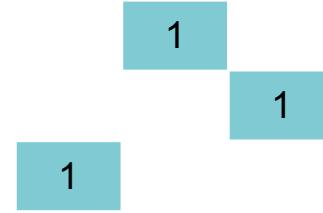
$$+ 1 \cdot 2 \cdot 2$$

# The determinant

1	1	1
1	2	1
1	1	2



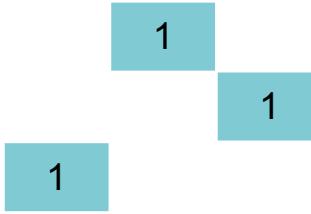
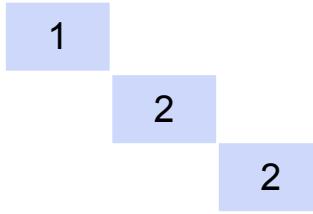
$$+ 1 \cdot 2 \cdot 2$$



$$+ 1 \cdot 1 \cdot 1$$

# The determinant

1	1	1
1	2	1
1	1	2

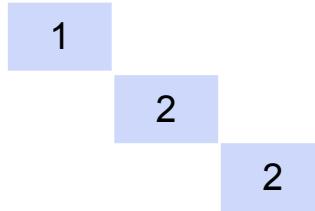


$$+ 1 \cdot 2 \cdot 2$$

$$+ 1 \cdot 1 \cdot 1$$

# The determinant

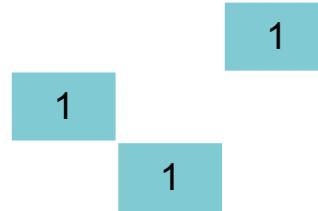
1	1	1
1	2	1
1	1	2



$$+ 1 \cdot 2 \cdot 2$$



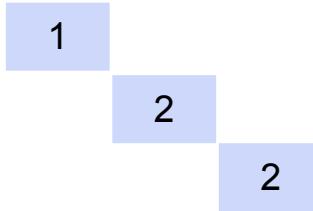
$$+ 1 \cdot 1 \cdot 1$$



$$+ 1 \cdot 1 \cdot 1$$

# The determinant

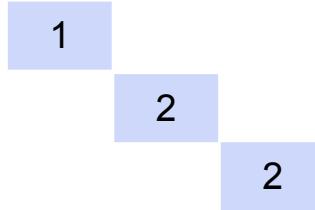
1	1	1
1	2	1
1	1	2



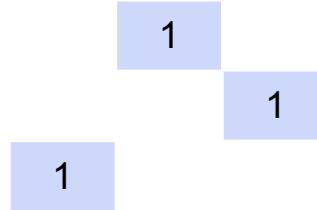
$$+ 1 \cdot 2 \cdot 2$$
$$+ 1 \cdot 1 \cdot 1$$
$$+ 1 \cdot 1 \cdot 1$$

# The determinant

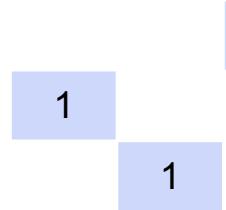
1	1	1
1	2	1
1	1	2



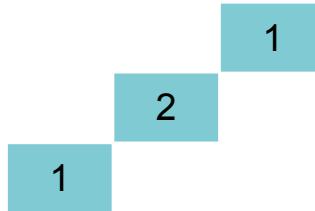
$$+ 1 \cdot 2 \cdot 2$$



$$+ 1 \cdot 1 \cdot 1$$



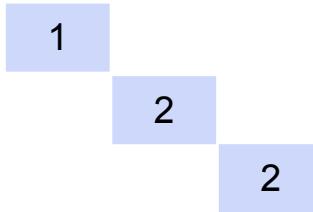
$$+ 1 \cdot 1 \cdot 1$$



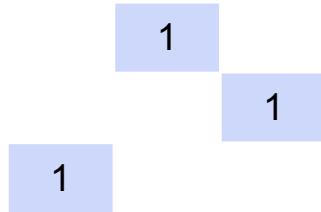
$$- 1 \cdot 2 \cdot 1$$

# The determinant

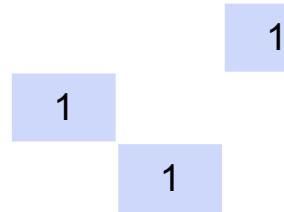
1	1	1
1	2	1
1	1	2



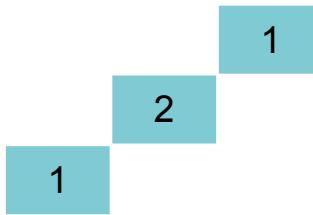
$$+ 1 \cdot 2 \cdot 2$$



$$+ 1 \cdot 1 \cdot 1$$



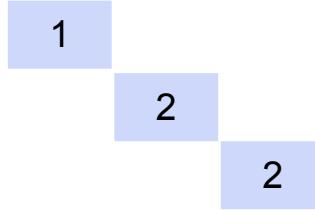
$$+ 1 \cdot 1 \cdot 1$$



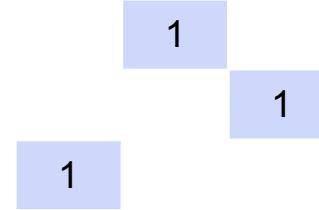
$$- 1 \cdot 2 \cdot 1$$

# The determinant

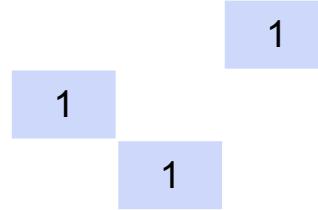
1	1	1
1	2	1
1	1	2



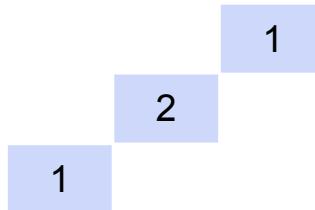
$$+ 1 \cdot 2 \cdot 2$$



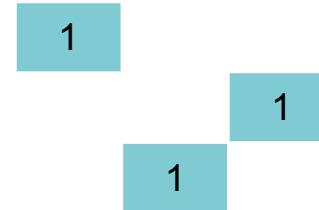
$$+ 1 \cdot 1 \cdot 1$$



$$+ 1 \cdot 1 \cdot 1$$



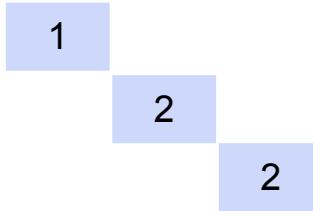
$$- 1 \cdot 2 \cdot 1$$



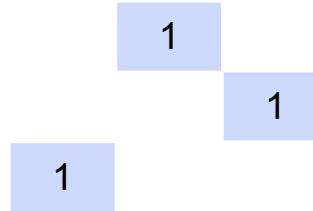
$$- 1 \cdot 1 \cdot 1$$

# The determinant

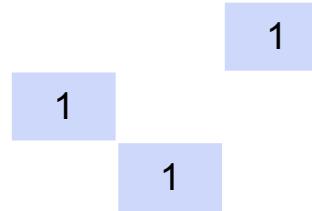
1	1	1
1	2	1
1	1	2



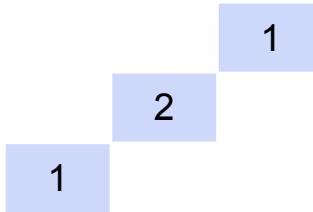
$$+ 1 \cdot 2 \cdot 2$$



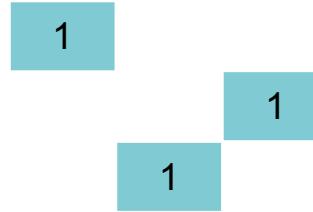
$$+ 1 \cdot 1 \cdot 1$$



$$+ 1 \cdot 1 \cdot 1$$



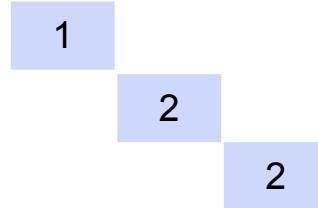
$$- 1 \cdot 2 \cdot 1$$



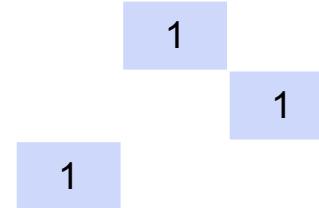
$$- 1 \cdot 1 \cdot 1$$

# The determinant

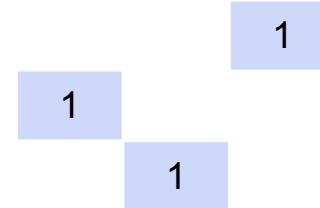
1	1	1
1	2	1
1	1	2



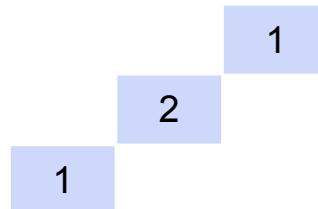
$$+ 1 \cdot 2 \cdot 2$$



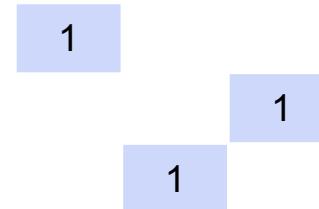
$$+ 1 \cdot 1 \cdot 1$$



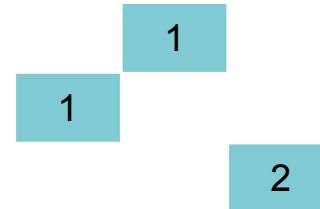
$$+ 1 \cdot 1 \cdot 1$$



$$- 1 \cdot 2 \cdot 1$$



$$- 1 \cdot 1 \cdot 1$$



$$- 1 \cdot 1 \cdot 2$$

# The determinant

1	1	1
1	2	1
1	1	2

$$+ 1 \cdot 2 \cdot 2$$

$$+ 1 \cdot 1 \cdot 1$$

$$+ 1 \cdot 1 \cdot 1$$

$$- 1 \cdot 2 \cdot 1$$

$$- 1 \cdot 1 \cdot 1$$

$$- 1 \cdot 1 \cdot 2$$

# The determinant

1	1	1
1	2	1
1	1	2

$$+ 1 \cdot 2 \cdot 2$$

$$+ 1 \cdot 1 \cdot 1$$

$$+ 1 \cdot 1 \cdot 1$$

$$\begin{aligned} \text{Det} = & 4 + 1 + 1 \\ & - 2 - 1 - 2 \end{aligned}$$

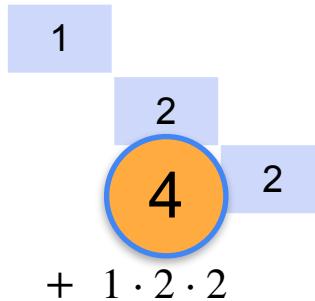
$$- 1 \cdot 2 \cdot 1$$

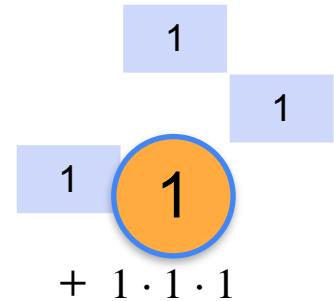
$$- 1 \cdot 1 \cdot 1$$

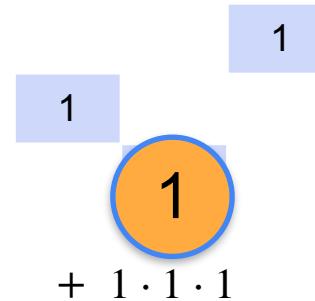
$$- 1 \cdot 1 \cdot 2$$

# The determinant

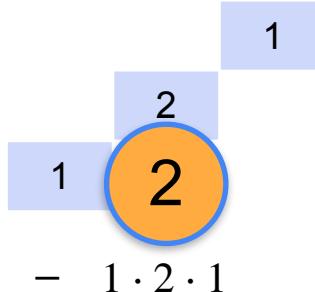
1	1	1
1	2	1
1	1	2

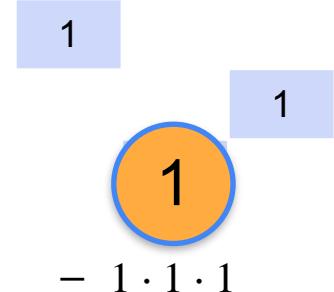
$$+ 1 \cdot 2 \cdot 2$$


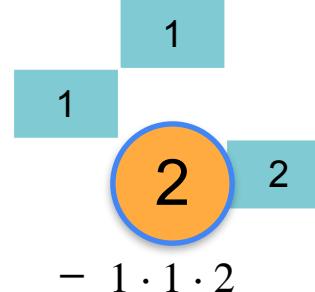
$$+ 1 \cdot 1 \cdot 1$$


$$+ 1 \cdot 1 \cdot 1$$


$$\begin{aligned} \text{Det} &= 4 + 1 + 1 \\ &\quad - 2 - 1 - 2 \\ &= 1 \end{aligned}$$

$$- 1 \cdot 2 \cdot 1$$


$$- 1 \cdot 1 \cdot 1$$


$$- 1 \cdot 1 \cdot 2$$


# Quiz: Determinants

**Problem:** Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

# Solution: Determinants

**Problem:** Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

# Solution: Determinants

**Problem:** Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

Determinant = 0

**Singular**

# Solution: Determinants

**Problem:** Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

Determinant = 0

**Singular**

1	1	1
1	1	2
0	0	-1

Determinant = 0

**Singular**

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

# Solution: Determinants

**Problem:** Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

Determinant = 0

**Singular**

1	1	1
1	1	2
0	0	-1

Determinant = 0

**Singular**

1	1	1
0	2	2
0	0	3

Determinant = 6

**Non-singular**

1	2	5
0	3	-2
2	4	10

# Solution: Determinants

**Problem:** Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

Determinant = 0

**Singular**

1	1	1
1	1	2
0	0	-1

Determinant = 0

**Singular**

1	1	1
0	2	2
0	0	3

Determinant = 6

**Non-singular**

1	2	5
0	3	-2
2	4	10

Determinant = 0

**Singular**

# The determinant

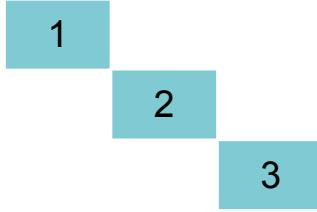
1	1	1
0	2	2
0	0	3

$$\text{Det} = 6+0+0-0-0-0$$

$$= 6$$

# The determinant

1	1	1
0	2	2
0	0	3



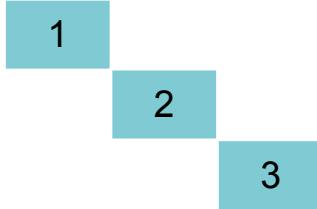
$$+ 1 \cdot 2 \cdot 3$$

$$\text{Det} = 6+0+0-0-0-0$$

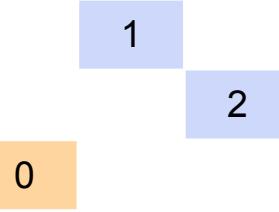
$$= 6$$

# The determinant

1	1	1
0	2	2
0	0	3



$$+ 1 \cdot 2 \cdot 3$$



$$+ 1 \cdot 2 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

# The determinant

1	1	1
0	2	2
0	0	3

1
2
3

$$+ 1 \cdot 2 \cdot 3$$

1
2
0

$$+ 1 \cdot 2 \cdot 0$$

1
0

$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

# The determinant

1	1	1
0	2	2
0	0	3

1
2
3

1
2
0

0
0

$$+ 1 \cdot 2 \cdot 3$$

$$+ 1 \cdot 2 \cdot 0$$

$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

1
2
0

$$- 1 \cdot 2 \cdot 0$$

# The determinant

1	1	1
0	2	2
0	0	3

1
2
3

1
2
0

0
0

$$+ 1 \cdot 2 \cdot 3$$

$$+ 1 \cdot 2 \cdot 0$$

$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

1
2
0

$$- 1 \cdot 2 \cdot 0$$

1
2
0

$$- 1 \cdot 2 \cdot 0$$

# The determinant

1	1	1
0	2	2
0	0	3

1
2
3

1
2
0

1
0
0

$$+ 1 \cdot 2 \cdot 3$$

$$+ 1 \cdot 2 \cdot 0$$

$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

1
2
0

$$- 1 \cdot 2 \cdot 0$$

1
2
0

$$- 1 \cdot 2 \cdot 0$$

1
0
3

$$- 1 \cdot 0 \cdot 3$$

# The determinant

1	1	1
0	2	2
0	0	3

1
2
3

1
2
0

1
0
0

$$+ 1 \cdot 2 \cdot 3$$

$$+ 1 \cdot 2 \cdot 0$$

$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

1
2
0

$$- 1 \cdot 2 \cdot 0$$

1
2
0

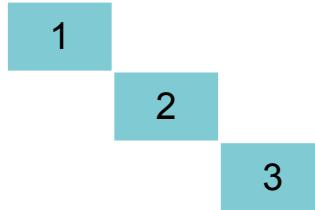
$$- 1 \cdot 2 \cdot 0$$

1
0
3

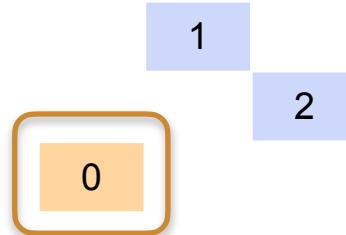
$$- 1 \cdot 0 \cdot 3$$

# The determinant

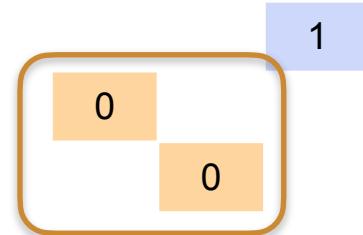
1	1	1
0	2	2
0	0	3



$$+ 1 \cdot 2 \cdot 3$$



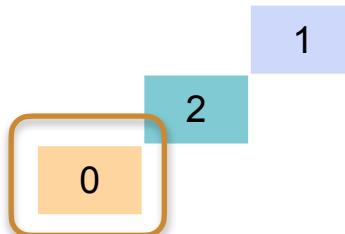
$$+ 1 \cdot 2 \cdot 0$$



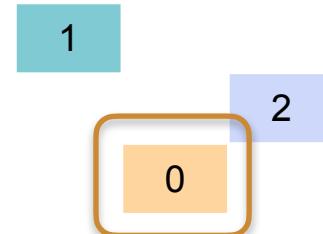
$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

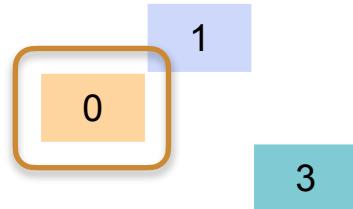
$$= 6$$



$$- 1 \cdot 2 \cdot 0$$



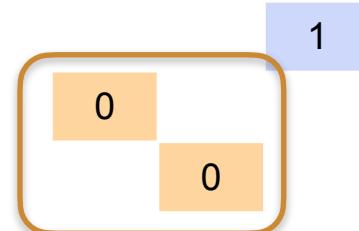
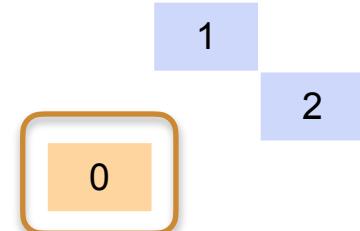
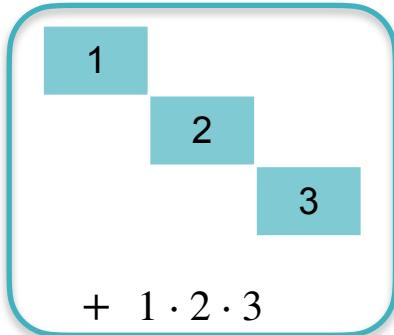
$$- 1 \cdot 2 \cdot 0$$



$$- 1 \cdot 0 \cdot 3$$

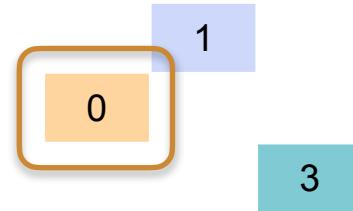
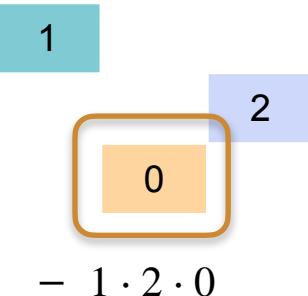
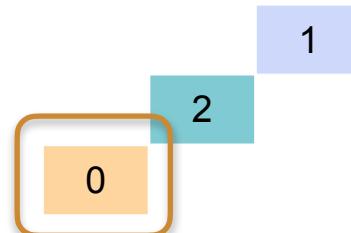
# The determinant

1	1	1
0	2	2
0	0	3

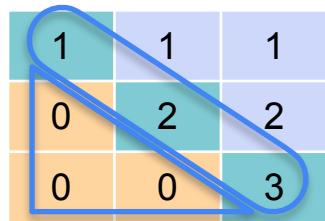


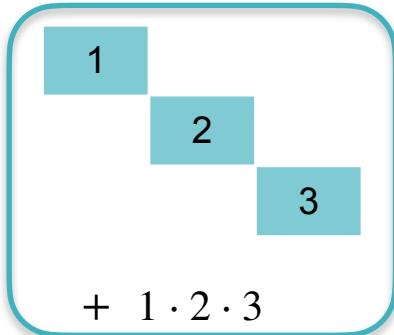
$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

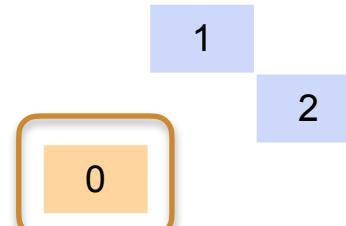
$$= 6$$

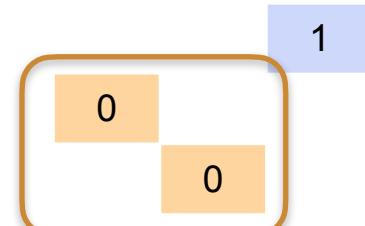


# The determinant


$$\begin{matrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{matrix}$$

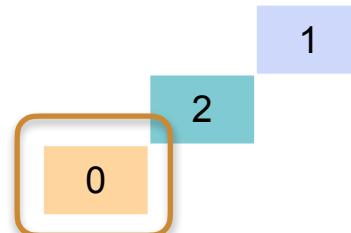

$$+ 1 \cdot 2 \cdot 3$$

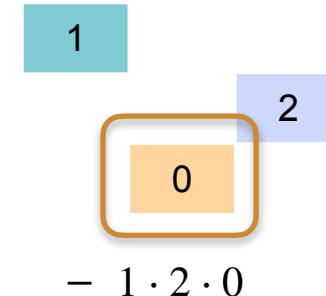

$$+ 1 \cdot 2 \cdot 0$$

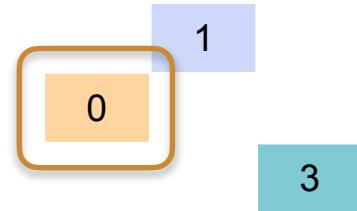

$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$


$$- 1 \cdot 2 \cdot 0$$


$$- 1 \cdot 2 \cdot 0$$


$$- 1 \cdot 0 \cdot 3$$

# The determinant

1	1	1
0	2	2
0	0	0

# The determinant

1	1	1
0	2	2
0	0	0

1
2
0

1
2
0

1
0
0

$$+ 1 \cdot 2 \cdot 0$$

$$+ 1 \cdot 2 \cdot 0$$

$$+ 1 \cdot 0 \cdot 0$$

1
2
0

1
2
0

1
0
0

$$- 1 \cdot 2 \cdot 0$$

$$- 1 \cdot 2 \cdot 0$$

$$- 1 \cdot 0 \cdot 0$$

# The determinant

1	1	1
0	2	2
0	0	0

1
2
0

1
2
0

1
0
0

$$+ 1 \cdot 2 \cdot 0$$

$$+ 1 \cdot 2 \cdot 0$$

$$+ 1 \cdot 0 \cdot 0$$

1
2
0

1
2
0

1
0
0

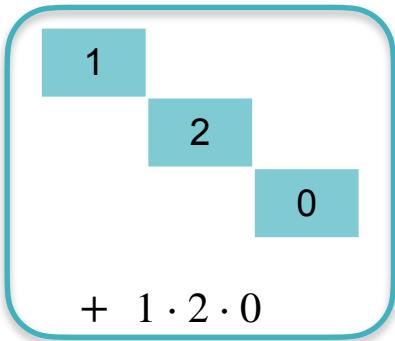
$$- 1 \cdot 2 \cdot 0$$

$$- 1 \cdot 2 \cdot 0$$

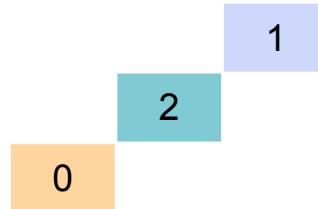
$$- 1 \cdot 0 \cdot 0$$

# The determinant

1	1	1
0	2	2
0	0	0



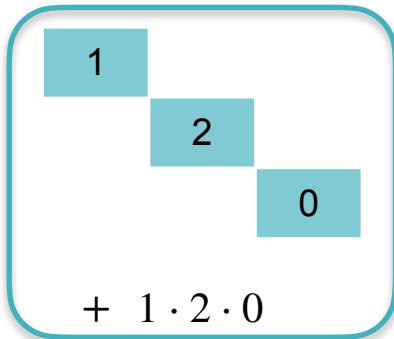
$$+ 1 \cdot 2 \cdot 0 + 1 \cdot 0 \cdot 0$$



$$- 1 \cdot 2 \cdot 0 - 1 \cdot 0 \cdot 0$$

# The determinant

1	1	1
0	2	2
0	0	0



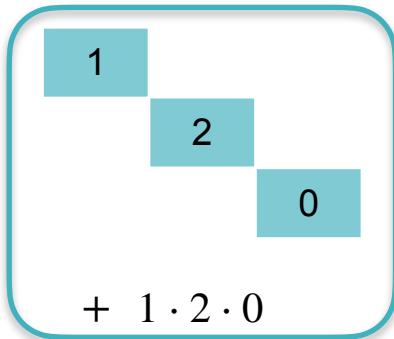
$$\text{Det} = 0+0+0-0-0-0$$

$$+ 1 \cdot 2 \cdot 0 + 1 \cdot 0 \cdot 0$$

$$- 1 \cdot 2 \cdot 0 - 1 \cdot 2 \cdot 0 - 1 \cdot 0 \cdot 0$$

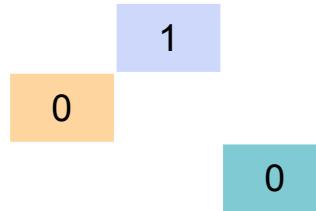
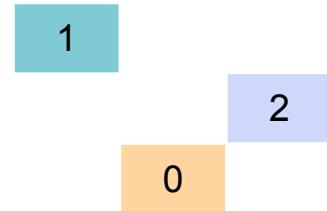
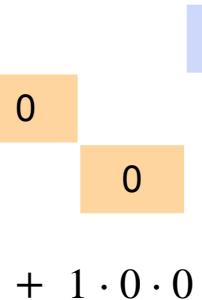
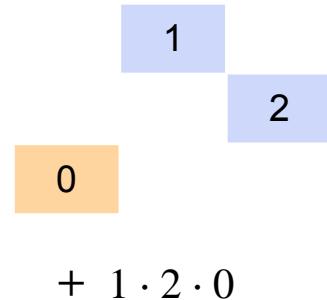
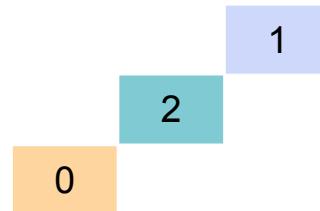
# The determinant

1	1	1
0	2	2
0	0	0



$$\text{Det} = 0+0+0-0-0-0$$

$$= 0$$





DeepLearning.AI

# System of Linear Equations

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## Conclusion